GLMs

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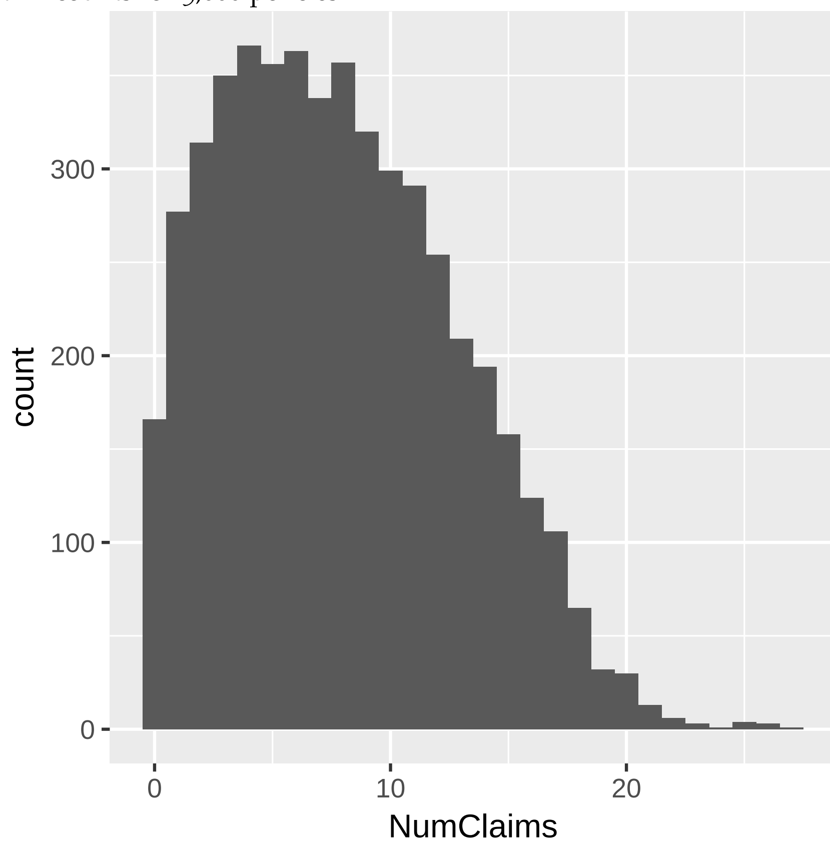
Fit a sample

Data

Claim counts for 5,000 policies.



How would you fit this data?

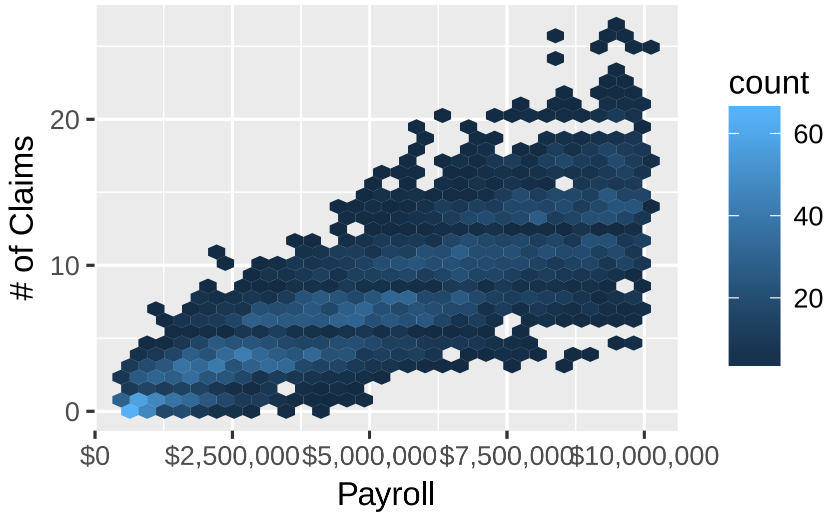
Things we can do when fitting a sample

- Pick a distribution
- Normal, lognormal, gamma, etc
- Transform data
- Often taking the log.
- Pick a fit method
- Maximum likelihood
- Least squares
- Minimum bias

- Assess quality of fit
- r-squared, penalized r-squared
- F-stat
- Likelihood, penalized likelihood

Add predictors

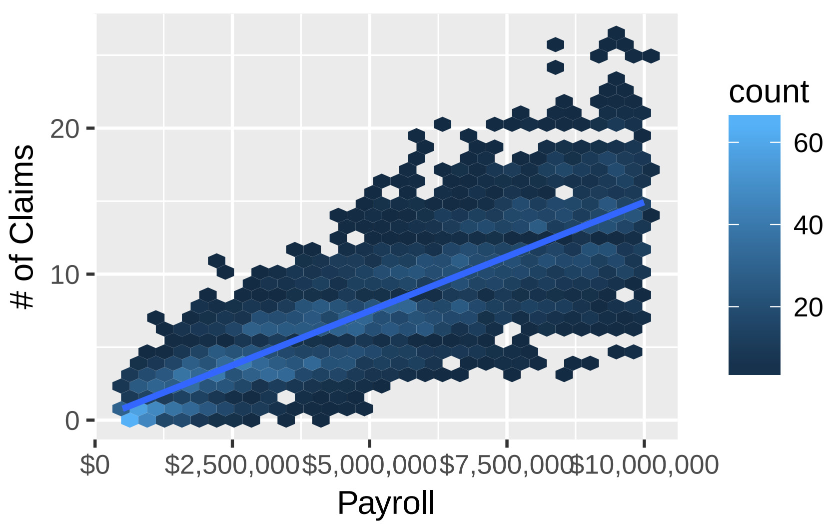
Number of claims ~ Payroll



What's wrong with a linear fit?

- Heteroskedastic
- Does it really capture the mean?

Number of claims ~ Payroll



Another distribution makes more sense

But how do we do that? If only we had a linear model that was a bit more general ...

GLMs

Recall OLS Assumptions

Warning: I play fast and loose with the difference between the response variable and the error term.

OLS Assumptions

- Linear relationship between response and predictors
- Errors are normally distributed
- Errors are uncorrelated
- Errors are homoskedastic

More general assumptions

- Relationship is between response and *transformed* linear combination of predictors
- Errors need not be normally distributed

Mathematically

$E[y] = g^{-1}(\beta_0 + \sum_{j=1}^{p} \beta_{ij})$

$g(x)$ is the "link" function.

The linear combination is often referred to by η. I don't know why it doesn't get a name.

I also don't know why the expectation is equal to the inverse of the link function. If talking about the transformed expectation doesn't make your head hurt, then you may like this formula better.

$g(E[y]) = \beta_0 + \sum_{j=1}^{p} \beta_{ij}$

Specify two things

1. The distribution
2. The "link" function

Distribution restrictions

Must be one of the exponential family of functions.

$f(y;\theta,\phi) = exp[\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)]$

Note this *doesn't* include the lognormal. That's OK; we can always perform a log transform of our data and fit a normal.

Lots of folks get very excited about this formula. I don't. I can never remember it and I never feel as though I need to. If you like this formula, you'll see it often, but you won't see it any more today.

Canonical links

Distribution	Link	
binomical	logit	$g(x) = \frac{exp(x)}{1+exp(x)}$
gaussian	identity	$g(x) = x$
poisson	log	$g(x) = ln(x)$
Gamma	inverse	$g(x) = 1/x$

Very easy to program

A linear model:

```
fit_lm <- lm(NumClaims ~ Payroll, data = dfGLM)
```

A GLM:

```
fit_glm <- glm(NumClaims ~ Payroll, data = dfGLM,
    family = "poisson")
```

Programmatic differences:

- Must indicate the family
- Must provide the link, though only if we're using something non-canonical

Offset

The offset is a kind of scaling factor that should not be included as a predictor. Comparable to the notion of exposure in insurance pricing.

Compare these two models

```
fit_1 <- glm(NumClaims ~ 1 + Payroll, data = dfGLM,
    family = "poisson")

fit_2 <- glm(NumClaims ~ 1, data = dfGLM, family = "poisson",
    offset = log(Payroll))

fit_1$aic
```

```
## [1] 24074.27
fit_2$aic
## [1] 23139.99
coef(fit_1)
##  (Intercept)      Payroll
## 8.662942e-01 1.998568e-07
coef(fit_2)
## (Intercept)
##    -13.4123
```

Fit for the second model is much better, because payroll isn't really a *predictor* of loss. It is a scaling element for exposure. Think the number of deaths by heart disease in Manhattan vs. number of deaths by heart disease in a rural town.

Measuring fit quality

Measuring fit quality

Comparing models typically involves comparison of the likelihood. Note that - comparable to r^2 - more parameters will *always* give better fit metrics, unless we're penalizing for extra parameters.

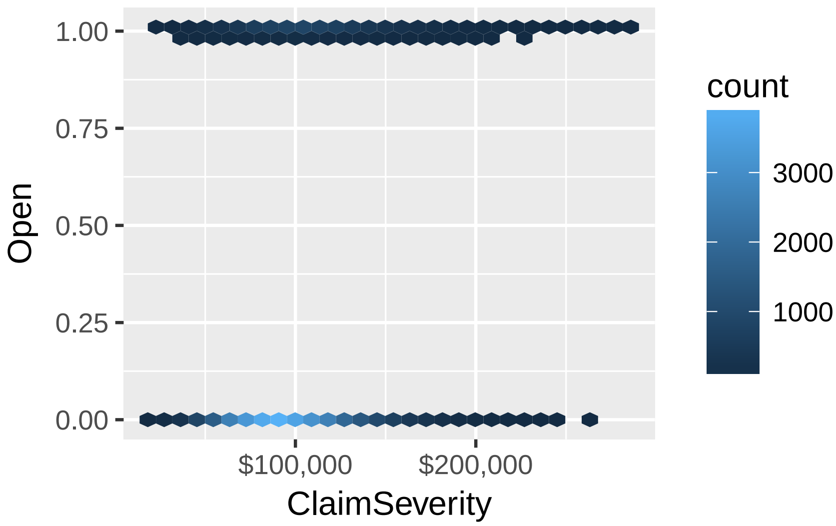
$AIC = 2[-l(\boldsymbol{y}; \boldsymbol{\theta}^M) + r]$ $BIC = 2[-l(\boldsymbol{y}; \boldsymbol{\theta}^M) + rln(n)]$

Deviance

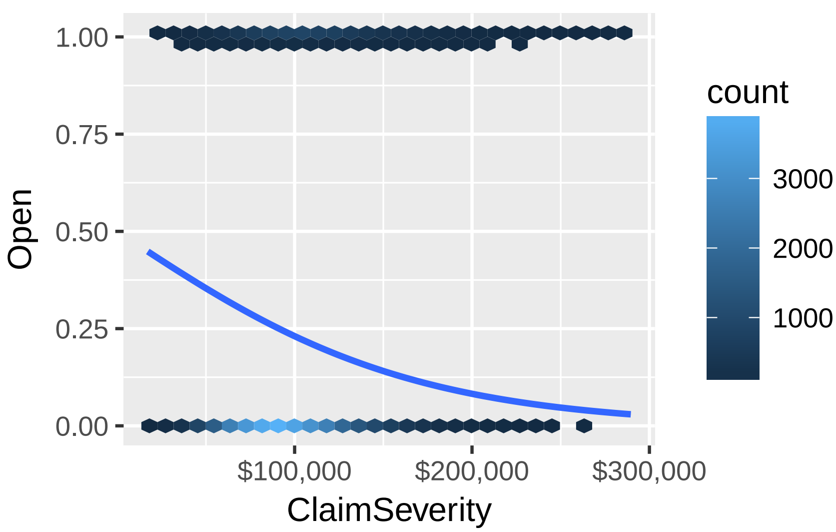
- Null deviance is comparable to sum of squares in OLS
- Reduction in residual deviance suggests a better model. However, trivial improvements may favor a more parsimonious model.

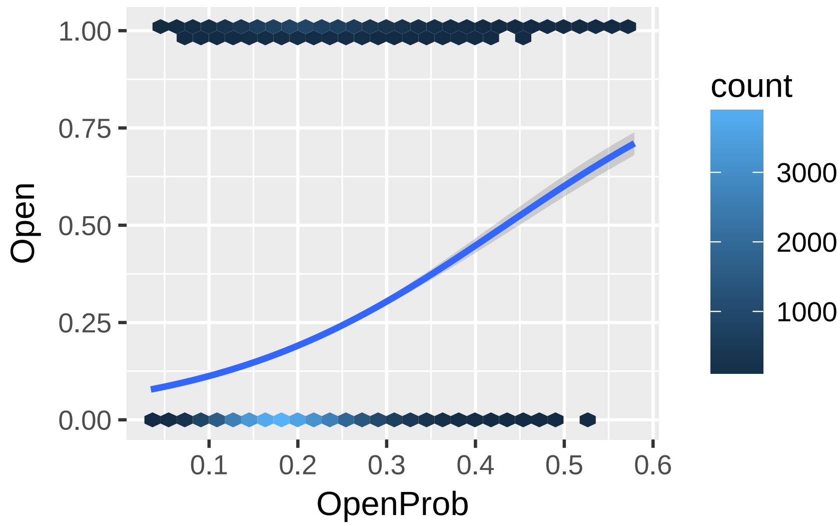
If time permits

Binomial



Binomial w/fit






```
##
## Call:
## glm(formula = Open ~ 0 + ClaimSeverity, family = "binomial",
##     data = dfBinomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0906  -0.8029  -0.7089  -0.5601   2.6528
##
## Coefficients:
##                 Estimate Std. Error z value
## ClaimSeverity -1.205e-05  1.220e-07  -98.81
##               Pr(>|z|)
## ClaimSeverity   <2e-16 ***
## ---
## Signif. codes:
##   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1  ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 54603  on 39388  degrees of freedom
## Residual deviance: 42270  on 39387  degrees of freedom
## AIC: 42272
##
## Number of Fisher Scoring iterations: 4
```

```
##
## Call:
```

```
## glm(formula = Open ~ 1 + ClaimSeverity, family = "binomial",
##     data = dfBinomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.4136  -0.6872  -0.5984  -0.5059   2.2119
##
## Coefficients:
##                 Estimate Std. Error z value
## (Intercept)   -2.683e+00  4.400e-02  -60.98
## ClaimSeverity  1.237e-05  3.937e-07   31.41
##               Pr(>|z|)
## (Intercept)     <2e-16 ***
## ClaimSeverity   <2e-16 ***
## ---
## Signif. codes:
##   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1  ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 39125  on 39387  degrees of freedom
## Residual deviance: 38133  on 39386  degrees of freedom
## AIC: 38137
##
## Number of Fisher Scoring iterations: 4
```

```
##
## Call:
## glm(formula = Open ~ 0 + ClaimSeverity, family = binomial(link = "identity"),
##     data = dfBinomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.2036  -0.7003  -0.6075  -0.4842   2.4287
##
## Coefficients:
##                Estimate Std. Error z value
## ClaimSeverity 1.978e-06  1.976e-08   100.1
##               Pr(>|z|)
## ClaimSeverity   <2e-16 ***
## ---
## Signif. codes:
##   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1  ' ' 1
##
```

```
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance:   Inf  on 39388  degrees of freedom
## Residual deviance: 38121  on 39387  degrees of freedom
## AIC: 38123
##
## Number of Fisher Scoring iterations: 3
```

```
##
## Call:
## glm(formula = Open ~ 1 + ClaimSeverity, family = binomial(link = "identity"),
##     data = dfBinomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.2133  -0.7008  -0.6063  -0.4803   2.4504
##
## Coefficients:
##                 Estimate Std. Error z value
## (Intercept)   -3.641e-03  6.035e-03  -0.603
## ClaimSeverity  2.013e-06  6.220e-08  32.366
##               Pr(>|z|)
## (Intercept)      0.546
## ClaimSeverity   <2e-16 ***
## ---
## Signif. codes:
##   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1  ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 39125  on 39387  degrees of freedom
## Residual deviance: 38121  on 39386  degrees of freedom
## AIC: 38125
##
## Number of Fisher Scoring iterations: 3
```

Resources

Good books

- Frees, Derrig, et al

Gelman, Andrew, and Jennifer Hill. 2006. *Data Analysis Using Regression and Multilevel/Hierarchical Models*. `http://www.stat.columbia.edu/~gelman/arm/`.