Algorithm HW2

1.

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, w, or Θ of B. Assume that k ≥ 1, ε > 0, and c >1 are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	*B*	*O*	*o*	Ω	*w*	Θ
a.	$\lg^k n$	n^{ε}					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\ln n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

2.

Given two non-negative function *f*, *g* (i.e. *f*, *g* : $\mathbf{N} \rightarrow \mathbf{R}^*$) such that $f \neq O(g)$, $f \neq \Theta(g)$, and $f \neq \Omega(g)$.

3.

a. Rank the following functions by order of growth; that is, find an arrangement g_1, g_2, …, g_{30} of the functions satisfying $g_1 = \Omega(g_2)$, $g2 = \Omega(g_3)$, …, $g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

$\lg(\lg^* n)$	$2^{\lg^* n}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$	$(\lg n)!$
$(\frac{3}{2})^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\ln \ln n$	$\lg^* n$	$n \cdot 2^n$	$n^{\lg \lg n}$	$\ln n$	1
$2^{\lg n}$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$\lg^*(\lg n)$	$2^{\sqrt{2 \lg n}}$	n	2^n	$n \lg n$	$2^{2^{n+1}}$

b. Give an example of a single nonnegative function $f(n)$ such that for all functions $g_i(n)$ in part (*a*), $f(n)$ is neither $O(g_i(n))$ nor $\Omega(g_i(n))$.

4.

Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.

a. *$f(n) = O(g(n))$ implies $g(n) = O(f(n))$.*

b. *$f(n) + g(n) = \Theta(\min(f(n), g(n))$.*

c. *$f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all* sufficiently large n.

d. *$f(n) = O(g(n))$ implies $2^{f(n)}= O(2^{g(n)})$.*

e. *$f(n) = O((f(n))^2)$.*

f. *$f(n) = O(g(n))$ implies $g(n)= \Omega(f(n))$.*

g. *$f(n) = \Theta(f(n/2))$.*

h. *$f(n) + o(f(n)) = \Theta(f(n))$.*

i. *$f(n) = \Theta(g(n))$ implies $f(n)/g(n)=c\neq 0$*

5.

Solve the recurrence $T(n) = 2T(\sqrt{n}) +1$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral.

6.

Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences.

Assume that $T(n)$ is constant for $n \leqq 2$. Make your bounds as tight as possible, and justify your answers.

a. $T(n) = 2T\left(\frac{n}{2}\right) + n^3$

b. $T(n) = T\left(\frac{9n}{10}\right) + n$

c. $T(n) = 16T\left(\frac{n}{4}\right) + n^2$

d. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

e. $T(n) = 7T\left(\frac{n}{2}\right) + n^2$

f. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

g. $T(n) = T(n-1) + n$

h. $T(n) = T(\sqrt{n}) + 1$

7.

Give asymptotic upper and lower bounds for T. n/ in each of the following recurrences.
Assume that *T*(*n*) is constant for sufficiently small *n*. Make your bounds
as tight as possible, and justify your answers.

a. $T(n) = 4T\left(\frac{n}{3}\right) + nlgn$

b. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{lgn}$

c. $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$

d. $T(n) = 3T\left(\frac{n}{3} - 2\right) + \frac{n}{2}$

e. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{lgn}$

f. $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$

g. $T(n) = T(n-1) + \frac{1}{n}$

h. $T(n) = T(n-1) + lgn$

i. $T(n) = T(n-2) + \frac{1}{lgn}$

j. $T(n) = \sqrt{n}T(\sqrt{n}) + n$

k. $T(n) = \sqrt{n}T(n-1) + n$