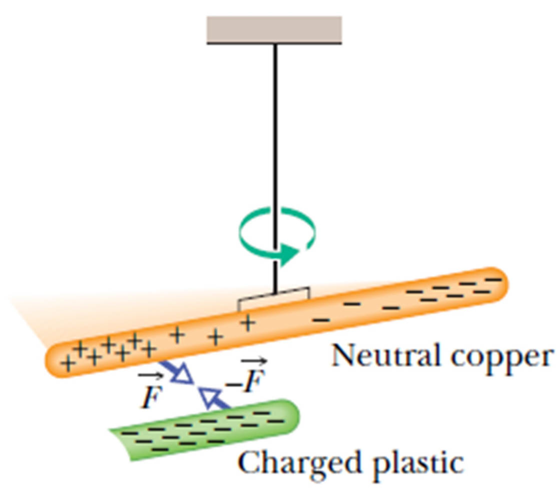
Contents

21 Electric Charge

21.1 Electric Charge:

Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.



21.2 Conductors and Insulators

Conductors are materials through which charge can move freely; examples include metals (such as copper in common lamp wire), the human body, and tap water.

Nonconductors—also called insulators—are materials through which charge cannot move freely; examples include rubber, plastic, glass, and chemically pure water.

Semiconductors are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips.

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance.

The properties of conductors and insulators are due to the structure and electrical nature of atoms.

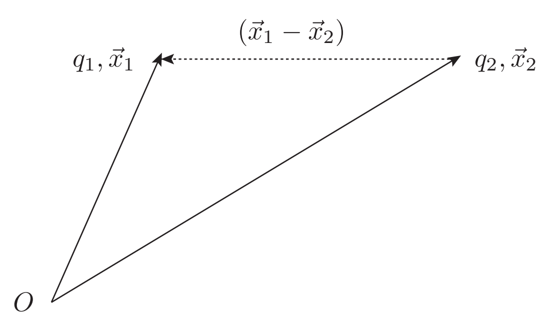
Atoms consist of positively charged protons, negatively charged electrons, and electrically neutral neutrons. The protons and neutrons are packed tightly together in a central nucleus.

When atoms of a conductor come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (positive ions).We call the mobile electrons conduction electrons.

There are few (if any) free electrons in a nonconductor.

21.3 Coulomb's Law

This force of repulsion or attraction due to the charge properties of objects is called an **electrostatic force**. The equation giving the force for charged particles is called **Coulomb's law**:



$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad \text{(Coulomb's law)},$$

where particle 1 has charge q_1 and particle 2 has charge q_2, k is a constant, and $\vec{F}_{12}$ is the force on particle 1 exerted by particle 2. Furthermore,

$$\hat{r}_{12} = \frac{\vec{x}_1 - \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|}, r = |\vec{x}_1 - \vec{x}_2|$$

where $\vec{x}_1$ is the position vector of q_1, and $\vec{x}_2$ is the position vector of q_2. $\hat{r}_{12}$ is a unit vector along an axis extending through the two particles (pointing from $\vec{x}_2$ to $\vec{x}_1$), r is the distance between them.

The SI unit of charge is the coulomb (C). The constant

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 N \cdot m^2/C^2.$$

Note that the odd appearance of 4π here has something to do with the surface area of the unit sphere. This 4π factor will simplify the expression of Gauss' Law that is to be discussed later.

The quantity ε_0 is called the permittivity constant.

$$\varepsilon_0 = 8.85 \times 10^{-12} C^2/(N \cdot m^2).$$

Coulomb's Law
1. varies directly as the magnitude of each charge,
2. varies inversely as the square of the distance,
3. is directed along the line of charge, and
4. is attractive if oppositely charged, repulsive if same type of charge

$$\vec{F} = \frac{qq_1}{4\pi\varepsilon_0}\frac{\vec{x} - \vec{x}_1}{|\vec{x} - \vec{x}_1|^3}$$

is the force on a point charge q, located at $\vec{x}$, due to another charge q_1, located at $\vec{x}_1$.

21.3.1 Linear Superposition

If there are n charged particles, they interact independently in pairs, and the force on any one of them, say particle 1, is given by the vector sum

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$$

in which, $\vec{F}_{14}$ is the force acting on particle 1 due to the presence of particle 4, etc.

The Coulomb force acting on a point charge q, located at $\vec{x}$, due to a system of point charges q_i, located at $\vec{x}_i$, $i = 1, 2, ..., n$, according to linear superposition, is equal to

$$\vec{F}(\vec{x}) = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{n} q_i \frac{\vec{x} - \vec{x}_i}{|\vec{x} - \vec{x}_i|^3}$$

21.3.2

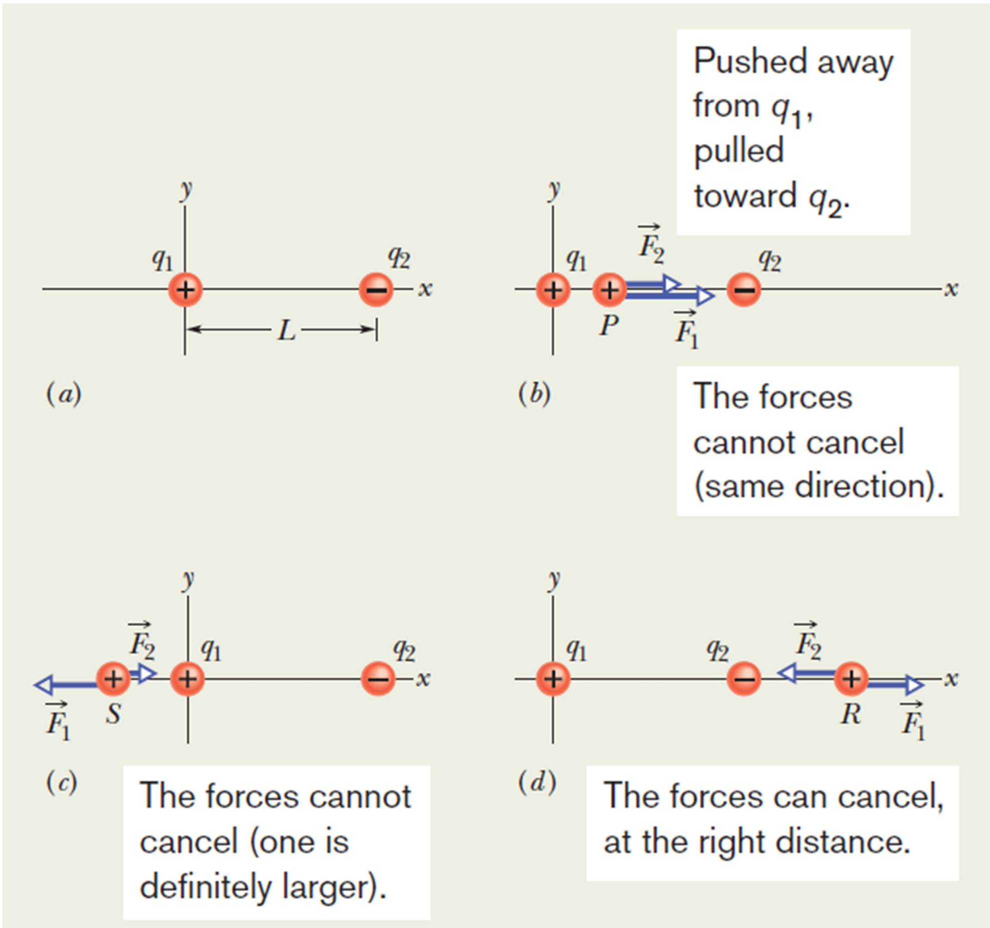
As with gravitational force law, the shell theorem (to be proved in Chapter 23) has analogs in electrostatics:

A Shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.

If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

21.3.3 Example, Equilibrium of two electrostatic forces

The following figure shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge of $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in equilibrium (the net force on it is zero)? Is that equilibrium stable or unstable? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it away?)



If $x > L$, $\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2}\hat{\imath}$ and $\vec{F}_2 = -\frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}\hat{\imath}$. The net force $\vec{F} = \vec{F}_1 + \vec{F}_2$ on proton located at x is equal to

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2}\hat{\imath} - \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}\hat{\imath}$$

At equilibrium, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}$$

$$\frac{(x-L)^2}{x^2} = \frac{1}{4}, \frac{(x-L)}{x} = \frac{1}{2}, x = 2L$$

Assume $x = 2L + \delta$ with $\delta \ll L$. The proton is slightly displace from $x = 2L$ by δ. Then the net force

$$\begin{aligned}
\vec{F} &= \frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{(2L+\delta)^2}\hat{\imath} - \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(L+\delta)^2}\hat{\imath} \\
&= \frac{2qq_p}{4\pi\varepsilon_0}\left(\frac{1}{\left(L+\frac{\delta}{2}\right)^2} - \frac{1}{(L+\delta)^2}\right)\hat{\imath} \\
&= \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{L^2}\left(\left(1+\frac{\delta}{2L}\right)^{-2} - \left(1+\frac{\delta}{L}\right)^{-2}\right)\hat{\imath} \\
&\simeq \delta\left(\frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{L^3}\right)\hat{\imath}
\end{aligned}$$

The equilibrium at $x = 2L$ is unstable if $qq_p > 0$; that is, if the proton is displaced leftward from point R ($\delta < 0$), then the net force $\vec{F}$ points to the left and will drive the proton farther leftward. If the proton is displaced rightward ($\delta > 0$), the net force $\vec{F}$ will then drive the proton farther rightward. In a stable equilibrium ($qq_p < 0$), if the proton is displaced slightly, it should return to the equilibrium position.

21.3.4 Current

Current i is the rate $\frac{dq}{dt}$ at which charge moves past a point or through a region

$$i = \frac{dq}{dt} \quad \text{(electrical current)},$$

in which i is the current (in amperes) and dq (in coulombs) is the amount of charge moving past a point or through a region in time dt (in seconds).

Therefore,

$$1C = (1A)\,(1s)\,.$$

21.4 Charge is Quantized

Since the days of Benjamin Franklin, our understanding of the nature of electricity has changed from being a type of 'continuous fluid' to a collection of smaller charged particles. The total charge was found to always be a multiple of a certain elementary charge, "e":

$$q = ne, \quad n = \pm 1, \pm 2, \pm 3, ...,$$

The value of this elementary charge is one of the fundamental constants of nature, and it is the magnitude of the charge of both the proton and the electron. The value of "e" is:

$$e = 1.602 \times 10^{-19} C$$

The Charges of Three Particles

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

Elementary particles either carry no charge, or carry a single elementary charge. When a physical quantity such as charge can have only discrete values, rather than any value, we say the quantity is quantized. It is possible, for example, to find a particle that has no charge at all, or a charge of $+10e$, or $-6e$, but not a particle with a charge of, say, $3.57e$.

Many descriptions of electric charge use terms that might lead you to the conclusion that charge is a substance. Phrases like:

"Charge on a sphere"

"Charge transferred"

"Charge carried on the electron"

However, charge is a property of particles, one of many properties, such as mass.

21.5 Charge is Conserved

If one rubs a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process.

This hypothesis of **conservation of charge** has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles.

Example 1: Radioactive decay of nuclei, in which a nucleus transforms into (becomes) a different type of nucleus.

A uranium-238 nucleus (^{238}U) transforms into a thorium- 234 nucleus (^{234}Th) by emitting an alpha particle. An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol ^{4}He. Here the net charge is 238.

$$^{238}U \rightarrow \ ^{234}Th \ + \ ^{4}He$$

Example 2: An electron e (charge $-e$) and its antiparticle, the positron e (charge $+e$), undergo an annihilation process, transforming into two gamma rays (high-energy light):. Here the net charge is zero.

$$e^- + e^+ \rightarrow \gamma + \gamma \quad \text{(annihilation)}.$$

Example 3: Gamma rays transforms into an electron and a positron. Here the net charge is again zero.

$$\gamma + \gamma \rightarrow e^- + e^+ \quad \text{(pair production)}.$$