B06505047 陳銘杰 數位控制系統 HW1

1. (a)

$$\mathrm{G(s)} = \frac{(s+1)(s+2)}{s(s+3)(s+4)} = \frac{s^2+3s+2}{s(s+3)(s+4)} = \frac{c_1}{s} + \frac{c_2}{s+3} + \frac{c_3}{s+4}$$

$$= \frac{(c_1+c_2+c_3)s^2 + (7c_1+4c_2+3c_3)s + 12c_1}{s(s+3)(s+4)}$$

$$c_1 = \frac{1}{6}, \begin{cases} c_2 + c_3 = \frac{5}{6} \\ 4c_2 + 3c_3 = \frac{11}{6} \end{cases}, c_2 = -\frac{2}{3}, c_3 = \frac{3}{2}$$

$$\mathrm{G(z)} = \frac{\frac{1}{6}z}{z-1} - \frac{\frac{2}{3}z}{z-e^{-0.3}} + \frac{\frac{3}{2}z}{z-e^{-0.4}}$$

(b)

$$\mathrm{G(s)} = \frac{27}{(s+2)(s^2+4s+13)} = \frac{c_1}{s+2} + \frac{c_2 s + c_3}{s^2+4s+13}$$

$$= \frac{(c_1+c_2)s^2 + (4c_1+2c_2+c_3)s + (13c_1+2c_3)}{(s+2)(s^2+4s+13)}$$

$$c_1 + c_2 = 0, 4c_1 + 2c_2 + c_3 = 0, 13c_1 + 2c_3 = 27$$

$$c_1 = 3, c_2 = -3, c_3 = -6$$

$$\mathrm{G(s)} = \frac{3}{s+2} + \frac{-3s-6}{s^2+4s+13} = \frac{3}{s+2} + \frac{-3(s+2)}{(s+2)^2+9}$$

$$\mathrm{G(z)} = \frac{3z}{z-e^{-0.2}} + \frac{z^2 - ze^{-0.2}cos0.3}{z^2 - 2ze^{-0.2}cos0.3 + e^{-0.4}}$$

2.

3. $\mathrm{F(z)} = \frac{(z+1)(z+0.3)(z+0.4)}{z(z-0.2)(z-0.5)(z-0.7)} = \frac{z^3+1.7z^2+0.82z+0.12}{z(z-0.2)(z-0.5)(z-0.7)}$

$$= \frac{A_1}{z} + \frac{B_1}{z-0.2} + \frac{C_1}{z-0.5} + \frac{D_1}{z-0.7}$$

$$= \frac{(A_1+B_1+C_1+D_1)z^3 + (-1.4A_1 - 1.2B_1 - 0.9C_1 - 0.7D_1)z^2}{z(z-0.2)(z-0.5)(z-0.7)}$$

$$+ \frac{(0.59A_1 + 0.35B_1 + 0.14C_1 + 0.1D_1)z - 0.07A_1}{z(z-0.2)(z-0.5)(z-0.7)}$$

$A_1 = -\frac{12}{7}$, $B_1 = 12$, $C_1 = -36$, $D_1 = \frac{187}{7}$

$$\mathrm{f(k)} = -\frac{12}{7}\delta(k-1) + 12 \times 0.2^{k-1} \times u(k-1) - 36 \times 0.5^{k-1} \times u(k-1) + \frac{187}{7} \times 0.7^{k-1} \times u(k-1)$$

4.

5. (a)

$$\mathrm{G(z)} = \frac{Y(z)}{U(z)} = \frac{z+1}{(z-1)(z-0.7)} = \frac{z+1}{z^2-1.7z+0.7}$$

$$\mathrm{Y(z)}(z^2 - 1.7z + 0.7) = U(z)(z+1)$$

$$\mathrm{Y(z)}(z^0 - 1.7z^{-1} + 0.7z^{-2}) = U(z)(z^{-1} + z^{-2})$$

$$\mathrm{y(k)} - 1.7\mathrm{y(k-1)} + 0.7\mathrm{y(k-2)} = \mathrm{u(k-1)} + \mathrm{u(k-2)}$$

(b)

(c)

$$\mathrm{T(z)} = \frac{G}{1+GH} = \frac{\frac{z+1}{(z-1)(z-0.7)}}{1 + \frac{z+1}{(z-1)(z-0.7)}} = \frac{z+1}{z^2-0.7z+1.7}$$

(d)