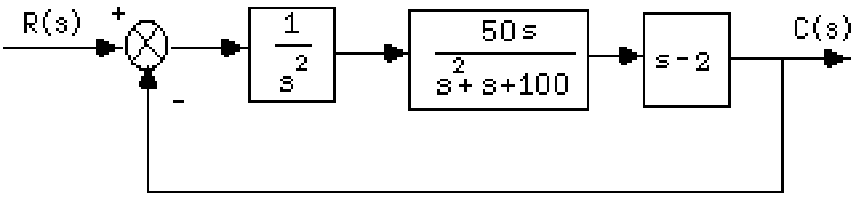
HW3 ANSWER

1.

a. Combine the inner feedback and the parallel pair.



Multiply the blocks in the forward path and apply the feedback formula to get,

$$T(s) = \frac{50(s-2)}{s^3+s^2+150s-100}$$

10.

$$T(s) = \frac{K}{s^2 + \alpha s + K};$$

$$\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = \frac{-\ln 0.1}{\sqrt{\pi^2 + \ln^2 0.1}} = 0.5912;$$

$$T_s = \frac{4}{\varsigma\omega_n} = 0.17.$$

Therefore, $\omega_n = 39.8$; $K = \omega_n^2 = 1584$; $\alpha = 2\zeta\omega_n = 47.06$.

11. We first find ξ, ω_n necessary for the specifications. We have $T_s = \frac{4}{\xi\omega_n} = 3$ and $T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1.5$. Eliminating ω_n from both equations we get $\frac{3\pi\xi}{4\sqrt{1-\xi^2}} = 1.5$. Cross-multiplying, squaring both sides and solving, we get $\xi = \sqrt{\frac{4}{4+\pi^2}} = 0.537$. $\omega_n = 2.4829$. The closed loop transfer function of the system is:

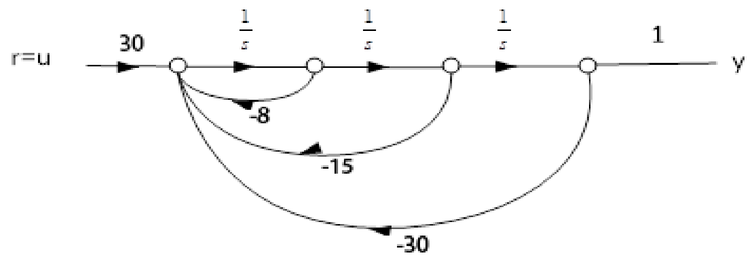
$$T(s) = \frac{\frac{30K_1}{s(s+2)}}{1 + \frac{30K_1}{s(s+2)} + \frac{30K_2 s}{s(s+2)}} = \frac{30K_1}{s^2 + (30K_2 + 2)s + 30K_1}$$

From which we get that $30K_1 = \omega_n^2$ or $K_1 = 0.2055$ and $30K_2 + 2 = 2\xi\omega_n = 2.667$ or $K_2 = 0.0222$.

25.

a. Phase Variable form

$$T(s) = \frac{30}{s^3 + 8s^2 + 15s + 30}$$



$\dot{x_1} = x_2$

$\dot{x_2} = x_3$

$\dot{x_3} = -30x_1 - 15x_2 - 8x_3 + 30r$

$y = x_1$

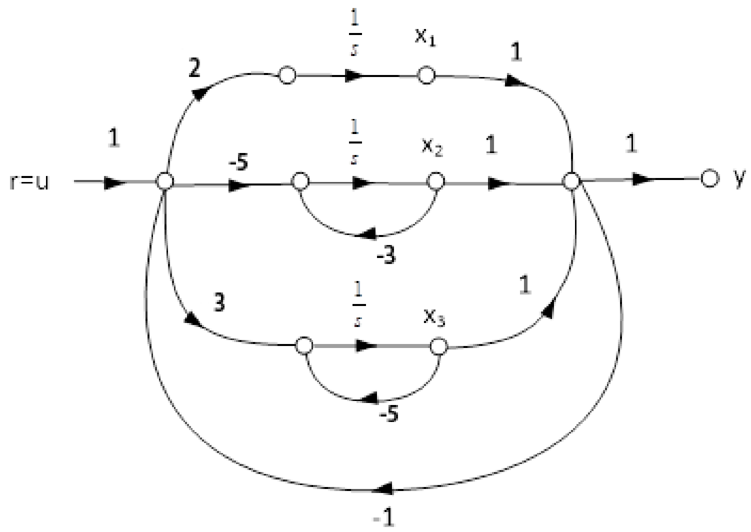
Or in matrix form

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -15 & -8 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]\boldsymbol{x}$$

b. Parallel form(不限定方法)

$$G(s) = \frac{30}{s(s+3)(s+5)} = \frac{2}{s} - \frac{5}{s+3} + \frac{3}{s+5}$$



The state equations are:

$\dot{x_1} = 2(u - x_1 - x_2 - x_3) = -2x_1 - 2x_2 - 2x_3 + 2u$

$\dot{x_2} = -5(u - x_1 - x_2 - x_3) - 3x_2 = +5x_1 + 2x_2 + 5x_3 - 5u$

$\dot{x_3} = 3(u - x_1 - x_2 - x_3) - 5x_3 = -3x_1 - 3x_2 - 8x_3 + 3u$

$y = x_1 + x_2 + x_3$

In matrix form:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & -2 & -2 \\ 5 & 2 & 5 \\ -3 & -3 & -8 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 1]\boldsymbol{x}$$