4.2.5 $\det A = 0$ (singular); $\det U = 16$; $\det U^{\mathrm{T}} = 16$; $\det U^{-1} = 1/16$; $\det M = 16$ (2 exchanges).

4.2.12 (a) False; $\det \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix} \neq 2 \det \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (b) False; $\det \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1$, its pivots are 1, 1. but there is a row exchange (c) False; $I + \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ is singular (d) True; $\det(AB) = \det(A)\det(B) = 0$ (e) False; $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and then $AB - BA = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

4.4.19 $C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$ and $AC^{\mathrm{T}} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Then $\det A = 3$. Cofactor of 100 is 0.

4.3.1 (1) True (product rule) (2) False (all 1's) (3) False ($\det[1\ \ 1\ \ 0;\ 0\ \ 1\ \ 1;\ 1\ \ 0\ \ 1] = 2$)

4.3.2 The 1, 1 cofactor is F_{n-1}. The 1, 2 cofactor has a 1 in column 1, with cofactor F_{n-2}. Multiply by $(-1)^{1+2}$ and also -1 to find $F_n = F_{n-1} + F_{n-2}$. So the determinants are Fibonacci numbers, except F_n is the usual F_{n-1}.

4.4.9 (a) $\det M = x_j$ (b) Look at column j of AM, it is $Ax = b$. All other columns of AM are the same as in A, so $AM = B_j$. (c) $\det A \det M = \det B_j \Rightarrow x_j = \det B_j / \det A$.

4.4.19 $C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$ and $AC^{\mathrm{T}} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Then $\det A = 3$. Cofactor of 100 is 0.

5.1.1 $u = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{3t} = 6 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} - 6 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{3t}$.

5.1.2 $\lambda = -5$ and $\lambda = -4$; both λ's are reduced by 7, with unchanged eigenvectors.

5.1.31 $\lambda(A) = 1, 4, 6$; $\lambda(B) = 2, \sqrt{3}, -\sqrt{3}$; $\lambda(C) = 0, 0, 6$.

5.2.2 (a) $\lambda = 1$ or -1 from $\lambda^2 = 1$ (b) trace $= 0$; det $= -1$ (c) Second row $8, -3$ from the trace and determinant.

5.2.12 (1) True; $\det A = 2 \neq 0$. (2) False; $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. (3) False; $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is diagonal!

5.2.38 If $A = S\Lambda S^{-1}$ then the product $(A-\lambda_1 I)\cdots(A-\lambda_n I)$ equals $S(\Lambda-\lambda_1 I)\cdots(\Lambda-\lambda_n I)S^{-1}$. The factor $\Lambda - \lambda_j I$ is zero in row j. *The product is zero in all rows* = zero matrix.

5.3.5 (a) $A = \begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}$ has $\lambda_1 = 1$ and $\lambda_2 = -\frac{1}{2}$ with $x_1 = (1,1)$ and $x_2 = (1,-2)$

(b) $A^n = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1^n & 0 \\ 0 & (-.5)^n \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$ approaches $A^\infty = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

(c) $\begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix} = A^k \begin{bmatrix} G_1 \\ G_0 \end{bmatrix}$ approaches $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$.

5.3.25 $B^k = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}^k \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$.

5.4.7 (a) $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$, $\lambda_1 = 3$ with $x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\lambda_2 = 2$ with $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; *unstable*

(b) $u = \begin{bmatrix} r \\ w \end{bmatrix} = 100e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 100e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (c) Ratio approaches $2/1$.

5.4.10 (a) $e^{A(t+T)} = Se^{\Lambda(t+T)}S^{-1} = Se^{\Lambda t}e^{\Lambda T}S^{-1} = Se^{\Lambda t}S^{-1}Se^{\Lambda T}S^{-1} = e^{At}e^{AT}$. (b) $e^A = I + A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $e^B = I + B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, $A + B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ gives $e^{A+B} = \begin{bmatrix} \cos 1 & -\sin 1 \\ \sin 1 & \cos 1 \end{bmatrix}$ from Example 1 in the text, at $t = 1$. This matrix is different from e^Ae^B.

5.4.20 $u(t) = \frac{1}{2}\cos 2t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{2}\cos\sqrt{6}t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

5.5.2 $C = \begin{bmatrix} 1 & -i \\ -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}$, $C^{\mathrm{H}} = C$ because $(A^{\mathrm{H}}A)^{\mathrm{H}} = A^{\mathrm{H}}A$.

5.5.9 (i) $\begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & i & 0 \\ 0 & 1 & 1 \end{bmatrix} = U$; $Ax = 0$ if x is a multiple of $\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix}$: this vector is orthogonal *not* to the columns of A^{T} (rows of A) but to the columns of A^{H}.

5.5.18 (1) True; the eigenvalues of A are real so $-i$ is not an eigenvalue and $A + iI$ is invertible.
(2) True; all $|\lambda(Q)| = 1$ so $-1/2$ is not an eigenvalue of Q and $Q + \frac{1}{2}I$ is invertible.
(3) False; real $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ has eigenvalues i and $-i$ and $A + iI$ is not invertible.

5.6.1 If B is invertible then $BA = B(AB)B^{-1}$ is similar to AB.

5.6.18 (i) $TT^{\mathrm{H}} = U^{-1}AUU^{\mathrm{H}}A^{\mathrm{H}}(U^{-1})^{\mathrm{H}} = I$ (ii) If T is triangular and unitary, then its diagonal entries (the eigenvalues) must have absolute value one. Then all off-diagonal entries are zero because the columns are to be unit vectors.

5.6.36 If $M^{-1}JM = K$ then $JM = \begin{bmatrix} m_{21} & m_{22} & m_{23} & m_{24} \\ 0 & 0 & 0 & 0 \\ m_{41} & m_{42} & m_{43} & m_{44} \\ 0 & 0 & 0 & 0 \end{bmatrix} = MK = \begin{bmatrix} 0 & m_{12} & m_{13} & 0 \\ 0 & m_{22} & m_{23} & 0 \\ 0 & m_{32} & m_{33} & 0 \\ 0 & m_{42} & m_{43} & 0 \end{bmatrix}$

That means $m_{21} = m_{22} = m_{23} = m_{24} = 0$ and M is not invertible.

6.1.13 The second derivative matrix is $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$, so f doesn't have a minimum at $(1, 1)$.

6.1.21 $A = \begin{bmatrix} 4 & -4 & 8 \\ -4 & 4 & -8 \\ 8 & -8 & 16 \end{bmatrix}$ has only one pivot = 4, rank = 1, eigenvalues $24, 0, 0$, $\det A = 0$.

6.2.12 B is positive definite, C is negative definite, A and D are indefinite. $x^{\mathrm{T}}Ax = -1$ has a real solution because the quadratic takes negative values and x can be scaled.

6.2.15 False (Q must contain eigenvectors of A); True (same eigenvalues as A); True ($Q^{\mathrm{T}}AQ = Q^{-1}AQ$ is similar to A); True (eigenvalues of e^{-A} are $e^{-\lambda} > 0$).

6.2.34 A is indefinite: $x^{\mathrm{T}}Ax = -1$ for $x = (0, 1, -1)$ (zero on diagonal) (determinants $1, 0, 0$ but not semidefinite!). B is positive semidefinite (determinants $2, 1, 0$) (pivots $2, \frac{1}{2}$, _____) ($x^{\mathrm{T}}Bx = 2(x_1 + \frac{1}{2}x_2 + x_3)^2 + \frac{1}{2}x_2^2$: only two squares).

6.3.3 $AA^{\mathrm{T}} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has $\sigma_1^2 = 3$ with $u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ and $\sigma_2^2 = 1$ with $u_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$.

$A^{\mathrm{T}}A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ has $\sigma_1^2 = 3$ with $v_1 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, $\sigma_2^2 = 1$ with $v_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$; and

nullvector $v_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$. Then $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^{\mathrm{T}}$.

6.3.14 $A^+ = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $B^+ = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$, $C^+ = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 0 \end{bmatrix}$.

A^+ is the right-inverse of A; B^+ is the left-inverse of B.