Data Structures: 505 22240 / ESOE 2012

Homework Assignment 5: Sorting and Sets
<u>Due</u>: the week after next in class, 11:10am

<u>Total score: 100</u>

1. Given an array containing the digits 7   1   8   0   8   2   9   4, show how the order of the digits changes during each step of (30%)
   a. insertion sort. (10%)
   b. selection sort. (10%)
   c. heapsort (using the backward min-heap version discussed in lecture). (10%)
   Show the array after each swap, except in insertion sort. For insertion sort, show the array after each insertion.

2. Another way to analyze randomized quick-sort is to use a **recurrence equation**. In this case, we let $T(n)$ denote the expected running time of randomized quick-sort, and we observe that, because of the worst-case partitions of good and bad splits, we can write (20%)

$$T(n) \leq \tfrac{1}{2}\big(T(3n/4) + T(n/4)\big) + \tfrac{1}{2}\big(T(n-1)\big) + bn,$$

   where bn is the time needed to partition a list for a given pivot and concatenate the result sublists after the recursive calls return. Show, by induction, that $T(n)$ is $O(n \log n)$.
   Hint: You can show that $T(n) \leq cn \log n$ for some constant $c > 0$, by induction, i.e., show that the right hand side is less than or equal to $cn \log n$.

3. Karen has a new way to do path compression in a tree-based union/find partition data structure starting at a node *v*. She puts all the nodes that are on the path from *v* to the root in a set *S*. Then she scans through *S* and sets the parent pointer of each node in *S* to its parent's parent pointer (assume that the parent pointer of the root points to itself). If this pass changed the value of any node's parent pointer, then she repeats this process, and goes on repeating this process until she makes a scan through *S* that does not change any node's parent value. Show that Karen's algorithm is correct and analyze its running time for a path of length *h*. (20%)

4. Suppose we have an array-based disjoint sets data structure, for which the array is (30%)

$$[2 \quad 5 \quad -5 \quad 5 \quad 3 \quad -5 \quad 2 \quad 2 \quad 3 \quad 0]$$

   a. Draw the forest that this array represents. (Each node in the forest represents an item in the range 0…9.) (10%)

   b. Given a sequence of union and find operations whose execution will convert the array to

$$[2 \quad 5 \quad -10 \quad 2 \quad 5 \quad 2 \quad 2 \quad 2 \quad 2 \quad 0]. (10\%)$$

   c. Draw the tree that corresponds to the converted array in (b). (10%)