B06505047 陳銘杰 HW2

6.5 (a)

$$h_1(t) = \frac{sin\omega_c t}{\pi t} \text{ }, H_1(j\omega) = \begin{cases} 1, for|\omega| \le \omega_c \\ 0, otherwise \end{cases}$$

$$\mathrm{H(j\omega)} = H_1\big(j(\omega + 2\omega_c)\big) + H_1\big(j(\omega - 2\omega_c)\big)$$

$$\mathrm{h(t)} = \mathrm{h_1(t)}e^{-j2\omega_c t} + h_1(t)e^{j2\omega_c t}$$

$$= 2\mathrm{h_1(t)}\frac{e^{-j2\omega_c t} + e^{j2\omega_c t}}{2}$$

$$= h_1(t)2\cos(2\omega_c t)$$

$$\mathrm{g(t)} = 2\cos(2\omega_c t)$$

6.5 (b)

More concentrated about the origin

6.14

$$\mathrm{H(j\omega)} = \frac{A(j\omega + 0.2)^2}{(j\omega + 10)(j\omega + 50)}$$

$$\mathrm{H(j0)} = 10^{\frac{12}{20}} = \frac{A(j0 + 0.2)^2}{(j0 + 10)(j0 + 50)} = \frac{0.04A}{10 \times 50}$$

$$\mathrm{A} = 50000$$

$$\mathrm{H(j\omega)} = \frac{50000(j\omega + 0.2)^2}{(j\omega + 10)(j\omega + 50)}$$

$$H_1(j\omega) = \frac{(j\omega + 10)(j\omega + 50)}{50000(j\omega + 0.2)^2}$$

6.26 (a)

$$\mathrm{H}(\mathrm{j}\omega) = 1 - H_0(j\omega)$$

$$\mathrm{h}(\mathrm{t}) = \delta(\mathrm{t}) - h_0(t)$$

$$h_0(t) = \frac{sin\omega_c t}{\pi t}$$

$$\mathrm{h}(\mathrm{t}) = \delta(\mathrm{t}) - \frac{sin\omega_c t}{\pi t}$$

6.26 (b)

More concentrated about the origin

6.26 (c)

$$\mathrm{s}(\mathrm{t}) = \mathrm{h}(\mathrm{t}) * \mathrm{u}(\mathrm{t})$$

$$= [\delta(\mathrm{t}) - h_0(t)] * u(t)$$

$$= \mathrm{u}(\mathrm{t}) - h_0(t) * u(t)$$

$$= \mathrm{u}(\mathrm{t}) - s_0(t)$$

$$\begin{cases} s_0(0) = 0 \\ s_0(0+) = \dfrac{1}{2} \\ s_0(\infty) = 1 \end{cases}$$

$$\mathrm{s}(0+) = \mathrm{u}(0+) - s_0(0+) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\mathrm{s}(\infty) = \mathrm{u}(\infty) - s_0(\infty) = 1 - 1 = 0$$

6.42 (a)

$$H_1(e^{j\omega}) = \frac{1+\frac{1}{2}e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}} = \frac{1+\frac{1}{2}\cos(\omega) - j\frac{1}{2}\sin(\omega)}{1+\frac{1}{4}\cos(\omega) - j\frac{1}{4}\sin(\omega)}$$

$$\left|H_1(e^{j\omega})\right| = \frac{\left(1+\frac{1}{2}\cos(\omega)\right)^2 + \left(\frac{1}{2}\sin(\omega)\right)^2}{\left(1+\frac{1}{4}\cos(\omega)\right)^2 + \left(\frac{1}{4}\sin(\omega)\right)^2}$$

$$= \frac{1+\frac{1}{4}cos^2(\omega) + \cos(\omega) + \frac{1}{4}sin^2(\omega)}{1+\frac{1}{16}cos^2(\omega) + \frac{1}{2}\cos(\omega) + \frac{1}{16}sin^2(\omega)}$$

$$= \frac{\frac{5}{4} + \cos(\omega)}{\frac{17}{16} + \frac{1}{2}\cos(\omega)}$$

$$\angle H_1(e^{j\omega})$$

$$= -\arctan\left(\frac{\frac{1}{2}\sin(\omega)}{1+\frac{1}{2}\cos(\omega)}\right) - \left(-\arctan\left(\frac{\frac{1}{4}\sin(\omega)}{1+\frac{1}{4}\cos(\omega)}\right)\right)$$

$$= -\arctan\left(\frac{\frac{1}{2}\sin(\omega)}{1+\frac{1}{2}\cos(\omega)}\right) + \arctan\left(\frac{\frac{1}{4}\sin(\omega)}{1+\frac{1}{4}\cos(\omega)}\right)$$

$$H_2(e^{j\omega}) = \frac{\frac{1}{2}+e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}} = \frac{\frac{1}{2}+\cos(\omega) - j\sin(\omega)}{1+\frac{1}{4}\cos(\omega) - j\frac{1}{4}\sin(\omega)}$$

$$\left|H_2(e^{j\omega})\right| = \frac{\left(\frac{1}{2}+\cos(\omega)\right)^2 + (\sin(\omega))^2}{\left(1+\frac{1}{4}\cos(\omega)\right)^2 + \left(\frac{1}{4}\sin(\omega)\right)^2}$$

$$=\frac{\frac{1}{4}+cos^2(\omega)+\cos(\omega)+sin^2(\omega)}{1+\frac{1}{16}cos^2(\omega)+\frac{1}{2}\cos(\omega)+\frac{1}{16}sin^2(\omega)}$$

$$=\frac{\frac{5}{4}+\cos(\omega)}{\frac{17}{16}+\frac{1}{2}\cos(\omega)}$$

$$\angle H_2\left(e^{j\omega}\right)=-\arctan\left(\frac{\sin(\omega)}{\frac{1}{2}+\cos(\omega)}\right)-\left(-\arctan\left(\frac{\frac{1}{4}\sin(\omega)}{1+\frac{1}{4}\cos(\omega)}\right)\right)$$

$$=-\arctan\left(\frac{\sin(\omega)}{\frac{1}{2}+\cos(\omega)}\right)+\arctan\left(\frac{\frac{1}{4}\sin(\omega)}{1+\frac{1}{4}\cos(\omega)}\right)$$

$$\left|H_1\left(e^{j\omega}\right)\right|=\left|H_2\left(e^{j\omega}\right)\right|$$

$$\begin{cases}\tau_1=-\frac{\partial H_1\left(e^{j\omega}\right)}{\partial\omega}\\ \tau_2=-\frac{\partial H_2\left(e^{j\omega}\right)}{\partial\omega}\end{cases},\tau_2>\tau_1$$

6.42 (b)

$$H_1\left(e^{j\omega}\right)=\frac{1+\frac{1}{2}e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}}=\frac{1}{1+\frac{1}{4}e^{-j\omega}}+\frac{1}{2}e^{-j\omega}\frac{1}{1+\frac{1}{4}e^{-j\omega}}$$

$$h_1[n]=\left(-\frac{1}{4}\right)^n u[n]+\frac{1}{2}\left(-\frac{1}{4}\right)^{n-1} u[n-1]$$

$$H_2\left(e^{j\omega}\right)=\frac{\frac{1}{2}+e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}}=\frac{1}{2}\frac{1}{1+\frac{1}{4}e^{-j\omega}}+e^{-j\omega}\frac{1}{1+\frac{1}{4}e^{-j\omega}}$$

$$h_1[n]=\frac{1}{2}\left(-\frac{1}{4}\right)^n u[n]+\left(-\frac{1}{4}\right)^{n-1} u[n-1]$$

6.42 (c)

$$\mathrm{G}\left(e^{j\omega}\right) = \frac{H_2\left(e^{j\omega}\right)}{H_1\left(e^{j\omega}\right)} = \frac{\frac{1}{2} + e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

$$\left|\mathrm{G}\left(e^{j\omega}\right)\right| = \frac{\frac{1}{4} + cos^2(\omega) + \cos(\omega) + sin^2(\omega)}{1 + \frac{1}{4}cos^2(\omega) + \cos(\omega) + \frac{1}{4}sin^2(\omega)} = \frac{\frac{5}{4} + \cos(\omega)}{\frac{5}{4} + \cos(\omega)} = 1$$