Contents

14 Fluids

In this chapter we will explore the behavior of fluids. In particular we will study the following:

Static fluids: Pressure exerted by a static fluid
Methods of measuring pressure
Pascal's principle Archimedes' principle, buoyancy
Real versus ideal Fluids in motion: fluids
Equation of continuity
Bernoulli's equation

14.1 Fluids

As the name implies a fluid is defined as a substance that can flow. Fluids conform to the boundaries of any container in which they are placed. A fluid cannot exert a force tangential to its surface. It can only exert a force perpendicular to its surface. Liquids and gases are classified together as fluids to contrast them from solids. In crystalline solids the constituent atoms are organized in a rigid three dimensional regular array known as the lattice.

14.2 Density:

Consider the fluid with a mass Δm and volume ΔV. The density (symbol ρ) is defined as the ratio of the mass over the volume.

$$\rho = \frac{\Delta m}{\Delta V}$$

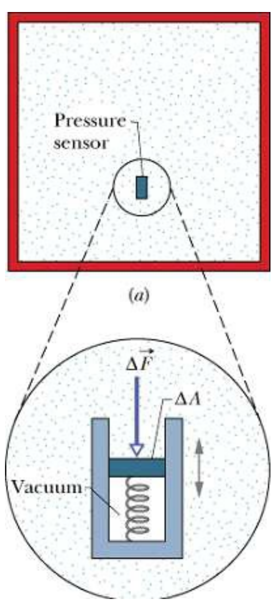
SI unit: kg/m^3

If the fluid is homogeneous the above equation has the form:

$$\rho = \frac{m}{V}$$

14.3 Pressure

Consider the device shown in the insert of the following figure which is immersed in a fluid filled vessel.



The device can measure the normal force F exerted on its piston from the compression of the spring attached to the piston. We assume that the piston has an area A. The pressure p exerted by the fluid on the piston is defined as:

$$p = \frac{F}{A}$$

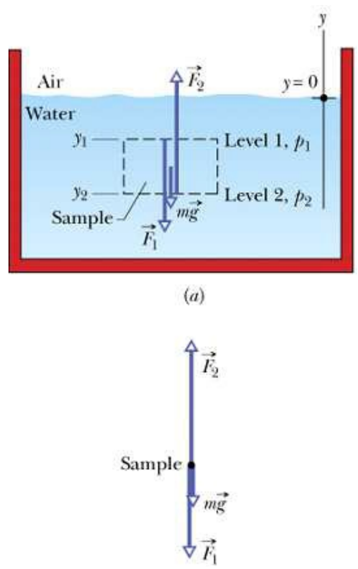
The SI unit for pressure is $\frac{N}{m^2}$ is known as the *pascal* (symbol: Pa). Other units are the *atmosphere* (atm), the *torr*, and the lb/in^2. The atm is defined as the average pressure of the atmosphere at sea level.

$$1\ atm = 1.01 \times 10^5\ Pa = 760\ Torr = 14.7\ lb/in^2$$

Experimentally it is found that the pressure p at any point inside the fluid has the same value regardless of the orientation of the cylinder. The assumption is made that the fluid is at rest.

14.4 Fluids at rest

Consider the tank shown in the figure below.



It contains a fluid of density ρ at rest. We will determine the pressure difference $p_2 - p_1$ between point 2 and point 1 whose y-coordinates are y_2 and y_1, respectively. Consider a part of the fluid in the form of a cylinder indicated by the dashed lines in the figure. This is our system and its is at equilibrium. The equilibrium condition is:

$$F_{ynet} = F_2 - F_1 - mg = 0$$

Here F_2 and F_1 are the forces exerted by the rest of the fluid on the bottom and top faces of the cylinder, respectively. Each face has an area A.

$$F_1 = p_1 A, F_2 = p_2 A, m = \rho V = \rho A (y_1 - y_2)$$

If we substitute into the equilibrium condition we get:

$$p_2 A - p_1 A - \rho g A (y_1 - y_2) = 0 \rightarrow (p_2 - p_1) = \rho g (y_1 - y_2)$$

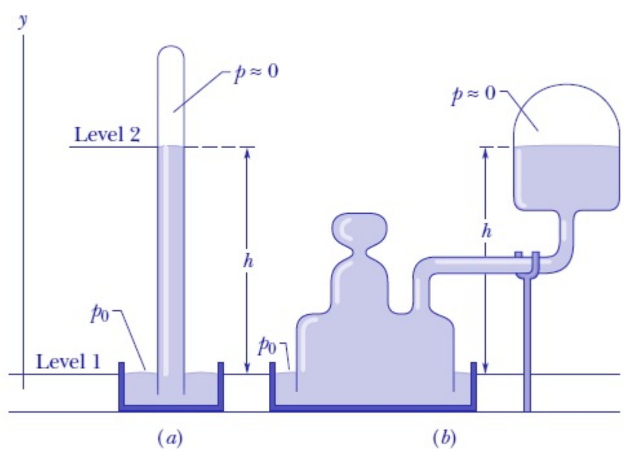
If we take $y_1 = 0$ and $h = -y_2$ then $p_1 = p_0$ and $p_2 = p$. The equation above takes the form:

$$p = p_0 + \rho g h$$

Note: The difference $p - p_0$ is known as gauge pressure.

14.5 The Mercury Barometer

The mercury barometer shown in fig (a) below has been constructed for the first time by Evangelista Toricelli.



It consists of a glass tube of length approximately equal to 1 meter. The tube is filled with mercury and then it is inverted with its open end immersed in a dish filled also with mercury. Toricelli observed that the mercury column drops so that its length is equal to h. The space in the tube above the mercury can be considered as empty. If we take $y_1 = 0$ and $y_2 = -h$ then $p_1 = p_o$ and

$$(p_2 - p_1) = \rho g (y_1 - y_2) \rightarrow p_0 = \rho g h$$

We note that the height h does no depend on the cross sectional area A of the tube. This is illustrated in fig (b) above. The average height of the mercury column at sea level is equal to 760 mm.

14.6 The open tube manometer

The open tube manometer consists of a U-tube that contains a liquid.

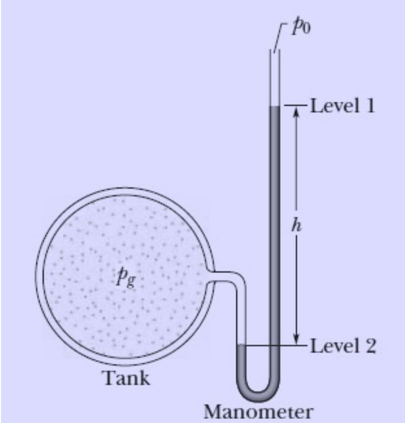


Fig. 14-6 An open-tube manometer, connected to measure the gauge pressure of the gas in the tank on the left. The right arm of the U-tube is open to the atmosphere.

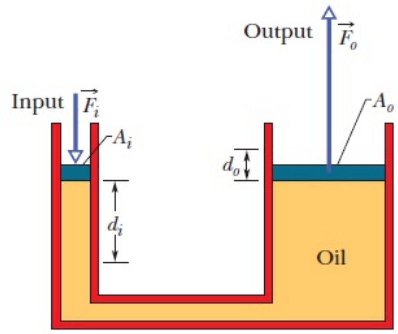
One end is connected to the space for which we wish to measure the gauge pressure. The other end is open to the atmosphere. At level 1: $y_1 = 0$ and $p_1 = p_o$ At level 2: $y_2 = -h$ and $p_2 = p$.

$$p_2 = p_1 + \rho g h \to p - p_0 = \rho g h \to p_g = \rho g h$$

If we measure the length h and if we assume that g is known, we can determine p_g. The gauge pressure can take either positive or negative values.

14.7 Pascal's Principle and the hydraulic lever

Pascal's principle can be formulated as follows: A change in the pressure applied to an enclosed incompressible liquid is transmitted undiminished to every portion of the fluid and to the walls of the container. Consider the enclosed vessel shown in the following figure which contains a liquid.



A force F_i is applied downwards to the left piston of area A_i. As a result, an upward force F_0 appears on the right piston which has area A_0. Force F_i produces a change in pressure $\Delta p = \frac{F_i}{A_i}$ This change will also appear on the right piston. Thus we have:

$$\Delta p = \frac{F_i}{A_i} = \frac{F_0}{A_0} \rightarrow F_0 = \frac{A_0}{A_i} F_i$$

If $A_o > A_i$, $F_o > F_i$.

14.7.1 The hydraulic lever; energy considerations.

The hydraulic lever shown in the above figure is filled with an incompressible liquid. We assume that under the action of force F_i the piston to the left travels downwards by a distance d_i. At the same time the piston to the right travels upwards by a distance d_0. During the motion we assume that a volume V of the liquid is displaced at both pistons

$$V = A_i d_i = A_o d_o \rightarrow d_0 = d_i \frac{A_i}{A_0}$$

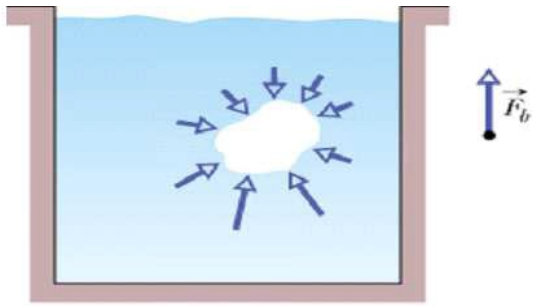
Note: Since $A_0 > A_i \rightarrow d_0 < d_i$, the output work

$$W_0 = F_0 d_0 = \left(F_i \frac{A_0}{A_i} \right) \left(d_i \frac{A_i}{A_0} \right)$$

Thus $W_0 = F_i d_i = W_i$. The work done on the left piston by F_i is equal to the work done by the piston to the right in lifting a load placed on it. With a hydraulic lever a given force F_i applied over a distance d_i can be transformed into a larger force F_0 applied over a smaller distance d_0

14.8 Buoyant Force

Consider a very thin plastic bag which is filled with water.



The bag is at equilibrium thus the net force acting on it must be zero. In addition to the gravitational force $\vec{F}_g$ there exists a second force $\vec{F}_b$ known as buoyant force which balances

$$\left|\vec{F}_b\right| = \left|\vec{F}_g\right| = m_f g$$

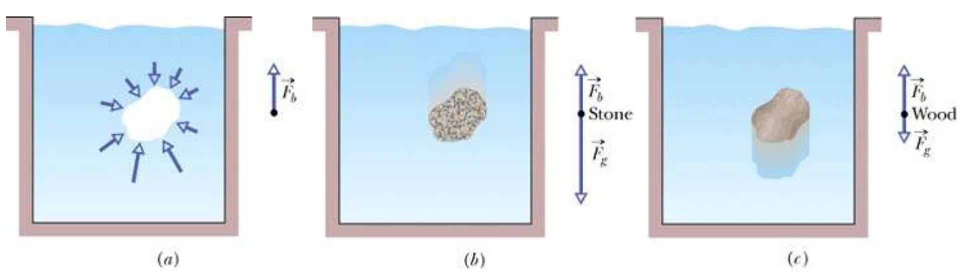
Here m_f is the mass of the water in the bag. If V is the bag volume we have:

$$m_f = \rho_f g V$$

Thus the magnitude of the buoyant force $\left|\vec{F}_b\right| = \rho_f g V$. $\vec{F}_b$ exists because the pressure on the bag exerted by the surrounding water increases with depth. The vector sum of all the forces points upwards, as shown in the above figure.

14.9 Archimedes' principle

Consider the following three figures.



They show three objects that have the same volume (V) and shape but are made of different materials. The first is made of water, the second of stone, and the third of wood. The buoyant force F_b in all cases is the same: $\left|\vec{F_b}\right| = \rho_f g V$. This result is summarized in what is known as Archimedes' Principle. When a body is fully or partially submerged in a fluid a buoyant force $\vec{F_b}$ is exerted on the body by the surrounding fluid. This force is directed upwards and its magnitude is equal to the weight $m_f g$ of the fluid that has been displaced by the body. We note that the submerged body is fig. (a) is at equilibrium with $\left|\vec{F_g}\right| = \left|\vec{F_b}\right|$. In fig. (b) $\left|\vec{F_g}\right| > \left|\vec{F_b}\right|$ and the stone accelerates downwards. In fig. (c) $\left|\vec{F_b}\right| > \left|\vec{F_g}\right|$ and the wood accelerates upwards.

14.9.1 Ideal Fluids:

The motion of real fluids is very complicated and not fully understood. For this reason we shall discuss the motion of an ideal fluid which is simpler to describe. Below we describe the characteristics of an ideal fluid.

1. Steady flow. The velocity $\vec{v}$ of the moving fluid at any fixed point does not change with time. This type of flow is known as laminar.

2. Incompressible flow. The assumption is made that the moving fluid is incompressible i.e. its density is uniform and constant.

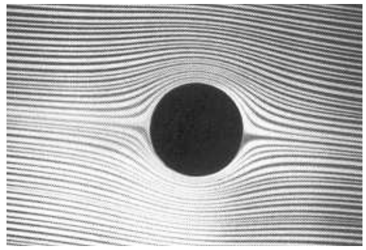
3. Nonviscous flow. Viscosity in fluids is a measure of how resistive the fluid is to flow. Viscosity in fluids is the analog of friction between solids. Both mechanisms convert kinetic energy into thermal energy (heat). An object moving in a non-viscous fluid experiences no drag force.

4. Irrotational flow. A small particle that moves with the fluid will not rotate about an axis through its center of mass.

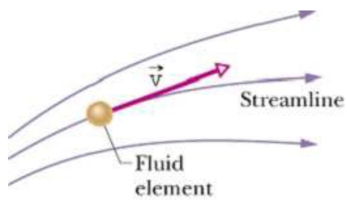
14.9.2 Streamlines

The flow of a fluid can be made visible by adding a tracer. In the case of a liquid the tracer can be a dye. An example

is given in the following picture.



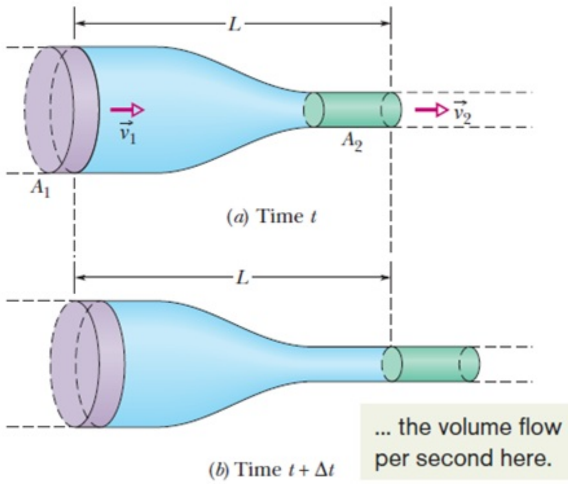
In the case of gas smoke particles can be used as a tracer. Each visible tracer particle follows a streamline which is a path that a fluid element would take. Three such streamlines are shown below.



The velocity $\vec{v}$ of a fluid element is always tangent to a streamline, in the same way that the velocity of a moving object is tangent to the path at any point. Two streamlines cannot intersect. If they did then two different velocities, each corresponding to the two streamlines at the intersection point, could be defined. This would be physically meaningless.

14.10 Equation of Continuity

In this section we consider the flow of a fluid through a tube whose cross-sectional area A is not constant. We will find the equation that connects the area A with the fluid speed v. Consider an fluid element e that moves with speed v through a tube of cross sectional area A. In a time interval Δt the element travels a distance $\Delta x = v\Delta t$ as below.



The fluid volume ΔV is given by the equitation:

$$\Delta V = A\Delta x = Av\Delta t$$

Assume that the fluid of volume ΔV and speed v_1 enters the tube from its left end. The cross-sectional area on the left is A_1. The same volume exits at the right end of the tube that has cross - sectional area A_2.

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t \rightarrow A_1 v_1 = A_2 v_2$$

The equation of continuity is based on the assumption that the fluid is incompressible. If we solve the equation of continuity for v_2 we get:

$$v_2 = v_1 \frac{A_1}{A_2}$$

If $A_2 < A_1$ then $v_2 > v_1$. In other words if the tube narrows the fluid speeds up. The opposite is also true: If $A_2 > A_1$ then $v_2 < v_1$. At points where the tube becomes wider, the fluid slows down. The equation of continuity is also true for a tube of flow which is a section of the fluid bounded by streamlines. This is so because streamlines cannot cross and therefore all the fluid inside the tube remains within its boundary. We refine the volume flow rate

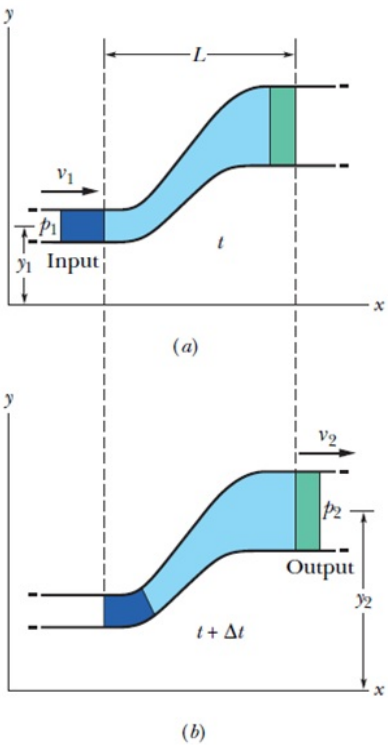
$$R_V = \frac{\Delta V}{\Delta t} = \frac{vA\Delta t}{\Delta t} = vA$$

In a similar fashion we define the mass flow rate

$$R_m = \frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \rho R_V = \rho A v$$

14.11 Bernoulli's Equation

Consider an ideal fluid flowing through the tube shown in the figure.



A fluid volume ΔV enters to the left at height y_1 with speed v_1 under pressure p_1. The same volume exits at the right end at height y_2 with speed v_2 under pressure p_2. We apply the work-kinetic energy theorem.

$$W = \Delta K \tag{1}$$

The change in kinetic energy

$$\Delta K = \frac{\Delta m}{2} v_2^2 - \frac{\Delta m}{2} v_1^2 = \frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right) \tag{2}$$

The work W has two terms. One term W_g from the gravitational force and a second W_p from the pressure force.

$$W = W_g + W_p$$

$$W_g = -\Delta m g\left(y_2 - y_1\right) = -\rho g \Delta V\left(y_2 - y_1\right)$$

$$W_p = p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2 = p_1 \Delta V - p_2 \Delta V = -\left(p_2 - p_1\right)\Delta V$$

$$W = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V \tag{3}$$

14.11.1 Bernoulli's Equation

Combining the three equations,(1) , (2) and (3):

$$W = \Delta K.$$
$$\Delta K = \frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right)$$
$$W = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V$$

we get:

$$\frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right) = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V$$

If we rearrange the terms we have:

$$p_1 + \frac{\rho}{2}v_1^2 + \rho g y_1 = p_2 + \frac{\rho}{2}v_2^2 + \rho g y_2$$

For the special case in which $y_1 = y_2$ we have:

$$p_1 + \frac{\rho}{2}v_1^2 = p_2 + \frac{\rho}{2}v_2^2$$

This equation states that: If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure must decrease.