Mechanics of Materials, Brief Edition

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SI Version

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Chapter 1
Tension, Compression, and Shear

1.1 Introduction to Mechanics of Materials
1.2 Normal Stress and Strain
1.3 Mechanical Properties of Materials
1.4 Elasticity, Plasticity, and Creep
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1.6 Shear Stress and Strain
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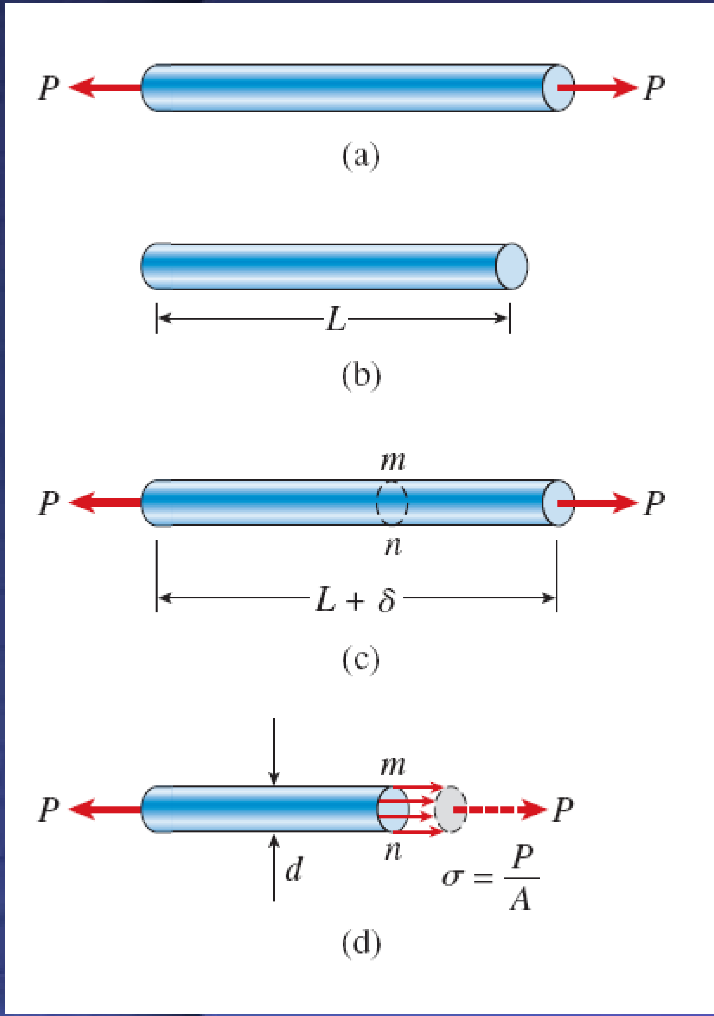
1.1 INTRODUCTION TO MECHANICS OF MATERIALS

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. Other names for this field of study are *strength of materials and mechanics of deformable bodies.*

The principal objective of mechanics of materials is to determine the stresses, strains, and displacements in structures and their components due to the loads acting on them.

1.2 NORMAL STRESS AND STRAIN



$$\sigma = \frac{P}{A} \qquad (1\text{-}1)$$

FIG. 1-2 Prismatic bar in tension:
(a) free-body diagram of a segment of the bar,
(b) segment of the bar before loading,
(c) segment of the bar after loading, and
(d) normal stresses in the bar

- Normal stresses: The stresses act in a direction perpendicular to the cut surface.

- Shear stresses: The stresses that acts parallel to the surface is called.

- Normal stresses:
  - tensile stresses
  - compressive stresses

In SI units: N/mm^2, or N/m^2 (pascal, Pa)

Normal Strain

Strain: Elongation per unit length, and is denoted by the Greek letter ε (epsilon).

$$\in = \frac{\delta}{L} \qquad (1\text{-}2)$$

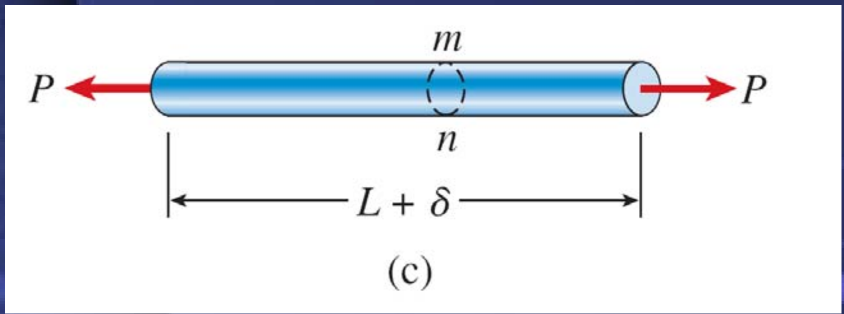


FIG. 1-2 Prismatic bar in tension:
(c) segment of the bar after loading, and

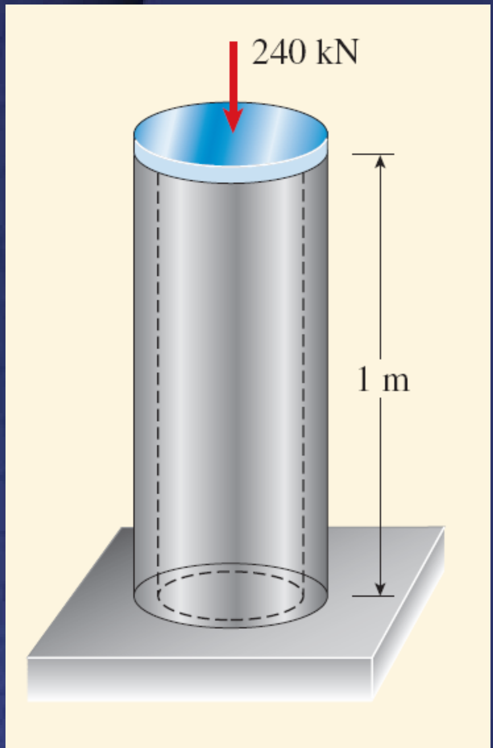
The strain ε is called a normal strain because it is associated with normal stresses.

It is a dimensionless quantity.

Consider a steel bar having length L equal to 2.0 m. When heavily loaded in tension, this bar might elongate by 1.4 mm, which means that the strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.4\text{mm}}{2.0\text{m}} = 0.0007 = 700 \times 10^{-6}$$

Example 1-1



A short post constructed from a hollow circular tube of aluminum supports a compressive load of 240 kN (Fig. 1-5). The inner and outer diameters of the tube are *d_1 = 90 mm and d_2 = 130 mm, respectively, and its length is 1 m. The* shortening of the post due to the load is measured as 0.55 mm.

Determine the compressive stress and strain in the post. (Disregard the weight of the post itself, and assume that the post does not buckle under the load.)

FIG. 1-5 Example 1-1. Hollow aluminum post in compression

Solution

The force P equals 240 kN (or 240,000 N), and the cross-sectional area A is

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\left|\left(130\text{ mm}\right)^2 - \left(90\text{ mm}\right)^2\right| = 6.912\text{ mm}^2$$

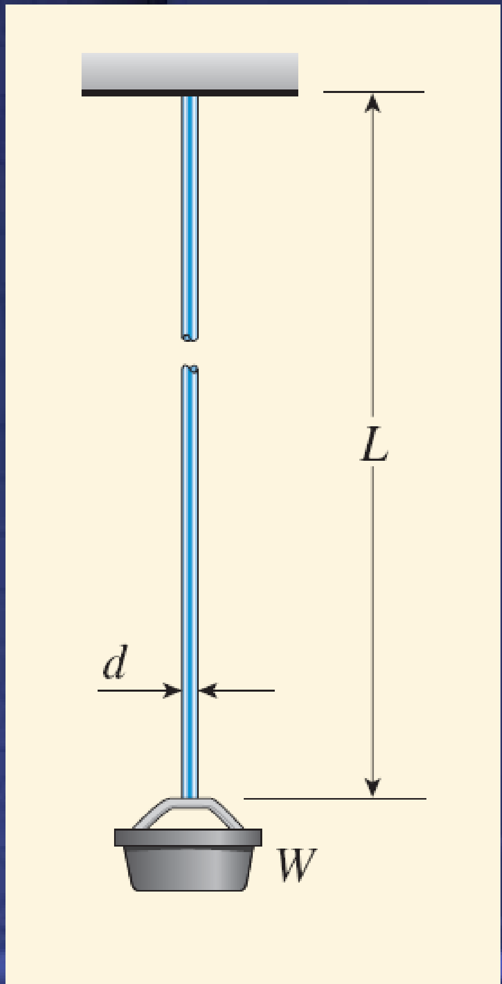
Therefore, the compressive stress in the post is

$$\sigma = \frac{P}{A} = \frac{240{,}000\text{ N}}{6912\text{ mm}^2} = 34.7\text{ MPa}$$

The compressive strain (from Eq. 1-2) is

$$\in = \frac{\delta}{L} = \frac{0.55\text{ mm}}{1.000\text{ mm}} = 550 \times 10^{-6}$$

Example 1-2



A circular steel rod of length L and diameter d hangs in a mine shaft and holds an ore bucket of weight W at its lower end (Fig. 1-6).

(a) Obtain a formula for the maximum stress σ_{max} in the rod, taking into account the weight of the rod itself.

(b) Calculate the maximum stress if L = 40 m, d = 8 mm, and W = 1.5 kN.

FIG. 1-6 Example 1-2. Steel rod supporting a weight W

Solution

(a) The weight W_0 of the rod itself. The latter is equal to the weight density γ of the steel times the volume V of the rod, or

$$W_0 = \gamma V = \gamma AL \qquad (1\text{-}4)$$

The maximum stress (from Eq. 1-1) becomes

$$\sigma_{max} = \frac{F_{max}}{A} = \frac{W + \gamma AL}{A} = \frac{W}{A} + \gamma L \qquad (1\text{-}5)$$

(b) the maximum stress is obtained

$$\sigma_{max} = \frac{1.5 \text{ kN}}{\pi (8 \text{ mm})^2 / 4} + \left(77.0 \text{ kN/m}^3\right)\left(40 \text{ m}\right)$$

$$= 29.84 \text{ MPa} + 3.11 \text{ MPa} = 33.0 \text{ MPa}$$

1.3 MECHANICAL PROPERTIES OF MATERIALS

FIG. 1-7 Tensile-test machine with automatic data-processing system. (Courtesy of MTS Systems Corporation)

The ASTM standard tension specimen has a diameter of 12.8 mm and a gage length of 50.8 mm between the gage marks.

In a static test, the load is applied slowly and the precise rate of loading is not of interest because it does not affect the behavior of the specimen.

FIG. 1-8 Typical tensile-test specimen with extensometer attached; the specimen has just fractured in tension. (Courtesy of MTS Systems Corporation)

(1) Proportional limit
(2) Modulus of elasticity
(3) Yield stress
(4) Perfectly plastic
(5) Strain harden
(6) Ultimate stress
(7) Lateral contraction
(8) Necking
(9) True stress-strain curve (*CE'*)
(10) Conventional stress-strain curve(*CE*)
(11) Ductile

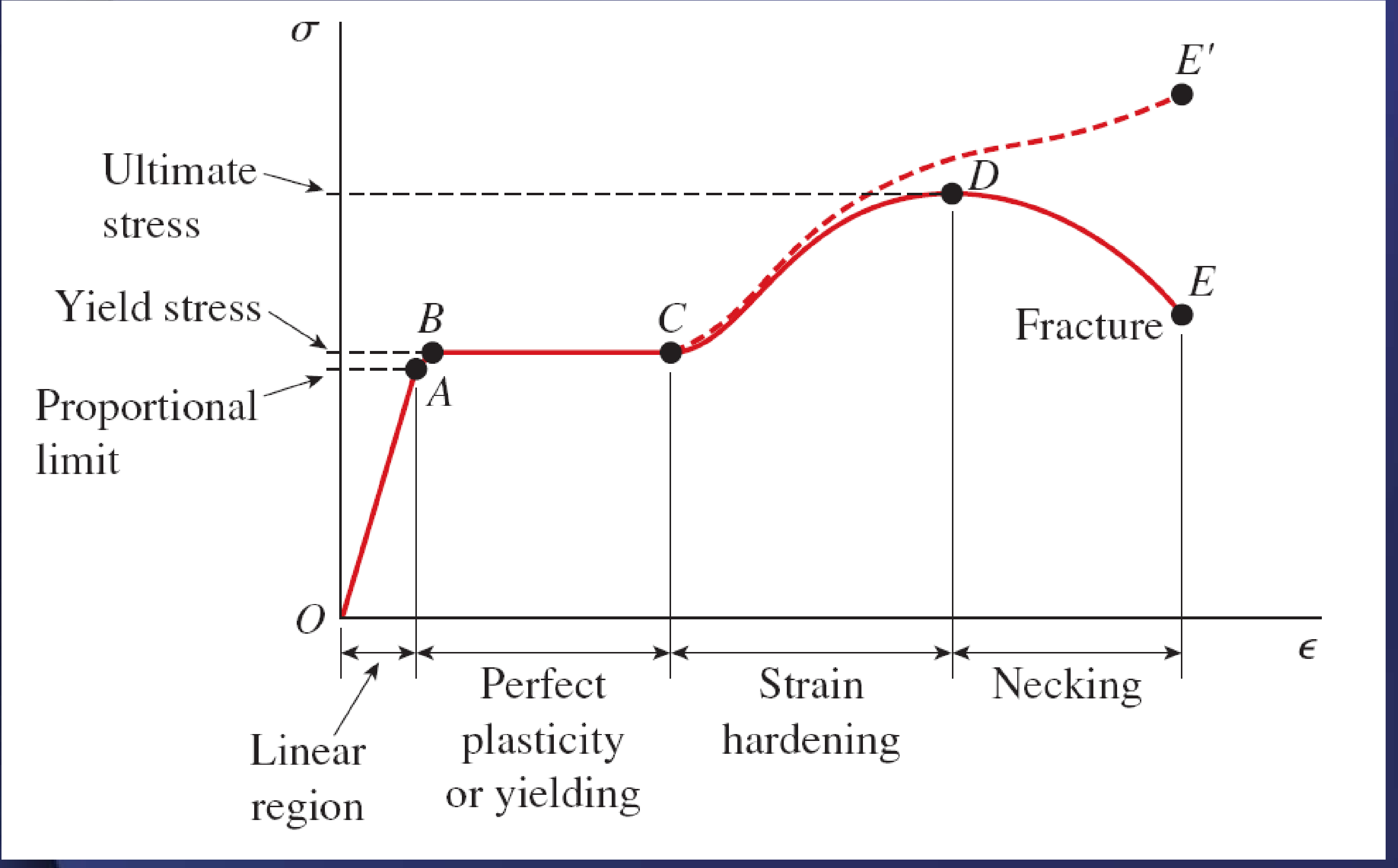


FIG. 1-10 Stress-strain diagram for a typical structural steel in tension (not to scale)

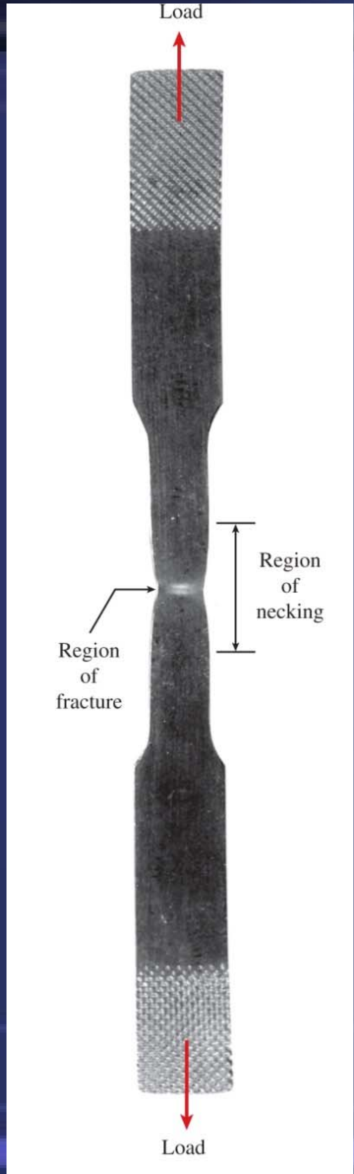
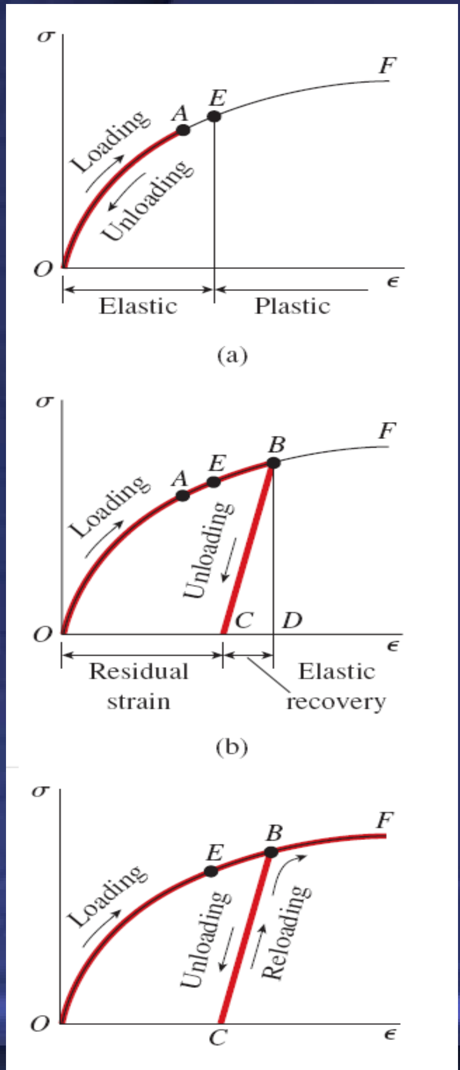


FIG. 1-11 Necking of a mild-steel bar in tension

1.4 ELASTICITY, PLASTICITY, AND CREEP



(1) Elasticity
(2) Residual strain
(3) Permanent set
(4) Partially elastic
(5) Elastic limit

FIG. 1-18 Stress-strain diagrams illustrating (a) elastic behavior, and (b) partially elastic behavior

FIG. 1-19 Reloading of a material and raising of the elastic and proportional limits

Creep

(1) Creep
(2) Relaxation

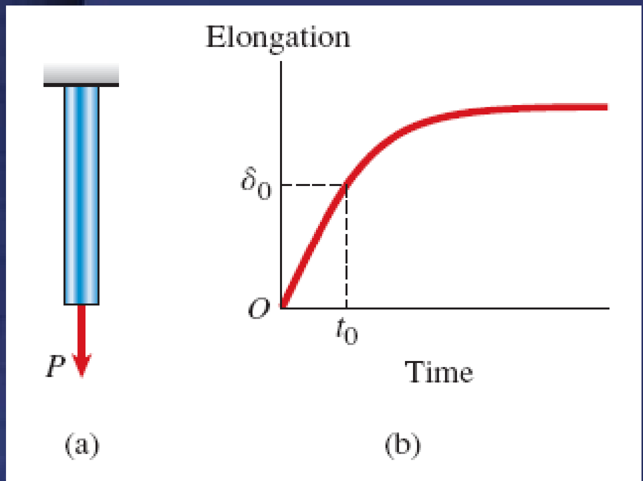


FIG. 1-20 Creep in a bar under constant load

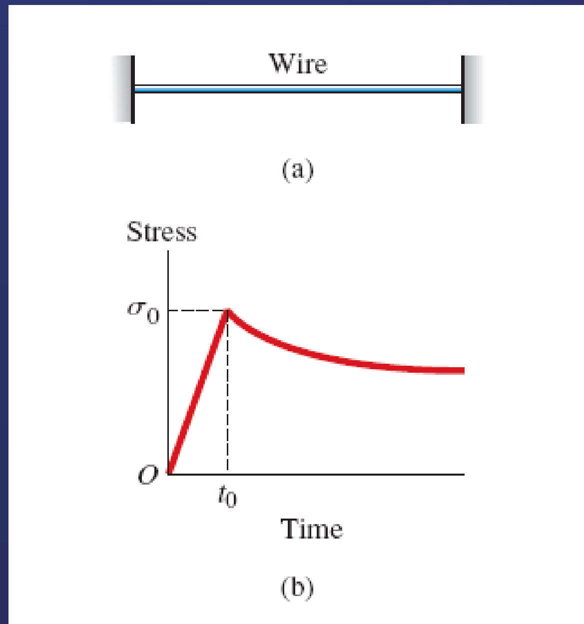


FIG. 1-21 Relaxation of stress in a wire under constant strain

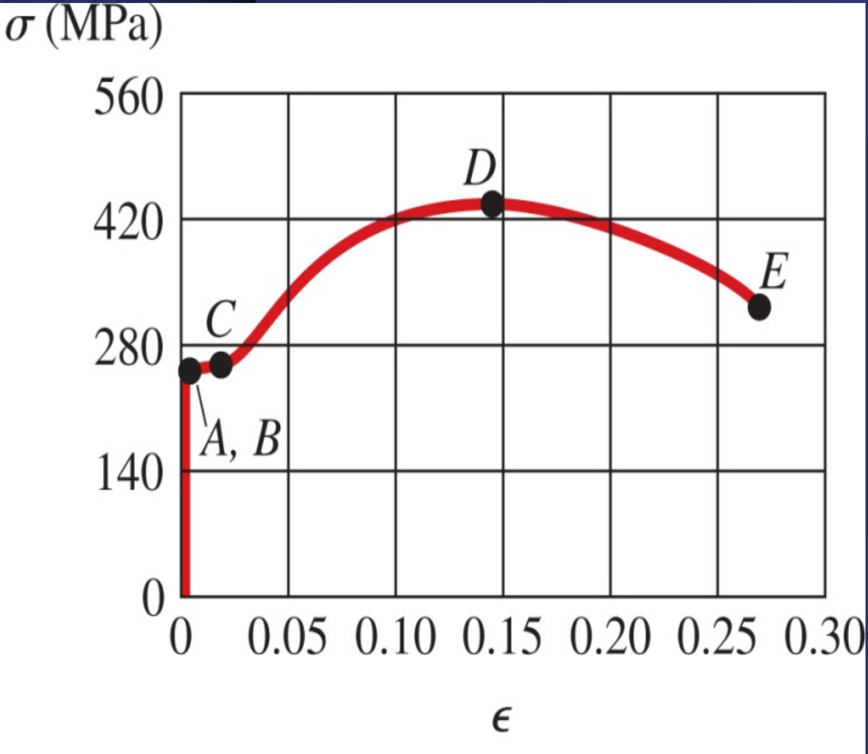


FIG. 1-12 Stress-strain diagram for a typical structural steel in tension (drawn to scale)

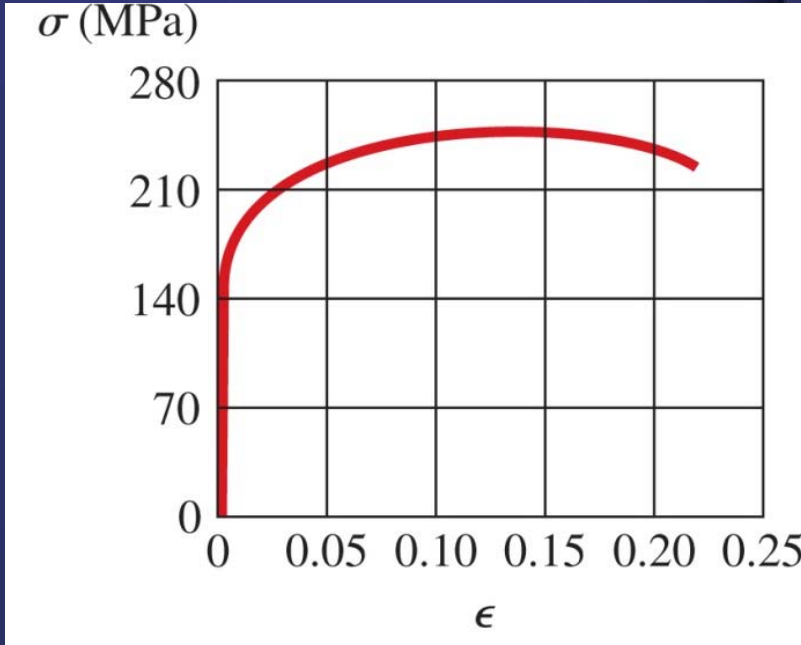


FIG. 1-13 Typical stress-strain diagram for an aluminum alloy

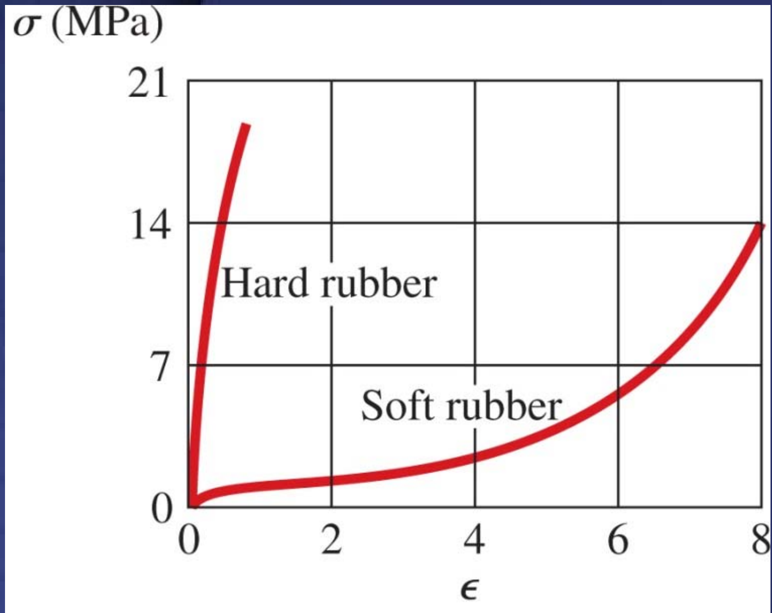


FIG. 1-15 Stress-strain curves for two kinds of rubber in tension

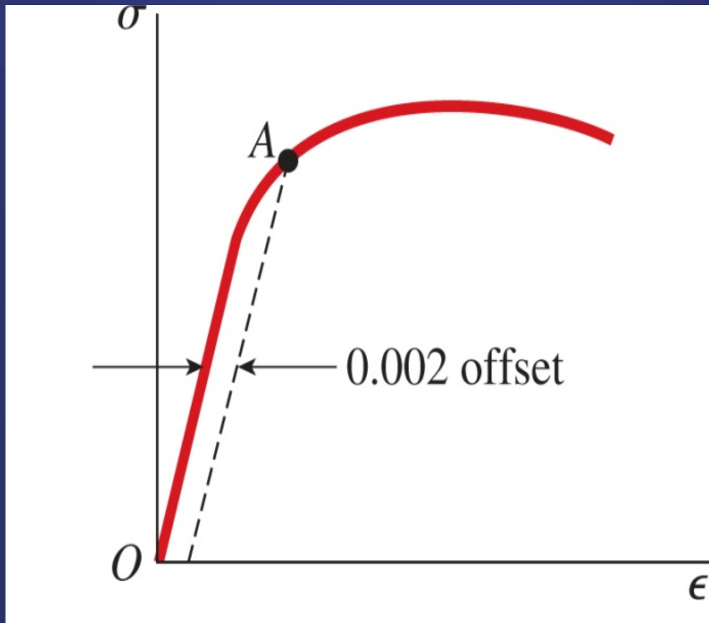


FIG. 1-14 Arbitrary yield stress determined by the offset method

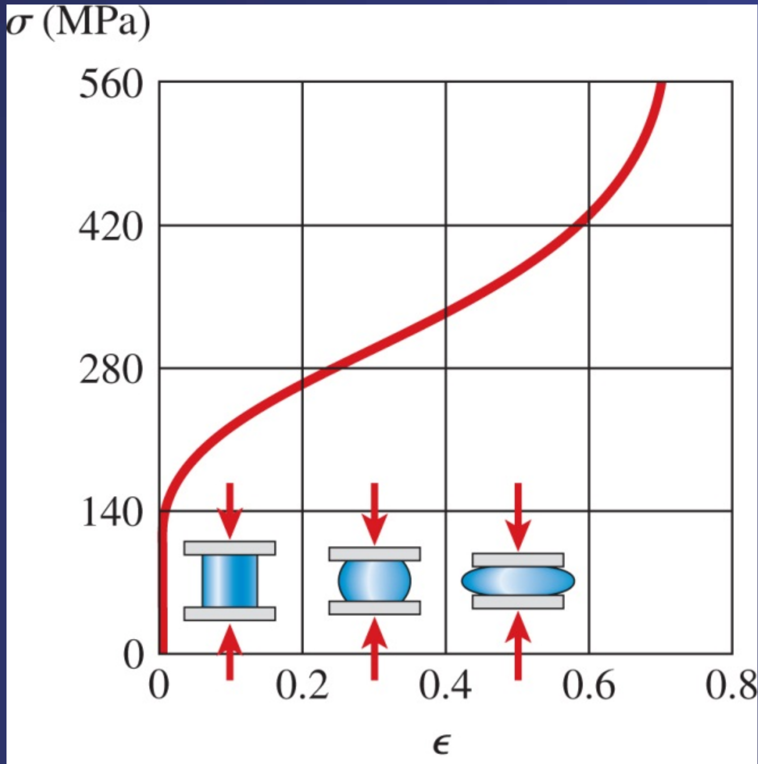


FIG. 1-17 Stress-strain diagram for copper in compression

1.5 LINEAR ELASTICITY, HOOKE'S LAW, AND POISSON'S RATIO

When a material behaves elastically and also exhibits a linear relationship between stress and strain, it is said to be linearly elastic.

The linear relationship between stress and strain for a bar in simple tension or compression is expressed by the equation

$$\sigma = E \in \qquad (1\text{-}8)$$

E is a constant of proportionality known as the modulus of elasticity for the material.

Modulus of elasticity is often called Young's modulus.

Poisson's Ratio

When a prismatic bar is loaded in tension, the axial elongation is accompanied by lateral contraction.

The lateral strain ϵ' at any point in a bar is proportional to the axial strain ϵ at that same point if the material is linearly elastic.

The ratio of the lateral strain ϵ' to the axial strain ϵ is known as Poisson's ratio and Is denoted by the Greek letter ν *(nu)*,

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon'}{\epsilon} \qquad (1\text{-}9)$$

In this book, all examples and problems are under the assumption that the material is linearly elastic, homogeneous, and isotropic

(1)Homogeneous
(2) Isotropic

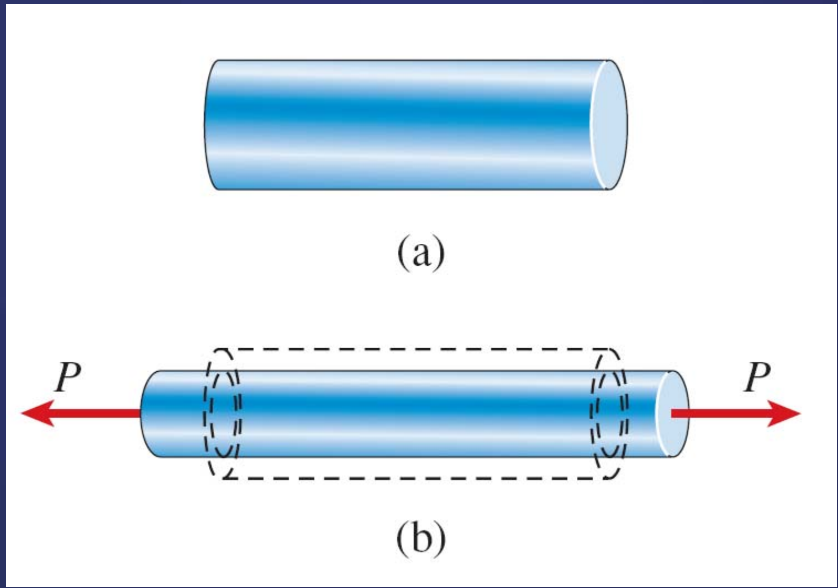
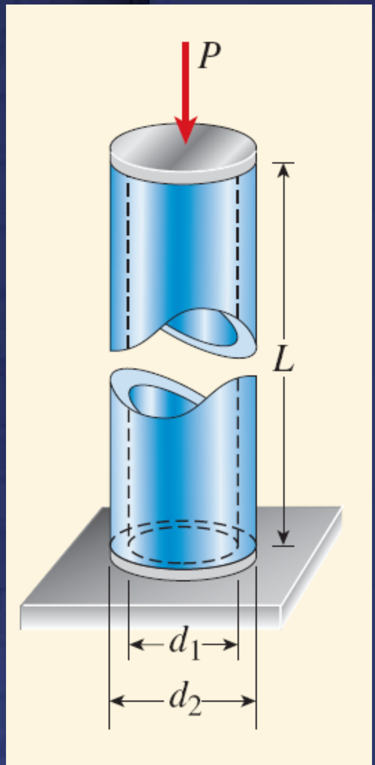


FIG. 1-22 Axial elongation and lateral contraction of a prismatic bar in tension:
(a) bar before loading, and
(b) bar after loading. (The deformations of the bar are highly exaggerated.)

Example 1-3



A steel pipe of length L = 1.2 m, outside diameter d_2 =150 mm, and inside diameter d_1 =110 mm is compressed by an axial force P = 620 kN (Fig. 1-23). The material has modulus of elasticity E = 200 GPa and Poisson's ratio v = 0.30.

Determine the following quantities for the pipe:

(a) the shortening δ,

(b) the lateral strain ϵ',

(c) the increase Δd_2 in the outer diameter and the increase Δd_1 in the inner diameter, and

(d) the increase Δt in the wall thickness.

FIG. 1-23 Example 1-3. Steel pipe in compression

Solution

The cross-sectional area A and longitudinal stress σ are determined as follows:

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\left|(150\text{ mm})^2 - (110\text{ mm})^2\right| = 8.168\text{ mm}^2$$

$$\sigma = -\frac{P}{A} = \frac{620\text{ kN}}{8168\text{ mm}^2} = -75.9\text{ MPa} \quad \text{(compression)}$$

$$\in = \frac{\sigma}{E} = \frac{-75.9\text{ MPa}}{200\text{ GPa}} = -379.5\times 10^{-6}$$

(a) The negative sign again indicates a shortening of the pipe.

(b) The lateral strain is obtained from Poisson's ratio (see Eq. 1-10):

$$\epsilon' = -\nu\,\epsilon = -(0.30)\left(-379.5\times10^{-6}\right) = 113.9\times10^{-6}$$

(c) The increase in outer diameter equals the lateral strain times the diameter:

$$\Delta d_2 = \epsilon'\,d_2 = \left(113.9\times10^{-6}\right)(150\text{ mm}) = 0.0171\text{ mm}$$

The increase in inner diameter is

$$\Delta d_1 = \epsilon'\,d_1 = \left(113.9\times10^{-6}\right)(110\text{ mm}) = 0.0125\text{ mm}$$

(d) The increase in wall thickness is

$$\Delta t = \epsilon' t = \left(113.9 \times 10^{-6}\right)\left(20 \text{ mm}\right) = 0.00228 \text{ mm}$$

The increase in wall thickness is equal to half the difference of the increases in diameters:

$$\Delta t = \frac{\Delta d_2 - \Delta d_1}{2} = \frac{1}{2}\left(0.0171 \text{ mm} - 0.0125 \text{ mm}\right) = 0.00228 \text{ mm}$$

1.6 SHEAR STRESS AND STRAIN

Shear stress acts tangential to the surface of the material.

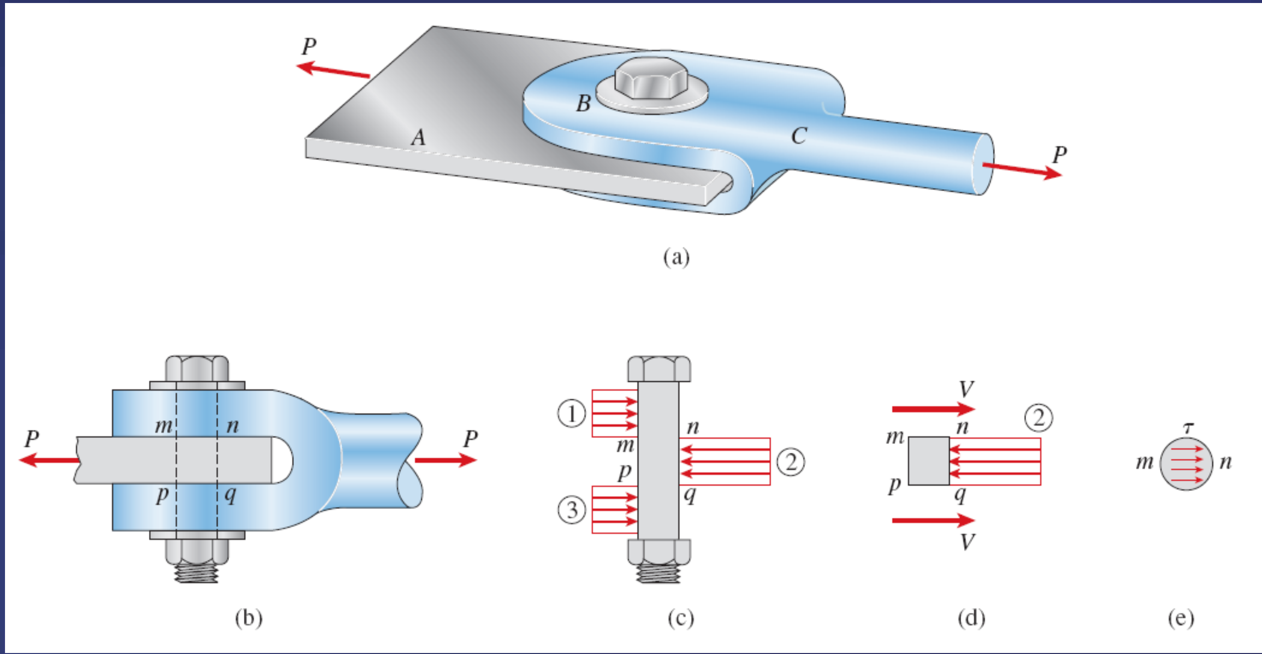


FIG. 1-24 Bolted connection in which the bolt is loaded in double shear

The average shear stress on the cross section of a bolt is obtained by dividing the total shear force V by the area A of the cross section on which it acts, as follows:

$$\tau_{\text{aver}} = \frac{V}{A} \qquad \text{(1-12)}$$

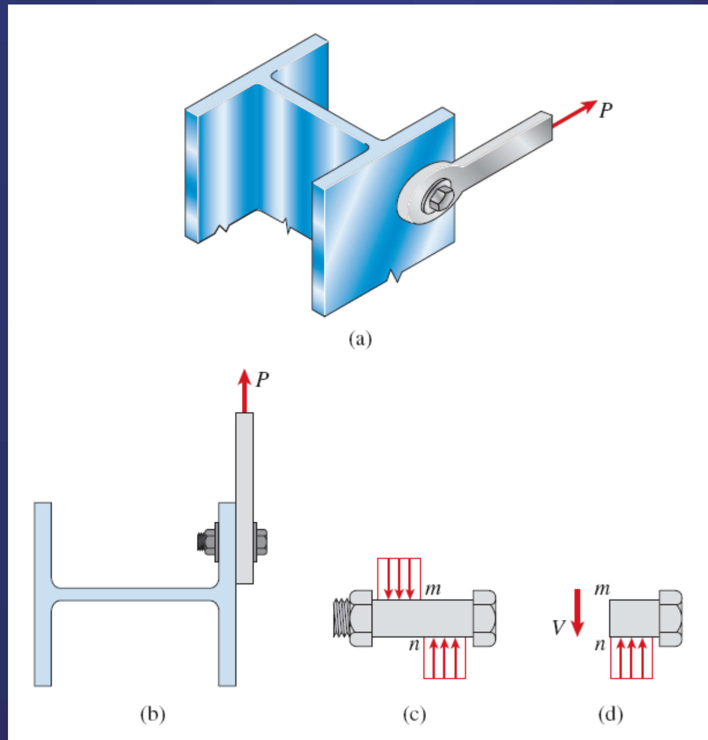


FIG. 1-25 Bolted connection in which the bolt is loaded in single shear

Equality of Shear Stresses on Perpendicular Planes

1. Shear stresses on opposite (and parallel) faces of an element are equal in magnitude and opposite in direction.
2. Shear stresses on adjacent (and perpendicular) faces of an element are equal in magnitude and have directions such that both stresses point toward, or both point away from, the line of intersection of the faces.

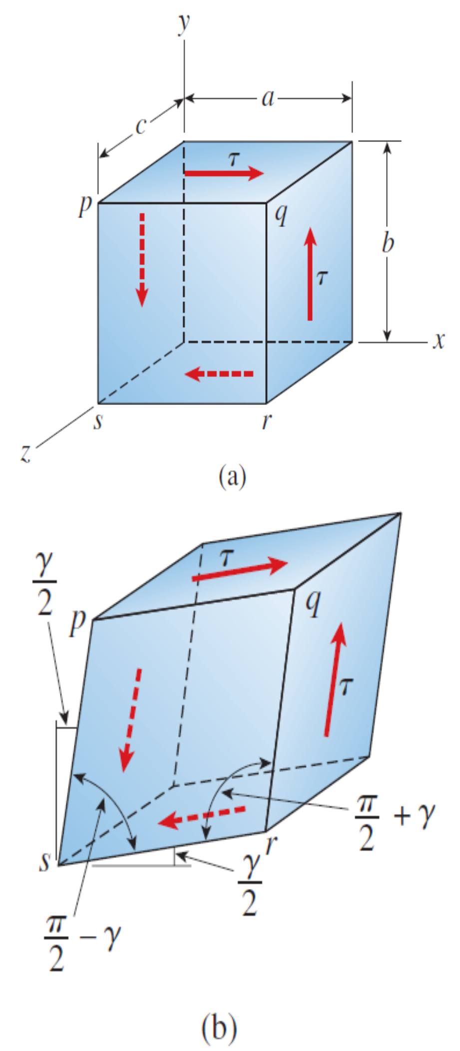


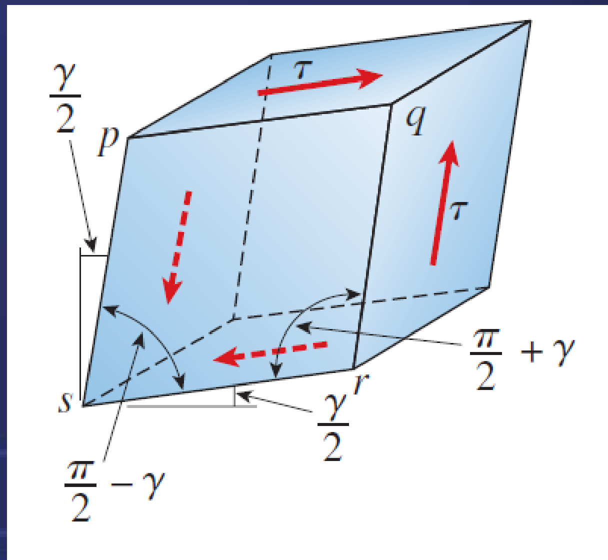
FIG. 1-28 Element of material subjected to shear stresses and strains

Shear Strain

The angle γ is a measure of the **distortion**, or change in shape, of the element and is called the **shear strain**.

Sign Conventions for Shear Stresses and Strains

Shear strain in an element is positive when the angle between two positive faces (or two negative faces) is reduced. The strain is negative when the angle between two positive (or two negative) faces is increased.
Or + face, + force direction
- face, - force direction



(b)

Hooke's Law in Shear

$$\tau = G\gamma \qquad (1\text{-}14)$$

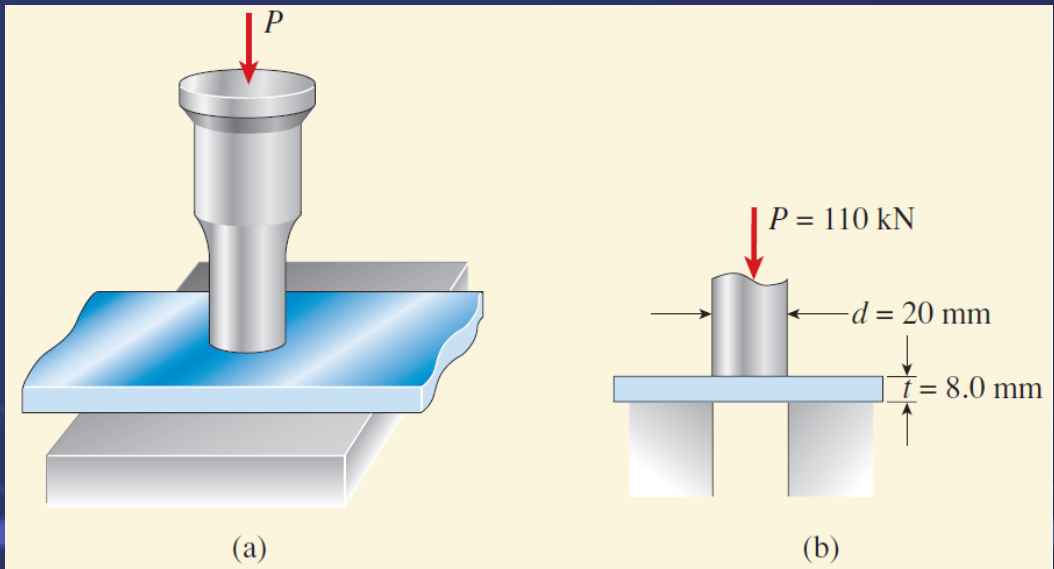
G is the shear modulus of elasticity (also called the modulus of rigidity).

$$G = \frac{E}{2(1+v)} \qquad (1\text{-}15)$$

Example 1-4

A punch for making holes in steel plates is shown in Fig. 1-29a. Assume that a punch having diameter d = 20 mm is used to punch a hole in an 8-mm plate, as shown in the cross-sectional view (Fig. 1-29b).

If a force P = 110 kN is required to create the hole, what is the average shear stress in the plate and the average compressive stress in the punch?



Solution

The shear area A_s *is*

$$A_s = \pi dt = \pi(20 \text{ mm})(8 \text{ mm}) = 502.7 \text{ mm}^2$$

The average shear stress in the plate is

$$\sigma_c = \frac{P}{A_{\text{punch}}} = \frac{P}{\pi d^2/4} = \frac{110 \text{ kN}}{\pi(20 \text{ mm})^2/4} = 350 \text{ MPa}$$

1.7 ALLOWABLE STRESSES AND ALLOWABLE LOADS

The ability of a structure to resist loads is called strength;

The actual strength of a structure must exceed the required strength.

The ratio of the actual strength to the required strength is called the factor of safety *n*:

$$\text{Factor of safety } n = \frac{\text{Actual strength}}{\text{Required strength}} \qquad (1\text{-}20)$$

An allowable stress (or *working stress) that* must not be exceeded anywhere in the structure. Thus,

$$\text{Allowable stress} = \frac{\text{Yield strength}}{\text{Factor of safety}} \qquad (1\text{-}22)$$

For tension and shear, respectively,

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{n_1} \quad \text{and} \quad \tau_{\text{allow}} = \frac{\tau_Y}{n_1} \qquad (1\text{-}23a,b)$$

in which σ_y *and* τ_y are the yield stresses and n_1 and n_2 are the corresponding factors of safety.

Allowable Loads

The allowable stress times the area over which it acts:

Allowable load = (Allowable stress)(Area)

For bars in direct tension and compression (no buckling), this equation becomes

$$P_{\text{allow}} = \sigma_{\text{allow}} A \qquad (1\text{-}26)$$

1.8 DESIGN FOR AXIAL LOADS AND DIRECT SHEAR

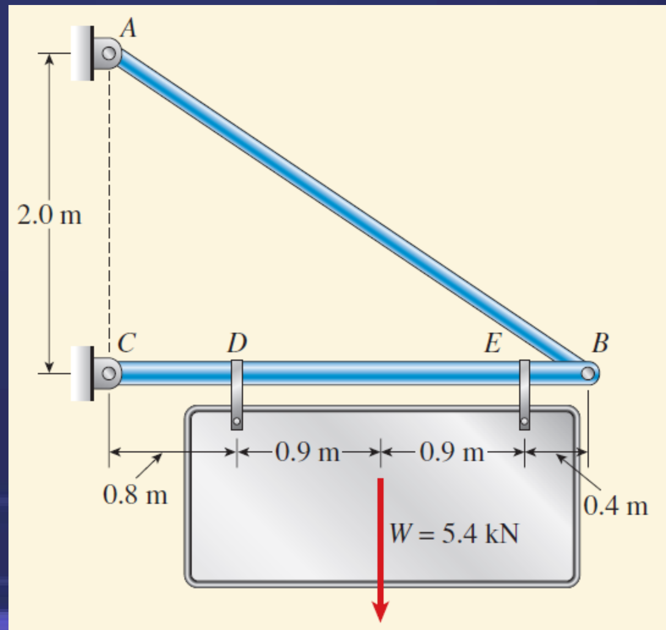
$$\text{Required area} = \frac{\text{Load to be transmitted}}{\text{Allowable stress}} \tag{1-29}$$

Example 1-8

The two-bar truss *ABC* shown in Fig. 1-33 has pin supports at points *A* and *C*, which are 2.0 m apart. Members *AB and BC* are steel bars, pin connected at joint *B*. The length of bar BC is 3.0 m.

A sign weighing 5.4 kN is suspended from bar *BC* at points *D* and *E*, which are located 0.8 m and 0.4 m, respectively, from the ends of the bar.

Determine the required cross-sectional area of bar *AB* and the required diameter of the pin at support *C* *if the allowable stresses in tension and shear are* 125 MPa and 45 MPa, respectively.
(*Note:* The pins at the supports are in double shear.
Also, disregard the weights of members *AB and BC*.)



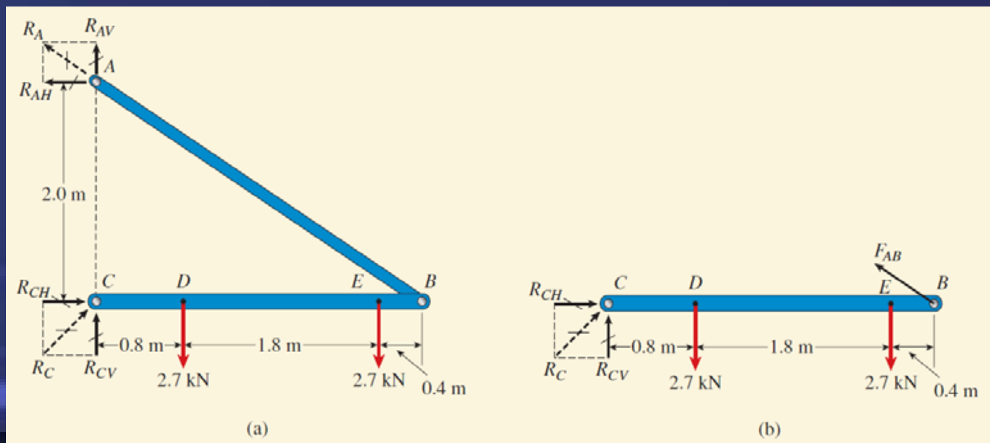
Solution

Reactions , forces in the bars , and shear force in the pin.

The horizontal component R_{AH} of the reaction at support A is obtained by summing moments about point C,

$$\sum M_C = 0$$

$$R_{AH}(2.0\ \text{m}) - (2.7\ \text{kN})(0.8\ \text{m}) - (2.7\ \text{kN})(2.6\ \text{m}) = 0$$



Solving this equation, we get

$$R_{AH} = 4.590 \text{ kN}$$

$$\sum F_{\text{horiz}} = 0 \qquad R_{CH} = R_{AH} = 4.590 \text{ kN}$$

Summing moments about joint *B* gives the desired reaction component :

$$\sum M_B = 0$$

$$-R_{CV}(3.0 \text{ m}) + (2.7 \text{ kN})(2.2 \text{ m}) + (2.7 \text{ kN})(0.4 \text{ m}) = 0$$

$$R_{CV} = 2.340 \text{ kN}$$

We return to the free-body diagram of the entire truss

$$\sum F_{\text{vert}} = 0 \qquad R_{AV} + R_{CV} - 2.7 \text{ kN} - 2.7 \text{ kN} = 0$$

$$R_{AV} = 3.060 \text{ kN}$$

The reaction at *A* is

$$R_A = \sqrt{(R_{AH})^2 + (R_{AV})^2} = 5.516 \text{ kN}$$

The shear force V_C acting on the pin at C is

$$R_C = V_C = \sqrt{(R_{CH})^2 + (R_{CV})^2} = 5.152 \text{ kN}$$

Required area . The required area of bar AB is

$$A_{AB} = \frac{V_{AB}}{\sigma_{\text{allow}}} = \frac{5.516 \text{ kN}}{125 \text{ MPa}} = 44.1 \text{ mm}^2$$

The required area of the pin at *C* (double shear) is

$$A_{\text{pin}} = \frac{V_C}{2\tau_{\text{allow}}} = \frac{5.152\ \text{kN}}{2(45\ \text{MPa})} = 57.2\ \text{mm}^2$$

The required diameter :

$$d_{\text{pin}} = \sqrt{4A_{\text{pin}} / \pi} = 8.54\ \text{mm}$$

The End of Chap. 1