1. $G(s) = \frac{(s+1)(s+2)}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$

A(1 7 12)
B(1 4 0)
C(1 3 0)
1 3 2

$= \frac{1/6}{s} + \frac{-2/3}{s+3} + \frac{3/2}{s+4}$

$H_s = \frac{C}{s+\alpha} \rightarrow H_z = \frac{Cz}{z - e^{-\alpha T}}$

$\rightarrow G(z) = \frac{1}{6}\frac{z}{z-1} - \frac{2}{3}\frac{z}{z-e^{-3T}} + \frac{3}{2}\frac{z}{z-e^{-4T}}, \quad T = 0.1$ ※

$G(s) = \frac{27}{(s+2)(s^2+4s+13)} = \frac{27}{(s+2)((s+2)^2+3^2)}$

9(1 4 13)
A(1 2 0)
B(0 1 2)

$= \frac{3}{s+2} + \frac{-3s-6}{(s+2)^2+3^2}$

$\Rightarrow \frac{3z}{z-e^{-2T}} - 3\cdot\frac{z^2 - ze^{-2T}\cos(3t)}{z^2 - 2ze^{-2T}\cos(3t)e^{-4T}}, \quad T = 0.1$ ※

2.

a)

```
>> clear
>> syms s
>> Ts=(s+1)*(s+2)/s/(s+3)/(s+4)

Ts =

((s + 1)*(s + 2))/(s*(s + 3)*(s + 4))

>> Td=ztrans(ilaplace(Ts))

Td =

z/(6*(z - 1)) - (2*z)/(3*(z - exp(-3))) + (3*z)/(2*(z - exp(-4)))
```

b)

```
>> clear
>> syms s
>> Ts=27/((s+2)*(s^2+4*s+13))

Ts =

27/((s + 2)*(s^2 + 4*s + 13))

>> Td=ztrans(ilaplace(Ts))

Td =

(3*z)/(z - exp(-2)) + (3*z*exp(2)*(cos(3) - z*exp(2)))/(exp(4)*z^2 - 2*cos(3)*exp(2)*z + 1)
```

3.

$$F(z) = \frac{(z+1)(z+.3)(z+.4)}{z(z-.2)(z-.5)(z-.7)} = \frac{-1.7143}{z} + \frac{12}{z-0.2} + \frac{-36}{z-0.5} + \frac{26.7143}{z-0.7}$$

$$= z^{-1}\left(-1.7143 + \frac{12z}{z-0.2} + \cdots\right)$$

$$\longrightarrow f(k) = -1.7143\delta(k-1) + 12\times 0.2^{(k-1)}u(k-1) - 36\times 0.5^{(k-1)}u(k-1) + 26.7143\times 0.7^{(k-1)}u(k-1)$$

$$= -1.7143\delta(k-1) + (60\times 0.2^k - 72\times 0.5^k + 38.1633\times 0.7^k)u(k-1) \quad \#$$

4.

```
>> syms z
>> F = ((z+1)*(z+0.3)*(z+0.4))/(z*(z-0.2)*(z-0.5)*(z-0.7))

F =

((z + 1)*(z + 2/5)*(z + 3/10))/(z*(z - 1/2)*(z - 1/5)*(z - 7/10))

>> iztrans(F)

ans =

60*(1/5)^n - 72*(1/2)^n - (12*kroneckerDelta(n - 1, 0))/7 + (1870*(7/10)^n)/49 - (1282*kroneckerDelta(n, 0))/49
```

5.

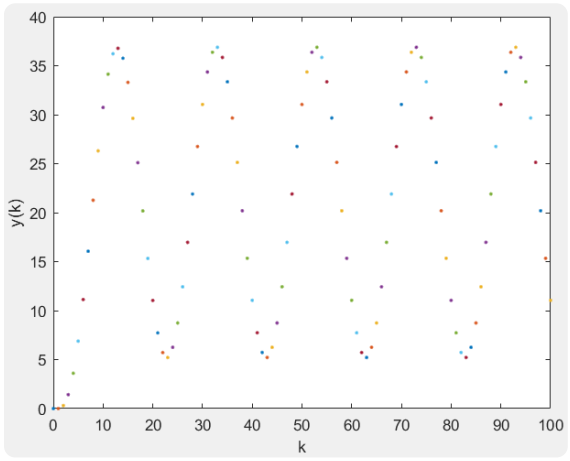
a. $$G(z) = \frac{Y(z)}{U(z)} = \frac{z+1}{(z-1)(z-0.7)} = \frac{z+1}{z^2-1.7z+0.7}$$

$$Y(z)(z^2 - 1.7z^{-1} + 0.7z^{-2}) = U(z)(z^{-1} + z^{-2})$$

$$\longrightarrow y(k) - 1.7y(k-1) + 0.7y(k-2) = u(k-1) + u(k-2) \quad \#$$

b.

```
>> clear
>> syms k z;
>> T=0.01;
>> u=sin(10*pi*k*T);
>> Uz=ztrans(u);
>> Yz=(z+1)/((z-1)*(z-0.7))*Uz;
>> y(k)=iztrans(Yz,k);
>> for k=0:100
yd=y(k);
plot(k,yd,'.')
hold on
end
>> xlabel('k');
>> ylabel('y(k)');
```

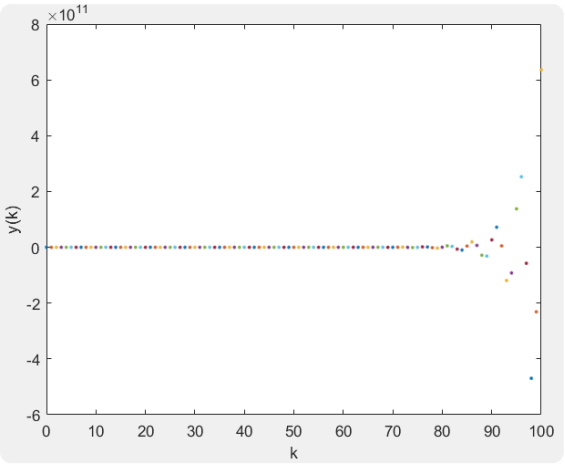


c.

$$T(z) = \frac{G}{1+GH} = \frac{\frac{z+1}{(z-1)(z-0.7)}}{1+\frac{z+1}{(z-1)(z-0.7)}} = \frac{z+1}{z^2-0.7z+1.7}$$ ✗

d.

```
>> clear
>> syms k z;
>> T=0.01;
>> u=sin(10*pi*k*T);
>> Uz=ztrans(u);
>> Yz=(10*(z+1)/(z^2-0.7*z+1.7))*Uz;
>> y(k)=iztrans(Yz,k);
>> for k=0:100
yd=y(k);
plot(k,yd,'.')
hold on
end
>> xlabel('k');
ylabel('y(k)');
```



6.

$$G(z) = (1-z^{-1})\mathcal{Z}\left(\frac{G(s)}{s}\right) = (1-z^{-1})\left(\frac{Tz^{-1}}{(1-z^{-1})^2} - \frac{1}{1-z^{-1}} + \frac{1}{1-e^{-T}z^{-1}}\right)$$

$$\frac{G(s)}{s} = \frac{1}{s^2(s+1)}, \quad y(t) = t - 1 + e^{-t}$$

$$\therefore G(z) = (z-1)\left(\frac{T}{(z-1)^2} + \frac{1}{z-e^{-T}} - \frac{1}{z-1}\right)$$

by matlab

$$\rightarrow T(z) = \frac{G}{1+G} = -\frac{ze^T - Tze^T - z + T - e^T + 1}{e^T z^2 + (Te^T - 2e^T)z + e^T - T}$$

jury array:

e^T	$Te^T - 2e^T$	$e^T - T$
$e^T - T$	$Te^T - 2e^T$	e^T
$2T - T^2e^{-T}$	$T(T-2)$	
$T(T-2)$	$2T - T^2e^{-T}$	
$2T - T^2e^{-T} - \frac{T^2(T-2)^2}{2T-T^2e^{-T}}$		

$\Rightarrow e^T > 0 \rightarrow$ unboundary

$\Rightarrow 2 > Te^{-T} \rightarrow$ unboundary

$\Rightarrow 1 > \left(\frac{T(T-2)}{2T-T^2e^{-T}}\right)^2$

$0 < T < 3.922$ ✗

7.

1	2.6	-0.56	-2.05	0.0775	0.35
0.35	0.0775	-2.05	-0.56	2.6	1

0.8775	2.5729	0.1575	-1.8540	-0.8325
-0.8325	-1.8540	0.1575	2.5729	0.8775

0.0877	0.8140	0.3069	0.5869
0.5869	0.3069	0.8140	0.0877

-3.8407

→ system is not stable ✱

8.

a) $$G(z) = \frac{4z^{-1} - 4z^{-2} + 8.4z^{-3}}{1 + 12z^{-1} + 7.2z^{-2} + 3.2z^{-3}}$$

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k)$$

controllable canonical form : $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3.2 & -7.2 & -12 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} 8.4 \\ -4 \\ 4 \end{pmatrix}^T$

observable .... : $A = \begin{pmatrix} 0 & 0 & -3.2 \\ 1 & 0 & -7.2 \\ 0 & 1 & -12 \end{pmatrix}$ $B = \begin{pmatrix} 8.4 \\ -4 \\ 4 \end{pmatrix}$ $C = (0\ 0\ 1)$

b) $$z^3 + 12z^2 + 7.2z + 3.2 = 0$$

c)

1	12	7.2	3.2
3.2	7.2	12	1

-9.24

→ negative, not stable