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3.11

$$a_k = a_{-k}, \quad a_k = a_{k\pm 10}, \quad a_1 = a_{-1} = a_{11} = 5$$

$$\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = \sum_{k=\langle N\rangle}|a_k|^2 = 50$$

$$|a_{-1}|^2 + |a_0|^2 + |a_1|^2 + \sum_{k=2}^{8}|a_k|^2 = 50$$

$$25 + |a_0|^2 + 25 + \sum_{k=2}^{8}|a_k|^2 = 50$$

$$|a_0|^2 + \sum_{k=2}^{8}|a_k|^2 = 0 \quad \Rightarrow \quad a_0 = 0,\ a_k = 0 \text{ for } k = 2 \sim 8$$

$$x[n] = \sum_{-1}^{8} a_k e^{jk\frac{2\pi}{N}n}$$

$$= 5e^{-j\frac{2\pi}{10}n} + 5e^{j\frac{2\pi}{10}n}$$

$$= 10 \cdot \frac{e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}}{2}$$

$$= 10\cos\frac{\pi}{5}n \quad \Rightarrow \quad A = 10,\ B = \frac{\pi}{5},\ C = 0$$

3.16 (a)

$$x_1[n] = (-1)^n$$

$$= \sum_{k=\langle 2\rangle} a_k e^{jk\left(\frac{2\pi}{2}\right)n}$$

$$= a_0 + a_1 e^{j\pi n}$$

$$= a_0 + a_1(-1)^n$$

$$\Rightarrow a_0 = 0,\ a_1 = 1$$

$$y_1[n] = \sum_{k=\langle 2\rangle} a_k H\left(e^{j\frac{2\pi}{2}}\right) e^{jk\frac{2\pi}{2}n}$$

$$= 0 + H\left(e^{j\pi}\right)e^{j\pi n}$$

$$= 0 \times (-1)^n$$

$$= 0$$

(b)

$$x_2[n] = 1 + \sin\left(\frac{3\pi}{4}n + \frac{\pi}{4}\right)$$

$$x_2[n+N] = 1 + \sin\left(\frac{3\pi}{4}n + \frac{3\pi}{4}N + \frac{\pi}{4}\right)$$

$$\frac{3\pi}{4}N = 2\pi m, \quad N = 16,\ m = 3$$

$$x_2[n] = e^{j\frac{2\pi}{16}0\cdot n} + \frac{1}{2j}\left(e^{j\left(\frac{2\pi}{16}\right)3n}e^{\frac{j\pi}{4}} - e^{-j\frac{2\pi}{16}3n}e^{-\frac{j\pi}{4}}\right)$$

$$= e^{j\frac{2\pi}{16}0\cdot n} - \frac{j}{2}\left(e^{j\frac{2\pi}{16}3n}e^{\frac{j\pi}{4}} - e^{j\frac{2\pi}{16}13n}e^{-\frac{j\pi}{4}}\right)$$

$$\Rightarrow a_0 = 1,\ a_3 = -\frac{je^{\frac{j\pi}{4}}}{2},\ a_{13} = \frac{je^{-\frac{j\pi}{4}}}{2}$$

$$y_2[n] = \sum_{k=0}^{15} a_k H\left(e^{j\frac{2\pi}{16}k}\right)e^{jk\frac{2\pi}{16}n}$$

$$= a_0 H(0) + a_3 H\left(e^{j\frac{3}{8}\pi}\right)e^{j\frac{3\pi}{8}n} + a_{13} H\left(e^{j\frac{13}{8}\pi}\right)e^{j\frac{13\pi}{8}n}$$

$$= -\frac{je^{\frac{j\pi}{4}}}{2}e^{j\frac{3\pi}{8}n} + \frac{je^{-\frac{j\pi}{4}}}{2}e^{j\frac{13\pi}{8}n}$$

$$= -\frac{je^{\frac{j\pi}{4}}}{2}e^{j\frac{3\pi}{8}n} + \frac{je^{-\frac{j\pi}{4}}}{2}e^{-j\frac{3\pi}{8}n}$$

$$= \sin\left(\frac{3}{8}\pi n + \frac{\pi}{4}\right)$$

3.16 (c)

$$x_3[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-4k} u[n-4k]$$

$$= \left(\frac{1}{2}\right)^{n} u[n] * \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$$a_k = \frac{1}{4} \sum_{n=0}^{3} \sum_{k=-\infty}^{\infty} \delta[n-4k] e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{1}{4} \sum_{k=-\infty}^{\infty} \sum_{n=0}^{3} \delta[n-4k] e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{1}{4}$$

$$r[n] = \sum_{k=0}^{3} a_k H(e^{j\frac{2\pi k}{4}}) e^{jk(\frac{2\pi}{4})n}$$

$$= \frac{1}{4}\left[H(e^{j0})e^{j0} + H(e^{j\frac{2\pi}{4}}) e^{j\cdot\frac{2\pi}{4}n} + H(e^{j\frac{4\pi}{4}}) e^{jn\frac{4\pi}{4}} + H(e^{j\frac{6\pi}{4}}) e^{j\frac{6\pi}{4}n}\right]$$

$$= \frac{1}{4}(0+0+0+0)$$

$$= 0$$

$$\Rightarrow y_3[n] = \left[\left(\frac{1}{2}\right)^n u[n]\right] * \left[\sum_{k=-\infty}^{\infty} \delta[n-4k]\right] * h[n]$$

$$= \left[\left(\frac{1}{2}\right)^n u[n]\right] * r[n]$$

$$= \left[\left(\frac{1}{2}\right)^n u[n]\right] * 0$$

$$= 0 \quad \#$$

3.28 (a)

Figure 3.28 a) : $a_k = \frac{1}{7}\sum_{k=0}^{6} x[n]\, e^{-jk\frac{2\pi}{7}n}$

$$= \frac{1}{7}\left[\sum_{k=0}^{4} e^{-jk\frac{2\pi}{7}n} + \sum_{n=5}^{6} 0\, e^{-jk\frac{2\pi}{7}n}\right]$$

$$= \frac{1}{7}\sum_{k=0}^{4}\left(e^{-jk\frac{2\pi}{7}}\right)^n$$

$$= \frac{1}{7}\,\frac{1-\left(e^{-jk\frac{2\pi}{7}}\right)^{4+1}}{1-e^{-jk\frac{2\pi}{7}}}$$

$$= \frac{1}{7}\,\frac{1-e^{-jk\frac{10}{7}\pi}}{1-e^{-jk\frac{2}{7}\pi}}$$

$$= \frac{1}{7}\,\frac{e^{-jk\frac{5}{7}\pi}\left(e^{jk\frac{5}{7}\pi}-e^{-jk\frac{5}{7}\pi}\right)}{e^{-jk\frac{1}{7}\pi}\left(e^{jk\frac{1}{7}\pi}-e^{-jk\frac{1}{7}\pi}\right)}$$

$$= \frac{1}{7}\, e^{-jk\frac{4}{7}\pi}\,\frac{\sin\left(\frac{5\pi}{7}k\right)}{\sin\left(\frac{\pi}{7}k\right)} \quad , k \neq 0$$

$$a_0 = \frac{1}{7}\sum_{k=0}^{6} x[n] = \frac{5}{7}$$

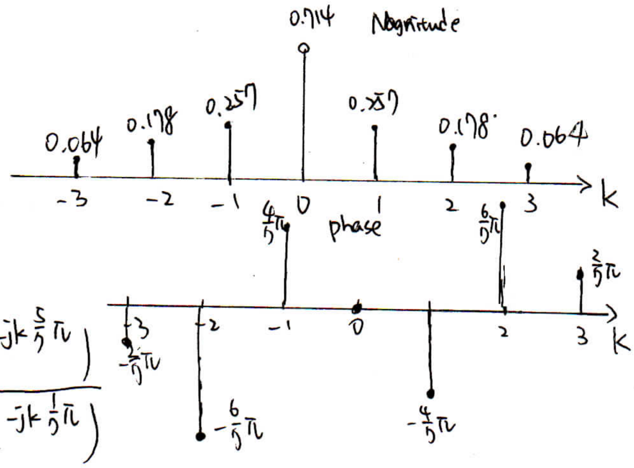


Figure 3.28 b) : $a_k = \frac{1}{6}\sum_{k=0}^{5} x[n]\, e^{-jk\frac{2\pi}{6}n}$

$$= \frac{1}{6}\left[\sum_{k=0}^{3} e^{-jk\frac{2\pi}{6}n} + \sum_{k=4}^{5} 0 \cdot e^{-jk\frac{2\pi}{6}n}\right]$$

$$= \frac{1}{6}\sum_{k=0}^{3}\left(e^{-jk\frac{2\pi}{6}}\right)^n$$

$$= \frac{1}{6}\,\frac{1-e^{-jk\frac{8\pi}{6}}}{1-e^{-jk\frac{2}{6}\pi}}$$

$$= \frac{1}{6}\, e^{-jk\frac{3}{6}\pi}\,\frac{\sin\frac{4\pi}{6}k}{\sin\frac{\pi}{6}k} \quad , k \neq 0$$

$$a_0 = \frac{1}{6}\sum_{k=0}^{5} x[n] = \frac{4}{6} = \frac{2}{3}$$

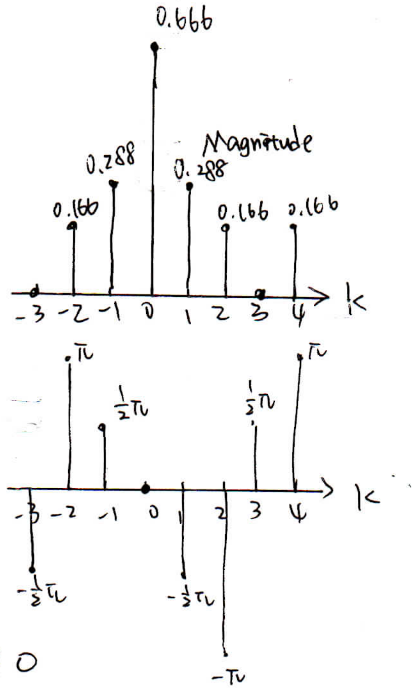
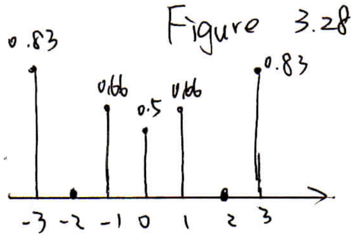
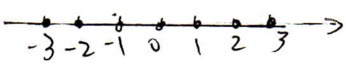


Figure 3.28 c) : $a_k = \frac{1}{6}\sum_{k=0}^{5} x[n]\, e^{-jk\frac{2\pi}{6}n}$

$$= \frac{1}{6}\left[e^0 + 2e^{-j\frac{2\pi}{6}n} - e^{-j\frac{4\pi}{6}n} - e^{-j\frac{8\pi}{6}n} + 2e^{-j\frac{10\pi}{6}n}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi k}{3}\right) - 2\cos\left(\frac{2\pi k}{3}\right)\right]$$





3.28 (b) $x[n] = \frac{1}{2}\left[\sin\left(\frac{2}{3}\pi + \frac{\pi}{2}\right)n + \sin\left(\frac{2}{3}\pi - \frac{\pi}{2}\right)n\right]$

$$= \frac{1}{2}\left[\sin\left(\frac{7\pi}{6}n\right) + \sin\left(\frac{\pi}{6}n\right)\right]$$

$$= \frac{e^{j\frac{7}{6}\pi n}}{4j} - \frac{e^{-j\frac{7}{6}\pi n}}{4j} + \frac{e^{j\frac{\pi}{6}n}}{4j} - \frac{e^{-j\frac{\pi}{6}n}}{4j}$$

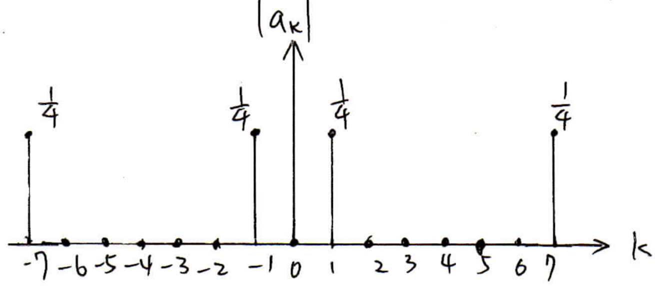
$$= \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$$

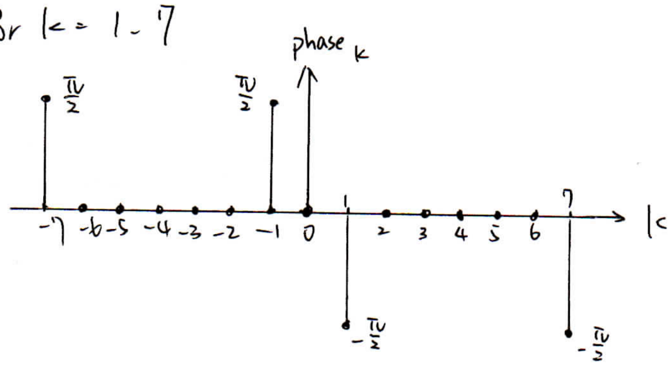
$$\Rightarrow \omega_0 = \frac{1}{6}, \quad a_{-7} = a_{-1} = -\frac{1}{4j} = \frac{j}{4}, \quad a_1 = a_7 = \frac{1}{4j} = \frac{-j}{4}$$

$$|a_k| = \frac{1}{4} \quad \text{for } k = \pm 7, \pm 1$$

$$\theta = \arctan\left(\frac{\frac{j}{4}}{0}\right) = \frac{\pi}{2} \quad \text{for } k = -7, -1$$

$$\theta = \arctan\left(\frac{-\frac{j}{4}}{0}\right) = -\frac{\pi}{2} \quad \text{for } k = 1, 7$$





3.28 (c)

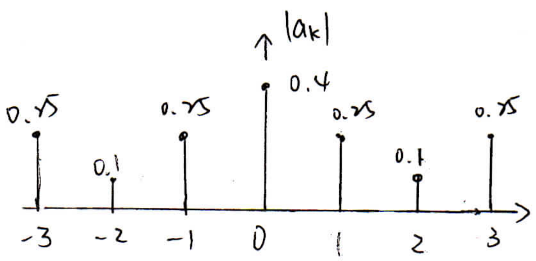
$$a_k = \frac{1}{4}\sum_{n=0}^{3}\left(1 - \sin\frac{n\pi}{4}\right)e^{-jk\frac{2\pi}{4}n}$$

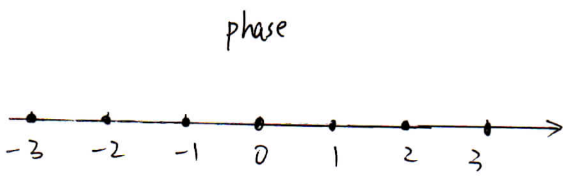
$$= \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)e^{-jk\frac{\pi}{2}} + 0 + \left(1 - \frac{1}{\sqrt{2}}\right)e^{-jk\frac{3}{2}\pi}\right]$$

$$= \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)e^{-jk\pi}\left(e^{jk\frac{\pi}{2}} + e^{-jk\frac{\pi}{2}}\right)\right]$$

$$= \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)(-1)^k \times 2\cos\frac{\pi k}{2}\right]$$

$$= \frac{1}{4}\left[1 + 2(-1)^k\left(1 - \frac{1}{\sqrt{2}}\right)\cos\frac{\pi}{2}k\right]$$





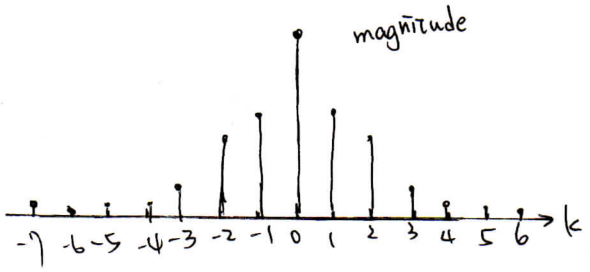
3.28 (d)

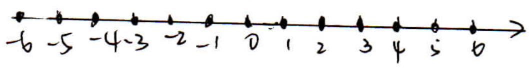
$$a_k = \frac{1}{12}\sum_{n=0}^{11}\left(1-\sin\frac{n\pi}{4}\right)e^{-jk\frac{2\pi}{12}n}$$

$$= \frac{1}{12}\left[1 + \left(1-\sin\frac{\pi}{4}\right)e^{-jk\frac{2\pi}{12}} + 0 + \left(1-\sin\frac{3\pi}{4}\right)e^{-jk\frac{6\pi}{12}} + e^{-jk\frac{8\pi}{12}} + \left(1-\sin\frac{5}{4}\pi\right)e^{-jk\frac{10\pi}{12}}\right]$$

$$+ \frac{1}{12}\left[(1+1)e^{-jk\frac{12}{12}\pi} + \left(1-\sin\frac{7}{4}\pi\right)e^{-jk\frac{7}{6}\pi}, + e^{-jk\frac{8}{6}\pi} + \left(1-\sin\frac{9}{4}\pi\right)e^{-jk\frac{9}{6}\pi} + \left(1-\sin\frac{11}{4}\pi\right)e^{-jk\frac{11}{6}\pi}\right]$$

$$= \frac{1}{12}\left[1 + 2\left(1-\frac{1}{\sqrt{2}}\right)\cos\frac{\pi k}{6} + 2\left(1\frac{1}{\sqrt{2}}\right)\cos\frac{\pi k}{2} + 2\cos\frac{2\pi k}{3} + 2\left(1+\frac{1}{\sqrt{2}}\right)\cos\frac{5\pi k}{6} + 2(-1)^k\right]$$





3.51

$N=8$, $a_k = -a_{k-4}$, $y[n] = \frac{1+(-1)^n}{2}x[n-1]$

$x[n] \xrightarrow{FS} a_k$, $e^{j4\frac{2\pi}{8}n}x[n] \xrightarrow{FS} a_{k-4}$

$\Rightarrow e^{j\pi n}x[n] \xrightarrow{FS} a_{k-4}$

$\Rightarrow (-1)^n x[n] \xrightarrow{FS} a_{k-4}$

$$y[n] = \begin{cases} 0 & , n \text{ odd} \\ x[n-1], & n \text{ even} \end{cases}$$

$y[n] \xrightarrow{FS} b_k$

$x[n-1] \xrightarrow{FS} a_k e^{-jn\frac{2\pi}{8}} = a_k e^{-jn\frac{\pi}{4}}$

$b_k = a_k e^{-jn\frac{\pi}{4}}$

$f[k] = e^{-jn\frac{\pi}{4}}$

#

3.51) (a)

$$x[n]=\sum_{k=0}^{N-1} a_k e^{\frac{j2\pi kn}{N}}$$

$$y[n]=\sum_{k=0}^{N-1} b_k e^{\frac{j2\pi kn}{N}}$$

$$x[n]y[n]=\sum_{k=0}^{N-1} a_k e^{\frac{2\pi jkn}{N}} \sum_{l=0}^{N-1} b_l e^{\frac{j2\pi ln}{N}}$$

$$=\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_k b_l \, e^{\frac{j2\pi(k+l)n}{N}}$$

$$=\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} a_k b_{m-k} \, e^{\frac{j2\pi mn}{N}}$$

$$C_m=\sum_{k=0}^{N-1} a_k b_{m-k} \quad \Rightarrow \quad x[n]y[n]=\sum_{m=0}^{N-1} C_m e^{\frac{2\pi jmn}{N}}$$

(b)

$$C_m=\sum_{k=\langle N\rangle} a_k b_{m-k} = \sum_{k=\langle N\rangle} b_k a_{m-k}$$

(c) i:

$$y[n]=\cos\frac{6\pi n}{N}=\frac{1}{2}\left(e^{\frac{j2\pi 3n}{N}}+e^{\frac{-j2\pi 3n}{N}}\right)$$

$$b_k=\frac{1}{2} \text{ for } k=\pm 3$$

$$\Rightarrow b_k=\frac{1}{2}\delta(k-3)+\frac{1}{2}\delta(k+3)$$

$$\Rightarrow C_k=\sum_{l=0}^{N-1} b_l a_{k-l} = \sum_{l=0}^{N-1}\frac{1}{2}[\delta(l-3)+\delta(l+3)]a_{k-l}$$

$$=\frac{1}{2}a_{k-3}+\frac{1}{2}a_{k+3} \ \#$$

ii:

$$y[n]=\sum_{r=-\infty}^{\infty}[n-rN]$$

$$b_k=\frac{1}{N}\sum_{n=0}^{N-1} y[n] e^{\frac{-j2\pi kn}{N}}$$

$$=\frac{1}{N}\sum_{n=0}^{N-1}\sum_{r=-\infty}^{\infty}[n-rN] \, e^{\frac{-j2\pi kn}{N}}$$

$$b_k=\frac{1}{N}\sum_{n=0}^{N-1}[n] e^{\frac{-j2\pi kn}{N}} \quad \text{as } 0\le k\le N-1$$

$$b_k=\frac{1}{N}$$

$$\Rightarrow C_k=\frac{1}{N}\sum_{l=0}^{N-1} a_l$$

3.57 (c) iii : $y[n] = \sum_{r=-\infty}^{\infty} [n-r]\frac{N}{3}$

$$b_k = \frac{1}{N}\sum_{n=0}^{N-1} y[n]\, e^{\frac{-j2\pi kn}{N}}$$

$$b_k = \frac{1}{N}\sum_{n=0}^{N-1} [n]+[n-\tfrac{N}{3}]+[n-\tfrac{2N}{3}]\, e^{\frac{-j2\pi kn}{N}} \quad \text{as } 0 \le k \le N-1$$

$$b_k = \frac{1}{N}\left[1+e^{-\frac{j2\pi k}{N}\frac{N}{3}} + e^{-\frac{j2\pi k}{N}\frac{2N}{3}}\right]$$

$$= \frac{1}{N}\left[1+e^{\frac{-j2\pi k}{3}} + e^{\frac{-j4\pi k}{3}}\right]$$

$$C_k = \sum_{l=0}^{N-1} b_l a_{k-l}$$

$$= \sum_{l=0}^{N-1} \frac{1}{N}\left[1+e^{-\frac{j2\pi k}{3}} + e^{\frac{-j4\pi k}{3}}\right] a_{k-l}$$

(d)

$x[n] = \cos(\frac{n}{3})$

$$y[n] = \begin{cases} 1 & |n| \le 3 \\ 0 & 4 < n \le 6 \end{cases}, \qquad N = 12$$

$x[n] = \frac{1}{2}\left(e^{\frac{j\pi n}{3}} + e^{\frac{-j\pi n}{3}}\right)$, $a_k = \frac{1}{2}$, for $k = \pm 2$

$a_{-2} = a_{10} = \frac{1}{2}$

$$b_k = \frac{1}{N}\sum_{n=0}^{N-1} y[n]e^{\frac{-j2\pi kn}{N}} = \frac{1}{12}\sum_{n=-3}^{3} e^{\frac{-j2\pi kn}{12}}$$

$$= \frac{1}{12}\,\frac{\sin\frac{7\pi k}{N}}{\sin\frac{\pi k}{N}}$$

$$C_k = \sum_{l=0}^{N-1} a_l b_{k-l} = \frac{1}{24}\left[\frac{\sin\frac{7\pi(k-2)}{N}}{\sin\frac{\pi(k-2)}{N}} + \frac{\sin\frac{7\pi(k-10)}{N}}{\sin\frac{\pi(k-10)}{N}}\right]$$

(e) $C_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]y[n]\, e^{\frac{-j2\pi kn}{N}}$, $C_k = \sum_{l=0}^{N-1} a_l b_{k-l}$, $\frac{1}{N}\sum_{n=0}^{N-1} x[n]y[n]e^{\frac{-j2\pi kn}{N}} = \sum_{l=0}^{N-1} a_l b_{k-l}$

$$\sum_{n=0}^{N-1} x[n]y[n] = N\sum_{l=0}^{N-1} a_l b_{k-l}$$

$$\Rightarrow \sum_{n=0}^{N-1} x[n]x^*[n] = N\sum_{l=0}^{N-1} a_l a^*_{k-l} \Rightarrow \sum_{k=\langle N\rangle} |x[n]|^2 = N\sum_{k=\langle N\rangle} |a_l|^2$$