



# Chapter 8

## Communication Systems

## 8.0 Introduction

Communication systems play a key role in our modern world in transmitting information between people, systems, and computers. In general terms, in all communication systems the information at the source is first processed by a transmitter or modulator to change it into a form suitable for transmission over the communication channel. At the receiver, the signal is then recovered through appropriate processing.

## 8.0 Introduction

Many of the concepts and techniques we have developed in the earlier chapters of this text play a central role in the analysis and design of communication systems. As with any concept that is closely tied to a wide variety of important applications, there are a large number of detailed issues to be considered, and, as indicated in the bibliography, there are many excellent texts on the subject.

## 8.0 Introduction

所有的通訊系統均需將訊號源的資訊，經傳送器或調變器變換成適合傳送的形式，接收後再將訊號做還原的處理。這種處理的需求有許多原因，較典型的原因是傳輸通道具有最適合的頻率範圍，在此範圍之外將使通訊嚴重惡化，甚至不可能。

對一種帶有資訊的訊號，轉變成另一種訊號的過程，稱為「調變」。而將原來帶有資訊的訊號抽取回來的過程，則稱為「解調」。

## 8.0 Introduction

One large class of modulation methods relies on the concept of amplitude modulation or AM in which the signal we wish to transmit is used to modulate the amplitude of another signal.

Another important class of AM systems involves the modulation of the amplitude of a pulsed signal, and in Sections 8.5 and 8.6 we examine this form of modulation as well as the concept of time-division multiplexing.

## 8.0 Introduction

振幅調變(AM)是很廣泛使用的，其中最常用的是「弦波振幅調變」。

另一種常用的AM方式是「脈波振幅調變」。

「弦波頻率調變」則是完全不同的方式，它利用帶有資訊的訊號來改變弦波訊號的頻率。

## 8.1 Complex Exponential And Sinusoidal Amplitude Modulation

Many communication systems rely on the concept of sinusoidal amplitude modulation, in which a complex exponential or sinusoidal signal  $c(t)$  has its amplitude multiplied (modulated ) by the information-bearing signal  $x(t)$ .

$$y(t) = x(t)c(t)$$

帶有資訊的訊號 $x(t)$ 通常稱為「調變訊號」， $c(t)$ 訊號在此為「載波訊號」，則調變後的訊號 $y(t) = x(t)c(t)$ 。

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

There are two common forms of sinusoidal amplitude modulation, one in which the carrier signal is a complex exponential of the form

$$c(t) = e^{j(\omega_c t + \theta_c)} \quad (8.1)$$

弦波振幅調變有兩種型式：

一為載波訊號為複指數  $c(t) = e^{j(\omega_c t + \theta_c)}$



## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

and the second in which the carrier signal is sinusoidal and the of the form

$$c(t) = \cos(\omega_c t + \theta_c). \quad (8.2)$$

二為載波型式為弦波  $c(t) = \cos(\omega_c t + \theta_c)$ .

let us choose  $\theta_c = 0$  , so that the modulated signal is

$$y(t) = x(t)e^{j\omega_c t}. \quad (8.3)$$

$\omega_c$  稱為「載波頻率」 設  $\theta_c = 0$  則  $y(t) = x(t)e^{j\omega_c t}$ .

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

From the multiplication property (Section 4.5), and with  $X(j\omega)$ ,  $Y(j\omega)$ , and  $C(j\omega)$  denoting the Fourier transforms of  $x(t)$ ,  $y(t)$ , and  $c(t)$ , respectively,

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) C(j(\omega - \theta)) d\theta. \quad (8.4)$$

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

For  $c(t)$  a complex exponential as given in eq. (8.1),

$$C(j\omega) = 2\pi\delta(\omega - \omega_c), \quad (8.5)$$

and hence,

$$Y(j\omega) = X(j\omega - j\omega_c). \quad (8.6)$$

調變後的訊號頻譜為原訊號頻譜在頻率軸上移位  $\omega_c$  (恰等於載波頻率)。

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

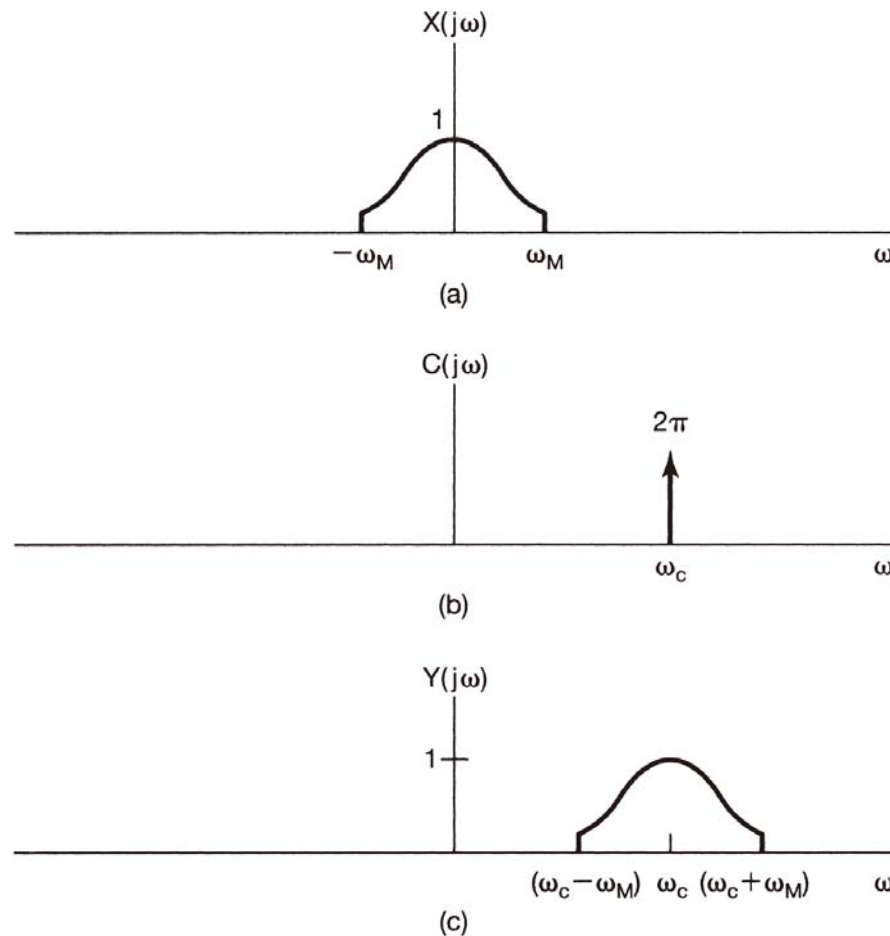


圖 8.1 說明原訊號、複指數載波訊號及調變後訊號的頻譜關係。

**Figure 8.1** Effect in the frequency domain of amplitude modulation with a complex exponential carrier: (a) spectrum of modulating signal  $x(t)$ ; (b) spectrum of carrier  $c(t) = e^{j\omega_C t}$ ; (c) spectrum of amplitude-modulated signal  $y(t) = x(t)e^{j\omega_C t}$ .

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

From eq. (8.3), it is clear that  $x(t)$  can be recovered from the modulated signal  $y(t)$  by multiplying by the complex exponential  $e^{-j\omega_c t}$ ; that is,

$$x(t) = y(t)e^{-j\omega_c t}. \quad (8.7)$$

將 $y(t)$ 乘上  $e^{-j\omega_c t}$  可還原為 $x(t)$ 。

Since  $e^{-j\omega_c t}$  is a complex signal, eq. (8.3) can be rewritten as

$$y(t) = x(t) \cos \omega_c t + jx(t) \sin \omega_c t. \quad (8.8)$$

## 8.1.1 Amplitude Modulation with a Complex Exponential Carrier

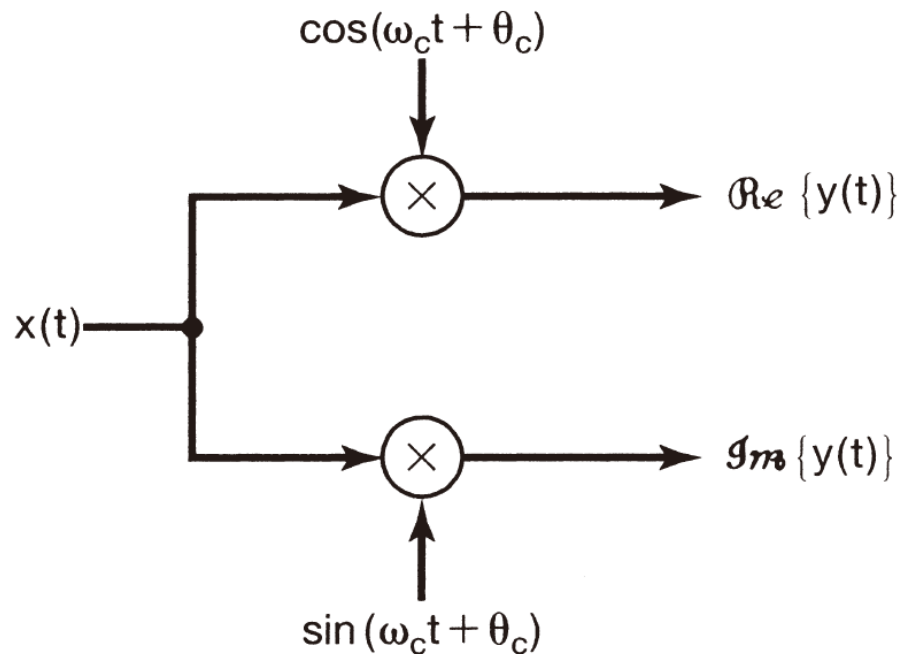


圖 8.2 為以  $e^{j\omega_c t}$  為載波的振幅調變的實現。

**Figure 8.2** Implementation of amplitude modulation with a complex exponential carrier  $c(t) = e^{j(\omega_c t + \theta_c)}$ .

## 8.1.2 Amplitude Modulation with a Sinusoidal Carrier

設  $c(t) = \cos(\omega_c t + \theta_c)$

且令  $y(t) = x(t)c(t)$

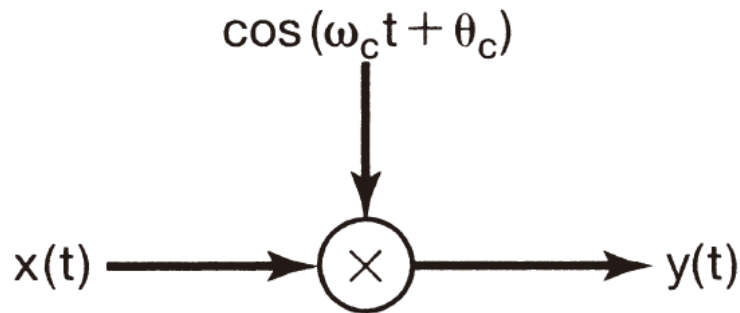


圖 8.3 為以  $\cos(\omega_c t + \theta_c)$  為載波的振幅調變的實現。

**Figure 8.3** Amplitude modulation with a sinusoidal carrier.

## 8.1.2 Amplitude Modulation with a Sinusoidal Carrier

Again, for convenience we choose  $\theta_c = 0$ . In this case, the spectrum of the carrier signal is

$$C(j\omega) = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)], \quad (8.9)$$

and thus, from eq. (8.4),

$$Y(j\omega) = \frac{1}{2}[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)]. \quad (8.10)$$



## 8.1.2 Amplitude Modulation with a Sinusoidal Carrier

Note that there is now a replication of the spectrum of the original signal, centered around both  $+\omega_c$  and  $-\omega_c$ . As a consequence,  $x(t)$  is recoverable from  $y(t)$  only if  $\omega_c > \omega_M$ , since otherwise the two replications will overlap in frequency.

調變後的訊號頻譜為原訊號頻譜複製到各以 $+\omega_c$ 及 $-\omega_c$ 為中心且大小衰減一半。

若 $\omega_c > \omega_M$ ，沒有重疊現象；若 $\omega_c < \omega_M$ ，則有重疊現象。

## 8.1.2 Amplitude Modulation with a Sinusoidal Carrier

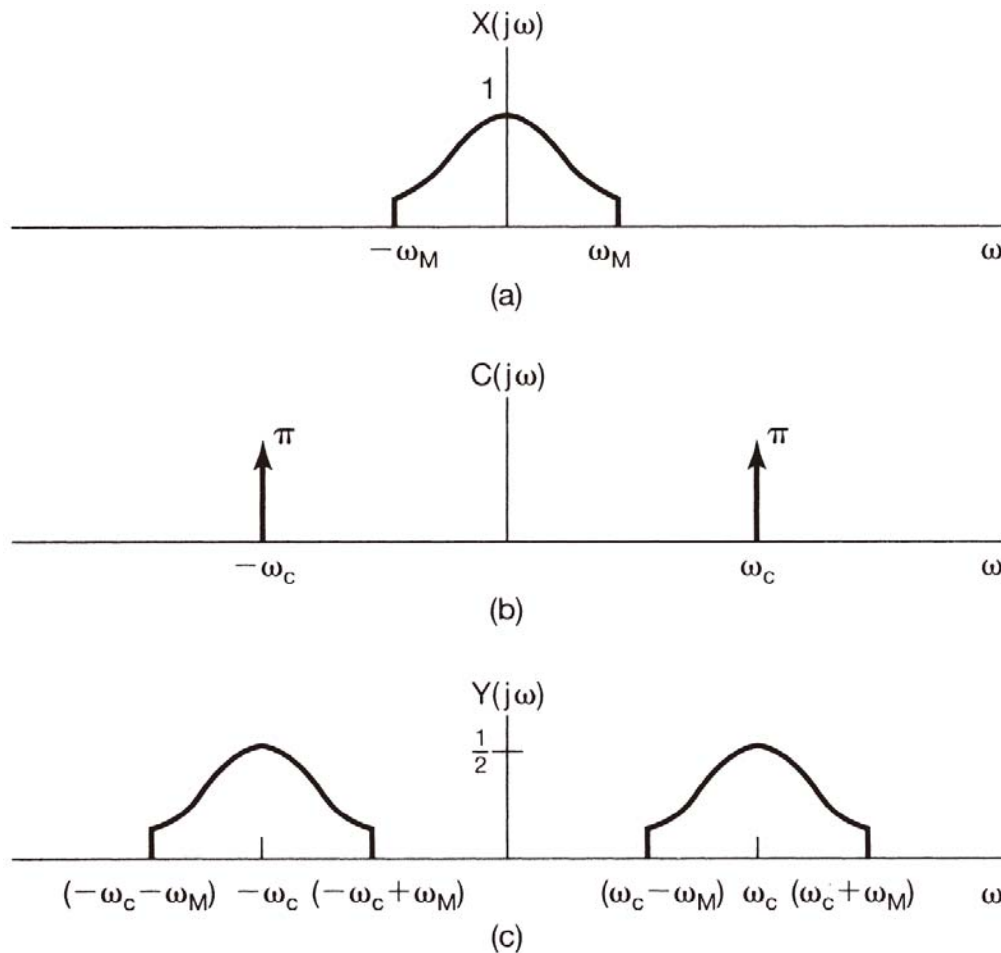
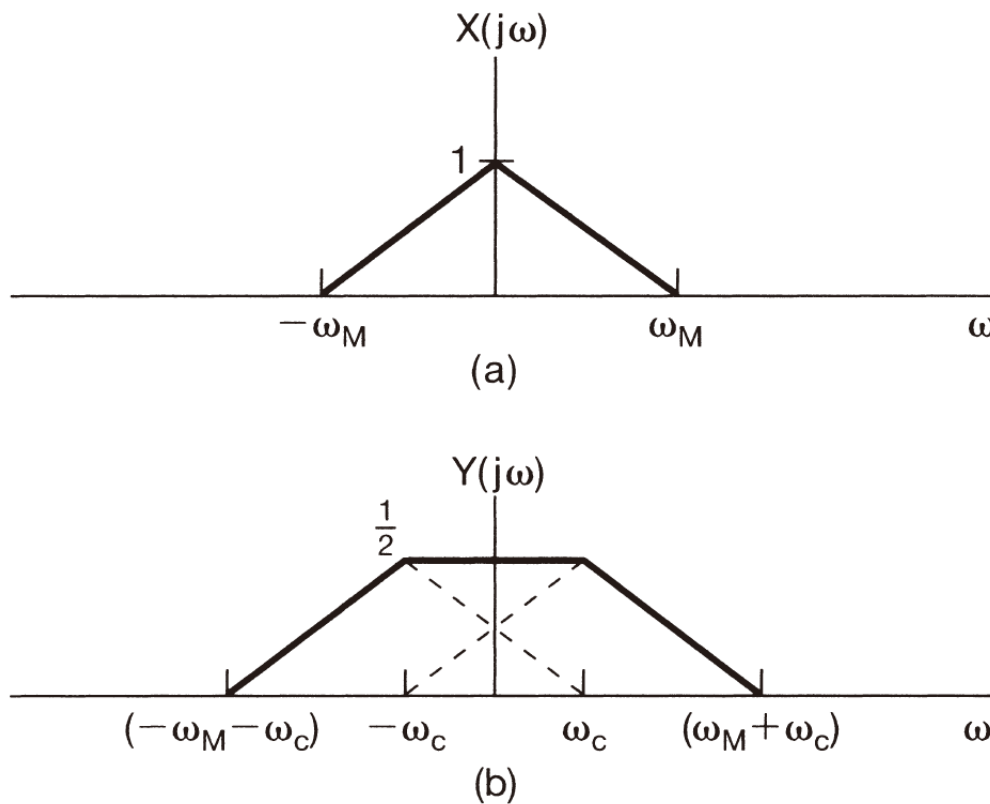


圖 8.4 說明了原訊號、弦波載波及調變後訊號的頻譜關係。

**Figure 8.4** Effect in the frequency domain of amplitude modulation with a sinusoidal carrier: (a) spectrum of modulating signal  $x(t)$ ; (b) spectrum of carrier  $c(t) = \cos \omega_c t$ ; (c) spectrum of amplitude-modulated signal.

## 8.1.2 Amplitude Modulation with a Sinusoidal Carrier



**Figure 8.5** Sinusoidal amplitude modulation with carrier  $\cos \omega_c t$  for which  $\omega_c = \omega_M/2$ : (a) spectrum of modulating signal; (b) spectrum of modulated signal.

## 8.2 Demodulation for Sinusoidal AM

There are two commonly used methods for demodulation, each with its own advantages and disadvantages.

常用的解調方法有二：

- 一為同步解調(傳送器與接收器相位同步)；
- 二為非周步解調。

## 8.2.1 Synchronous Demodulation

Assuming that  $\omega_c > \omega_M$ , demodulation of a signal that was modulated with a sinusoidal carrier is relatively straightforward. Specifically, consider the signal

設  $\omega_c > \omega_M$

調變後訊號

$$y(t) = x(t) \cos \omega_c t. \quad (8.11)$$

## 8.2.1 Synchronous Demodulation

As was suggest in Example 4.21, the original signal can be recovered by modulating  $y(t)$  with the same sinusoidal carrier and applying a lowpass filter to the result. To see this, consider

利用相同的調變方式處理 $y(t)$ 。

即

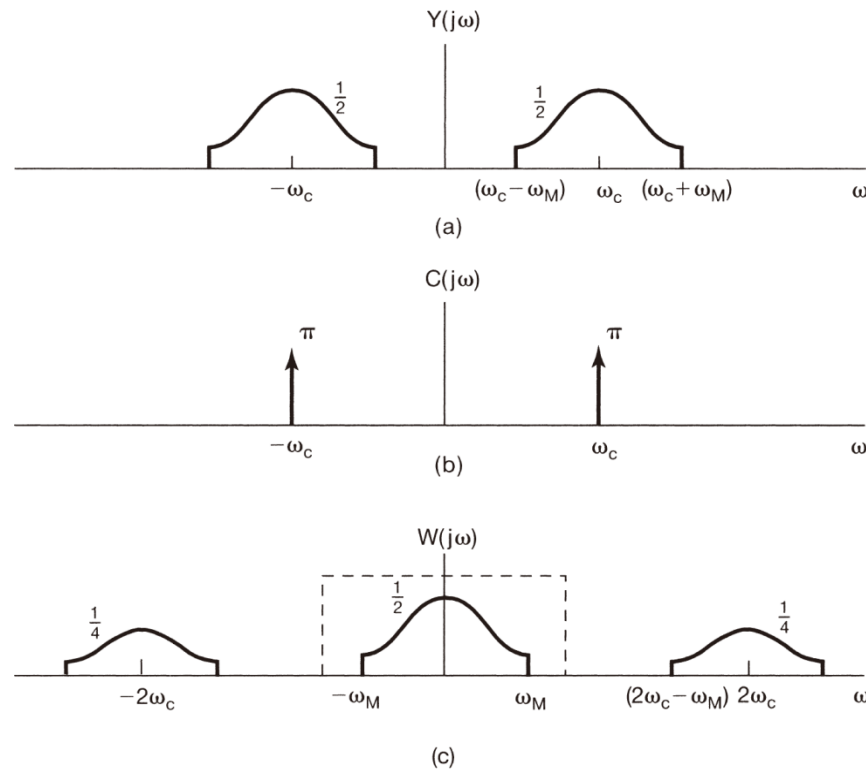
$$w(t) = y(t) \cos \omega_c t. \quad (8.12)$$

## 8.2.1 Synchronous Demodulation

The basis for using eq. (8.12) and a lowpass filter to demodulate  $y(t)$  can also be seen algebraically. From eqs. (8.11) and (8.12), it follows that

$$w(t) = x(t) \cos^2 \omega_c t,$$

## 8.2.1 Synchronous Demodulation



**Figure 8.6** Demodulation of an amplitude-modulated signal with a sinusoidal carrier: (a) spectrum of modulated signal; (b) spectrum of carrier signal; (c) spectrum of modulated signal multiplied by the carrier. The dashed line indicates the frequency response of a lowpass filter used to extract the demodulated signal.



## 8.2.1 Synchronous Demodulation

or, using the trigonometric identity

$$\cos^2 \omega_c t = \frac{1}{2} + \frac{1}{2} \cos 2\omega_c t,$$

We can rewrite  $w(t)$  as

$$w(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\omega_c t. \quad (8.13)$$

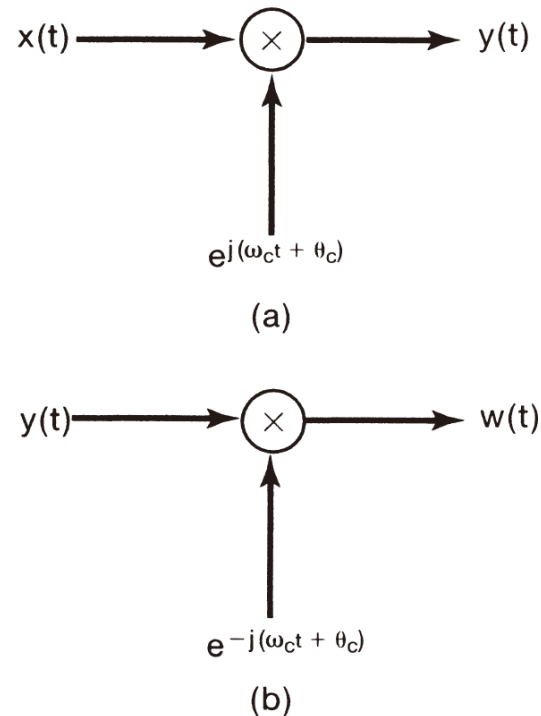
化簡得  $w(t)$  含兩項，一為原訊號之半，一為較高頻的訊號。

## 8.2.1 Synchronous Demodulation

Thus,  $w(t)$  consists of the sum of two terms, namely one-half the original signal and one-half the original signal modulated with a sinusoidal carrier at twice the original carrier frequency  $\omega_c$ .

經低通濾波器濾除第二項可得第一項(原訊號)。

## 8.2.1 Synchronous Demodulation



**Figure 8.7** System for amplitude modulation and demodulation using a complex exponential carrier: (a) modulation; (b) demodulation.

## 8.2.1 Synchronous Demodulation

For the case of the complex exponential carrier, with  $\theta_c$  denoting the phase of the modulating carrier and  $\phi_c$  the phase of the demodulating carrier,

考慮載波相角

$\theta_c \neq 0$

$$y(t) = e^{j(\omega_c t + \theta_c)} x(t), \quad (8.14)$$

$$w(t) = e^{-j(\omega_c t + \phi_c)} y(t), \quad (8.15)$$

and consequently,

$$w(t) = e^{j(\theta_c - \phi_c)} x(t). \quad (8.16)$$

## 8.2.1 Synchronous Demodulation

The input to the lowpass filter is now

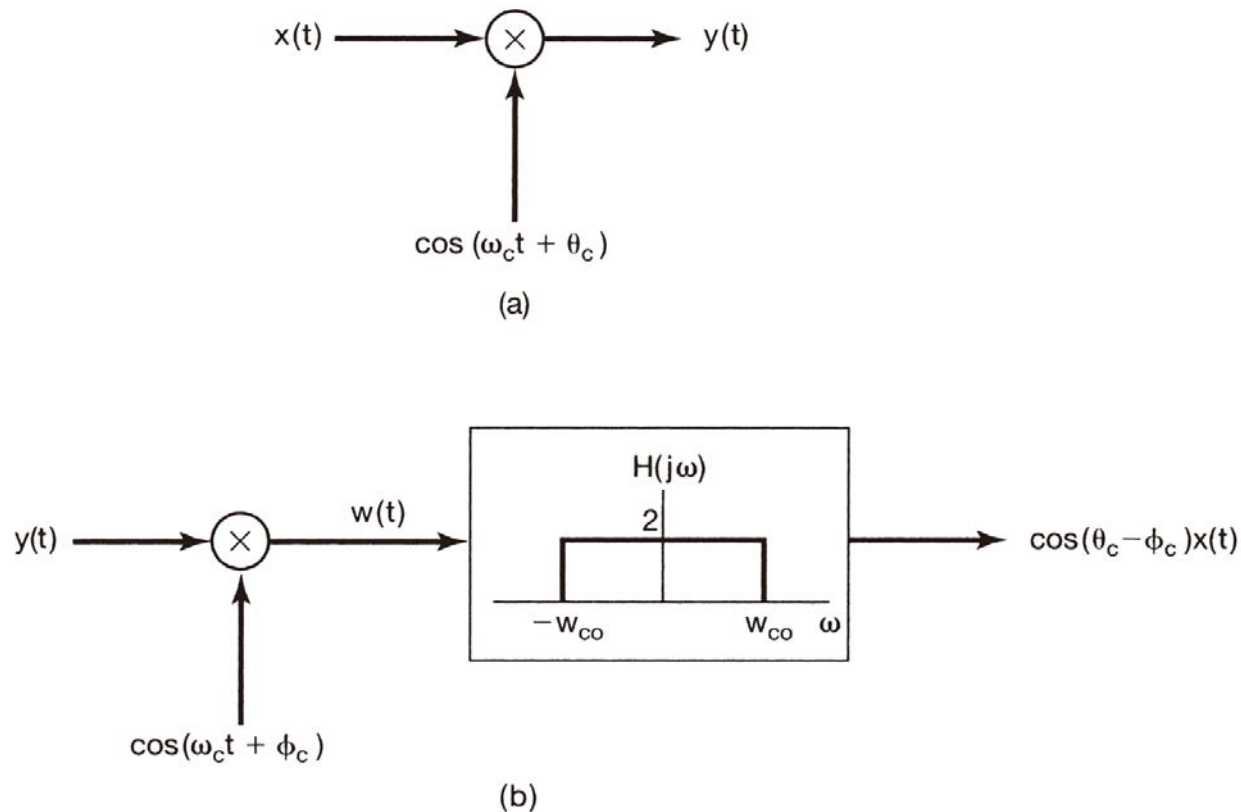
$$w(t) = x(t) \cos(\omega_c t + \theta_c) \cos(\omega_c t + \phi_c), \quad (8.17)$$

or, using the trigonometric identity

$$\cos(\omega_c t + \theta_c) \cos(\omega_c t + \phi_c) = \frac{1}{2} \cos(\theta_c - \phi_c) + \frac{1}{2} \cos(2\omega_c t + \theta_c + \phi_c), \quad (8.18)$$

且  $\theta_c$  與  $\phi_c$  可能不相等。

## 8.2.1 Synchronous Demodulation



**Figure 8.9** Sinusoidal amplitude modulation and demodulation system for which the carrier signals and the modulator and demodulator are not synchronized: (a) modulator; (b) demodulator.

## 8.2.1 Synchronous Demodulation

we have

$$w(t) = \frac{1}{2} \cos(\theta_c - \phi_c) x(t) + \frac{1}{2} x(t) \cos(2\omega_c t + \theta_c + \phi_c),$$

and the output of the lowpass filter is then  $x(t)$  multiplied by the amplitude factor  $\cos(\theta_c - \phi_c)$ . If the oscillators in the modulator and demodulator are in phase,  $\theta_c = \phi_c$ , and the output of the lowpass filter is  $x(t)$ . On the other hand, if these oscillators have a phase difference of  $\pi/2$ , the output will be zero. In general, for a maximum output signal, the oscillators should be in phase.

## 8.2.1 Synchronous Demodulation

$w(t)$ 經低通濾波器可得  $\frac{1}{2}\cos(\theta_c - \phi_c)x(t)$  。

若  $\theta_c = \phi_c$ ，則可得  $x(t)$ ；若  $\theta_c$  與  $\phi_c$  相差  $\pi/2$ ，則濾波器輸出為0。

故調變與調解器的同步須小心為之。



## 8.2.2 Asynchronous Demodulation

In particular, the envelope of  $y(t)$ —that is, a smooth curve connecting the peaks in  $y(t)$ —would appear to be a reasonable approximation to  $x(t)$ . Thus,  $x(t)$  could be approximately recovered through the use of a system that tracks these peaks to extract the envelope.

$y(t)$ 的包絡線為將 $y(t)$ 的尖峰連接而得的平滑曲線，且近似 $x(t)$ 。故可利用包絡線檢測器還原得近似的 $x(t)$ 。

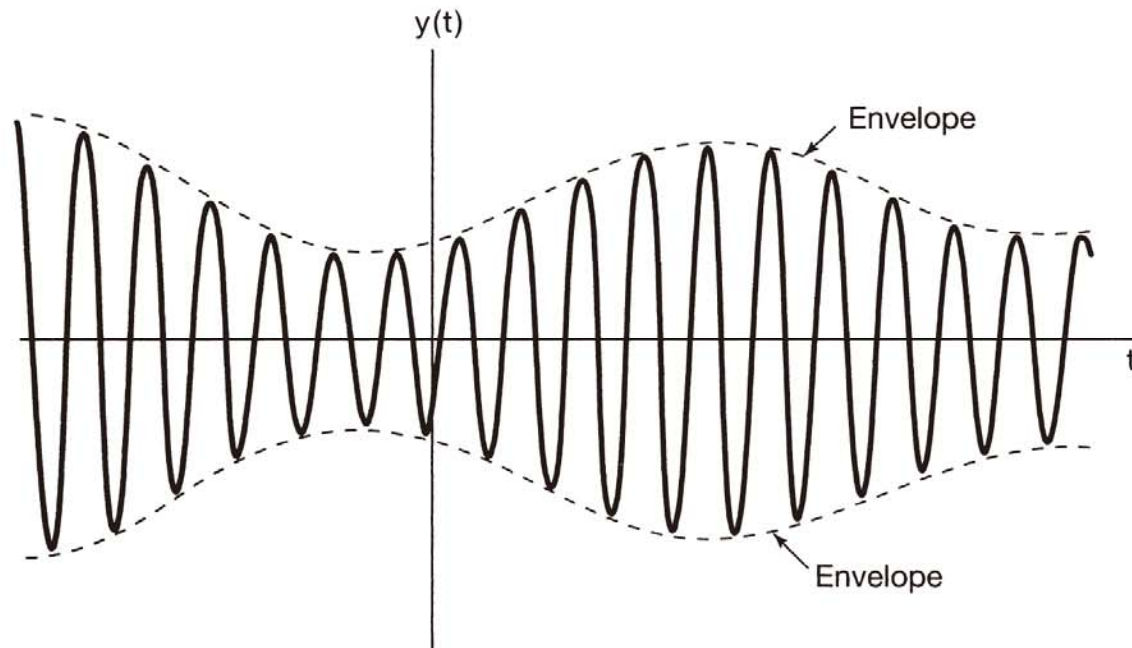
## 8.2.2 Asynchronous Demodulation

To use the envelope detector for demodulation, we require that  $A$  be sufficiently large so that  $x(t) + A$  is positive. Let  $K$  denote the maximum amplitude of  $x(t)$ ; that is,  $|x(t)| \leq k$ . For  $x(t) + A$  to be positive, we require that  $A > K$ . The ratio  $K/A$  is commonly referred to as the *modulation index*  $m$ .

令  $A$  夠大，使得  $x(t) + A$  為正值；且 令  $k$  為  $x(t)$  的最大振幅，即： $|x(t)| \leq k$

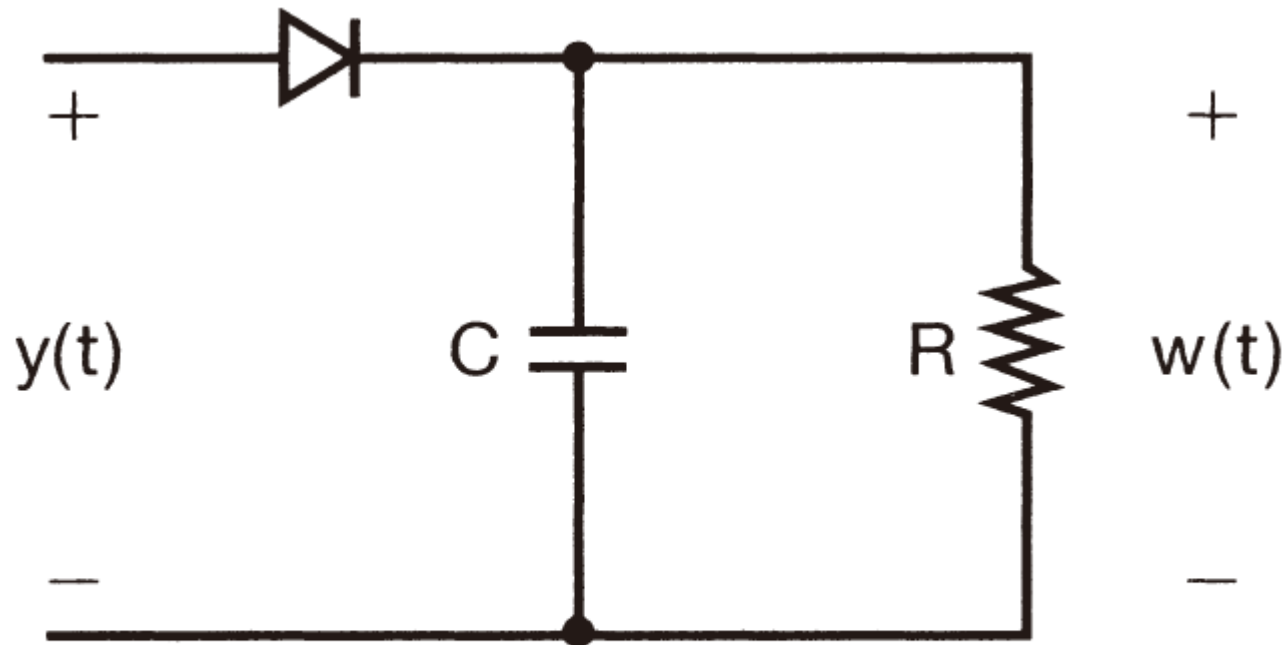
可得  $A > K$ ，令  $m = K/A$  稱為調變指數。

## 8.2.2 Asynchronous Demodulation



**Figure 8.10** Amplitude-modulated signal for which the modulating signal is positive. The dashed curve represents the envelope of the modulated signal.

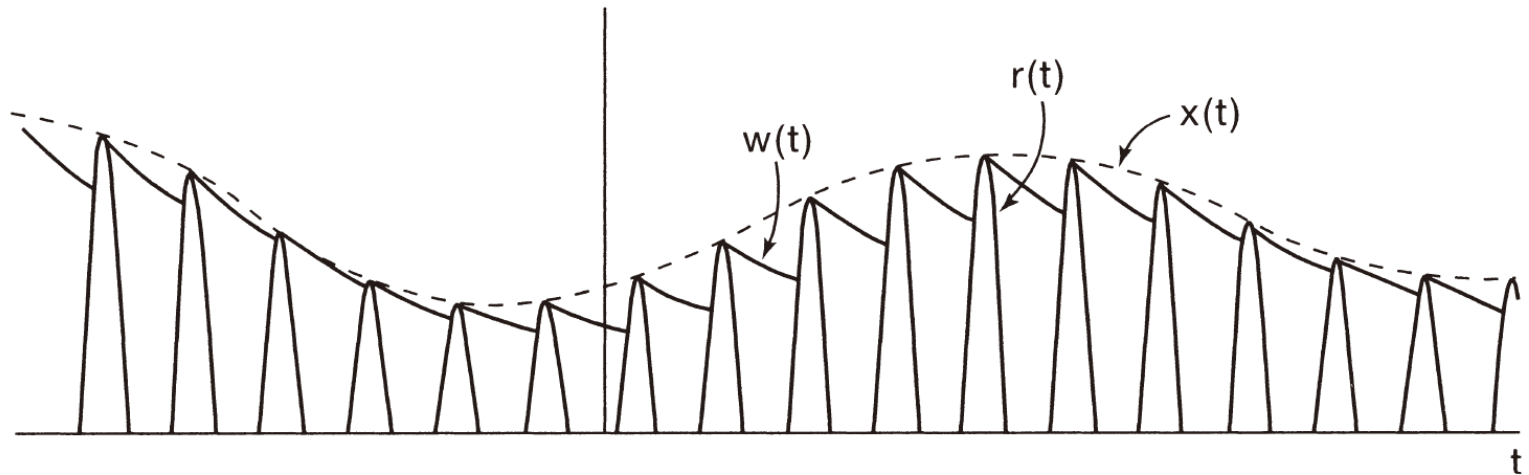
## 8.2.2 Asynchronous Demodulation



利用半波整流的包  
絡線檢測電路

(a)

## 8.2.2 Asynchronous Demodulation

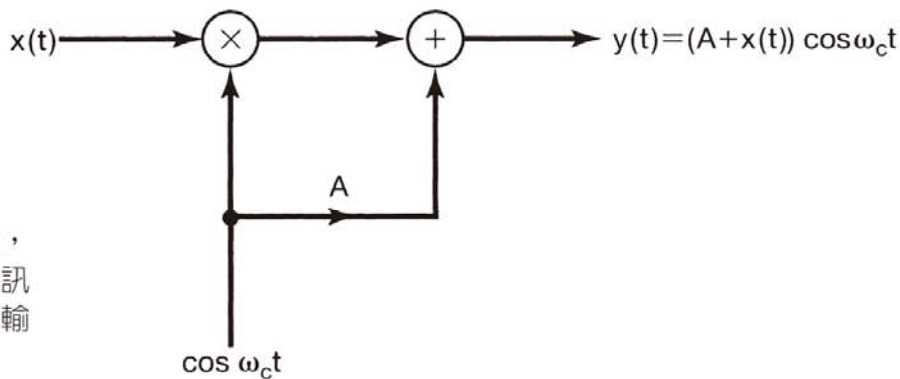


(b)

$r(t)$  為半波整流訊號； $x(t)$  為包絡線； $w(t)$  為電路所得的包絡線，近似於  $x(t)$ 。

**Figure 8.11** Demodulation by envelope detection: (a) circuit for envelope detection using half-wave rectification; (b) waveforms associated with the envelope detector in (a):  $r(t)$  is the half-wave rectified signal,  $x(t)$  is the true envelope, and  $w(t)$  is the envelope obtained from the circuit in (a). The relationship between  $x(t)$  and  $w(t)$  has been exaggerated in (b) for purposes of illustration. In a practical asynchronous demodulation system,  $w(t)$  would typically be a much closer approximation to  $x(t)$  than depicted here.

## 8.2.2 Asynchronous Demodulation



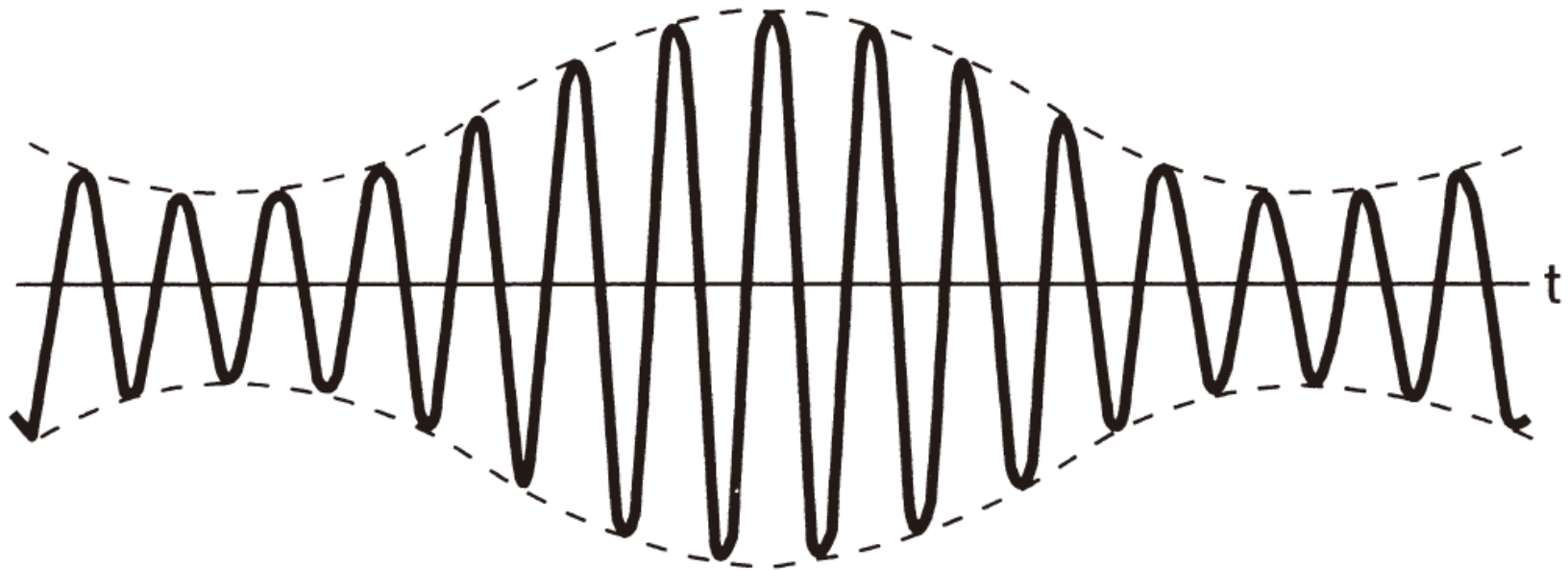
調變指數  $K/A$  愈大，  
則可傳輸較大的訊  
號功率，提高傳輸  
效率。

**Figure 8.12** Modulator for an asynchronous modulation-demodulation system.

調變指數  $K/A$  愈大，則可傳輸較大的訊號功率，提高傳輸效率。

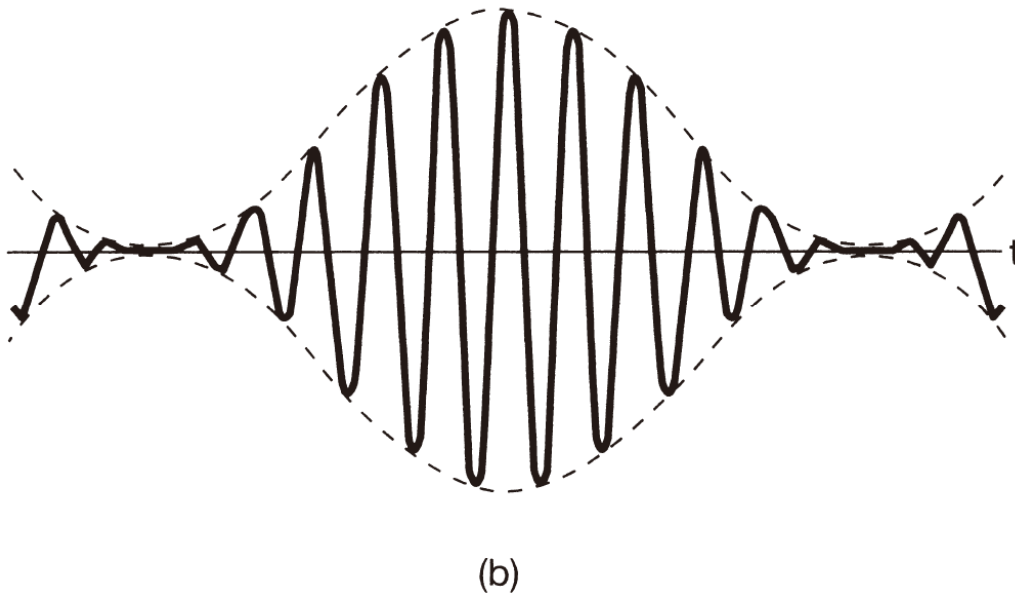
調變指數  $K/A$  愈小，則可改善  $x(t)$  的取得。故功率的傳輸效率與訊號解調的品質，是必須有所取捨的。

## 8.2.2 Asynchronous Demodulation



(a)

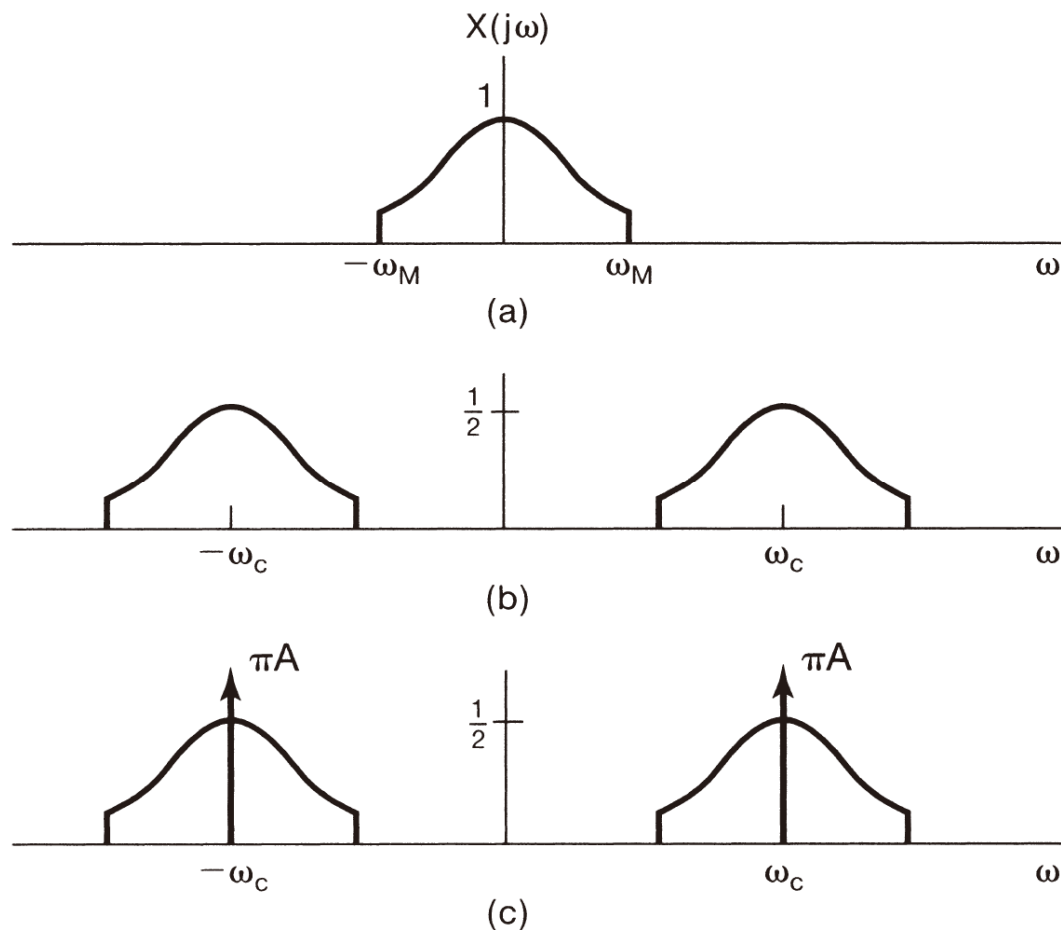
## 8.2.2 Asynchronous Demodulation



**Figure 8.13** Output of the amplitude modulation system of Figure 8.12: (a) modulation index  $m = 0.5$ ; (b) modulation index  $m = 1.0$ .



## 8.2.2 Asynchronous Demodulation



**Figure 8.14** Comparison of spectra for synchronous and asynchronous sinusoidal amplitude modulation systems: (a) spectrum of modulating signal; (b) spectrum of  $x(t) \cos \omega_c t$  representing modulated signal in a synchronous system; (c) spectrum of  $[x(t) + A] \cos \omega_c t$  representing modulated signal in an asynchronous system.

## 8.3 Frequency-Division Multiplexing

Many systems used for transmitting signals provide more bandwidth than is required for any one signal. For example, a typical microwave link has a total bandwidth of several gigahertz, which is considerably greater than the bandwidth required for one voice channel.

將許多頻譜重疊的有限頻帶訊號，分別利用不同頻率的弦波振幅調變，使頻譜不再重疊，則可同時在單一的寬頻通道上傳輸，稱為「分頻多工」。

## 8.3 Frequency-Division Multiplexing

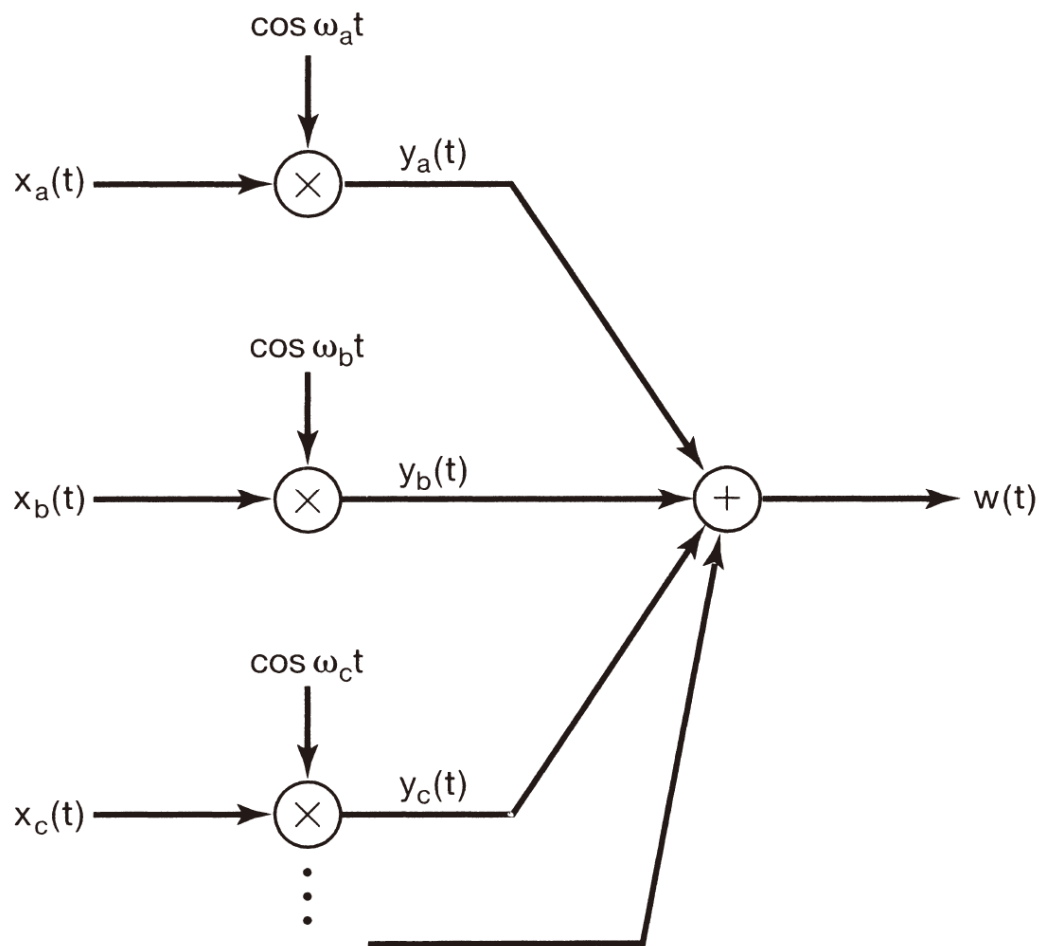


圖 8.15 為分頻多工結構圖。

**Figure 8.15** Frequency-division multiplexing using sinusoidal amplitude modulation.

## 8.3 Frequency-Division Multiplexing

To recover the individual channels in the demultiplexing process requires two basic steps: bandpass filtering to extract the modulated signal corresponding to a specific channel, followed by demodulation to recover the original signal.

解多工的過程有兩個基本階段：先以帶通濾波器抽取個別的調變訊號，再透過解調器還原出原訊號。

## 8.3 Frequency-Division Multiplexing

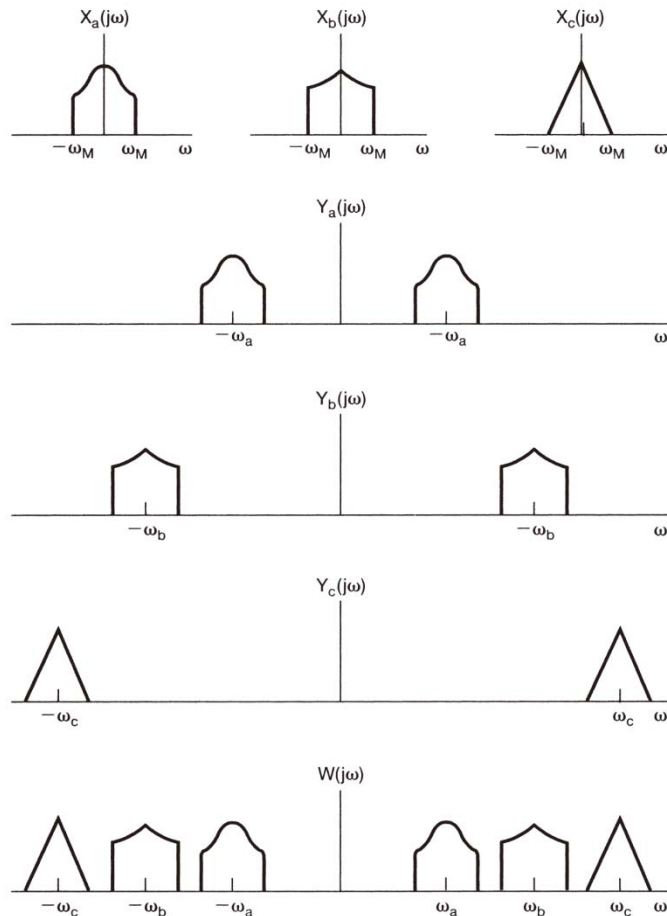
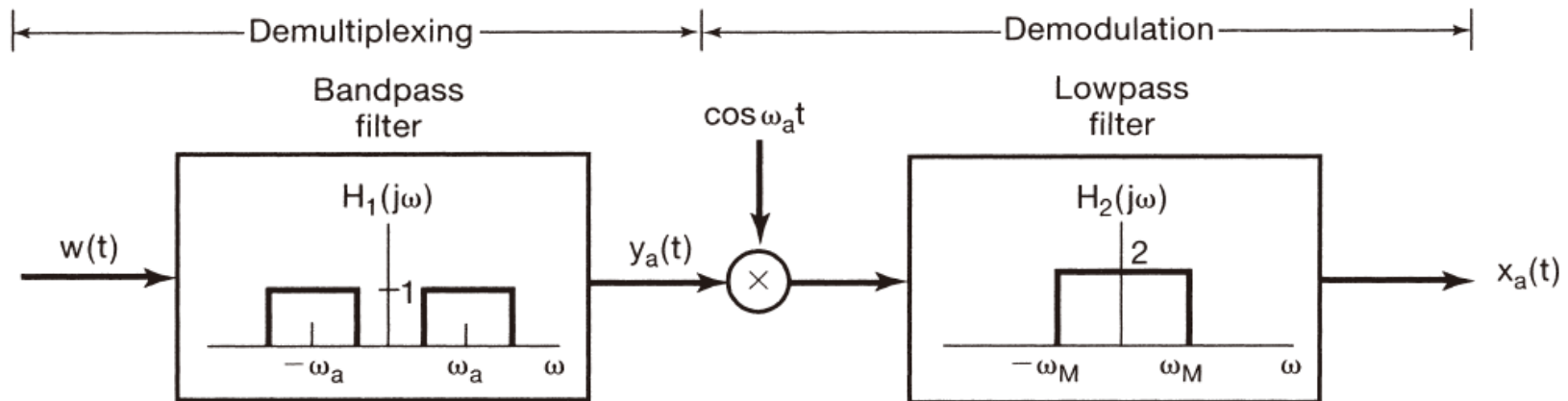


圖 8.16 為分頻多工的頻譜關係示意圖。

**Figure 8.16** Spectra associated with the frequency-division multiplexing system of Figure 8.15.

## 8.3 Frequency-Division Multiplexing



**Figure 8.17** Demultiplexing and demodulation for a frequency-division multiplexed signal.

## 8.3 Frequency-Division Multiplexing

Telephone communication is one important application of frequency-division multiplexing. Another is the transmission of signals through the atmosphere in the RF band.

電話通訊及透過大氣層在射頻（RF）頻帶中做訊號傳輸，都是分頻多工的重要應用。

## 8.3 Frequency-Division Multiplexing

As illustrated in Figure 8.16, in the frequency-division multiplexing system of Figure 8.15 the spectrum of each individual signal is replicated at both positive and negative frequencies, and thus the modulated signal occupies twice the bandwidth of the original.

圖8.15的分頻多工系統，在圖8.16中可見各訊號的頻譜是正負頻對稱的，故調變訊號的頻寬為原訊號的兩倍，使頻寬的使用效率不好。



## 8.3 Frequency-Division Multiplexing

Frequency range	Designation	Typical uses	Propagation method	Channel features
30–300 Hz	ELF (extremely low frequency)	Macrowave, submarine communication	Megametric waves	Penetration of conducting earth and seawater
0.3–3 kHz	VF (voice frequency)	Data terminals, telephony	Copper wire	
3–30 kHz	VLF (very low frequency)	Navigation, telephone, telegraph, frequency and timing standards	Surface ducting (ground wave)	Low attenuation, little fading, extremely stable phase and frequency, large antennas
30–300 kHz	LF (low frequency)	Industrial (power line) communication, aeronautical and maritime long-range navigation, radio beacons	Mostly surface ducting	Slight fading, high atmospheric pulse
0.3–3 MHz	MF (medium frequency)	Mobile, AM broadcasting, amateur, public safety	Ducting and ionospheric reflection (sky wave)	Increased fading, but reliable
3–30 MHz	HF (high frequency)	Military communication, aeronautical mobile, international fixed, amateur and citizen's band, industrial	Ionospheric reflecting sky wave, 50–400 km layer altitudes	Intermittent and frequency-selective fading, multipath
30–300 MHz	VHF (very high frequency)	FM and TV broadcast, land transportation (taxis, buses, railroad)	Sky wave (ionospheric and tropospheric scatter)	Fading, scattering, and multipath
0.3–3 GHz	UHF (ultra high frequency)	UHF TV, space telemetry, radar, military	Transhorizon tropospheric scatter and line-of-sight relaying	
3–30 GHz	SHF (super high frequency)	Satellite and space communication, common carrier (CC), microwave	Line-of-sight ionosphere penetration	Ionospheric penetration, extraterrestrial noise, high directly
30–300 GHz	EHF (extremely high frequency)	Experimental, government, radio astronomy	Line of sight	Water vapor and oxygen absorption
$10^3$ – $10^7$ GHz	Infrared, visible light, ultraviolet	Optical communications	Line of sight	

**Figure 8.18** Allocation of frequencies in the RF spectrum.

圖 8.18 為射頻頻譜各頻段的介紹。

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

With a sinusoidal carrier, on the other hand, the spectrum of the signal is shifted to  $+\omega_c$  and  $-\omega_c$ , and thus, twice the bandwidth is required. This suggests that there is a basic redundancy in the modulated signal with a sinusoidal carrier.

使用弦波載波將使頻譜移到以  $+\omega_c$  及  $-\omega_c$  為中心之處，必須用到兩倍頻寬。為了去除這種多餘頻寬的情況，可利用「單邊帶調變」。

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

In Figure 8.19(b) results from modulation with a sinusoidal carrier, where we identify an upper and lower sideband for the portion of the spectrum centered at  $+\omega_c$  and that centered at  $-\omega_c$ .

Comparing Figures 8.19(a) and (b), we see that  $X(j\omega)$  can be recovered if only the upper sidebands at positive and negative frequencies are retained, or alternatively, if only the lower sidebands at positive and negative frequencies are retained.

以圖8.19為例，(b)圖為弦波調變後的頻譜，若只保留正負頻率雙方的上邊帶(或下邊帶)，應可還原 $X(j\omega)$ 。

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

The resulting spectrum if only the upper sidebands are retained is shown in Figure 8.19(c), and the resulting spectrum if only the lower sidebands are retained is shown in Figure 8.19(d). The conversion of  $x(t)$  to the form corresponding to Figure 8.19(c) or (d) is referred to as *single-sideband modulation* (SSB), in contrast to the *double-sideband modulation* (DSB) of Figure 8.19(b), in which both sidebands are retained.

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

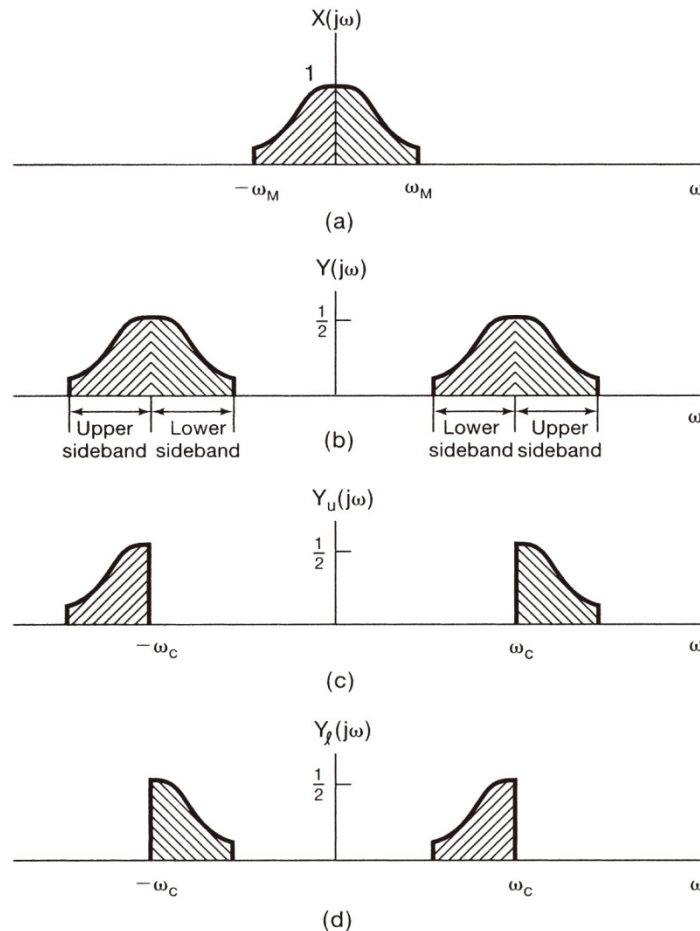
此種將 $x(t)$ 變換至圖8.19(c)或(d)的方式，稱為「單邊帶調變」(SSB)；(b)圖則稱為「雙邊帶調變」(DSB)。

單邊帶調變的方法：

一為利用有陡峭的截止區的帶通濾波器（保留下邊帶）或高通濾波器（保留上邊帶）；

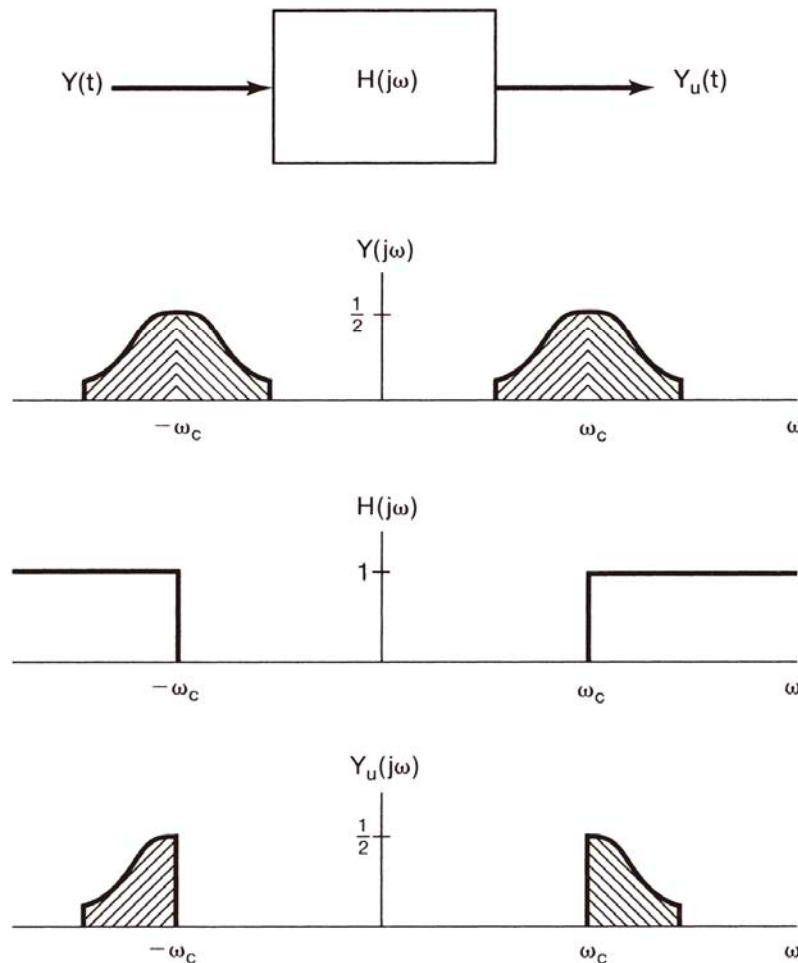
另一為相位移的方法。

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation



**Figure 8.19** Double- and single-sideband modulation: (a) spectrum of modulating signal; (b) spectrum after modulation with a sinusoidal carrier; (c) spectrum with only the upper sidebands; (d) spectrum with only the lower sidebands.

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation



**Figure 8.20** System for retaining the upper sidebands using ideal high-pass filtering.

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

Figure 8.21 depicts a system designed to retain the lower sidebands. The system  $H(j\omega)$  in the figure is referred to as a “90° phase-shift network,” for which the frequency response is of the form

$$H(j\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}. \quad (8.20)$$

圖8.21中用以保留下邊帶達成SSB的相位移系統  $H(j\omega)$ 。



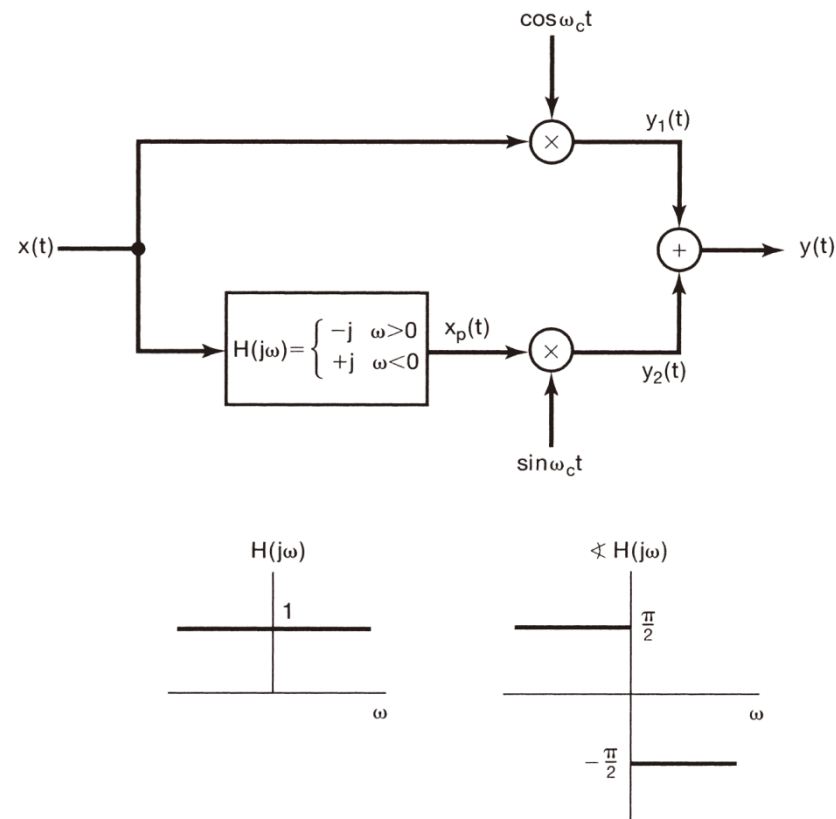
## 8.4 Single-Sideband Sinusoidal Amplitude Modulation

As is examined in Problem 8.28, to retain the upper sidebands instead of the lower sidebands, the phase characteristic of  $H(j\omega)$  is reversed so that

$$H(j\omega) = \begin{cases} j, & \omega > 0 \\ -j, & \omega < 0 \end{cases} \quad (8.21)$$

圖8.21中用以保留上邊帶達成SSB的相位移系統  $H(j\omega)$ 。

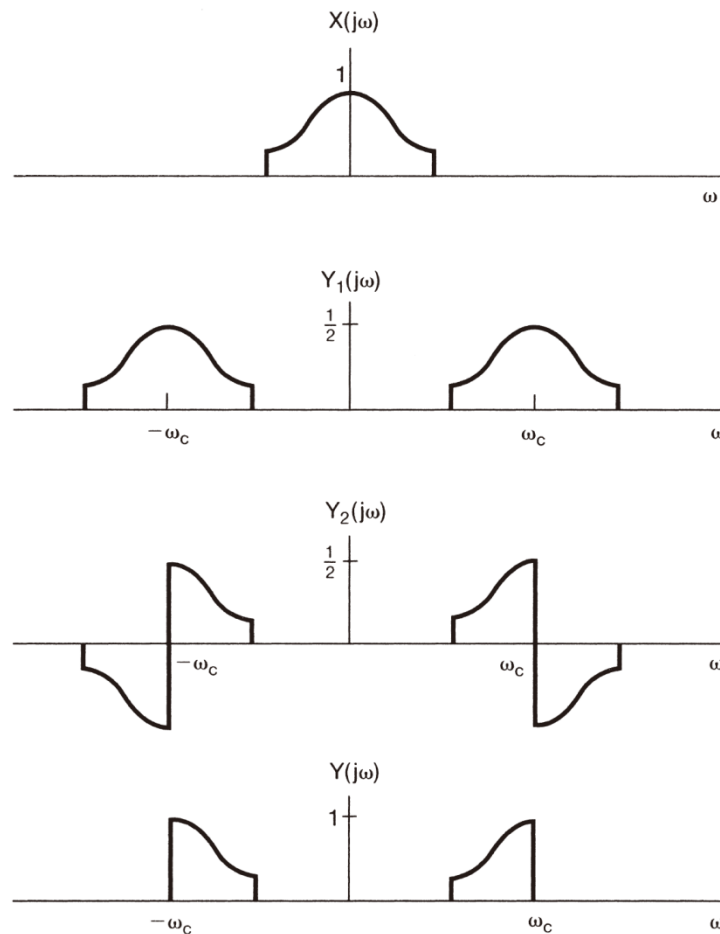
# 8.4 Single-Sideband Sinusoidal Amplitude Modulation



**Figure 8.21** System for single-sideband amplitude modulation, using a  $90^\circ$  phase-shift network, in which only the lower sidebands are retained.

圖 8.21 為利用相位移達成 SSB 的系統（保留下邊帶）。

## 8.4 Single-Sideband Sinusoidal Amplitude Modulation



**Figure 8.22** Spectra associated with the single-sideband system of Figure 8.21.

## 8.5.1 Modulation of a Pulse-Train Carrier

若載波為脈波串，則振幅調變將是等時距的 $x(t)$ 的時間片段。一般而言，任意訊號是無法由此一時間片段的組合來還原的。

From Figure 8.23,

$$y(t) = x(t)c(t); \quad (8.22)$$

振幅調變的時域關係式

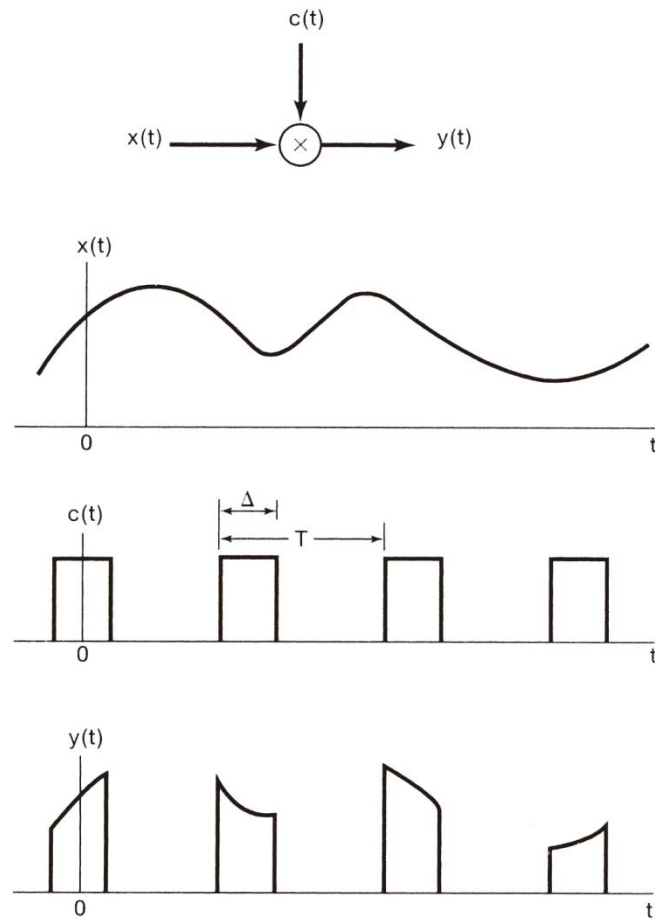
## 8.5.1 Modulation of a Pulse-Train Carrier

i.e., the modulated signal  $y(t)$  is the product of  $x(t)$  and the carrier  $c(t)$ . With  $Y(j\omega)$ ,  $X(j\omega)$ , and  $C(j\omega)$  representing the Fourier transforms of each of these signals, it follows from the multiplication property that

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) C(j(\omega - \theta)) d\theta. \quad (8.23)$$

振幅調變的頻域關係式

## 8.5.1 Modulation of a Pulse-Train Carrier



**Figure 8.23** Amplitude modulation of a pulse train.

## 8.5.1 Modulation of a Pulse-Train Carrier

Since  $c(t)$  is periodic with period  $T$ ,  $C(j\omega)$  consists of impulses in frequency spaced by  $2\pi/T$ ; that is,

因 $c(t)$ 為週期函數(脈波串)

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c),$$

(8.24)

則 $c(t)$ 的傅立葉轉換如(8.24)式。

## 8.5.1 Modulation of a Pulse-Train Carrier

where  $\omega_c = 2\pi / T$  and the coefficients  $a_k$  are the Fourier series coefficients of  $c(t)$ , which, from Example 3.5, are

$$a_k = \frac{\sin(k\omega_c \Delta / 2)}{\pi k}. \quad (8.25)$$

其中  $\omega_c = 2\pi / T$  且  $a_k$  為(8.25)式的傅立葉係數。



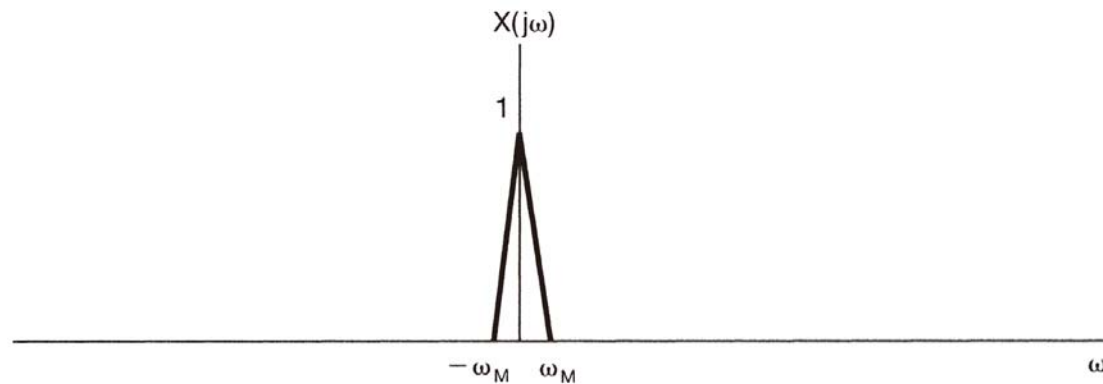
## 8.5.1 Modulation of a Pulse-Train Carrier

From eqs. (8.23) and (8.24),  $Y(j\omega)$  is sum of scaled and shifted replicas of  $X(j\omega)$ :

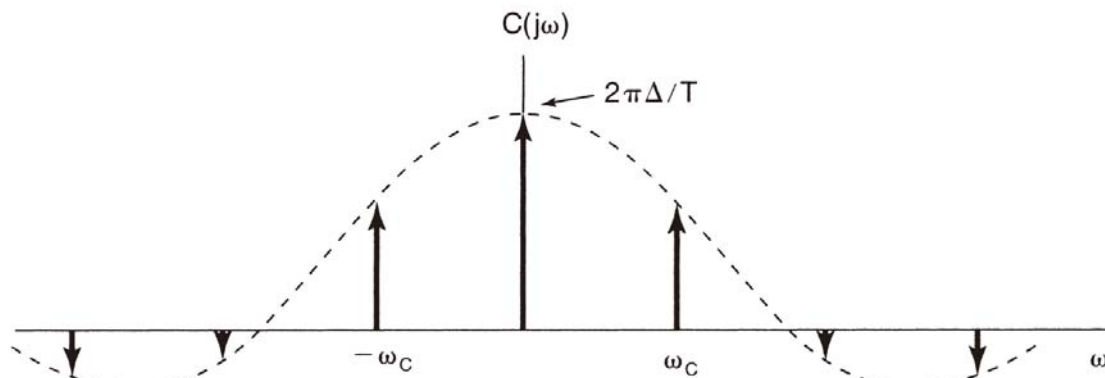
$$Y(j\omega) = \sum_{k=-\infty}^{+\infty} a_k X(j(\omega - k\omega_c)). \quad (8.26)$$

脈波振幅調變的傅立葉轉換關係式

## 8.5.1 Modulation of a Pulse-Train Carrier

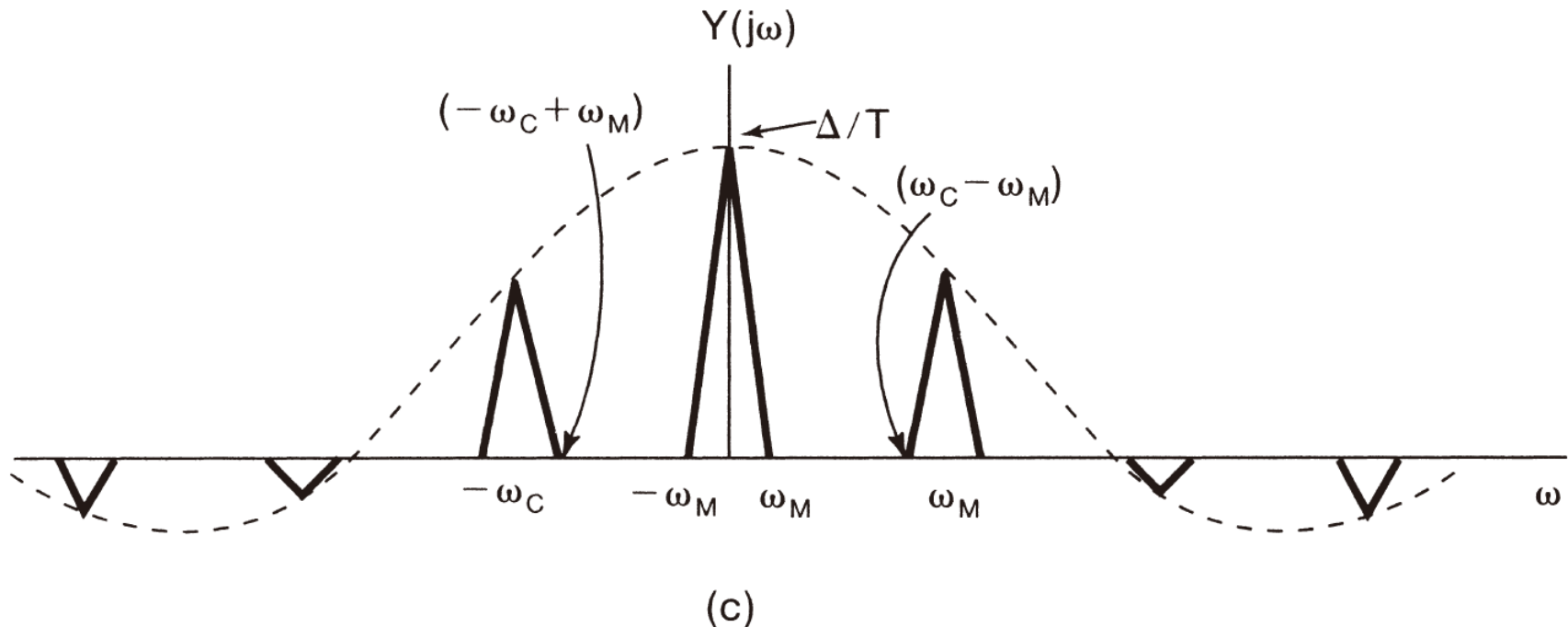


(a)



(b)

## 8.5.1 Modulation of a Pulse-Train Carrier



**Figure 8.24** Spectra associated with amplitude modulation of a pulse train: (a) spectrum of a bandlimited-signal  $x(t)$ ; (b) spectrum of the pulse carrier signal  $c(t)$  in Figure 8.23; (c) spectrum of the modulated pulse train  $y(t)$ .

## 8.5.1 Modulation of a Pulse-Train Carrier

Comparing eq. (8.26) with eq. (7.6) and Figure 8.24 with Figure 7.3(c), we see that the spectrum of  $y(t)$  is very similar in form to the spectrum resulting from sampling with a periodic impulse train, the only difference being the values of the Fourier coefficients of the pulse train.

比較圖8.24及圖7.3(c)可知，脈波振幅調變與脈衝振幅調變的頻譜極為相似，僅傅立葉係數不同而已。

## 8.5.1 Modulation of a Pulse-Train Carrier

Consequently, the replicas of  $X(j\omega)$  do not overlap as long as  $\omega_c > 2\omega_M$ , which corresponds to the condition of the Nyquist sampling theorem.

若  $\omega_c > 2\omega_M$ ，則  $x(t)$  可還原。

## 8.5.2 Time-Division Multiplexing

Amplitude modulation with a pulse-train carrier is often used to transmit several signals over a single channel.

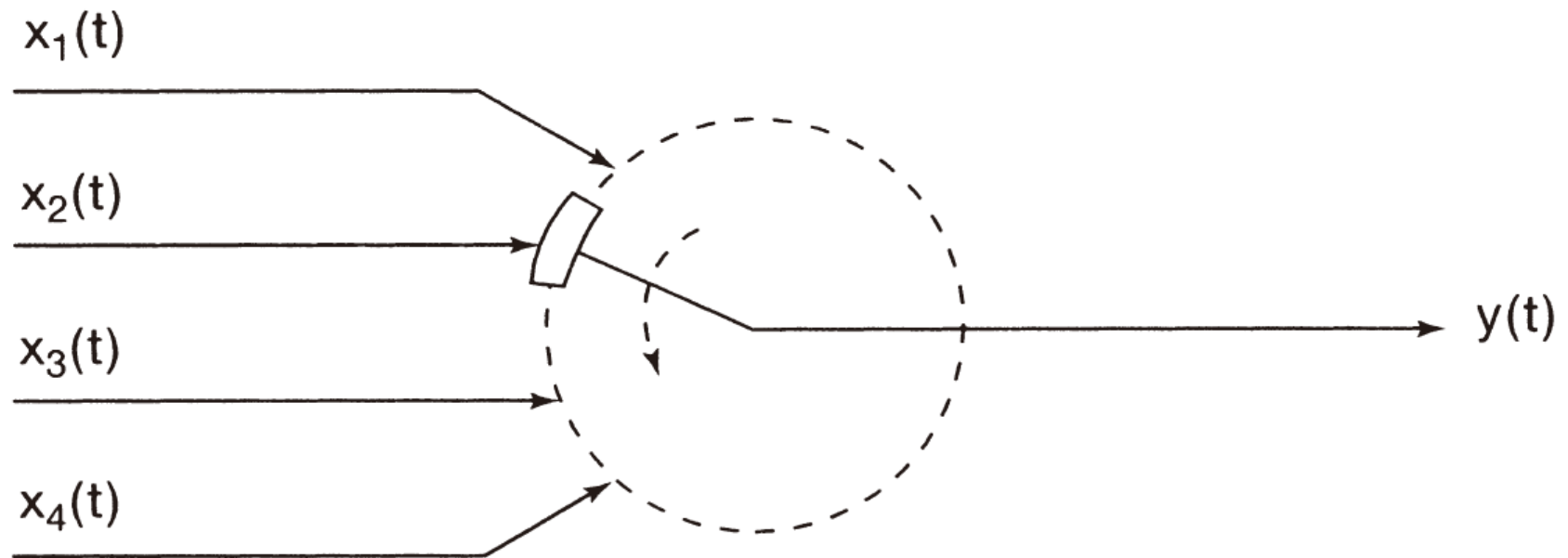
利用脈波串載波的振幅調變，常用於在單一通道上傳輸多個訊號。

## 8.5.2 Time-Division Multiplexing

In this technique for transmitting several signals over a single channel, each signal is in effect assigned a set of time slots of duration  $\Delta$  that repeat every  $T$  seconds and that do not overlap with the slots assigned to other signals.

若脈波載波頻率為  $T$ ，脈波寬度為  $\Delta$ ，只要每個訊號使用通道的時間（每  $T$  秒佔用  $\Delta$  秒）不重疊，則可在同一通道上傳輸多個訊號，稱為「分時多工」(TDM)。

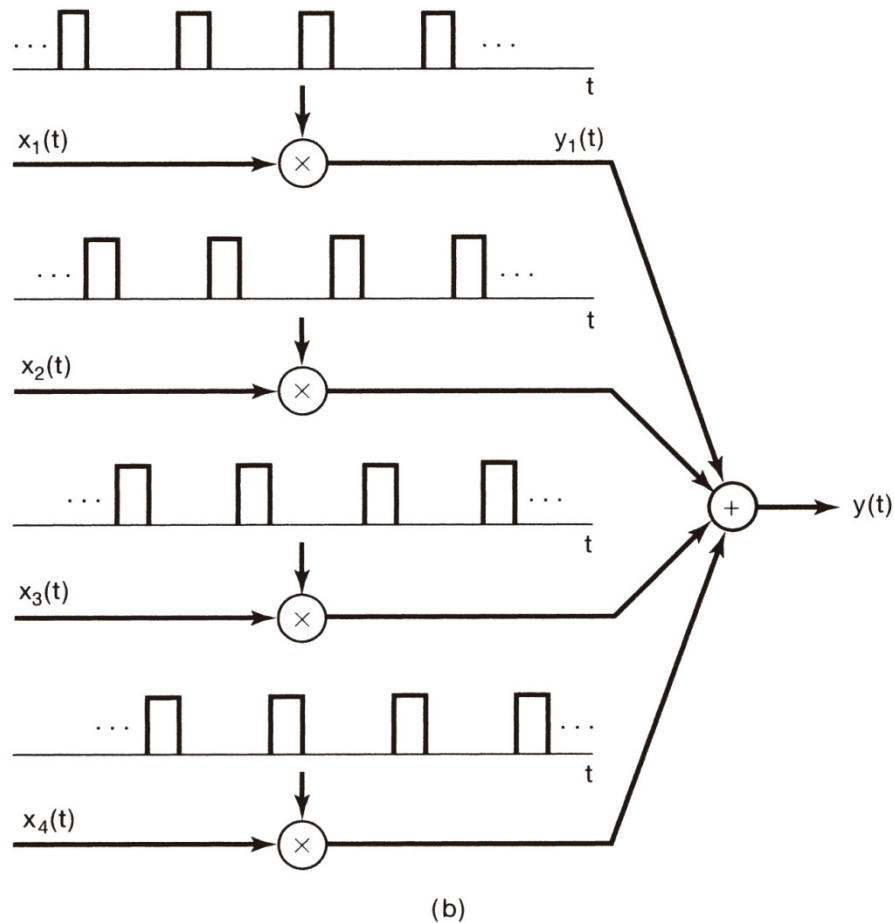
## 8.6.1 Pulse-Amplitude Modulation



(a)



## 8.6.1 Pulse-Amplitude Modulation



**Figure 8.25** Time-division multiplexing.

## 8.6.1 Pulse-Amplitude Modulation

在實際通訊系統中，一般傳輸的是訊號的取樣值，不是片段值。

In fact, in modern communication systems, sampled values of the information-bearing signal  $x(t)$ , rather than time slices are more typically transmitted. For practical reasons, there are limitations on the maximum amplitude that can be transmitted over a communication channel, so that transmitting impulse-sampled versions of  $x(t)$  is not practical.

## 8.6.1 Pulse-Amplitude Modulation

Instead, the samples  $x(nT)$  are used to modulate the amplitude of a sequence of pulses, resulting in what is referred to as a pulse-amplitude modulation (PAM) system.

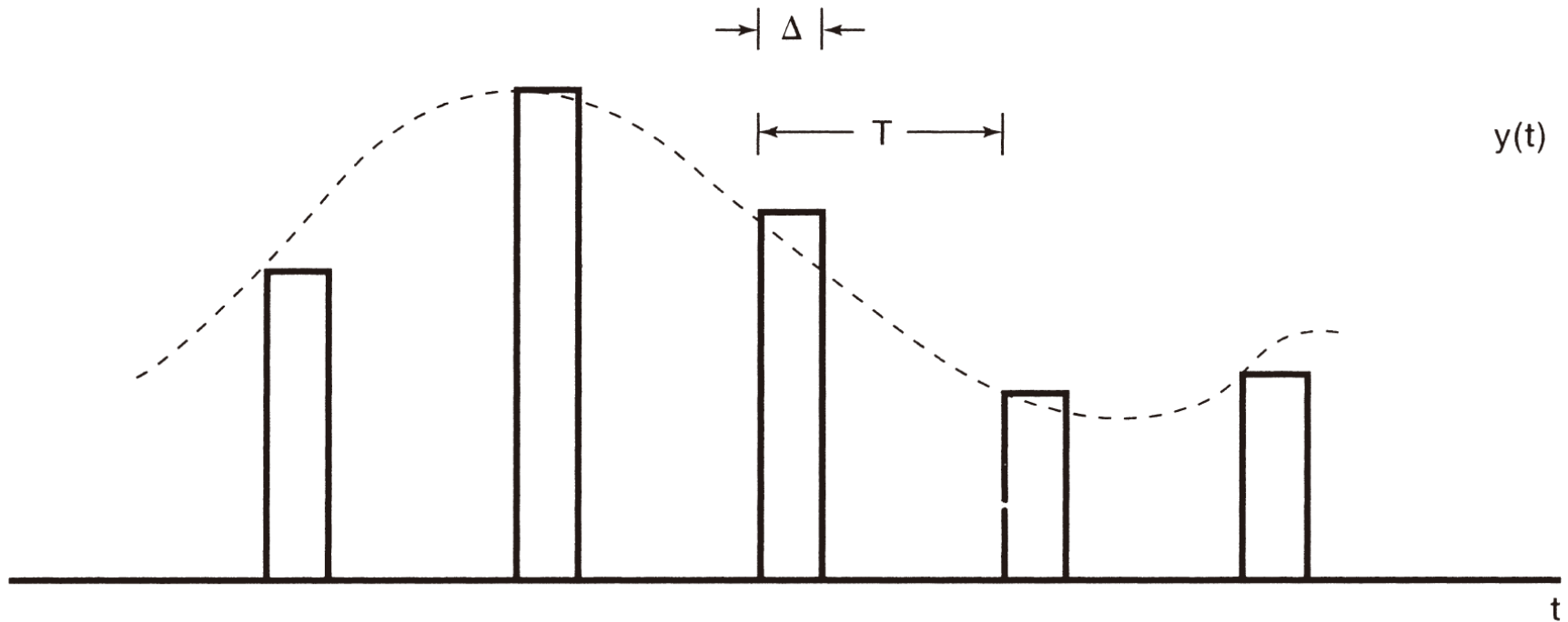
因傳輸通道對訊號大小的限制，無法用脈衝取樣調，則可利用取樣值 $x(nT)$ 為大小，調變出一組脈波序列，稱為「脈波振幅調變」（PAM）。

## 8.6.1 Pulse-Amplitude Modulation

In addition to energy considerations, a number of other issues must be addressed in designing a PAM signal.

只要取樣頻率超過奈奎士速率，則取樣值可重建出原訊號。

## 8.6.1 Pulse-Amplitude Modulation



**Figure 8.26** Transmitted waveform for a single PAM channel. The dotted curve represents the signal  $x(t)$ .

圖 8.26 為脈波振幅調變的波形。

## 8.6.1 Pulse-Amplitude Modulation

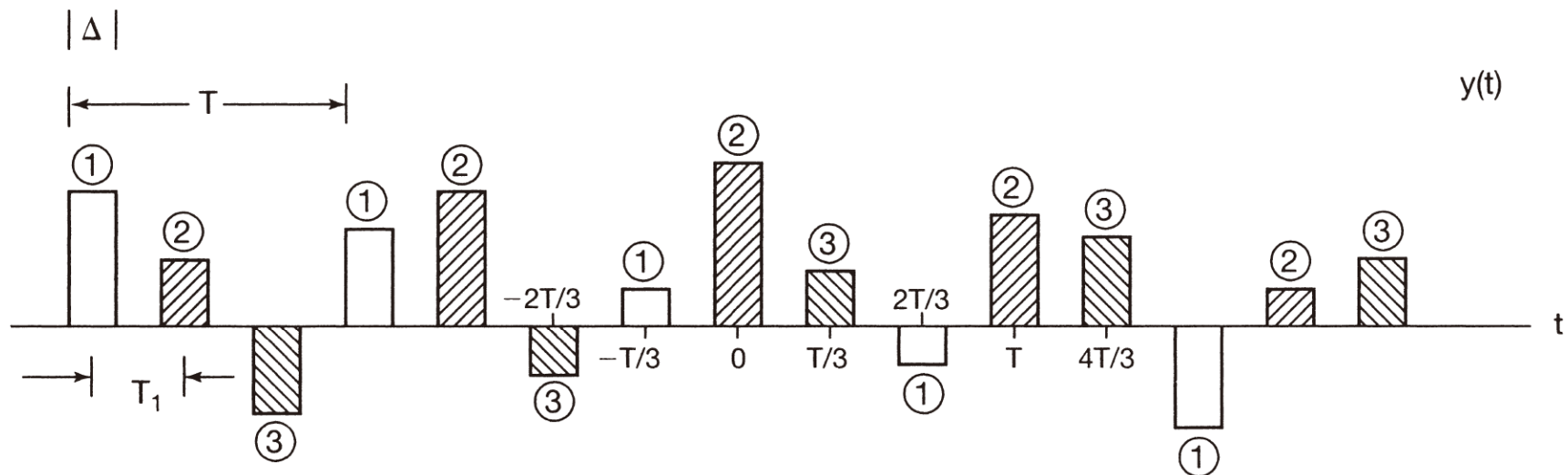


圖 8.27 為三個分時 PAM 通道的波形。 $T_1 = T/3$  為各訊號取樣的時間間隔。

**Figure 8.27** Transmitted waveform with three time-multiplexed PAM channels. The pulses associated with each channel are distinguished by shading, as well as by the channel number above each pulse. Here, the intersymbol spacing is  $T_1 = T/3$ .

## 8.6.2 Intersymbol Interference in PAM Systems

For example, to the midpoints of each pulse, we can separate the samples of the three signals. That is,

$$y(t) = Ax_1(t), \quad t = 0, \pm 3T_1, \pm 6T_1, \dots, \quad (8.27)$$

$$y(t) = Ax_2(t), \quad t = T_1, T_1 \pm 3T_1, T_1 \pm 6T_1, \dots,$$

$$y(t) = Ax_3(t), \quad t = 2T_1, 2T_1 \pm 3T_1, T_1 \pm 6T_1, \dots,$$

## 8.6.2 Intersymbol Interference in PAM Systems

In other words, samples of  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$  can be obtained by appropriate sampling of the received time-multiplexed PAM signal.

理想上，由接收到的分時PAM訊號經適當取樣，可還原出各訊號。

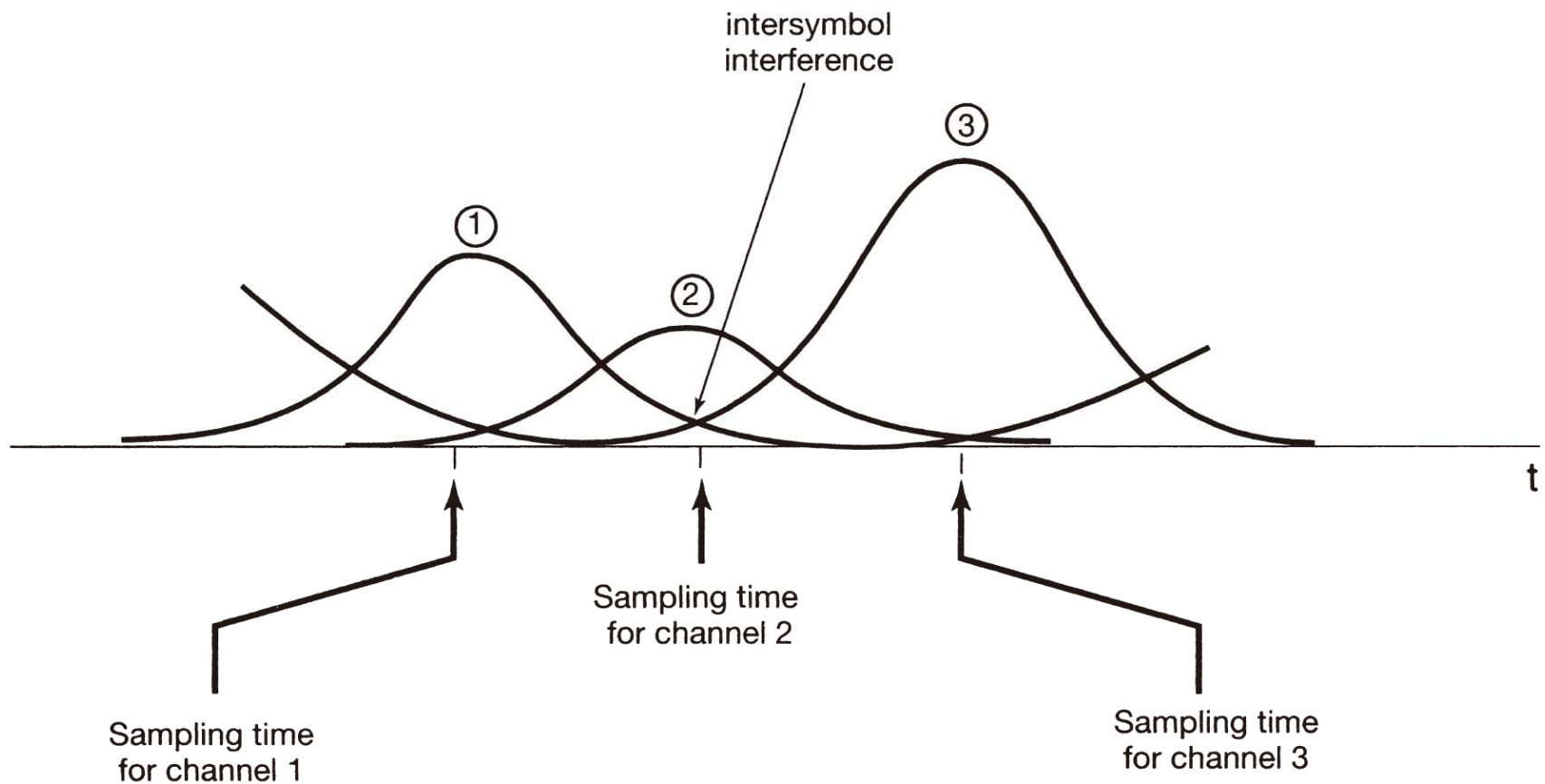


## 8.6.2 Intersymbol Interference in PAM Systems

Additive noise in the channel will, of course, introduce amplitude errors at the sampling times. Filtering due to the nonideal frequency response of a channel causes a smearing of the individual pulses that can cause the received pulses to overlap in time. This interference is illustrated in Figure 8.28 and is referred to as intersymbol interference.

因通道的非理想頻率響應的濾波暈染了個別的脈波，而使接收到的脈波在時間上重疊(如圖8.28)稱為碼際干擾」。

## 8.6.2 Intersymbol Interference in PAM Systems



**Figure 8.28** Intersymbol interference.

## 8.6.2 Intersymbol Interference in PAM Systems

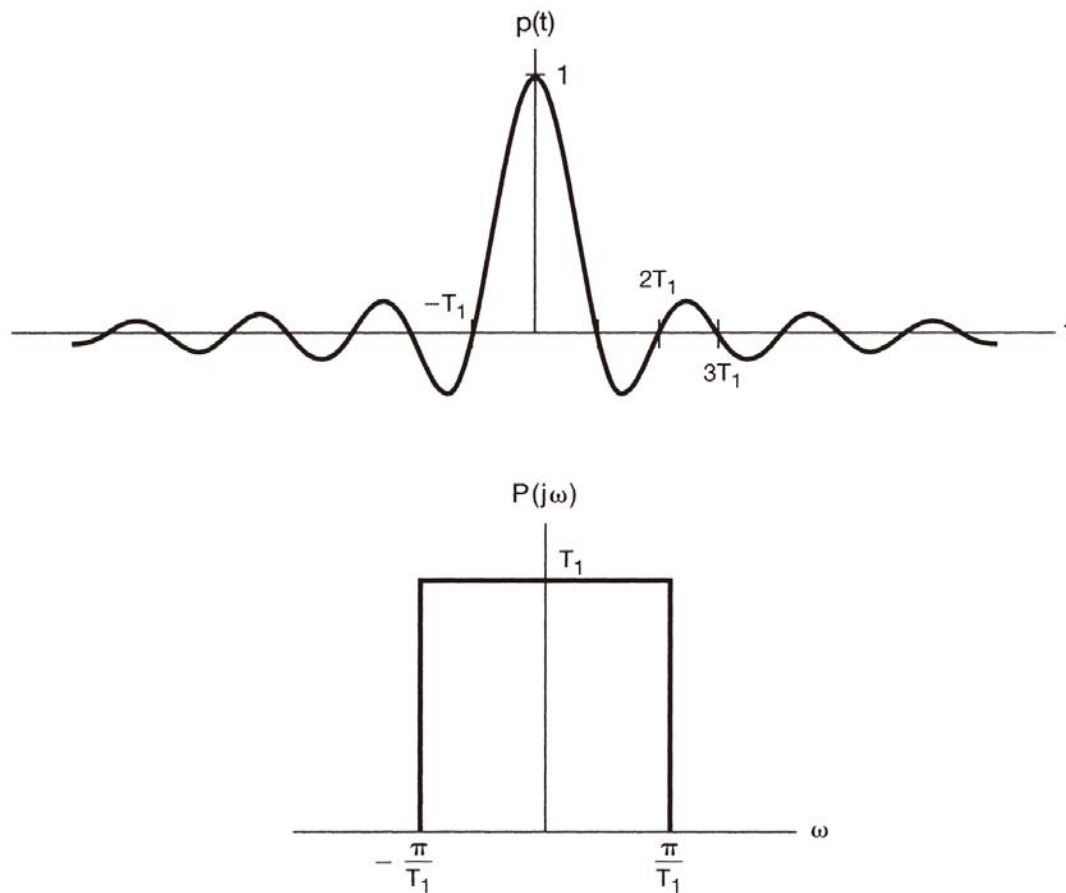
如果使用的脈波為有限頻帶，則每一個PAM訊號接收時將不失真。

For example, consider the sinc pulse

$$p(t) = \frac{T_1 \sin(\pi t / T_1)}{\pi t}$$

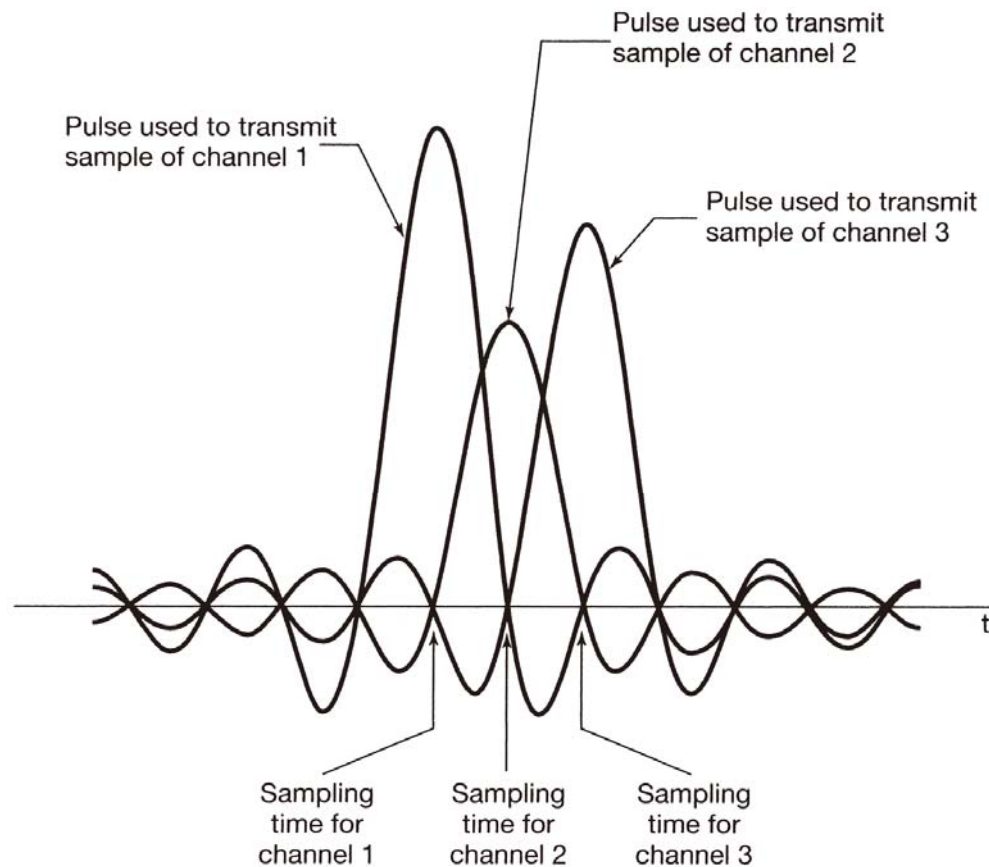
考慮sinc脈波，其頻譜限制在  $-\pi / T_1 \leq \omega \leq \pi / T_1$ ，則在  $T_1$  時間整數倍時，脈波值為0，故在取樣點上不會造成碼際干擾。

## 8.6.2 Intersymbol Interference in PAM Systems



**Figure 8.29** A sinc pulse and its corresponding spectrum.

## 8.6.2 Intersymbol Interference in PAM Systems



**Figure 8.30** Absence of intersymbol interference when sinc pulses with correctly chosen zero-crossings are used.

## 8.6.2 Intersymbol Interference in PAM Systems

The sinc pulse is only one of many band-limited pulses with time-domain zero-crossing at  $\pm T_1$ ,  $\pm 2T_1$ , etc. More generally, consider a pulse  $p(t)$  with spectrum of the form

$$P(j\omega) = \begin{cases} 1 + P_1(j\omega), & |\omega| \leq \frac{\pi}{T_1}, \\ P_1(j\omega), & \frac{\pi}{T_1} < |\omega| \leq \frac{2\pi}{T_1} \\ 0, & \text{otherwise} \end{cases} \quad (8.28)$$

## 8.6.2 Intersymbol Interference in PAM Systems

and with  $P_1(j\omega)$  having odd symmetry around  $\pi/T_1$ , so that

$$P_1\left(-j\omega + j\frac{\pi}{T_1}\right) = -P_1\left(j\omega + j\frac{\pi}{T_1}\right) \quad 0 \leq \omega \leq \frac{\pi}{T_1},$$

(8.29)

sinc脈波只是其一，一般而言，脈波 $p(t)$ 的頻譜只要滿足(8.28)及(8.29)式即可。

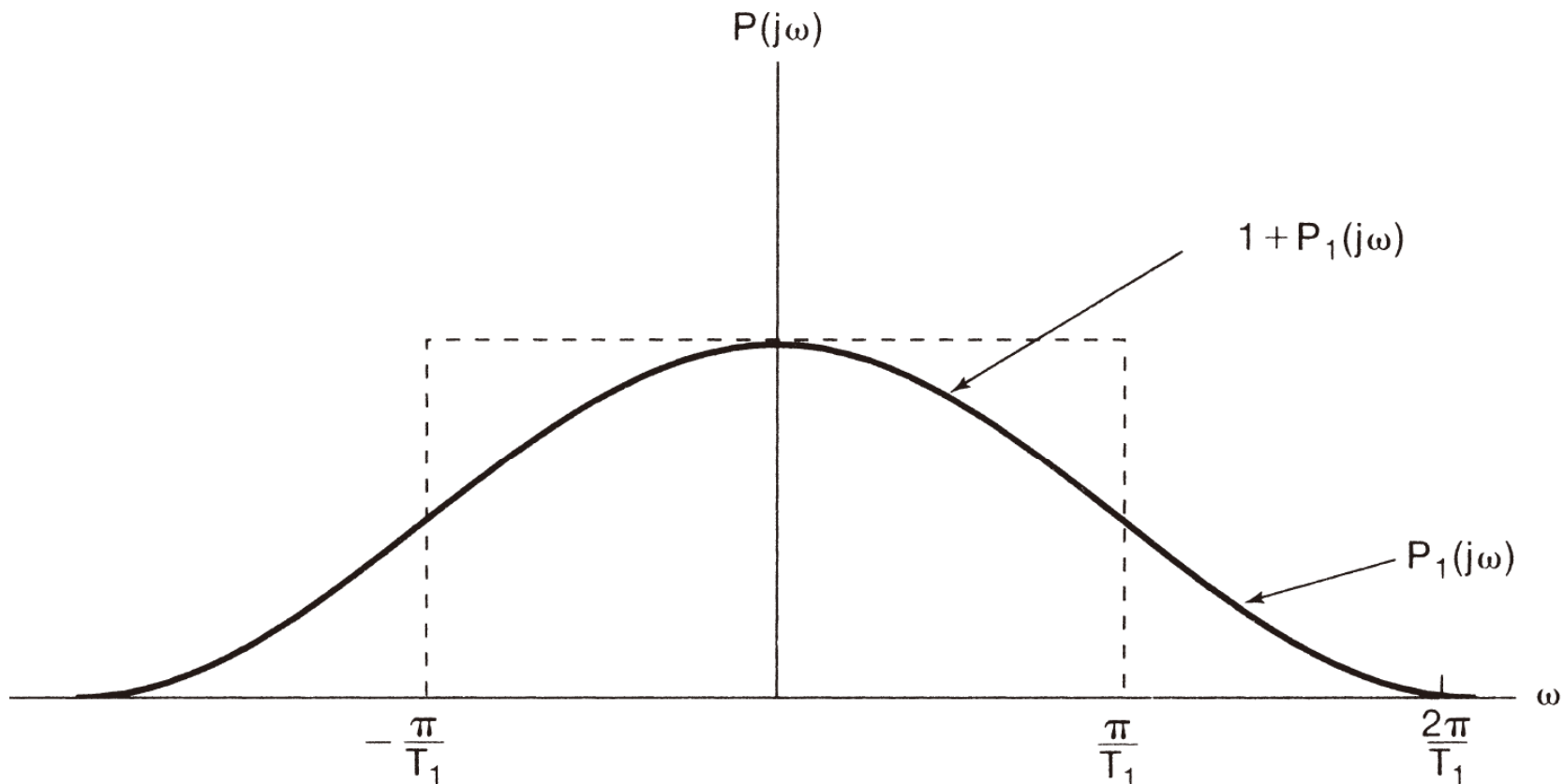
## 8.6.2 Intersymbol Interference in PAM Systems

if  $|H(j\omega)|$  is not constant over the passband, there maybe a need to perform channel equalization—i.e., filtering of the received signal to correct for the nonconstant channel gain..

若通道在通帶上的  $|H(j\omega)|$  不為常數，則可利用通道等化來解決。



## 8.6.2 Intersymbol Interference in PAM Systems



**Figure 8.31** Odd symmetry around  $\pi/T_1$  as defined in eq. (8.29).

## 8.6.3 Digital Pulse-Amplitude and Pulse-Code Modulation

This set of samples can be thought of as a discrete-time signal  $x[n]$ , and in many applications  $x[n]$  is in fact stored in or generated by a digital system.

許多應用中， $x[n]$ 常是儲存或產生於數位系統中。

## 8.6.3 Digital Pulse-Amplitude and Pulse-Code Modulation

More generally, in order to protect against transmission errors or provide secure communication, the sequence of binary digits representing  $x[n]$  might first be *transformed* or *encoded* into another sequence of 0's and 1's before transmission.

為求免於傳輸誤差，在 $x[n]$ 傳輸前可將 $x[n]$ 轉換（編碼）成由0或1組成的序列。

## 8.6.3 Digital Pulse-Amplitude and Pulse-Code Modulation

For obvious reasons, a PAM system modulated by an encoded sequence of 0's and 1's is referred to as a *pulse-code modulation* (PCM) system.

上述將PAM系統再經由0或1的編序列的調變，稱為「脈波編碼調變」(PCM)。

## 8.7 Sinusoidal Frequency Modulation

In the preceding sections, we discussed a number of specific amplitude modulation systems in which the modulating signal was used to vary the amplitude of a sinusoidal or a pulse carrier.

In another very important class of modulation techniques referred to as frequency modulation (FM), the modulating signal is used to control the frequency of a sinusoidal carrier. Modulation systems of this type have a number of advantages over amplitude modulation systems.

## 8.7 Sinusoidal Frequency Modulation

As suggested by Figure 8.10, with sinusoidal amplitude modulation the peak amplitude of the envelope of the carrier is directly dependent on the amplitude of the modulating signal  $x(t)$ , which can have a large dynamic range—i.e., can vary significantly.

## 8.7 Sinusoidal Frequency Modulation

將要調變的訊號用來控制弦波載波的頻率，稱為「頻率調變」(FM)。

頻率調變有多項優點：

一為**FM**的載波包絡線是常數，故永遠在峰值功率操作；二為在**FM**系統中，由外加的干擾或衰減所引起的振幅改變，可以在接收器消除。

## 8.7 Sinusoidal Frequency Modulation

For this reason, in public broadcasting and a variety of other contexts, FM reception is typically better than AM reception.

典型而言，FM接收比AM接收效果更好。FM通常需要較大的頻寬。

FM系統為高度非線性，無法如AM一樣直接分析。



## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

As we discussed in Section 5.5, there is a corresponding property for discrete-time signals which we can use to analyze discrete-time amplitude modulation. Specifically, consider

$$y[n] = x[n]c[n].$$

離散時間弦波振幅調變的時域關係

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

With  $X(e^{j\omega})$ ,  $Y(e^{j\omega})$ , and  $C(e^{j\omega})$  denoting the Fourier transforms of  $x[n]$ ,  $y[n]$ , and  $c[n]$ , respectively,  $Y(e^{j\omega})$  is proportional to the periodic convolution of  $X(e^{j\omega})$  and  $C(e^{j\omega})$ ; that is,

$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) C(e^{j(\omega-\theta)}) d\theta. \quad (8.54)$$

離散時間弦波振幅調變的頻域關係

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

Let us first consider sinusoidal amplitude modulation with a complex exponential carrier, so that

若載波為複指數

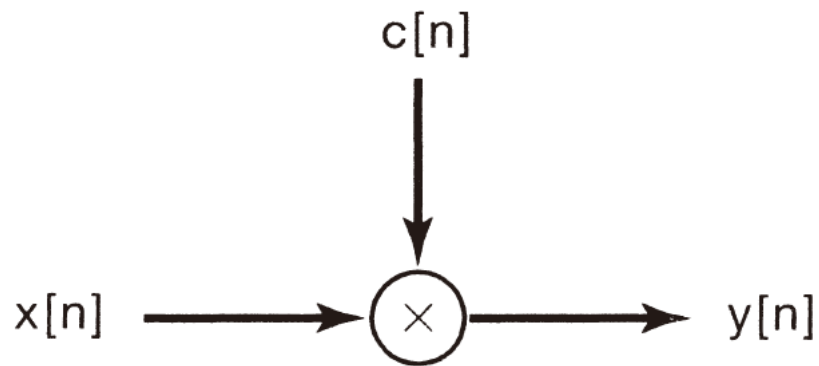
$$c[n] = e^{j\omega_c n}. \quad (8.55)$$

As we saw in Section 5.2, the Fourier transform of  $c[n]$  is a periodic impulse train; that is,

$$C(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_c + k2\pi), \quad (8.56)$$

載波的傅立葉轉換為週期性脈衝串

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation



**Figure 8.40** Discrete-time amplitude modulation.

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

Demodulation is accomplished by multiplying by  $e^{-j\omega_c n}$  to translate the spectrum back to its original location on the frequency axis, so that

$$x[n] = y[n]e^{-j\omega_c n}. \quad (8.57)$$

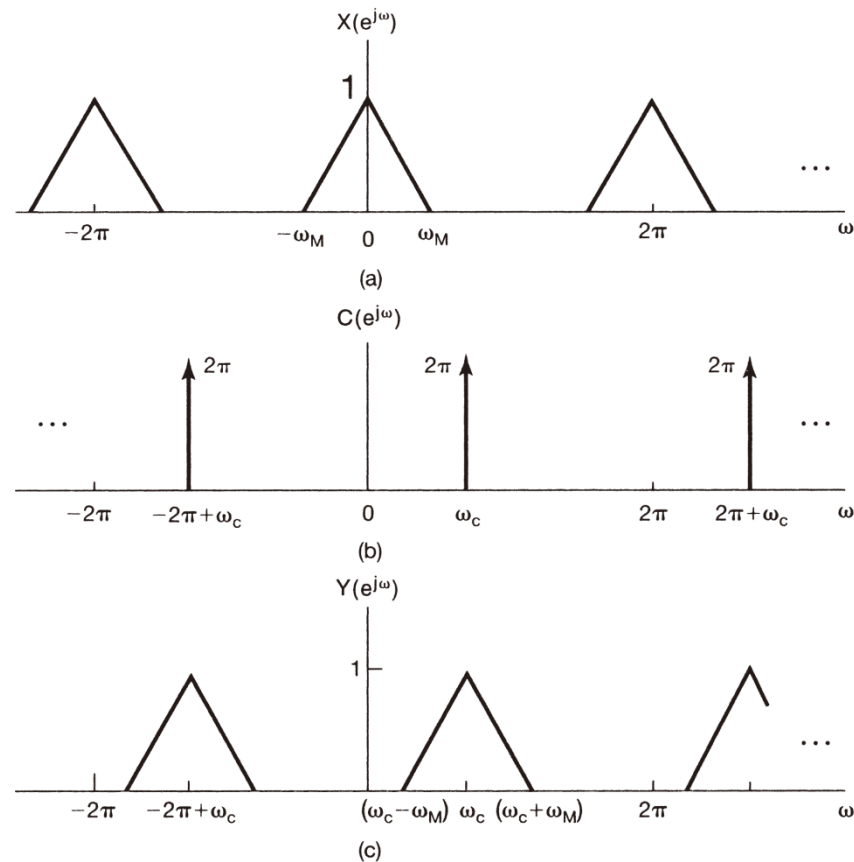
解調只需將 $y[n]$ 除以  $e^{-j\omega_c n}$  而得 $x[n]$ 。

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

With  $c[n] = \cos \omega_c n$ , the spectrum of the carrier consists of periodically repeated pairs of impulses at

若載波為弦波  $c[n] = \cos \omega_c n$

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation



**Figure 8.41** (a) Spectrum of  $x[n]$ ; (b) spectrum of  $c[n] = e^{j\omega_c n}$ ; (c) spectrum of  $y[n] = x[n]c[n]$ .

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

With  $X(e^{j\omega})$  as shown in Figure 8.42(a), the resulting spectrum for the modulated signal is shown in Figure 8.42(c) and corresponds to replicating  $X(e^{j\omega})$  at the frequencies  $\omega = \pm\omega_c + k2\pi$ . In order that the individual replications of  $X(e^{j\omega})$  do not overlap, we require that

$$\omega_c > \omega_M \quad (8.58)$$

載波頻譜在  $\omega = \pm\omega_c + k2\pi$ ，為使調變後各個  $X(e^{j\omega})$  在  $\omega = \pm\omega_c + k2\pi$  不重疊，則  $\omega_c > \omega_M$ 。



## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

and

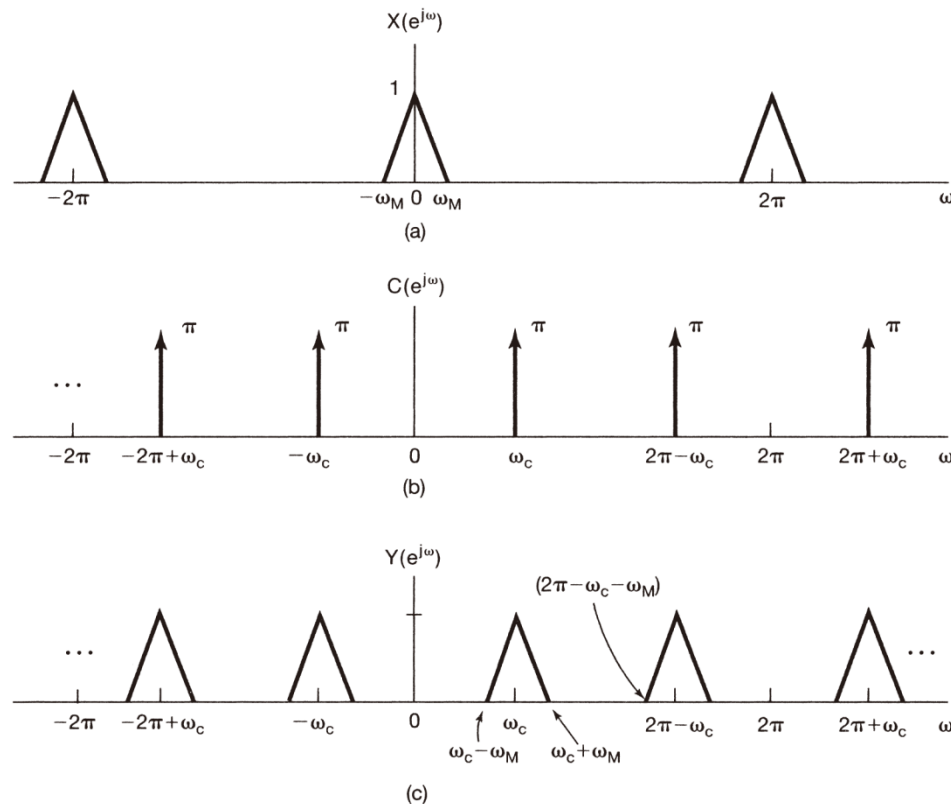
$$2\pi - \omega_c - \omega_m > \omega_c + \omega_M$$

or, equivalently,

即

$$\omega_c < \pi - \omega_M. \quad (8.59)$$

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation



**Figure 8.42** Spectra associated with discrete-time modulation using a sinusoidal carrier: (a) spectrum of a bandlimited-signal  $x[n]$ ; (b) spectrum of a sinusoidal carrier signal  $c[n] = \cos \omega_c n$ ; (c) spectrum of the modulated signal  $y[n] = x[n]c[n]$ .

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

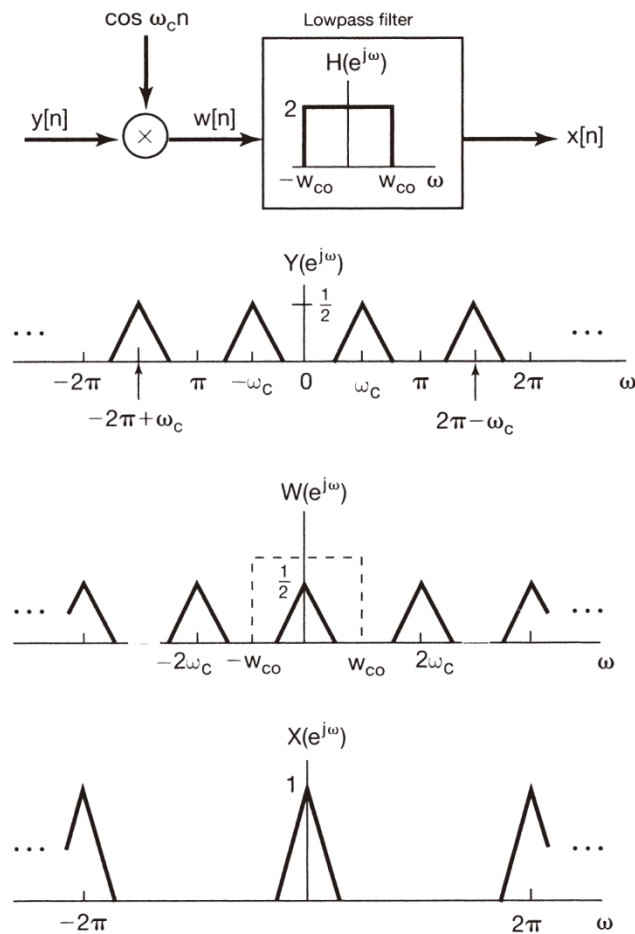
Combining eqs. (8.58) and (8.59), we see that for amplitude modulation with a sinusoidal carrier, we must restrict  $\omega_c$  so that

合併得 
$$\omega_M < \omega_c < \pi - \omega_M.$$

By lowpass filtering to eliminate the unwanted replications of  $X(e^{j\omega})$ , the demodulated signal is obtained.

利用低通濾波去除不必要的  $X(e^{j\omega})$  的複製部分，可得解調訊號。

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation



**Figure 8.43** System and associated spectra for discrete-time synchronous demodulation.

## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

如同連續時間，我們亦可利用離散時間訊號來調變脈波串，使得離散時間訊號的分時多工可行。

The implementation of discrete-time multiplexing systems provides an excellent example of the flexibility of discrete-time processing in general and the importance of the operation of upsampling (see Section 7.5.2) in particular.

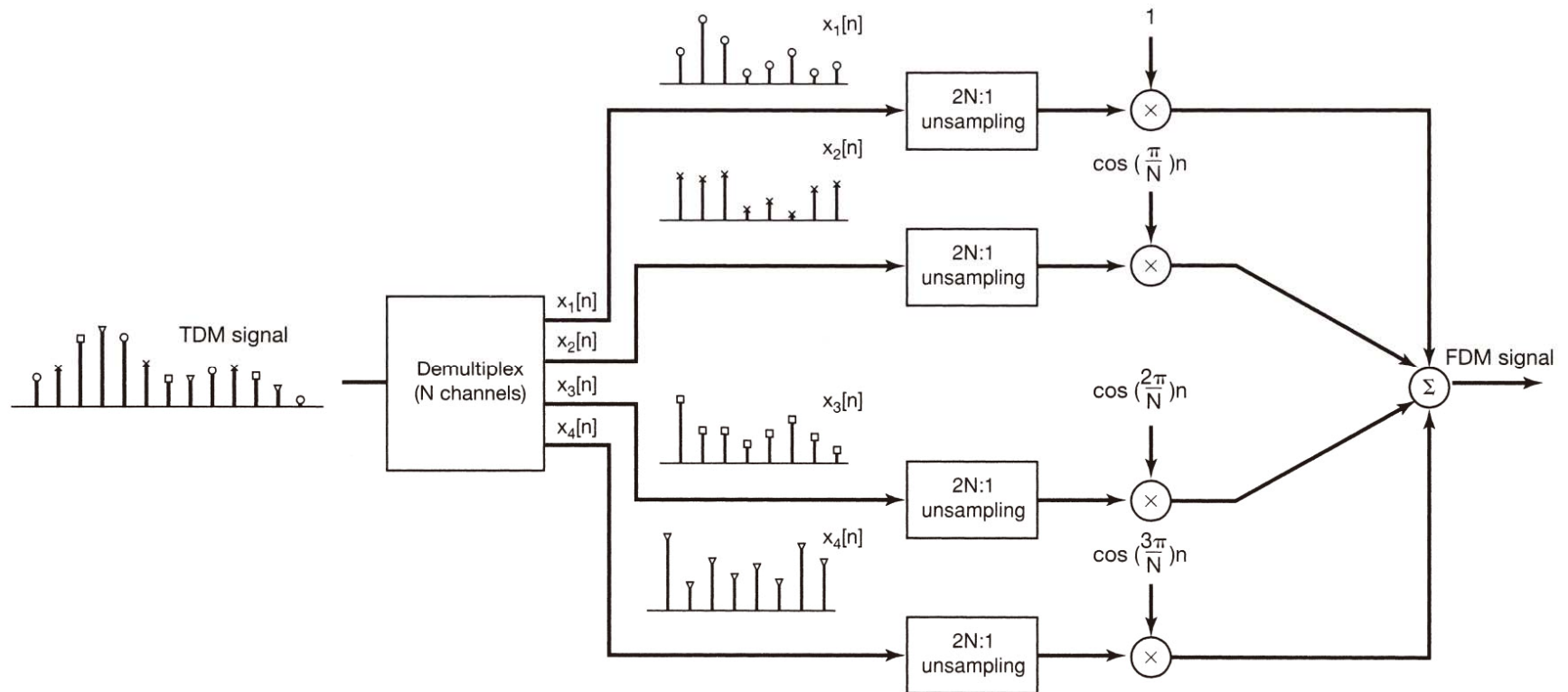
## 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation

FDM中輸入訊號的頻譜條件

$$X_i(\omega) = 0, \quad \frac{\pi}{M} < |\omega| < \pi. \quad (8.61)$$

對於 $M$ 通道的離散時間分頻多工而言，每個輸入通道  $x_i[n]$  必須是有限頻帶。

## 8.9 Summary



**Figure 8.44** Block diagram for TDM-to-FDM transmultiplexing.