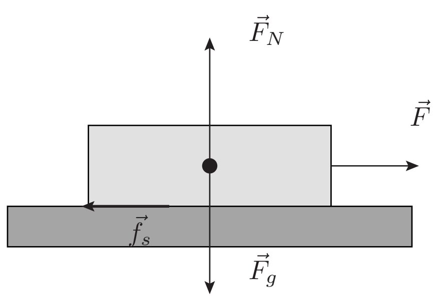
Contents

6 Friction, Drag, and Centripetal Force

6.1 Friction

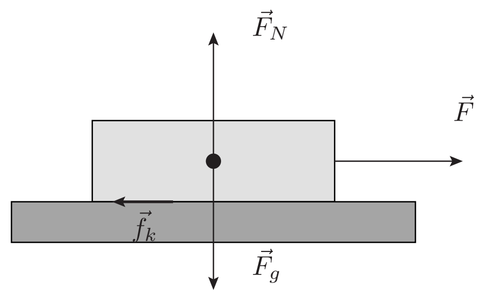


Static friction force:

In the above figure, $\vec{F}_N$ is the normal force and $\vec{F}$ is the horizontal external force acting on the body. The friction force $\vec{f}_s$ is equal to $-\vec{F}$ when $\left|\vec{F}\right|$, the magnitude of the external force is less than

$$\left|\vec{F}\right| < f_{s,\max} = \mu_s F_N$$

where μ_s is the **coefficient of static friction**, otherwise the body begins to accelerate.
Kinetic friction force:



If the body begins to move, the friction force becomes

$$\vec{f}_k = -\mu_k F_N \frac{\vec{F}}{\left|\vec{F}\right|}$$

where μ_k is the **coefficient of kinetic friction**. Note that the magnitude of $\vec{f}_k$ is proportional to the normal force.

$$f_k = \mu_k F_N$$

ABS: anti-lock braking system ? Usually $\mu_k < \mu_s$, if tires are locked, μ_k comes into play and lengthens the braking distance.

6.2 The Drag Force and Terminal Speed

When there is a relative motion between a fluid and a body, the body experiences a drag force $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body.

As a simple example, let us assume that the fluid is the air and the drag force is proportional to the speed of the body.

$$\vec{D} = -\gamma \vec{v}$$

Let's also assume the positive z-axis points in the vertical direction. The gravitational force on the body is $\vec{F}_g = -mg\hat{k}$. The position vector is $\vec{r}\,(t) = z\,(t)\,\hat{k}$, the velocity is

$$\vec{v} = \frac{dz}{dt}\hat{k} = v\hat{k}$$

where $v = \frac{dz}{dt}$. The acceleration is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{k} = \frac{d^2z}{dt^2}\hat{k}$$

Newton's 2nd law gives

$$m\vec{a} = \vec{F}_g + \vec{D} = -mg\hat{k} - \gamma\vec{v}$$

$$m\frac{dv}{dt}\hat{k} = -mg\hat{k} - \gamma v\hat{k}$$

or

$$\frac{dv}{dt} + \frac{\gamma}{m}v = -g \tag{1}$$

This is actually a differential equation for function $v(t)$ and is called a first order differential equation because only the first derivative $\frac{dv}{dt}$ is involved. It is easy to see that the constant function

$$v_p(t) = -\frac{gm}{\gamma} \tag{2}$$

satisfying

$$\frac{dv_p}{dt} + \frac{\gamma}{m}v_p = -g \tag{3}$$

is a particular solution for (1). Knowing that

$$\frac{d}{dt}e^t = e^t,$$

we have

$$\frac{d}{dt}e^{at} = \frac{d(at)}{dt}\frac{d}{d(at)}e^{at} = ae^{at}$$

and thus

$$\frac{d}{dt}Ce^{-\frac{\gamma}{m}t} = -\frac{\gamma}{m}Ce^{-\frac{\gamma}{m}t}$$

where C can be any constant. The function

$$v_h(t) \equiv Ce^{-\frac{\gamma}{m}t} \tag{4}$$

satisfies

$$\frac{dv_h}{dt} + \frac{\gamma}{m}v_h = 0 \tag{5}$$

Combining the solution (2) for (3) and (4) for (5), the function

$$v(t) = v_h(t) + v_p(t) = Ce^{-\frac{\gamma}{m}t} - \frac{gm}{\gamma} \tag{6}$$

satisfies (1) with the terminal speed

$$v_t = v(\infty) = -\frac{gm}{\gamma}$$

The function (6) is the general solution for (1). This can be seen by noting that, for $n > 1$

$$\frac{d^n v(t)}{dt^n} = -\frac{\gamma}{m}\frac{d^{n-1}v(t)}{dt^{n-1}}$$

and thus $\frac{dv(0)}{dt}, \frac{d^2v(0)}{dt^2}, ...\frac{d^nv(0)}{dt^n}...$ are determined from $v(0)$. Expanding $v(t)$ in a Taylor series

$$v(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!}\frac{d^n v(0)}{dt^n}$$

we see that the solution $v(t)$ is uniquely determined once $v(0)$ is defined. Thus choosing $C = v(0) + \frac{gm}{\gamma}$ in (6) gives us the solution we want. In particular, if $v(0) = 0$, then $C = \frac{gm}{\gamma}$ and

$$v(t) = \left(e^{-\frac{\gamma}{m}t} - 1\right)\frac{gm}{\gamma}$$

In general, the drag force is usually related to the speed of the body by

$$D = \beta v^2$$

where v is the speed of the body relative to the air and β is some constant that may depend on the type of fluid, the effective cross-sectional area and the air density.

For a body falls from rest through the air,

$$\vec{F}_g + \vec{D} = m\vec{a}$$

or

$$-mg + D = ma$$

The body accelerates with initial $a = -g$ and its speed v increases from zero. Its acceleration decreases until $D = -F_g = mg$, $a = 0$, and the speed v reaches the terminal speed v_t:

$$\beta v_t^2 = mg, v_t = -\sqrt{\frac{mg}{\beta}}$$

6.3 Uniform Circular Motion

Recall that a body in uniform motion has a centripetal acceleration of constant magnitude given by

$$a = \frac{v^2}{R}$$

Let us examine two examples of uniform circular motion:

6.3.1 Rounding a curve in a car

You are sitting in the center of the rear seat of a car moving at constant speed v. When the driver suddenly turns left, rounding a corner in a circular arc, you slide across the seat toward the right and then jam against the car wall for the rest of the turn. What is going on?

When the car moves in the circular arc, it is in uniform circular motion and has an acceleration toward the center of the circle. By newton's second law, a force must cause this acceleration. Moreover, this force must also be directed toward the center of the circle. In this example, the centripetal force is the friction force on the tires from the road; it makes the turn possible.

If you are to move in uniform circular motion along with the car, there must also be a centripetal force on you. However, apparently the frictional force on you from the seat was not great enough to make you go on in a circle with the car. Thus the seat slid beneath you, until the right wall of the car jammed into you. Then its push on you provided the needed centripetal forces on you, and you joined the car's uniform circular motion.

6.3.2 Orbiting Earth

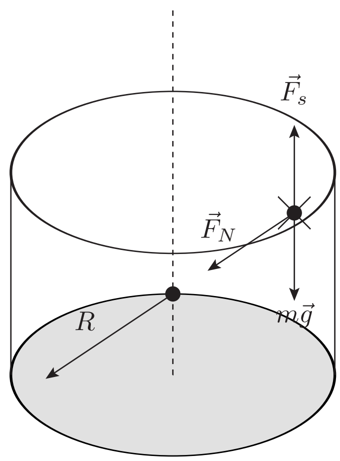
This time you are a passenger in a space shuttle. As it and you orbit Earth, you float through your cabin. What is going on?

Both you and the shuttle are in uniform circular motion and have accelerations directed toward the center of the circle. Again, the Newton's 2nd law, centripetal forces must cause the accelerations. This time the centripetal forces are gravitational pulls exerted by Earth and directed radially inward. toward the center of Earth.

In both car and shuttle you are in uniform circular motion, acted on by a centripetal force—yet your sensations in the two situations are quite different. In the car, jammed up against the wall, you are award of being compressed by the wall. In the orbiting shuttle, however, you are floating around with no sensation of any force acting on you. Why the difference?

The difference is due to the nature of the two centripetal forces. In the car, the centripetal force is the push on the part of your body touching the car wall. You can sense the compression on that part of your body. In the shuttle, the centripetal force is Earth's gravitational pull on every atom of your body. Thus there is no compression or pull on any one part of your body and no sensation of a force acting on you.

6.3.3 More Examples



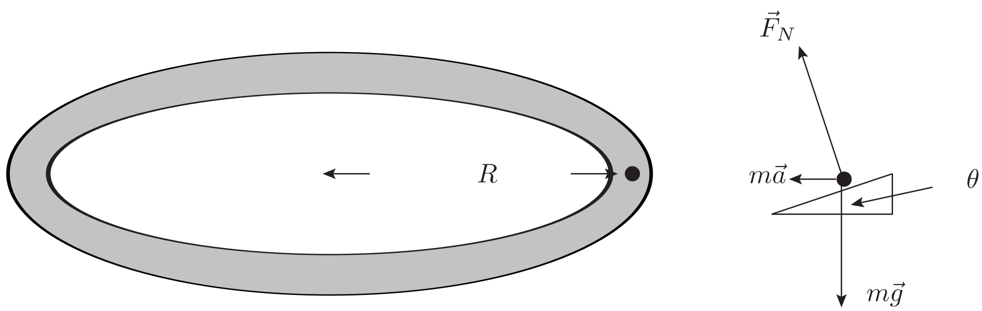
A Rotor is essentially a large hollow cylinder that is rotated rapidly around its central axis. Before the ride begins, a rider enters the cylinder through a door on the side and stands on a floor, up against a canvas-covered wall. The door is closed, and as the cylinder begins to turn, the rider, wall, and floor move in unison. When the rider's speed reaches some predetermined value, the floor abruptly and alarmingly falls away. The rider does not fall with it but instead is pinned to the wall while the cylinder rotates.

$$\vec{f}_s + m\vec{g} = 0$$

$$\left|\vec{F}_N\right| = m\frac{v^2}{R}$$

$$\mu_s \left|\vec{F}_N\right| = \left|\vec{f}_s\right| = |m\vec{g}| = mg$$

$$v^2 = \frac{gR}{\mu_s}$$



Curved portions of highway are always banked (tilted) to prevent cars from sliding off the highway. If the friction between the tires and the road surface is negligible, what bank angle θ prevents sliding?

$$m\,|\vec{a}| = m\frac{v^2}{R} = \left|\vec{F}_N\right| \sin\theta$$

$$m\,|\vec{g}| = mg = \left|\vec{F}_N\right| \cos\theta$$

The ratio of the last two equations becomes

$$\frac{v^2}{Rg} = \tan\theta$$