HW2

Chap 3

3.

Equations of motion in Laplace:

$$(s^2+3s+2)X_1(s)-(s+2)X_2(s)-sX_3(s)=0$$
$$-(s+2)X_1(s)+(2s^2+2s+2)X_2(s)-sX_3(s)=F(s)$$
$$-sX_1(s)-sX_2(s)+(2s^2+3s)X_3(s)=0$$

Equations of motion in the time domain:

$$\frac{d^2x_1}{dt^2}+3\frac{dx_1}{dt}+2x_1-\frac{dx_2}{dt}-2x_2-\frac{dx_3}{dt}=0$$
$$-\frac{dx_1}{dt}-2x_1+2\frac{d^2x_2}{dt^2}+2\frac{dx_2}{dt}+2x_2-\frac{dx_3}{dt}=f(t)$$
$$-\frac{dx_1}{dt}-\frac{dx_2}{dt}+2\frac{d^2x_3}{dt^2}+3\frac{dx_3}{dt}=0$$

Define state variables:

$$z_1=x_1 \quad \text{or} \quad x_1=z_1 \tag{1}$$
$$z_2=\frac{dx_1}{dt} \quad \text{or} \quad \frac{dx_1}{dt}=z_2 \tag{2}$$
$$z_3=x_2 \quad \text{or} \quad x_2=z_3 \tag{3}$$
$$z_4=\frac{dx_2}{dt} \quad \text{or} \quad \frac{dx_2}{dt}=z_4 \tag{4}$$
$$z_5=x_3 \quad \text{or} \quad x_3=z_5 \tag{5}$$
$$z_6=\frac{dx_3}{dt} \quad \text{or} \quad \frac{dx_3}{dt}=z_6 \tag{6}$$

Substituting Eq. (1) in (2), (3) in (4), and (5) in (6), we obtain, respectively:

$$\frac{dz_1}{dt}=z_2 \tag{7}$$
$$\frac{dz_3}{dt}=z_4 \tag{8}$$
$$\frac{dz_5}{dt}=z_6 \tag{9}$$

Substituting Eqs. (1) through (6) into the equations of motion in the time domain and solving for the derivatives of the state variables and using Eqs. (7) through (9) yields the state equations:

$$\frac{dz_1}{dt} = z_2$$
$$\frac{dz_2}{dt} = -2z_1 - 3z_2 + 2z_3 + z_4 + z_6$$
$$\frac{dz_3}{dt} = z_4$$
$$\frac{dz_4}{dt} = z_1 + \frac{1}{2}z_2 - z_3 - z_4 + \frac{1}{2}z_6 + \frac{1}{2}f(t)$$
$$\frac{dz_5}{dt} = z_6$$
$$\frac{dz_6}{dt} = \frac{1}{2}z_2 + \frac{1}{2}z_4 - \frac{3}{2}z_6$$

The output is $x_1 = z_1$.

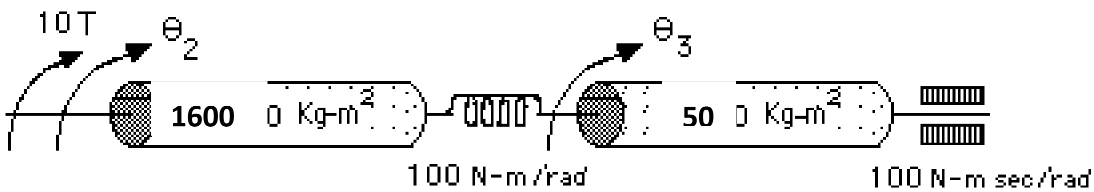
In vector-matrix form:

4 T

$$\dot{\mathbf{Z}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & -3 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0.5 & -1 & -1 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 0.5 & 0 & -1.5 \end{bmatrix} \mathbf{Z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} f(t)$$
$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{Z}$$

***請不要將 $\dot{x} = [A]x + [B]u$ 寫成 $\dot{x} = [A]x + [Bu]$

5.



Writing the equations of motion,

$$(1600s^2 + 100)\theta_2 - 100\theta_3 = 4T$$
$$-100\theta_2 + (50s^2 + 100s + 100)\theta_3 = 0$$

Taking the inverse Laplace transform and simplifying,

$$\ddot{\theta}_2 + 0.0625\theta_2 - 0.0625\theta_3 = 0.0025T$$
$$-2\theta_2 + \ddot{\theta}_3 + 2\dot{\theta}_3 + 2\theta_3 = 0$$

Defining the state variables as

$$x_1 = \theta_2,\ x_2 = \dot{\theta}_2,\ x_3 = \theta_3,\ x_4 = \dot{\theta}_3$$

Writing the state equations using the equations of motion and the definitions of the state variables

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \ddot{\theta}_2 = -0.0625\theta_2 + 0.0625\theta_3 + 0.0025T = -0.0625x_1 + 0.0625x_3 + 0.0025T \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \ddot{\theta}_3 = 2\theta_2 - 2\theta_3 - 2\dot{\theta}_3 = 2x_1 - 2x_3 - 2x_4 \\
y &= 4\theta_2 = 4x_1
\end{aligned}$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.0625 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -2 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0.0025 \\ 0 \\ 0 \end{bmatrix}T$$
$$y = \begin{bmatrix} 4 & 0 & 0 & 0 \end{bmatrix}\mathbf{x}$$

13.(a)

a. G(s)=**C**(s**I**-**A**)$^{-1}$**B**

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix};\ \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 23 \end{bmatrix};\ \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$(s\boldsymbol{I} - \boldsymbol{A})^{-1} = \frac{1}{s^3 + 2s^2 + 3s + 1}\begin{bmatrix} s^2 + 2s + 3 & s + 2 & 1 \\ -1 & s(s+2) & s \\ -s & -(3s+1) & s^2 \end{bmatrix}$$

Therefore, $G(s) = \frac{23}{s^3+2s^2+3s+1}$. Note that in this case the result could have also been obtained by inspection.

17.

Since $T_m = J_{eq}\frac{d\omega_m}{dt} + D_{eq}\omega_m$, and $T_m = K_t\, i_a$,

$$J_{eq}\frac{d\omega_m}{dt} + D_{eq}\omega_m = K_t\, i_a \qquad (1)$$

Or,

$$\frac{d\omega_m}{dt} = -\frac{D_{eq}}{J_{eq}}\omega_m + \frac{K_t}{J_{eq}} i_a$$

But, $\omega_m = \frac{N_2}{N_1}\omega_L$.

Substituting in (1) and simplifying yields the first state equation,

$$\frac{d\omega_L}{dt} = -\frac{D_{eq}}{J_{eq}}\omega_L + \frac{K_t}{J_{eq}}\frac{N_1}{N_2} i_a$$

The second state equation is:

$$\frac{d\theta_L}{dt} = \omega_L$$

Since

$$e_a = R_a i_a + L_a\frac{di_a}{dt} + K_b\omega_m = R_a i_a + L_a\frac{di_a}{dt} + K_b\frac{N_2}{N_1}\omega_L,$$

the third state equation is found by solving for $\frac{di_a}{dt}$. Hence,

$$\frac{di_a}{dt} = -\frac{K_b}{L_a}\frac{N_2}{N_1}\omega_L - \frac{R_a}{L_a} i_a + \frac{1}{L_a} e_a$$

Thus the state variables are: $x_1 = \omega_L$, $x_2 = \theta_L$, and $x_3 = i_a$.

Finally, the output is $y = \theta_m = \frac{N_2}{N_1}\theta_L$.

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{D_{eq}}{J_{eq}} & 0 & \frac{K_t}{J_{eq}}\frac{N_1}{N_2} \\ 1 & 0 & 0 \\ -\frac{K_b}{L_a}\frac{N_2}{N_1} & 0 & -\frac{R_a}{L_a} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} e_a \; ; \; y = \begin{bmatrix} 0 & \frac{N_2}{N_1} & 0 \end{bmatrix}\mathbf{x}$$

where,

$$\mathbf{x} = \begin{bmatrix} \omega_L \\ \theta_L \\ i_a \end{bmatrix}$$

Chap 4

13. (a)

$\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \dfrac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8472$ s; %OS = $e^{-\zeta\pi / \sqrt{1-\zeta^2}}$ x 100 = 28.06 %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.

16.

a. $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.517$, $\omega_n = \dfrac{4}{\zeta T_s} = 15.474$. Therefore, poles = $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

= -8 ± j13.246.

18.

a. The impedance equations are:

$$(2s^2 + s)\theta_1 - s\theta_2 = T$$
$$-s\theta_1 + (s+1)\theta_2 = 0$$

Solving for θ_2

$$\theta_2 = \frac{\begin{vmatrix} 2s^2 + s & T \\ -s & 0 \end{vmatrix}}{\begin{vmatrix} 2s^2 + s & -s \\ -s & s+1 \end{vmatrix}} = \frac{T}{2s^2 + 2s + 1}$$

So

$$\frac{\theta_2(s)}{T(s)} = \frac{\frac{1}{2}}{s^2 + s + \frac{1}{2}}$$

b. $\omega_n = \frac{1}{\sqrt{2}}$, $2\xi\omega_n = 1$ or $\xi = \frac{1}{\sqrt{2}}$. $T_s = \frac{4}{\xi\omega_n} = 8$ sec. $T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 6.28$sec and %OS = $100e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 4.32\ \%$.

22.

a. $$\frac{s+5}{s(s+1)(s^2+3s+10)}=\frac{0.5}{s}-\frac{0.5}{s+1}-\frac{1}{\sqrt{31}}\frac{\frac{\sqrt{31}}{2}}{(s+1.5)^2+7.75}$$

The amplitude of residue of the pole at -1 is larger than the amplitude of the sinusoid, so a pole-zero cancellation cannot be assumed.

b. $$\frac{s+5}{s(s+2)(s^2+4s+15)}=\frac{0.167}{s}-\frac{0.136}{s+2}-0.0303\frac{(s+2)+3.32\sqrt{11}}{(s+2)^2+11}$$

Since the amplitudes of the sinusoids are of the same order of magnitude as the residue of the pole at -2, pole-zero cancellation cannot be assumed.

c.

$$C(s)=\frac{(s+5)}{s(s+4.5)(s^2+2s+20)}=\frac{0.055}{s}-\frac{0.0036}{s+4.5}-\frac{0.052(s+1)+0.3\sqrt{19}}{(s+1)^2+19}$$

Since the amplitudes of the sinusoids much larger than the residue of the pole at -4.5, a pole-zero cancellation can be assumed. Since $2\zeta\omega_n = 2$, and $\omega_n = \sqrt{20} = 4.47211$, $\zeta = 0.224$,

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}\text{x}100 = 48.64\%,\ T_s = \frac{4}{\zeta\omega_n} = 4 \text{ sec},\ T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.72 \text{ sec};\ \omega_n T_r = 1.23,$$

therefore, $T_r = 0.275$ sec.

d.

$$C(s)=\frac{(s+5)}{s(s+4.9)(s^2+5s+20)}=\frac{0.051}{s}-\frac{0.0010}{s+4.9}-\frac{0.05(s+2.5)+0.702\sqrt{13.75}}{(s+2.5)^2+13.75}$$

Since the amplitude of the sinusoids are several orders of magnitude larger than the residue of the pole at -4.9, a pole-zero cancellation can be assumed. Since $2\zeta\omega_n = 5$, and $\omega_n = \sqrt{20} = 4.47211$, $\zeta = 00.56$,

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}\text{x}100 = 12.03\%,\ T_s = \frac{4}{\zeta\omega_n} = 0.39 \text{ sec},\ T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.847 \text{ sec},\ T_r = 0.39 \text{ sec}.$$

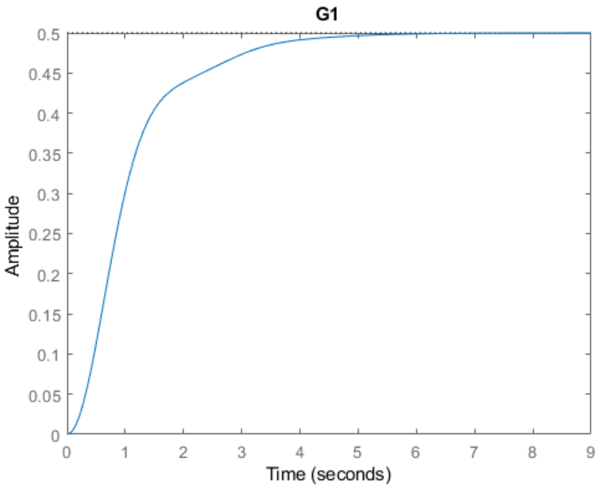
26.

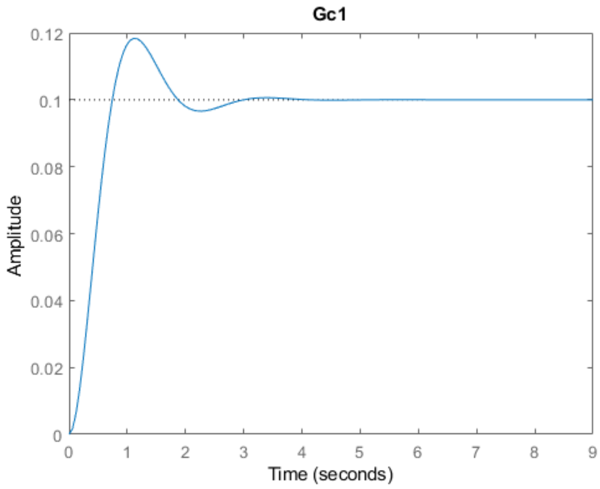
a. $$|s\boldsymbol{I}-\boldsymbol{A}| = \left|\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}\right| = \begin{vmatrix} s-3 & -2 & -1 \\ -1 & s-1 & 0 \\ -1 & -5 & s \end{vmatrix} = s^3-4s^2-4$$

b. Solving $s^3-4s^2-4=0$ gives poles at 4.2242 and -0.1121±j0.967

第 7 題(4-22)，滿多同學 c 小題寫錯，因此跑一次 matlab 給大家看結果，下次記得確實判斷再寫答案。G 開頭為原始轉移函數，Gc 為極零點對消後的轉移函數，大家可以觀察一下前後差別。

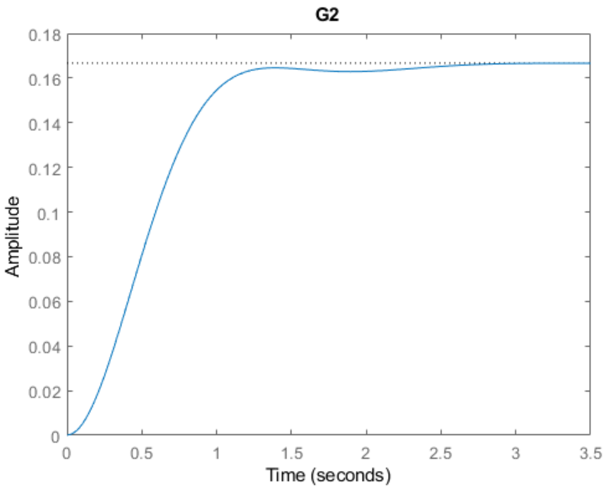
7.a.

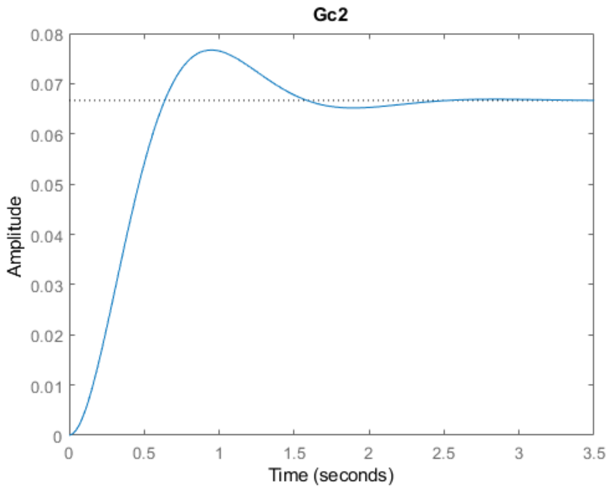





```
info_G1 = struct with fields:

       RiseTime: 2.0048
   SettlingTime: 3.8604
    SettlingMin: 0.4509
    SettlingMax: 0.5000
      Overshoot: 0
     Undershoot: 0
           Peak: 0.5000
       PeakTime: 10.5458

info_Gc1 = struct with fields:

       RiseTime: 0.5029
   SettlingTime: 2.6100
    SettlingMin: 0.0924
    SettlingMax: 0.1184
      Overshoot: 18.3964
     Undershoot: 0
           Peak: 0.1184
       PeakTime: 1.1359
```

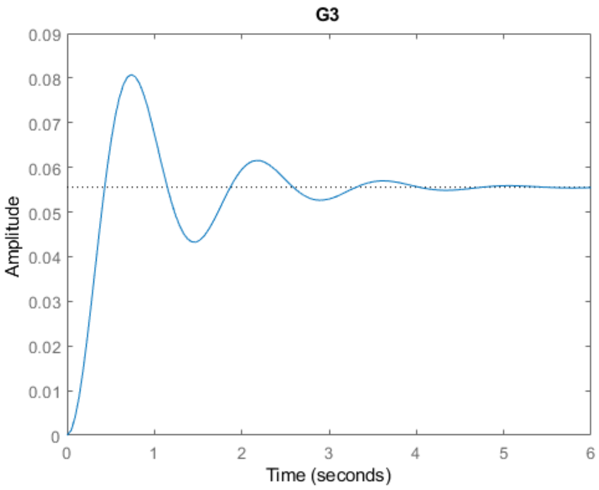
7.b.

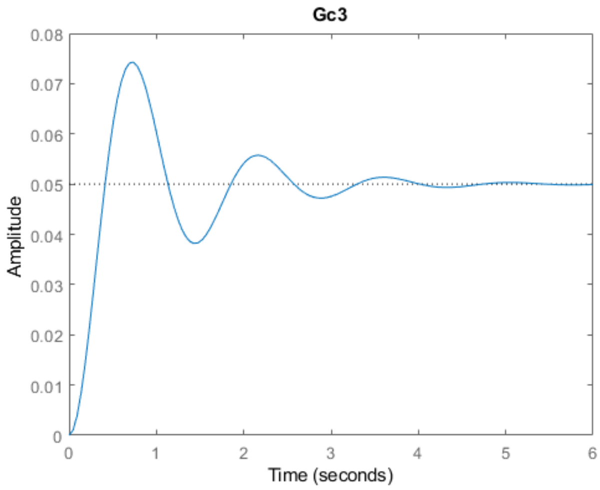





```
info_G2 = struct with fields:

        RiseTime: 0.7459
    SettlingTime: 2.1035
     SettlingMin: 0.1505
     SettlingMax: 0.1666
       Overshoot: 0
      Undershoot: 0
            Peak: 0.1666
        PeakTime: 3.2927

info_Gc2 = struct with fields:

        RiseTime: 0.4316
    SettlingTime: 2.0314
     SettlingMin: 0.0617
     SettlingMax: 0.0767
       Overshoot: 15.0389
      Undershoot: 0
            Peak: 0.0767
        PeakTime: 0.9441
```

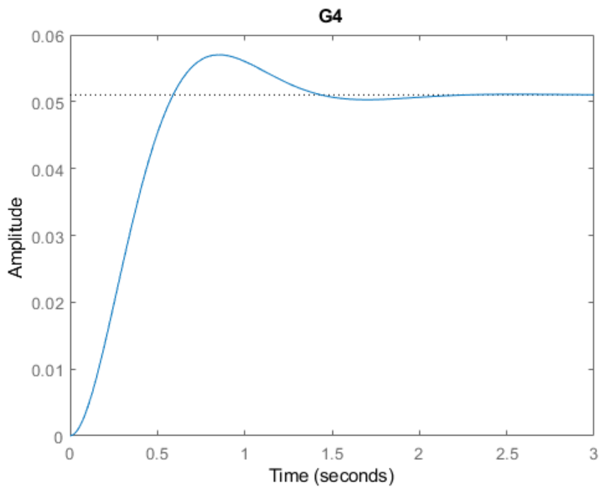
7.c.

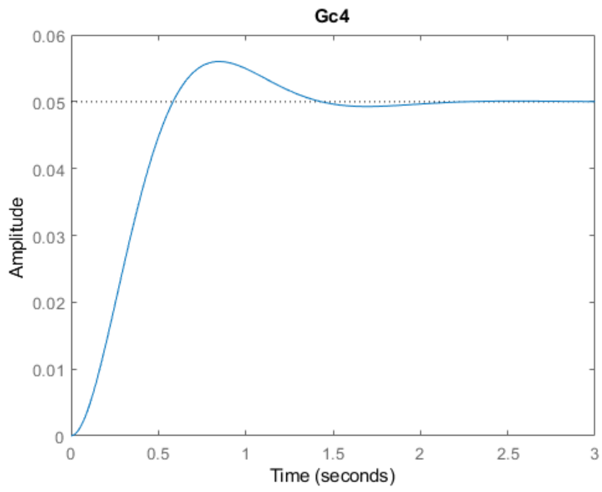





```
info_G3 = struct with fields:

        RiseTime: 0.2887
    SettlingTime: 3.7770
     SettlingMin: 0.0433
     SettlingMax: 0.0808
       Overshoot: 45.3994
      Undershoot: 0
            Peak: 0.0808
        PeakTime: 0.7368

info_Gc3 = struct with fields:

        RiseTime: 0.2767
    SettlingTime: 3.7804
     SettlingMin: 0.0382
     SettlingMax: 0.0743
       Overshoot: 48.5150
      Undershoot: 0
            Peak: 0.0743
        PeakTime: 0.7368
```

7.d.






```
info_G4 = struct with fields:

        RiseTime: 0.3974
    SettlingTime: 1.3117
     SettlingMin: 0.0464
     SettlingMax: 0.0570
       Overshoot: 11.7478
      Undershoot: 0
            Peak: 0.0570
        PeakTime: 0.8474

info_Gc4 = struct with fields:

        RiseTime: 0.3936
    SettlingTime: 1.3088
     SettlingMin: 0.0459
     SettlingMax: 0.0560
       Overshoot: 12.0265
      Undershoot: 0
           Peak: 0.0560
        PeakTime: 0.8474
```