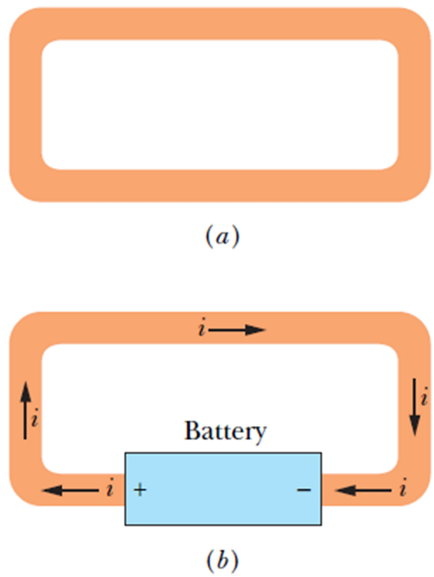
Contents

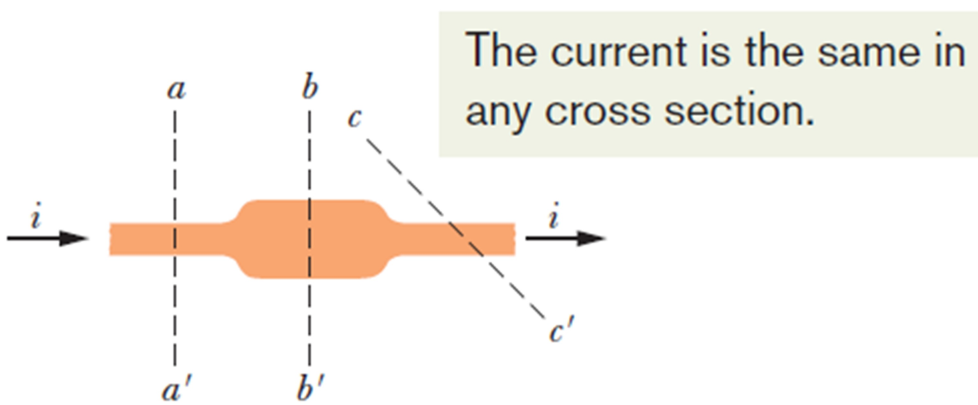
26 Current and Resistance

26.1 Electric Current

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface. Two examples are given.
1. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of $10^6 m/s$. If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second—but there is no net transport of charge and thus no current through the wire. However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a net transport of charge and thus an electric current through the wire.
2. The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second. There is no net transport of charge, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.



(a) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (b) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current i.



The figure shows a section of a conductor, part of a conducting loop in which current has been established. If charge dq passes through a hypothetical plane (such as aa') in time dt, then the current i through that plane is defined as:

$$i = \frac{dq}{dt} \qquad \text{(definition of current)}$$

The charge that passes through the plane in a time interval extending from 0 to t is:

$$q = \int dq = \int_0^t i\left(t'\right) dt'$$

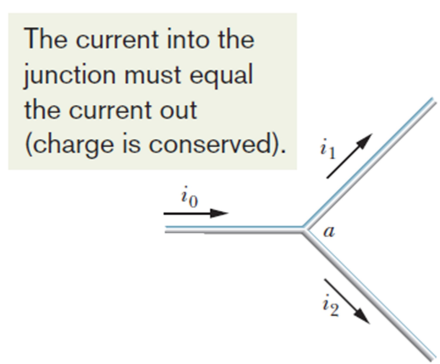
Under steady-state conditions, the current is the same for planes aa' bb', and cc' and for all planes that pass completely through the conductor, no matter what their location or orientation.

The SI unit for current is the coulomb per second, or the ampere (A):

$$1 \; ampere \; = 1A = 1 \text{ coulomb per second} = \; 1C/s$$

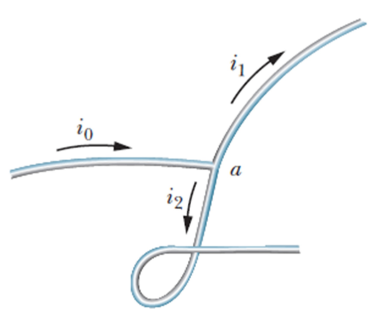
26.2 Electric Current, Conservation of Charge, and Direction of Current

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.



(a)

The relation $i_0 = i_1 + i_2$ is true at junction a no matter what the orientation in space of the three wires. Currents are scalars, not vectors.



(b)

26.3 Current Density

The magnitude of current density, J, is equal to the current per unit area through any element of cross section. It has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

$$i = \int \vec{J} \cdot d\vec{A}.$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$.

$$i = \int J dA = J \int dA = JA.$$

$$J = \frac{i}{A}$$

Here, A is the total area of the surface.

The SI unit for current density is the ampere per square meter (A/m^2).

Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call streamlines.

The current, which is toward the right, makes a transition from the wider conductor at the left to the narrower conductor at the right. Since charge is conserved during the transition, the amount of charge and thus the amount of current cannot change.

However, the current density changes—it is greater in the narrower conductor.

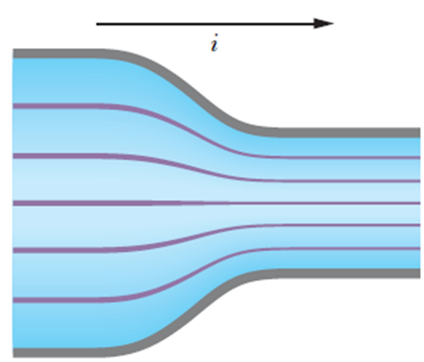


Fig. 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

26.4 Drift Speed

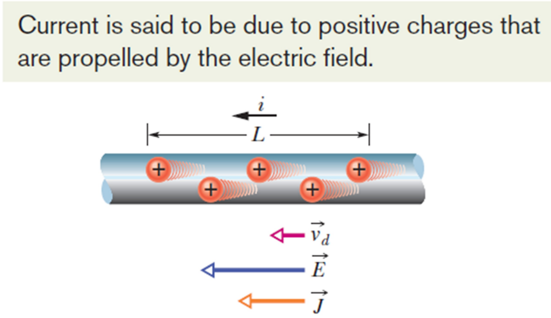


Fig. 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$. By convention, the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.

When a conductor has a current passing through it, the electrons move randomly, but they tend to drift with a drift speed $\vec{v}_d$ in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion.

In the figure, the equivalent drift of positive charge carriers is in the direction of the applied electric field, $\vec{E}$. If we assume that these charge carriers all move with the same drift speed $\vec{v}_d$ and that the current density $\vec{J}$ is uniform across the wire's cross-sectional area A, then the number of charge carriers in a length L of the wire is nAL. Here n is the number of carriers per unit volume.

The total charge of the carriers in the length L, each with charge e, is then

$$q = (nAL)\, e$$

The total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}$$

$\Rightarrow$

$$i = \frac{q}{t} = \frac{(nAL)\, e}{\frac{L}{v_d}} = nAev_d$$

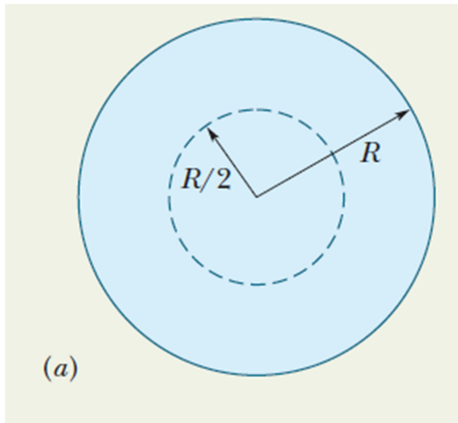
$\Rightarrow$

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$\Rightarrow$

$$\vec{J} = (ne)\, \vec{v}_d$$

26.5 Example, Current Density, Uniform and Nonuniform



(a) The current density in a cylindrical wire of radius R is uniform across a cross section of the wire and is J. What is the current through the outer portion of the wire between the radial distance $\frac{R}{2}$ and R?

We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi \left(\frac{R}{2}\right)^2 = \pi \left(\frac{3}{4}R^2\right)$$

$$i = JA'$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$. What now is the current through the same outer portion of the wire?

The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus

$$\vec{J} \cdot d\vec{A} = JdA$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = \frac{R}{2}$ and $r = R$. The simplest replacement (because J is a function of r) is the area $2\pi r dr$ of a thin ring of circumference

$2\pi r$ and width dr. We can then integrate

$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J dA = \int_{\frac{R}{2}}^{R} ar^2 2\pi r dr \\ &= 2\pi a \int_{\frac{R}{2}}^{R} r^3 dr = 2\pi a \left[\frac{r^4}{4}\right]_{\frac{R}{2}}^{R} \\ &= \frac{\pi}{2} a \left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4 \end{aligned}$$

26.5.1 In a current, the conduction electrons move very slowly

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\mu m$ when it has a uniform current $i = 17mA$? Assume that each copper atom contributes one conduction electron to the current and the current density is uniform across the wires's cross section.

Let us start with $n =$ atoms per unit volume.

$$\begin{aligned} n &= N_A \left(\frac{1}{M}\right) \rho_{mass} = \frac{(6.02 \times 10^{23}/mol)\left(8.96 \times 10^3 \frac{kg}{m^3}\right)}{63.54 \times 10^{-3} \frac{kg}{mole}} \\ &= 8.49 \times 10^{28} m^{-3} \end{aligned}$$

where $N_A = 6.02 \times 10^{23}/mol$, the number of atoms per mole is the Avogadro's number. $M = 63.54 \times 10^{-3} \frac{kg}{mole}$ is the molar mass of copper. $\rho_{mass} = 8.96 \times 10^3 \frac{kg}{m^3}$ is the mass density of copper.

Next according to

$$\frac{i}{A} = nev_d,$$

we get

$$\begin{aligned} v_d &= \frac{i}{ne\pi r^2} = \frac{17 \times 10^{-3} A}{(8.49 \times 10^{28} m^{-3})(1.6 \times 10^{-19} C)\pi (9 \times 10^{-4} m)^2} \\ &= 4.9 \times 10^{-7} m/s \end{aligned}$$

which is only $1.8mm/h$.

26.6 Resistance and Resistivity

We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i}$$

The SI unit for resistance is the volt per ampere. This has a special name, the *ohm* (symbol Ω):

$$1ohm = 1\Omega = 1 \text{ volt per ampere} = 1V/A$$

In a circuit diagram, we represent a resistor and a resistance with the symbol  .

Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega\cdot$m)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

The resistivity, ρ, of a resistor is defined as:

$$\vec{E} = \rho \vec{J}$$

The SI unit for ρ is $\Omega \cdot m$.

The conductivity σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho}$$

$$\vec{j} = \sigma \vec{E}$$

26.6.1 Calculating Resistance from Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

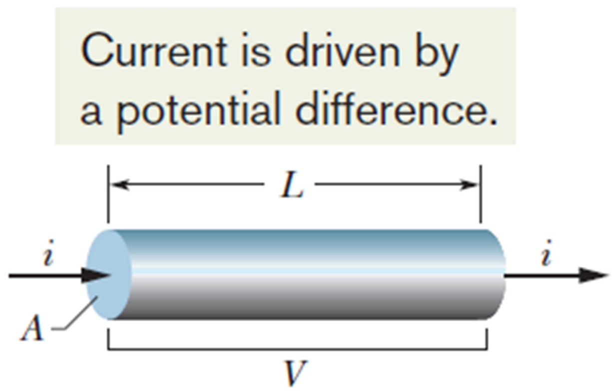


Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

If the streamlines representing the current density are uniform throughout the wire, the electric field, $\vec{E}$, and the current density, $\vec{J}$, will be constant for all points within the wire.

$$E = \frac{V}{L}$$

and

$$J = \frac{i}{A}$$

$\Longrightarrow$

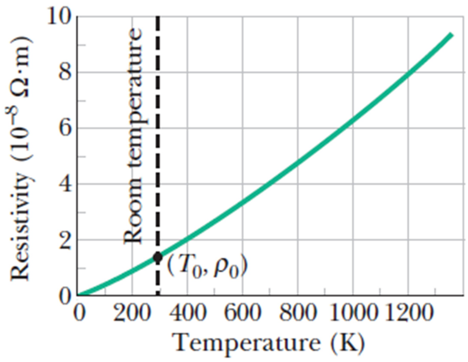
$$\rho = \frac{E}{J} = \frac{\frac{V}{L}}{\frac{i}{A}} = \frac{A}{L}\frac{V}{i}$$

$\Longrightarrow$

$$R = \frac{V}{i} = \rho\frac{L}{A}$$

26.6.2 Variation with Temperature

Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \mathrm{m}$.

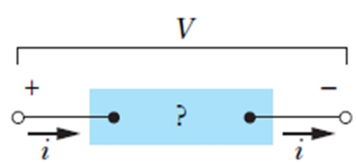


Resistivity can depend on temperature.

The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes.

$$\rho - \rho_0 = \rho_0 \alpha \left(T - T_0\right)$$

26.7 Ohm's Law



(*a*)

Ohm's law is an assertion that the current through a device is always directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

26.8 A Macroscopic View of Ohm's Law

It is often assumed that the conduction electrons in a metal move with a single effective speed v_{eff}. For copper, $v_{eff} = 1.6 \times 10^6 m/s$.

When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly—in a direction opposite that of the field—with an average drift speed v_d. The drift speed in a typical metallic conductor is about $5 \times 10^{-7} m/s$, less than the effective speed ($1.6 \times 10^6 m/s$) by many orders of magnitude.

The motion of conduction electrons in an electric field is a combination of the motion due to random collisions and that due to $\vec{E}$.

If an electron of mass m is placed in an electric field of magnitude E, the electron will experience an acceleration:

$$a = \frac{F}{m} = \frac{eE}{m}.$$

In the average time τ between collisions, the average electron will acquire a drift speed of

$$v_d = a\tau = \frac{eE\tau}{m}.$$

$$\vec{J} = ne\vec{v}_d$$

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

$$E = \left(\frac{m}{e^2 n\tau}\right) J$$

$$\rho = \frac{m}{e^2 n\tau}$$

26.8.1 Example, Mean Free Time and Mean Free Distance

(a) What is the mean free time τ between collisions for the conduction in copper?

The mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ, we find the mean free time τ from

$$\rho = \frac{m}{e^2 n\tau}$$

That equation gives us

$$\tau = \frac{m}{ne^2\rho}$$

The number of conduction electron per unit volume in copper is $8.49 \times 10^{28} m^{-3}$. For copper, $\rho = 1.69 \times 10^{-8} \Omega \cdot m$. Thus

$$\begin{aligned}\tau &= \frac{9.1 \times 10^{-31} kg}{(8.49 \times 10^{28} m^{-3})\,(1.6 \times 10^{-19} C)^2\,(1.69 \times 10^{-8} \Omega \cdot m)} \\ &= 2.5 \times 10^{-14} s.\end{aligned}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. What is λ for the conduction electrons in copper, assuming that their effective speed $v_{eff} = 1.6 \times 10^6 m/s$?

The distance d any particle travels in a certain time t at a constant speed v is $d = vt$.

For the electrons in copper, this gives us

$$\begin{aligned}\lambda &= v_{eff}\tau = \left(1.6 \times 10^6 m/s\right)\left(2.5 \times 10^{-14} s\right) \\ &= 4.0 \times 10^{-8} m = 40 nm\end{aligned}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electrons passes many copper atoms before finally hitting one.

26.9 Power in Electric Circuits

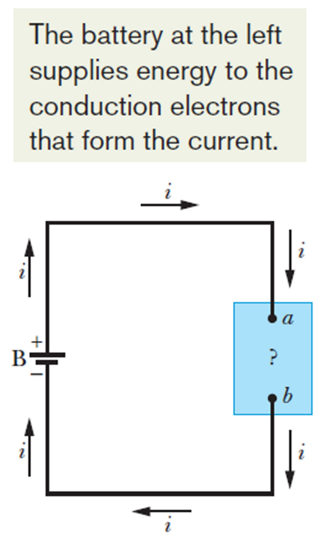


Fig. 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

In the figure, there is an external conducting path between the two terminals of the battery. A steady current i is produced in the circuit, directed from terminal a to terminal b. The amount of charge dq that moves between those terminals in time interval dt is equal to idt.

This charge dq moves through a decrease in potential of magnitude V, and thus its electric potential energy decreases in magnitude by the amount

$$dU = dqV = idtV$$

The power P associated with that transfer is the rate of transfer $\frac{dU}{dt}$, given by

$$P = iV$$
$$P = i^2 R$$
$$P = \frac{V^2}{R}$$

The unit of power is the volt-ampere (VA).

$$1VA = 1\left(\frac{J}{C}\right)\left(\frac{C}{s}\right) = 1\frac{J}{s} = 1W$$