Problem 4.3.4

For a constant parameter $a > 0$, a Rayleigh random variable X has PDF

$$f_X(x) = \begin{cases} a^2 x e^{-a^2 x^2/2} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

What is the CDF of X?

Problem 4.4.7

The cumulative distribution function of random variable U is

$$F_U(u) = \begin{cases} 0 & u < -5, \\ \frac{u+5}{8} & -5 \le u < -3, \\ \frac{1}{4} & -3 \le u < 3, \\ \frac{3u-7}{8} & 3 \le u < 5, \\ 1 & u \ge 5. \end{cases}$$

(a) What are E[U] and Var[U]?

(b) What is E[2^U]?

Problem 4.5.12

X is a uniform random variable with expected value $\mu_X = 7$ and variance $\text{Var}[X] = 3$. What is the PDF of X?

Problem 4.6.14

At time $t = 0$, the price of a stock is a constant k dollars. At time $t > 0$ the price of a stock is a Gaussian random variable X with $\mathrm{E}[X] = k$ and $\mathrm{Var}[X] = t$. At time t, a *Call Option at Strike* k has value $V = (X - k)^+$, where the operator $(\cdot)^+$ is defined as $(z)^+ = \max(z, 0)$.

(a) Find the expected value $\mathrm{E}[V]$.

(b) Suppose you can buy the call option for d dollars at time $t = 0$. At time t, you can sell the call for V dollars and earn a profit (or loss perhaps) of $R = V - d$ dollars. Let d_0 denote the value of d such that $\mathrm{P}[R > 0] = 1/2$. Your strategy is that you buy the option if $d \leq d_0$ so that your probability of a profit is $\mathrm{P}[R > 0] \geq 1/2$. Find d_0.

(c) Let d_1 denote the value of d such that $\mathrm{E}[R] = 0.01 \times d$. Now your strategy is to buy the option if $d \leq d_1$ so that your expected return is at least one percent of the option cost. Find d_1.

(d) Are the strategies "Buy the option if $d \leq d_0$" and "Buy the option if $d \leq d_1$" reasonable strategies?

Problem 4.7.6

When you make a phone call, the line is busy with probability 0.2 and no one answers with probability 0.3. The random variable X describes the conversation time (in minutes) of a phone call that is answered. X is an exponential random variable with $\mathrm{E}[X] = 3$ minutes. Let the random variable W denote the conversation time (in seconds) of all calls ($W = 0$ when the line is busy or there is no answer.)

(a) What is $F_W(w)$?

(b) What is $f_W(w)$?

(c) What are $\mathrm{E}[W]$ and $\mathrm{Var}[W]$?