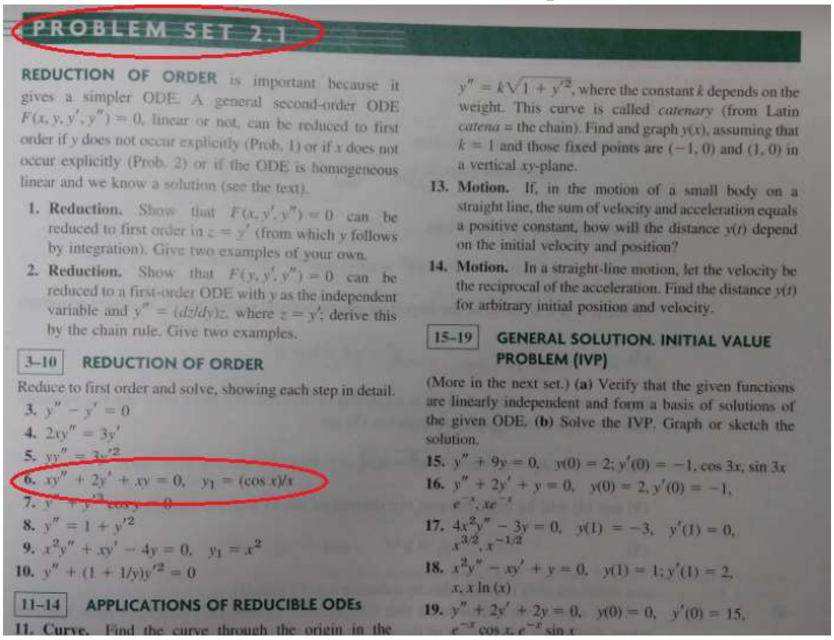
106 學年度 工程數學（一）作業題號

Chapter1 Review question	18，20，22，28	Sec 6.1	4,30,40
		Sec 6.2	8,24
		Sec 6.3	5,20
		Sec 6.4	12
		Sec 6.5	6,22
		Sec 6.6	6,20
		Sec 6.7	19
Sec 2.1	6	Sec 7.1	5,9
Sec 2.2	6,23	Sec 7.2	14
Sec 2.3	12	Sec 7.3	18
Sec 2.4	7,19	Sec 7.4	10,22
Sec 2.5	14	Sec 7.7	12,24
Sec 2.6	6,14	Sec 7.8	6
Sec 2.7	11		
Sec 2.8	4,10,18		
Sec 2.9	18		
Sec 2.10	10		
Sec 3.1	13		
Sec 3.2	2,8		
Sec 3.3	1,10		
Sec 4.1	4,12		
Sec 4.3	12,18		
Sec 4.4	9,12		
Sec 4.5	5,10		
Sec 4.6	3,20		
Sec 5.1	12		
Sec 5.2	2		
Sec 5.3	3,12		
Sec 5.4	7,22		
Sec 5.5	9		

注意事項：

*作業量會比第一章出題方式增加許多，請同學掌握時間

*以各章節作單位出題，非個章 review question



PROBLEM SET 2.1

REDUCTION OF ORDER is important because it gives a simpler ODE. A general second-order ODE $F(x, y, y', y'') = 0$, linear or not, can be reduced to first order if y does not occur explicitly (Prob. 1) or if x does not occur explicitly (Prob. 2) or if the ODE is homogeneous linear and we know a solution (see the text).

1. **Reduction.** Show that $F(x, y', y'') = 0$ can be reduced to first order in $z = y'$ (from which y follows by integration). Give two examples of your own.
2. **Reduction.** Show that $F(y, y', y'') = 0$ can be reduced to a first-order ODE with y as the independent variable and $y'' = (dz/dy)z$, where $z = y'$; derive this by the chain rule. Give two examples.

3–10 REDUCTION OF ORDER

Reduce to first order and solve, showing each step in detail.

3. $y'' - y' = 0$
4. $2xy'' = 3y'$
5. $yy'' = 3y'^2$
6. $xy'' + 2y' + xy = 0, \quad y_1 = (\cos x)/x$
7. $y'' + y'^3 \cos y = 0$
8. $y'' = 1 + y'^2$
9. $x^2y'' + xy' - 4y = 0, \quad y_1 = x^2$
10. $y'' + (1 + 1/y)y'^2 = 0$

11–14 APPLICATIONS OF REDUCIBLE ODEs

11. **Curve.** Find the curve through the origin in the

$y'' = k\sqrt{1 + y'^2}$, where the constant k depends on the weight. This curve is called *catenary* (from Latin *catena* = the chain). Find and graph $y(x)$, assuming that $k = 1$ and those fixed points are $(-1, 0)$ and $(1, 0)$ in a vertical xy-plane.

13. **Motion.** If, in the motion of a small body on a straight line, the sum of velocity and acceleration equals a positive constant, how will the distance $y(t)$ depend on the initial velocity and position?
14. **Motion.** In a straight-line motion, let the velocity be the reciprocal of the acceleration. Find the distance $y(t)$ for arbitrary initial position and velocity.

15–19 GENERAL SOLUTION. INITIAL VALUE PROBLEM (IVP)

(More in the next set.) **(a)** Verify that the given functions are linearly independent and form a basis of solutions of the given ODE. **(b)** Solve the IVP. Graph or sketch the solution.

15. $y'' + 9y = 0, \quad y(0) = 2; y'(0) = -1, \cos 3x, \sin 3x$
16. $y'' + 2y' + y = 0, \quad y(0) = 2, y'(0) = -1, e^{-x}, xe^{-x}$
17. $4x^2y'' - 3y = 0, \quad y(1) = -3, \quad y'(1) = 0, x^{3/2}, x^{-1/2}$
18. $x^2y'' - xy' + y = 0, \quad y(1) = 1; y'(1) = 2, x, x \ln (x)$
19. $y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 15, e^{-x} \cos x, e^{-x} \sin x$

example：Sec 2.1　第 6 題

*僅適用第十版工程數學課本：Erwin Kreyszig - Advanced Engineering Mathematics 10th Edition