Problem 5.2.4

For two independent flips of a fair coin, let X equal the total number of tails and let Y equal the number of heads on the last flip. Find the joint PMF $P_{X,Y}(x,y)$.

Problem 5.3.6

Random variables N and K have the joint PMF

$$P_{N,K}(n,k) = \begin{cases} \frac{100^n e^{-100}}{(n+1)!} & k=0,1,\ldots,n; \\ & n=0,1,\ldots \\ 0 & \text{otherwise.} \end{cases}$$

Find the marginal PMF $P_N(n)$. Show that the marginal PMF $P_K(k)$ satisfies $P_K(k) = \mathrm{P}[N > k]/100$.

Problem 5.6.3

Observe 100 independent flips of a fair coin. Let X equal the number of heads in the first 75 flips. Let Y equal the number of heads in the remaining 25 flips. Find $P_X(x)$ and $P_Y(y)$. Are X and Y independent? Find $P_{X,Y}(x,y)$.

Problem 5.9.4

X_1 and X_2 are identically distributed Gaussian $(0, 1)$ random variables. Moreover, they are jointly Gaussian. Under what conditions are X_1, X_2 and $X_1 + X_2$ identically distributed?

Problem 5.10.9

Random variables $X_1, X_2, \ldots, X_n$ are iid; each X_j has CDF $F_X(x)$ and PDF $f_X(x)$. Consider

$$L_n = \min(X_1, \ldots, X_n)$$
$$U_n = \max(X_1, \ldots, X_n).$$

In terms of $F_X(x)$ and/or $f_X(x)$:

(a) Find the CDF $F_{U_n}(u)$.

(b) Find the CDF $F_{L_n}(l)$.

(c) Find the joint CDF $F_{L_n,U_n}(l,u)$.