Contents

8 Conservation of Energy

One general type of energy is **potential energy** U, which can be associated with the configuration.

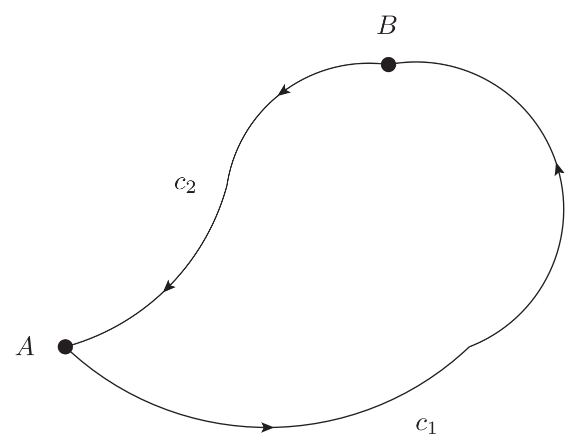
8.1 Path Independence and Conservative Forces

In Chapter 7, we discussed the relation between work and a change in kinetic energy. The work-kinetic energy theorem says that

$$\Delta K = \Delta W$$

Here, we discuss the relation between work and a change in potential energy. A force field $\vec{F}(x, y, z)$ is called conservative if *the net work* $\Delta W\left(\vec{A} \to \vec{B}\right)$ *done on a particle in moving between any two points* $\vec{A}$ *to* $\vec{B}$ *does not depend on the path from* $\vec{A}$ *to* $\vec{B}$ *taken by the particle.* Otherwise, $\vec{F}$ is called non-conservative.

A necessary and sufficient condition for a force field to be conservative is: *The net work done by a conservative force on a particle moving around any closed path is zero.*



To show that the necessary condition follows, for the closed path in the above, the work done can be split into two parts: one along the path c_1 and the other along c_2.

$$\Delta W\left(\vec{A} \to \vec{A}\right) = \Delta W\left(\vec{A} \to \vec{B}\right)_{c_1} + \Delta W\left(\vec{B} \to \vec{A}\right)_{c_2}$$

Since reversing the path taken by the particle between any two prescribed points also reverses the work done on the particle, we have

$$\Delta W\left(\vec{B} \to \vec{A}\right)_{c_2} = -\Delta W\left(\vec{A} \to \vec{B}\right)_{c_2}$$

Thus

$$\Delta W\left(\vec{A} \to \vec{A}\right) = \Delta W\left(\vec{A} \to \vec{B}\right)_{c_1} - \Delta W\left(\vec{A} \to \vec{B}\right)_{c_2} = 0$$

where in the last step, we have utilized the property that $\vec{F}$ is conservative and $\Delta W\left(\vec{A} \to \vec{B}\right)$ is path independent. On the other hand, suppose $\Delta W\left(\vec{A} \to \vec{A}\right) = 0$ for any closed path, and c_1 and c_2 are two paths from $\vec{A}$ to $\vec{B}$. Then

$$\begin{aligned}
&\Delta W\left(\vec{A} \to \vec{B}\right)_{c_1} - \Delta W\left(\vec{A} \to \vec{B}\right)_{c_2} \\
&= \Delta W\left(\vec{A} \to \vec{B}\right)_{c_1} + \Delta W\left(\vec{B} \to \vec{A}\right)_{c_2} = \Delta W\left(\vec{A} \to \vec{A}\right) = 0
\end{aligned}$$

Thus, the path independent condition $\Delta W\left(\vec{A} \to \vec{B}\right)_{c_1} = \Delta W\left(\vec{A} \to \vec{B}\right)_{c_2}$ can be derived from the condition of vanishing $\Delta W\left(\vec{A} \to \vec{A}\right)$ for any closed path.

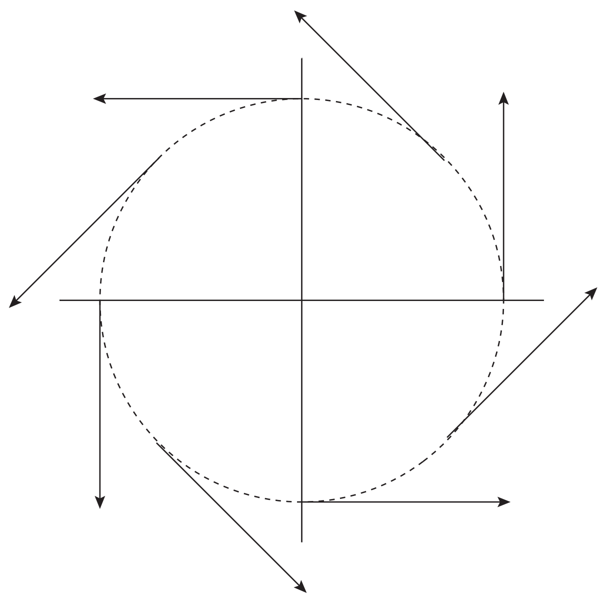
8.1.1 Counter Examples

i. friction force The friction force on an object is always opposite to the direction of the motion of the object and the work done on the object by friction is always negative. Thus friction force is non-conservative.

ii. force that is always tangent to a circle centered at the origin. The force field is

$$\vec{F} = -y\hat{\imath} + x\hat{\jmath} = r\hat{e}_\theta. \tag{1}$$

Consider the closed path on the unit circle:



The force (1) at every point on the unit circle is tangent to the circle. Thus, the work done in moving the particle around the unit circle is 2π and this force is not conservative.

8.2 Work and Potential Energy

For a particle in a conservative force $\vec{F}$, the work done in moving the particle depends only on the change of the position of the particle. Therefore we may associate a change of potential energy ΔU that depends only on the change of position. In particular, we want to find a potential energy $U(\vec{x})$ such that the change of potential energy $\Delta U\left(\vec{A} \to \vec{B}\right) = U\left(\vec{B}\right) - U\left(\vec{A}\right)$ to be the negative of the work $\Delta W\left(\vec{A} \to \vec{B}\right)$ done by $\vec{F}$.

$$U\left(\vec{B}\right) - U\left(\vec{A}\right) = \Delta U\left(\vec{A} \to \vec{B}\right) = -\Delta W\left(\vec{A} \to \vec{B}\right)$$

Replace $\vec{B}$ in the above by $\vec{x}$, we then get

$$U(\vec{x}) = -\Delta W\left(\vec{A} \to \vec{x}\right) + U\left(\vec{A}\right) \tag{2}$$

For fixed $\vec{A}$, the potential U so defined is a function of position $\vec{x}$ because $\Delta W\left(\vec{A} \to \vec{x}\right)$ depends only on $\vec{x}$ and not on the path taken from $\vec{A}$ to $\vec{x}$. If we had chosen another point $\vec{A}'$ as the starting point for defining the potential energy U', we then have

$$\begin{aligned}
U'(\vec{x}) &= -\Delta W\left(\vec{A}' \rightarrow \vec{x}\right) + U'\left(\vec{A}'\right) \\
&= -\Delta W\left(\vec{A} \rightarrow \vec{x}\right) + U'\left(\vec{A}'\right) - \Delta W\left(\vec{A}' \rightarrow \vec{A}\right) \\
&= -\Delta W\left(\vec{A} \rightarrow \vec{x}\right) + U\left(\vec{A}\right) + \left(U'\left(\vec{A}'\right) - U\left(\vec{A}\right) - \Delta W\left(\vec{A}' \rightarrow \vec{A}\right)\right) \\
&= U(\vec{x}) + \left(U'\left(\vec{A}'\right) - U\left(\vec{A}\right) - \Delta W\left(\vec{A}' \rightarrow \vec{A}\right)\right)
\end{aligned}$$

$U'(\vec{x})$ differs from $U(\vec{x})$ by a constant $C = \left(U'\left(\vec{A}'\right) - U\left(\vec{A}\right) - \Delta W\left(\vec{A}' \rightarrow \vec{A}\right)\right)$. In fact, it is straightforward to see that the changes are the same for two potential functions differing from each other only by a constant.

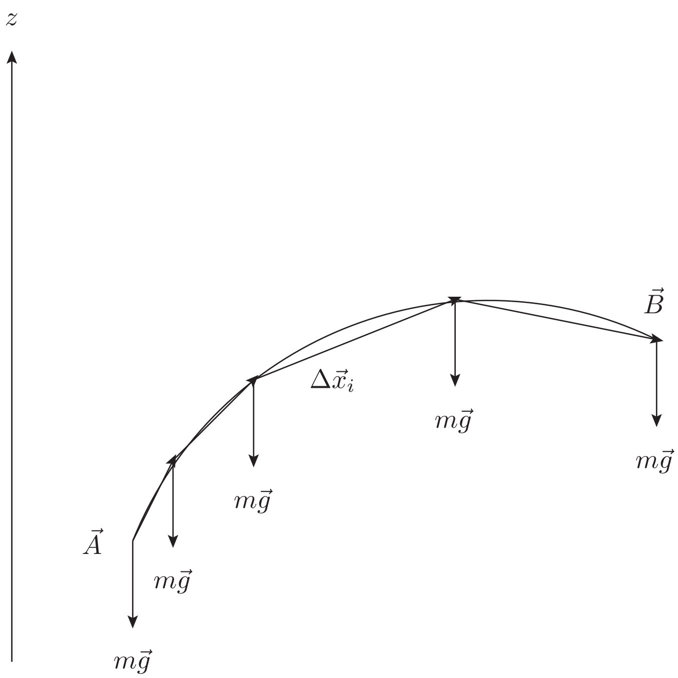
$$\Delta U(\vec{x}) = \Delta\left(U(\vec{x}) + C\right)$$

8.3 Determining Potential Energy Values

8.3.1 Gravitational Potential Energy

Assume positive z-axis points in the vertical direction. The gravitational force field is a constant vector field with

$$\vec{F}(x, y, z) = m\vec{g} = -mg\hat{k}$$



The work ΔW_i done in moving the point particle of mass m from $\vec{x}_i = (x_i, y_i, z_i)$ to $\vec{x}_i + \Delta\vec{x}_i = (x_i + \Delta x_i, y_i + \Delta y_i, z_i + \Delta z_i)$ is equal to

$$\Delta W_i = m\vec{g} \cdot \Delta\vec{x}_i = -mg\Delta z_i$$

The net work ΔW in moving the particle from $\vec{A} = (x_A, y_A, z_A)$ to $\vec{B} = (x_B, y_B, z_B)$ is

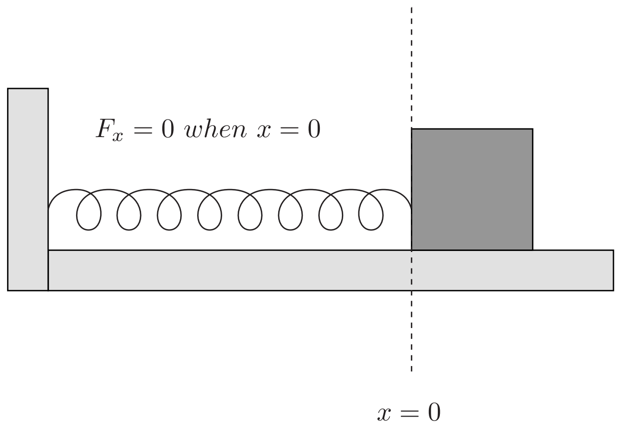
$$\begin{aligned} \Delta W\left(\vec{A} \to \vec{B}\right) &= \sum_i \Delta W_i = -mg \sum_i \Delta z_i = -mg\Delta z \\ &= -mg\left(z_B - z_A\right) \end{aligned}$$

$\Delta W\left(\vec{A} \to \vec{B}\right)$ depends only on the z coordinates of $\vec{A}$ to $\vec{B}$ and is independent of the path. Thus $m\vec{g}$ is a conservative force field and by (2) the potential $U(x, y, z) = U(\vec{x})$ is

$$\begin{aligned} U(\vec{x}) &= -\Delta W\left(\vec{A} \to \vec{x}\right) + U\left(\vec{A}\right) = mg\left(z - z_A\right) + U\left(\vec{A}\right) \\ &= mgz + C \end{aligned}$$

8.3.2 Elastic Potential Energy

We next consider the one dimensional block-spring system.



As the block moves from point x_A to x_B, the spring force $F_x = -kx$ does work on the block. The spring force depends only on the x position of the block which is constrained to move on the x-axis. The spring force F_x must be conservative (why?) and its potential is given by

$$U(x) = -\Delta W(x_A \to x) + U_A$$

Now the work

$$\Delta W(x_A \to x) = \int_{x_A}^{x} dW = \int_{x_A}^{x} F_x dx = -k \int_{x_A}^{x} x dx = -\frac{1}{2}kx^2 + \frac{1}{2}kx_A^2$$

Thus

$$U(x) = \frac{1}{2}kx^2 + C$$

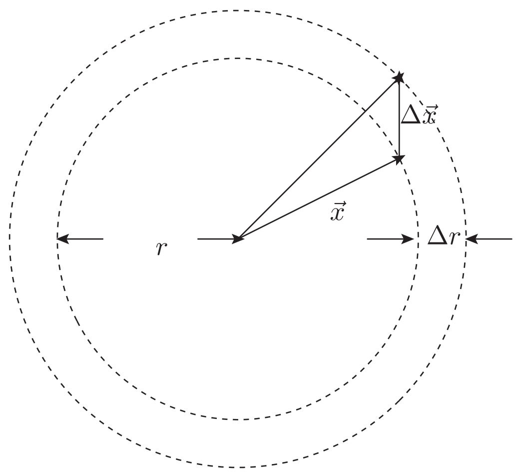
In practice, it is often convenient to choose $C = 0$.

8.3.3 Spherically Symmetric Central Force

The magnitude of force $\left|\vec{F}\right|$ is constant on a spherical surface centered at the origin. In addition, the direction of $\vec{F}$ is in the radial direction.

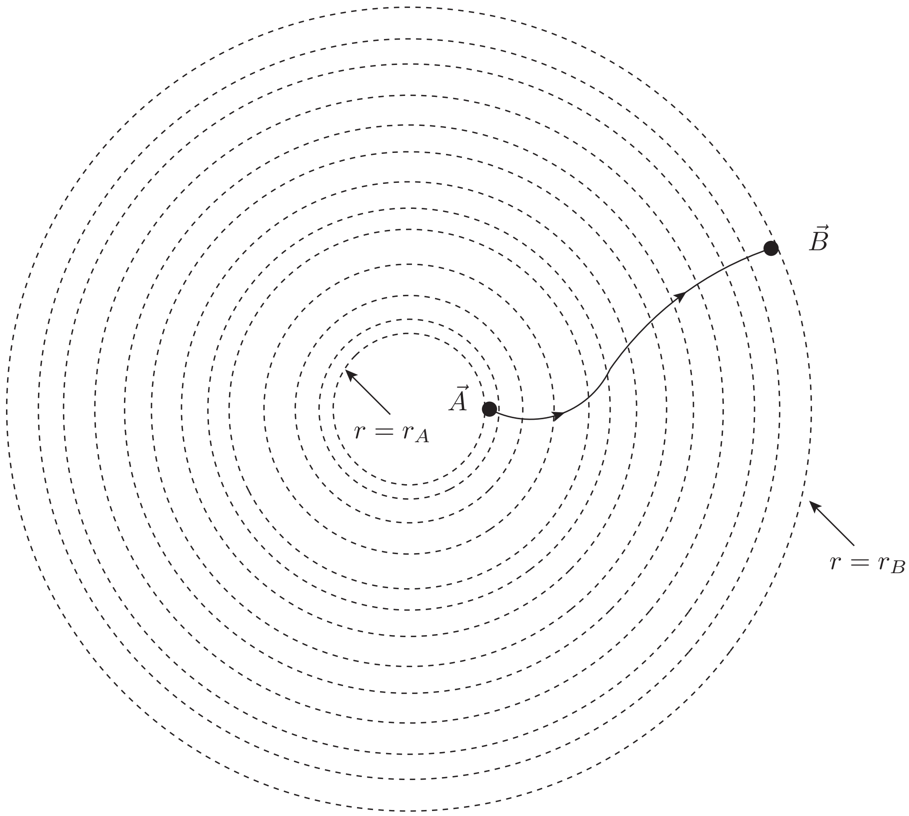
$$\vec{F}(\vec{x}) = f(r)\hat{r}$$

where $r = |\vec{x}|$ is the radial distance of the point $\vec{x}$ from the origin and $\hat{r} = \frac{\vec{x}}{r}$ is the unit vector in the radial direction. We shall show immediately that this $\vec{F}$ is conservative. To begin with, consider the work ΔW done for an infinitesimal displacement from $\vec{x}$ to $\vec{x} + \Delta\vec{x}$



$$\begin{aligned}
\Delta W = \vec{F} \cdot \Delta\vec{x} = f(r)\, \hat{r} \cdot \Delta\vec{x} &= \frac{f(r)}{r} \vec{x} \cdot \Delta\vec{x} \\
&\simeq \frac{f(r)}{2r} \left((\vec{x} + \Delta\vec{x}) \cdot (\vec{x} + \Delta\vec{x}) - \vec{x} \cdot \vec{x} \right) \\
&= \frac{f(r)}{2r} \Delta \left(\vec{x} \cdot \vec{x} \right) = \frac{f(r)}{2r} \Delta \left(|\vec{x}|^2 \right) \\
&= \frac{f(r)}{2r} \Delta \left(r^2 \right) = f(r)\, \Delta r
\end{aligned}$$

Thus for a path from $\vec{A}$ to $\vec{B}$ as shown below:



$$\Delta W\left(\vec{A} \to \vec{B}\right) = \sum_i \Delta W_i = \int_{r_A=|\vec{x}_A|}^{r_B=|\vec{x}_B|} f(r)\, dr$$

The work done depends only on the radial distances $r_A = |\vec{x}_A|$ and $r_B = |\vec{x}_B|$. Thus $\vec{F}$ is conservative and the potential energy is

$$U(x, y, z) = U(r) = -\int f(r)\, dr$$

$$f(r) = -\frac{dU(r)}{dr}$$

8.4 Determining Force from Potential Energy

A conservative force $\vec{F}(\vec{x})$ field allows us to find the potential with (2). It is also possible to find the force $\vec{F}$ from a known potential $U(\vec{x})$.

8.4.1 One-dimensional $U(x)$

Since $dU = -dW = -F_x dx$, we have

$$F_x = -\frac{dU(x)}{dx}$$

8.4.2 Three-dimensional $U(\vec{x})$

Knowing that $\Delta U = -\Delta W = -\vec{F} \cdot \Delta\vec{x}$, we have

$$\vec{F} \cdot \Delta\vec{x} = -\Delta U = -\left(U(\vec{x} + \Delta\vec{x}) - U(\vec{x})\right)$$

Let us assume that $\Delta\vec{x} = (\Delta x, 0, 0)$ only points in the x direction. Then

$$\vec{F} \cdot \Delta\vec{x} = F_x \Delta x = -\left(U(x + \Delta x, y, z) - U(x, y, z)\right)$$

or express in terms of **partial derivative**:

$$F_x = -\lim_{\Delta x \to 0} \frac{U(x + \Delta x, y, z) - U(x, y, z)}{\Delta x} = -\frac{\partial U(x, y, z)}{\partial x}$$

where, by treating y and z as parameters and considering $U(x, y, z)$ as a function of x, $\frac{\partial U(x,y,z)}{\partial x}$ is just like an ordinary derivative with respect to the variable x. Similarly, choosing $\Delta\vec{x} = (0, \Delta y, 0)$ and $\Delta\vec{x} = (0, 0, \Delta z)$ give us

$$F_y = -\lim_{\Delta y \to 0} \frac{U(x, y + \Delta y, z) - U(x, y, z)}{\Delta y} = -\frac{\partial U(x, y, z)}{\partial y}$$

and

$$F_z = -\lim_{\Delta z \to 0} \frac{U(x, y, z + \Delta z) - U(x, y, z)}{\Delta z} = -\frac{\partial U(x, y, z)}{\partial z}$$

Define the bookkeeping notation: gradient $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$. The force $\vec{F}$ can be written as

$$\begin{aligned}
\vec{F} &= F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k} \\
&= -\frac{\partial U(x, y, z)}{\partial x}\hat{\imath} - \frac{\partial U(x, y, z)}{\partial y}\hat{\jmath} - \frac{\partial U(x, y, z)}{\partial z}\hat{k} \\
&= -\vec{\nabla} U(x, y, z)
\end{aligned}$$

$\vec{F} = -\vec{\nabla} U$ in three dimensions is the counter part of $F_x = -\frac{dU}{dx}$ in one dimension.

As an example, if $U = x^2 + y^2 + z^2$, then

$$F_x = -\frac{\partial}{\partial x}\left(x^2 + y^2 + z^2\right) = -\frac{\partial}{\partial x}\left(x^2\right) = -\frac{d}{dx}\left(x^2\right) = -2x$$

Similarly

$$\begin{aligned} F_y &= -\frac{\partial}{\partial y}\left(x^2 + y^2 + z^2\right) = -\frac{\partial}{\partial y}\left(y^2\right) = -2y, \\ F_z &= -\frac{\partial}{\partial z}\left(x^2 + y^2 + z^2\right) = -2z \end{aligned}$$

and

$$\vec{F} = -2\vec{x}$$

Let us consider another example of $U = xyz^2$.

$$\begin{aligned} F_x &= -\frac{\partial}{\partial x}\left(xyz^2\right) = -yz^2\frac{\partial}{\partial x}\left(x\right) = -yz^2 \\ F_y &= -\frac{\partial}{\partial y}\left(xyz^2\right) = -xz^2\frac{\partial}{\partial y}\left(y\right) = -xz^2 \\ F_z &= -\frac{\partial}{\partial z}\left(xyz^2\right) = -xy\frac{\partial}{\partial x}\left(z^2\right) = -2xyz \\ \vec{F} &= -yz^2\hat{\imath} - xz^2\hat{\jmath} - 2xyz\hat{k} \end{aligned}$$

8.5 Conservation of Mechanical Energy

We have defined the potential energy so that

$$\Delta U = -\Delta W$$

By work-kinetic energy theorem, we also have

$$\Delta K = \Delta W$$

Thus

$$\Delta\left(K + U\right) = 0$$

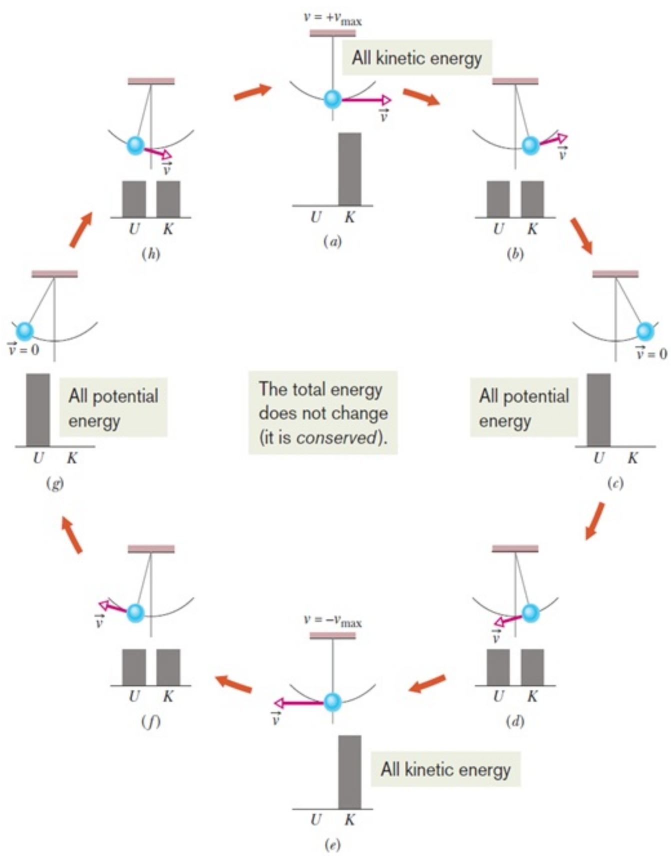
and the mechanical energy E_{mec} defined as the sum of the potential energy U and the kinetic energy K,

$$E_{mec} = K + U \tag{3}$$

is a constant of motion. In other words, *in an isolated system where only conservative forces cause energy changes, the kinetic energy and the potential energy can change, but their sum the mechanical energy E_{mec} of the system, cannot change.* This result is called the **principle of conservation of mechanical energy**.

The principle of conservation of mechanical energy allows us to solve problems that would be quite difficult to solve using Newton's laws: When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion and without finding the work done by the forces involved.*

An example in which the principle of conservation of mechanical energy can be applied is a pendulum. As a pendulum swings, the energy of the pendulum-Earth system is transferred back and forth between kinetic energy K and gravitational potential energy U, with the sum $K + U$ being constant.

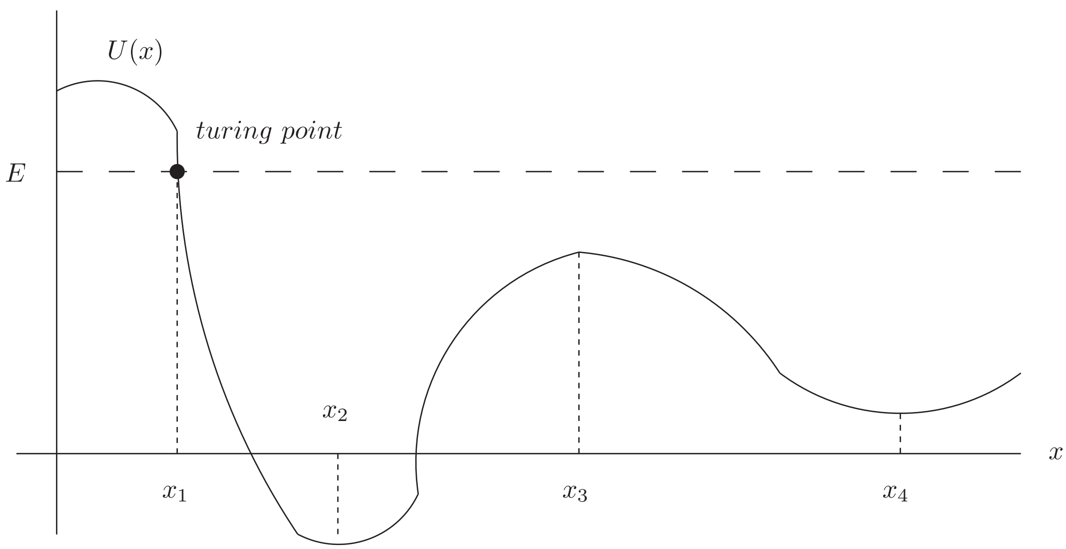


If we know the gravitational energy when the pendulum bob is at its highest point, (3) gives us the kinetic energy of the bob at its lowest point. For example, let us choose the lowest point as the reference point, with the potential energy $U_2 = 0$. Suppose then the potential energy at the highest point is U_1 relative to the reference point, Because the bob momentarily stops at its highest point, the kinetic energy there is $K_1 = 0$. The kinetic energy K_2 at the lowest point, by (3), satisfies

$$K_2 = K_1 + U_1 - U_2 = U_1$$

Note we get the above result without considering the motion between the highest and the lowest points and without finding the work done by any forces involved in the motion.

8.6 Reading a Potential Energy Curve



8.6.1 Turning Points

Turing points are the points at which the point particle reverses the direction of its velocity. Thus the velocity v or kinetic energy K must vanish at a turning point and the intersections of the potential curve with the horizontal line $U(x) = E$ give the locations of possible turning points.

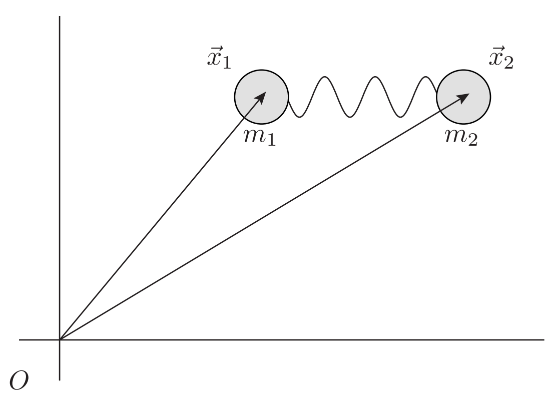
8.6.2 Equilibrium Points

An equilibrium point is the position where the particle can remain stationary. The particle has no kinetic energy and no force acts on it. In the above figure, x_2, x_3 and x_4 are possible equilibrium points because $\frac{dU}{dx}$ vanishes there (with total mechanical energy being $U(x_2)$, $U(x_3)$, and $U(x_4)$ respectively). e.g., if a particle is located exactly at x_3 with $E = U(x_3)$ and $K = 0$, the force on it is zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it father in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium**. At particle placed at x_2 or x_4 will continue to be stuck there. It cannot move left or right on its own because to do so would require a negative energy. If we push it slightly left or right, a restoring force that moves it back appears A particle at such a position is said to be in **stable equilibrium**.

8.7 Work Done on a System by an External Force

8.7.1 Conservative System

Let us consider a system composed of two particles connected by a spring and located at $\vec{x}_1$ and $\vec{x}_2$ with masses m_1 and m_2.



The force acting on m_1 is $\vec{F}_1$ and on m_2 is $\vec{F}_2$, and the work done on this system of two particles is

$$\Delta W = \Delta W_1 + \Delta W_2 = \vec{F}_1 \cdot \Delta \vec{x}_1 + \vec{F}_2 \cdot \Delta \vec{x}_2 = \Delta K_1 + \Delta K_2 \tag{4}$$

Assume there is an internal force $\vec{F}_{12} = -k(\vec{x}_1 - \vec{x}_2)$ acting on m_1 by m_2 and m_1 exerts force $\vec{F}_{21} = -\vec{F}_{12}$ on m_2. Let $\vec{F}_1^{(ext)}$ and $\vec{F}_2^{(ext)}$ be the external forces on m_1 and m_2.

$$\vec{F}_1 = \vec{F}_1^{(ext)} + \vec{F}_{12}$$
$$\vec{F}_2 = \vec{F}_2^{(ext)} + \vec{F}_{21}$$

The net force is also the net external force.

$$\vec{F}_1 + \vec{F}_2 = \vec{F}_1^{(ext)} + \vec{F}_2^{(ext)}$$

The net work ΔW can be divided into net external work $\Delta W^{(ext)}$ and internal work $\Delta W^{(int)}$.

$$\begin{aligned} \Delta W &= \vec{F}_1^{(ext)} \cdot \Delta\vec{x}_1 + \vec{F}_2^{(ext)} \cdot \Delta\vec{x}_2 + \vec{F}_{12} \cdot \Delta\vec{x}_1 + \vec{F}_{21} \cdot \Delta\vec{x}_2 \\ &= \Delta W^{(ext)} + \Delta W^{(int)} \end{aligned} \tag{5}$$

where $\Delta W^{(ext)}$ is the work done by the external forces

$$\Delta W^{(ext)} = \vec{F}_1^{(ext)} \cdot \Delta\vec{x}_1 + \vec{F}_2^{(ext)} \cdot \Delta\vec{x}_2$$

and

$$\begin{aligned} \Delta W^{(int)} &= \vec{F}_{12} \cdot \Delta\vec{x}_1 + \vec{F}_{21} \cdot \Delta\vec{x}_2 = \vec{F}_{12} \cdot \Delta(\vec{x}_1 - \vec{x}_2) \\ &= -k(\vec{x}_1 - \vec{x}_2) \cdot \Delta(\vec{x}_1 - \vec{x}_2) = -\Delta\left(\frac{k}{2}(\vec{x}_1 - \vec{x}_2) \cdot (\vec{x}_1 - \vec{x}_2)\right) \\ &= -\Delta\left(\frac{k}{2}|\vec{x}_1 - \vec{x}_2|^2\right) = -\Delta U(\vec{x}_1, \vec{x}_2) \end{aligned} \tag{6}$$

In this case, the work done by the internal forces depends only on the change of configuration of the system and does not depend on the path. This kind of internal interactions is said to be conservative. Combining $(4), (5)$ and (6), we get

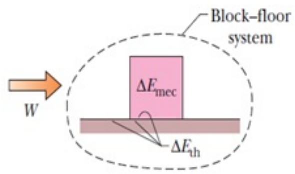
$$\Delta K = \Delta K_1 + \Delta K_2 = \Delta W^{(ext)} + \Delta W^{(int)} = \Delta W^{(ext)} - \Delta U$$

or

$$\Delta W^{(ext)} = \Delta(K + U) = \Delta E_{mech}$$

where $E_{mech} = K + U$ is the mechanical energy.

8.7.2 Friction Involved



We next consider the example of block-floor system in which an external force $\vec{F}$ pulls a block through a displacement $\Delta\vec{x}$. During the motion, a kinetic friction force $\vec{f_k}$ from the floor acts on the block. We then have

$$\Delta K = \left(\vec{F} + \vec{f_k}\right) \cdot \Delta\vec{x} = \Delta W^{(ext)} + \Delta W^{(int)} \tag{7}$$

where the external work done is

$$\Delta W^{(ext)} = \vec{F} \cdot \Delta\vec{x}$$

and the internal work done by the friction force is

$$\Delta W^{(int)} = \vec{f_k} \cdot \Delta\vec{x}$$

By experiment we find that the block and the portion of the floor along which it slides become warmer as the block slides. As we shall discuss in Chapter 18, the temperature of an object is related to the object's thermal energy E_{th}. Here, the thermal energy of the block and floor increases because (1) there is a friction between them and (2) there is a sliding. As the block slides over the floor, the sliding causes repeated tearing and re-forming of the welds between the block and floor, which makes the block and floor warmer. Thus the sliding increases their thermal energy E_{th}. Through experiment, we find that the increase ΔE_{th} is equal to the work done by the force $-\vec{f_k}$ that opposes the friction $\vec{f_k}$:

$$\Delta E_{th} = -\vec{f_k} \cdot \Delta\vec{x} = -\Delta W^{(int)}$$

(7) can be rewritten as

$$\Delta W^{(ext)} = \Delta K + \Delta E_{th} = \Delta E_{mec} + \Delta E_{th}$$

Sometimes, the work done by the internal forces $\Delta W^{(int)}$ can not be attributed to the change of a certain potential energy. Some portion of

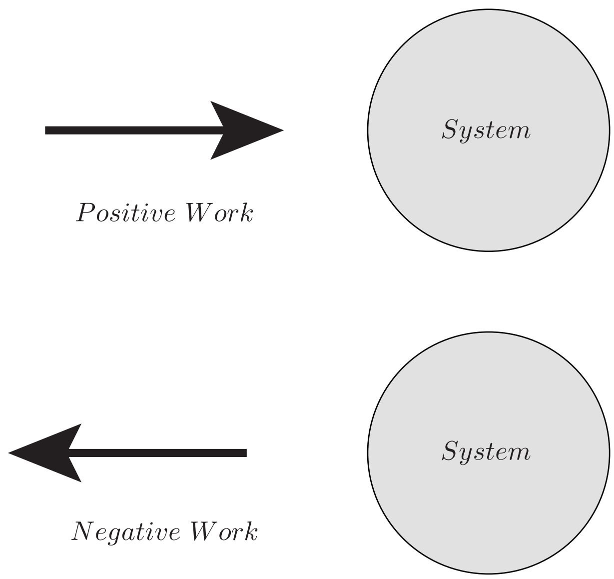
$\Delta W^{(int)}$ may appear as the change of the thermal energy $-\Delta E_{th}$ such as when there are frictions involved in the internal interactions and we have

$$\Delta W^{(int)} = -\Delta U - \Delta E_{th} \tag{8}$$

Since $\Delta K = \Delta W^{(ext)} + \Delta W^{(int)}$, (8) leads to

$$\Delta W^{(ext)} = \Delta (K + U) + \Delta E_{th} = \Delta E_{mec} + \Delta E_{th}$$

In general, work is the energy transferred to or from a system by means of an external force acting on the system.



8.8 Conservation of Energy

We have discussed several situations in which energy is transferred to or from objects and systems. In each situation, we assume that the energy that was involved could always be accounted for; that is, energy could not magically appear or disappear. The energy obeys a law called the **law of conservation of energy**.

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

The only type of energy transfer that we have considered in the work ΔW done on a system. Thus at this point, this law states that

$$\Delta W^{(ext)} = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

where ΔE_{mec} is the change in the mechanical energy of the system, ΔE_{th} is the change in the thermal energy of the system, and ΔE_{int} is the change of any other type of internal energy of the system.

Note that the law of conservation of energy is *not* something we have derived from basic physics principles. Rather, it is a law based on countless experiments. Scientists and engineers have never found an exception to it.

8.8.1 Isolated System

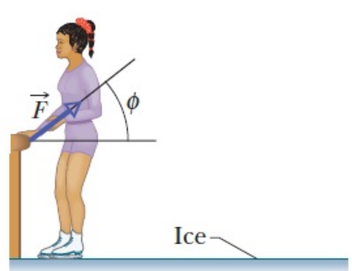
If a system is isolated from its environment, there can be no energy transfer to or from it. For that case, the law of conservation of energy says: **The total energy E of an isolated system cannot change**.

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

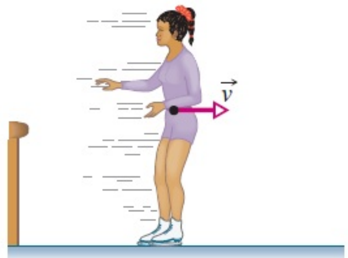
In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

8.8.2 External Force and Internal Energy Transfer

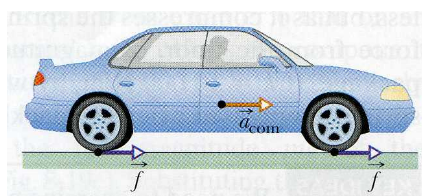
An external force can change the kinetic energy or potential energy of an object without doing work on the object — that is, without transferring energy to the object. Instead, the force is responsible for the transfers of energy from one type to another inside the object. The figure below shows an example. An initially stationary ice-skater pushes away from a railing and then slides over the ice.



Her kinetic energy increases because of an external force on her from the rail. However, that force does not transfer energy from the rail to her. Thus the force does no work on her. Rather, her kinetic energy increases as a result of internal transfers from the biochemical energy in her muscle.



The following figure shows another example. An engine increases the speed of a car with four-wheel drive. During the acceleration, the engine causes the tires to push backward on the road surface. This push produces frictional forces $\vec{f}$ that act on each tire in the forward direction. The net external force $\vec{F}$ from the road, which is the sum of these frictional forces, accelerates the car, increasing its kinetic energy. However, $\vec{F}$ does not transfer energy from the road to the car and so does no work on the car. (This is because the contact point between a tire and the road is motionless) Rather, the car's kinetic energy increases as a result of internal transfers from the energy stored in the fuel.



8.8.3 Power

If an amount of energy ΔE is transferred in an amount of time Δt, the average power due to the force is

$$P_{avg} = \frac{\Delta E}{\Delta t}$$

The instantaneous power due to the force is

$$P = \frac{dE}{dt}$$