HW4 ANSWER

CH6

6.

s^6	1	-6	1	-6
s^5	1	0	1	
s^4	-6	0	-6	
s^3	-24	0	0	ROZ
s^2	ε	-6		
s^1	$-144/\varepsilon$	0		
s^0	-6			

Even (4): 2 rhp; 2 lhp; Rest (2): 1 rhp; 1 lhp; Total: 3 rhp; 3 lhp

9.

The characteristic equation is:

$$1+\frac{K(s-1)}{s(s+2)(s+3)}=0$$

Or

$$s^3+5s^2+(6+K)s-K=0$$

s^3	1	$6+K$
s^2	5	$-K$
s^1	$\frac{6K+30}{5}$	0
s^0	$-K$	0

Therefore $-5<K<0$.

25.

The characteristic equation is $1+\frac{K(s+5)}{s(s+1)(s+3)}=s^3+4s^2+(K+3)s+5K=0$

s^3	1	3 + K
s^2	4	5K
s^1	$\frac{12-K}{4}$	0
s^0	5K	0

a. Stable for 0 < K < 12.

b. The system will oscillate when K=12.

c. When K=12 the third row becomes a row of zeros, the auxiliary equation is $Q_a(s) = 4s^2 + 60$. The poles on the $j\omega$ axis are $s = \pm j\sqrt{15}$ so the oscillation frequency is $\sqrt{15}$ rad/sec.

38.

$$T(s) = \frac{K(s+1)(s+10)}{s^3 + (5.45+K)s^2 + (11.91+11K)s + (43.65+10K)}$$

s^3	1	11.91+11K
s^2	5.45+K	43.65+10K
s^1	$\frac{11K^2 + 61.86K + 21.26}{5.45+K}$	0
s^0	43.65+10K	0

For stability, $-0.36772 < K < \infty$. Stable for all positive K.

CH7

4.

Reduce the system to an equivalent unit feedback system by first moving 1/s to the left past the summing junction. This move creates a forward path consisting of a parallel pair $\left(\frac{1}{s}+1\right)$ in cascade with a feedback loop consisting of $G(s) = \frac{3}{s+4}$ and $H(s) = 2$. Thus,

$$G_e(s) = \left(\frac{s+1}{s}\right)\left(\frac{3/(s+4)}{1+24/(s+4)}\right)$$

Hence the system is Type 1, and the steady-state errors are as follows:

Steady state error for $10u(t) = 0$

Steady state error for $10tu(t) = \frac{10}{K_v} = \frac{10}{3/28} = 93.33$

Steady state error for $10t^2u(t) = \infty$

12.

One way to solve the problem is obtain $T(s) = \frac{6}{s^3+6s^2+11s+18}$ using any method. Then

$E(s) = (1 - T(s))R(s) = \frac{s^3+6s^2+11s+12}{s^3+6s^2+11s+18}R(s).$

When $r(t) = 20u(t), e_{ss} = \lim_{s\to 0} sE(s) = \frac{12}{18}(20)$=13.3333=$\frac{20}{1+K_P}$.Solving we get $K_P = 0.5$

When $r(t) = 20tu(t), e_{ss} = \lim_{s\to 0} sE(s) = \infty$. So $K_v = 0$.

When $r(t) = 20t^2u(t), e_{ss} = \lim_{s\to 0} sE(s) = \infty$. So $K_a = 0$.

Since $K_P = 0.5,\ K_v = 0,\ K_a = 0$ this is a type 0 system.

19.

a. Type 0

b. $E(s) = \frac{R(s)}{1 + G(s)}$. Thus, $e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{12/s}{1+\frac{K(s^2+6s+6)}{(s+5)^2(s+3)}} = \frac{12}{1+0.08K}$.

c. $e(\infty) = \infty$, since the system is Type 0.

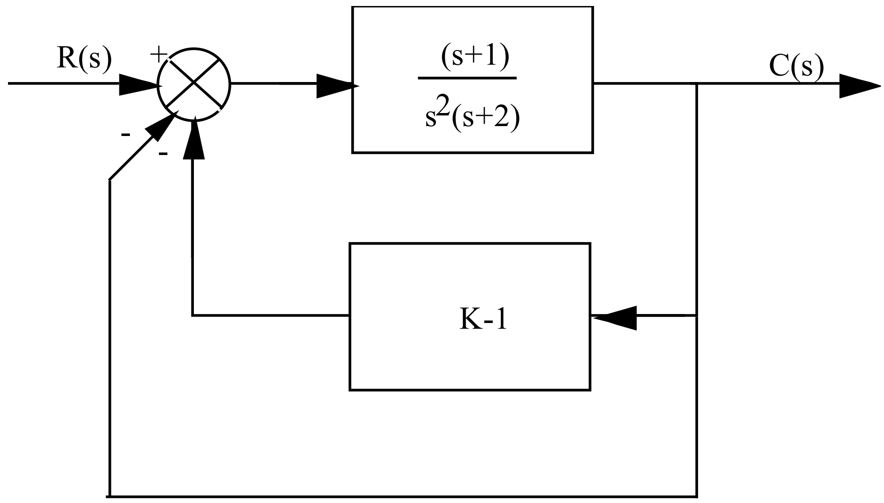
25.

a. For 20% overshoot, ζ = 0.456. Also, $K_v = 1000 = \frac{K}{a}$. Since $T(s) = \frac{K}{s^2+as+K}$, $2\zeta\omega_n = a$, and $\omega_n = \sqrt{K}$. Hence, $a = 0.912\sqrt{K}$. Solving for a and K, K = 831,744, and a = 831.744.

b. For 10% overshoot, ζ = 0.591. Also, $\frac{1}{K_v}$ = 0.01. Thus, $K_v = 100 = \frac{K}{a}$. Since $T(s) = \frac{K}{s^2+as+K}$, $2\zeta\omega_n = a$, and $\omega_n = \sqrt{K}$. Hence, $a = 1.182\sqrt{K}$. Solving for a and K, K = 13971 and a = 139.71.

33.

Produce a unity-feedback system:



Thus, $G_e(s) = \dfrac{\dfrac{(s+1)}{s^2(s+2)}}{1+\dfrac{(s+1)(K-1)}{s^2(s+2)}} = \dfrac{s+1}{s^3+2s^2+(K-1)s+(K-1)}$. Error = 0.001 = $\dfrac{1}{1+K_p}$.

Therefore, $K_p = 999 = \dfrac{1}{K-1}$. Hence, K = 1.001001.

Check stability: Using original block diagram, $T(s) = \dfrac{\dfrac{(s+1)}{s^2(s+2)}}{1+\dfrac{K(s+1)}{s^2(s+2)}} = \dfrac{s+1}{s^3+2s^2+Ks+K}$.

Making a Routh table:

s^3	1	K
s^2	2	K
s^1	$\frac{K}{2}$	0
s^0	K	0

Therefore, system is stable and steady-state error calculations are valid.