

CHAPTER 10

PHASE TRANSFORMATIONS IN METALS

PROBLEM SOLUTIONS

The Kinetics of Phase Transformations

10.1 *Name the two stages involved in the formation of particles of a new phase. Briefly describe each.*

Solution

The two stages involved in the formation of particles of a new phase are *nucleation* and *growth*. The nucleation process involves the formation of normally very small particles of the new phase(s) which are stable and capable of continued growth. The growth stage is simply the increase in size of the new phase particles.

10.2 (a) Rewrite the expression for the total free energy change for nucleation (Equation 10.1) for the case of a cubic nucleus of edge length a (instead of a sphere of radius r). Now differentiate this expression with respect to a (per Equation 10.2) and solve for both the critical cube edge length, a^* , and also ΔG^* .

(b) Is ΔG^* greater for a cube or a sphere? Why?

Solution

(a) This problem first asks that we rewrite the expression for the total free energy change for nucleation (analogous to Equation 10.1) for the case of a cubic nucleus of edge length a . The volume of such a cubic nucleus is a^3 , whereas the total surface area is $6a^2$ (since there are six faces each of which has an area of a^2). Thus, the expression for ΔG is as follows:

$$\Delta G = a^3 \Delta G_v + 6a^2 \gamma$$

Differentiation of this expression with respect to a is as

$$\begin{aligned} \frac{d\Delta G}{da} &= \frac{d(a^3 \Delta G_v)}{da} + \frac{d(6a^2 \gamma)}{da} \\ &= 3a^2 \Delta G_v + 12a \gamma \end{aligned}$$

If we set this expression equal to zero as

$$3a^2 \Delta G_v + 12a \gamma = 0$$

and then solve for a ($= a^*$), we have

$$a^* = -\frac{4\gamma}{\Delta G_v}$$

Substitution of this expression for a in the above expression for ΔG yields an equation for ΔG^* as

$$\begin{aligned} \Delta G^* &= (a^*)^3 \Delta G_v + 6(a^*)^2 \gamma \\ &= \left(-\frac{4\gamma}{\Delta G_v}\right)^3 \Delta G_v + 6\gamma \left(-\frac{4\gamma}{\Delta G_v}\right)^2 \end{aligned}$$

$$= \frac{32\gamma^3}{(\Delta G_v)^2}$$

(b) ΔG_v for a cube—i.e., $(32) \left[\frac{\gamma^3}{(\Delta G_v)^2} \right]$ —is greater than for a sphere—i.e., $\left(\frac{16\pi}{3} \right) \left[\frac{\gamma^3}{(\Delta G_v)^2} \right] =$

(16.8) $\left[\frac{\gamma^3}{(\Delta G_v)^2} \right]$. The reason for this is that surface-to-volume ratio of a cube is greater than for a sphere.

10.3 If copper (which has a melting point of 1085°C) homogeneously nucleates at 849°C, calculate the critical radius given values of $-1.77 \times 10^9 \text{ J/m}^3$ and 0.200 J/m^2 , respectively, for the latent heat of fusion and the surface free energy.

Solution

This problem states that copper homogeneously nucleates at 849°C, and that we are to calculate the critical radius given the latent heat of fusion ($-1.77 \times 10^9 \text{ J/m}^3$) and the surface free energy (0.200 J/m^2). Solution to this problem requires the utilization of Equation 10.6 as

$$r^* = \left(-\frac{2\gamma T_m}{\Delta H_f} \right) \left(\frac{1}{T_m - T} \right)$$

$$= \left[-\frac{(2)(0.200 \text{ J/m}^2)(1085 + 273 \text{ K})}{-1.77 \times 10^9 \text{ J/m}^3} \right] \left(\frac{1}{1085^\circ\text{C} - 849^\circ\text{C}} \right)$$

$$= 1.30 \times 10^{-9} \text{ m} = 1.30 \text{ nm}$$

10.4 (a) For the solidification of iron, calculate the critical radius r^* and the activation free energy ΔG^* if nucleation is homogeneous. Values for the latent heat of fusion and surface free energy are $-1.85 \times 10^9 \text{ J/m}^3$ and 0.204 J/m^2 , respectively. Use the supercooling value found in Table 10.1.

(b) Now calculate the number of atoms found in a nucleus of critical size. Assume a lattice parameter of 0.292 nm for solid iron at its melting temperature.

Solution

(a) This portion of the problem asks that we compute r^* and ΔG^* for the homogeneous nucleation of the solidification of Fe. First of all, Equation 10.6 is used to compute the critical radius. The melting temperature for iron, found inside the front cover is 1538°C ; also values of ΔH_f ($-1.85 \times 10^9 \text{ J/m}^3$) and γ (0.204 J/m^2) are given in the problem statement, and the supercooling value found in Table 10.1 is 295°C (or 295 K). Thus, from Equation 10.6 we have

$$\begin{aligned} r^* &= \left(-\frac{2\gamma T_m}{\Delta H_f} \right) \left(\frac{1}{T_m - T} \right) \\ &= \left[-\frac{(2)(0.204 \text{ J/m}^2)(1538 + 273 \text{ K})}{-1.85 \times 10^9 \text{ J/m}^3} \right] \left(\frac{1}{295 \text{ K}} \right) \\ &= 1.35 \times 10^{-9} \text{ m} = 1.35 \text{ nm} \end{aligned}$$

For computation of the activation free energy, Equation 10.7 is employed. Thus

$$\begin{aligned} \Delta G^* &= \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2} \right) \frac{1}{(T_m - T)^2} \\ &= \left[\frac{(16)(\pi)(0.204 \text{ J/m}^2)^3 (1538 + 273 \text{ K})^2}{(3)(-1.85 \times 10^9 \text{ J/m}^3)^2} \right] \left[\frac{1}{(295 \text{ K})^2} \right] \\ &= 1.57 \times 10^{-18} \text{ J} \end{aligned}$$

(b) In order to compute the number of atoms in a nucleus of critical size (assuming a spherical nucleus of radius r^*), it is first necessary to determine the number of unit cells, which we then multiply by the number of atoms per unit cell. The number of unit cells found in this critical nucleus is just the ratio of critical nucleus and unit cell volumes. Inasmuch as iron has the BCC crystal structure, its unit cell volume is just a^3 where a is the unit cell length

(i.e., the lattice parameter); this value is 0.292 nm, as cited in the problem statement. Therefore, the number of unit cells found in a radius of critical size is just

$$\begin{aligned}\# \text{ unit cells /particle} &= \frac{\frac{4}{3}\pi r^3}{a^3} \\ &= \frac{\left(\frac{4}{3}\right)(\pi)(1.35 \text{ nm})^3}{(0.292 \text{ nm})^3} = 414 \text{ unit cells}\end{aligned}$$

Inasmuch as 2 atoms are associated with each BCC unit cell, the total number of atoms per critical nucleus is just

$$(414 \text{ unit cells/critical nucleus})(2 \text{ atoms/unit cell}) = 828 \text{ atoms/critical nucleus}$$

10.5 (a) Assume for the solidification of iron (Problem 10.4) that nucleation is homogeneous, and the number of stable nuclei is 10^6 nuclei per cubic meter. Calculate the critical radius and the number of stable nuclei that exist at the following degrees of supercooling: 200 K and 300 K.

(b) What is significant about the magnitudes of these critical radii and the numbers of stable nuclei?

Solution

(a) For this part of the problem we are asked to calculate the critical radius for the solidification of iron (per Problem 10.4), for 200 K and 300 K degrees of supercooling, and assuming that there are 10^6 nuclei per meter cubed for homogeneous nucleation. In order to calculate the critical radii, we replace the $T_m - T$ term in Equation 10.6 by the degree of supercooling (denoted as ΔT) cited in the problem.

For 200 K supercooling,

$$\begin{aligned} r_{200}^* &= \left(-\frac{2\gamma T_m}{\Delta H_f} \right) \left(\frac{1}{\Delta T} \right) \\ &= \left[-\frac{(2)(0.204 \text{ J/m}^2)(1538 + 273 \text{ K})}{-1.85 \times 10^9 \text{ J/m}^3} \right] \left(\frac{1}{200 \text{ K}} \right) \\ &= 2.00 \times 10^{-9} \text{ m} = 2.00 \text{ nm} \end{aligned}$$

And, for 300 K supercooling,

$$\begin{aligned} r_{300}^* &= \left[-\frac{(2)(0.204 \text{ J/m}^2)(1538 + 273 \text{ K})}{-1.85 \times 10^9 \text{ J/m}^3} \right] \left(\frac{1}{300 \text{ K}} \right) \\ &= 1.33 \times 10^{-9} \text{ m} = 1.33 \text{ nm} \end{aligned}$$

In order to compute the number of stable nuclei that exist at 200 K and 300 K degrees of supercooling, it is necessary to use Equation 10.8. However, we must first determine the value of K_1 in Equation 10.8, which in turn requires that we calculate ΔG^* at the homogeneous nucleation temperature using Equation 10.7; this was done in Problem 10.4, and yielded a value of $\Delta G^* = 1.57 \times 10^{-18} \text{ J}$. Now for the computation of K_1 , using the value of n^* for at the homogenous nucleation temperature (10^6 nuclei/m^3):

$$K_1 = \frac{n^*}{\exp\left(-\frac{\Delta G^*}{kT}\right)}$$

$$= \frac{10^6 \text{ nuclei/m}^3}{\exp \left[- \frac{1.57 \times 10^{-18} \text{ J}}{(1.38 \times 10^{-23} \text{ J/atom-K})(1538^\circ\text{C} - 295^\circ\text{C})} \right]}$$

$$= 5.62 \times 10^{45} \text{ nuclei/m}^3$$

Now for 200 K supercooling, it is first necessary to recalculate the value ΔG^* of using Equation 10.7, where, again, the $T_m - T$ term is replaced by the number of degrees of supercooling, denoted as ΔT , which in this case is 200 K.

Thus

$$\Delta G_{200}^* = \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2} \right) \frac{1}{(\Delta T)^2}$$

$$= \left[\frac{(16)(\pi)(0.204 \text{ J/m}^2)^3 (1538 + 273 \text{ K})^2}{(3)(-1.85 \times 10^9 \text{ J/m}^3)^2} \right] \left[\frac{1}{(200 \text{ K})^2} \right]$$

$$= 3.41 \times 10^{-18} \text{ J}$$

And, from Equation 10.8, the value of n^* is

$$n_{200}^* = K_1 \exp \left(- \frac{\Delta G_{200}^*}{kT} \right)$$

$$= (5.62 \times 10^{45} \text{ nuclei/m}^3) \exp \left[- \frac{3.41 \times 10^{-18} \text{ J}}{(1.38 \times 10^{-23} \text{ J/atom-K})(1538 \text{ K} - 200 \text{ K})} \right]$$

$$= 3.5 \times 10^{-35} \text{ stable nuclei}$$

Now, for 300 K supercooling the value of ΔG^* is equal to

$$\Delta G_{300}^* = \left[\frac{(16)(\pi)(0.204 \text{ J/m}^2)^3 (1538 + 273 \text{ K})^2}{(3)(-1.85 \times 10^9 \text{ J/m}^3)^2} \right] \left[\frac{1}{(300 \text{ K})^2} \right]$$

$$= 1.51 \times 10^{-18} \text{ J}$$

from which we compute the number of stable nuclei at 300 K of supercooling as

$$n_{300}^* = K_1 \exp\left(-\frac{\Delta G_{300}^*}{kT}\right)$$

$$n^* = (5.62 \times 10^{45} \text{ nuclei/m}^3) \exp\left[-\frac{1.51 \times 10^{-18} \text{ J}}{(1.38 \times 10^{-23} \text{ J/atom-K})(1538 \text{ K} - 300 \text{ K})}\right]$$

$$= 2.32 \times 10^7 \text{ stable nuclei}$$

(b) Relative to critical radius, r^* for 300 K supercooling is slightly smaller than for 200 K (1.33 nm versus 2.00 nm). [From Problem 10.4, the value of r^* at the homogeneous nucleation temperature (295 K) was 1.35 nm.] More significant, however, are the values of n^* at these two degrees of supercooling, which are dramatically different— 3.5×10^{-35} stable nuclei at $\Delta T = 200 \text{ K}$, versus 2.32×10^7 stable nuclei at $\Delta T = 300 \text{ K}$!

10.6 For some transformation having kinetics that obey the Avrami equation (Equation 10.17), the parameter n is known to have a value of 1.7. If, after 100 s, the reaction is 50% complete, how long (total time) will it take the transformation to go to 99% completion?

Solution

This problem calls for us to compute the length of time required for a reaction to go to 99% completion. It first becomes necessary to solve for the parameter k in Equation 10.17. In order to do this it is best manipulate the equation such that k is the dependent variable. We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln(1 - y)$$

Now solving for k gives

$$k = -\frac{\ln(1 - y)}{t^n}$$

And, from the problem statement, for $y = 0.50$ when $t = 100$ s and given that $n = 1.7$, the value of k is equal to

$$k = -\frac{\ln(1 - 0.5)}{(100 \text{ s})^{1.7}} = 2.76 \times 10^{-4}$$

We now want to manipulate Equation 10.17 such that t is the dependent variable. The above equation may be written in the form:

$$t^n = -\frac{\ln(1 - y)}{k}$$

And solving this expression for t leads to

$$t = \left[-\frac{\ln(1 - y)}{k} \right]^{1/n}$$

Now, using this equation and the value of k determined above, the time to 99% transformation completion is equal to

$$t = \left[-\frac{\ln (1 - 0.99)}{2.76 \times 10^{-4}} \right]^{1/1.7} = 305 \text{ s}$$

10.7 Compute the rate of some reaction that obeys Avrami kinetics, assuming that the constants n and k have values of 3.0 and 7×10^{-3} , respectively, for time expressed in seconds.

Solution

This problem asks that we compute the rate of some reaction given the values of n and k in Equation 10.17. Since the reaction rate is defined by Equation 10.18, it is first necessary to determine $t_{0.5}$, or the time necessary for the reaction to reach $y = 0.5$. We must first manipulate Equation 10.17 such that t is the dependent variable. We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln(1 - y)$$

which may be rearranged so as to read

$$t^n = -\frac{\ln(1 - y)}{k}$$

Now, solving for t from this expression leads to

$$t = \left[-\frac{\ln(1 - y)}{k} \right]^{1/n}$$

For $t_{0.5}$ this equation takes the form

$$t_{0.5} = \left[-\frac{\ln(1 - 0.5)}{k} \right]^{1/n}$$

And, incorporation of values for n and k given in the problem statement (3.0 and 7×10^{-3} , respectively), then

$$t_{0.5} = \left[-\frac{\ln(1 - 0.5)}{7 \times 10^{-3}} \right]^{1/3.0} = 4.63 \text{ s}$$

Now, the rate is computed using Equation 10.18 as

$$\text{rate} = \frac{1}{t_{0.5}} = \frac{1}{4.63 \text{ s}} = 0.216 \text{ s}^{-1}$$

10.8 It is known that the kinetics of recrystallization for some alloy obey the Avrami equation and that the value of n in the exponential is 2.5. If, at some temperature, the fraction recrystallized is 0.40 after 200 min, determine the rate of recrystallization at this temperature.

Solution

This problem gives us the value of y (0.40) at some time t (200 min), and also the value of n (2.5) for the recrystallization of an alloy at some temperature, and then asks that we determine the rate of recrystallization at this same temperature. It is first necessary to calculate the value of k . We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln(1 - y)$$

Now solving for k gives

$$k = -\frac{\ln(1 - y)}{t^n}$$

which, using the values cited above for y , n , and t yields

$$k = -\frac{\ln(1 - 0.40)}{(200 \text{ min})^{2.5}} = 9.0 \times 10^{-7}$$

At this point we want to compute $t_{0.5}$, the value of t for $y = 0.5$, which means that it is necessary to establish a form of Equation 10.17 in which t is the dependent variable. From one of the above equations

$$t^n = -\frac{\ln(1 - y)}{k}$$

And solving this expression for t leads to

$$t = \left[-\frac{\ln(1 - y)}{k} \right]^{1/n}$$

For $t_{0.5}$, this equation takes the form

$$t_{0.5} = \left[-\frac{\ln (1 - 0.5)}{k} \right]^{1/n}$$

and incorporation of the value of k determined above, as well as the value of n cited in the problem statement (2.5), then $t_{0.5}$ is equal to

$$t_{0.5} = \left[-\frac{\ln (1 - 0.5)}{9.0 \times 10^{-7}} \right]^{1/2.5} = 226.3 \text{ min}$$

Therefore, from Equation 10.18, the rate is just

$$\text{rate} = \frac{1}{t_{0.5}} = \frac{1}{226.3 \text{ min}} = 4.42 \times 10^{-3} \text{ (min)}^{-1}$$

10.9 The kinetics of the austenite-to-pearlite transformation obey the Avrami relationship. Using the fraction transformed–time data given here, determine the total time required for 95% of the austenite to transform to pearlite:

<i>Fraction Transformed</i>	<i>Time (s)</i>
0.2	12.6
0.8	28.2

Solution

The first thing necessary is to set up two expressions of the form of Equation 10.17, and then to solve simultaneously for the values of n and k . In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp(-kt^n)$$

Now taking natural logarithms

$$\ln(1 - y) = -kt^n$$

Or

$$-\ln(1 - y) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1 - y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1 - y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. Using values cited in the problem statement, the two equations are thus

$$\ln \left\{ \ln \left[\frac{1}{1 - 0.2} \right] \right\} = \ln k + n \ln (12.6 \text{ s})$$

$$\ln \left\{ \ln \left[\frac{1}{1 - 0.8} \right] \right\} = \ln k + n \ln (28.2 \text{ s})$$

Solving these two expressions simultaneously for n and k yields $n = 2.453$ and $k = 4.46 \times 10^{-4}$.

Now it becomes necessary to solve for the value of t at which $y = 0.95$. One of the above equations—viz

$$-\ln (1 - y) = kt^n$$

may be rewritten as

$$t^n = -\frac{\ln (1 - y)}{k}$$

And solving for t leads to

$$t = \left[-\frac{\ln (1 - y)}{k} \right]^{1/n}$$

Now incorporating into this expression values for n and k determined above, the time required for 95% austenite transformation is equal to

$$t = \left[-\frac{\ln (1 - 0.95)}{4.64 \times 10^{-4}} \right]^{1/2.453} = 35.7 \text{ s}$$

10.10 The fraction recrystallized–time data for the recrystallization at 600°C of a previously deformed steel are tabulated here. Assuming that the kinetics of this process obey the Avrami relationship, determine the fraction recrystallized after a total time of 22.8 min.

<i>Fraction Recrystallized</i>	<i>Time (min)</i>
0.20	13.1
0.70	29.1

Solution

The first thing necessary is to set up two expressions of the form of Equation 10.17, and then to solve simultaneously for the values of n and k . In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp(-kt^n)$$

Now taking natural logarithms

$$\ln(1 - y) = -kt^n$$

Or

$$-\ln(1 - y) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1 - y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1 - y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. The two equations are thus

$$\ln \left\{ \ln \left[\frac{1}{1 - 0.20} \right] \right\} = \ln k + n \ln (13.1 \text{ min})$$

$$\ln \left\{ \ln \left[\frac{1}{1 - 0.70} \right] \right\} = \ln k + n \ln (29.1 \text{ min})$$

Solving these two expressions simultaneously for n and k yields $n = 2.112$ and $k = 9.75 \times 10^{-4}$.

Now it becomes necessary to solve for y when $t = 22.8 \text{ min}$. Application of Equation 10.17 leads to

$$\begin{aligned} y &= 1 - \exp(-kt^n) \\ &= 1 - \exp[-(9.75 \times 10^{-4})(22.8 \text{ min})^{2.112}] = 0.51 \end{aligned}$$

10.11 (a) From the curves shown in Figure 10.11 and using Equation 10.18, determine the rate of recrystallization for pure copper at the several temperatures.

(b) Make a plot of $\ln(\text{rate})$ versus the reciprocal of temperature (in K^{-1}), and determine the activation energy for this recrystallization process. (See Section 5.5.)

(c) By extrapolation, estimate the length of time required for 50% recrystallization at room temperature, 20°C (293 K).

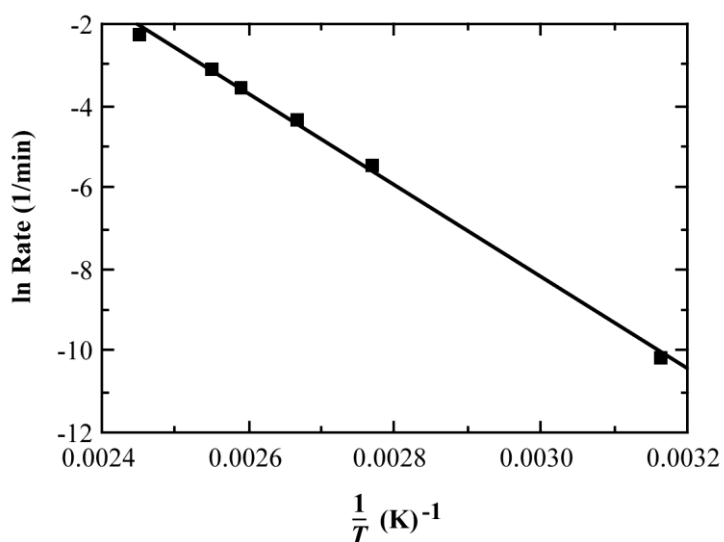
Solution

This problem asks us to consider the percent recrystallized versus logarithm of time curves for copper shown in Figure 10.11.

(a) The rates at the different temperatures are determined using Equation 10.18, which rates are tabulated below:

Temperature ($^\circ\text{C}$)	Rate (min^{-1})
135	0.105
119	4.4×10^{-2}
113	2.9×10^{-2}
102	1.25×10^{-2}
88	4.2×10^{-3}
43	3.8×10^{-5}

(b) These data are plotted below.



The activation energy, Q , is related to the slope of the line drawn through the data points as

$$Q = -\text{Slope}(R)$$

where R is the gas constant. The slope of this line is equal to

$$\text{Slope} = \frac{\Delta \ln \text{rate}}{\Delta \left(\frac{1}{T}\right)} = \frac{\ln \text{rate}_1 - \ln \text{rate}_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Let us take $1/T_1 = 0.0025 \text{ K}^{-1}$ and $1/T_2 = 0.0031 \text{ K}^{-1}$; the corresponding $\ln \text{rate}$ values are $\ln \text{rate}_1 = -2.6$ and $\ln \text{rate}_2 = -9.4$. Thus, using these values, the slope is equal to

$$\text{Slope} = \frac{-2.6 - (-9.4)}{0.0025 \text{ K}^{-1} - 0.0031 \text{ K}^{-1}} = -1.133 \times 10^4 \text{ K}$$

And, finally the activation energy is

$$\begin{aligned} Q &= -(\text{Slope})(R) = -(-1.133 \times 10^4 \text{ K}^{-1})(8.31 \text{ J/mol} \cdot \text{K}) \\ &= 94,150 \text{ J/mol} \end{aligned}$$

(c) At room temperature (20°C), $1/T = 1/(20 + 273 \text{ K}) = 3.41 \times 10^{-3} \text{ K}^{-1}$. Extrapolation of the data in the plot to this $1/T$ value gives

$$\ln(\text{rate}) \cong -12.8$$

which leads to

$$\text{rate} \cong \exp(-12.8) = 2.76 \times 10^{-6} (\text{min})^{-1}$$

But since

$$\text{rate} = \frac{1}{t_{0.5}}$$

$$t_{0.5} = \frac{1}{\text{rate}} = \frac{1}{2.76 \times 10^{-6} \text{ (min)}^{-1}}$$

$$= 3.62 \times 10^5 \text{ min} = 250 \text{ days}$$

10.12 Determine values for the constants n and k (Equation 10.17) for the recrystallization of copper (Figure 10.11) at 102°C .

Solution

In this problem we are asked to determine, from Figure 10.11, the values of the constants n and k (Equation 10.17) for the recrystallization of copper at 102°C . One way to solve this problem is to take two values of percent recrystallization (which is just $100y$, Equation 10.17) and their corresponding time values, then set up two simultaneous equations, from which n and k may be determined. In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp(-kt^n)$$

Now taking natural logarithms

$$\ln(1 - y) = -kt^n$$

Or

$$-\ln(1 - y) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1 - y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1 - y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. From the 102°C curve of Figure 10.11, let us arbitrarily choose two percent recrystallized values, 20% and 80% (i.e., $y_1 = 0.20$ and $y_2 = 0.80$). Their corresponding time values are $t_1 = 50$ min and $t_2 = 100$ min (realizing that the time axis is scaled logarithmically). Thus, our two simultaneous equations become

$$\ln \left[\ln \left(\frac{1}{1 - 0.2} \right) \right] = \ln k + n \ln (50)$$

$$\ln \left[\ln \left(\frac{1}{1 - 0.8} \right) \right] = \ln k + n \ln (100)$$

from which we obtain the values $n = 2.85$ and $k = 3.21 \times 10^{-6}$.

Metastable Versus Equilibrium States

10.13 *In terms of heat treatment and the development of microstructure, what are two major limitations of the iron–iron carbide phase diagram?*

Solution

Two limitations of the iron-iron carbide phase diagram are:

- (1) The nonequilibrium martensite does not appear on the diagram; and
- (2) The diagram provides no indication as to the time-temperature relationships for the formation of pearlite, bainite, and spheroidite, all of which are composed of the equilibrium ferrite and cementite phases.

10.14 (a) *Briefly describe the phenomena of superheating and supercooling.*

(b) *Why do these phenomena occur?*

Solution

(a) Superheating and supercooling correspond, respectively, to heating or cooling above or below a phase transition temperature without the occurrence of the transformation.

(b) These phenomena occur because right at the phase transition temperature, the driving force is not sufficient to cause the transformation to occur. The driving force is enhanced during superheating or supercooling.

Isothermal Transformation Diagrams

10.15 Suppose that a steel of eutectoid composition is cooled to 550°C (1020°F) from 760°C (1400°F) in less than 0.5 s and held at this temperature.

(a) How long will it take for the austenite-to-pearlite reaction to go to 50% completion? To 100% completion?

(b) Estimate the hardness of the alloy that has completely transformed to pearlite.

Solution

We are called upon to consider the isothermal transformation of an iron-carbon alloy of eutectoid composition.

(a) From Figure 10.22, a horizontal line at 550°C intersects the 50% and reaction completion curves at about 2.5 and 6 seconds, respectively; these are the times asked for in the problem statement.

(b) The pearlite formed will be fine pearlite. From Figure 10.30a, the hardness of an alloy of composition 0.76 wt% C that consists of fine pearlite is about 265 HB (27 HRC).

10.16 *Briefly cite the differences between pearlite, bainite, and spheroidite relative to microstructure and mechanical properties.*

Solution

The microstructures of pearlite, bainite, and spheroidite all consist of α -ferrite and cementite phases. For pearlite, the two phases exist as layers which alternate with one another. Bainite consists of very fine and parallel needle-shaped particles of cementite that are surrounded an α -ferrite matrix. For spheroidite, the matrix is ferrite, and the cementite phase is in the shape of sphere-shaped particles.

Bainite is harder and stronger than pearlite, which, in turn, is harder and stronger than spheroidite.

10.17 *What is the driving force for the formation of spheroidite?*

Solution

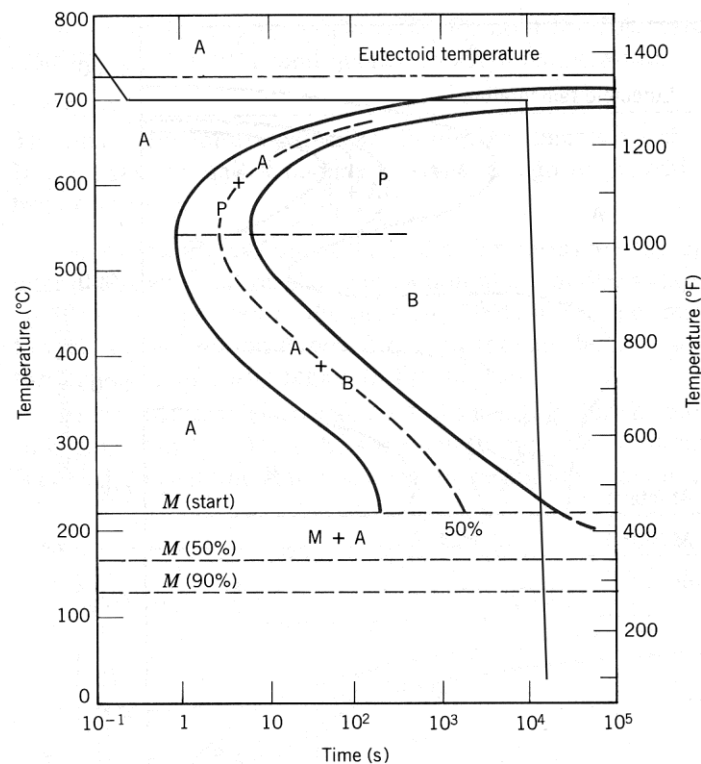
The driving force for the formation of spheroidite is the net reduction in ferrite-cementite phase boundary area.

10.18 Using the isothermal transformation diagram for an iron–carbon alloy of eutectoid composition (Figure 10.22), specify the nature of the final microstructure (in terms of microconstituents present and approximate percentages of each) of a small specimen that has been subjected to the following time–temperature treatments. In each case assume that the specimen begins at 760°C (1400°F) and that it has been held at this temperature long enough to have achieved a complete and homogeneous austenitic structure.

(a) Cool rapidly to 700°C (1290°F), hold for 10^4 s, then quench to room temperature.

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.



After cooling and holding at 700°C for 10^4 s, approximately 50% of the specimen has transformed to coarse pearlite. Upon cooling to room temperature, the remaining 50% transforms to martensite. Hence, the final microstructure consists of about 50% coarse pearlite and 50% martensite.

(b) Reheat the specimen in part (a) to 700°C (1290°F) for 20 h.

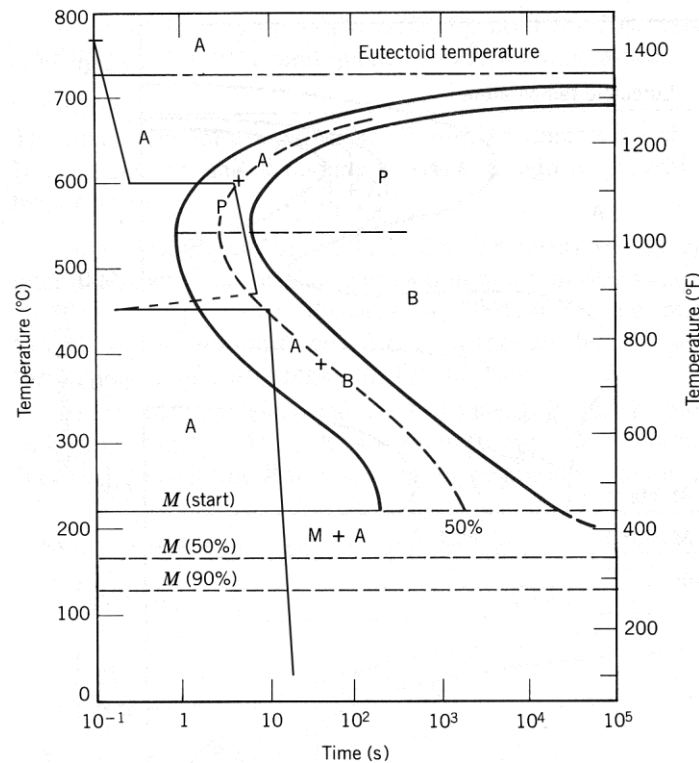
Solution

Heating to 700°C for 20 h the specimen in part (a) will transform the coarse pearlite and martensite to spheroidite.

(c) *Rapidly cool to 600°C (1110°F), hold for 4 s, rapidly cool to 450°C (840°F), hold for 10 s, then quench to room temperature.*

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.

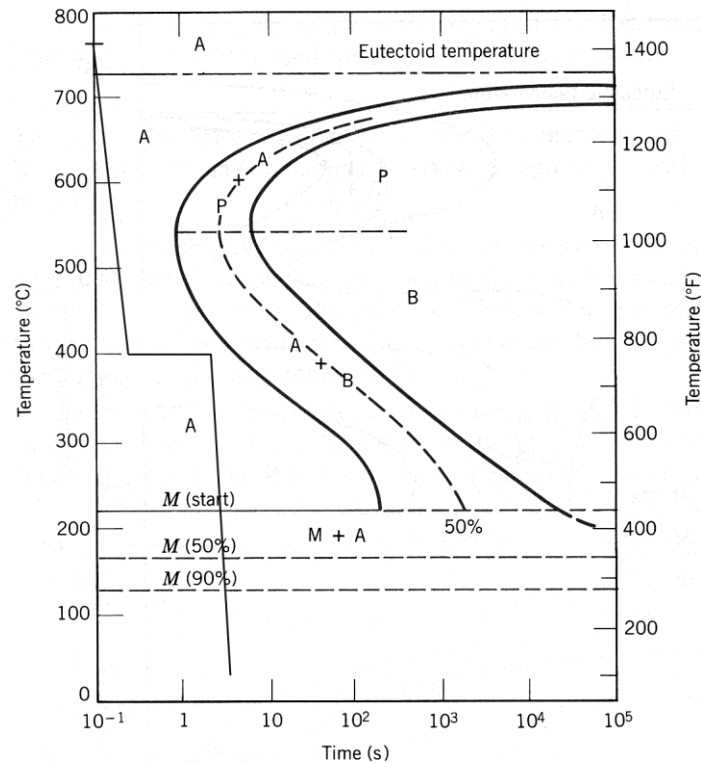


After cooling to and holding at 600°C for 4 s, approximately 50% of the specimen has transformed to pearlite (medium). During the rapid cooling to 450°C no transformations occur. At 450°C we start timing again at zero time; while holding at 450°C for 10 s, approximately 50 percent of the remaining unreacted 50% (or 25% of the original specimen) will transform to bainite. And upon cooling to room temperature, the remaining 25% of the original specimen transforms to martensite. Hence, the final microstructure consists of about 50% pearlite (medium), 25% bainite, and 25% martensite.

(d) *Cool rapidly to 400°C (750°F), hold for 2 s, then quench to room temperature.*

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.

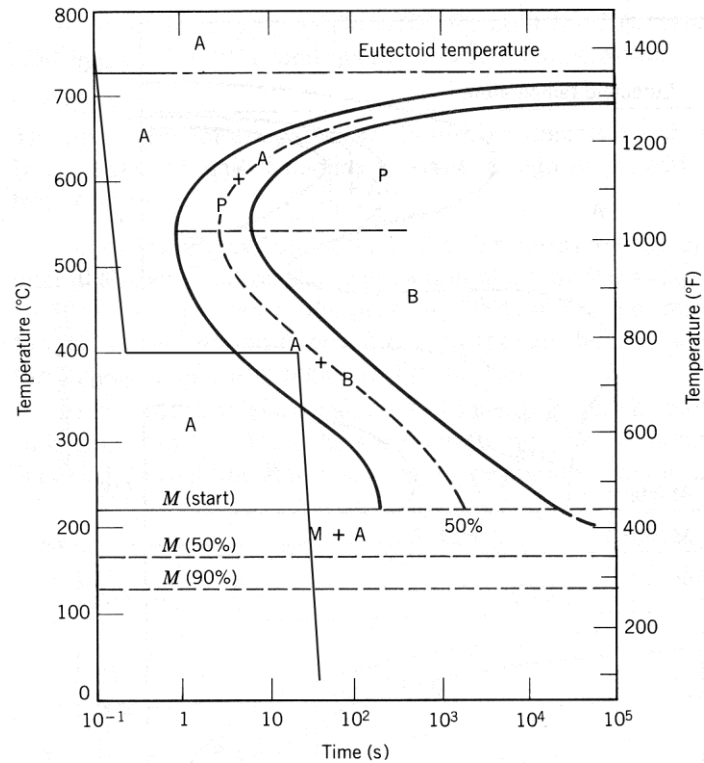


After cooling to and holding at 400°C for 2 s, no of the transformation begin lines have been crossed, and therefore, the specimen is 100% austenite. Upon cooling rapidly to room temperature, all of the specimen transforms to martensite, such that the final microstructure is 100% martensite.

(e) Cool rapidly to 400°C (750°F), hold for 20 s, then quench to room temperature.

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.

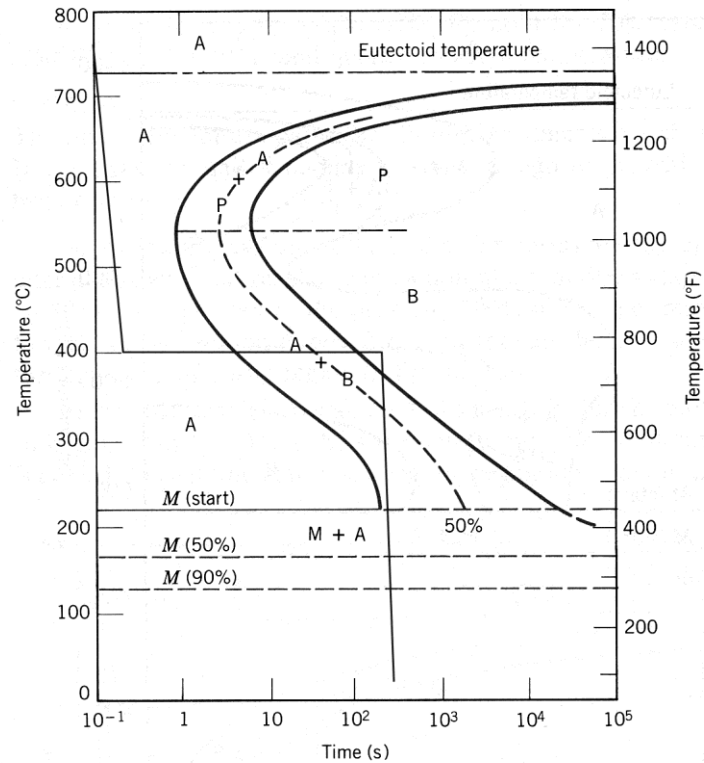


After cooling and holding at 400°C for 20 s, approximately 40% of the specimen has transformed to bainite. Upon cooling to room temperature, the remaining 60% transforms to martensite. Hence, the final microstructure consists of about 40% bainite and 60% martensite.

(f) Cool rapidly to 400°C (750°F), hold for 200 s, then quench to room temperature.

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.

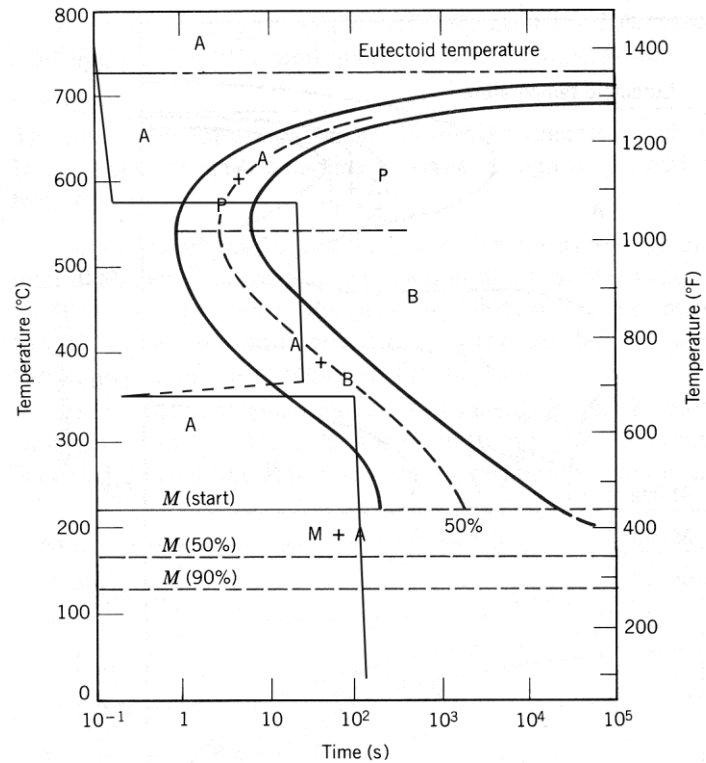


After cooling and holding at 400°C for 200 s, the entire specimen has transformed to bainite. Therefore, during the cooling to room temperature no additional transformations will occur. Hence, the final microstructure consists of 100% bainite.

(g) *Rapidly cool to 575°C (1065°F), hold for 20 s, rapidly cool to 350°C (660°F), hold for 100 s, then quench to room temperature.*

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.

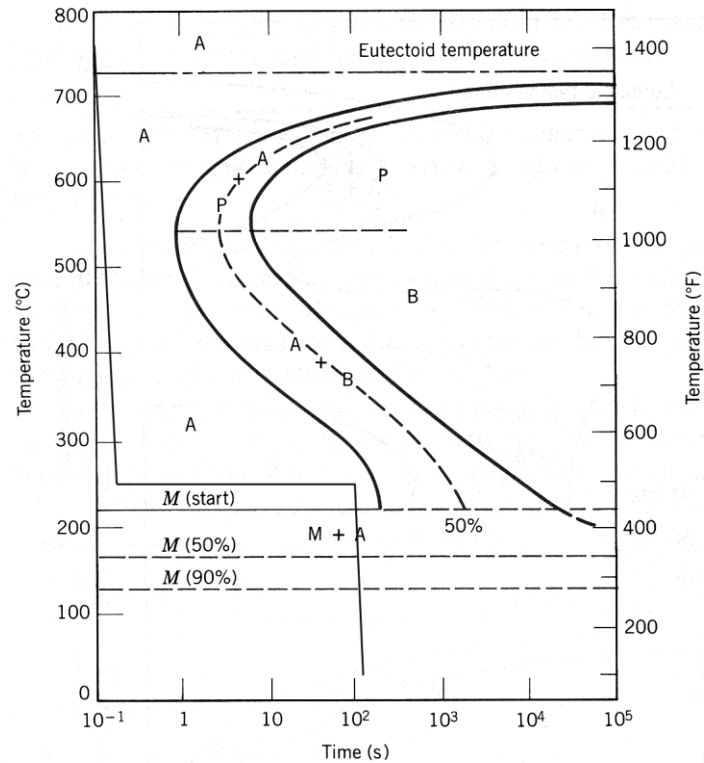


After cooling and holding at 575°C for 20 s, the entire specimen has transformed to fine pearlite. Therefore, during the second heat treatment at 350°C no additional transformations will occur. Hence, the final microstructure consists of 100% fine pearlite.

(h) Rapidly cool to 250°C (480°F), hold for 100 s, then quench to room temperature in water. Reheat to 315°C (600°F) for 1 h and slowly cool to room temperature.

Solution

Below is Figure 10.22 upon which is superimposed the above heat treatment.



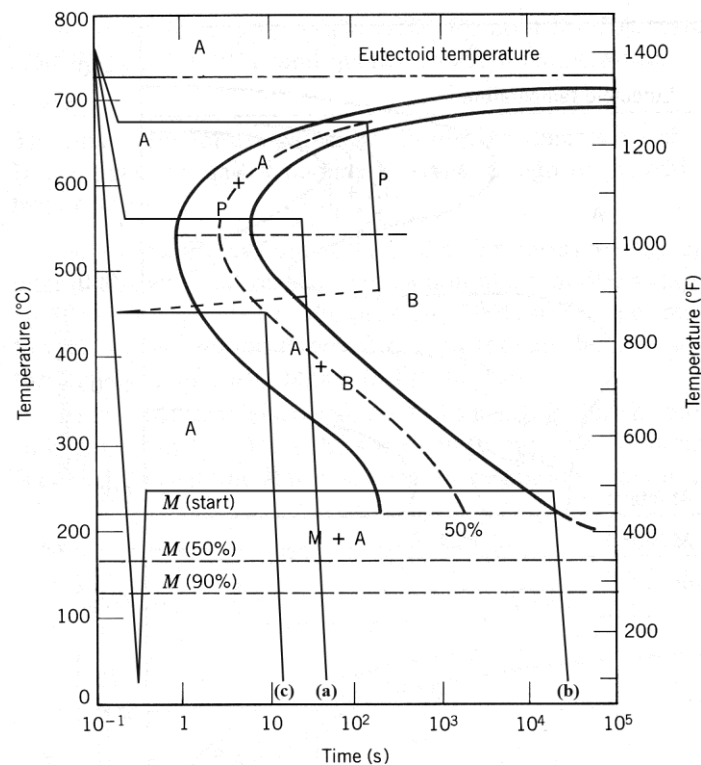
After cooling and holding at 250°C for 100 s, no transformations will have occurred—at this point, the entire specimen is still austenite. Upon rapidly cooling to room temperature in water, the specimen will completely transform to martensite. The second heat treatment (at 315°C for 1 h)—not shown on the above plot—will transform the material to tempered martensite. Hence, the final microstructure is 100% tempered martensite.

10.19 Make a copy of the isothermal transformation diagram for an iron–carbon alloy of eutectoid composition (Figure 10.22) and then sketch and label time–temperature paths on this diagram to produce the following microstructures:

- (a) 100% fine pearlite
- (b) 100% tempered martensite
- (c) 50% coarse pearlite, 25% bainite, and 25% martensite

Solution

Below is shown the isothermal transformation diagram for a eutectoid iron-carbon alloy, with time-temperature paths that will yield (a) 100% fine pearlite; (b) 100% tempered martensite; and (c) 50% coarse pearlite, 25% bainite, and 25% martensite.

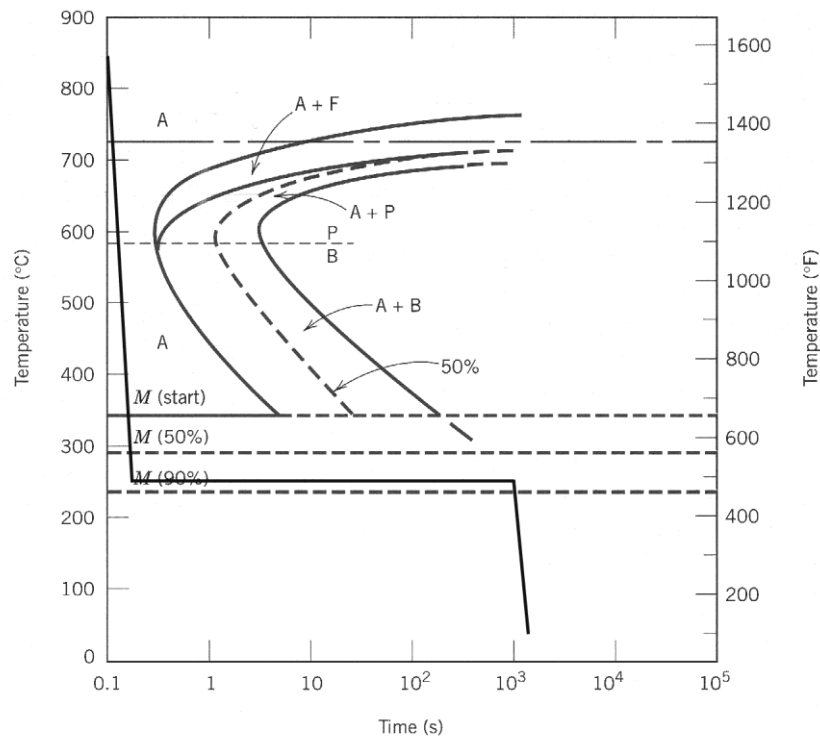


10.20 Using the isothermal transformation diagram for a 0.45 wt% C steel alloy (Figure 10.39), determine the final microstructure (in terms of just the microconstituents present) of a small specimen that has been subjected to the following time-temperature treatments. In each case assume that the specimen begins at 845 °C (1550 °F), and that it has been held at this temperature long enough to have achieved a complete and homogeneous austenitic structure.

(a) Rapidly cool to 250 °C (480 °F), hold for 10^3 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

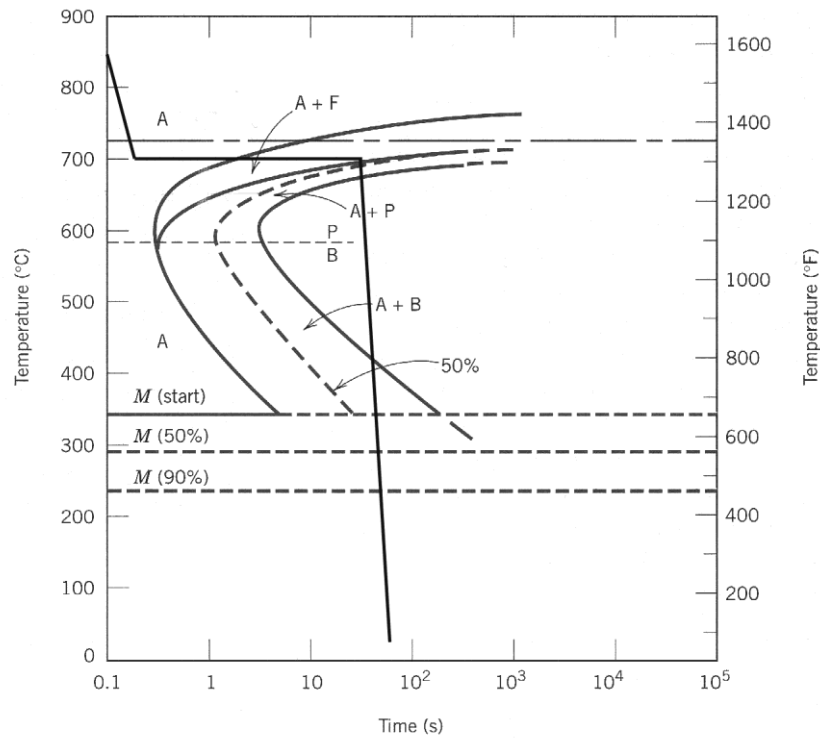


While rapidly cooling to 250 °C about 80% of the specimen transforms to martensite; during the 1000 s isothermal treatment at 250 °C no additional transformations occur. During the final cooling to room temperature, the untransformed austenite also transforms to martensite. Hence, the final microstructure consists of 100% martensite.

(b) Rapidly cool to 700 °C (1290 °F), hold for 30 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

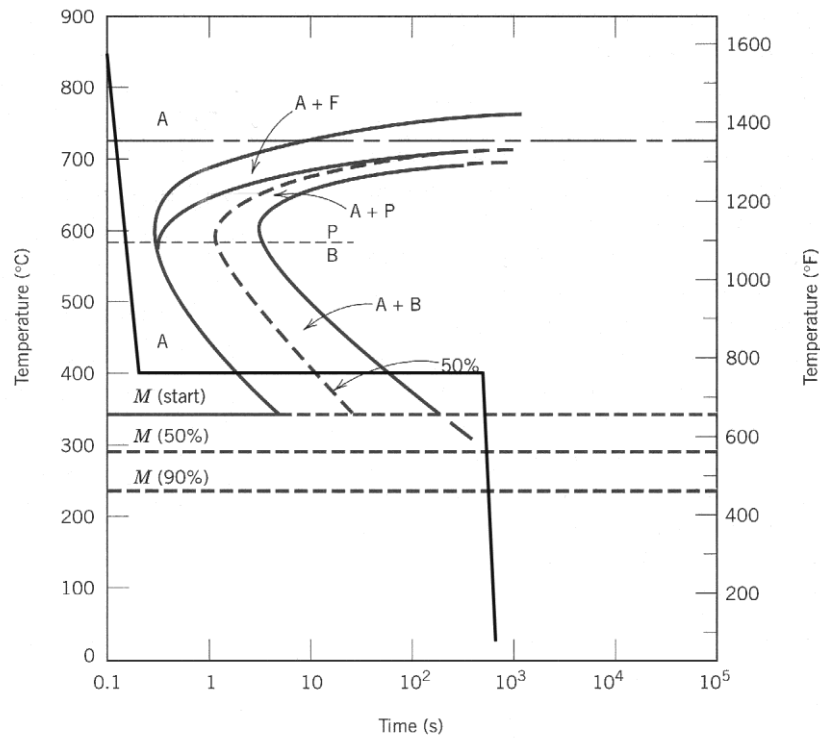


After cooling to and holding at 700°C for 30 s, a portion of specimen has transformed to proeutectoid ferrite. While cooling to room temperature, the remainder of the specimen transforms to martensite. Hence, the final microstructure consists proeutectoid ferrite and martensite.

(c) Rapidly cool to 400 °C (750 °F), hold for 500 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

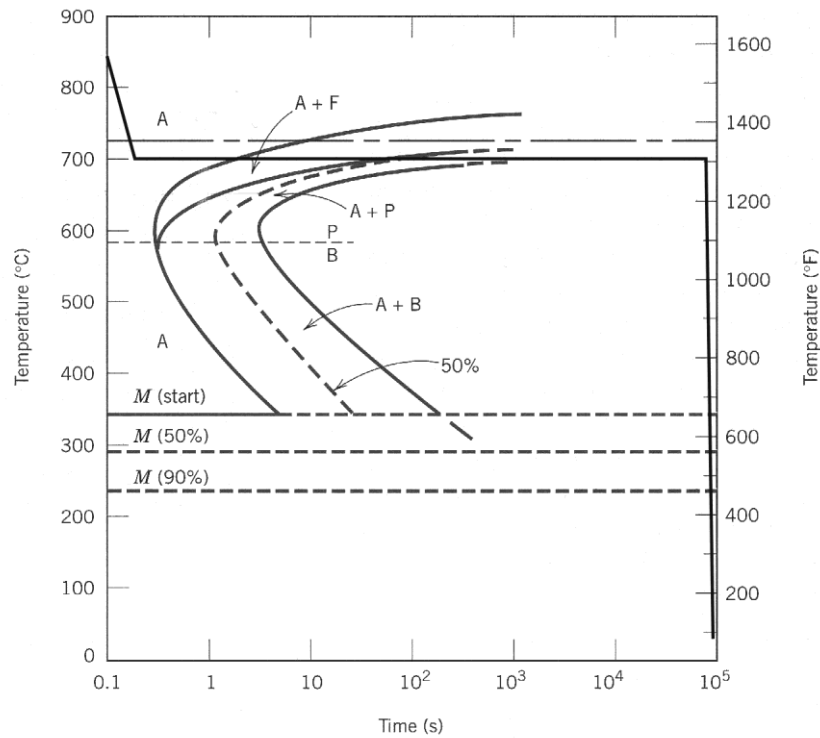


After cooling to and holding at 400°C for 500 s, all of the specimen has transformed to bainite. Hence, the final microstructure consists of 100% bainite.

(d) Rapidly cool to 700 °C (1290 °F), hold at this temperature for 10⁵ s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

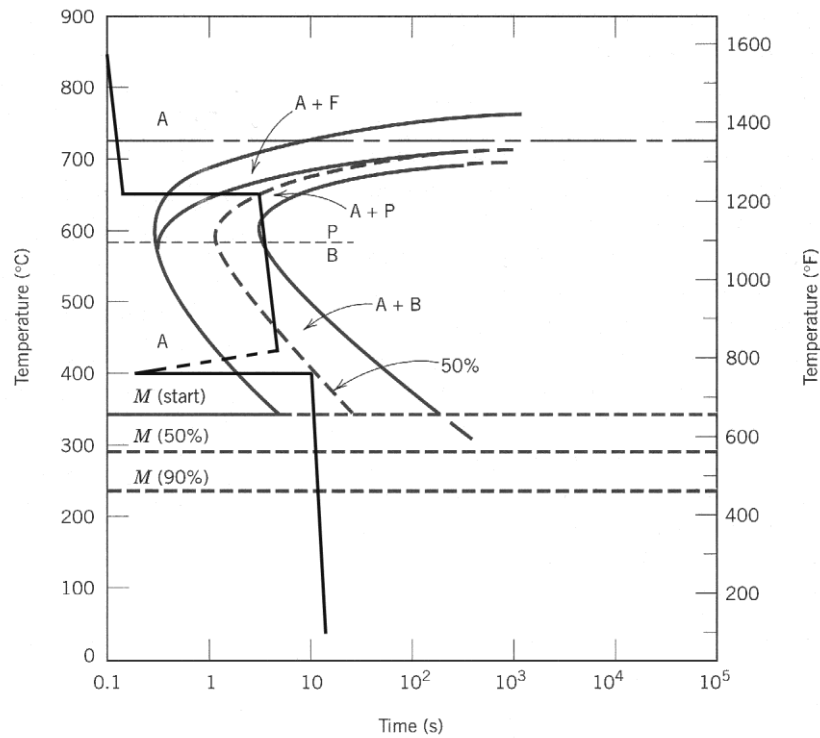


After cooling to and while holding at 700°C the specimen first transforms to proeutectoid ferrite and coarse pearlite. Continued heat treating at 700°C for 10⁵ s results in a further transformation into spheroidite. Hence, the final microstructure consists of 100% spheroidite.

(e) Rapidly cool to 650 °C (1200 °F), hold at this temperature for 3 s, rapidly cool to 400 °C (750 °F), hold for 10 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

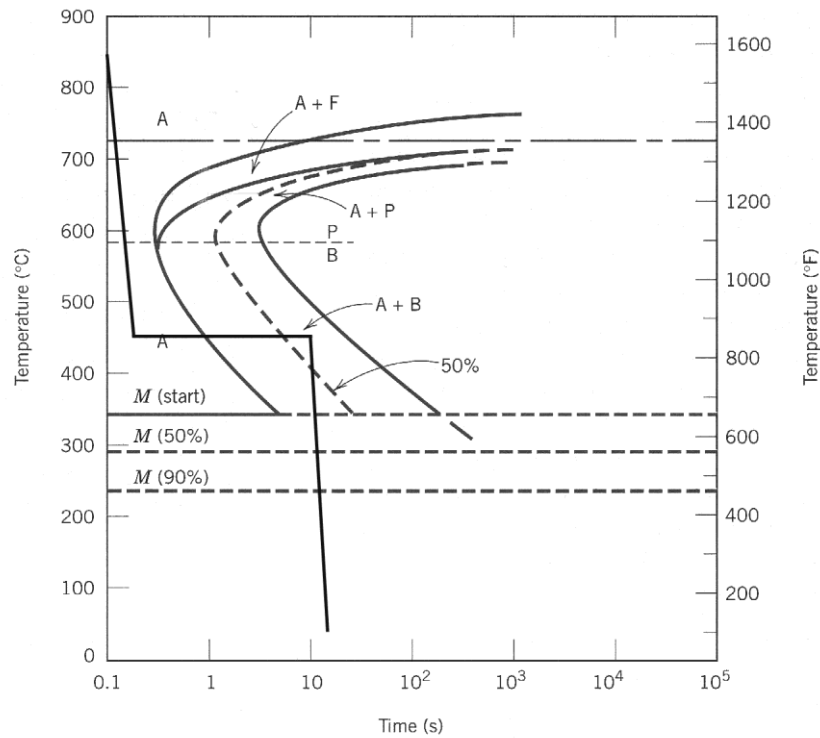


After cooling to and holding at 650°C for 3 s, some of the specimen first transforms to proeutectoid ferrite and then to pearlite (medium). During the second stage of the heat treatment at 400°C, some (but not all) of the remaining unreacted austenite transforms to bainite. As a result of the final quenching, all of the remaining austenite transforms to martensite. Hence, the final microstructure consists of ferrite, pearlite (medium), bainite, and martensite.

(f) Rapidly cool to 450 °C (840 °F), hold for 10 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

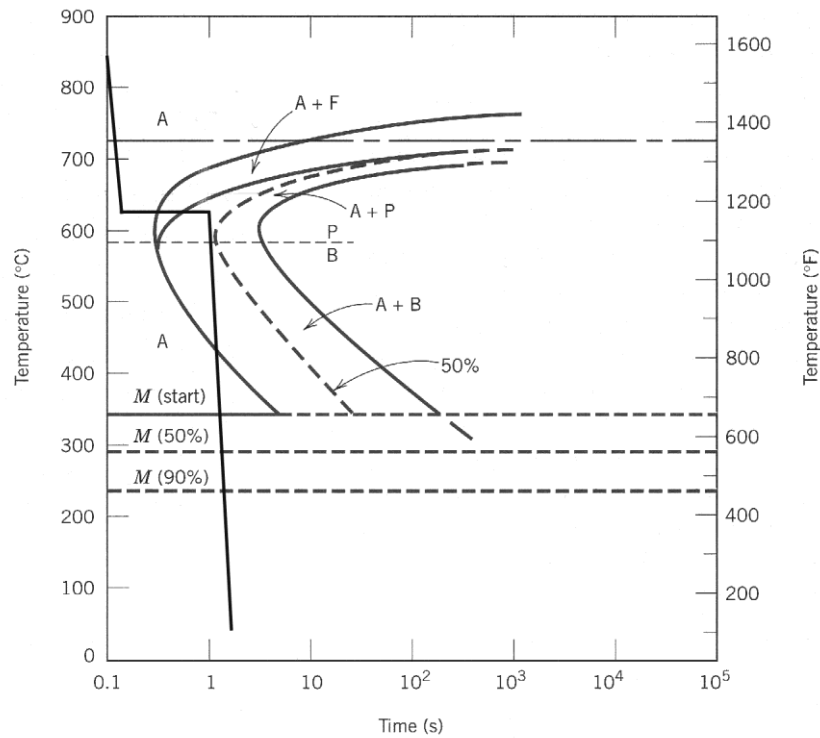


After cooling to and holding at 450°C for 10 s, a portion of the specimen first transforms to bainite. During the quenching to room temperature, the remainder of the specimen transforms to martensite. Hence, the final microstructure consists of bainite and martensite.

(g) Rapidly cool to 625 °C (1155 °F), hold for 1 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

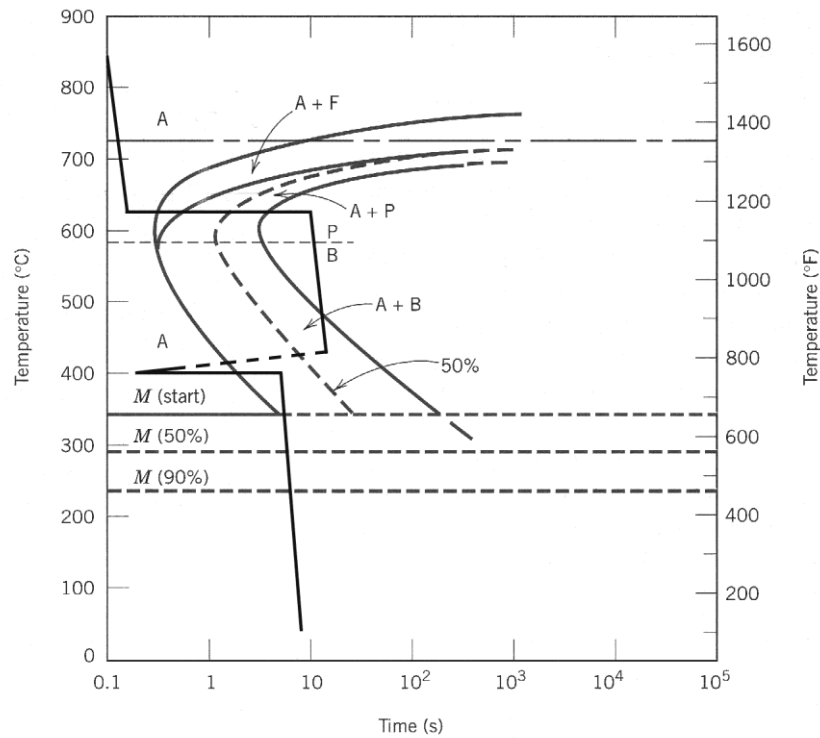


After cooling to and holding at 625°C for 1 s, a portion of the specimen first transforms to proeutectoid ferrite and pearlite. During the quenching to room temperature, the remainder of the specimen transforms to martensite. Hence, the final microstructure consists of ferrite, pearlite, and martensite.

(h) Rapidly cool to 625 °C (1155 °F), hold at this temperature for 10 s, rapidly cool to 400 °C (750 °F), hold at this temperature for 5 s, then quench to room temperature.

Solution

Below is Figure 10.39 upon which is superimposed the above heat treatment.

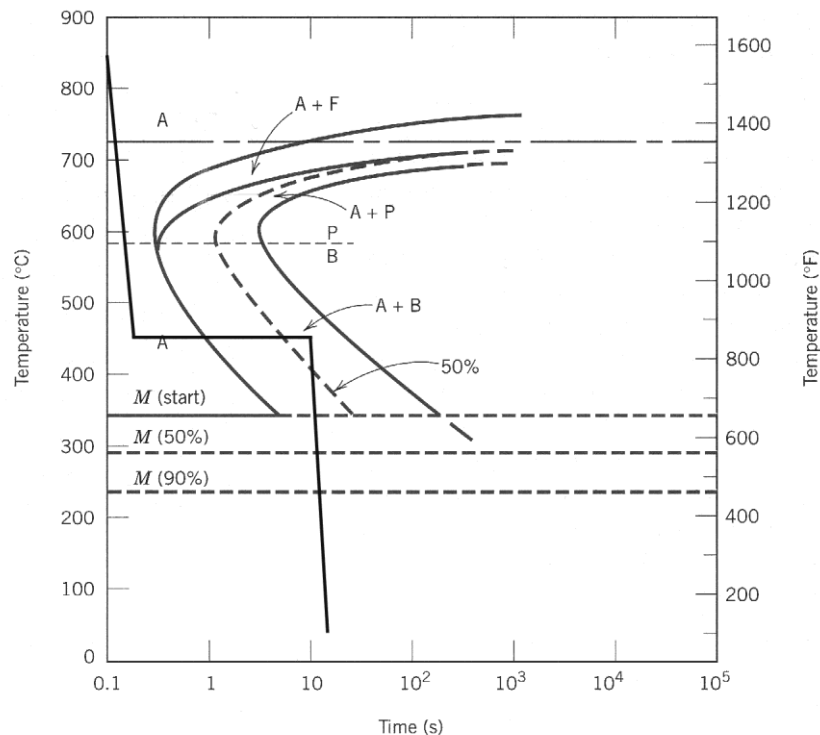


After cooling to and holding at 625°C for 10 s, all of the specimen transforms to proeutectoid ferrite and pearlite. During the second part of the heat treatment at 400°C no additional transformation will occur. Hence, the final microstructure consists of ferrite and pearlite.

10.21 For parts (a), (c), (d), (f), and (h) of Problem 10.20, determine the approximate percentages of the microconstituents that form.

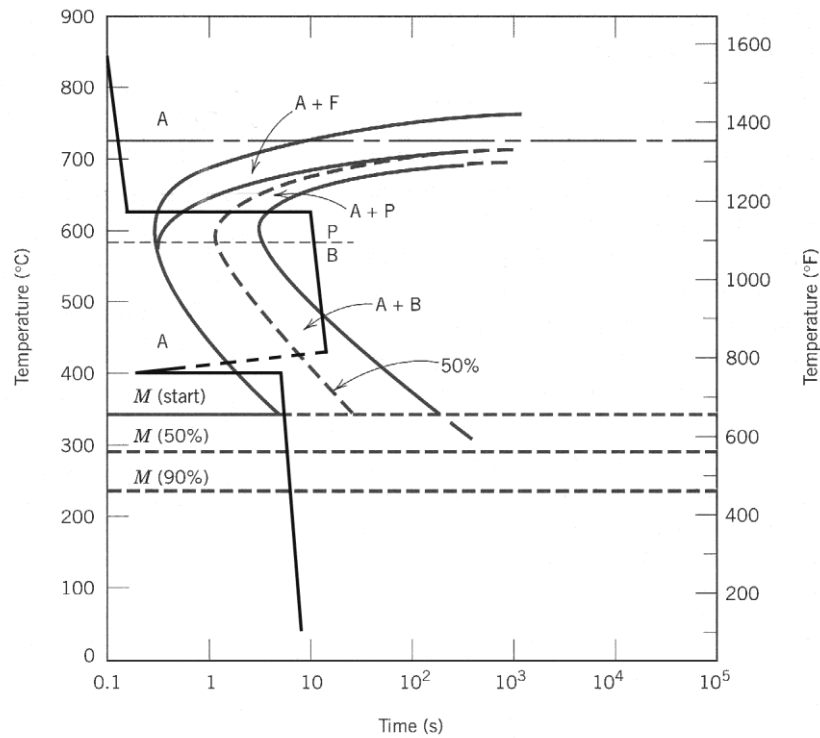
Solution

- (a) From Problem 10.20(a) the microstructure consists of 100% martensite.
- (c) From Problem 10.20(c) the microstructure consists of 100% bainite.
- (d) From Problem 10.20(d) the microstructure consists of 100% spheroidite.
- (f) Figure 10.39 onto which the heat treatment for Problem 10.20(f) has been constructed is shown below.



From this diagram, for the isothermal heat treatment at 450°C, the horizontal line constructed at this temperature and that ends at the 10 s point spans approximately 70% of the distance between the bainite reaction start and reaction completion curves. Therefore, the final microstructure consists of about 70% bainite and 30% martensite (the martensite forms while cooling to room temperature after 10 s at 450°C).

- (h) Figure 10.39 onto which the heat treatment for Problem 10.20(h) has been constructed is shown below.



After holding for 10 s at 625°C, the specimen has completely transformed to proeutectoid ferrite and fine pearlite; no further reaction will occur at 400°C. Therefore, we can calculate the mass fractions using the appropriate lever rule expressions, Equations 9.20 and 9.21, as follows:

$$W_p = \frac{C_0' - 0.022}{0.74} = \frac{0.45 - 0.022}{0.74} = 0.58 \text{ or } 58\%$$

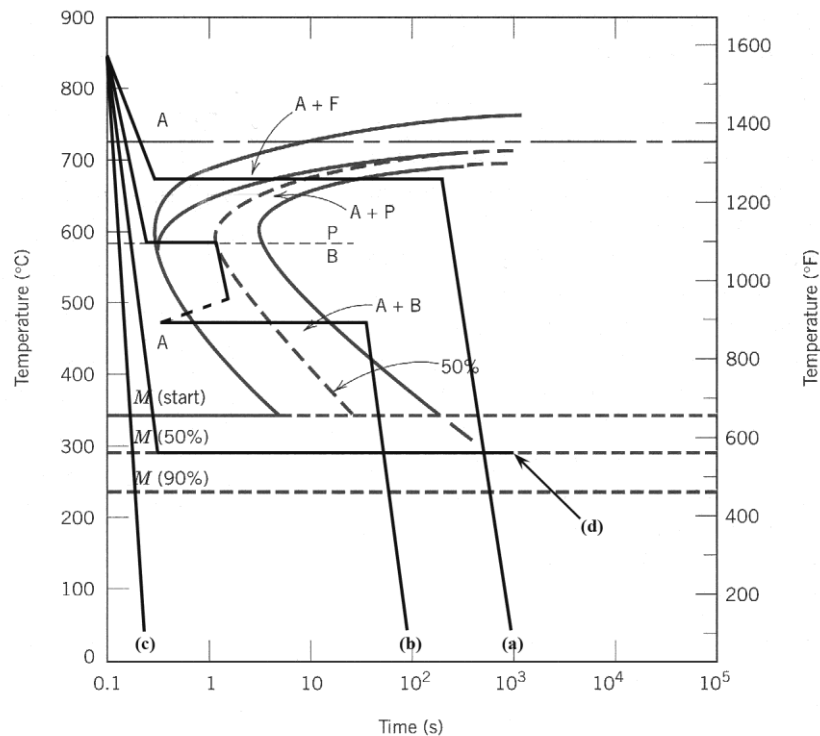
$$W_{\alpha'} = \frac{0.76 - C_0'}{0.74} = \frac{0.76 - 0.45}{0.74} = 0.42 \text{ or } 42\%$$

10.22 Make a copy of the isothermal transformation diagram for a 0.45 wt% C iron-carbon alloy (Figure 10.39), and then sketch and label on this diagram the time-temperature paths to produce the following microstructures:

- (a) 42% proeutectoid ferrite and 58% coarse pearlite
- (b) 50% fine pearlite and 50% bainite
- (c) 100% martensite
- (d) 50% martensite and 50% austenite

Solution

Below is shown an isothermal transformation diagram for a 0.45 wt% C iron-carbon alloy, with time-temperature paths that will produce (a) 42% proeutectoid ferrite and 58% coarse pearlite; (b) 50% fine pearlite and 50% bainite; (c) 100% martensite; and (d) 50% martensite and 50% austenite.



Continuous Cooling Transformation Diagrams

10.23 *Name the microstructural products of eutectoid iron–carbon alloy (0.76 wt% C) specimens that are first completely transformed to austenite, then cooled to room temperature at the following rates:*

- (a) 200°C/s ,
- (b) 100°C/s , and
- (c) 20°C/s .

Solution

We are called upon to name the microstructural products that form for specimens of an iron-carbon alloy of eutectoid composition that are continuously cooled to room temperature at a variety of rates. Figure 10.27 is used in these determinations.

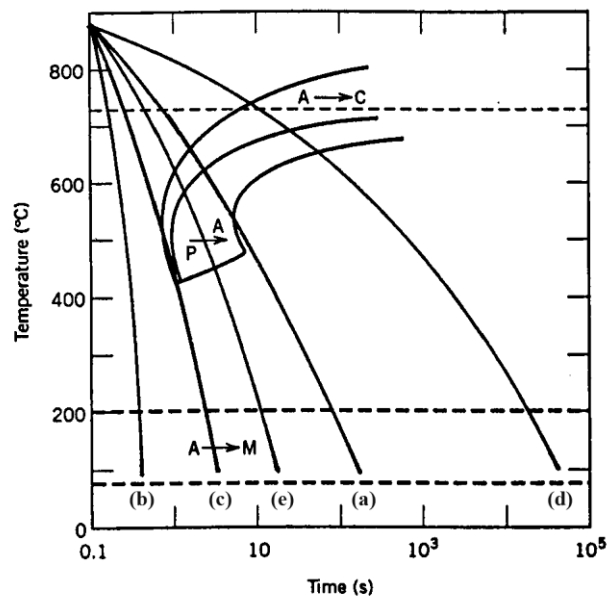
- (a) At a rate of 200°C/s , only martensite forms.
- (b) At a rate of 100°C/s , both martensite and pearlite form.
- (c) At a rate of 20°C/s , only fine pearlite forms.

10.24 Figure 10.40 shows the continuous cooling transformation diagram for a 1.13 wt% C iron-carbon alloy. Make a copy of this figure and then sketch and label continuous cooling curves to yield the following microstructures:

- (a) Fine pearlite and proeutectoid cementite
- (b) Martensite
- (c) Martensite and proeutectoid cementite
- (d) Coarse pearlite and proeutectoid cementite
- (e) Martensite, fine pearlite, and proeutectoid cementite

Solution

Below is shown a continuous cooling transformation diagram for a 1.13 wt% C iron-carbon alloy, with continuous cooling paths that will produce (a) fine pearlite and proeutectoid cementite; (b) martensite; (c) martensite and proeutectoid cementite; (d) coarse pearlite and proeutectoid cementite; and (e) martensite, fine pearlite, and proeutectoid cementite.



10.25 *Cite two important differences between continuous cooling transformation diagrams for plain carbon and alloy steels.*

Solution

Two important differences between continuous cooling transformation diagrams for plain carbon and alloy steels are: (1) for an alloy steel, a bainite nose will be present, which nose will be absent for plain carbon alloys; and (2) the pearlite-proeutectoid noses for plain carbon steel alloys are positioned at shorter times than for the alloy steels.

10.26 *Briefly explain why there is no bainite transformation region on the continuous cooling transformation diagram for an iron–carbon alloy of eutectoid composition.*

Solution

There is no bainite transformation region on the continuous cooling transformation diagram for an iron–carbon alloy of eutectoid composition (Figure 10.25) because by the time a cooling curve has passed into the bainite region, the entirety of the alloy specimen will have transformed to pearlite.

10.27 Name the microstructural products of 4340 alloy steel specimens that are first completely transformed to austenite, then cooled to room temperature at the following rates:

- (a) 10°C/s ,
- (b) 1°C/s ,
- (c) 0.1°C/s , and
- (d) 0.01°C/s .

Solution

This problem asks for the microstructural products that form when specimens of a 4340 steel are continuously cooled to room temperature at several rates. Figure 10.28 is used for these determinations.

- (a) At a cooling rate of 10°C/s , only martensite forms.
- (b) At a cooling rate of 1°C/s , both martensite and bainite form.
- (c) At a cooling rate of 0.1°C/s , martensite, proeutectoid ferrite, and bainite form.
- (d) At a cooling rate of 0.01°C/s , martensite, proeutectoid ferrite, pearlite, and bainite form.

10.28 *Briefly describe the simplest continuous cooling heat treatment procedure that would be used in converting a 4340 steel from one microstructure to another.*

- (a) *(Martensite + bainite) to (ferrite + pearlite)*
- (b) *(Martensite + bainite) to spheroidite*
- (c) *(Martensite + bainite) to (martensite + bainite + ferrite)*

Solution

This problem asks that we briefly describe the simplest continuous cooling heat treatment procedure that would be used in converting a 4340 steel from one microstructure to another. Solutions to this problem require the use of Figure 10.28.

(a) In order to convert from (martensite + bainite) to (ferrite + pearlite) it is necessary to heat above about 720°C, allow complete austenitization, then cool to room temperature at a rate slower than 0.006°C/s.

(b) To convert from (martensite + bainite) to spheroidite the alloy must be heated to about 700°C for several hours.

(c) In order to convert from (martensite + bainite) to (martensite + bainite + ferrite) it is necessary to heat to above about 720°C, allow complete austenitization, then cool to room temperature at a rate between 0.3°C/s and 0.02°C/s.

10.29 *On the basis of diffusion considerations, explain why fine pearlite forms for the moderate cooling of austenite through the eutectoid temperature, whereas coarse pearlite is the product for relatively slow cooling rates.*

Solution

For moderately rapid cooling, the time allowed for carbon diffusion is not as great as for slower cooling rates. Therefore, the diffusion distance is shorter, and thinner layers of ferrite and cementite form (i.e., fine pearlite forms).

Mechanical Behavior of Iron-Carbon Alloys

Tempered Martensite

10.30 *Briefly explain why fine pearlite is harder and stronger than coarse pearlite, which in turn is harder and stronger than spheroidite.*

Solution

The hardness and strength of iron-carbon alloys that have microstructures consisting of α -ferrite and cementite phases depend on the boundary area between the two phases. The greater this area, the harder and stronger the alloy inasmuch as (1) these boundaries impede the motion of dislocations, and (2) the cementite phase restricts the deformation of the ferrite phase in regions adjacent to the phase boundaries. Fine pearlite is harder and stronger than coarse pearlite because the alternating ferrite-cementite layers are thinner for fine, and therefore, there is more phase boundary area. The phase boundary area between the sphere-like cementite particles and the ferrite matrix is less in spheroidite than for the alternating layered microstructure found in coarse pearlite.

10.31 *Cite two reasons why martensite is so hard and brittle.*

Solution

Two reasons why martensite is so hard and brittle are: (1) there are relatively few operable slip systems for the body-centered tetragonal crystal structure, and (2) virtually all of the carbon is in solid solution, which produces a solid-solution hardening effect.

10.32 Rank the following iron–carbon alloys and associated microstructures from the highest to the lowest tensile strength:

- (a) 0.25 wt%C with spheroidite,
- (b) 0.25 wt%C with coarse pearlite,
- (c) 0.60 wt%C with fine pearlite, and
- (d) 0.60 wt%C with coarse pearlite.

Justify this ranking.

Solution

This problem asks us to rank four iron-carbon alloys of specified composition and microstructure according to hardness. This ranking is as follows:

0.60 wt% C, fine pearlite
0.60 wt% C, coarse pearlite
0.25 wt% C, coarse pearlite
0.25 wt% C, spheroidite

The 0.25 wt% C, coarse pearlite is stronger than the 0.25 wt% C, spheroidite since coarse pearlite is stronger than spheroidite; the composition of the alloys is the same. The 0.60 wt% C, coarse pearlite is stronger than the 0.25 wt% C, coarse pearlite, since increasing the carbon content increases the strength. Finally, the 0.60 wt% C, fine pearlite is stronger than the 0.60 wt% C, coarse pearlite inasmuch as the strength of fine pearlite is greater than coarse pearlite because of the many more ferrite-cementite phase boundaries in fine pearlite.

10.33 *Briefly explain why the hardness of tempered martensite diminishes with tempering time (at constant temperature) and with increasing temperature (at constant tempering time).*

Solution

This question asks for an explanation as to why the hardness of tempered martensite diminishes with tempering time (at constant temperature) and with increasing temperature (at constant tempering time). The hardness of tempered martensite depends on the ferrite-cementite phase boundary area; since these phase boundaries are barriers to dislocation motion, the greater the area the harder the alloy. The microstructure of tempered martensite consists of small sphere-like particles of cementite embedded within a ferrite matrix. As the size of the cementite particles increases, the phase boundary area diminishes, and the alloy becomes softer. Therefore, with increasing tempering time, the cementite particles grow, the phase boundary area decreases, and the hardness diminishes. As the tempering temperature is increased, the rate of cementite particle growth also increases, and the alloy softens, again, because of the decrease in phase boundary area.

10.34 Briefly describe the simplest heat treatment procedure that would be used in converting a 0.76 wt% C steel from one microstructure to the other, as follows:

- (a) Spheroidite to tempered martensite
- (b) Tempered martensite to pearlite
- (c) Bainite to martensite
- (d) Martensite to pearlite
- (e) Pearlite to tempered martensite
- (f) Tempered martensite to pearlite
- (g) Bainite to tempered martensite
- (h) Tempered martensite to spheroidite

Solution

In this problem we are asked to describe the simplest heat treatment that would be required to convert a eutectoid steel from one microstructure to another. Figure 10.27 is used to solve the several parts of this problem.

(a) For spheroidite to tempered martensite, austenitize at a temperature of about 760°C, quench to room temperature at a rate greater than about 140°C/s, then isothermally heat at a temperature between 250 and 650°C.

(b) For tempered martensite to pearlite, austenitize at a temperature of about 760°C, then cool to room temperature at a rate less than about 35°C/s.

(c) For bainite to martensite, first austenitize at a temperature of about 760°C, then quench to room temperature at a rate greater than about 140°C/s.

(d) For martensite to pearlite, first austenitize at a temperature of about 760°C, then cool to room temperature at a rate less than about 35°C/s.

(e) For pearlite to tempered martensite, first austenitize at a temperature of about 760°C, then rapidly quench to room temperature at a rate greater than about 140°C/s, then isothermally heat treat (temper) at a temperature between 250 and 650°C.

(f) For tempered martensite to pearlite, first austenitize at a temperature of about 760°C, then cool to room temperature at a rate less than about 35°C/s.

(g) For bainite to tempered martensite, first austenitize at a temperature of about 760°C, then rapidly quench to room temperature at a rate greater than about 140°C/s, then isothermally heat treat (temper) at a temperature between 250 and 650°C.

(h) For tempered martensite to spheroidite simply heat at about 700°C for approximately 20 h.

- 10.35 (a) *Briefly describe the microstructural difference between spheroidite and tempered martensite.*
(b) *Explain why tempered martensite is much harder and stronger.*

Solution

(a) Both tempered martensite and spheroidite have sphere-like cementite particles within a ferrite matrix; however, these particles are much larger for spheroidite.

(b) Tempered martensite is harder and stronger inasmuch as there is much more ferrite-cementite phase boundary area for the smaller particles; thus, there is greater reinforcement of the ferrite phase, and more phase boundary barriers to dislocation motion.

10.36 *Estimate the Rockwell hardnesses for specimens of an iron–carbon alloy of eutectoid composition that have been subjected to the heat treatments described in parts (b), (d), (f), (g), and (h) of Problem 10.18.*

Solution

This problem asks for estimates of Rockwell hardness values for specimens of an iron-carbon alloy of eutectoid composition that have been subjected to some of the heat treatments described in Problem 10.18.

(b) The microstructural product of this heat treatment is 100% spheroidite. According to Figure 10.30a, the hardness of a 0.76 wt% C alloy with spheroidite is about 87 HRB.

(d) The microstructural product of this heat treatment is 100% martensite. According to Figure 10.32, the hardness of a 0.76 wt% C alloy consisting of martensite is about 64 HRC.

(f) The microstructural product of this heat treatment is 100% bainite. From Figure 10.31, the hardness of a 0.76 wt% C alloy consisting of bainite is about 385 HB. And, conversion from Brinell to Rockwell hardness using Figure 6.18 leads to a hardness of 36 HRC.

(g) The microstructural product of this heat treatment is 100% fine pearlite. According to Figure 10.30a, the hardness of a 0.76 wt% C alloy consisting of fine pearlite is about 27 HRC.

(h) The microstructural product of this heat treatment is 100% tempered martensite. According to Figure 10.35, the hardness of a water-quenched eutectoid alloy that was tempered at 315°C for one hour is about 57 HRC.

10.37 *Estimate the Brinell hardnesses for specimens of a 0.45 wt% C iron-carbon alloy that have been subjected to the heat treatments described in parts (a), (d), and (h) of Problem 10.20.*

Solution

This problem asks for estimates of Brinell hardness values for specimens of an iron-carbon alloy of composition 0.45 wt% C that have been subjected to some of the heat treatments described in Problem 10.20.

(a) The microstructural product of this heat treatment is 100% martensite. According to Figure 10.32, the hardness of a 0.45 wt% C alloy consisting of martensite is about 630 HB.

(d) The microstructural product of this heat treatment is 100% spheroidite. According to Figure 10.30a the hardness of a 0.45 wt% C alloy with spheroidite is about 150 HB.

(h) The microstructural product of this heat treatment is proeutectoid ferrite and fine pearlite. According to Figure 10.30a, the hardness of a 0.45 wt% C alloy consisting of fine pearlite is about 200 HB.

10.38 *Determine the approximate tensile strengths for specimens of a eutectoid iron–carbon alloy that have experienced the heat treatments described in parts (a) and (c) of Problem 10.23.*

Solution

This problem asks for estimates of tensile strength values for specimens of an iron-carbon alloy of eutectoid composition that have been subjected to some of the heat treatments described in Problem 10.23.

(a) The microstructural product of this heat treatment is 100% martensite. According to Figure 10.32, the hardness of a 0.76 wt% C alloy is about 690 HB. For steel alloys, hardness and tensile strength are related through Equation 6.20a, and therefore

$$TS(\text{MPa}) = 3.45 \times HB = (3.45)(690 \text{ HB}) = 2380 \text{ MPa} \quad (345,000 \text{ psi})$$

(c) The microstructural product of this heat treatment is 100% fine pearlite. According to Figure 10.30a, the hardness of a 0.76 wt% C alloy consisting of fine pearlite is about 265 HB. Therefore, the tensile strength is

$$TS(\text{MPa}) = 3.45 \times HB = (3.45)(265 \text{ HB}) = 915 \text{ MPa} \quad (132,500 \text{ psi})$$

10.39 For a eutectoid steel, describe isothermal heat treatments that would be required to yield specimens having the following Rockwell hardnesses:

- (a) 93 HRB,
- (b) 40 HRC, and
- (c) 27 HRC.

Solution

For this problem we are asked to describe isothermal heat treatments required to yield specimens having several Brinell hardnesses.

(a) From Figure 10.30a, in order for a 0.76 wt% C alloy to have a Rockwell hardness of 93 HRB, the microstructure must be coarse pearlite. Thus, utilizing the isothermal transformation diagram for this alloy, Figure 10.22, we must rapidly cool to a temperature at which coarse pearlite forms (i.e., to about 675°C), allow the specimen to isothermally and completely transform to coarse pearlite. At this temperature an isothermal heat treatment for at least 200 s is required.

(b) This portion of the problem asks for a hardness of 40 HRC the microstructure could consist of either (1) about 75% fine pearlite and 25% martensite (Figure 10.32), or (2) tempered martensite (Figure 10.35).

For case (1), after austenitizing, rapidly cool to about 580°C (Figure 10.22), hold at this temperature for about 4 s (to obtain 75% fine pearlite), and then rapidly quench to room temperature.

For case (2), after austenitizing, rapidly cool to room temperature in order to achieve 100% martensite. Then temper this martensite for about 2000 s at 535°C (Figure 10.35).

(c) From Figure 10.30a, in order for a 0.76 wt% C alloy to have a Rockwell hardness of 27 HRC, the microstructure must be fine pearlite. Thus, utilizing the isothermal transformation diagram for this alloy, Figure 10.22, we must rapidly cool to a temperature at which fine pearlite forms (i.e., at about 580°C), allow the specimen to isothermally and completely transform to fine pearlite. At this temperature an isothermal heat treatment for at least 7 s is required.

DESIGN PROBLEMS

Continuous Cooling Transformation Diagrams Mechanical Behavior of Iron-Carbon Alloys

10.D1 *Is it possible to produce an iron-carbon alloy of eutectoid composition that has a minimum hardness of 90 HRB and a minimum ductility of 35%RA? If so, describe the continuous cooling heat treatment to which the alloy would be subjected to achieve these properties. If it is not possible, explain why.*

Solution

This problem inquires as to the possibility of producing an iron-carbon alloy of eutectoid composition that has a minimum hardness of 90 HRB and a minimum ductility of 35%RA. If the alloy is possible, then the continuous cooling heat treatment is to be stipulated.

According to Figures 10.30*a* and *b*, the following is a tabulation of Rockwell B hardnesses and percents area reduction for fine and coarse pearlites and spheroidite for a 0.76 wt% C alloy.

<u>Microstructure</u>	<u>HRB</u>	<u>%RA</u>
Fine pearlite	> 100	20
Coarse pearlite	93	28
Spheroidite	88	67

Therefore, none of the microstructures meets both of these criteria. Both fine and coarse pearlites are hard enough, but lack the required ductility. Spheroidite is sufficiently ductile, but does not meet the hardness criterion.

10.D2 *Is it possible to produce an iron-carbon alloy that has a minimum tensile strength of 690 MPa (100,000 psi) and a minimum ductility of 40%RA? If so, what will be its composition and microstructure (coarse and fine pearlites and spheroidite are alternatives)? If this is not possible, explain why.*

Solution

This problem asks if it is possible to produce an iron-carbon alloy that has a minimum tensile strength of 690 MPa (100,000 psi) and a minimum ductility of 40%RA. If such an alloy is possible, its composition and microstructure are to be stipulated.

From Equation 6.20a, this tensile strength corresponds to a Brinell hardness of

$$HB = \frac{TS \text{ (MPa)}}{3.45} = \frac{690 \text{ MPa}}{3.45} = 200$$

According to Figures 10.30a and b, the following is a tabulation of the composition ranges for fine and coarse pearlites and spheroidite that meet the stipulated criteria.

<u>Microstructure</u>	Compositions for <u>HB \geq 200</u>	Compositions for <u>%RA \geq 40%</u>
Fine pearlite	> 0.45 %C	< 0.47 %C
Coarse pearlite	> 0.7 %C	< 0.54 %C
Spheroidite	not possible	0-1.0 %C

Therefore, only fine pearlite has a composition range overlap for both of the hardness and ductility restrictions; the fine pearlite would necessarily have to have a carbon content between 0.45 and 0.47 wt% C.

10.D3 *It is desired to produce an iron-carbon alloy that has a minimum hardness of 175 HB and a minimum ductility of 52%RA. Is such an alloy possible? If so, what will be its composition and microstructure (coarse and fine pearlites and spheroidite are alternatives)? If this is not possible, explain why.*

Solution

This problem inquires as to the possibility of producing a iron-carbon alloy having a minimum hardness of 175 HB and a minimum ductility of 52%RA. The composition and microstructure are to be specified; possible microstructures include fine and coarse pearlites and spheroidite.

To solve this problem, we must consult Figures 10.30*a* and *b*. The following is a tabulation of the composition ranges for fine and coarse pearlites and spheroidite that meet the stipulated criteria.

<u>Microstructure</u>	Compositions for <u>HB \geq 175</u>	Compositions for <u>%RA \geq 52%</u>
Fine pearlite	> 0.36 %C	< 0.33 %C
Coarse pearlite	> 0.43 %C	< 0.40 %C
Spheroidite	> 0.70	<0-1.0 %C

Thus, only spheroidite has a composition overlap for both of hardness and ductility restrictions; the spheroidite would necessarily have to have a carbon content greater than 0.70 wt% C.

Tempered Martensite

10.D4 (a) *For a 1080 steel that has been water quenched, estimate the tempering time at 425°C (800°F) to achieve a hardness of 50 HRC.*

(b) *What will be the tempering time at 315°C (600°F) necessary to attain the same hardness?*

Solution

This problem asks us to consider the tempering of a water-quenched 1080 steel to achieve a hardness of 50 HRC. It is necessary to use Figure 10.35.

(a) The time necessary at 425°C is about 500 s.

(b) At 315°C, the time required (by extrapolation) is approximately 4×10^6 s (about 50 days).

10.D5 *An alloy steel (4340) is to be used in an application requiring a minimum tensile strength of 1380 MPa (200,000 psi) and a minimum ductility of 43%RA. Oil quenching followed by tempering is to be used. Briefly describe the tempering heat treatment.*

Solution

We are to consider the tempering of an oil-quenched 4340 steel. From Figure 10.34, for a minimum tensile strength of 1380 MPa (200,000 psi) a tempering temperature of less than 450°C (840°F) is required. Also, for a minimum ductility of 43%RA, tempering must be carried out at a temperature greater than about 400°C (750°F). Therefore, tempering must occur at between 400 and 450°C (750 and 840°F) for 1 h.

10.D6 *Is it possible to produce an oil-quenched and tempered 4340 steel that has a minimum yield strength of 1400 MPa (203,000 psi) and a ductility of at least 42%RA? If this is possible, describe the tempering heat treatment. If it is not possible, explain why.*

Solution

This problem asks if it is possible to produce an oil-quenched and tempered 4340 steel that has a minimum yield strength of 1400 MPa (203,000 psi) and a minimum ductility of 42%RA, and, if possible, to describe the tempering heat treatment. In Figure 10.34 is shown the tempering characteristics of this alloy. According to this figure, in order to achieve a minimum yield strength of 1400 MPa a tempering temperature of less than about 410°C is required. On the other hand, tempering must be carried out at greater than about 360°C for a minimum ductility of 42%RA. Therefore, an oil-quenched and tempered 4340 alloy possessing these characteristics is possible; tempering would be carried out at between 360°C and 410°C for 1 h.