

All-Pairs Shortest Path: Floyd-Warshall Algorithm

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Shortest-Paths Algorithms

- Several variants of shortest-path algorithms
 - Single-source all-destination
 - Dijkstra's algorithm
 - Bellman-Ford algorithm
 - All-pairs shortest paths
 - Floyd-Warshall algorithm

Floyd-Warshall Algorithm

○ Characteristics

- All-pairs shortest paths
- Based on DP
- Supports negative edge-weights, but not negative-weight cycles
- $O(n^3)$

Three-step DP Formula for Floyd-Warshall Algorithm

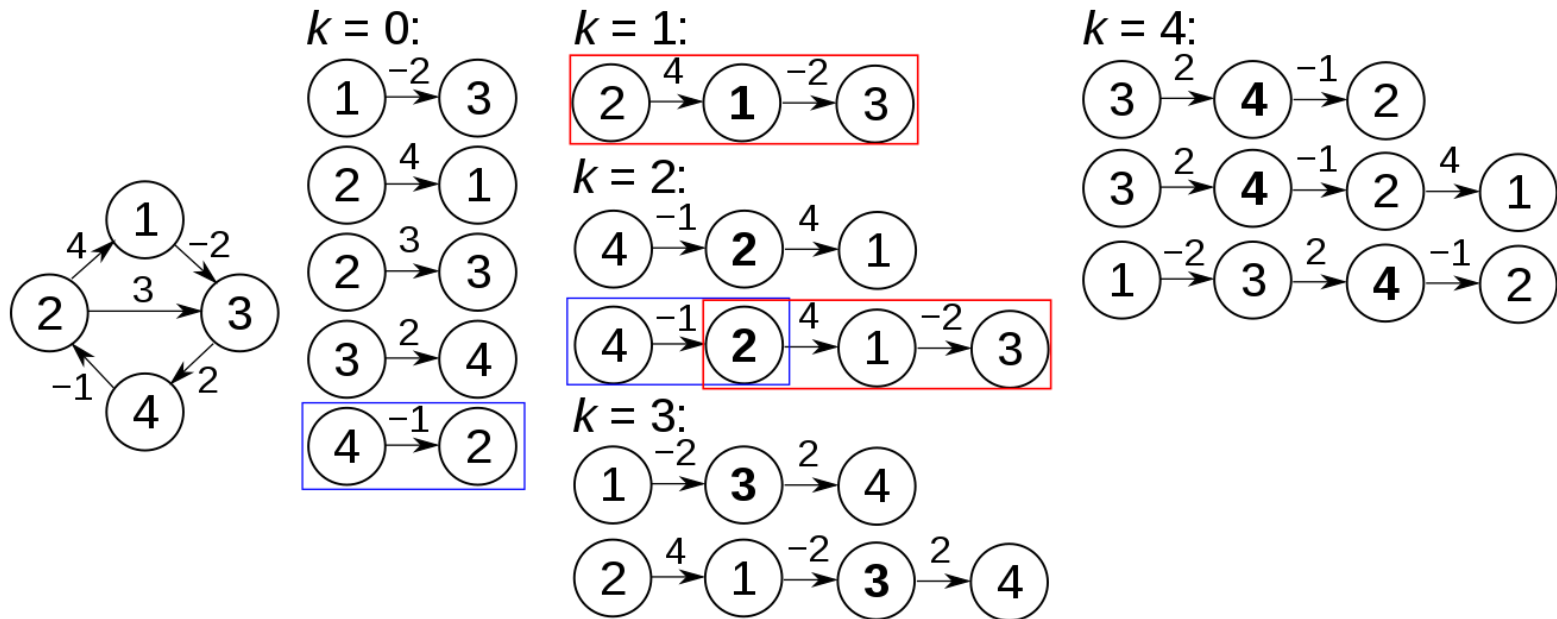
- Optimum-value function:** $d_{i,j}^k$ is the shortest path from nodes i to j using nodes only from $\{1, 2, \dots, k\}$ ($k \leq n$) as intermediate points
- Recurrent equation:**

$$d_{i,j}^k = \min \begin{cases} d_{i,j}^{k-1} \\ d_{i,k}^{k-1} + d_{k,j}^{k-1} \end{cases}, \text{ with } d_{i,j}^0 = w(i, j).$$

Weight between i and j

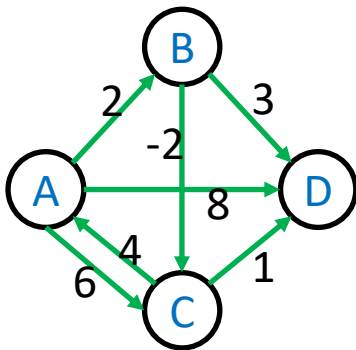
- Final answer:** $d_{i,j}^n$

An Illustrative Example



Example 1 (1/5)

Quiz!



Distance: {}

D^0

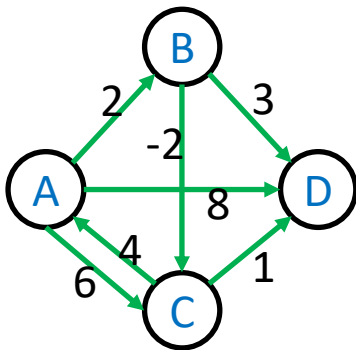
	A	B	C	D
A	0	2	6	8
B	∞	0	-2	3
C	4	∞	0	1
D	∞	∞	∞	0

Predecessor

π_0

	A	B	C	D
A	-	A	A	A
B	-	-	B	B
C	C	-	-	C
D	-	-	-	-

Example 1 (2/5)



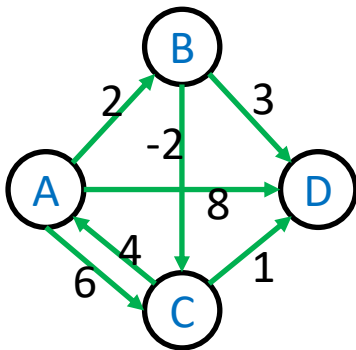
D^1 Distance: {A}

	A	B	C	D
A	0	2	6	8
B	∞	0	-2	3
C	4	6	0	1
D	∞	∞	∞	0

π_1 Predecessor

	A	B	C	D
A	-	A	A	A
B	-	-	B	B
C	C	A	-	C
D	-	-	-	-

Example 1 (3/5)



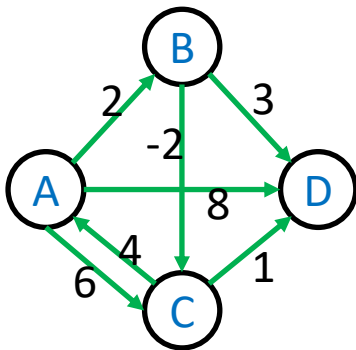
D^2 Distance: {A,B}

	A	B	C	D
A	0	2	0	5
B	∞	0	-2	3
C	4	6	0	1
D	∞	∞	∞	0

π_2 Predecessor

	A	B	C	D
A	-	A	B	B
B	-	-	B	B
C	C	A	-	C
D	-	-	-	-

Example 1 (4/5)



D^3 Distance: {A,B,C}

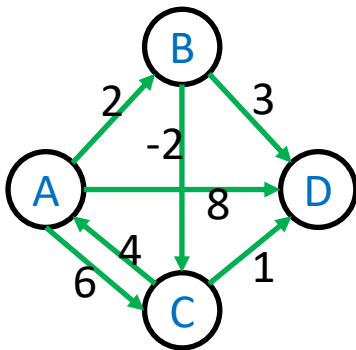
	A	B	C	D
A	0	2	0	1
B	2	0	-2	-1
C	4	6	0	1
D	∞	∞	∞	0

π_3

Predecessor

	A	B	C	D
A	-	A	B	C
B	C	-	B	C
C	C	A	-	C
D	-	-	-	-

Example 1 (5/5)



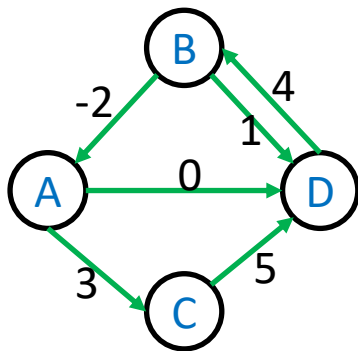
D^4 Distance: {A,B,C,D} π_4 Predecessor

	A	B	C	D
A	0	2	0	1
B	2	0	-2	-1
C	4	6	0	1
D	∞	∞	∞	0

	A	B	C	D
A	-	A	B	C
B	C	-	B	C
C	C	A	-	C
D	-	-	-	-

Example 2

Quiz!



Distance: {}

D^0

	A	B	C	D
A	0	∞	3	0
B	-2	0	∞	1
C	∞	∞	0	5
D	∞	4	∞	0

Predecessor

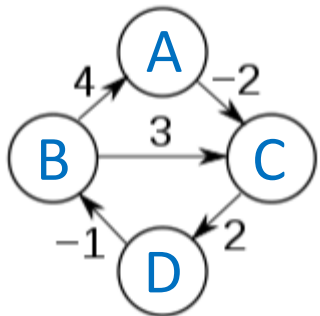
π_0

	A	B	C	D
A	-	-	A	A
B	B	-	-	B
C	-	-	-	C
D	-	D	-	-

Solution

Example 3

Quiz!



Distance: {}

D^0

	A	B	C	D
A	0	∞	-2	∞
B	4	0	3	∞
C	∞	∞	0	2
D	∞	-1	∞	0

Predecessor

π_0

	A	B	C	D
A	-	-	A	-
B	B	-	B	-
C	-	-	-	C
D	-	D	-	-

Solution

Reference

- Youtube tutorials
 - [Abdul Bari](#): Basics
 - [Joe James](#): Basics plus predecessors
 - [Erik Demaine](#): Comprehensive
- Web resources
 - [Chiu CC](#) (in Chinese): Detailed descriptions