B06505047 陳銘杰 HW6

7.6 $\omega(t) = x_1(t)x_2(t)$

$$W(j\omega) = \frac{1}{2\pi}[X_1(j\omega) * X_2(j\omega)]$$

$$\omega_M = \omega_1 + \omega_2 \, , \omega_N = 2\omega_M = 2(\omega_1 + \omega_2)$$

$$T = \frac{2\pi}{\omega_N} = \frac{2\pi}{2(\omega_1 + \omega_2)} = \frac{\pi}{\omega_1 + \omega_2}$$

7.23 $y(t) = x_1(t) * x_2(t)$

$$Y(j\omega) = X_1(j\omega)X_2(j\omega)$$

$$\omega_s > 2\omega_M \, , \omega_s > 2(1000\pi) = 2000\pi$$

$$\frac{2\pi}{T} > 2000\pi \;\; , T < 10^{-3}s$$

7.37 (a)

$$P(t) = p_1(t) + p_1(t - \Delta) \, , p_1(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{2\pi k}{W}\right)$$

$$P(j\omega) = \left(1 + e^{-j\Delta\omega}\right)\left(W \sum_{k=-\infty}^{\infty} \delta(\omega - kW)\right)$$

$$g(t) = p(t)f(t) = p_1(t)f(t) + p_1(t - \Delta)f(t) = ap_1(t) + bp_1(t - \Delta)$$

$$G(j\omega) = \left(a + be^{-j\Delta\omega}\right)P_1(j\omega) = W \sum_{k=-\infty}^{\infty} \left(a + be^{-jkW\Delta}\right)\delta(\omega - kW)$$

$$y_1(t) = x(t)p(t)f(t)$$

$$Y_1(j\omega) = \frac{1}{2\pi}\left(G(j\omega) * X(j\omega)\right) = \frac{W}{2\pi} \sum_{k=-\infty}^{\infty} \left(a + be^{-jkW\Delta}\right)X\left(j(\omega - W)\right)$$

$$Y_1(j\omega) = \frac{W}{2\pi}\left((a + b)X(j\omega) + \left(a + be^{-jW\Delta}\right)X\left(j(\omega - W)\right)\right)$$

$$Y_2(j\omega) = Y_1(j\omega)H_1(j\omega)$$

$$= \frac{jW}{2\pi}\Big((a+b)X(j\omega) + \big(\mathrm{a} + \mathrm{b}e^{-jW\Delta}\big)X\big(j(\omega - W)\big)\Big)$$

$$y_3(t) = x(t)p(t)$$

$$Y_3(j\omega) = \frac{W}{2\pi}\left(2X(j\omega) + \left(1 + e^{-j\Delta\omega}\right)X(j(\omega - W))\right)$$

7.37 (b)

$$\frac{jW}{2\pi}\Big((a+b)X(j\omega) + \big(\mathrm{a} + \mathrm{b}e^{-jW\Delta}\big)X\big(j(\omega - W)\big)\Big)$$

$$+\frac{W}{2\pi}\Big(2X(j\omega) + \left(1 + e^{-j\Delta\omega}\right)X\big(j(\omega - W)\big)\Big) = kX(j\omega)$$

$$\frac{W}{2\pi}\big((2 + \mathrm{ja} + \mathrm{jb})X(j\omega)\big)$$

$$+\frac{W}{2\pi}\Big(\left(1 + e^{-j\Delta\omega} + ja + jbe^{-j\Delta\omega}\right)X\big(j(\omega - W)\big)\Big) = kX(j\omega)$$

$$1 + e^{-j\Delta\omega} + ja + jbe^{-j\Delta\omega} = 0$$

$$\mathrm{W}\Delta = \frac{\pi}{2}\ , a = 1\ , b = -1$$

$$\mathrm{a} = \sin(\mathrm{W}\Delta) + \frac{1 + \cos(W\Delta)}{\tan(W\Delta)}$$

$$\mathrm{b} = -\frac{1 + \cos(W\Delta)}{\sin(W\Delta)}$$

$$\mathrm{k} = \frac{2\pi}{W}\left(\frac{1}{2 + ja + jb}\right)$$

7.50(a)

$$h_0[n] = u[n] - u[n - N]$$

7.50(b)

$$\mathrm{H}\left(e^{j\omega}\right)H_0\left(e^{j\omega}\right) = N \quad for|\omega| < \frac{\omega_s}{2}$$

$$\frac{\omega_s}{2} = \frac{\pi}{N}$$

$$H_0(e^{j\omega}) = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}$$

$$\mathrm{H}(e^{j\omega}) = \begin{cases} N\dfrac{1 - e^{-j\omega}}{1 - e^{-j\omega N}} & for\ |\omega| < \dfrac{\pi}{N} \\ 0 & for \dfrac{\pi}{N} < |\omega| \leq \pi \end{cases}$$

7.50(c)

$$h_1[n] = \frac{1}{N}\left[h_0[n] * h_0[-n]\right]$$

7.50(d)

$$H_1(e^{j\omega}) = \frac{1}{N}\left|H_0(e^{j\omega})\right|^2$$

$$\mathrm{H}(e^{j\omega}) = \begin{cases} N^2\left|\dfrac{1 - e^{-j\omega}}{1 - e^{-j\omega N}}\right|^2 & for\ |\omega| < \dfrac{\pi}{N} \\ 0 & for \dfrac{\pi}{N} < |\omega| \leq \pi \end{cases}$$