機率與統計 Exercise2 Chapter2 B06505047 陳銘杰

Problem 2.1.7

A : first photo detector is acceptable

B : second photo detector is acceptable

$P[A] = \frac{3}{5}$, $P[B|A] = \frac{4}{5}$, $P[B|A^C] = \frac{2}{5}$

$$=> P[A \cap B] = P[B|A] \times P[A] = \frac{12}{25}, P[A^C \cap B] = P[B|A^C] \times P[A^C] = \frac{4}{25}$$

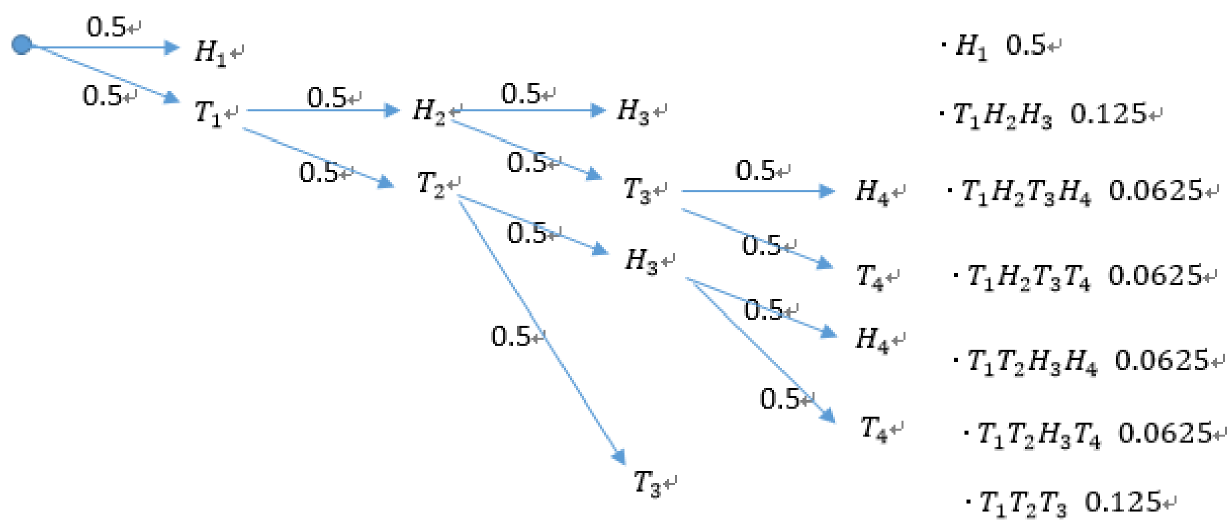
$$P[B] = P[A \cap B] + P[A^C \cap B] = \frac{16}{25}$$

(a) $P[A \cap B^C] + P[A^C \cap B] = P[A] - P[A \cap B] + \frac{4}{25} = \frac{3}{5} - \frac{12}{25} + \frac{4}{25} = \frac{7}{25}$

(b) $P[A^C \cap B^C] = 1 - P[A \cup B] = 1 - P[A] - P[B] + P[A \cap B] = 1 - \frac{3}{5} - \frac{16}{25} + \frac{12}{25} = \frac{6}{25}$

Problem 2.1.10

(a)



(b) $P[H_3] = P[T_1H_2H_3] + P[T_1T_2H_3] = 0.125 + 0.125 = 0.25$

$P[T_3] = P[T_1H_2T_3] + P[T_1T_2T_3] = 0.125 + 0.125 = 0.25$

(c) $P[D] = P[H_1] + P[T_1H_2H_3] + P[T_1H_2T_3H_4] + P[T_1T_2H_3H_4]$

$= 0.5 + 0.125 + 0.0625 + 0.0.625 = 0.75$

$P[H_1|D] = \frac{P[H_1 \cap D]}{P[D]} = \frac{P[H_1]}{P[D]} = \frac{0.5}{0.75} = \frac{2}{3}$

(d) $P[H_3] = 0.25$

$P[H_2] = 0.5 \times 0.5 = 0.25$

$P[H_3 \cap H_2] = P[T_1H_2H_3] = 0.5 \times 0.5 \times 0.5 = 0.125 \neq 0.25 \times 0.25$

=>H_3 and H_2 are not independent events

Problem 2.2.12

假設全部共有 n 個箱子，其中 5 個含有特殊標記，刮五顆球，若全部皆有標記則勝利。

令 W 為勝利的事件$P[W]=\frac{1}{\binom{n}{5}}=\frac{5!(n-5)!}{n!}=\frac{5\times4\times3\times2\times1}{n\times(n-1)\times(n-2)\times(n-3)\times(n-4)}$

$P[W]=0.01 \Rightarrow n\times(n-1)\times(n-2)\times(n-3)\times(n-4)=12000 \Rightarrow n=8.697$

n=8 時，勝利機率為 0.01786，n=9 時，勝利機率為 0.00794，故將遊戲設計為 9 個箱子裡，5 個有特殊標記，可以讓勝利機率最靠近 0.01。

Problem 2.3.3

$P[G=2,Y=1,R=2]=\left(\frac{7}{16}\right)^2\times\left(\frac{7}{16}\right)^2\times\frac{1}{8}\times\binom{5}{2}\times\binom{3}{2}\times\binom{1}{1}=\frac{2401}{524288}\times10\times3\times1=0.13739$

$P[G=R]=P[G=0,Y=5,R=0]+P[G=1,Y=3,G=1]+P[G=2,Y=1,R=2]$

$=\left(\frac{1}{8}\right)^5\times\binom{5}{5}+\left(\frac{7}{16}\right)\times\left(\frac{7}{16}\right)\times\left(\frac{1}{8}\right)^3\times\binom{5}{1}\times\binom{4}{1}\times\binom{3}{3}+\frac{72030}{524288}$

$=\frac{1}{32768}+\frac{980}{131072}+\frac{72030}{524288}=0.14489$

Problem 2.3.5

(a) $P[K]=P[G_1K]+P[G_2K]=\frac{1}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{3}=\frac{7}{18}$

(b) $P[K_1]=P[K]=\frac{7}{18}$

$P[K_2]=P[O_1T_1K_2]+P[O_1T_2K_2]+P[O_2T_1K_2]+P[O_2T_2K_2]$

(O_i代表第一個人從 i 組選出，T_i代表第二個人從 i 組選出)

$=\frac{1}{3}\times\frac{2}{8}\times\frac{1}{2}+\frac{1}{3}\times\frac{6}{8}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{8}\times\frac{1}{2}+\frac{2}{3}\times\frac{5}{8}\times\frac{1}{3}$

$=\frac{1}{24}+\frac{1}{12}+\frac{1}{8}+\frac{5}{36}=\frac{3+6+9+10}{72}=\frac{28}{72}=\frac{7}{18}=0.38888\ldots$

$P[K_1K_2]=P[O_1K_1T_1K_2]+P[O_1K_1T_2K_2]+P[O_2K_1T_1K_2]+P[O_2K_1T_2K_2]$

$=\frac{1}{3}\times\frac{1}{2}\times\frac{2}{8}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{2}\times\frac{6}{8}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{3}\times\frac{3}{8}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{3}\times\frac{5}{8}\times\frac{1}{3}$

$=\frac{1}{48}+\frac{1}{24}+\frac{1}{24}+\frac{5}{108}=\frac{9+18+18+20}{432}=\frac{65}{432}\neq\frac{7}{18}\times\frac{7}{18}$

K_1 and K_2 are not independent

(c) $P[M=5]=P[M=5|G_1]P[G_1]+P[M=5|G_2]P[G_2]$

$=\frac{3}{9}\times\binom{10}{5}\times\left(\frac{1}{2}\right)^5\times\binom{5}{5}\times\left(\frac{1}{2}\right)^5+\frac{6}{9}\times\binom{10}{5}\times\left(\frac{1}{3}\right)^5\times\binom{5}{5}\times\left(\frac{2}{3}\right)^5$

$=\frac{1}{3}\times252\times\frac{1}{32}\times1\times\frac{1}{32}+\frac{2}{3}\times252\times\frac{1}{243}\times1\times\frac{32}{243}$

$=\frac{252}{3072}+\frac{16128}{177147}=0.1731$