§7 Sample rate selection

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References:

1. Franklin, Powell, Workman, “Digital Control of Dynamic Systems”,3rd Ed., Adison-Wesley, 1998
2. R. Isermann, “Digital Control System”, Springer, 1989.

§7.1 Introduction

※ In general, performance is improved and cost increases with the sampling rate.

※ To maintain a better performance, word-length I/O transferring rate must be increased with the sampling rate.

In this chapter we shall investigate the following factors which determine a suitable sampling rate：

(1) fundamental limit imposed by Nyquist sampling theorem and time delay due to DAC

(2) required smoothness of time response

(3) required performance on disturbance rejection

(4) sensitivity to plant parameter variations

(5) noise and anti-aliasing filter

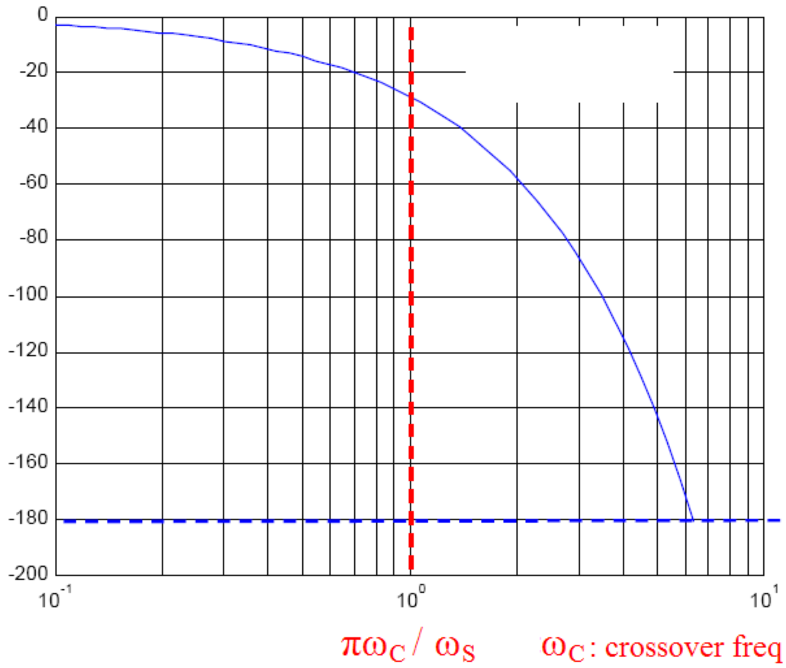
(6) advantages of multiple-sampling rate

7.2 The sampling rate theorem and delay due to DAC

Assuming that the higher harmonic components, in the command $r(t)$ for $\omega > \omega_r$ (resonance freq.) is negligible and can be treated as noise (high or low frequency) without causing any serious trouble, then Nyquist theorem requires

$$\omega_s > 2\omega_r \approx 2\omega_B \qquad \omega_B : \text{Bandwidth of closed-loop}$$

Furthermore, DAC causes a delay time of $T/2$, corresponding to additional phase lag shown below



It is clear from the above diagram that the phase delay introduced by ZOH at the crossover frequency is

$\omega_S \approx 12\omega_C$	$\omega_S \approx 6\omega_C$	$\omega_S \approx 3\omega_C$
5^o	13^o	30^o

Therefore, if $\omega_S > 12\ \omega_C$, the loss in phase margin is negligible. However, if $\omega_S < 6\ \omega_C$, significant loss in phase margin will occur causing difficulties in further compensation.

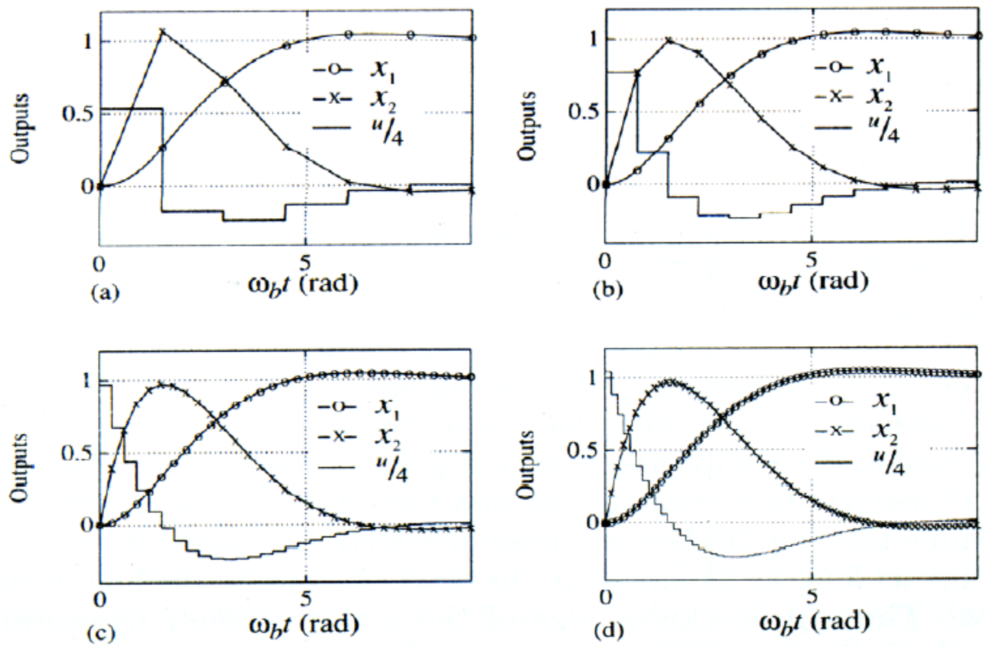
7.3 Time response and smoothness

In experience, to yield a smooth transient response it requires

$$6\omega_B \leqq \omega_S \leqq 40\omega_B$$

depending on the applications.

The following shows the smoothness for different ω_S



In addition, a pilot flying an airplane with digital stability- augmentation system will complain if the sampling time delay is about 0.1 second. To keep this delay to be within 10% of the rise time, it requires

$$0.1\times 1.8/\omega_N \approx 0.1\times t_r = T = \frac{\pi}{\omega_S}$$

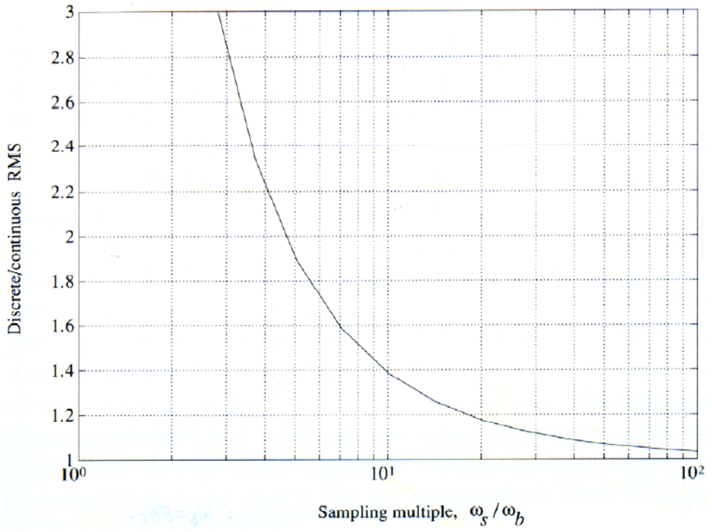
$$\omega_s \approx 17.5\omega_N \quad \text{or} \quad \omega_s \approx 20\,\omega_B$$

7.4 Errors due to random plant disturbance

Berman and Gran (1974) suggested that ω_S of aircraft control should be selected primary on the basis of disturbance rejection. It is indeed the same case for many applications since disturbance rejection is the primary concern in feedback control, not command following.

Let the disturbance be white noise, y_A and y_D be, respectively, the output due to disturbance for analog system and digital system, $1 > y_D/y_A$ reflects performance degradation due to digital implementation

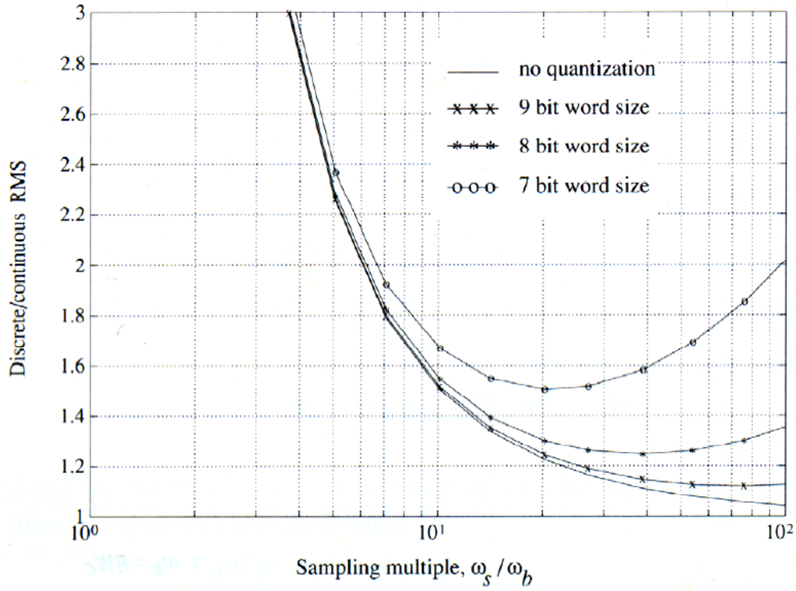
ω_S/ω_C	$\omega_S \approx 4\omega_C$	$\omega_S \approx 7\omega_C$	$\omega_S \approx 10\omega_C$	$\omega_S \approx 20\omega_C$	$\omega_S \approx 40\omega_C$
y_D/y_A	2.20	1.60	1.40	1.20	1.08



The above diagram shows identical to the previous table. It is clear that, to maintain a degradation below 20%, it requires

$\omega_s \approx 20\omega_B$ (distrubance rejection)

When ω_S is smaller than $20\omega_B$, degradation increases rapidly, whereas it gains only very little to increase ω_S further. It should be notice that disturbance rejection is improved by increasing ω_S only when the word length is sufficiently large as is shown below.



However, if a 16-bits or 32-bits microprocessor is used, the word length is generally sufficient. Hence the performance degradation due to quantization is negligible.

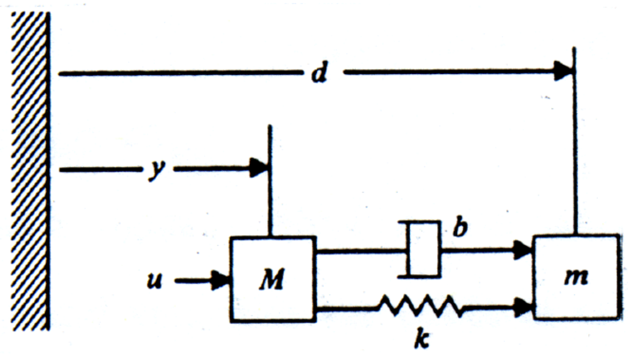
Based on the above discussions, a proper ω_S regarding disturbance rejection is suggested as followings：

$$6\ \omega_B \le \omega_S \le 20\ \omega_B \text{ (industry)}$$

$$20\ \omega_B \le \omega_S \le 40\ \omega_B \text{ (university Lab)}$$

■ Effect of plant resonance poles

Consider the two mass-spring system shown below



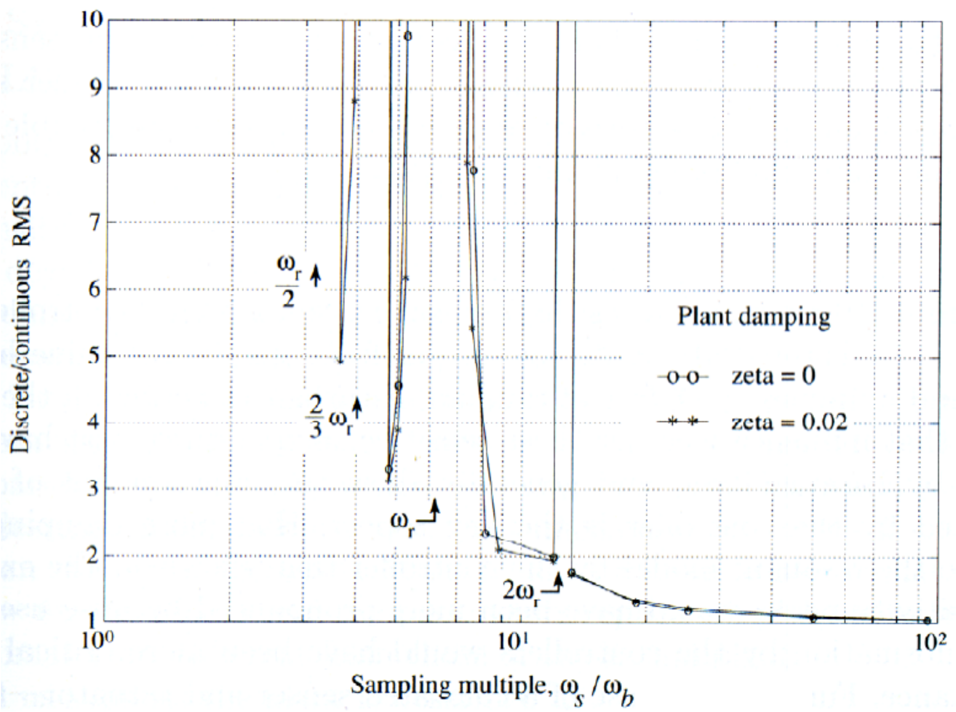
whose transfer function between input u(t) and the output d(t) is

$$\frac{d(s)}{u(s)} = \frac{1}{M}\,\frac{(bs+k)/m}{s^2\left[s^2+\frac{Mb+mb}{Mm}s+\frac{Mk+mk}{Mm}\right]}$$

There is a pair of lightly damped pole at

$$\omega_r^2 = \frac{(M+m)k}{Mm}$$

The analog control system has a bandwidth $\omega_B = \omega_r/6$ to add a slight damping ratio to dominant poles of the close-loop system. The following diagram compares the performance of the analog system with digital systems of different ω_S



Serious performance degradation occurs when

$$\omega_S = \omega_r /2,\ \omega_S = 2\omega_r /3,\ \omega_S = \omega_r, \text{ and } \omega_S = 2\omega_r$$

which means：at the above sampling rates, the resonance mode becomes unobservable in the digital control system, hence it fails.

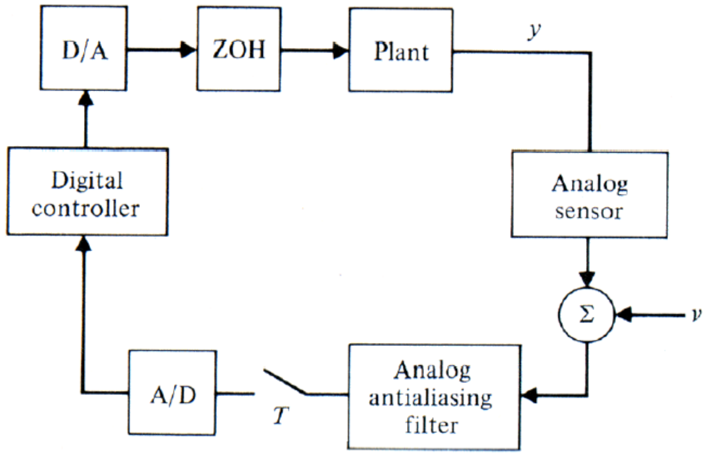
One may try to pick up a sampling rate the valley of the above curve, but it is very difficult to precisely identify the true resonance frequency, hence such an approach will not work.

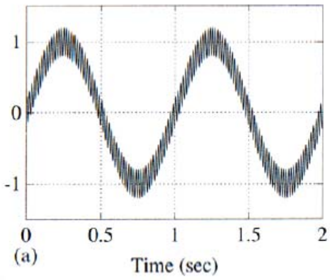
As a result, the only safe way is to require

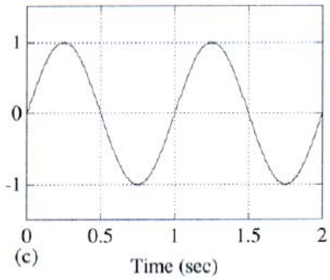
$$\omega_s > 2\omega_r$$

7.5：Anti-aliasing filters

The need for an anti-aliasing filter has been discussed before in Chpater 3. The following diagram shows the usual location of an anti-aliasing filter

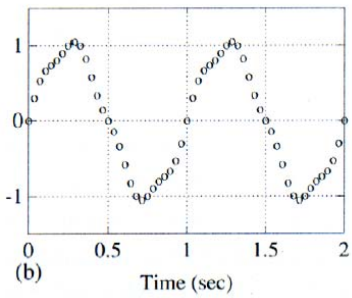


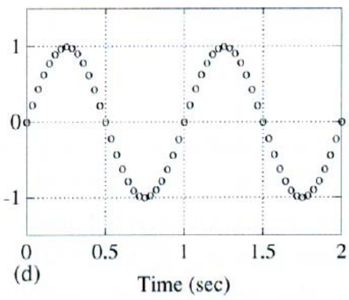




(a) continuous signal corrupted with noise

(b) signal after anti-aliasing filter





(b) sampled data without anti-aliasing filter

(d) sampled data after anti-aliasing filter

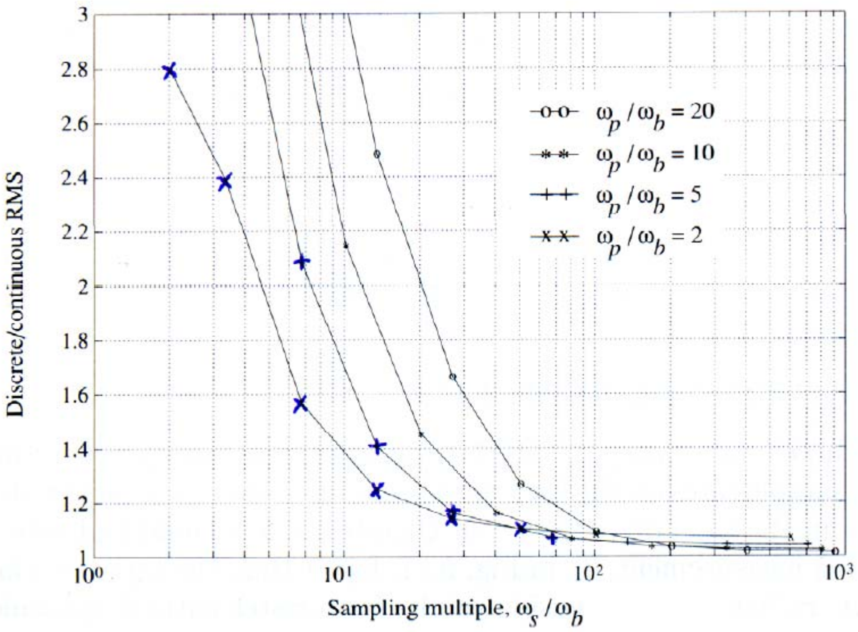
It is clear that an anti-aliasing is useful to eliminate high frequency noise in the sampled data. However, an anti-aliasing filter will introduce into the control loop additional phase lag. A conservative approach for remedy is to select an anti-aliasing filter of a corner frequency ω_P sufficiently large such that

$$\omega_s/2 >> \omega_p >> \omega_B$$

An alternative approach is to include the anti-aliasing filter into the plant model G(s) and to compensate the loss of phase in designing the controller. This approach allows us to select a ω_S

$$10\omega_B \leqq \omega_s \leqq 30\omega_B \qquad \text{(anti-aliasing)}$$

The following diagram compares the performance of an analog system and digital systems with various ω_p/ω_B and ω_s/ω_B



It is clear from the above diagram that a design with

$$2\omega_B \leq \omega_p \leq 5\omega_B \text{ and } \omega_s=20\omega_B$$

■ Conclusion

It is better to maintain $\omega_s=20\omega_B$ as long as hardware can support. 16bits to 32 bits are preferable. An anti-aliasing filter with a corner frequency $2\omega_B \leq \omega_p \leq 5\omega_B$ is also preferable.