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## Diffusion Mechanisms

- 5.3 (a) Compare interstitial and vacancy atomic mechanisms for diffusion.  
(b) Cite two reasons why interstitial diffusion is normally more rapid than vacancy diffusion.

### Solution

(a) With vacancy diffusion, atomic motion is from one lattice site to an adjacent vacancy. Self-diffusion and the diffusion of substitutional impurities proceed via this mechanism. On the other hand, atomic motion is from interstitial site to adjacent interstitial site for the interstitial diffusion mechanism.

(b) Interstitial diffusion is normally more rapid than vacancy diffusion because: (1) interstitial atoms, being smaller, are more mobile; and (2) the probability of an empty adjacent interstitial site is greater than for a vacancy adjacent to a host (or substitutional impurity) atom.

2(數字不同)

5.7 A sheet of steel 1.5 mm thick has nitrogen atmospheres on both sides at 1200°C and is permitted to achieve a steady-state diffusion condition. The diffusion coefficient for nitrogen in steel at this temperature is  $6 \times 10^{-11} \text{ m}^2/\text{s}$ , and the diffusion flux is found to be  $1.2 \times 10^{-7} \text{ kg/m}^2\text{-s}$ . Also, it is known that the concentration of nitrogen in the steel at the high-pressure surface is  $4 \text{ kg/m}^3$ . How far into the sheet from this high-pressure side will the concentration be  $2.0 \text{ kg/m}^3$ ? Assume a linear concentration profile.

### Solution

This problem is solved by using Equation 5.3 in the form

$$J = -D \frac{C_A - C_B}{x_A - x_B}$$

If we take  $C_A$  to be the point at which the concentration of nitrogen is  $4 \text{ kg/m}^3$ , then it becomes necessary to solve for  $x_B$ , as

$$x_B = x_A + D \left[ \frac{C_A - C_B}{J} \right]$$

Assume  $x_A$  is zero at the surface, in which case

$$\begin{aligned} x_B &= 0 + (6 \times 10^{-11} \text{ m}^2/\text{s}) \left[ \frac{4 \text{ kg/m}^3 - 2 \text{ kg/m}^3}{1.2 \times 10^{-7} \text{ kg/m}^2\text{-s}} \right] \\ &= 1 \times 10^{-3} \text{ m} = 1 \text{ mm} \end{aligned}$$

5.29 A diffusion couple similar to that shown in Figure 5.1a is prepared using two hypothetical metals A and B. After a 30-h heat treatment at 1000 K (and subsequently cooling to room temperature) the concentration of A in B is 3.2 wt% at the 15.5-mm position within metal B. If another heat treatment is conducted on an identical diffusion couple, only at 800 K for 30 h, at what position will the composition be 3.2 wt% A? Assume that the preexponential and activation energy for the diffusion coefficient are  $1.8 \times 10^{-5} \text{ m}^2/\text{s}$  and 152,000 J/mol, respectively.

#### Solution

In order to determine the position within the diffusion couple at which the concentration of A in B is 3.2 wt%, we must employ Equation 5.6b with  $t$  constant. That is

$$\frac{x^2}{D} = \text{constant}$$

Or

$$\frac{x_{800}^2}{D_{800}} = \frac{x_{1000}^2}{D_{1000}}$$

It is first necessary to compute values for both  $D_{800}$  and  $D_{1000}$ ; this is accomplished using Equation 5.8 as follows:

$$\begin{aligned} D_{800} &= (1.8 \times 10^{-5} \text{ m}^2/\text{s}) \exp \left[ -\frac{152,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(800 \text{ K})} \right] \\ &= 2.12 \times 10^{-15} \text{ m}^2/\text{s} \end{aligned}$$

$$\begin{aligned} D_{1000} &= (1.8 \times 10^{-5} \text{ m}^2/\text{s}) \exp \left[ -\frac{152,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1000 \text{ K})} \right] \\ &= 2.05 \times 10^{-13} \text{ m}^2/\text{s} \end{aligned}$$

Now, solving the above expression for  $x_{800}$  yields

$$\begin{aligned} x_{800} &= x_{1000} \sqrt{\frac{D_{800}}{D_{1000}}} \\ &= (15.5 \text{ mm}) \sqrt{\frac{2.12 \times 10^{-15} \text{ m}^2/\text{s}}{2.05 \times 10^{-13} \text{ m}^2/\text{s}}} \\ &= 1.6 \text{ mm} \end{aligned}$$

## 4

5.31 An FCC iron-carbon alloy initially containing 0.20 wt% C is carburized at an elevated temperature and in an atmosphere wherein the surface carbon concentration is maintained at 1.0 wt%. If after 49.5 h the concentration of carbon is 0.35 wt% at a position 4.0 mm below the surface, determine the temperature at which the treatment was carried out.

### Solution

This problem asks us to compute the temperature at which a nonsteady-state 49.5 h diffusion anneal was carried out in order to give a carbon concentration of 0.35 wt% C in FCC Fe at a position 4.0 mm below the surface. From Equation 5.5

$$\frac{C_x - C_0}{C_s - C_0} = \frac{0.35 - 0.20}{1.0 - 0.20} = 0.1875 = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Or

$$\operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 0.8125$$

Now it becomes necessary, using the data in Table 5.1 and linear interpolation, to determine the value of  $\frac{x}{2\sqrt{Dt}}$ .

Thus

$z$	$\text{erf}(z)$
0.90	0.7970
$y$	0.8125
0.95	0.8209

$$\frac{y - 0.90}{0.95 - 0.90} = \frac{0.8125 - 0.7970}{0.8209 - 0.7970}$$

From which

$$y = 0.9324$$

Thus,

$$\frac{x}{2\sqrt{Dt}} = 0.9324$$

And since  $t = 49.5 \text{ h}$  (178,200 s) and  $x = 4.0 \text{ mm}$  ( $4.0 \times 10^{-3} \text{ m}$ ), solving for  $D$  from the above equation yields

$$D = \frac{x^2}{(4t)(0.9324)^2}$$

$$= \frac{(4.0 \times 10^{-3} \text{ m})^2}{(4)(178,200 \text{ s})(0.869)} = 2.58 \times 10^{-11} \text{ m}^2/\text{s}$$

Now, in order to determine the temperature at which  $D$  has the above value, we must employ Equation 5.9a; solving this equation for  $T$  yields

$$T = \frac{Q_d}{R (\ln D_0 - \ln D)}$$

From Table 5.2,  $D_0$  and  $Q_d$  for the diffusion of C in FCC Fe are  $2.3 \times 10^{-5} \text{ m}^2/\text{s}$  and 148,000 J/mol, respectively.

Therefore

$$T = \frac{148,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K}) [\ln (2.3 \times 10^{-5} \text{ m}^2/\text{s}) - \ln (2.58 \times 10^{-11} \text{ m}^2/\text{s})]}$$

$$= 1300 \text{ K} = 1027^\circ\text{C}$$

## 5

6.18 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

### Solution

This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain  $\epsilon_x$  as

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{20.025 \text{ mm} - 20.000 \text{ mm}}{20.000 \text{ mm}} = 1.25 \times 10^{-3}$$

In order to determine the longitudinal strain  $\epsilon_z$  we need Poisson's ratio, which may be computed using Equation 6.9; solving for  $\nu$  yields

$$\nu = \frac{E}{2G} - 1 = \frac{105 \times 10^3 \text{ MPa}}{(2)(39.7 \times 10^3 \text{ MPa})} - 1 = 0.322$$

Now  $\epsilon_z$  may be computed from Equation 6.8 as

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{1.25 \times 10^{-3}}{0.322} = -3.88 \times 10^{-3}$$

Now solving for  $l_0$  using Equation 6.2

$$\begin{aligned} l_0 &= \frac{l_i}{1 + \epsilon_z} \\ &= \frac{74.96 \text{ mm}}{1 - 3.88 \times 10^{-3}} = 75.25 \text{ mm} \end{aligned}$$

## 6

6.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than  $7.5 \times 10^{-3}$  mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

Material	Modulus of Elasticity (GPa)	Yield Strength (MPa)	Poisson's Ratio
Aluminum alloy	70	200	0.33
Brass alloy	101	300	0.34
Steel alloy	207	400	0.30
Titanium alloy	107	650	0.34

### Solution

This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 27,500 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left( \frac{d_0}{2} \right)^2} = \frac{27,500 \text{ N}}{\pi \left( \frac{10 \times 10^{-3} \text{ m}}{2} \right)^2} = 350 \times 10^6 \text{ N/m}^2 = 350 \text{ MPa}$$

Of the alloys listed, the Ti and steel alloys have yield strengths greater than 350 MPa.

Relative to the second criterion (i.e., that  $\Delta d$  be less than  $7.5 \times 10^{-3}$  mm), it is necessary to calculate the change in diameter  $\Delta d$  for these three alloys. From Equation 6.8

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}} = -\frac{E \Delta d}{\sigma d_0}$$

Now, solving for  $\Delta d$  from this expression,

$$\Delta d = -\frac{\nu \sigma d_0}{E}$$

For the steel alloy

$$\Delta d = -\frac{(0.30)(350 \text{ MPa})(10 \text{ mm})}{207 \times 10^3 \text{ MPa}} = -5.1 \times 10^{-3} \text{ mm}$$

Therefore, the steel is a candidate.

For the Ti alloy

$$\Delta d = -\frac{(0.34)(350 \text{ MPa})(10 \text{ mm})}{107 \times 10^3 \text{ MPa}} = -11.1 \times 10^{-3} \text{ mm}$$

Hence, the titanium alloy is *not* a candidate.