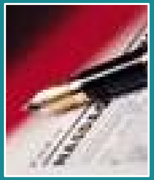
  

Chapter 1
Signals and Systems

1.1.1Examples and Mathematical Representation

- Signals may describe a wide variety of physical phenomena. Although signals can be represented in many ways, in all cases the information in a signal in contained in a pattern of variations of some form.

1.1.1Examples and Mathematical Representation

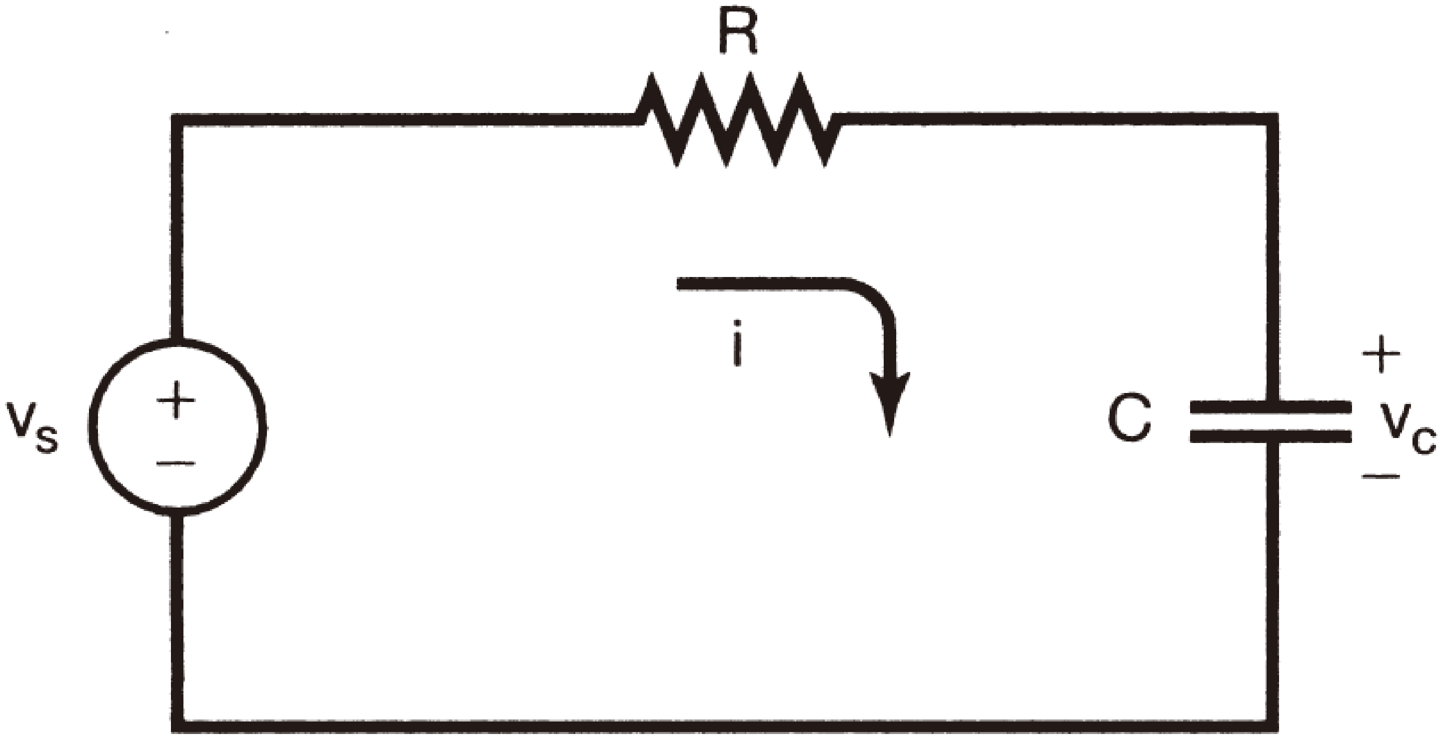


Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

1.1.1Examples and Mathematical Representation

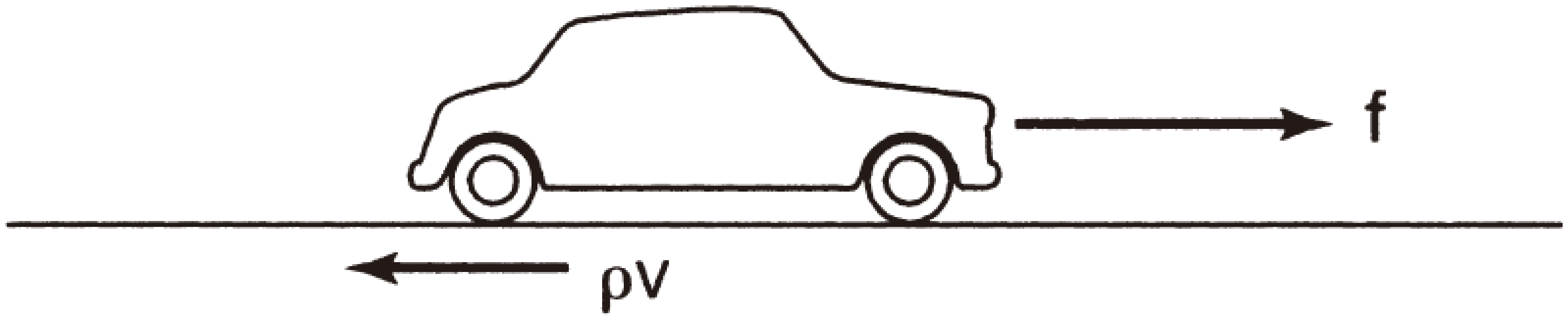


Figure 1.2 An automobile responding to an applied force f from the engine and to a retarding frictional force ρv proportional to the automobile's velocity v.

1.1.1Examples and Mathematical Representation

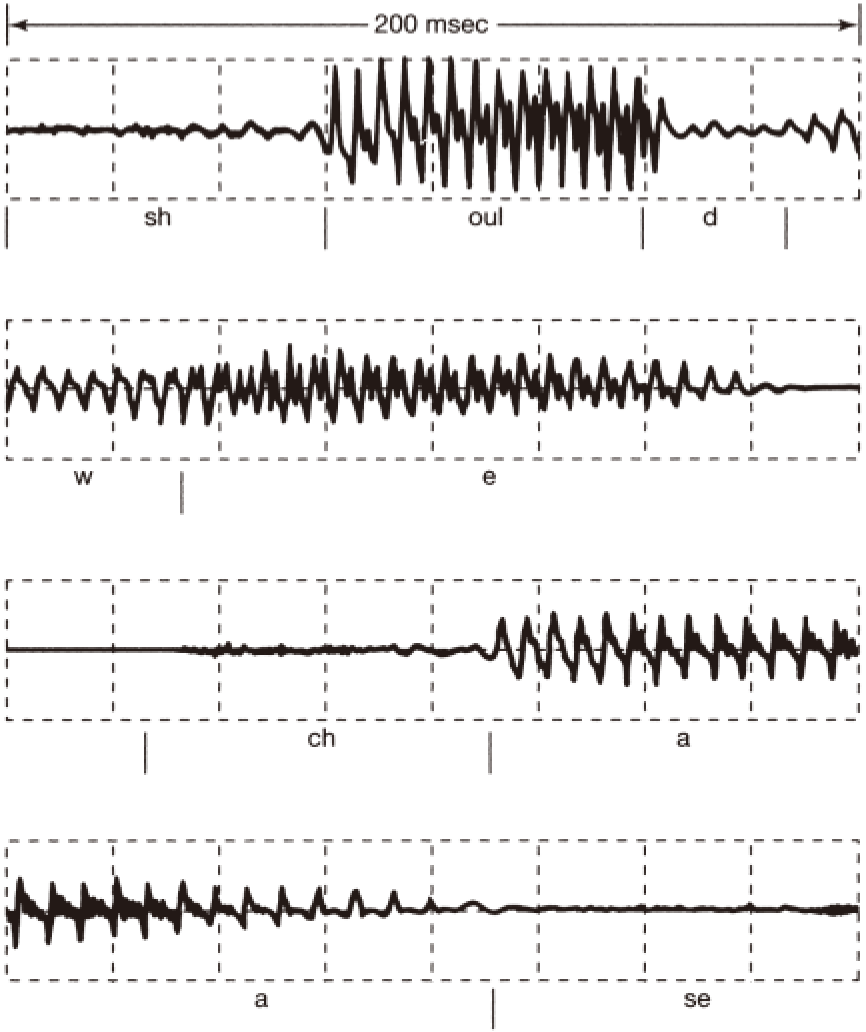


Figure 1.3 Example of a recording of speech. [Adapted from *Applications of Digital Signal Processing*, A.V. Oppenheim, ed. (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1978), p. 121.] The signal represents acoustic pressure variations as a function of time for the spoken words "should we chase." The top line of the figure corresponds to the word "should," the second line to the word "we," and the last two lines to the word "chase." (We have indicated the approximate beginnings and endings of each successive sound in each word.)

1.1.1Examples and Mathematical Representation

- Signals are represented mathematically as functions of one or more independent variables.
  訊號可用數學方式表示為具有一個或數個獨立變數的函數
- For example, a speech signal can be represented mathematically by acoustic pressure as a function of time, and a picture can be represented by brightness as a function of two spatial variables.

1.1.1Examples and Mathematical Representation

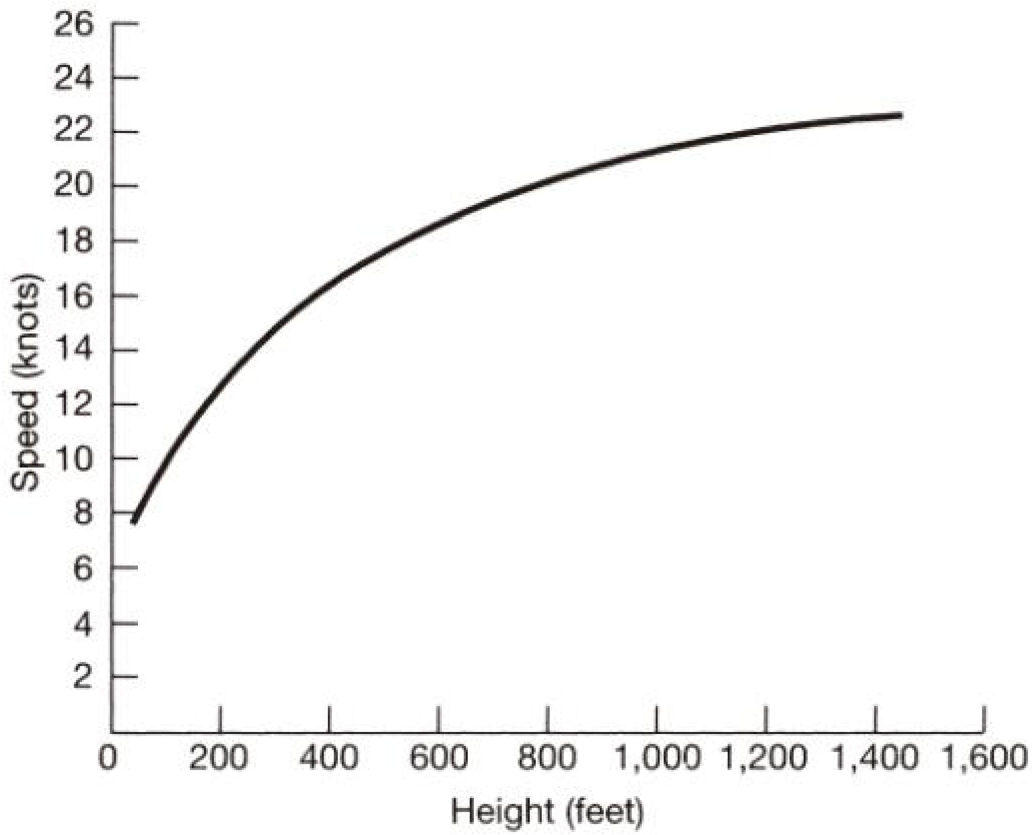


Figure 1.5 Typical annual vertical wind profile. (Adapted from Crawford and Hudson, National Severe Storms Laboratory Report, ESSA ERLTM-NSSL 48, August 1970.)

1.1.1Examples and Mathematical Representation

- Two basic types of signals
  - Continuous-time signals (連續時間訊號)
    - The independent variable is continuous, and thus these signals are defined for a continuum of values of the independent variable.
      連續時間的獨立變數為連續的，所以訊號的定義是在毒力變數軸(時間軸)上連續的數值
  - Discrete-time signals (離散時間訊號)
    - Fore these signals, the independent variable takes on only a discrete set of values.
      離散時間訊號只定義在離散的時間點上所得的一組離散的數值

1.1.1Examples and Mathematical Representation

Figure 1.6 An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

1.1.1Examples and Mathematical Representation

- To distinguish between continuous-time and discrete-time signals, we will use the symbol t to denote the continuous-time independent variable and n to denote the discrete-time independent variable.

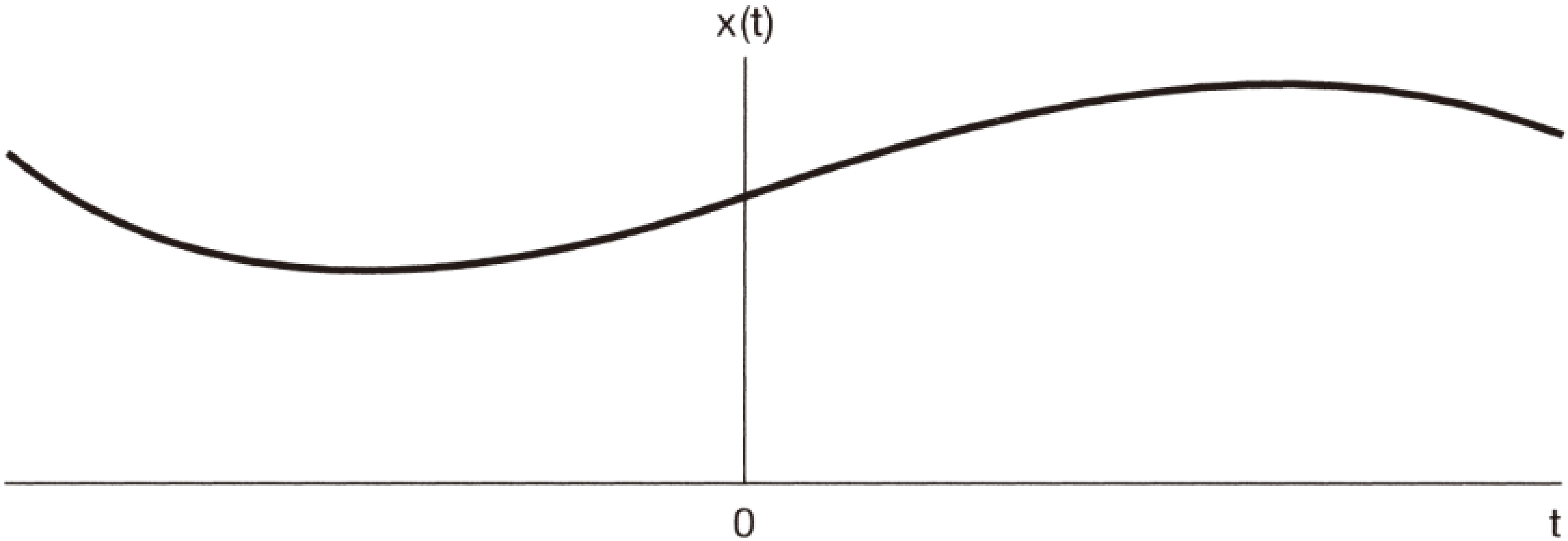
  為了有所區別，我們以t代表連續時間的獨立變數，離散時間的獨立變數為n

1.1.1Examples and Mathematical Representation

- For continuous-time signals we will enclose the independent variable in parentheses (・), whereas for discrete-time signals we will use brackets[・]

  連續時間的訊號以小括號(・)表示；離散時間的訊號則以中括號表示[・]
- A discrete-time signal x[n] may represent a phenomenon for which the independent variable is inherently discrete.

1.1.1Examples and Mathematical Representation



(a)

1.1.1Examples and Mathematical Representation

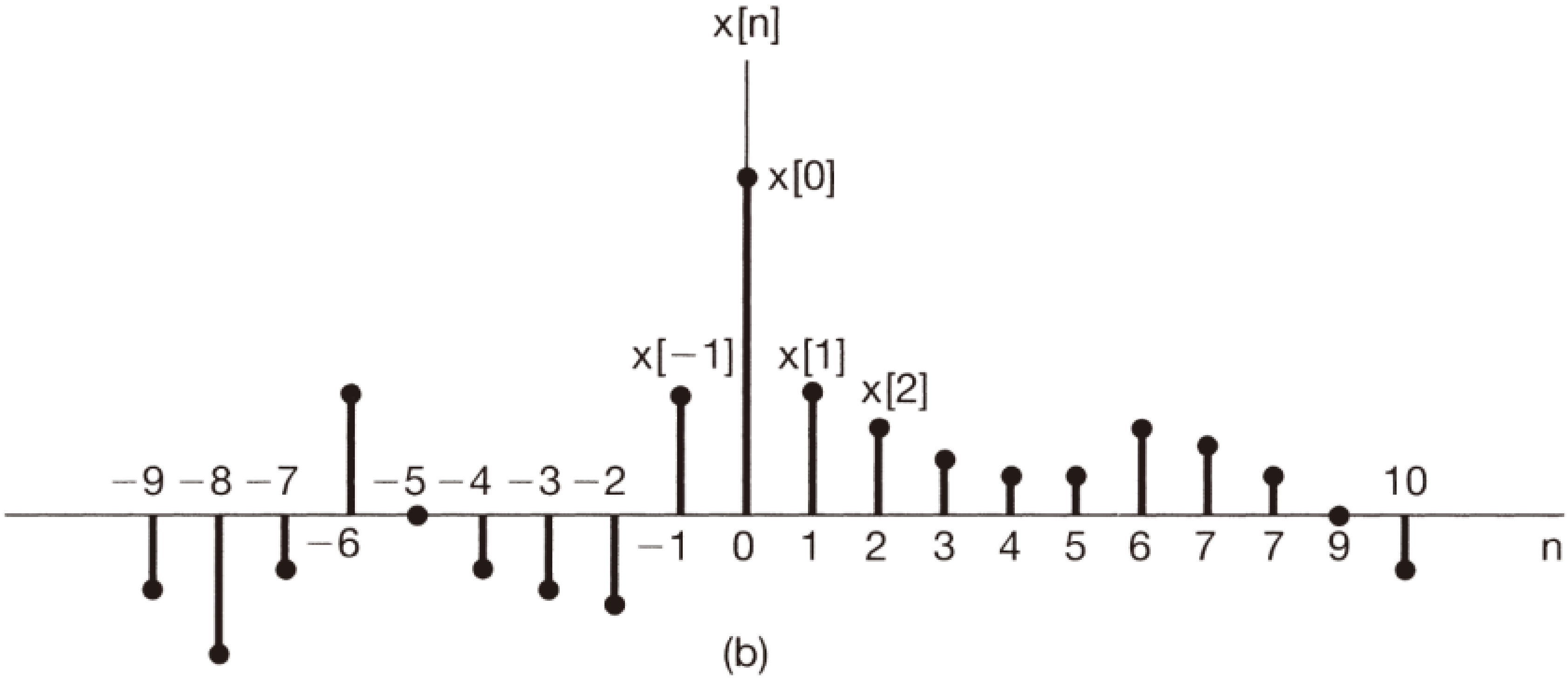


Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

1.1.2 Signal Energy and Power

- The signals we consider are directly related to physical quantities capturing power and energy in a physical system.
- If $v(t)$ and $i(t)$ are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t) \qquad (1.1)$$

1.1.2 Signal Energy and Power

The total energy expended over the time interval is $t_1 \le t \le t_2$

$$\int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt \qquad (1.2)$$

and the average power over this time interval is

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt \qquad (1.3)$$

1.1.2 Signal Energy and Power

The total energy over the time interval $t_1 \le t \le t_2$ in a continuous-time signal $x(t)$ is defined as

$$\int_{t_1}^{t_2} \left| x(t) \right|^2 dt \qquad (1.4)$$

where $|x|$ denotes the magnitude of number x.

1.1.2 Signal Energy and Power

The total energy in a discrete-time signal x[n] over the time interval $n_1 \le n \le n_2$ is defined as

$$\sum_{n=n_1}^{n_2} \left| x[n] \right|^2 \qquad (1.5)$$

and dividing by the number of points in the interval, $n_2 - n_1 + 1$, yields the average power over the interval.

1.1.2 Signal Energy and Power

We define the total energy as limits of eqs.(1.4) and (1.5) as the time interval increases without bound. In continuous time,

$$E_{\infty} \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad (1.6)$$

and in discrete time,

$$E_{\infty} \triangleq \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \qquad (1.7)$$

1.1.2 Signal Energy and Power

In an analogous fashion, we can define the time-averaged power over an infinite interval as

$$p_{\infty} \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| x(t) \right|^2 dt \qquad (1.8)$$

and

$$p_{\infty} \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} \left| x[n] \right|^2 \qquad (1.9)$$

1.1.2 Signal Energy and Power

We see from eq.(1.8) that

$$p_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0 \qquad (1.10)$$

An example of finite-energy signal is a signal that takes on the value 1 for $0 \le t \le 1$ and 0 otherwise. In this case,

$$E_\infty = 1 \text{ and } P_\infty = 0.$$

1.2 Transformations of the Independent Variable

- Focus on a very limited but important class of elementary signal transformations that involve simple modification of independent variable
- These elementary transformations allow us to introduce several basic properties of signals and system.
- We will find that they also play an important role in defining and characterizing far richer and important classes of systems.

1.2.1 Examples of Transformations of the Independent Variable

- A simple and very important example of transforming the independent variable of a signal
  - Time shift
  - Time reversal
  - Time scaling

1.2.1 Examples of Transformations of the Independent Variable

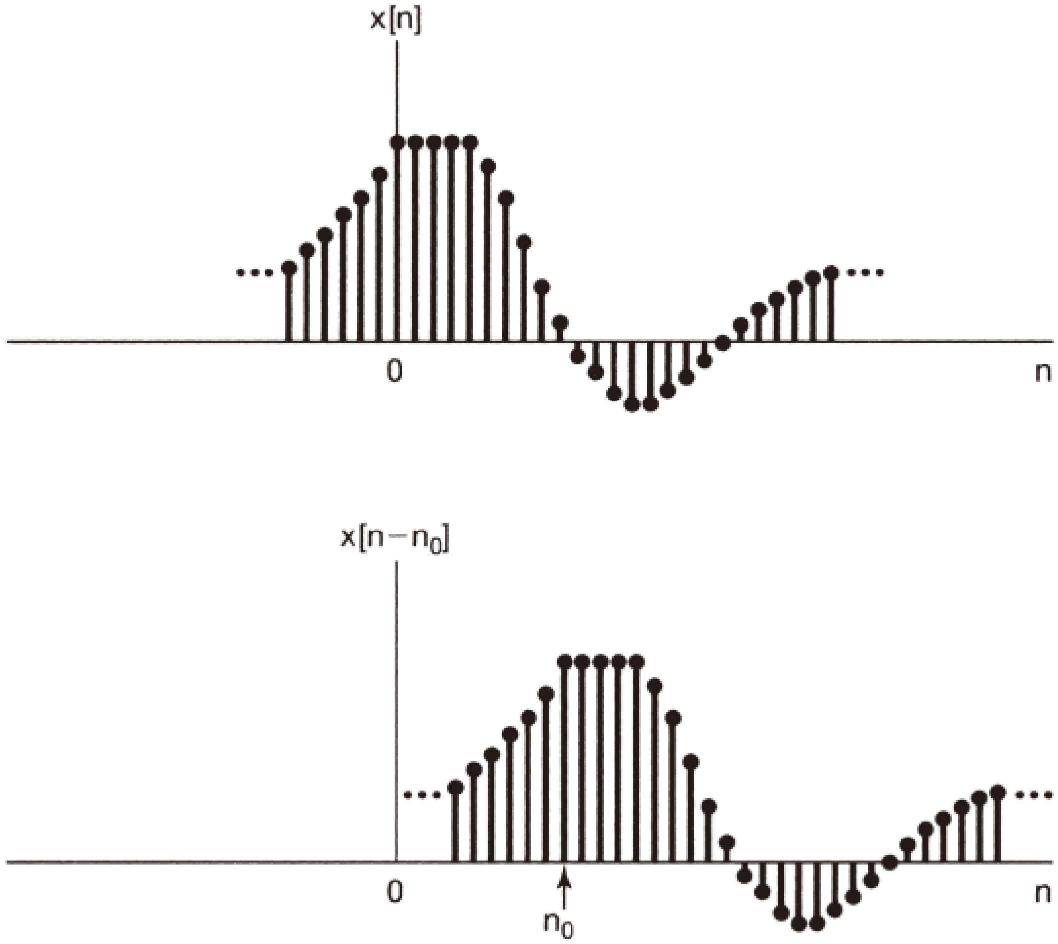


圖 1.8 爲離散時間訊號的時間移位。

Figure 1.8 Discrete-time signals related by a time shift. In this figure $n_0 > 0$, so that $x[n - n_0]$ is a delayed verson of $x[n]$ (i.e., each point in $x[n]$ occurs later in $x[n - n_0]$).

1.2.1 Examples of Transformations of the Independent Variable

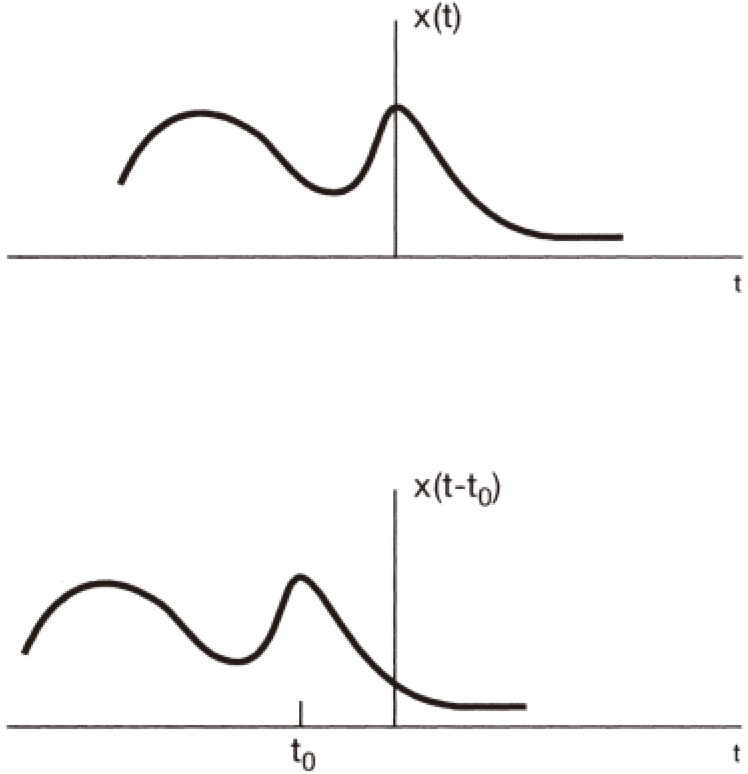


Figure 1.9 Continuous-time signals related by a time shift. In this figure $t_0 < 0$, so that $x(t - t_0)$ is an advanced version of $x(t)$ (i.e., each point in $x(t)$ occurs at an earlier time in $x(t - t_0)$).

1.2.1 Examples of Transformations of the Independent Variable

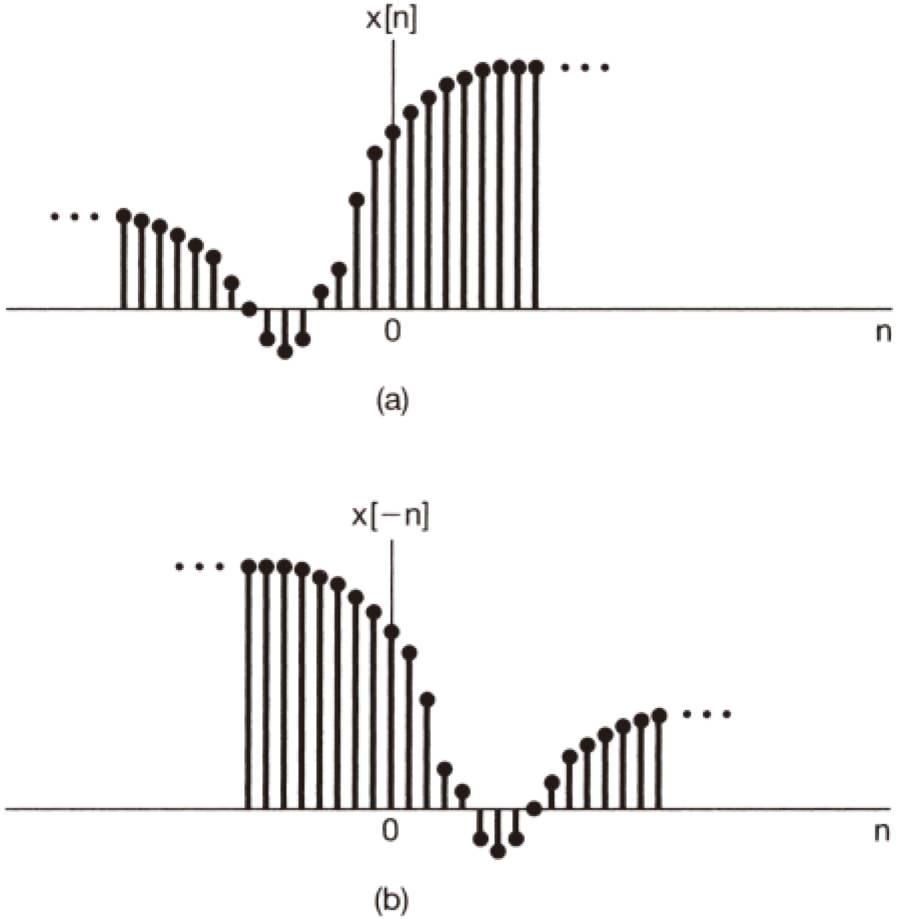


Figure 1.10 (a) A discrete-time signal $x[n]$; (b) its reflection $x[-n]$ about $n = 0$.

1.2.1 Examples of Transformations of the Independent Variable

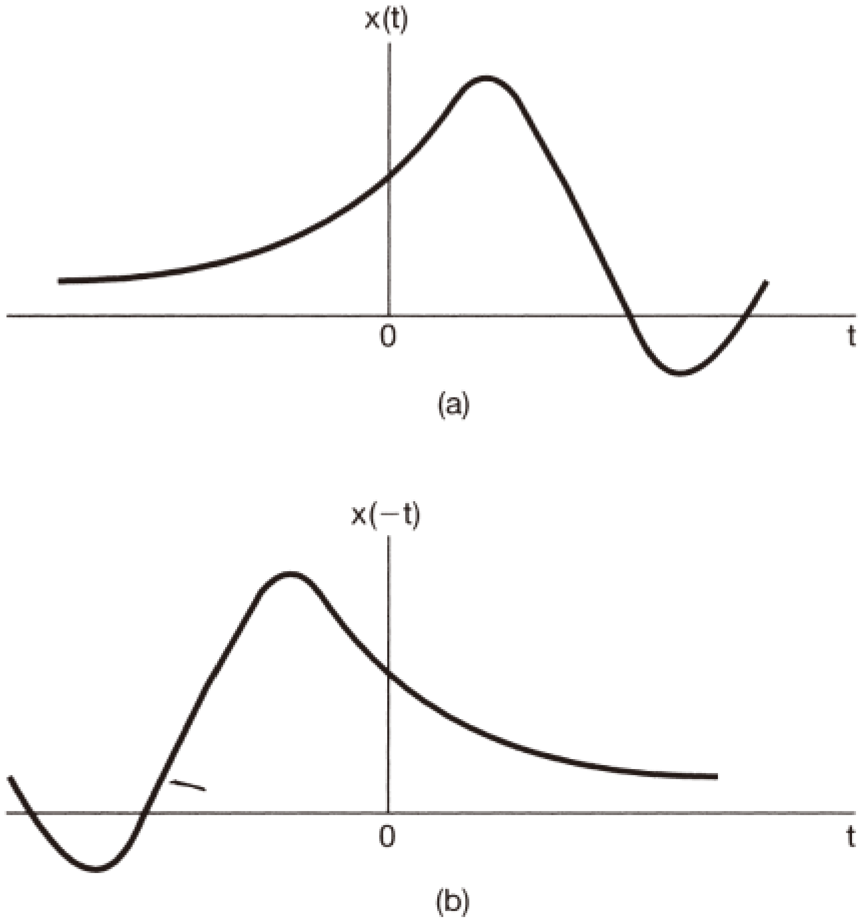


Figure 1.11 (a) A continuous-time signal $x(t)$; (b) its reflection $x(-t)$ about $t = 0$.

1.2.1 Examples of Transformations of the Independent Variable

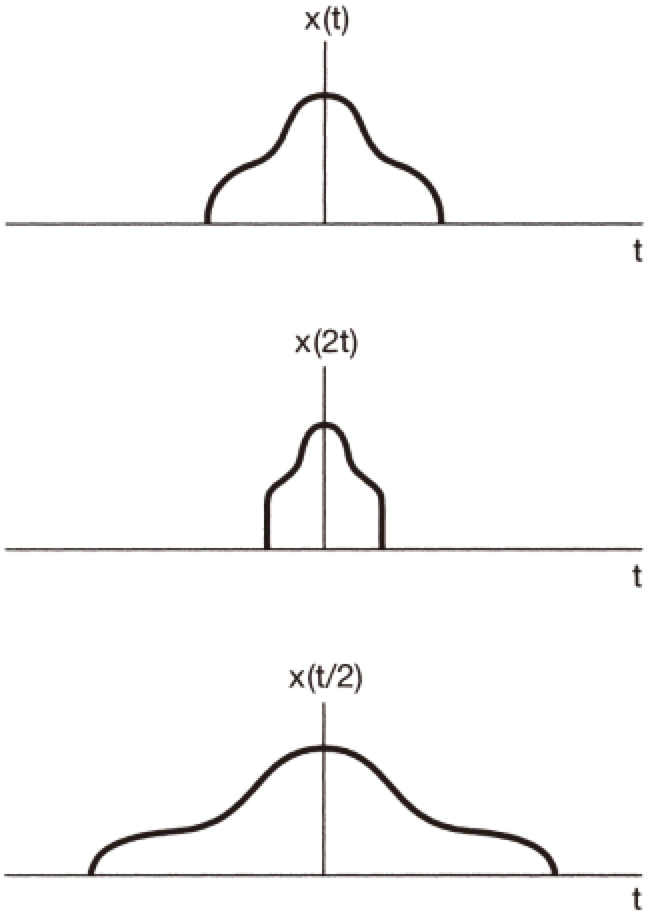


Figure 1.12 Continuous-time signals related by time scaling.

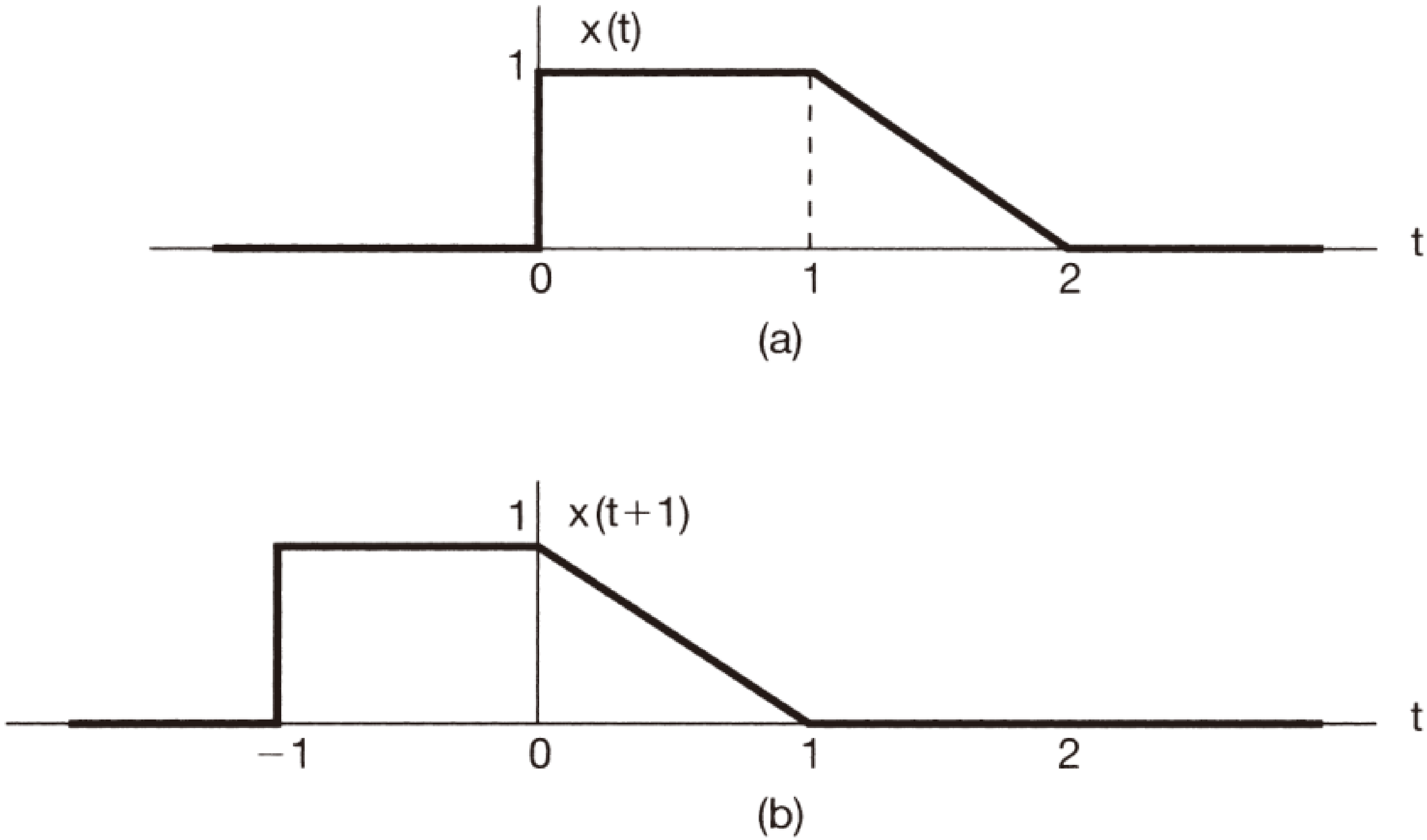
Example 1.1

Given the signal $x(t)$ shown in Figure 1.13(a), the signal $x(t+1)$ corresponds to an advance by one unit along the t axis as illustrated in Figure 1.13(b). Specifically, we note that the value of $x(t)$ at $t = t_0$ occurs in $x(t+1)$ at $t = t_0 - 1$. For example, the value of $x(t)$ at $t = 1$ is found in $x(t+1)$ at $t = 1-1=0$. Also, since $x(t)$ is zero for $t < 0$, we have $x(t+1)$ zero for $t < -1$. Similarly, since $x(t)$ is zero for $t > 2$, x(t+1) is zero for $t > 1$

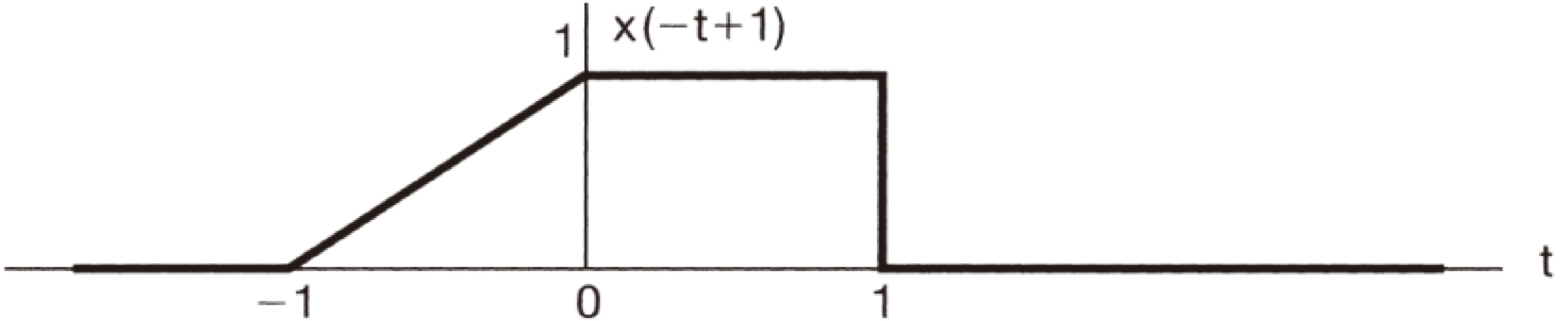
Example 1.1

Let us also consider the signal $x(-t+1)$, which may be obtained by replacing t with $-t$ in $x(t+1)$. That is, $x(-t+1)$ is the time reversed version of $x(t+1)$. Thus, $x(-t+1)$ may be obtained graphically be reflecting $x(t+1)$ about the t axis as shown in Figure 1.13(c)

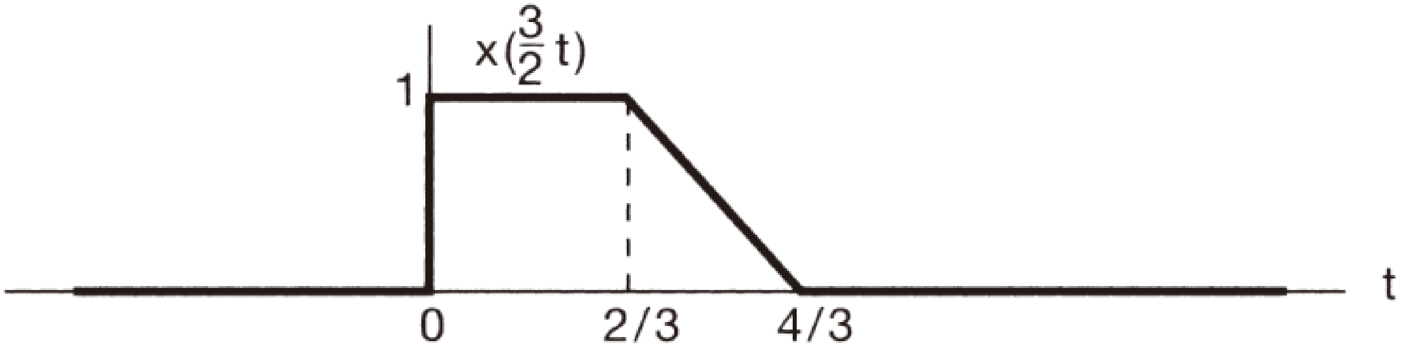
Example 1.1



Example 1.1



(c)



(d)

Example 1.1

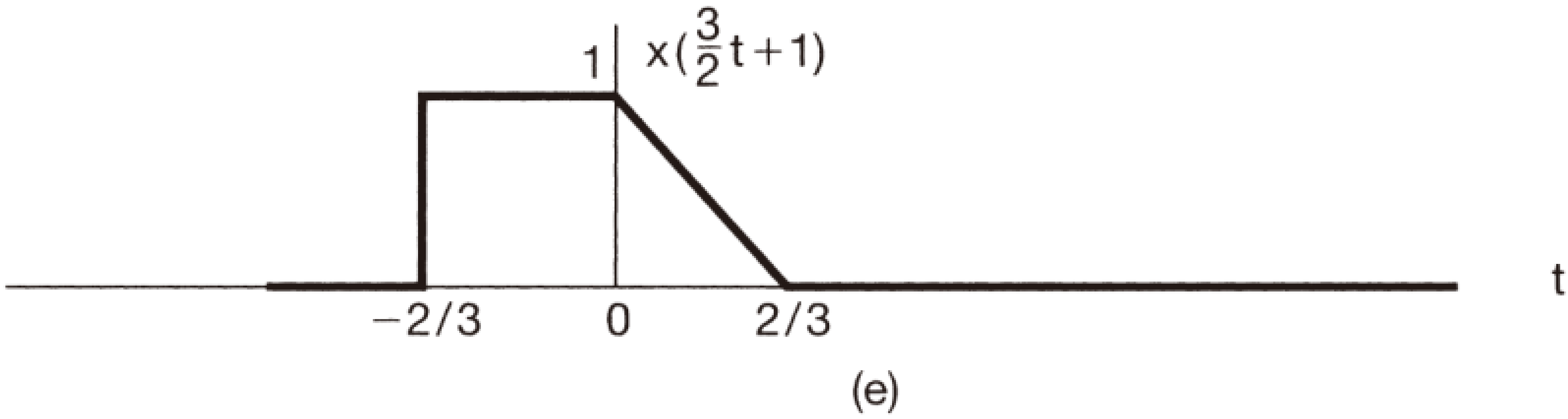


Figure 1.13 (a) The continuous-time signal $x(t)$ used in Examples 1.1–1.3 to illustrate transformations of the independent variable; (b) the time-shifted signal $x(t+1)$; (c) the signal $x(-t+1)$ obtained by a time shift and a time reversal; (d) the time-scaled signal $x(\frac{3}{2}t)$; and (e) the signal $x(\frac{3}{2}t+1)$ obtained by time-shifting and scaling.

1.2.2 Periodic Signals

- A periodic continuous-time signal $x(t)$ has the property that there is a positive value of T for which

$$x(t) = x(t+T) \quad (1.11)$$

for all values of t. In other words, a periodic signal has the property that it is unchanged by a time shift of T. We say that $x(t)$ is periodic with *period* T.
連續時間週期訊號可表示為在某一個正數T之下，對任何時間t可得$x(t)$ = x(t+T)。亦即週期訊號在時間軸上移位T時間其波形均不變。T稱為週期

1.2.2 Periodic Signals

- We can readily deduce that if x(t) is periodic with period T, then $x(t) = x(t+mT)$ for all t and for any integer m. Thus, x(t) is also periodic with period 2T, 3T, 4T,….The fundamental period T_0 of $x(t)$ is the smallest positive value of T for which eq. (1.11) holds.

  若$x(t)$為週期訊號，其週期為T，則對任何時間t及任意整數m，$x(t) = x(t+mT)$。基本週期T_0為可使(1.11)式成立的最小正數T。

1.2.2 Periodic Signals

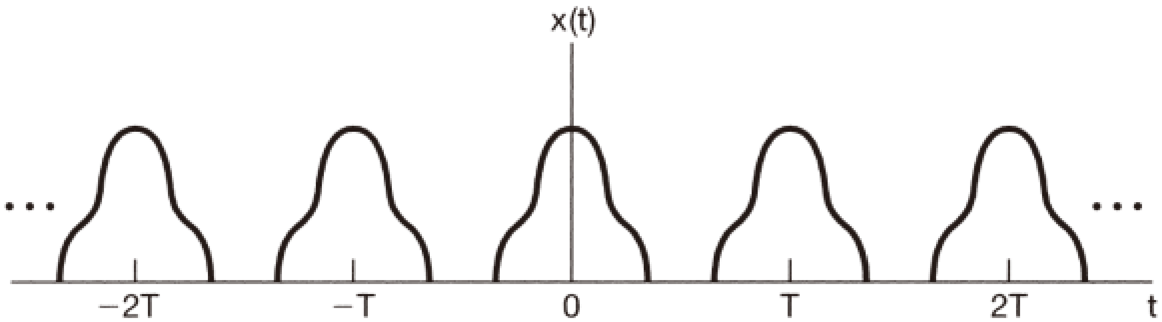


Figure 1.14 A continuous-time periodic signal.

1.2.2 Periodic Signals

- Periodic signals are defined analogously in discrete time. Specifically, a discrete-time signal $x[n]$ is periodic with period N, where N is a positive integer, if it is unchanged by a time shift of N, i.e., if

$$x[n] = x[n+N] \qquad (1.12)$$

for all values of n.
離散時間週期訊$x[n]$，其週期為一正整數N，且對任何時間n之下，可滿足$x[n]=x[n+N]$。基本週期N_0為可使(1.12)式成立最小正數N。

1.2.2 Periodic Signals

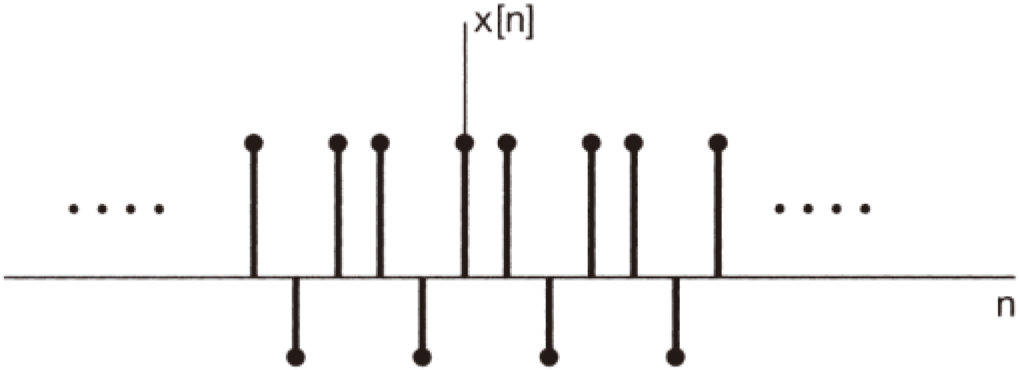


Figure 1.15 A discrete-time periodic signal with fundamental period $N_0 = 3$.

Example 1.4

Let us illustrate the type of problem solving that may be required in determining whether or not a given signal is periodic. The signal whose periodicity we wish to check is given by

$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \geq 0 \end{cases} \tag{1.13}$$

Example 1.4

We know that $\cos(t+2\pi) = \cos(t)$ and $\sin(t+2\pi) = \sin(t)$.

Thus, considering $t > 0$ and $t < 0$ separately, we see that $x(t)$ does repeat itself over every interval of length 2π. However, as illustrated in Figure 1.16, $x(t)$ also has a discontinuity at the time origin that does not recur at any other time. Since every feature in the shape of a periodic signal must recur periodically, we conclude that the signal $x(t)$ is not periodic.

Example 1.4

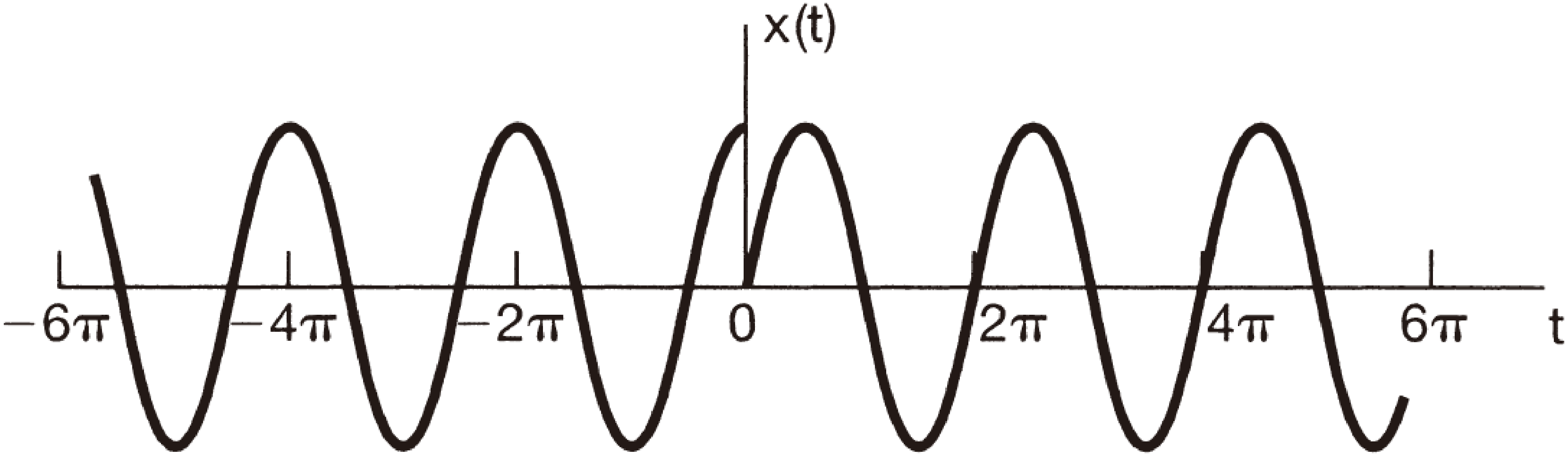


Figure 1.16 The signal $x(t)$ considered in Example 1.4.

1.2.3 Even and Odd Signals

In continuous time a signal is even if

$$x(-t) = x(t) \qquad (1.14)$$

while a discrete-time signal is even if

$$x[-n] = x[n] \qquad (1.15)$$

A signal is referred to as odd if

$$x(-t) = -x(t) \qquad (1.16)$$

$$x[-n] = -x[n] \qquad (1.17)$$

1.2.3 Even and Odd Signals

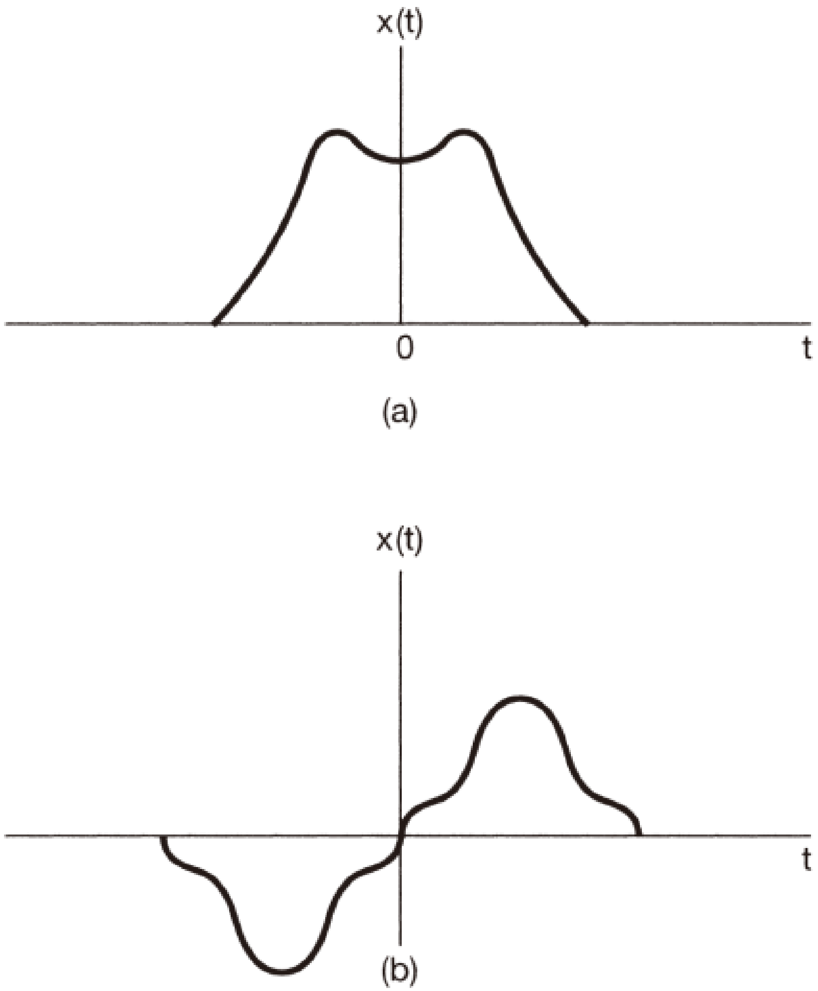


Figure 1.17 (a) An even continuous-time signal; (b) an odd continuous-time signal.

1.2.3 Even and Odd Signals

An important fact is that any signal can be broken into a sum of two signals, one of which is even and one of which is odd.

$$\mathcal{E}v\left\{ x(t) = \frac{1}{2}[x(t) + x(-t)] \right.$$

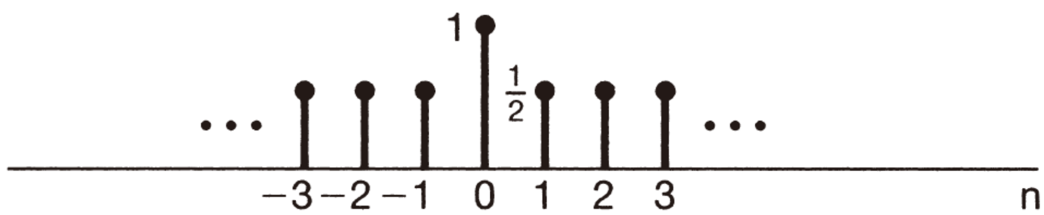
which is referred to as the even part of x(t). Similarly, the odd part of x(t) is given by

$$\mathcal{O}d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

1.2.3 Even and Odd Signals

$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\mathcal{E}v\{x[n]\} = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



1.2.3 Even and Odd Signals

$$\mathcal{O}d\{x[n]\} = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$

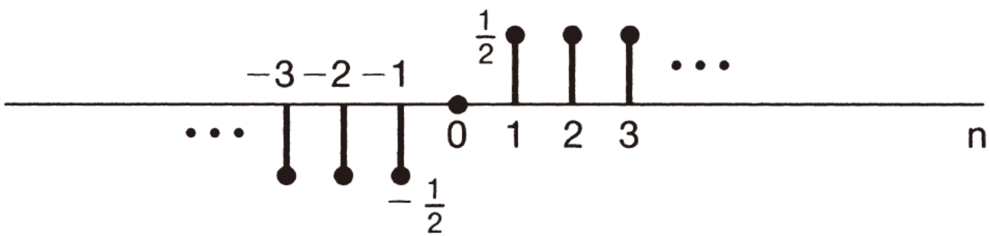


Figure 1.18 Example of the even-odd decomposition of a discrete-time signal.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- The continuous-time complex exponential signal is of the form

$$x(t) = Ce^{at} \qquad (1.20)$$

where C and a are, in general, complex numbers.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- Real Exponential Signals

  If C and a are real [in which case $x(t)$ is called a real exponential]

  - If a is positive, then as t increase $x(t)$ is a growing exponential
  - If a is negative, then $x(t)$ is a decaying exponential
  - a=0, $x(t)$ is constant

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

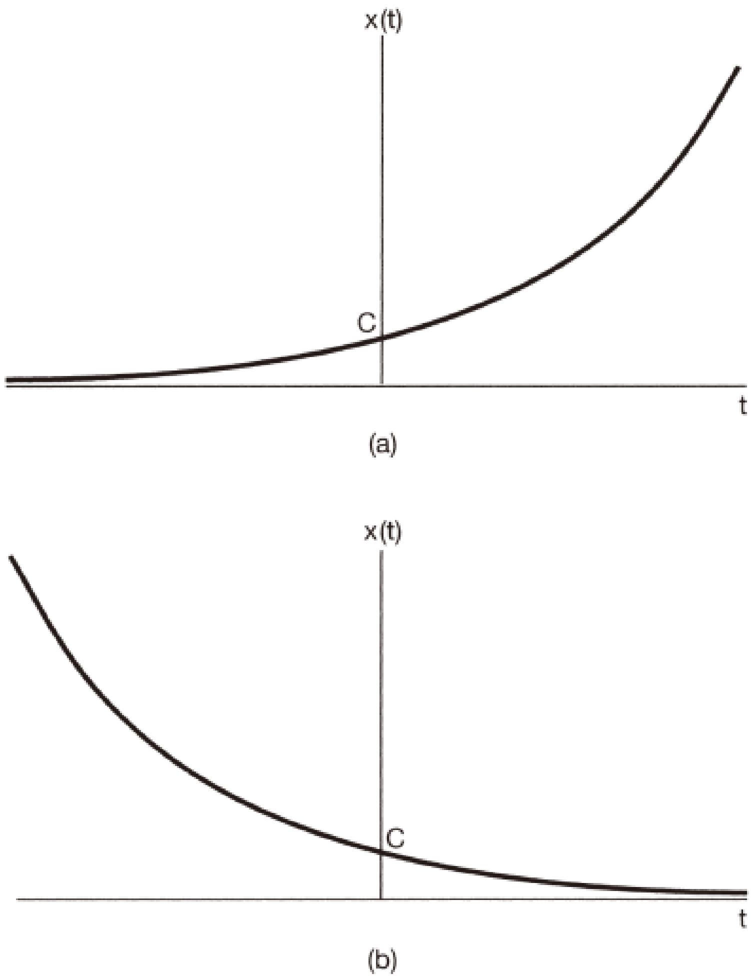


Figure 1.19 Continuous-time real exponential $x(t) = Ce^{at}$: (a) $a > 0$; (b) $a < 0$.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- Periodic Complex Exponential and Sinusoidal Signals
  A second important class of complex exponential is obtained by constraining a to be purely imaginary.

$$x(t) = e^{j\omega_0 t} \qquad (1.21)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)} \qquad (1.22)$$

$$e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

it follows that for periodicity, we must have

$$e^{j\omega_0 T} = 1 \qquad (1.23)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- If $\omega_0 = 0$, then $x(t)$ = 1, which is periodic for any value of T. If $\omega_0 \neq 0$, then the fundamental period T_0 of $x(t)$ is

$$T_0 = \frac{2\pi}{|\omega_0|} \qquad (1.24)$$

Thus, the signals $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have the same fundamental period.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

A signal closely related to the periodic complex exponential is the *sinusoidal signal*

$$x(t) = A\cos(\omega_0 t + \varphi) \qquad (1.25)$$

with seconds as the units of t, the units of φ and ω_0 are radians and radians per second. It is also common to write $\omega_0 = 2\pi f_0$, where f_0 has the units of cycles per second, or hertz (Hz).

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

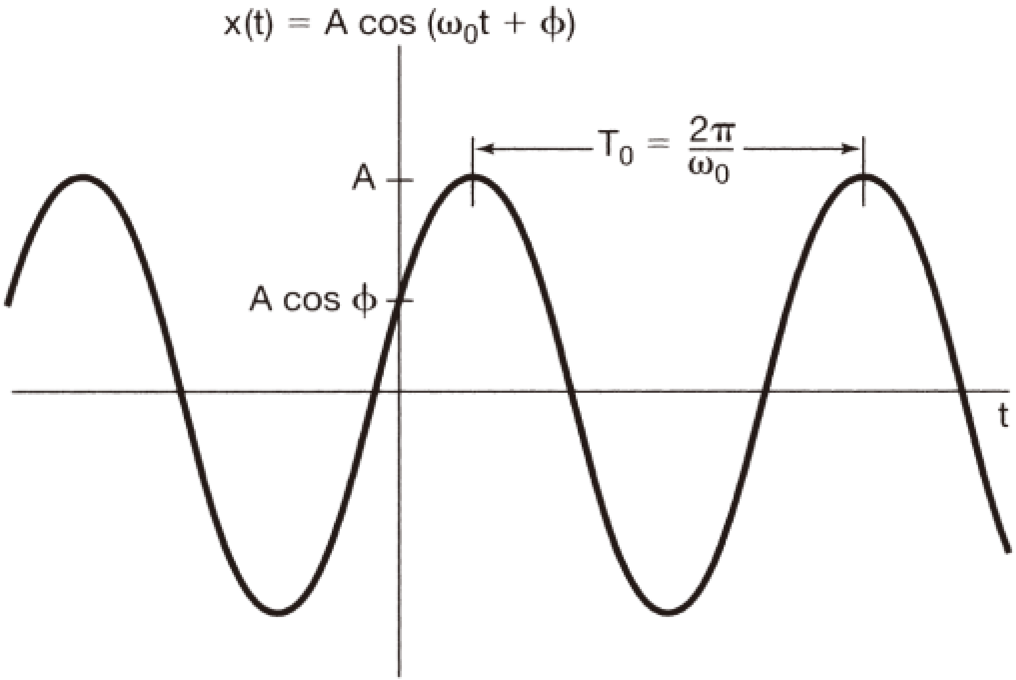


Figure 1.20 Continuous-time sinusoidal signal.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

By using Euler's relation, the complex exponential in eq.(1.21) can be written in terms of periodic complex exponentials, again with the same fundamental period:

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t \qquad (1.26)$$

the sinusoidal signal of eq. (1.25) can be written in terms of periodic complex exponentials

$$A\cos(\omega_0 t + \varphi) = \frac{A}{2}e^{j\varphi}e^{j\omega_0 t} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 t} \qquad (1.27)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

we can express a sinusoid in terms of a complex exponential signal as

$$A\cos(\omega_0 t+\varphi)=A\,\mathrm{Re}\left\{e^{j(\omega_0 t+\varphi)}\right\} \tag{1.28}$$

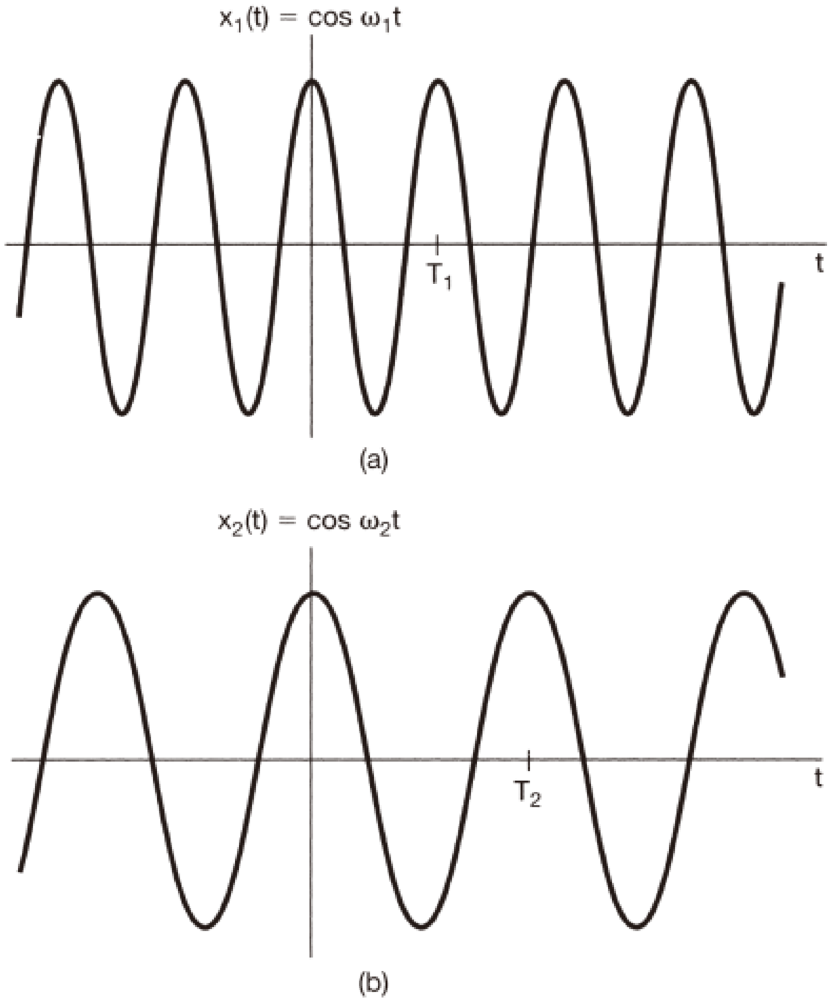
where, if c is a complex number, $\mathrm{Re}\{c\}$ denotes its real part. We will also use the notation $\mathrm{Jm}\{c\}$ for the imaginary part of c, so that, for example,

$$A\sin(\omega_0 t+\varphi)=A\,\mathrm{Jm}\left\{e^{j(\omega_0 t+\varphi)}\right\} \tag{1.29}$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- We see that the fundamental period T0 of a continuous-time sinusoidal signal or a periodic complex exponential is inversely proportional to $|\omega_0|$, which we will refer to as the *fundamental frequency*.
- $\omega_0 = 0$. We mentioned earlier, *x(t)* is constant and therefore is periodic with period T for any positive value of *T*.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals



1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

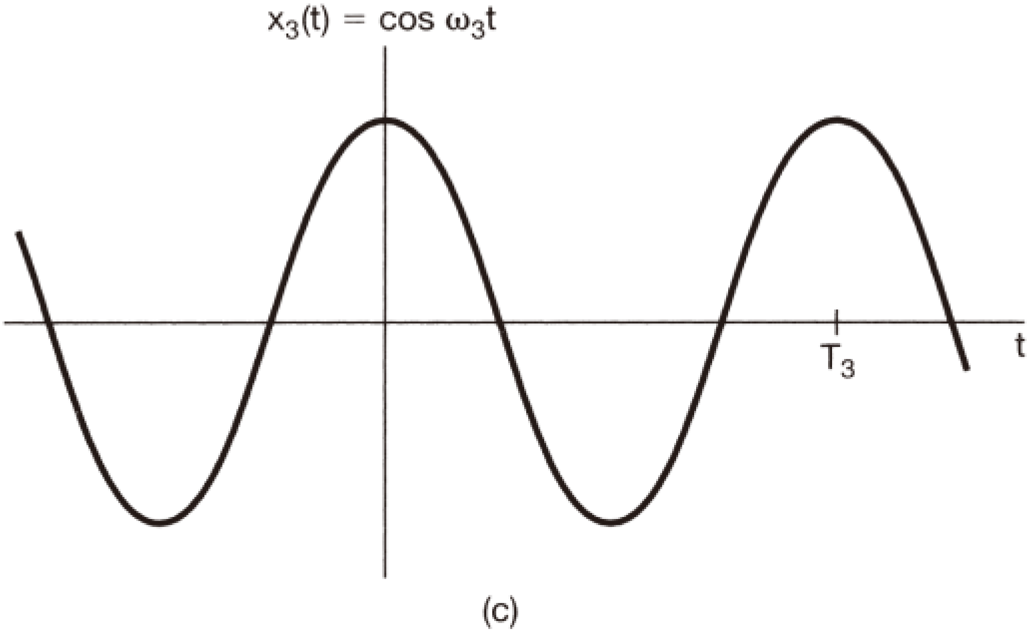


Figure 1.21 Relationship between the fundamental frequency and period for continuous-time sinusoidal signals; here, $\omega_1 > \omega_2 > \omega_3$, which implies that $T_1 < T_2 < T_3$.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

$$E_{\text{period}} = \int_0^{T_0} \left| e^{j\omega_0 t} \right|^2 dt$$

$$= \int_0^{T_0} 1 \cdot dt = T_0 \qquad (1.30)$$

$$P_{\text{period}} = \frac{1}{T_0} E_{\text{period}} = 1 \qquad (1.31)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

The complex periodic exponential signal has finite average power equal to

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|e^{j\omega_0 t}\right|^2 dt = 1 \qquad (1.32)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

a necessary condition for a complex exponential $e^{j\omega t}$ to be periodic with period T_0 is that

$$e^{j\omega T_0} = 1 \qquad (1.33)$$

which implies that ωT_0 is a multiple of 2π ,i.e.,

$$\omega T_0 = 2\pi k, \qquad k = 0, \pm 1, \pm 2, \ldots. \qquad (1.34)$$

if we define

$$\omega_0 = \frac{2\pi}{T_0} \qquad (1.35)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

Harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency ω_0 :

$$\varphi_k(t) = e^{jk\omega_0 t}, \qquad k = 0, \pm 1, \pm 2, \ldots. \qquad (1.36)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

For $k=0, \varphi_k(t)$ is a constant, while for any other of $k, \varphi_k(t)$ is periodic with fundamental frequency $|k|\omega_0$ and fundamental period

$$\frac{2\pi}{|k|\omega_0}=\frac{T_0}{|k|} \qquad (1.37)$$

Example 1.5

It is sometimes desirable to express the sum of two complex exponentials as the product of a single complex exponential and a single sinusoid. For example, suppose we wish to plot the magnitude of the Signal

$$x(t) = e^{j2t} + e^{j3t} \tag{1.38}$$

We obtain

$$x(t) = e^{j2.5t}\left(e^{-j0.5} + e^{j0.5t}\right) \tag{1.39}$$

Example 1.5

Which, because of Euler's relation, can be rewritten as

$$x(t) = 2e^{j2.5t}\cos(0.5t) \qquad (1.40)$$

We can directly obtain an expression for the magnitude of $x(t)$

$$|x(t)| = 2|\cos(0.5t)| \qquad (1.41)$$

Example 1.5

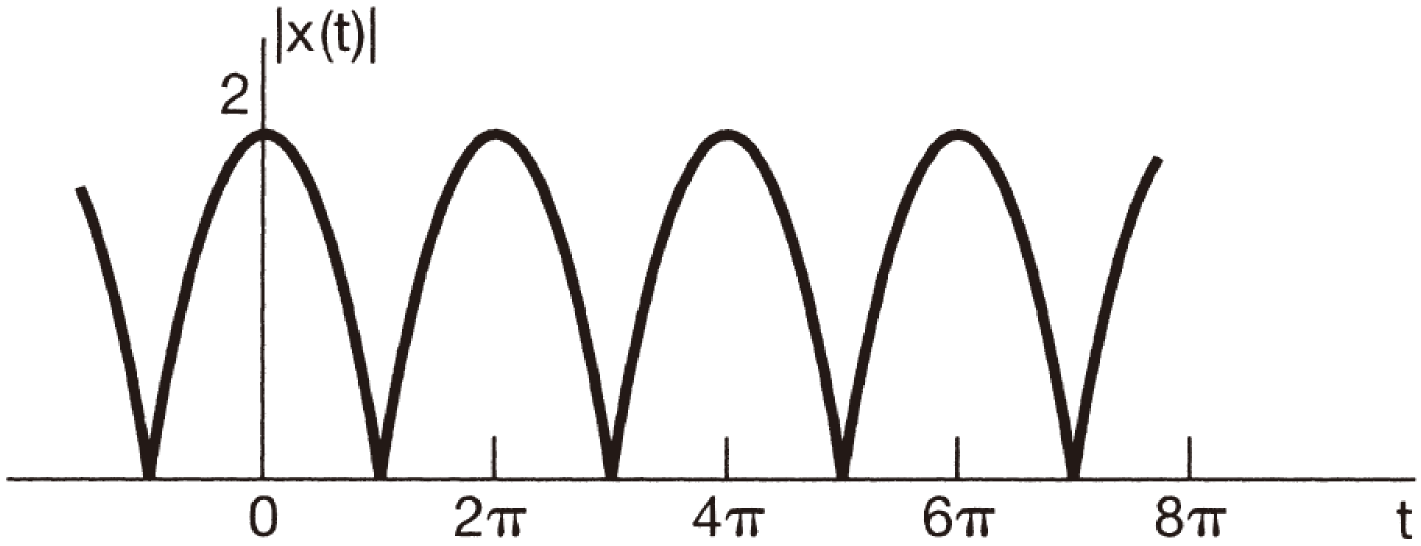


Figure 1.22 The full-wave rectified sinusoid of Example 1.5.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- General Complex Exponential Signals

$$C = |C| e^{j\theta}$$

$$a = r + j\omega_0$$

$$Ce^{at} = |C| e^{j\theta} e^{(r + j\omega_0)t} = |C| e^{rt} e^{j(\omega_0 t + \theta)} \tag{1.42}$$

Using Euler's relation, we can expand this further as

$$Ce^{at} = |C| e^{rt} \cos(\omega_0 t + \theta) + j|C| e^{rt} \sin(\omega_0 t + \theta) \tag{1.43}$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

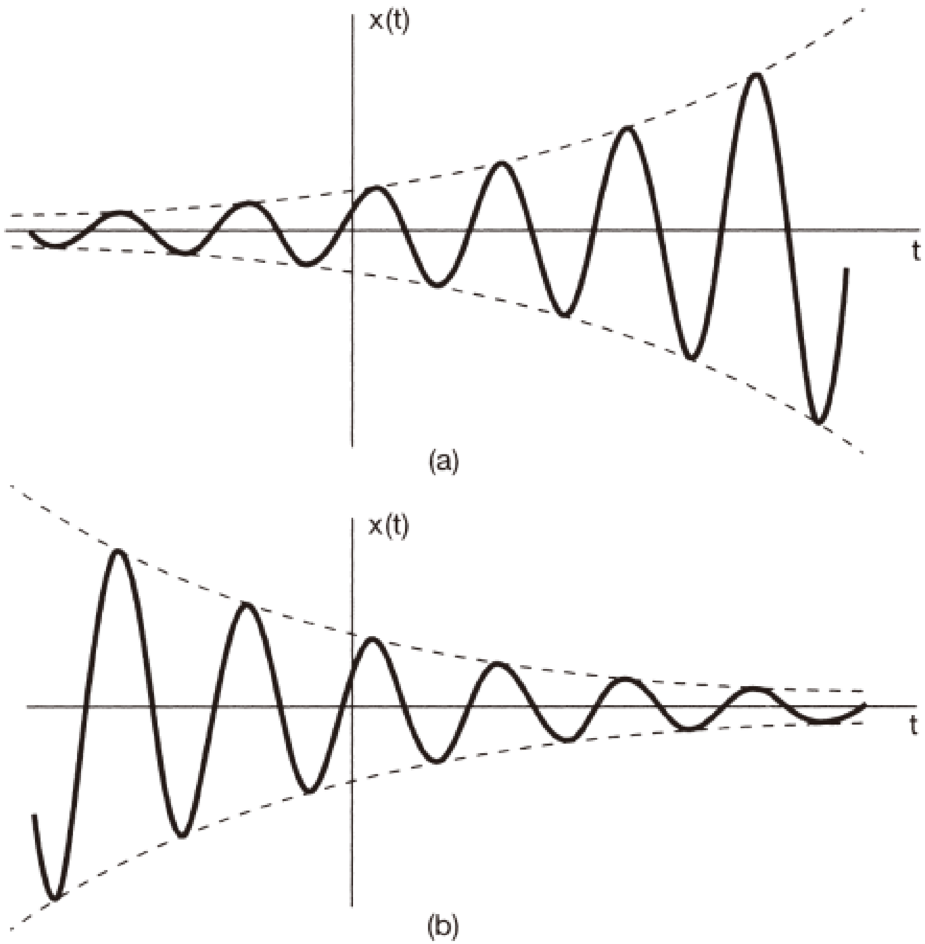


圖 1.23(a) 爲漸增弦波訊號 ($r > 0$)。
圖 1.23(b) 爲衰減弦波訊號 ($r < 0$)。

Figure 1.23 (a) Growing sinusoidal signal $x(t) = Ce^{rt}\cos(\omega_0 t + \theta)$, $r > 0$; (b) decaying sinusoid $x(t) = Ce^{rt}\cos(\omega_0 t + \theta)$, $r < 0$.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

As in continuous time, an important signal in discrete time is the complex exponential signal or sequence, defined by

$$x[n] = C\alpha^n \qquad (1.44)$$

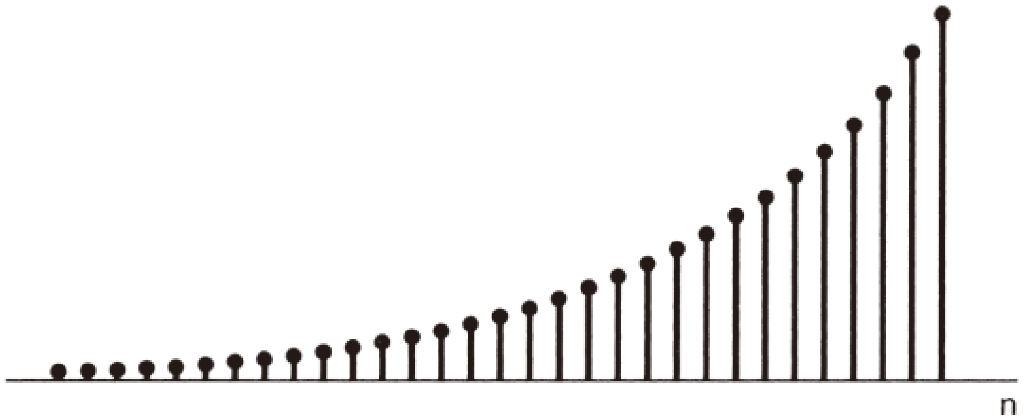
Where C and α are, in general, complex numbers. This could alternatively be expressed in the form

$$x[n] = Ce^{\beta n} \qquad (1.45)$$

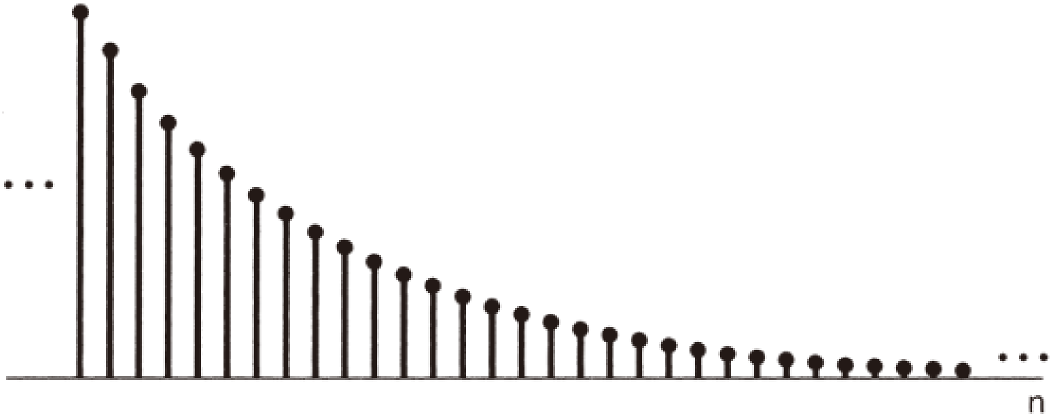
Where

$$\alpha = e^{\beta}$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals



(a)



(b)

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

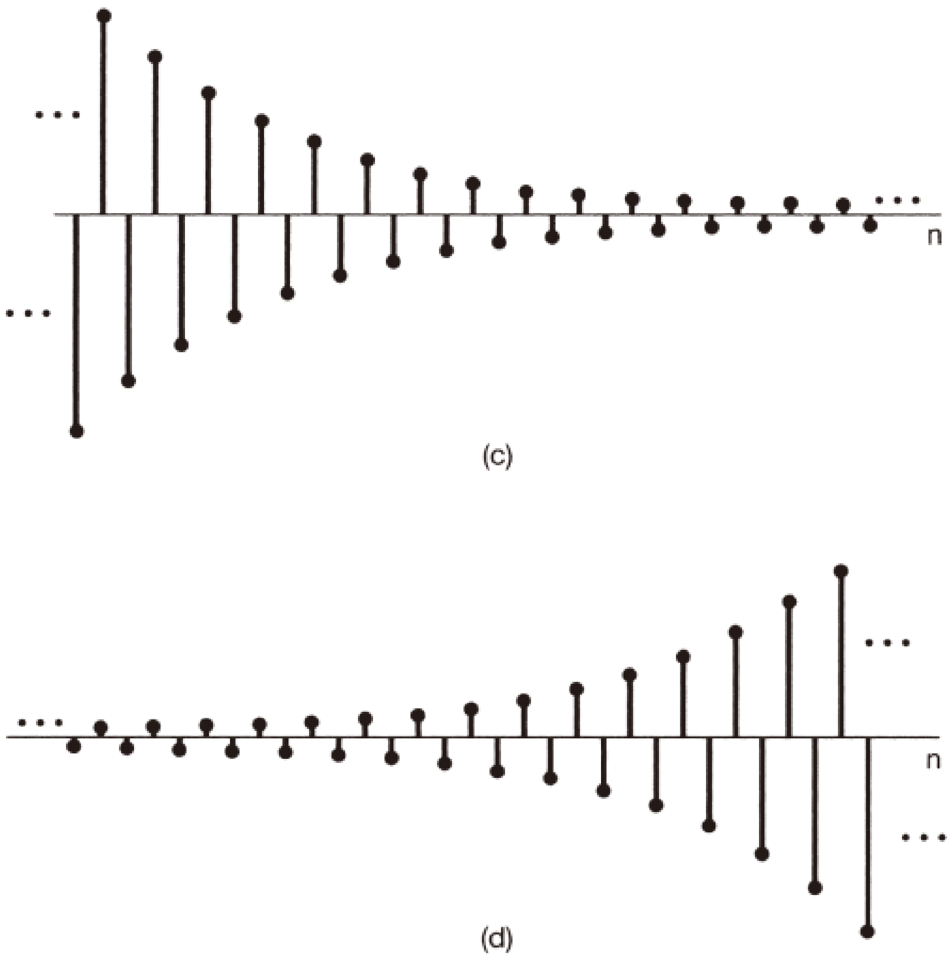


Figure 1.24 The real exponential signal $x[n] = C\alpha^n$:
(a) $\alpha > 1$; (b) $0 < \alpha < 1$;
(c) $-1 < \alpha < 0$; (d) $\alpha < -1$.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

- Sinusoidal Signals

$$x[n] = e^{j\omega_0 n} \qquad (1.46)$$

As in the continuous-time case, this signal is closely related to the sinusoidal signal

$$x[n] = A\cos(\omega_0 n + \varphi) \qquad (1.47)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

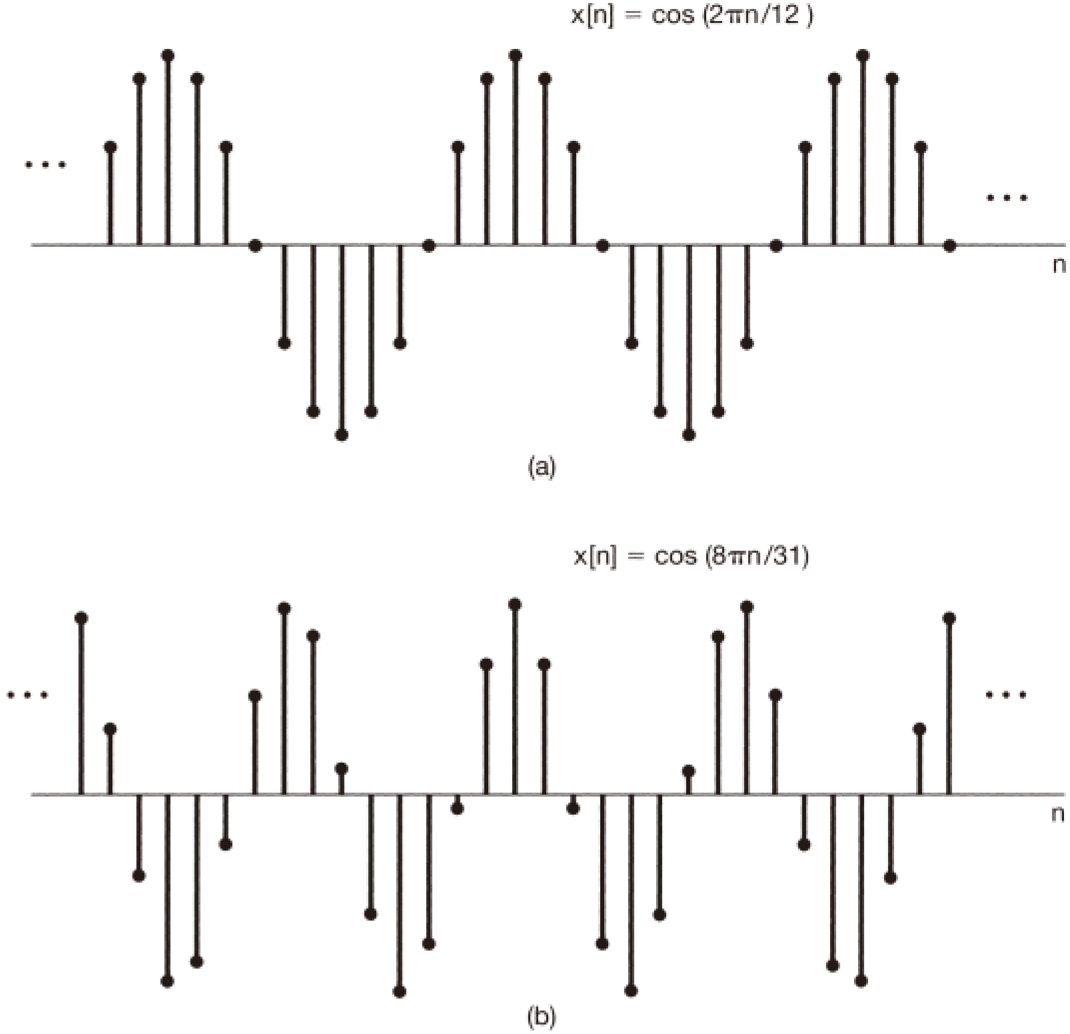
As before, Euler's relation allows us to relate complex exponentials and sinusoids:

$$e^{j\omega_0 n} = \cos\omega_0 n + j\sin\omega_0 n \qquad (1.48)$$

and

$$A\cos(\omega_0 n + \varphi) = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n} \qquad (1.49)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals



1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

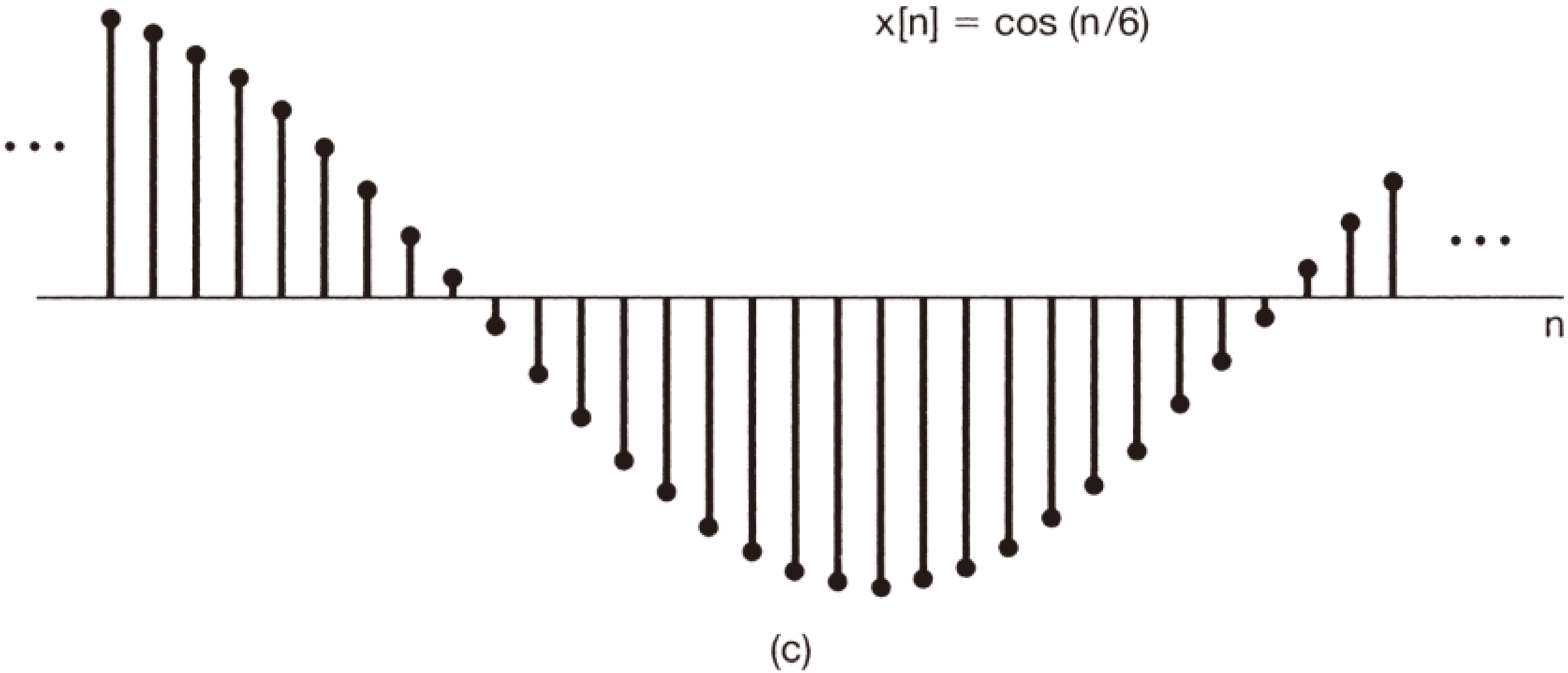


Figure 1.25 Discrete-time sinusoidal signals.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

- General Complex Exponential Signals

$$C = |C|e^{j\theta}$$

$$\alpha = |\alpha|e^{j\omega_0}$$

$$C\alpha^n = |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta) \qquad (1.50)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

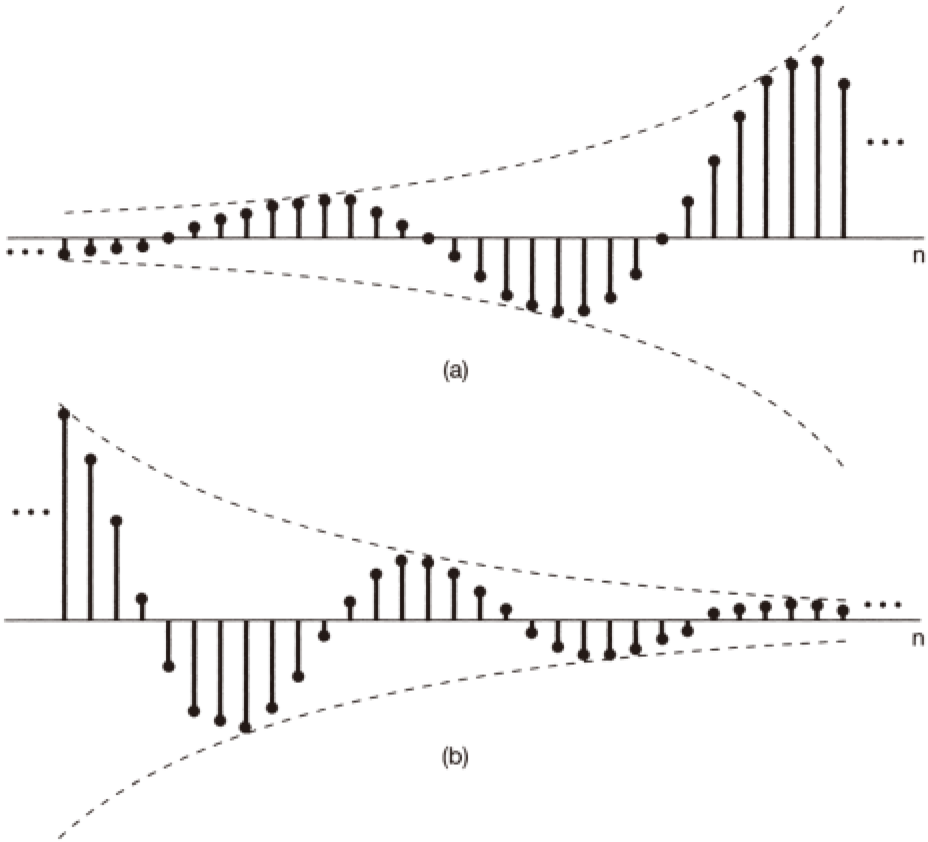


Figure 1.26 (a) Growing discrete-time sinusoidal signals; (b) decaying discrete-time sinusoid.

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

- We identified the two properties of its continuous-time counterpart $e^{j\omega_0 t}$:

1. The larger the magnitude of ω_0 , the higher is the rate of oscillation in the signal
2. $e^{j\omega_0 t}$ is periodic for any value of ω_0 .

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$$e^{j(\omega_0+2\pi)n} = e^{j2\pi n}e^{j\omega_0 n} = e^{j\omega_0 n} \qquad (1.51)$$

We see that the exponential at frequency $\omega_0 + 2\pi$ is the same as that at frequency ω_0 .

Any interval of length 2 π will do, on most occasions

we will use the interval $0 \le \omega_0 \le 2\pi$ or the interval $-\pi \le \omega_0 \le \pi$

$$e^{j\pi n} = \left(e^{j\pi}\right)^n = (-1)^n \qquad (1.52)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

The second property we wish to consider concerns the periodicity of the discrete-time complex exponential. In order for the signal $e^{j\omega_0 n}$ to be periodic with period $N>0$, we must have

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \qquad (1.53)$$

Or equivalently,

$$e^{j\omega_0 N} = 1 \qquad (1.54)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$\omega_0 N$ must be a multiple of 2π. That is, there must be integer m such that

$$\omega_0 N = 2\pi m \qquad (1.55)$$

or equivalently,

$$\frac{\omega_0}{2\pi} = \frac{m}{N} \qquad (1.56)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

We find that the fundamental frequency of the periodic signal $e^{j\omega_0 n}$ is

$$\frac{2\pi}{N} = \frac{\omega_0}{m} \qquad (1.57)$$

Note that the fundamental period can also be written as

$$N = m\left(\frac{2\pi}{\omega_0}\right) \qquad (1.58)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

TABLE 1.1 Comparison of the signals $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$. 表 1.1 $e^{j\omega_0 t}$ 與 $e^{j\omega_0 n}$ 的各種比較

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct values of ω_0	Identical signals for values of ω_0 separated by multiples of 2π
Periodic for any choice of ω_0	Periodic only if $\omega_0 = 2\pi m/N$ for some integers $N > 0$ and m.
Fundamental frequency ω_0	Fundamental frequency* ω_0/m
Fundamental period $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $\frac{2\pi}{\omega_0}$	Fundamental period* $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $m\left(\frac{2\pi}{\omega_0}\right)$

*Assumes that m and N do not have any factors in common.

Example 1.6

Suppose that we wish to determine the fundamental period of the discrete-time signal

$$x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n} \quad (1.59)$$

The first exponential on the right-hand side of eq.(1.59) has a fundamental period of 3. While this can be verified from eq.(1.58), there is a simpler way to obtain that answer. In particular, note that the angle $(2\pi/3)n$ of the first term must be incremented by a multiple of 2π for the values of this exponential to begin repeating.

Example 1.6

We then immediately see that if n is incremented by 3, the angle will be incrementing by a single multiple of 2π. With regard to the second term, we see that incrementing the angle $(3\pi/4)n$ by 2π would require n to be incremented by 8/3, which is impossible, since n is restricted to being an integer. Similarly, incrementing the angle by 4π would require a noninteger increment of 16/3 to n. However, incrementing the angle by 6π require an increment of 8 to n, and thus the fundamental period of the second term is 8.

Example 1.6

Now, for the entire signal $x[n]$ to repeat, each of the terms in eq.(1.59) must go through an integer number of its own fundamental period. The smallest increment of n that accomplishes this is 24. That is, over an interval of 24 points, the first term on the right-hand side of eq.(1.59) will have gone through eight of its fundamental periods, the second term through three of its fundamental periods, and the overall signal $x[n]$ through exactly one of its fundamental periods.

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$$\varphi_k[n] = e^{jk(2\pi/N)n}, \qquad k = 0, \pm 1, \ldots \qquad (1.60)$$

$$\varphi_{k+N}[n] = e^{j(k+N)(2\pi/N)n}$$

$$= e^{jk(2\pi/N)n} e^{j2\pi n} = \varphi_k[n] \qquad (1.61)$$

$$\varphi_0[n] = 1, \varphi_1[n] = e^{j2\pi n/N}, \varphi_2[n] = e^{j4\pi n/N}, \ldots, \varphi_{N-1}[n] = e^{j2\pi(N-1)n/N} \qquad (1.62)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

One of the simplest discrete-time signals is the unit impulse, which is defined as

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \qquad (1.63)$$

A second basic discrete-time signal is the discrete-time unit step, denoted by $u[n]$ and defined by

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \qquad (1.64)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

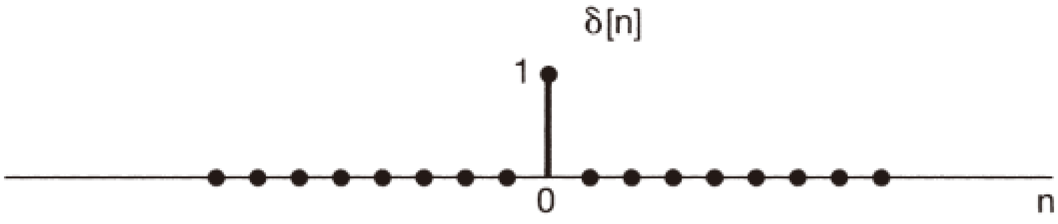


Figure 1.28 Discrete-time unit impulse (sample).

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

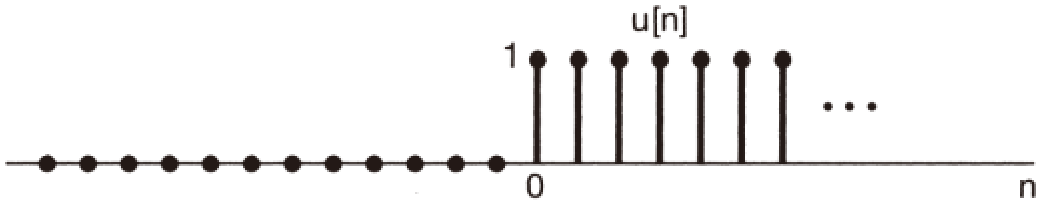


Figure 1.29 Discrete-time unit step sequence.

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

In particular, the discrete-time unit impulse is the first difference of the discrete-time step

$$\delta[n] = u[n] - u[n-1] \qquad (1.65)$$

Conversely, the discrete-time unit step is the running sum of the unit sample. That is,

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad (1.66)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

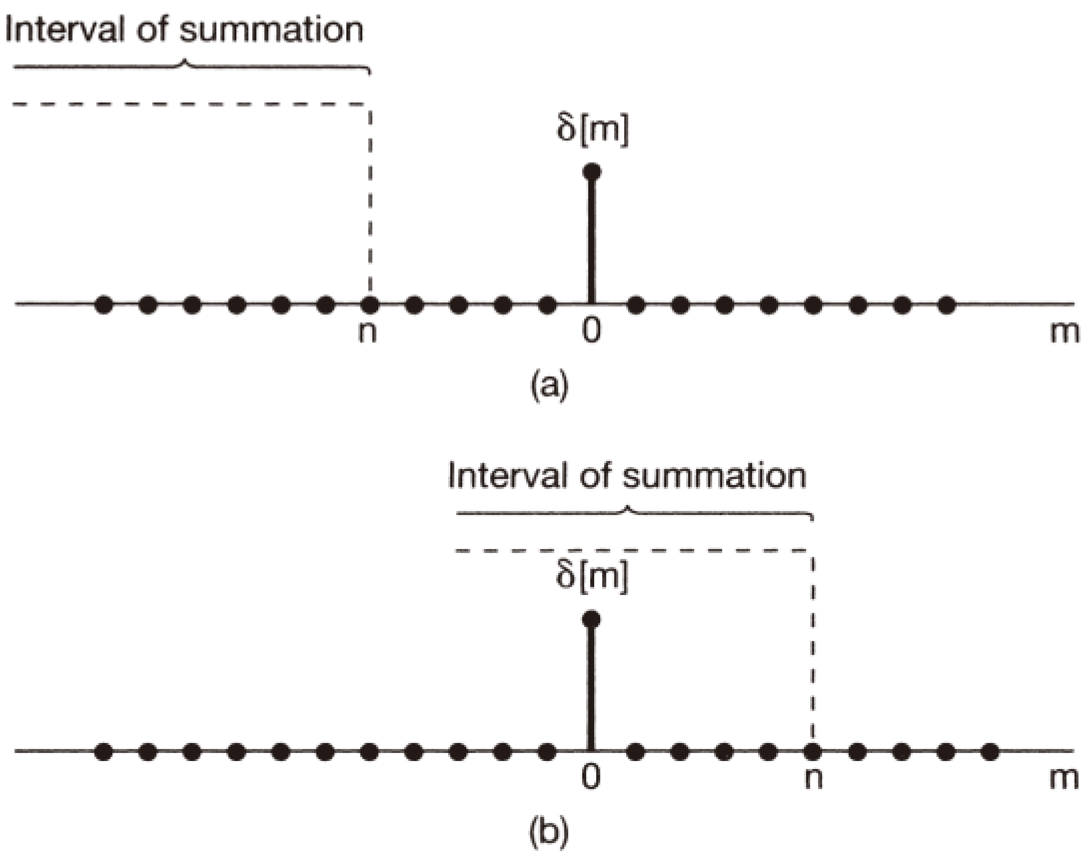


Figure 1.30 Running sum of eq. (1.66): (a) $n < 0$; (b) $n > 0$.

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

We find that the discrete-time unit step can also be written in terms of the unit sample as

$$u[n] = \sum_{k=\infty}^{0} \delta[n-k]$$

Or equivalently,

$$u[n] = \sum_{k=0}^{0} \delta[n-k] \qquad (1.67)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

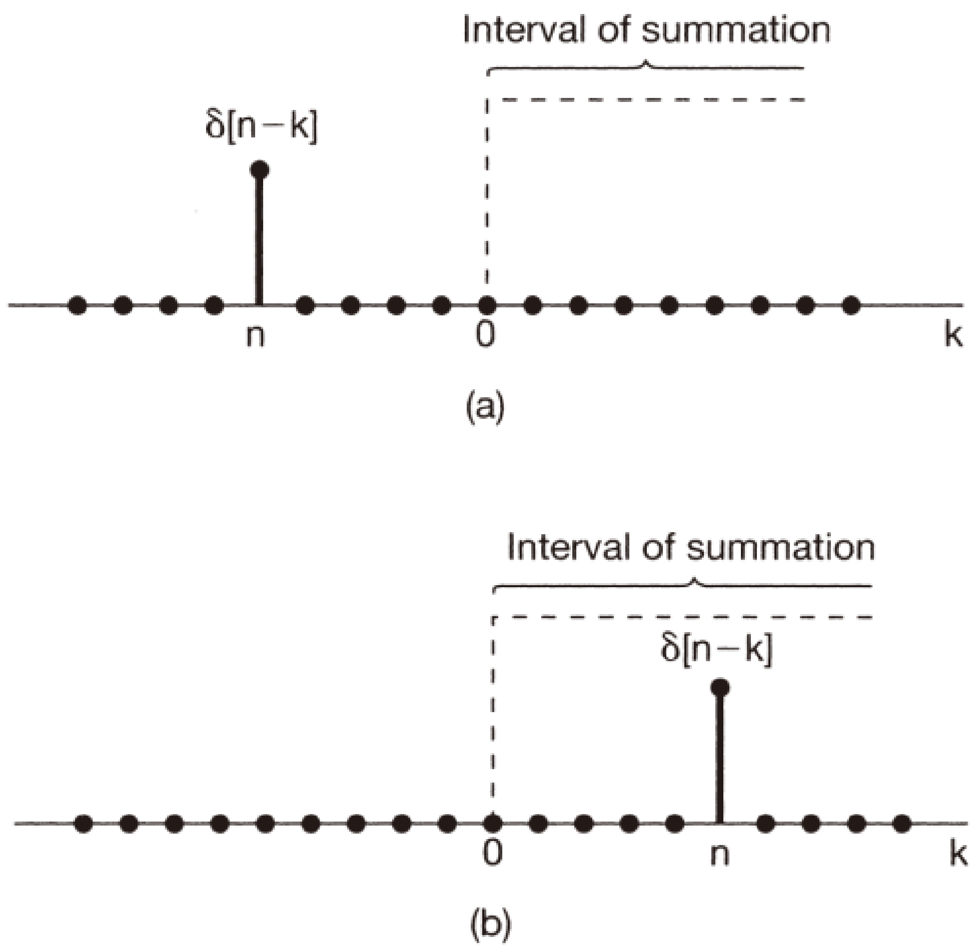


Figure 1.31 Relationship given in eq. (1.67): (a) $n < 0$; (b) $n > 0$.

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

In particular, since $\delta[n]$ is nonzero only for $n = 0$, it follows that

$$x[n]\delta[n] = x[0]\delta[n] \qquad (1.68)$$

More generally, if we consider a unit impulse $\delta[n-n_0]$ at $n = n_0$, then

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0] \qquad (1.69)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

The continuous-time unit step function u(t) is defined in a manner similar to its discrete-time counterpart. Specifically,

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \qquad (1.70)$$

In particular, the continuous-time unit step is the running integral of the unit impulse

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau \qquad (1.71)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

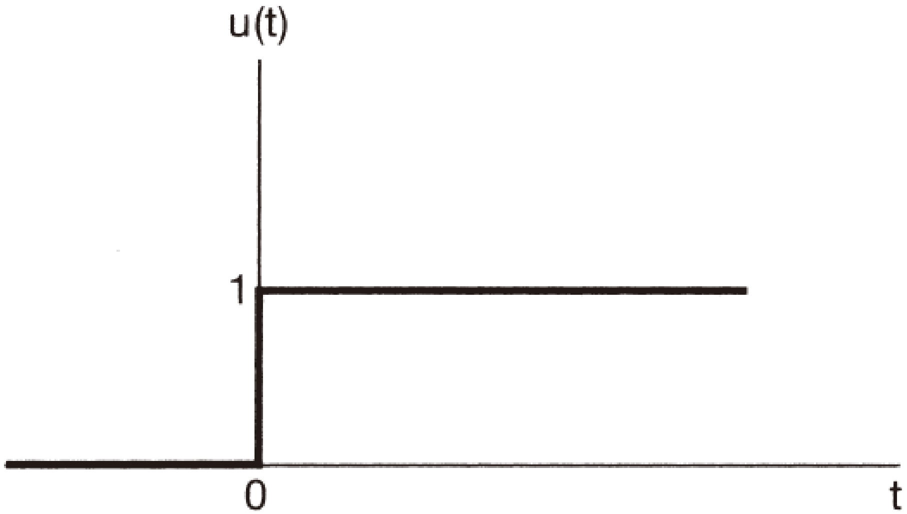


Figure 1.32 Continuous-time unit step function.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

In particular, it follows from eq.(1.71) that the continuous-time unit impulse can be thought of as the first derivative of the continuous-time step:

$$\delta(t)=\frac{du(t)}{dt} \qquad (1.72)$$

$$\delta_{\triangle}(t)=\frac{du_{\triangle}(t)}{dt} \qquad (1.73)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

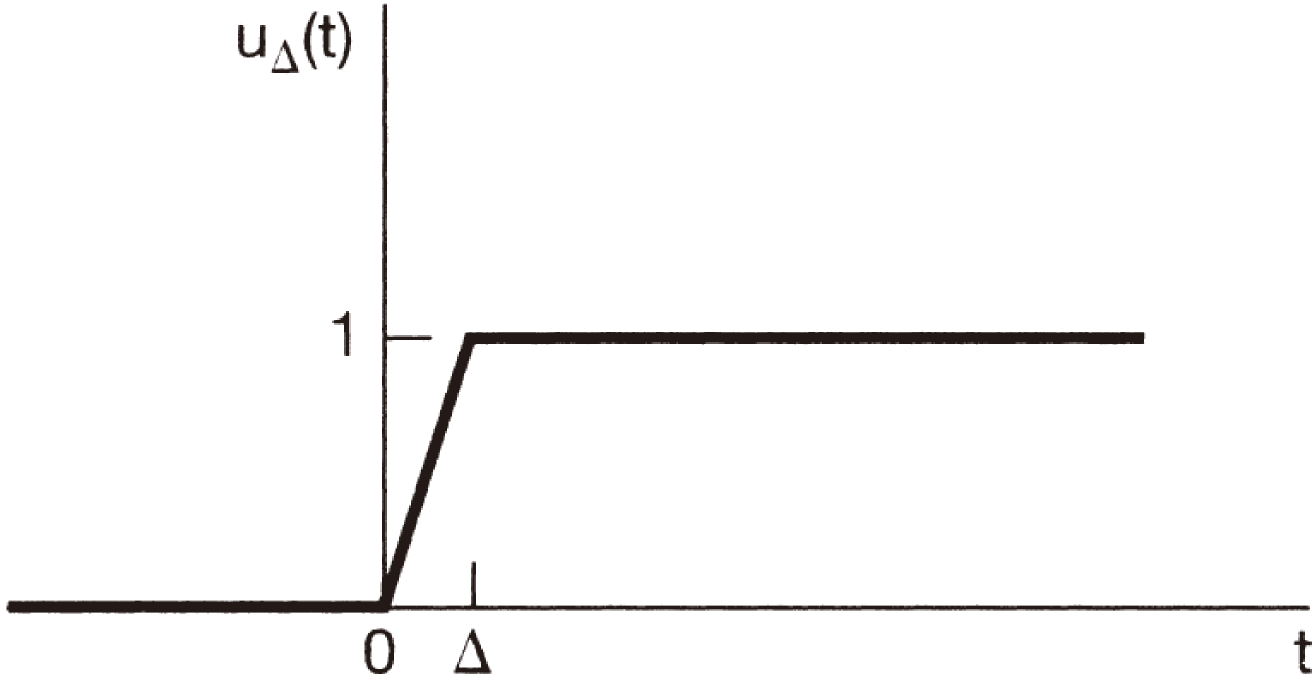


Figure 1.33 Continuous approximation to the unit step, $u_\Delta(t)$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

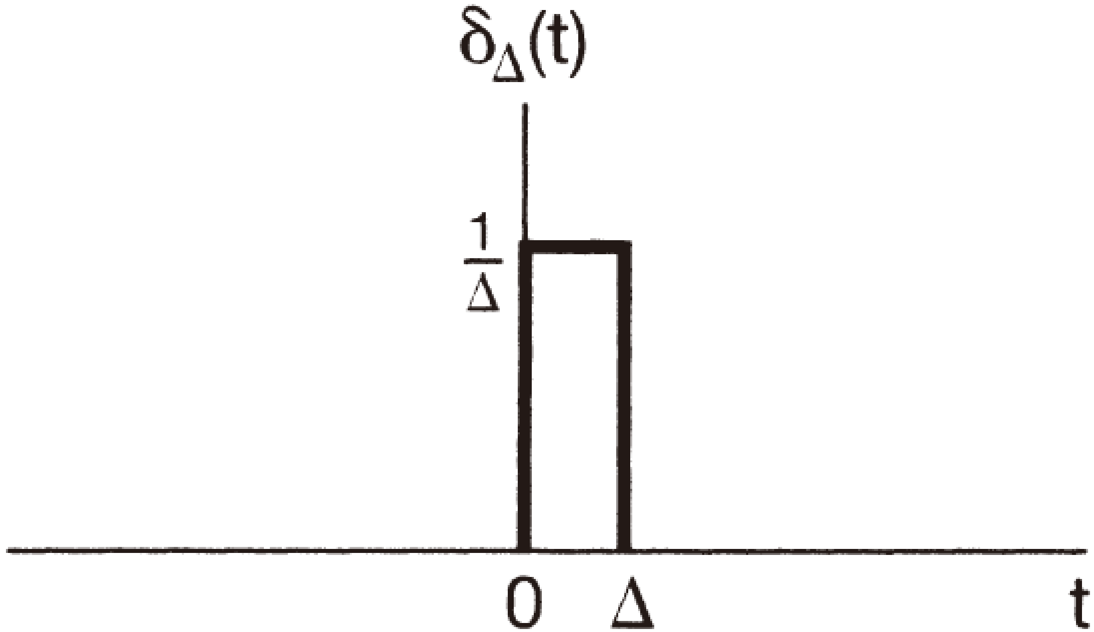


Figure 1.34 Derivative of $u_{\Delta}(t)$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

Note that $\delta_{\Delta}(t)$ is a short pulse, of duration Δ and with unit area for any value of Δ. As $\Delta \to 0$, $\delta_{\Delta}(t)$ becomes narrower and higher, maintaining its unit area. Its limiting form,

$$\delta(t) = \lim_{\Delta \to 0} \delta_{\Delta}(t) \qquad (1.74)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

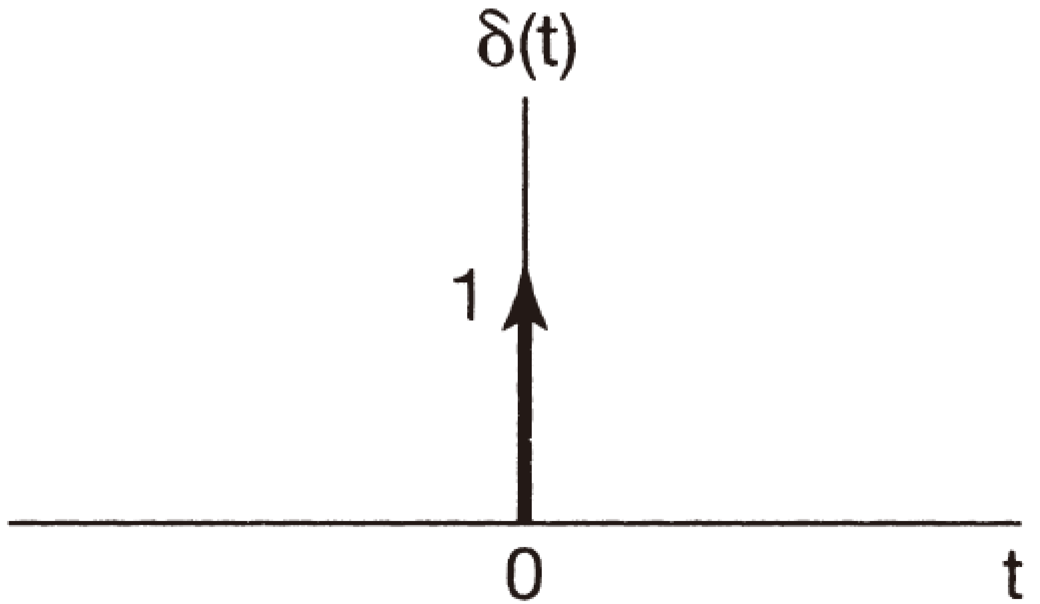


Figure 1.35 Continuous-time unit impulse.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

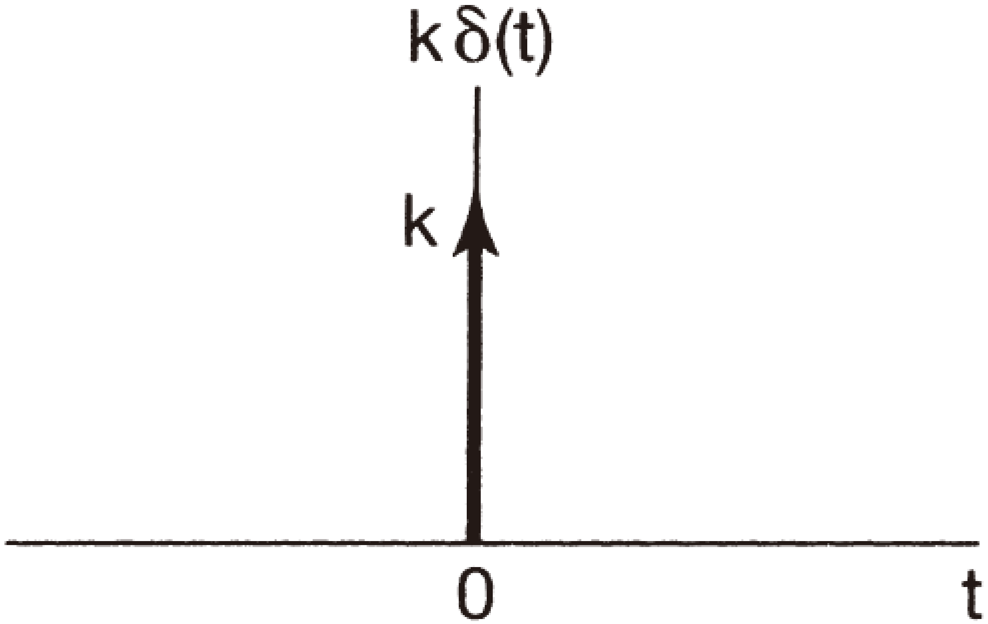


Figure 1.36 Scaled impulse.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

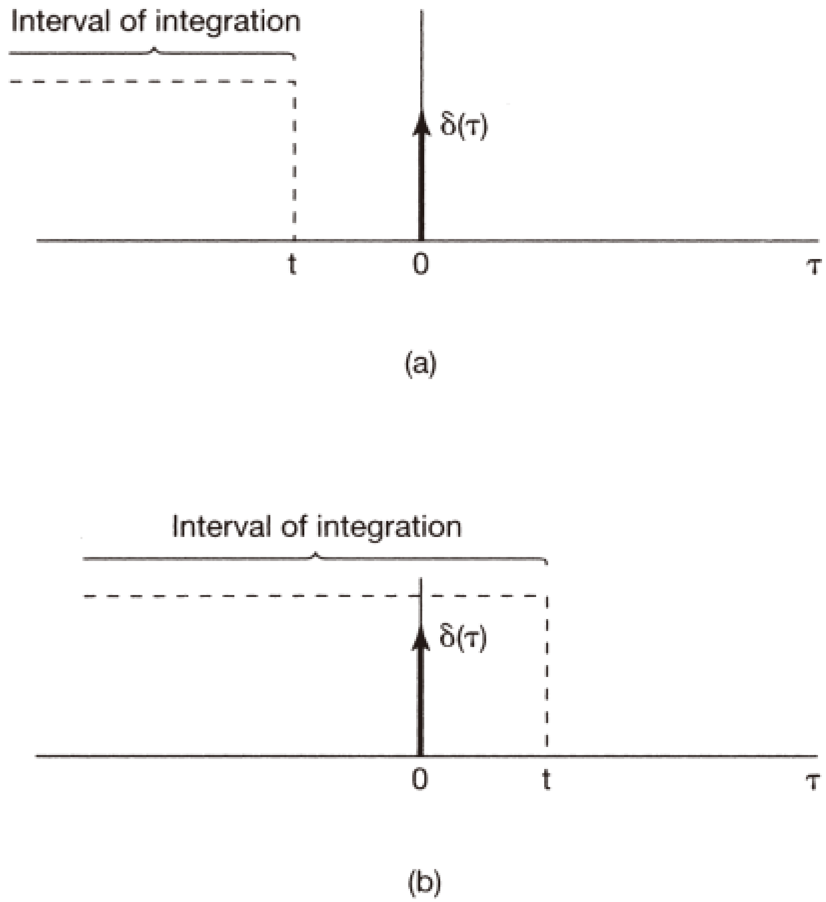


Figure 1.37 Running integral given in eq. (1.71): (a) $t < 0$; (b) $t > 0$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

Changing the variable of integration from τ to σ = t – τ:

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau = \int_{\infty}^{0} \delta(t-\sigma)(-d\sigma)$$

Or equivalently,

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma) d\sigma \qquad (1.75)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

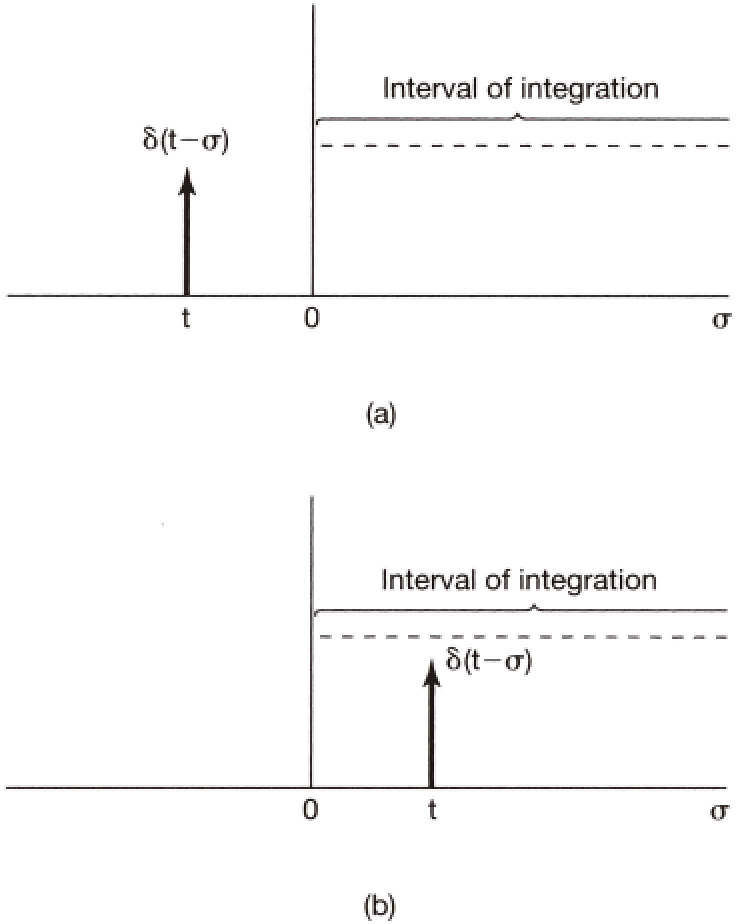


Figure 1.38 Relationship given in eq. (1.75): (a) $t < 0$; (b) $t > 0$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

For Δ sufficiently small so that $x(t)$ is approximately constant over this interval,

$$x(t)\delta_{\Delta}(t) \approx x(0)\delta_{\Delta}(t)$$

Since $\delta(t)$ is the limit as $\Delta \rightarrow 0$ of $\delta_{\Delta}(t)$, it follows that

$$x(t)\delta(t) = x(0)\delta(t) \qquad (1.76)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

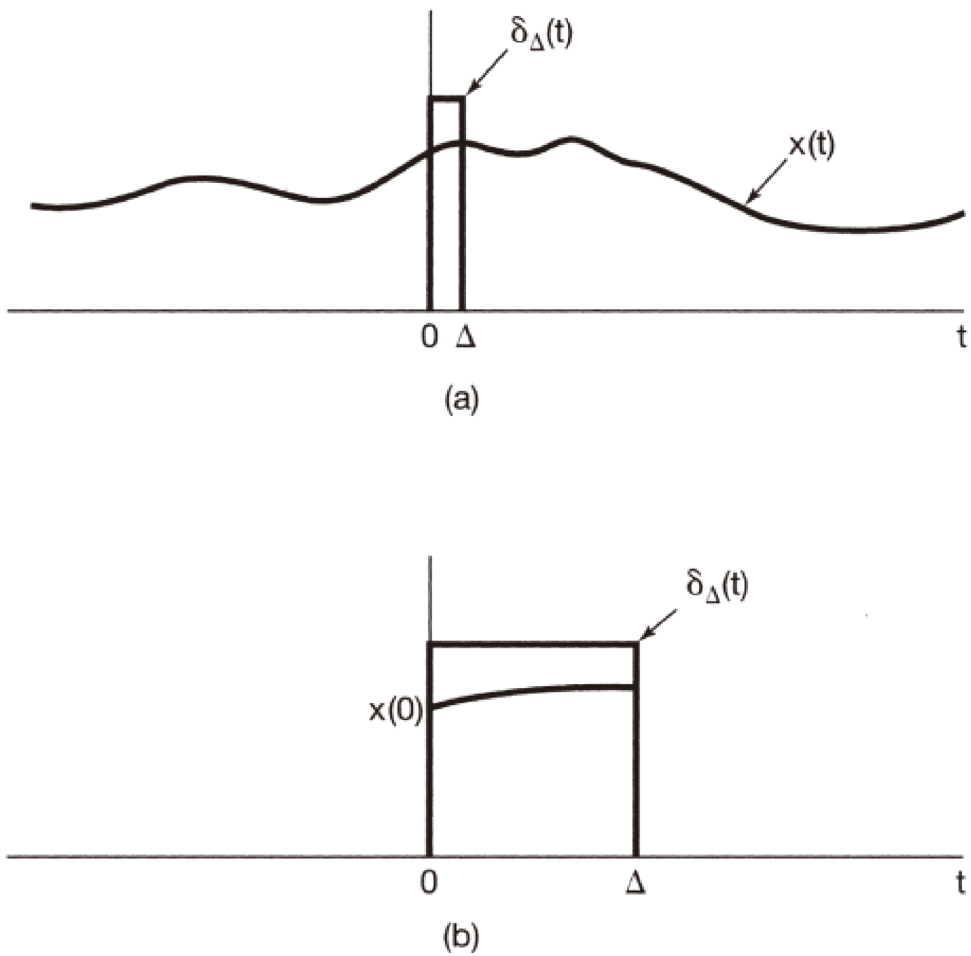


Figure 1.39 The product $x(t)\delta_\Delta(t)$: (a) graphs of both functions; (b) enlarged view of the nonzero portion of their product.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

By the same argument, we have an analogous expression for an impulse concentrated at an arbitrary point, say, t_0. That is,

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

Example 1.7

Consider the discontinuous signal *x(t)* depicted in Figure 1.40(a). Because of the relationship between the continuous-time unit impulse and unit step, we can readily calculate and graph the derivative of this signal. Specifically, the derivative of *x(t)* is clearly 0, except at the discontinuities. In the case of the unit step, we have seen [eq.(1.72)] that differentiation gives rise to a unit impulse located at the point of discontinuity.

Example 1.7

Furthermore, by multiplying both sides of eq.(1.72) by any number k, we see that the derivative of a unit step with a discontinuity of size k gives rise to an impulse of area k at the point of discontinuity. This rule also holds for any other signal with a jump discontinuity, such as $x(t)$ in Figure 1.40(a).

Example 1.7

Consequently, we can sketch its derivative $x(t)$, as in Figure 1.40(b), where an impulse is placed at each discontinuity of $x(t)$, with area equal to the size of the discontinuity. Note, for example, that the discontinuity in $x(t)$ at $t = 2$ has a value of -3, so that an impulse scaled by -3 is located at $t = 2$ in the signal $x(t)$.

Example 1.7

As a check of our result, we can verify that we can recover $x(t)$ from $\dot{x}(t)$. Specifically, since $x(t)$ and $\dot{x}(t)$ are both zero for $t \le 0$, we need only check that for $t > 0$,

$$x(t) = \int_0^t \dot{x}(\tau)\, d\tau. \tag{1.77}$$

As illustrated in Figure 1.40(c), for $t < 1$, the integral on the right-hand side of eq. (1.77) is zero, since none of the impulses that constitute $\dot{x}(t)$ are within the interval of integration. For $1 < t < 2$, the first impulse (located at $t = 1$) is the only one within the integration interval, and thus the integral in eq. (1.77) equals 2, the area of this impulse. For $2 < t < 4$, the first two impulses are within the interval of integration, and the integral accumulates the sum of both of their areas, namely, $2 - 3 = -1$. Finally, for $t > 4$, all three impulses are within the integration interval, so that the integral equals the sum of all three areas—that is, $2 - 3 + 2 = +1$. The result is exactly the signal $x(t)$ depicted in Figure 1.40(a).

Example 1.7

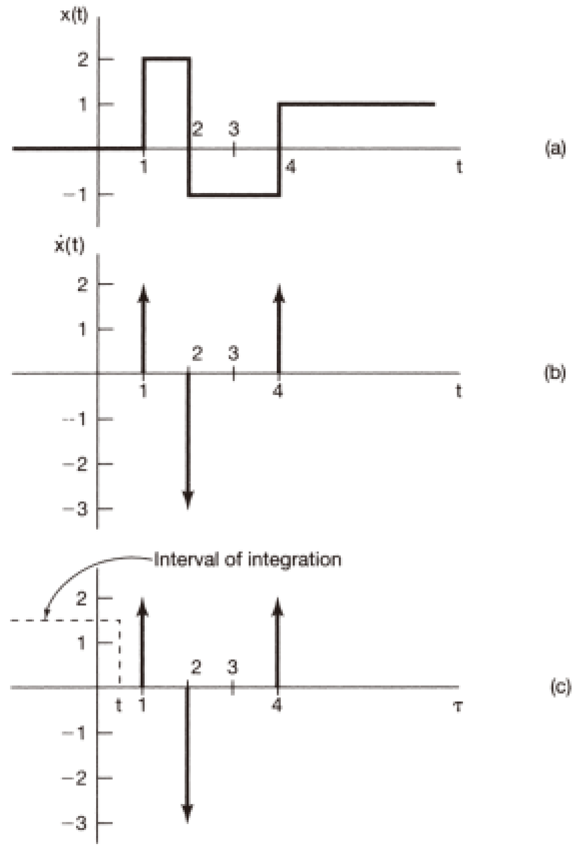


Figure 1.40 (a) The discontinuous signal $x(t)$ analyzed in Example 1.7; (b) its derivative $\dot{x}(t)$; (c) depiction of the recovery of $x(t)$ as the running integral of $\dot{x}(t)$, illustrated for a value of t between 0 and 1.

1.5 Continuous-Time and Discrete-Time System

- In contexts ranging from signal processing and communications to electromechanical motors, automotive vehicles, and chemical-processing plants, a system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs.

  「系統」可視為一種將輸入訊號轉換至輸出訊號的過程

1.5 Continuous-Time and Discrete-Time System

- A continuous-time system is a system in which continuous-time input signals are applied and result in continuous-time output signals.

  連續時間系統是一種以連續時間訊號為輸入，而得連續時間輸出訊號的系統

- We will often represent the input-output relation of a continuous-time system by the notation.

$$x(t) \to y(t) \qquad (1.78)$$

1.5 Continuous-Time and Discrete-Time System

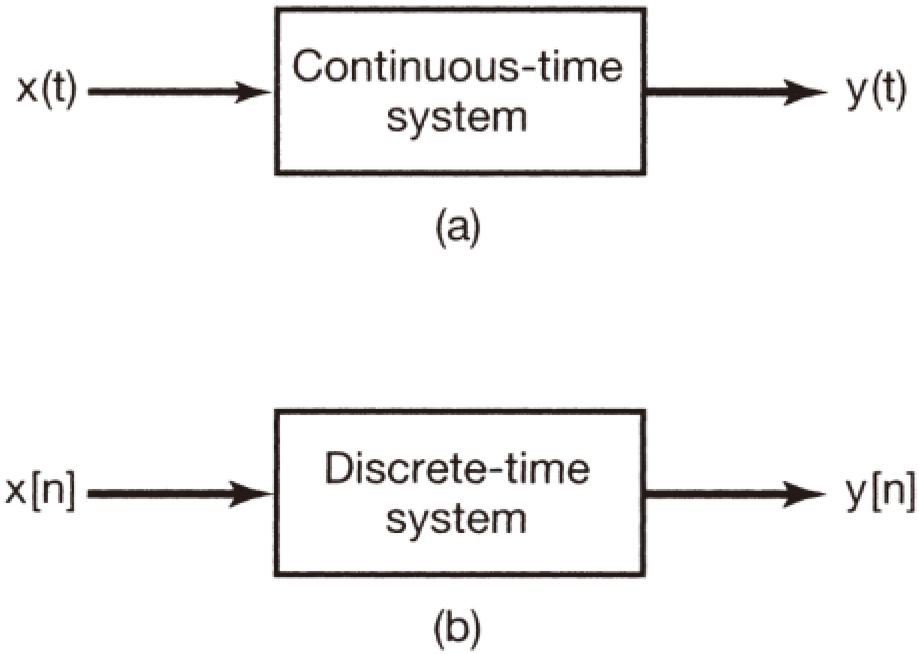


Figure 1.41 (a) Continuous-time system; (b) discrete-time system.

1.5 Continuous-Time and Discrete-Time System

- A discrete-time system – that is, a system that transforms discrete-time inputs into discrete-time output – will be depicted as in Figure 1.41(b) and will sometimes by represented symbolically as

$$x[n] \to y[n] \qquad (1.79)$$

離散時間系統為一種將離散時間輸入，轉換為離散時間輸出的系統

Example 1.8

$$i(t) = \frac{v_s(t) - v_c(t)}{R} \qquad (1.80)$$

We can relate *i(t)* to the rate of change with time of the voltage across the capacitor:

$$i(t) = C\frac{dv_c(t)}{dt} \qquad (1.81)$$

Example 1.8

Equating the right-hand sides of eq.(1.80) and (1.81), we obtain a differential equation describing the relationship between the input $v_s(t)$ and the output $v_c(t)$:

$$\frac{dv_c(t)}{dt}+\frac{1}{RC}v_c(t)=\frac{1}{RC}v_s(t) \qquad (1.82)$$

1.5 Continuous-Time and Discrete-Time System

$$\frac{dy(t)}{dt} + ay(t) = bx(t) \qquad (1.85)$$

Where $x(t)$ is the input, $y(t)$ is the output, and a and b are constants.

1.5 Continuous-Time and Discrete-Time System

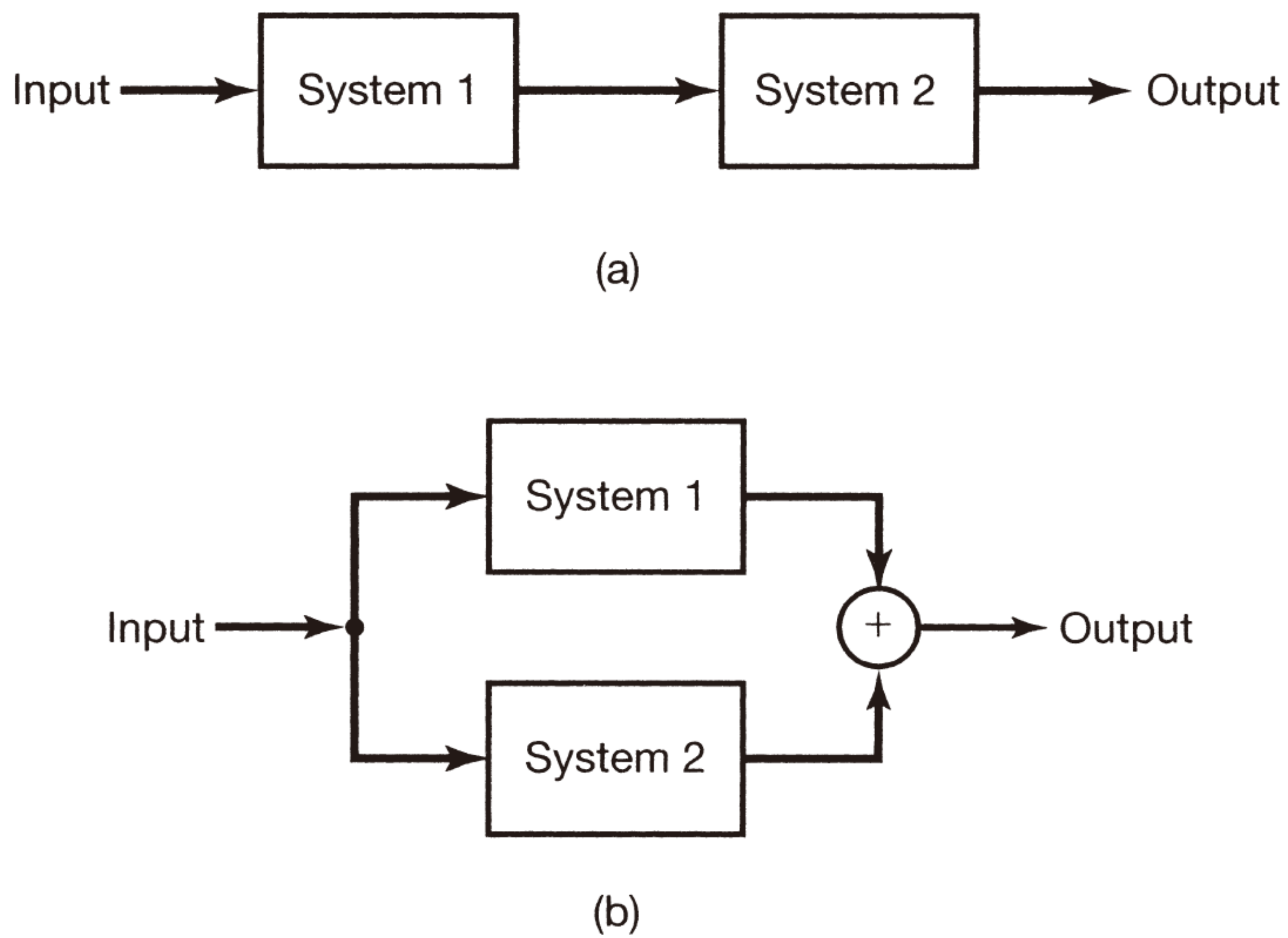
- The key to doing this successfully is identifying classes of systems that have two important characteristics:
  1. The systems in this class have properties and structures that we can exploit to gain insight into their behavior and to develop effective tools for their analysis.
  2. Many systems of practical importance can be accurately modeled using systems in this class.

1.5.2 Interconnections of Systems

- A series or cascade interconnection
  - Diagrams such as this are referred to as block diagrams. Here, the output of System 1 is the input to System 2, and the overall system transforms an input by processing it first by System 1 and then by System 2.

1.5.2 Interconnections of Systems

- A parallel interconnection
  - The same input signal is applied to System 1 and 2. The output of the parallel interconnection is the sum of the outputs of System 1 and 2.


Input
System 1
System 2
Output
(a)
System 1
Input
+
Output
System 2
(b)

1.5.2 Interconnections of Systems

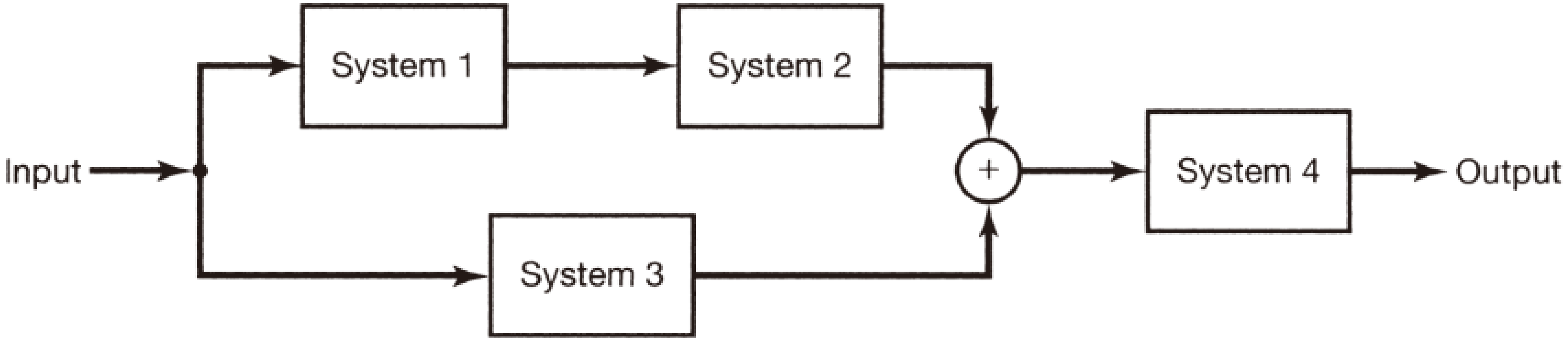


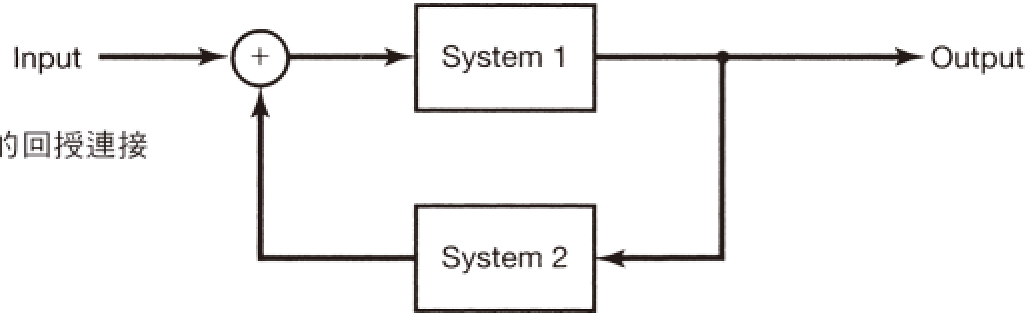
Figure 1.42 Interconnection of two systems: (a) series (cascade) interconnection; (b) parallel interconnection; (c) series-parallel interconnection.

1.5.2 Interconnections of Systems

- Feedback interconnection
  - The output of System 1 is the input to System 2, while the output of System 2 is fed back and added to the external input to produce the actual input to System 1.

    系統的回授連接為系統1的輸出作為系統2的輸入，並將系統2的輸出與外加輸入合成後作為系統1的輸入

1.5.2 Interconnections of Systems



系統的回授連接
圖

Figure 1.43 Feedback interconnection.

1.5.2 Interconnections of Systems

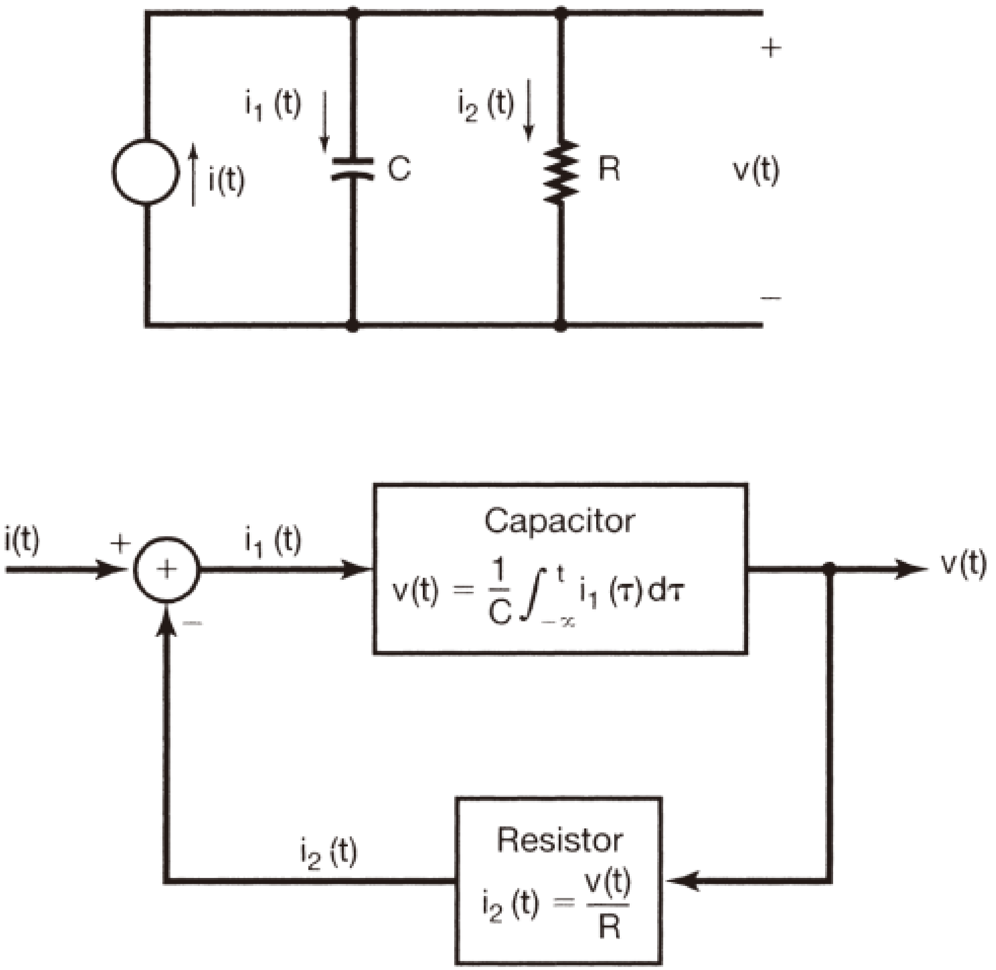


Figure 1.44 (a) Simple electrical circuit; (b) block diagram in which the circuit is depicted as the feedback interconnection of two circuit elements.

1.6.1 Systems With and Without Memory

- A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the input at only that same time.

無記憶系統為系統的輸出只與當時的輸入值有關

$$y[n] = \left(2x[n] - x^2[n]\right)^2 \qquad (1.90)$$

1.6.1 Systems With and Without Memory

A resistor is a memoryless system; with the input *x(t)* taken as the current and with the voltage taken as the output *y(t)*, the input-output relationship of a resistor is

$$y(t) = Rx(t) \tag{1.91}$$

Where *R* is the resistance.

1.6.1 Systems With and Without Memory

The input-output relationship for the continuous-time identity system is

$$y(t) = x(t)$$

and the corresponding relationship in discrete time is

$$y[n] = x[n]$$

An example of discrete-time system with memory is an accumulator or summer

$$y[n] = \sum_{k=-\infty}^{n} x[k] \tag{1.92}$$

1.6.1 Systems With and Without Memory

and a second example is a delay

$$y[n] = x[n-1] \qquad (1.93)$$

A capacitor is an example of a continuous-time system with memory, since if the input is taken to be the current and the output is the voltage, then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau \qquad (1.94)$$

where C is the capacitance.

1.6.1 Systems With and Without Memory

- The relationship between the input and output of an accumulator can be described as

$$y[n]=\sum_{k=-\infty}^{n-1}x[k]+x[n] \qquad (1.95)$$

or equivalently,

$$y[n]=y[n-1]+x[n] \qquad (1.96)$$

1.6.2 Invertibility and Inverse System

- An example of an invertible continuous-time system is

$$y(t) = 2x(t) \qquad (1.97)$$

for which the inverse system is

$$w(t) = \frac{1}{2}y(t) \qquad (1.98)$$

the inverse system is

$$w[n] = y[n] - y[n-1] \qquad (1.99)$$

1.6.2 Invertibility and Inverse System

Examples of noninvertible systems are

$$y[n] = 0 \qquad (1.100)$$

That is, the system that produces the zero output sequence for any input sequence, and

$$y(t) = x^2(t) \qquad (1.101)$$

1.6.2 Invertibility and Inverse System

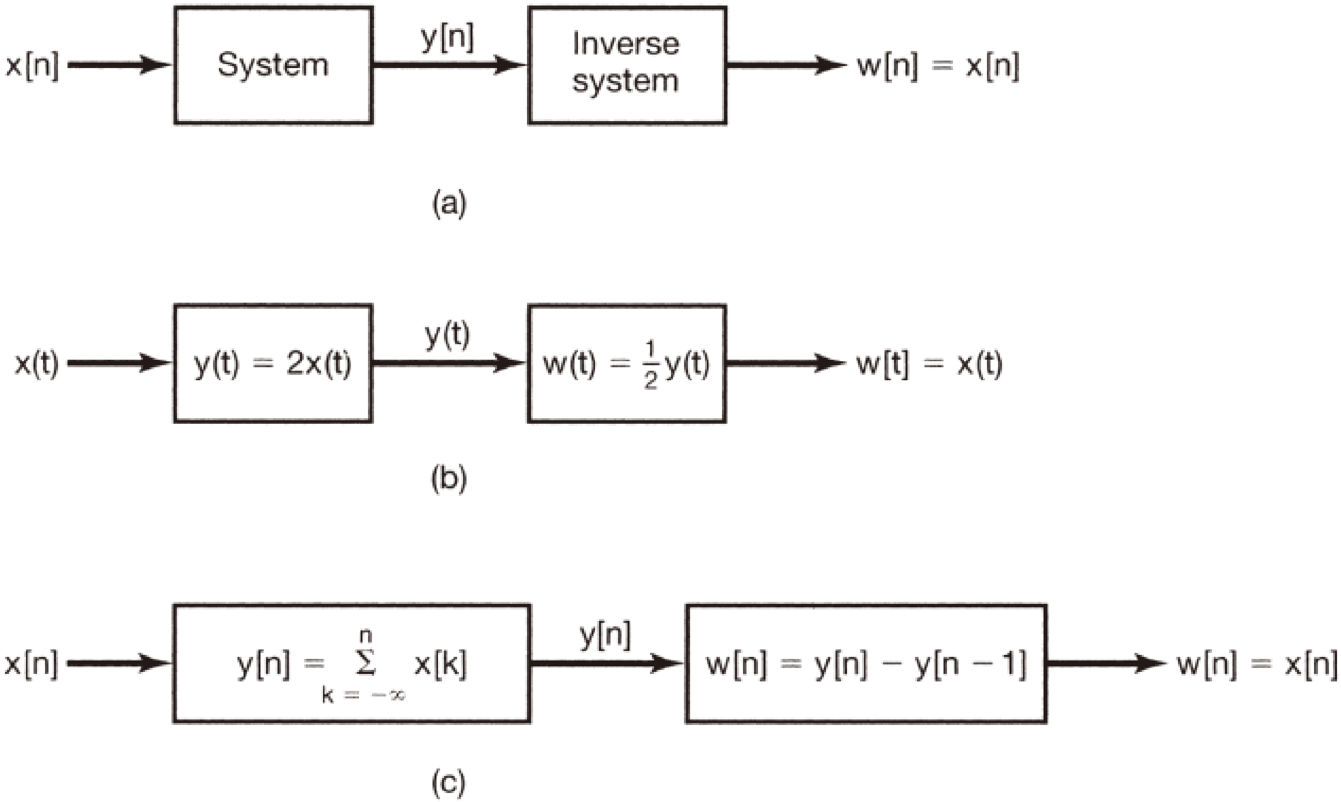


Figure 1.45 Concept of an inverse system for: (a) a general invertible system; (b) the invertible system described by eq. (1.97); (c) the invertible system defined in eq. (1.92).

1.6.3 Causality

- A system is causal if the output at any time depends on values of the input at only the present and past times.

  若一系統的輸出只與當時和過去的輸入有關，則稱為「因果系統」

1.6.3 Causality

$$y[n] = x[n] - x[n+1] \qquad (1.102)$$

$$y(t) = x(t+1) \qquad (1.103)$$

$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{+M} x[n-k] \qquad (1.104)$$

Example 1.12

The first system is defined by

$$y[n] = x[-n] \qquad (1.105)$$

In particular, for $n < 0$, e.g.n = -4, we see that y[-4] = x[4], so that the output at this time depends on a future value of the input.

$$y(t) = x(t)\cos(t+1) \qquad (1.106)$$

Example 1.12

In this system, the output at any time t equals the input at that same multiplied by a number that varies with time.

$$y(t) = x(t)g(t)$$

Where g(t) is a time-varying function, namely g(t) = cos(t+1).

1.6.4 Stability

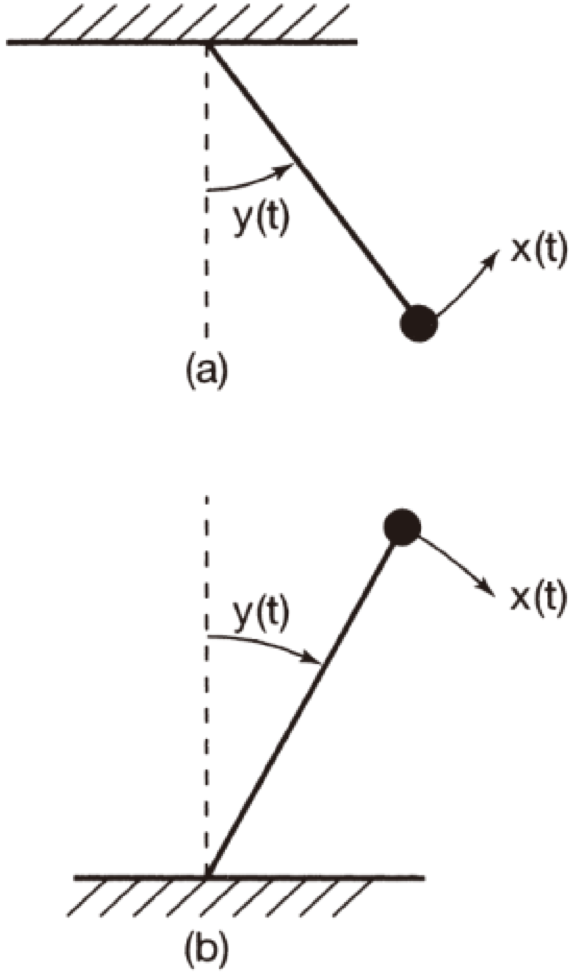


Figure 1.46 (a) A stable pendulum; (b) an unstable inverted pendulum.

1.6.4 Stability

- The preceding examples provide us with an intuitive understanding of the concept of stability. More formally, if the input to a stable system is bounded, then the output must also be bounded and therefore cannot diverge.

  「穩定性」的定義為當一個穩定系統的輸入為有界時，其輸出亦必為有界(不發散)

1.6.4 Stability

We see that this terminal velocity value V must satisfy

$$\frac{p}{m}V = \frac{1}{m}F \qquad (1.107)$$

$$V = \frac{F}{P} \qquad (1.108)$$

$$y[n] = \sum_{k=-\infty}^{n} u[k] = (n+1)u[n]$$

1.6.5 Time Invariance

- A system is time invariant if the behavior and characteristics of the system are fixed over time.
若系統的表現和特性在時間上是固定不變的，則稱為「非時變系統」
- A system is time invariant if a time shift in the input signal results in an identical time shift in the output signal
若系統的輸入訊號有時間移位時，其輸出訊號亦有相同的時間移位，則系統為非時變

1.6.5 Time Invariance

- If $y[n]$ is the output of a discrete-time, time-invariant system when $x[n]$ is the input, then $y[n\text{-}n_0]$ is the output when $x[n\text{-}n_0]$ is applied. In continuous time with *y(t)* the output corresponding to the input *x(t)*, a time-invariant system will have $y(t\text{-}t_0)$ as the output when $x(t\text{-}t_0)$ is the input.

1.6.6 Linearity

- A linear system, in continuous time or discrete time, is a system that possesses the important property of superposition: If an input consists of the weighted sum of several signals, then the output is the superposition – that is, the weighted sum – of the responses of the system to each of those signals.

1.6.6 Linearity

- The system is linear if
  1. The response to $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$.
  2. The response to $ax_1(t)$ is $ay_1(t)$, where a is any complex constant.
- The two properties defining a linear system can be combined into a single statement;

$$\text{continous time}: ax_1(t)+bx_2(t)\rightarrow ay_1(t)+by_2(t) \tag{1.121}$$

$$\text{discrete time}: ax_1[n]+bx_2[n]\rightarrow ay_1[n]+by_2[n] \tag{1.122}$$

1.6.6 Linearity

$$x[n]=\sum_{k} a_k x_k[n]=a_1x_1[n]+a_2x_2[n]+a_3x_3[n]+\ldots \qquad (1.123)$$

Is

$$y[n]=\sum_{k} a_k y_k[n]=a_1y_1[n]+a_2y_2[n]+a_3y_3[n]+\ldots \qquad (1.124)$$

if $x[n]\rightarrow y[n]$, then the homogeneity property tells us that

$$0=0\cdot x[n]\rightarrow 0\cdot y[n]=0 \qquad (1.125)$$

1.7 Summary

- 在本章中，我們發展出一些有關連續時間及離散時間的訊號與系統的基本概念。我們透過一些例子提出了訊號與系統的直覺概念，也提出了全書將使用的訊號與系統的數學表示法。特別是，我們引進了訊號的圖形式及數學式的表示法，且利用這些表示法在求取自變數的變換上。我們也定義並檢視了一些連續時尖及離散時間的一些基本的訊號。這些訊號包含了複指數訊號、弦波訊號，以及單為脈衝與步級函數。此外，我們也探討了連續時間與離散時間訊號的週期性的概念

1.7 Summary

- 在發展有關系統的一些基本概念中，我們引進了方塊圖，使我們對系統相互連接的討論更具洞察能力，而且我們定義了許多系統的重要性質，包括因果性、穩定性、非時變性及線性。
- 本書大部分內容主要焦點在於線性非時變(LTI)類型的系統，包含連續時間與離散時間兩方面的。這些系統在系統分析與設計上扮演了特別重要的角色，部分理由是因為在自然界中我們所面對的許多系統，可以很成功地模式化為線性非時變。此外，正如後續章節中將看到的，線性且非時變的性質允許我們更詳細去分析LTI系統的行為表現