

Material Science

Homework set 7 Solution

$$1. \quad \begin{cases} G_1 = V_s g_v^L + V_L g_v^L \\ G_2 = V_s g_v^S + V_L g_v^L + A_{ls} \gamma \end{cases}$$

$$\Delta G^{hom} = G_2 - G_1 = V_s (g_v^S - g_v^L) + A_{ls} \gamma = \frac{4}{3} \pi r^3 \Delta G_v + 4 \pi r^2 \gamma$$

$$\frac{d\Delta G^{hom}}{dr} = 4 \pi r^2 \Delta G_v + 8 \pi r \gamma = 0 \quad \xrightarrow{\text{solve}} \quad r^* = -\frac{2\gamma}{\Delta G_v}$$

The per unit volume free change can be express $\Delta G_v = \frac{H_f (T_m - T)}{T_m}$,

Therefore, substitution of this expression for ΔG_v into the critical radius yields

$$r^* = -\frac{2\gamma T_m}{H_f (T_m - T)} = -\frac{2\gamma T_m}{H_f \Delta T}$$

2. (1) There are two types of nucleation: homogeneous and heterogeneous. The distinction between them is made according to the site at which nucleating events occur. For the homogeneous type, nuclei of the new phase form uniformly throughout the parent phase, whereas for the heterogeneous type, nuclei form preferentially at structural inhomogeneities, such as container surfaces, insoluble impurities, grain boundaries, dislocations, and so on.

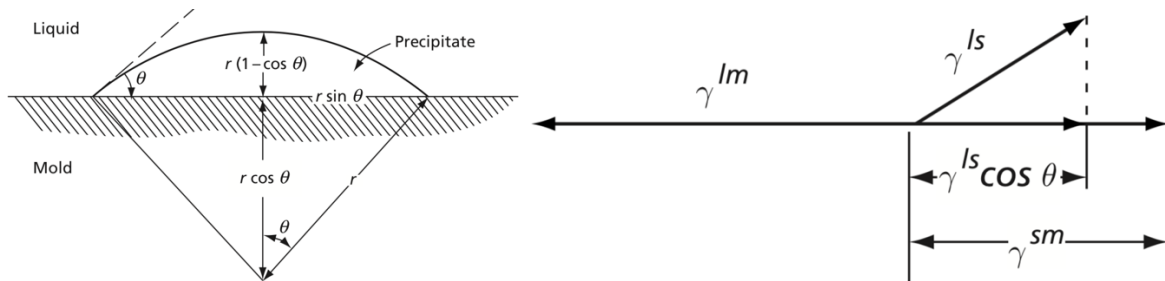
(2)

We can now write an equation giving the free energy of an embryo. In this case we shall express the equation in terms of the radius of the surface of the spherical cap rather than in terms of the number of atoms in the embryo. The equation in question is

$$\Delta G^{het} = V_c \Delta g_v + A_{ls} \gamma^{ls} + A_{sm} (\gamma^{sm} - \gamma^{lm})$$

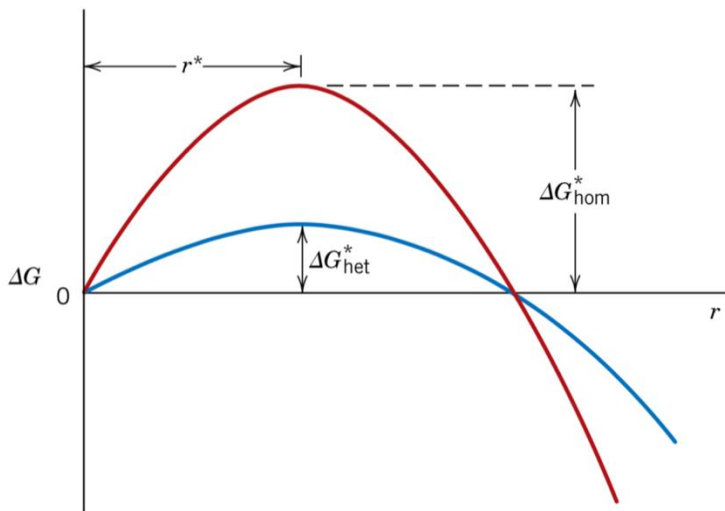
where ΔG^{het} is the free energy associated with the heterogeneously nucleated embryo, V_c the volume of the cap-shaped embryo, A_{ls} is the area of the cap that faces the liquid, A_{sm} is the area of the interface between the embryo and the mold wall, Δg_v is the free energy per unit volume associated with the freezing process, and γ^{ls} , γ^{sm} , and γ^{lm} are the surface energies as defined above. Note that A_{sm} is multiplied by the difference between γ^{sm} , and γ^{lm} . This is because the surface formed between the embryo and the mold replaces an equivalent area of liquid and mold interface.

$$\begin{aligned} \Delta G^{het} &= \frac{1}{3} \pi r^3 (2 - 3 \cos \theta + \cos^3 \theta) \Delta g_v + 2 \pi r^2 (1 - \cos \theta) \gamma^{ls} + \pi r^2 \sin^2 \theta (\gamma^{sm} - \gamma^{lm}) \\ &= \frac{4}{3} \pi r^3 \frac{(2 - 3 \cos \theta + \cos^3 \theta)}{4} \Delta g_v + 4 \pi r^2 \gamma^{ls} \frac{(2 - 3 \cos \theta + \cos^3 \theta)}{4} \\ &= \left(\frac{4}{3} \pi r^3 \Delta g_v + 4 \pi r^2 \gamma^{ls} \right) \frac{(2 - 3 \cos \theta + \cos^3 \theta)}{4} \\ &= \Delta G^{hom} \frac{(2 - 3 \cos \theta + \cos^3 \theta)}{4} \end{aligned}$$



The surface forces associated with the spherical cap embryo

This lower ΔG^{het} for heterogeneous nucleation means that a smaller energy must be overcome during the nucleation process (than for homogeneous nucleation), and, therefore, heterogeneous nucleation occurs more readily.



3. $y = 1 - \exp(-kt^n)$

$$t^n = -\frac{\ln(1-y)}{k} \rightarrow t^3 = -\frac{\ln(1-y)}{7 \times 10^{-3}}$$

For $t_{0.5}$ this equation takes the form $t_{0.5} = \sqrt[3]{-\frac{\ln(1-0.5)}{7 \times 10^{-3}}} = 4.626 \text{ sec}$

→ The rate is computed as $r = \frac{1}{t_{0.5}} = 0.216 \text{ sec}^{-1}$

4.

- (1) Superheating and supercooling correspond, respectively, to heating or cooling above or below a phase transition temperature without the occurrence of the transformation. These phenomena occur because right at the phase transition temperature, the driving force is not sufficient to cause the transformation to occur. The driving force is enhanced during superheating or supercooling.
- (2) The microstructures of pearlite, bainite, and spheroidite all consist of α -ferrite and cementite phases. For pearlite, the two phases exist as layers which alternate with one another. Bainite consists of very fine and parallel needle-shaped particles of cementite that are surrounded an α -ferrite matrix. For spheroidite, the matrix is ferrite, and the cementite phase is in the shape of sphere-shaped particles. Bainite is harder and stronger than pearlite, which, in turn, is harder and stronger than spheroidite.

- (3) Two important differences between continuous cooling transformation diagrams for plain carbon and alloy steels are: (a) for an alloy steel, a bainite nose will be present, which nose will be absent for plain carbon alloys; and (b) the pearlite-proeutectoid noses for plain carbon steel alloys are positioned at shorter times than for the alloy steels. There is no bainite transformation region on the continuous cooling transformation diagram for an iron-carbon alloy of eutectoid composition because by the time a cooling curve has passed into the bainite region, the entirety of the alloy specimen will have transformed to pearlite.
- (4) The hardness and strength of iron-carbon alloys that have microstructures consisting of α -ferrite and cementite phases depend on the boundary area between the two phases. The greater this area, the harder and stronger the alloy inasmuch as (a) these boundaries impede the motion of dislocations, and (b) the cementite phase restricts the deformation of the ferrite phase in regions adjacent to the phase boundaries. Fine pearlite is harder and stronger than coarse pearlite because the alternating ferrite-cementite layers are thinner for fine, and therefore, there is more phase boundary area. The phase boundary area between the sphere-like cementite particles and the ferrite matrix is less in spheroidite than for the alternating layered microstructure found in coarse pearlite.
- (5) Both tempered martensite and spheroidite have sphere-like cementite particles within a ferrite matrix; however, these particles are much larger for spheroidite. Tempered martensite is harder and stronger inasmuch as there is much more ferrite-cementite phase boundary area for the smaller particles; thus, there is greater reinforcement of the ferrite phase, and more phase boundary barriers to dislocation motion.
5. This ranking is as follows: (c) > (d) > (b) > (a).
The 0.25 wt% C, coarse pearlite is stronger than the 0.25 wt% C, spheroidite since coarse pearlite is stronger than spheroidite; the composition of the alloys is the same. The 0.60 wt% C, coarse pearlite is stronger than the 0.25 wt% C, coarse pearlite, since increasing the carbon content increases the strength. Finally, the 0.60 wt% C, fine pearlite is stronger than the 0.60 wt% C, coarse pearlite inasmuch as the strength of fine pearlite is greater than coarse pearlite because of the many more ferrite-cementite phase boundaries in fine pearlite.
6. (1) 50% bainite and 50% martensite
(2) 15% pearlite, 45% bainite and 40% martensite
(3) 100% tempered martensite
7. Sketch on the figure(next page) and label time below followings,
(1) Rapidly cool to 680°C, hold for 80 s, rapidly cool to 560°C, hold for 2 s, then quench to room temperature.
(2) Rapidly cool to 175°C, hold on.
(3) After cooling to and while holding at 680°C the specimen first transforms to proeutectoid ferrite and coarse pearlite. Continued heat treating at 680°C for 10^8 s results in a further transformation into spheroidite.
(4) Rapidly cool to 450°C, hold for 8 s, then quench to room temperature.

