

HW-CH37

1. An experimenter arranges to trigger two flashbulbs simultaneously, producing a big flash located at the origin of his reference frame and a small flash at $x = 30.0\text{km}$. An observer moving at a speed of $0.250c$ in the positive direction of x also views the flashes. (a) What is the time interval between them according to her? (b) Which flash does she say occurs first?
2. Relativistic reversal of events. Figures 37-25a and b show the (usual) situation in which a primed reference frame passes an unprimed reference frame, in the common positive direction of the x and x' axes, at a constant relative velocity of magnitude v . We are at rest in the unprimed frame; Bullwinkle, an astute student of relativity in spite of his cartoon upbringing, is at rest in the primed frame. The figures also indicate events A and B that occur at the following space-time coordinates as measured in our unprimed frame and in Bullwinkle's primed frame:

<i>Event</i>	<i>Unprimed</i>	<i>Primed</i>
A	(x_A, t_A)	(x'_A, t'_A)
B	(x_B, t_B)	(x'_B, t'_B)

In our frame, event A occurs before event B, with temporal separation $\delta t = t_B - t_A = 1.00\mu\text{s}$ and spatial separation $\delta x = x_B - x_A = 400\text{m}$. Let $\delta t'$ be the temporal separation of the events according to Bullwinkle. (a) Find an expression for $\delta t'$ in terms of the speed parameter $\beta (= v/c)$ and the given data. Graph $\delta t'$ versus β for the following two ranges of β :

(b) 0 to 0.01 (v is low, from 0 to $0.01c$) (c) 0.1 to 1 (v is high, from $0.1c$ to the limit c) (d) At what value of β is $\delta t' = 0$? For what range of β is the sequence of events A and B according to Bullwinkle (e) the same as ours and (f) the reverse of ours? (g) Can event A cause event B, or vice versa? Explain.

3. (a) If m is a particle's mass, p is its momentum magnitude, and K is its kinetic energy, show that

$$m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) For low particle speeds, show that the right side of the equation reduces to m . (c) If a particle has $K = 55.0\text{ MeV}$ when $p = 121\text{ MeV}/c$, what is the ratio m/m_e of its mass to the electron mass?

4. In Section 28-6, we showed that a particle of charge q and mass m will move in a circle of radius $r = mv/|q|B$ when its velocity \vec{v} is perpendicular to a uniform magnetic field \vec{B} . We also found that the period T of the motion is independent of speed v . These two results are approximately correct if $v \ll c$. For relativistic speeds, we must use the correct equation for the radius:

$$r = \frac{p}{|q|B} = \frac{\gamma mv}{|q|B}.$$

- (a) Using this equation and the definition of period ($T = 2\pi r/v$), find the correct expression for the period. (b) Is T independent of v ? If a 10.0 MeV electron moves in a circular path in a uniform magnetic field of magnitude 2.20 T, what are (c) the radius according to Chapter 28, (d) the correct radius, (e) the period according to Chapter 28, and (f) the correct period?
5. Another approach to velocity transformations. In Fig. 37-31, reference frames B and C move past reference frame A in the common direction of their x axes. Represent the x components of the velocities of one frame relative to another with a two-letter subscript. For example, v_{AB} is the x component of the velocity of A relative to B. Similarly, represent the corresponding speed parameters with two-letter subscripts. For example, $\beta_{AB}(= v_{AB}/c)$ is the speed parameter corresponding to v_{AB} . (a) Show that

$$\beta_{AC} = \frac{\beta_{AB} + \beta_{BC}}{1 + \beta_{AB}\beta_{BC}}.$$

Let M_{AB} represent the ratio $(1 - \beta_{AB})/(1 + \beta_{AB})$, and let M_{BC} and M_{AC} represent similar ratios. (b) Show that the relation

$$M_{AC} = M_{AB}M_{BC}$$

is true by deriving the equation of part (a) from it.

6. The car-in-the garage problem. Carman has just purchased the world's longest stretch limo, which has a proper length of $L_c = 30.5m$. In Fig. 37-32a, it is shown parked in front of a garage with a proper length of $L_g = 6.00m$. The garage has a front door (shown open) and a back door (shown closed). The limo is obviously longer than the garage. Still, Garageman, who owns the garage and knows something about relativistic length contraction, makes a bet with Carman that the limo can fit in the garage with both doors closed. Carman, who dropped his physics course before reaching special relativity, says such a thing, even in principle, is impossible.

To analyze Garageman's scheme, an x_c axis is attached to the limo. with $x_c = 0$ at the rear bumper, and an x_g axis is attached to the garage, with $x_g = 0$ at the (now open) front door. Then Carman is to drive the limo directly toward the front door at a velocity of 0.9980c (which is, of course, both technically and financially impossible). Carman is stationary in the x_c reference frame; Garageman is stationary in the x_g reference frame.// There are two events to consider. Event 1: When the rear bumper clears the front door. the front door is closed. Let the time of this event be zero to both Carman and Garageman: $t_{g1} = t_{c1} = 0$. The event occurs at $x_c = x_g = 0$. Figure 37-32b shows event 1 according to the x_g reference frame. Event 2: When the front bumper reaches the back door. that door opens. Figure 37-32c shows event 2 according to the x_g reference frame.

According to Garageman, (a) what is the length of the limo, and what are the space-time coordinates (b) x_{g2} and (c) t_{g2} of event 2? (d) For how

long is the limo temporarily "trapped" inside the garage with both doors shut? Now consider the situation from the x_c reference frame, in which the garage comes racing past the limo at a velocity of $-0.9980c$. According to Carman, (e) what is the length of the passing garage, what are the space-time coordinates (f) x_{c2} and (g) t_{c2} of event 2, (h) is the limo ever in the garage with both doors shut, and (i) which event occurs first? (j) Sketch events 1 and 2 as seen by Carman. (k) Are the events causally related; that is, does one of them cause the other? (l) Finally, who wins the bet?

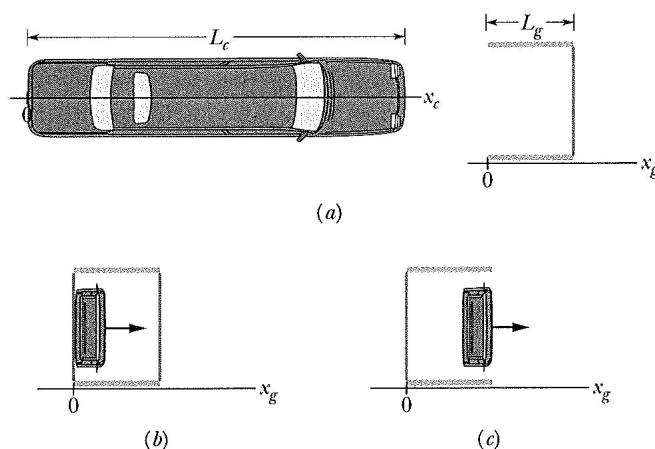


Fig. 37-32 Problem 69.

7. A radar transmitter T is fixed to a reference frame S' that is moving to the right with speed v relative to reference frame S . A mechanical timer (essentially a clock) in frame S' , having a period τ_0 (measured in S'), causes transmitter T to emit timed radar pulses, which travel at the speed of light and are received by R , a receiver fixed in frame S . (a) What is the period τ of the timer as detected by observer A , who is fixed in frame S ? (b) Show that at receiver R the time interval between pulses arriving from T is not τ or τ_0 , but

$$\tau_R = \tau_0 \sqrt{\frac{c+v}{c-v}}.$$

- (c) Explain why receiver R and observer A , who are in the same reference frame, measure a different period for the transmitter. (Hint: A clock and a radar pulse are not the same thing.)

