機率與統計 Exercise5 Chapter5 B06505047 陳銘杰

Problem 5.2.4

$S = \{tt, th, ht, hh\}, P(tt) = P(th) = P(ht) = P(hh) = 0.25$

$$P_{X,Y}(x,y) = \begin{cases} 0.25 & x = 2, y = 0 \\ 0.25 & x = 0, y = 2 \\ 0.5 & x = 1, y = 1 \end{cases}$$

Problem 5.3.6

$$P_N(n) = \sum_{k \in S_K} P_{N,K}(n,k) = \sum_{k=0}^{k=n} \frac{100^n e^{-100}}{(n+1)!}$$

$$= (n+1)\frac{100^n e^{-100}}{(n+1)!} = \frac{100^n e^{-100}}{n!} \quad (n = 0,1,2 \dots)$$

$$P_K(k) = \sum_{n \in S_N} P_{N,K}(n,k) \ when\ K = k, N \geq k$$

$$P_K(k) = \sum_{n=k}^{n=\infty} \frac{100^n e^{-100}}{(n+1)!} = \frac{1}{100}\sum_{n=k}^{n=\infty} \frac{100^{n+1} e^{-100}}{(n+1)!}$$

$$= \frac{1}{100}\sum_{n=k}^{n=\infty} P_N(n+1) = \frac{P[N > k]}{100}$$

Problem 5.6.3

$$P_X(x) = \binom{75}{x}\left(\frac{1}{2}\right)^{75}, P_Y(y) = \binom{25}{y}\left(\frac{1}{2}\right)^{25}$$

$X\ and\ Y\ are\ independent\ (100\ independent\ flips)$

$$P_{X,Y}(x,y) = \binom{75}{x}\binom{25}{y}\left(\frac{1}{2}\right)^{100}$$

Problem 5.9.4

$Y = X_1 + X_2$

$E[Y] = E[X_1 + X_2] = E[X_1] + E[X_2] = 0$

$Var[Y] = Var[X_1 + X_2] = E[(X_1 + X_2)^2]$

$$= \mathrm{E}\left[{X_1}^2 + 2X_1X_2 + {X_2}^2\right]$$

$$= 1 + 2\mathrm{E}[X_1X_2] + 1$$

$$\mathrm{E}[\mathrm{Y}] = 0, \mathrm{Var}[\mathrm{Y}] = 1, \rightarrow \mathrm{E}[X_1X_2] = Cov[X_1, X_2] = -\frac{1}{2}$$

Problem 5.10.9 (a)

$$F_{U_n}(u) = P[\max(X_1, \dots, X_n) \le u]$$

$$= \mathrm{P}[X_1 \le \mathrm{u}, \dots, X_n \le \mathrm{u}]$$

$$= \mathrm{P}[X_1 \le \mathrm{u}]\mathrm{P}[X_2 \le \mathrm{u}] \dots \mathrm{P}[X_n \le \mathrm{u}]$$

$$= (F_X(u))^n$$

Problem 5.10.9 (b)

$$F_{L_n}(l) = 1 - P[\min(X_1, \dots, X_n) > l]$$

$$= 1 - \mathrm{P}[X_1 > \mathrm{l}, \dots, X_n > l]$$

$$= 1 - \mathrm{P}[X_1 > l]\mathrm{P}[X_2 > l] \dots \mathrm{P}[X_n > l]$$

$$= 1 - (1 - F_X(l))^n$$

Problem 5.10.9 (c)

$$\mathrm{P}[L_n > \mathrm{l}, U_n \le \mathrm{u}] = P[\min(X_1, \dots, X_n) > l, \max(X_1, \dots, X_n) \le u]$$

$$= P[l < X_i \le u, i = 1,2, \dots, n]$$

$$= P[l < X_1 \le u] \dots P[l < X_n \le u]$$

$$= [F_X(u) - F_X(l)]^n$$

$$\mathrm{P}[U_n \le \mathrm{u}] = \mathrm{P}[L_n > \mathrm{l}, U_n \le \mathrm{u}] + \mathrm{P}[L_n \le \mathrm{l}, U_n \le \mathrm{u}]$$

$$F_{L_n,U_n}(l, u) = \mathrm{P}[U_n \le \mathrm{u}] - \mathrm{P}[L_n > \mathrm{l}, U_n \le \mathrm{u}]$$

$$= (F_X(u))^n - [F_X(u) - F_X(l)]^n$$