Chapter 8: **FLOW IN PIPES**

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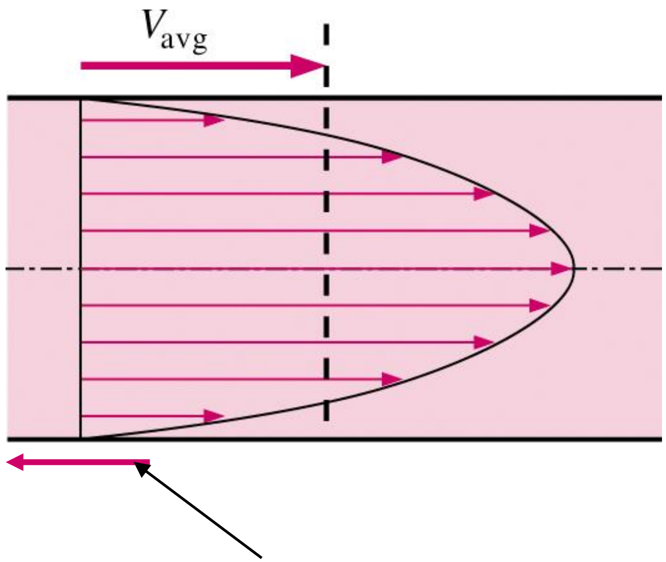
Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
- Understand the different velocity and flow rate measurement techniques and learn their advantages and disadvantages

Introduction



Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the <u>no-slip condition</u>, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and <u>friction</u> along the pipe walls

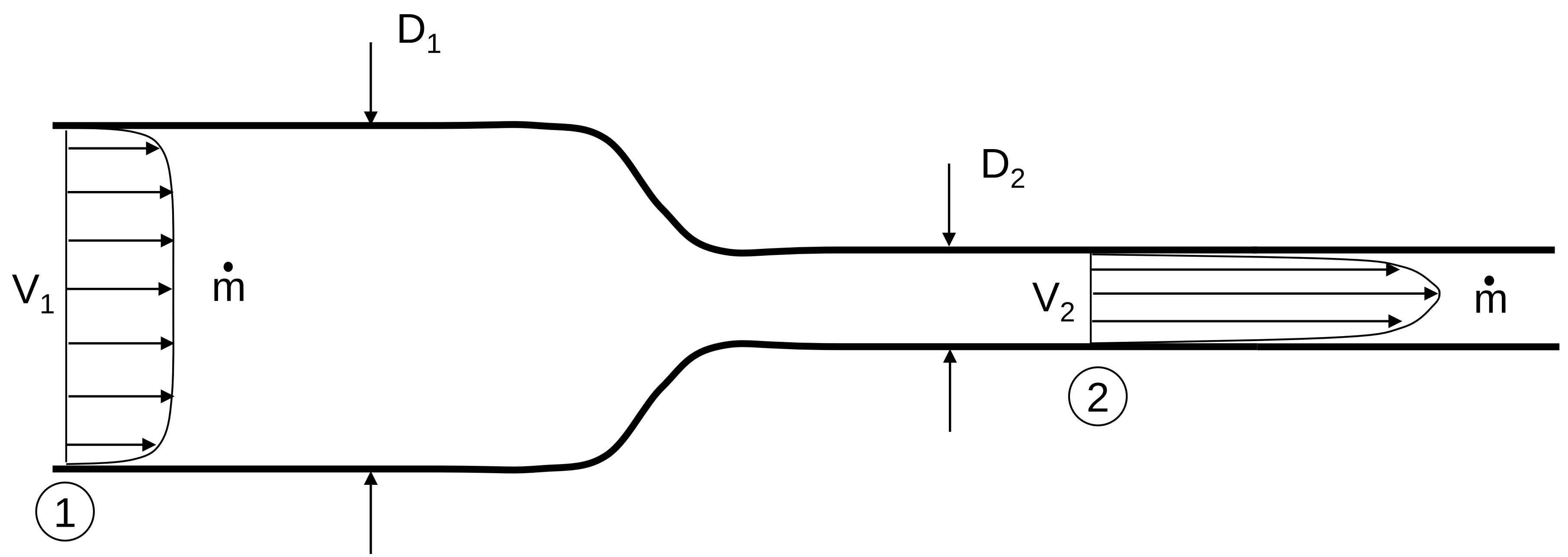
Introduction

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass

$$\dot{m} = \underset{\text{same}}{\rho}\, \underset{\text{same}}{V_{avg}}\, \underset{\text{same}}{A} = constant$$

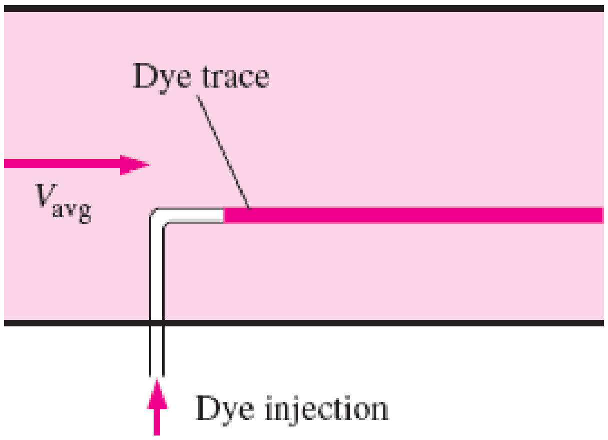
Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$

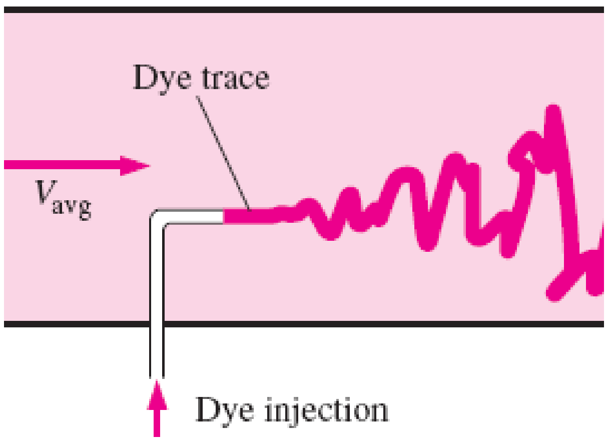


LAMINAR AND TURBULENT FLOWS

- **Laminar flow:** characterized by *smooth streamlines* and *highly ordered motion.*
- **Turbulent flow:** characterized by *velocity fluctuations* and *highly disordered motion.*
- The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.



(*a*) Laminar flow



(*b*) Turbulent flow

Reynolds Number

- The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid,* among other things.
- British engineer Osborne Reynolds (1842–1912) discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid.
- The ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$

Reynolds Number

- At large Reynolds numbers, the inertial forces are large relative to the viscous forces $\Rightarrow$ Turbulent Flow
- At *small* or *moderate* Reynolds numbers, the viscous forces are large enough to suppress these fluctuations $\Rightarrow$ Laminar Flow
- The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number,** Re_{cr}.
- The value of the critical Reynolds number is different for different geometries and flow conditions. For example, Re_{cr} = 2300 for internal flow in a circular pipe.

Reynolds Number

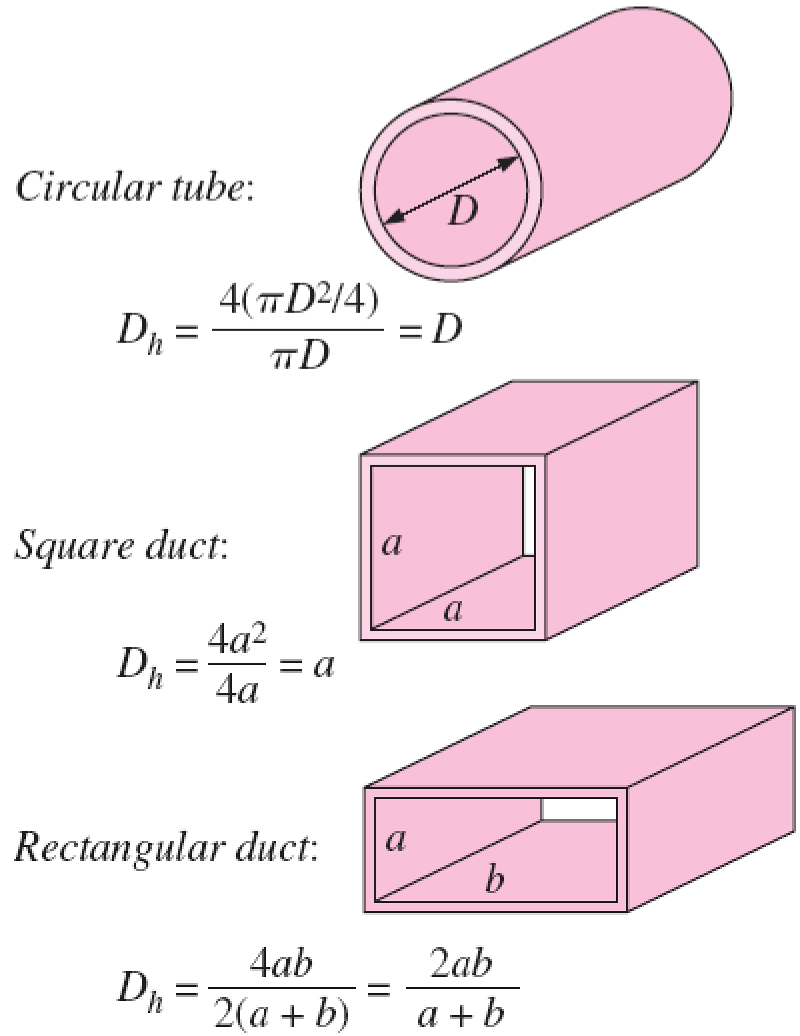
- For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as

$$D_h = \frac{4A_c}{p}$$

  A_c = cross-section area

  P = wetted perimeter

- The transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and *fluctuations in the flow.*



Reynolds Number

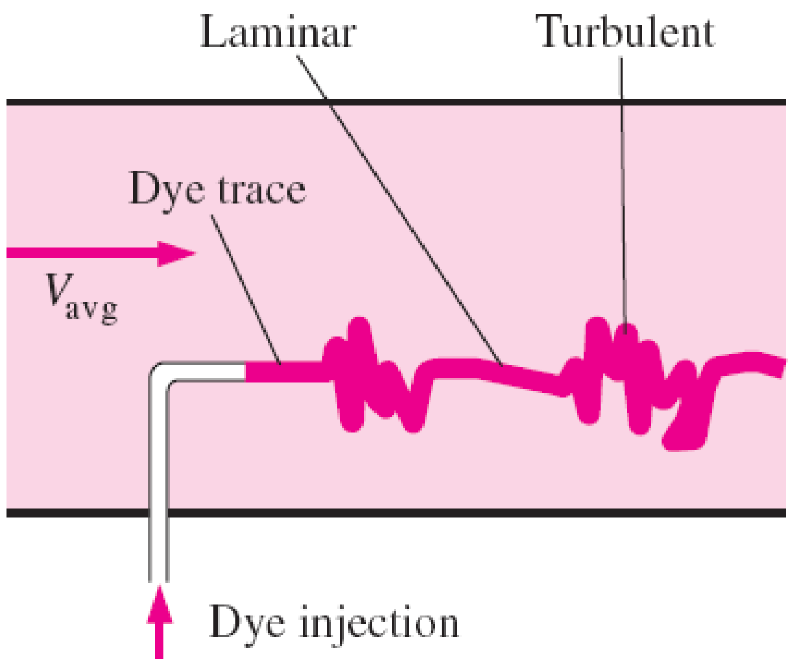
- Under most practical conditions, the flow in a circular pipe is

$$\mathrm{Re} \lesssim 2300 \qquad \text{laminar flow}$$

$$2300 \lesssim \mathrm{Re} \lesssim 4000 \qquad \text{transitional flow}$$

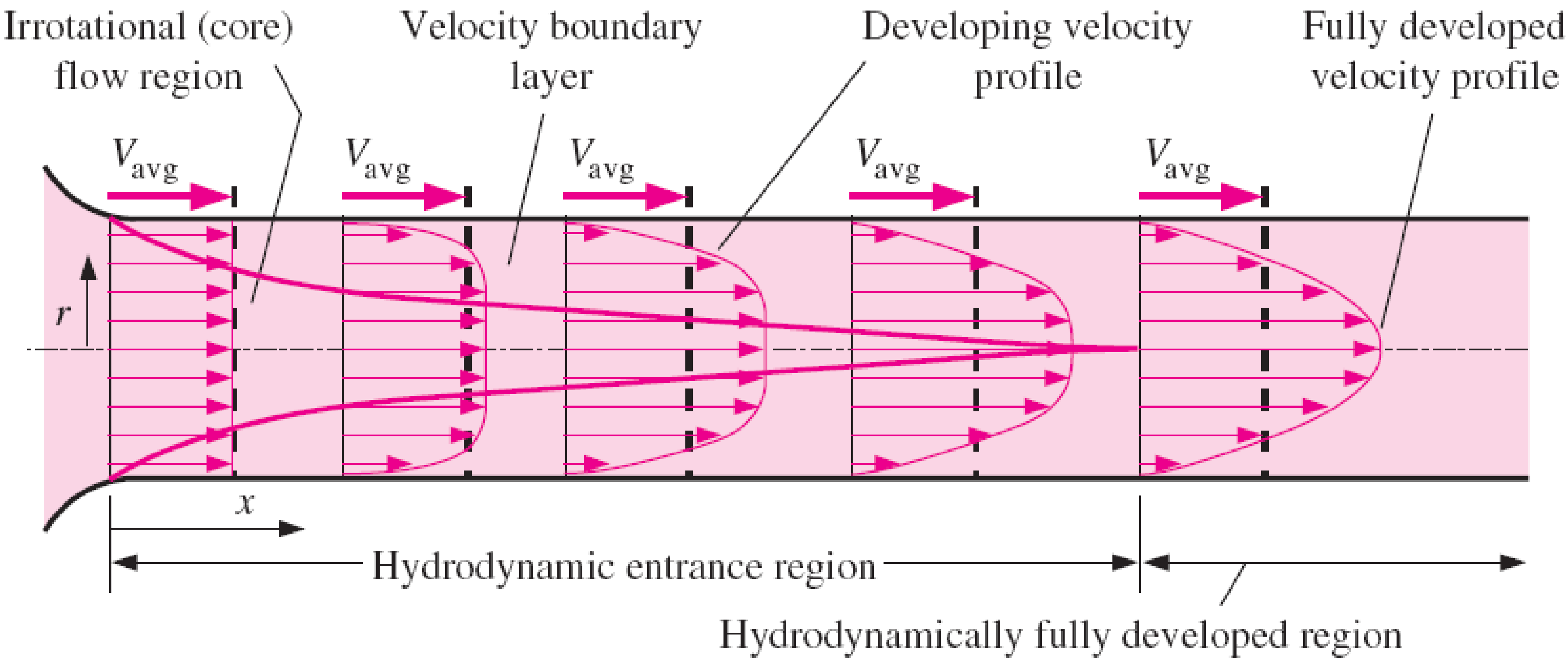
$$\mathrm{Re} \gtrsim 4000 \qquad \text{turbulent flow}$$

- In transitional flow, the flow switches between laminar and turbulent randomly.



THE ENTRANCE REGION

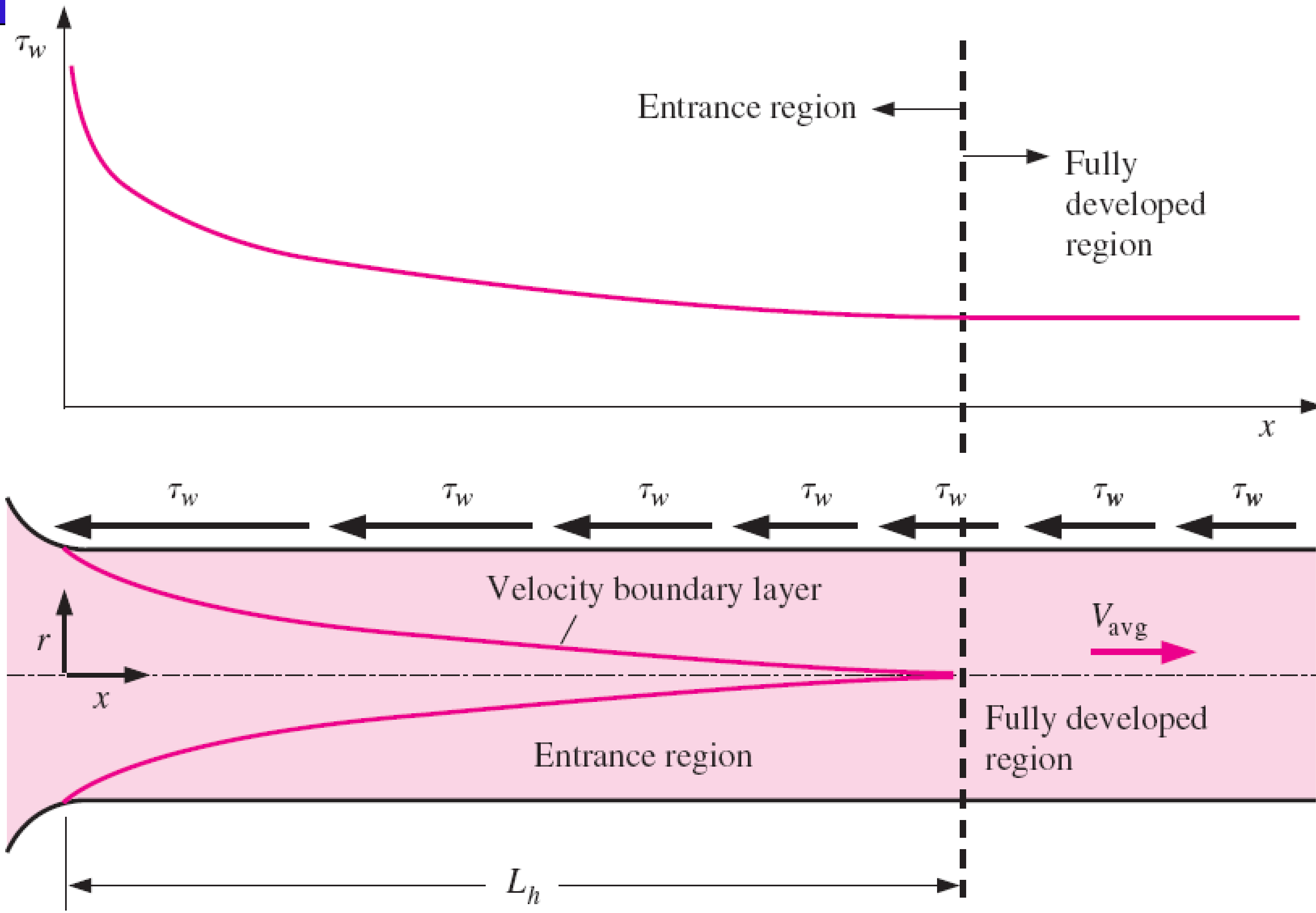
Consider a fluid entering a circular pipe at a uniform velocity.



THE ENTRANCE REGION

- The velocity profile in the fully developed region is *parabolic* in laminar flow and somewhat *flatter* (or *fuller*) in turbulent flow.
- The time-averaged velocity profile remains unchanged when the flow is fully developed, and thus $u = u(r)$ only.
- The velocity profile remains unchanged in the fully developed region, so does the wall shear stress.
- The wall shear stress is the *highest* at the pipe inlet where the thickness of the boundary layer is smallest, and decreases gradually to the fully developed value. Therefore, the pressure drop is *higher* in the entrance regions of a pipe.

THE ENTRANCE REGION



Entry Lengths

- The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.
- In *laminar flow,* the hydrodynamic entry length is given approximately as

$$L_{h,\,\text{laminar}} \cong 0.05\text{Re}D$$

- In *turbulent flow,* the hydrodynamic entry length for turbulent flow can be approximated as

$$L_{h,\,\text{turbulent}} = 1.359D\text{Re}_D^{1/4}$$

- The entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker.

Entry Lengths

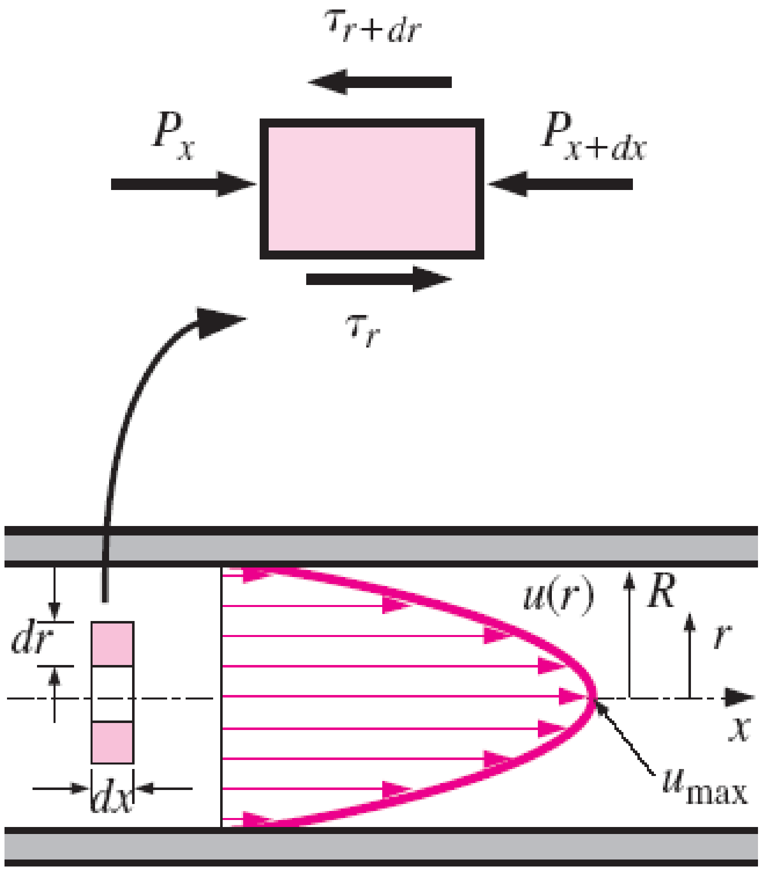
- In the limiting laminar case of Re 2300, the hydrodynamic entry length is 115*D.*
- In many pipe flows of practical engineering interest, the entrance effects for turbulent flow become insignificant beyond a pipe length of 10 diameters, and the hydrodynamic entry length is approximated as

$$L_{h,\ \text{turbulent}} \approx 10D$$

- In *turbulent flow,* it is reasonable to assume the flow is fully developed for a pipe whose length is several times longer than the length of its entrance region.

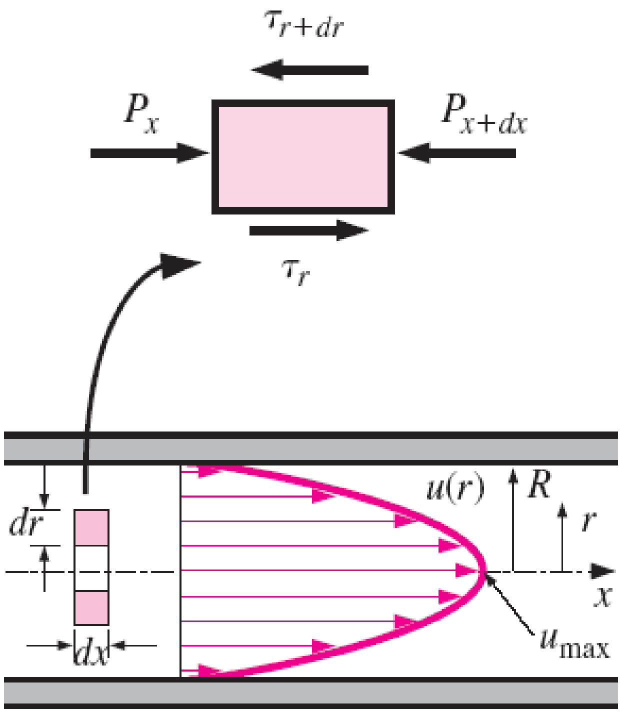
LAMINAR FLOW IN PIPES

- In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe.
- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and no motion in the radial direction such that no acceleration (since flow is steady and fully-developed).



LAMINAR FLOW IN PIPES

- Now consider a ring-shaped differential volume element of radius *r,* thickness *dr,* and length *dx* oriented coaxially with the pipe. A force balance on the volume element in the flow direction gives



$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

- Dividing by $2\pi drdx$ and rearranging,

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

LAMINAR FLOW IN PIPES

- Taking the limit as *dr, dx* → 0 gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

- Substituting $\tau = -\mu(du/dr)$ gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

- The left side of the equation is a function of *r*, and the right side is a function of *x*. The equality must hold for any value of *r* and *x*; therefore, $f(r) = g(x)$ = constant.

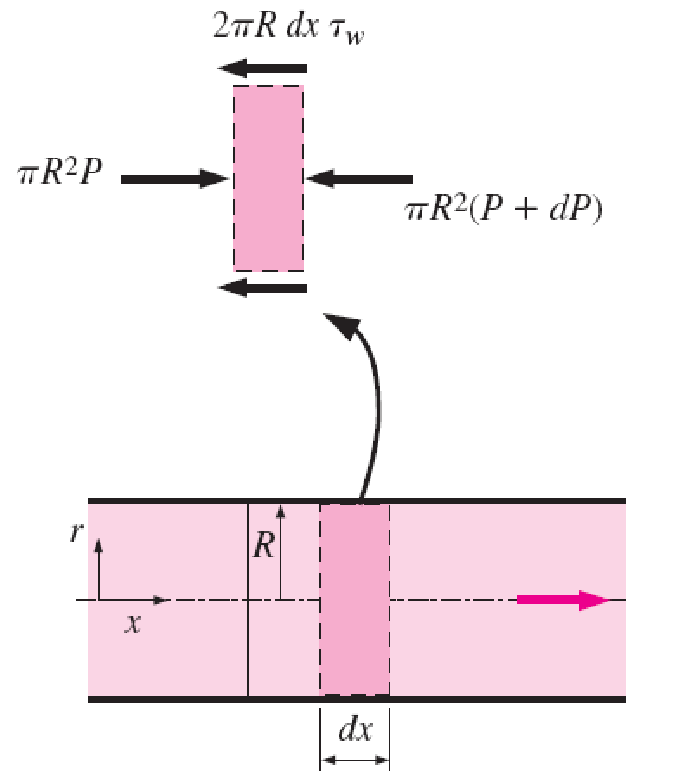
LAMINAR FLOW IN PIPES

- Thus we conclude that *dP*/*dx* = constant and we can verify that

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

- Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Then we solve the *u*(r) eq. by rearranging and integrating it twice to give

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

LAMINAR FLOW IN PIPES

- *Since* $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$, *then we can get* u(r)

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

- Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic.* Since u is positive for any r, and thus the dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).
- The average velocity is determined from

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

LAMINAR FLOW IN PIPES

- The velocity profile is rewritten as

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right)$$

- Thus we can get

$$u_{\max} = 2V_{\text{avg}}$$

- Therefore, *the average velocity in fully developed laminar pipe flow is one half of the maximum velocity*.

Pressure Drop and Head Loss

- The *pressure drop* ΔP of pipe flow is related to the power requirements of the fan or pump to maintain flow. Since dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

- The pressure drop for laminar flow can be expressed as

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

- ΔP due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss.*

Pressure Drop and Head Loss

- The pressure drop represents the pressure loss ΔP_L (No viscosity $\Rightarrow$ No loss)
- In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows as

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2}$$

where $\rho V_{\text{avg}}^2/2$ is the *dynamic pressure* and f is the **Darcy friction factor,**

$$f = \frac{8\tau_w}{\rho V_{\text{avg}}^2}$$

It is also called the **Darcy–Weisbach friction factor,** named after the Frenchman Henry Darcy (1803–1858) and the German Julius Weisbach (1806–1871)

Pressure Drop and Head Loss

- It should not be confused with the *friction coefficient* C_f, *Fanning friction factor,* which is defined as

  $C_f = 2\tau_w / (\rho V^2_{avg}) = f/4$.

- The friction factor for fully developed laminar flow in a circular pipe

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$ (*independent of the roughness*)

- In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L.

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2_{avg}}{2g}$$ (Frictional losses due to viscosity)

Pressure Drop and Head Loss

- Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{\text{pump},\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

- The average velocity for laminar flow in a horizontal pipe is

$$V_{\text{avg}} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

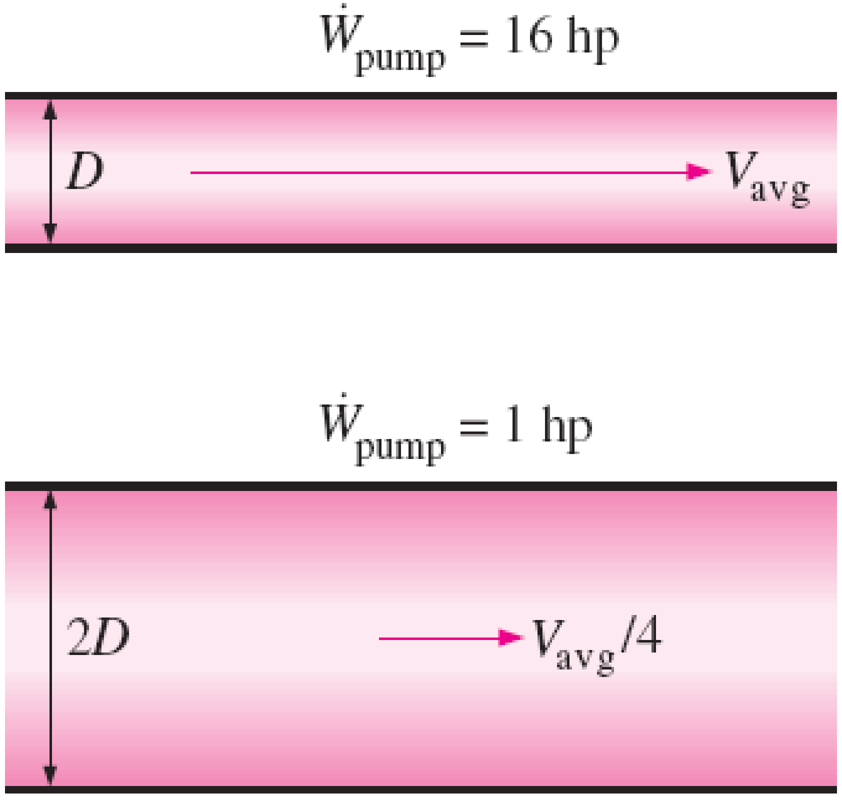
- The volume flow rate for laminar flow through a horizontal pipe becomes

$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

- This equation is known as **Poiseuille's law,** and this flow is called *Hagen–Poiseuille flow.*

Pressure Drop and Head Loss — Poiseuille's law

- *For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe.*



- Since

$$\dot{W} = \dot{V}\Delta p = \dot{V}(\dot{V}\frac{128\mu L}{\pi D^4})$$

Pressure Drop and Head Loss (Skipped)

- In the above cases, the pressure drop equals to the head loss, but this is not the case for inclined pipes or pipes with variable cross-sectional area.
- Let's examine the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},\,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},\,e} + h_L$$

Or

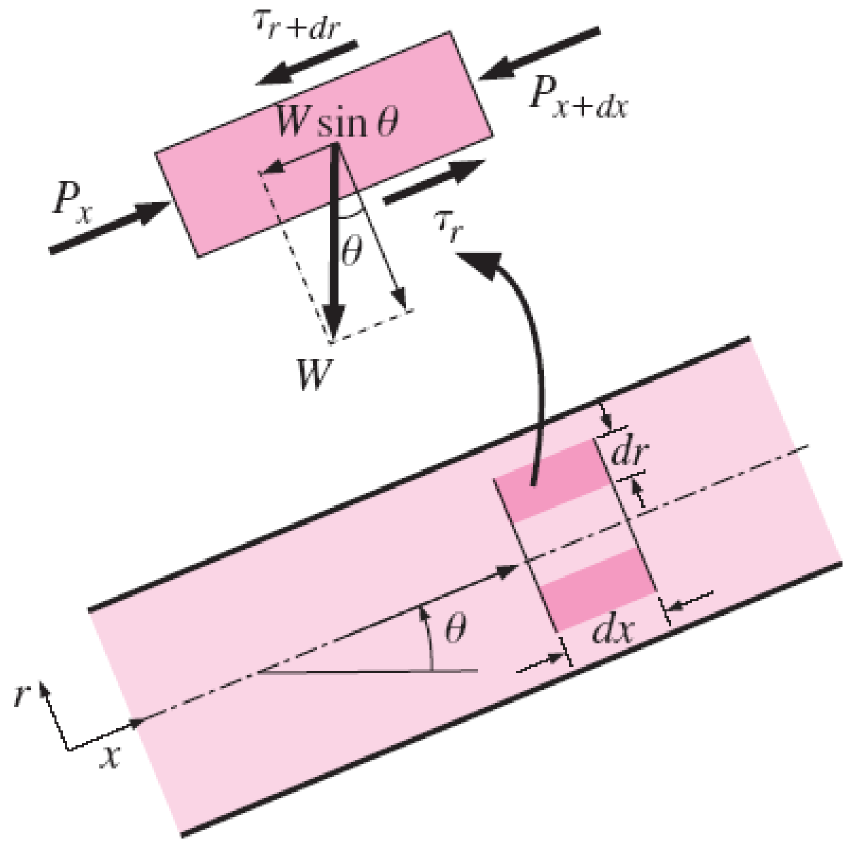
$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine},\,e} - h_{\text{pump},\,u} + h_L]$$

From the above eq., when the pressure drop = the head loss?

Pressure Drop and Head Loss — Inclined Pipes

Analogous to horizontal pipe. Read by yourself

- Similar to the horizontal pipe flow, except there is an additional force which is the weight component in the flow direction whose magnitude is



$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta$$

$$= \rho g (2\pi r\, dr\, dx) \sin\theta$$

Pressure Drop and Head Loss — Inclined Pipes

- The force balance now becomes

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r$$
$$- (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0$$

- which results in the differential equation

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

- The velocity profile can be shown to be

$$\boxed{u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)}$$

Pressure Drop and Head Loss — Inclined Pipes

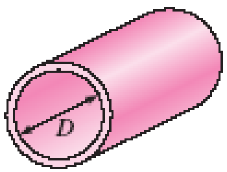
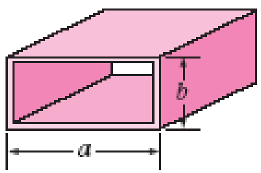
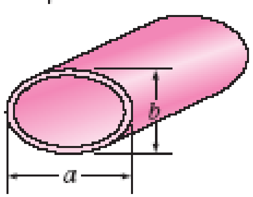
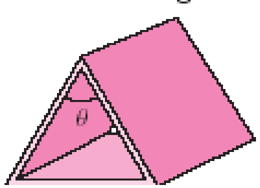
- The *average velocity* and the *volume flow rate* relations for laminar flow through inclined pipes are, respectively,

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L}$$

$$\text{and} \qquad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

- Note that $\theta > 0$ and thus $\sin\theta > 0$ for uphill flow, and $\theta < 0$ and thus $\sin\theta < 0$ for downhill flow.

Laminar Flow in Noncircular Pipes

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle	—	64.00/Re
		
Rectangle	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

Friction factor for fully developed *laminar flow* in pipes of various cross sections

$(D_h = 4A_c/p$ and $\mathrm{Re} = V_{avg} D_h/\nu)$

TURBULENT FLOW IN PIPES (Skipped)

- Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.
- However, turbulent flow is a complex mechanism. The theory of turbulent flow remains largely undeveloped.
- Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

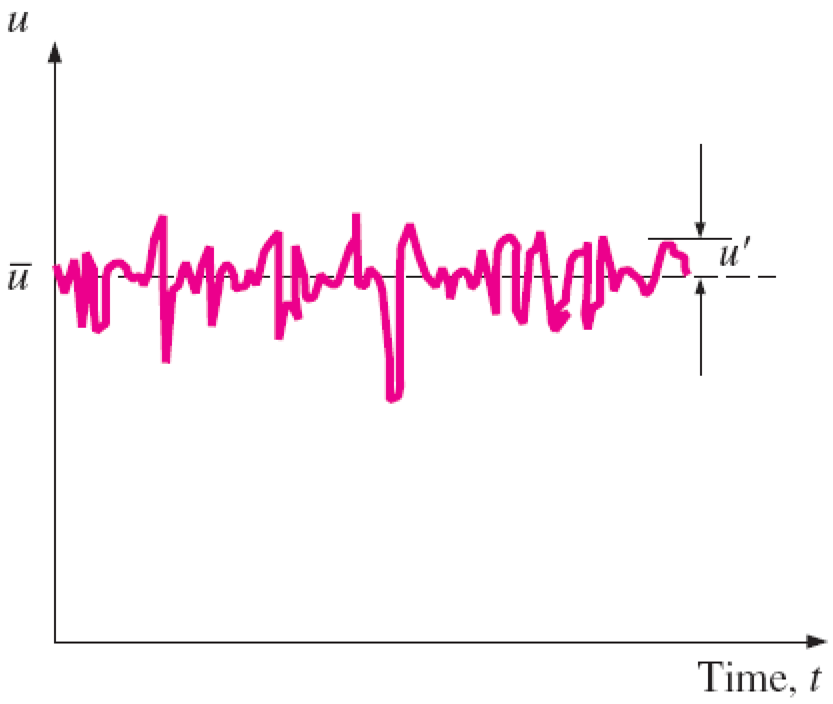
TURBULENT FLOW IN PIPES (Skipped)

- Turbulent flow is characterized by *random* and *rapid fluctuations* of swirling regions of fluid, called **eddies,** throughout the flow.
- These fluctuations provide an additional mechanism for momentum and energy transfer.
- In laminar flow, momentum and energy are transferred across streamlines by molecular diffusion.
- In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, such that associated with much higher values of friction, heat transfer, and mass transfer coefficients.

TURBULENT FLOW IN PIPES (Skipped)

- Even when the average flow is steady, the eddy motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow).
- We observe that the instantaneous velocity can be expressed as the sum of an *average value* $\overline{u}$ and a *fluctuating component* u',

$$u = \overline{u} + u'$$



TURBULENT FLOW IN PIPES (Skipped)

- The average value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the time average levels off to a constant. $\Rightarrow$ $\boxed{\overline{u'} = 0.}$
- The magnitude of u' is usually just a few percent of $\bar{u}$, but the high frequencies of eddies (in the order of a thousand per second) makes them very effective for the transport of momentum, thermal energy, and mass.
- The shear stress in turbulent flow can not be analyzed in the same manner as did in laminar flow. Experiments show it is much larger due to turbulent fluctuation.

TURBULENT FLOW IN PIPES (Skipped)

- The turbulent shear stress consists of two parts: the *laminar component,* and the *turbulent component,*

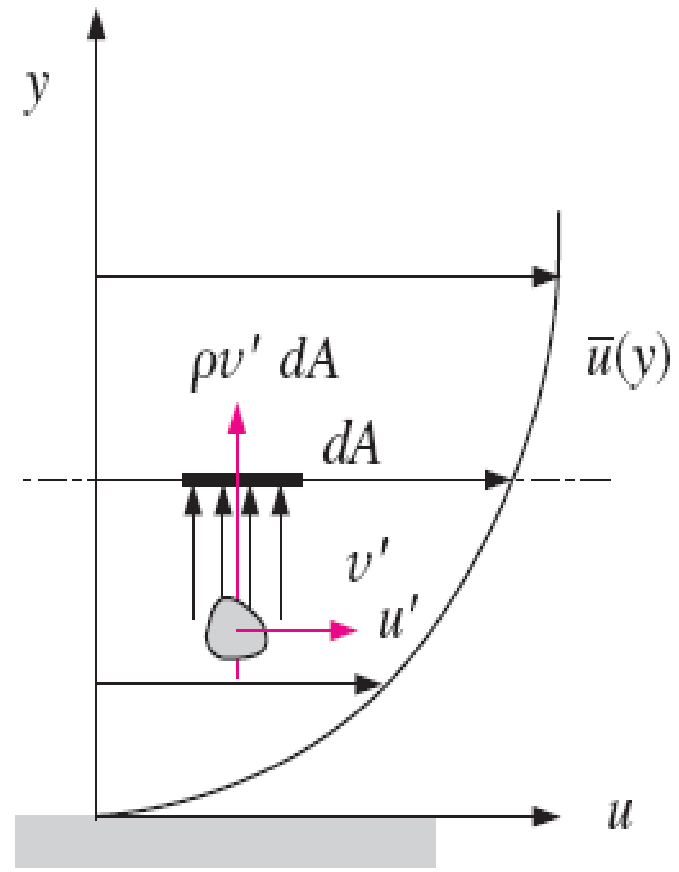
$$\boxed{\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}}$$

- The velocity profile is approximately parabolic in laminar flow, it becomes flatter or “fuller” in turbulent flow.
- The fullness increases with the Reynolds number, and the velocity profile becomes more nearly uniform, however, that the flow speed at the wall of a stationary pipe is always zero (no-slip condition).

Turbulent Shear Stress (Skipped)

- Consider turbulent flow in a horizontal pipe, and the upward eddy motion of fluid particles in a layer of lower velocity to an adjacent layer of higher velocity through a differential area *dA*
- Then the **turbulent shear stress** can be expressed as

$$\tau_{\text{turb}} = -\rho\overline{u'v'}$$



Note that $\overline{u'v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$

Turbulent Shear Stress (Skipped)

- Experimental results show that $\overline{u'v'}$ is usually a negative quantity.
- Terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are called **Reynolds stresses** or **turbulent stresses.**
- Many semi-empirical formulations have been developed that model the Reynolds stress in terms of average velocity gradients. Such models are called **turbulence models**.
- Momentum transport by eddies in turbulent flows is analogous to the molecular momentum diffusion.

Turbulent Shear Stress (Skipped)

- In many of the simpler turbulence models, turbulent shear stress is expressed as suggested by the French mathematician Joseph Boussinesq in 1877 as

$$\tau_{\text{turb}} = -\rho\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}$$

- where μ_t the **eddy viscosity** or **turbulent viscosity,** which accounts for momentum transport by turbulent eddies.
- The total shear stress can thus be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \overline{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \overline{u}}{\partial y}$$

- where $\nu_t = \mu_t/\rho$ is the **kinematic eddy viscosity** or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*).

Turbulent Shear Stress (Skipped)

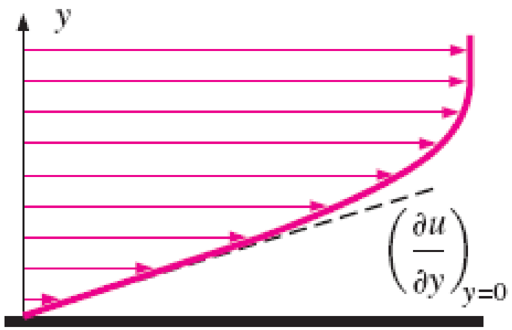
- For practical purpose, eddy viscosity must be modeled as a function of the average flow variables; we call this *eddy viscosity closure*.
- For example, L. Prandtl introduced the concept of **mixing length** l_m, which is related to the average size of the eddies that are primarily responsible for mixing, and expressed the turbulent shear stress as

$$\tau_{\text{turb}} = \mu_t \frac{\partial \overline{u}}{\partial y} = \rho l_m^2 \left(\frac{\partial \overline{u}}{\partial y} \right)^2$$

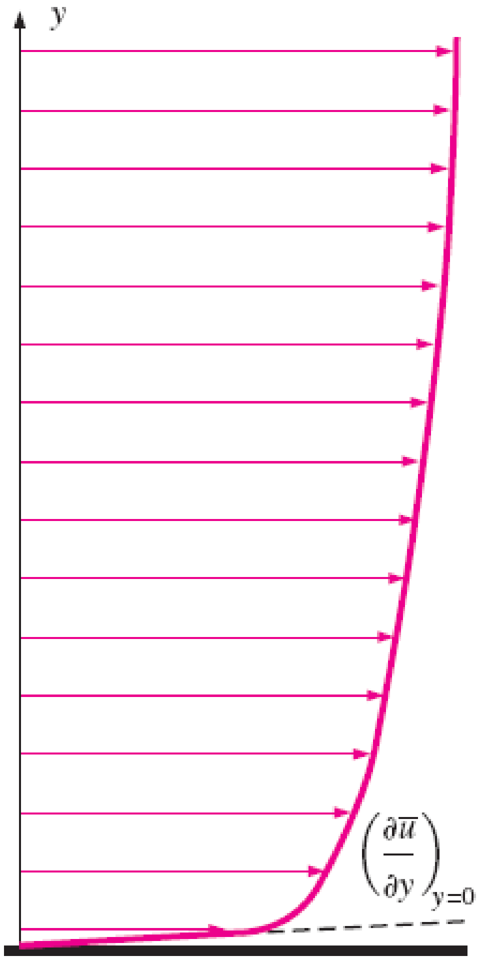
- l_m is not a constant for a given flow and its determination is not easy.

Turbulent Shear Stress (Skipped)

- Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer.
- The velocity profiles are shown in the figures. So it is no surprise that the wall shear stress is much larger in turbulent flow than it is in laminar flow.
- Molecular viscosity is a fluid property; however, eddy viscosity is a flow property.



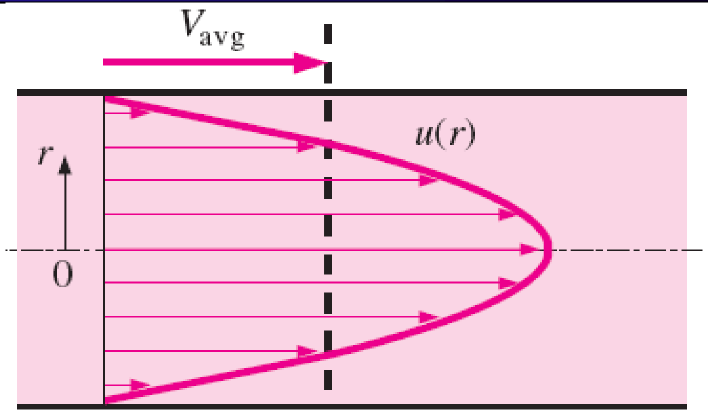
Laminar flow



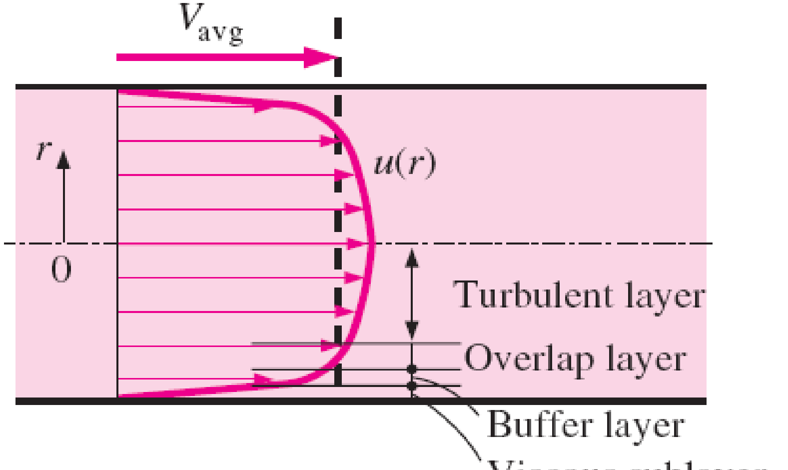
Turbulent flow

Turbulent Velocity Profile

- Typical velocity profiles for fully developed laminar and turbulent flows are given in Figures.
- Note that the velocity profile is parabolic in laminar flow but is much fuller in turbulent flow, with a sharp drop near the pipe wall.



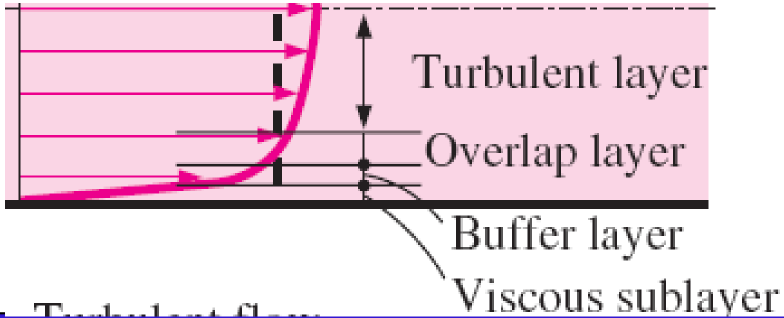
Laminar flow



Turbulent flow

Turbulent Velocity Profile

- Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall.
  - **Viscous** (or **laminar** or **linear** or **wall**) sublayer: where viscous effects are dominant and the velocity profile in this layer is very nearly *linear*, and the flow is streamlined.
  - **Buffer layer:** viscous effects are still dominant: however, turbulent effects are becoming significant.
  - **Overlap** (or **transition**) **layer (or** the **inertial sublayer):** the turbulent effects are much more significant, but still not dominant.
  - **Outer** (or **turbulent**) **layer:** turbulent effects dominate over molecular diffusion (viscous) effects.



Turbulent Velocity Profile (Skipped)

- The Viscous sublayer (next to the wall):
  - The thickness of this sublayer is very small (typically, much less than 1 % of the pipe diameter), but this thin layer plays a dominant role on flow characteristics because of the large velocity gradients it involves.
  - The wall dampens any eddy motion, and thus the flow in this layer is essentially laminar and the shear stress consists of laminar shear stress which is proportional to the fluid viscosity.
  - The velocity profile in this layer to be very nearly linear, and experiments confirm that.

Turbulent Velocity Profile (Viscous sublayer) (Skipped)

- The velocity gradient in the viscous sublayer remains nearly constant at $du/dy = u/y$, and the wall shear stress can be expressed as

$$\tau_w = \mu \frac{u}{y} = \rho\nu \frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

- where y is the distance from the wall. The square root of τ_w / ρ has the dimensions of velocity, and thus it is viewed as a fictitious velocity called the **friction velocity** expressed as $u_* = \sqrt{\tau_w/\rho}$.
- The velocity profile in the viscous sublayer can be expressed in dimensionless form as

$$\frac{u}{u_*} = \frac{y u_*}{\nu}$$

Turbulent Velocity Profile (Viscous sublayer) (Skipped)

- This equation is known as the **law of the wall,** and it is found to satisfactorily correlate with experimental data for smooth surfaces for $0 \leq yu_*/\nu \leq 5$.
- Therefore, the thickness of the viscous sublayer is roughly

$$y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

- where u_δ is the flow velocity at the edge of the viscous sublayer, which is closely related to the average velocity in a pipe. Thus we conclude the viscous sublayer is suppressed and it gets thinner as the velocity (and thus the Reynolds number) increases. Consequently, the velocity profile becomes nearly flat and thus the velocity distribution becomes more uniform at very high Reynolds numbers.

Turbulent Velocity Profile (Viscous sublayer) (Skipped)

- The quantity ν/u_* is called the **viscous length;** it is used to nondimensionalize the distance y; then we can get nondimensionalized velocity defined as

$$\textit{Nondimensionalized variables:} \qquad y^+ = \frac{yu_*}{\nu} \qquad \text{and} \qquad u^+ = \frac{u}{u_*}$$

- Then the normalized law of wall becomes simply

$$u^+ = y^+$$

- Note that y^+ resembles the Reynolds number expression.

Turbulent Velocity Profile (Overlap layer) (Skipped)

- In the overlap layer, experiments confirm that the velocity is proportional to the logarithm of distance, and the velocity profile can be expressed as

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y u_*}{\nu} + B \quad (8\text{-}46)$$

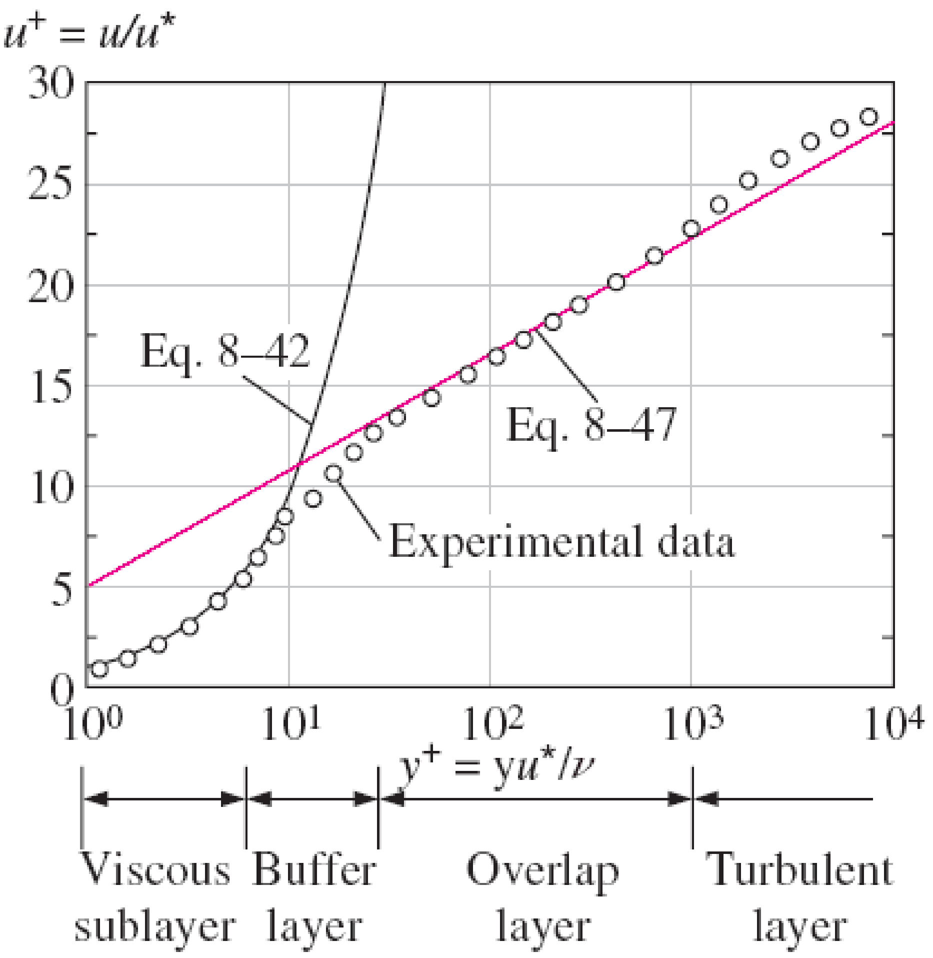
- where κ and B are constants and determined experimentally to be about 0.40 and 5.0, respectively. Equation 8–46 is known as the **logarithmic law.** Thus the velocity profile is

$$\frac{u}{u_*} = 2.5 \ln \frac{y u_*}{\nu} + 5.0 \quad \text{or} \quad u^+ = 2.5 \ln y^+ + 5.0 \quad (8\text{-}47)$$

- It is viewed as a *universal velocity profile* for turbulent flow in pipes or over surfaces.

Turbulent Velocity Profile (Overlap layer) (Skipped)

- Note from the figure that the logarithmic-law velocity profile is quite accurate for $y^+ > 30$, but neither velocity profile is accurate in the buffer layer, i.e., the region $5 < y^+ < 30$. Also, the viscous sublayer appears much larger in the figure.



Turbulent Velocity Profile (Turbulent layer) (Skipped)

- A good approximation for the outer turbulent layer of pipe flow can be obtained by evaluating the constant *B* by setting $y = R - r = R$ and $u = u_{max}$, an substituting it back into Eq. 8–46 together with $\kappa = 0.4$ gives

$$\frac{u_{\max} - u}{u_*} = 2.5 \ln \frac{R}{R - r} \qquad (8\text{-}48)$$

- The deviation of velocity from the centerline value $u_{max} - u$ is called the **velocity defect,** and Eq. 8–48 is called the **velocity defect law.** It shows that the normalized velocity profile in the turbulent layer for a pipe is independent of the viscosity of the fluid. This is not surprising since the eddy motion is dominant in this region, and the effect of fluid viscosity is negligible.

Turbulent Velocity Profile (Skipped)

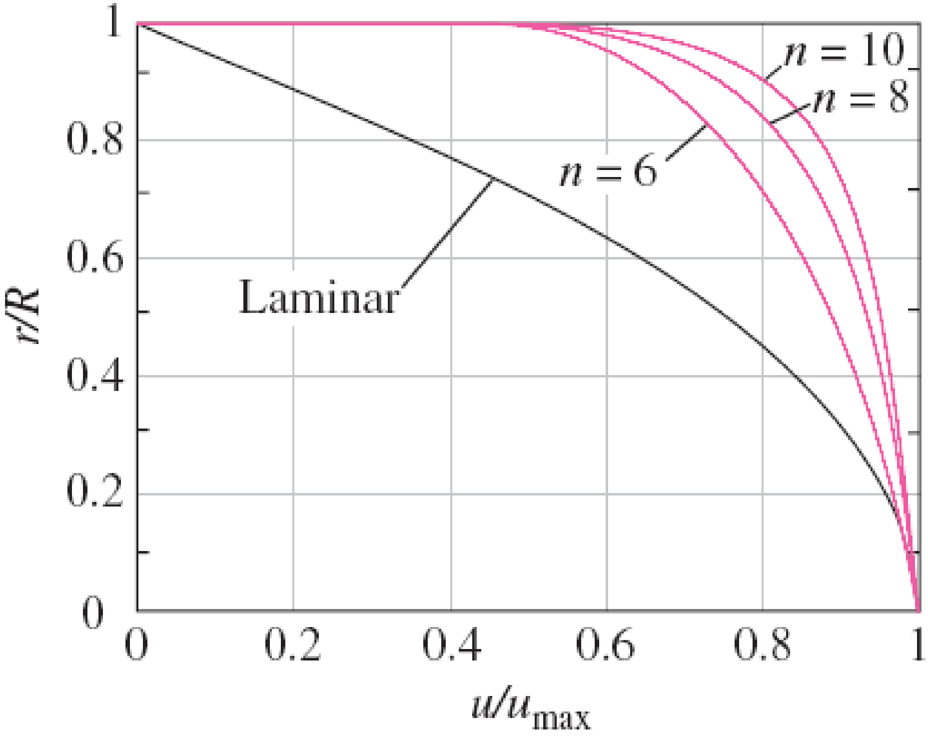
- Numerous other empirical velocity profiles exist for turbulent pipe flow. Among those, the simplest and the best known is the **power-law velocity profile** expressed as

$$\frac{u}{u_{\max}} = \left(\frac{y}{R}\right)^{1/n} \qquad \text{or} \qquad \frac{u}{u_{\max}} = \left(1 - \frac{r}{R}\right)^{1/n} \qquad \textbf{(8–49)}$$

- where the exponent n is a constant whose value depends on the Reynolds number. The value of n increases with increasing Reynolds number. The value $n = 7$ generally approximates many flows in practice, giving rise to the term *one-seventh power-law velocity profile*.

Turbulent Velocity Profile (Skipped)

- Note that the power-law profile cannot be used to calculate wall shear stress since it gives a velocity gradient of infinity there, and it fails to give zero slope at the centerline. But these regions of discrepancy constitute a small portion of flow, and the power-law profile gives highly accurate results for turbulent flow through a pipe.



Turbulent Velocity Profile (Skipped)

- The characteristics of the flow in viscous sublayer are very important since they set the stage for flow in the rest of the pipe. Any irregularity or roughness on the surface disturbs this layer and affects the flow. Therefore, unlike laminar flow, the friction factor in turbulent flow is a strong function of surface roughness.
- The roughness is a relative concept, and it has significance when its height ε is comparable to the thickness of the laminar sublayer (which is a function of the Reynolds number). All materials appear "rough" under a microscope with sufficient magnification. In fluid mechanics, a surface is characterized as being rough when $\varepsilon > \delta_{sublayer}$ and is said to be smooth when $\varepsilon < \delta_{sublayer}$. Glass and plastic surfaces are generally considered to be hydrodynamically smooth.

The Moody Chart

- The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.
- It is no way to find a mathematical closed form for friction factor by theoretical analysis; therefore, all the available results are obtained from painstaking experiments.
- Most such experiments were conducted by Prandtl's student J. Nikuradse in 1933, followed by the works of others. The friction factor was calculated from the measurements of the flow rate and the pressure drop.
- Functional forms were obtained by curve-fitting experimental data.

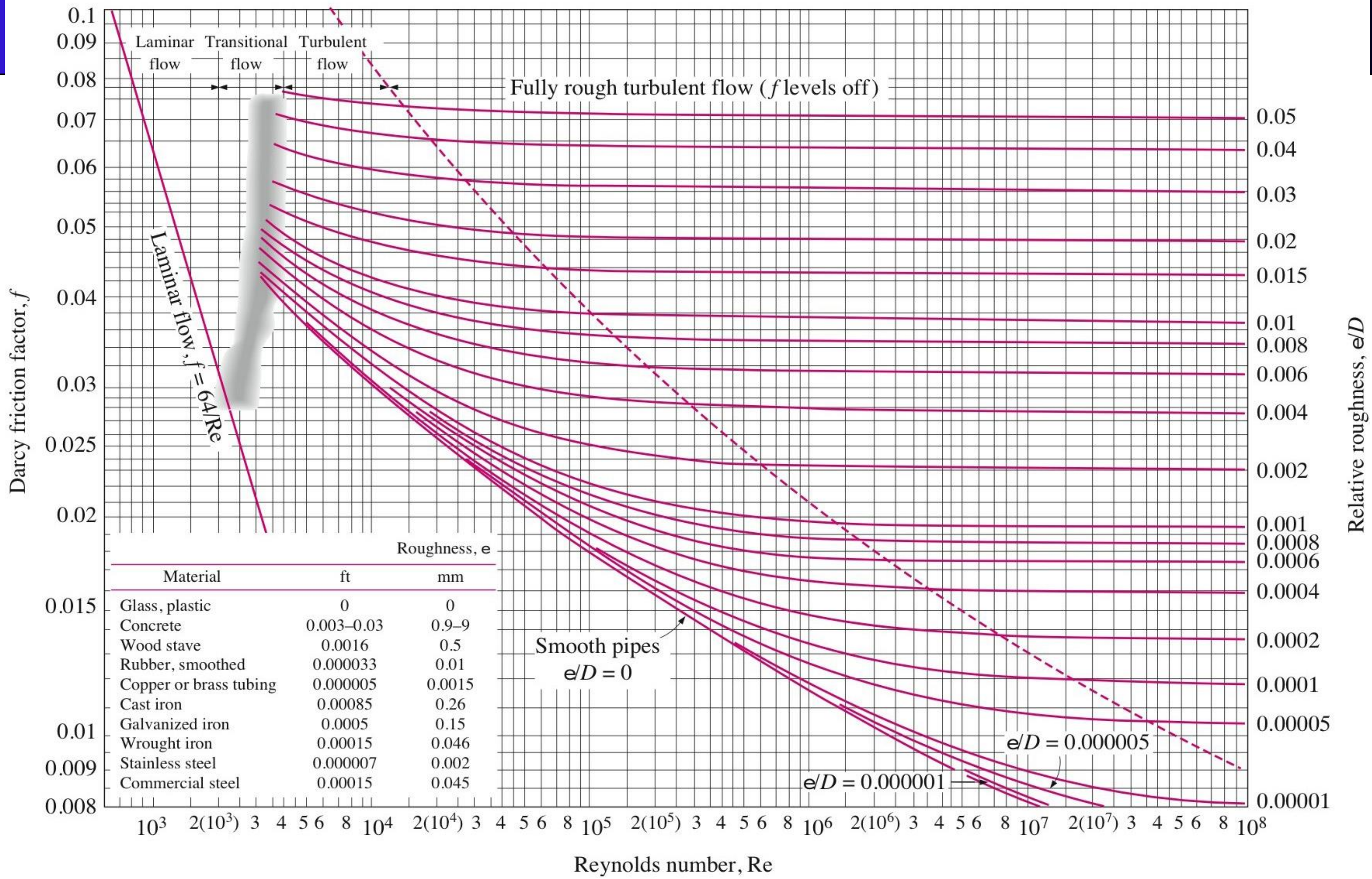
The Moody Chart

- In 1939, Cyril F. Colebrook combined the available data for transition and turbulent flow in smooth as well as rough pipes into the **Colebrook equation**:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

- In 1942, the American engineer Hunter Rouse verified Colebrook's equation and produced a graphical plot of *f.*
- In 1944, Lewis F. Moody redrew Rouse's diagram into the form commonly used today, called Moody chart given in the appendix as Fig. A–12.

The Moody Chart



Material	Roughness, e ft	Roughness, e mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The Moody Chart

- The Moody chart presents the Darcy friction factor for pipe flow as a function of the Reynolds number and ε/D over a wide range. It is probably one of the most widely accepted and used charts in engineering. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter.

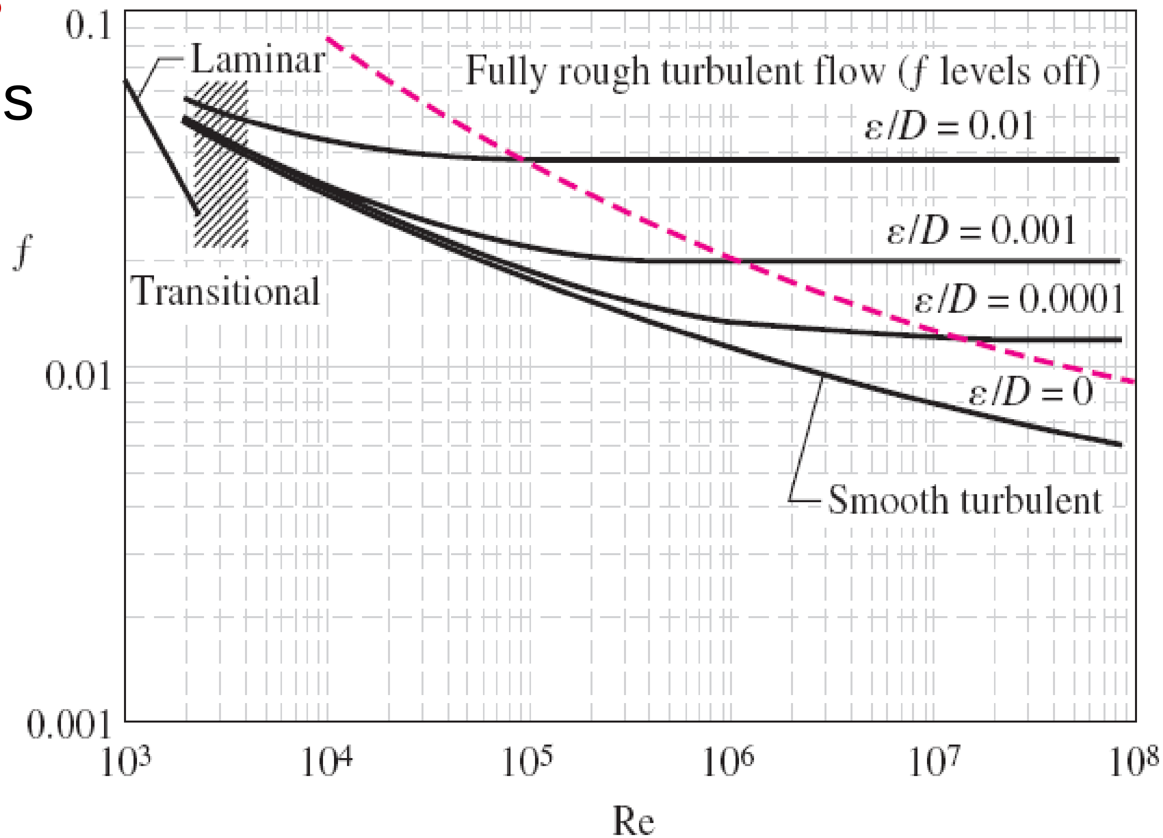
- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc

Equivalent roughness values for new commercial pipes

	Roughness, ε	
Material	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Observations from the Moody chart

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness
- The friction factor is a minimum for a smooth pipe and increases with roughness
- The data in the transition region are the least reliable.



Observations from the Moody chart

- In the transition region, at small relative roughnesses, the friction factor increases and approaches the value for smooth pipes.
- At very large Reynolds numbers, the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called *fully rough turbulent flow* or just *fully rough flow*

von Kármán equation $$1/\sqrt{f} = -2.0 \log[(\varepsilon/D)/3.7] \quad (\mathrm{Re} \rightarrow \infty)$$

Prandtl equation $$1/\sqrt{f} = 2.0 \log(\mathrm{Re}\sqrt{f}) - 0.8 \quad (\varepsilon = 0)$$

Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered
  - Determine Δp (or h_L) given L, D, $\dot{V}$ (or flow rate)
    - Can be solved directly using Moody chart and Colebrook equation
  - Determine $\dot{V}$, given L, D, Δp
  - Determine D, given L, Δp, $\dot{V}$ (or flow rate)
- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs
- However, iterative approach required since both *V* and *D* are in the Reynolds number.

Types of Fluid Flow Problems

- Explicit relations have been developed which eliminate iteration. They are useful for quick, direct calculation, but introduce an additional 2% error

$$h_L = 1.07 \frac{\dot{\mathcal{V}}^2 L}{g D^5} \left\{ \ln \left[\frac{\epsilon}{3.7D} + 4.62 \left(\frac{\nu D}{\dot{\mathcal{V}}} \right)^{0.9} \right] \right\}^{-2} \qquad \begin{gathered} 10^{-6} < \epsilon/D < 10^{-2} \\ 3000 < Re < 3 \times 10^8 \end{gathered}$$

$$\dot{\mathcal{V}} = -0.965 \left(\frac{g D^5 h_L}{L} \right)^{0.5} \ln \left[\frac{\epsilon}{3.7D} + \left(\frac{3.17 \nu^2 L}{g D^3 h_L} \right)^{0.5} \right] \qquad Re > 2000$$

$$D = 0.66 \left[\epsilon^{1.25} \left(\frac{L \dot{\mathcal{V}}^2}{g h_L} \right)^{4.75} + \nu \dot{\mathcal{V}}^{9.4} \left(\frac{L}{g h_L} \right)^{5.2} \right]^{0.04} \qquad \begin{gathered} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3 \times 10^8 \end{gathered}$$

EXAMPLE 8–3: Determining the Head Loss in a Water Pipe

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0174$.

Therefore,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = \mathbf{11.8\ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = \mathbf{27.3\ ft}$$

$$\dot{W}_{\text{pump}} = \dot{V}\,\Delta P = \mathbf{461\ W}$$

EXAMPLE 8–4 Determining the Diameter of an Air Duct

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) = -2.0\log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20 = f\frac{150\text{ m}}{D}\frac{V^2}{2(9.81\text{ m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 8–2). Solving for the four equations, then we can get

$$D = \mathbf{0.267\ m}, \qquad f = 0.0180, \qquad V = 6.24\text{ m/s}, \qquad \text{and} \qquad \mathrm{Re} = 100{,}800$$

Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee–Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} = \mathbf{0.271\ m}$$

EXAMPLE 8–5 Determining the Flow Rate of Air in a Duct

The new flow rate can also be determined directly from the second Swamee–Jain formula to be

$$\begin{aligned}
\dot{V} &= -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D}+\left(\frac{3.17\nu^2L}{gD^3h_L}\right)^{0.5}\right] \\
&= -0.965\left(\frac{(9.81\text{ m/s}^2)(0.267\text{ m})^5(20\text{ m})}{300\text{ m}}\right)^{0.5} \\
&\quad\times\ln\left[0+\left(\frac{3.17(1.655\times10^{-5}\text{ m}^2\text{/s})^2(300\text{ m})}{(9.81\text{ m/s}^2)(0.267\text{ m})^3(20\text{ m})}\right)^{0.5}\right] \\
&= 0.24\text{ m}^3\text{/s}
\end{aligned}$$

Minor Losses

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing.
- The head loss introduced by a completely open valve may be negligible. But a partially closed valve may cause the largest head loss in the system which is evidenced by the drop in the flow rate.
- We introduce a relation for the minor losses associated with these components as follows.

Minor Losses

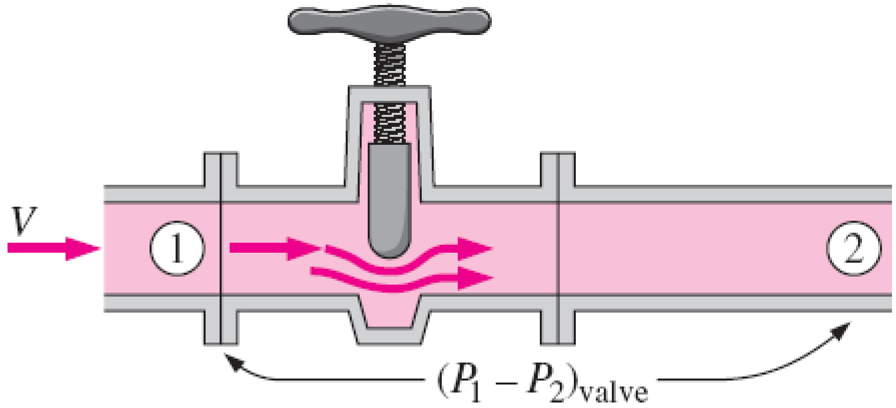
$$h_L = \Delta P_L / \rho g$$

$$K_L = \Delta P_L / (\tfrac{1}{2}\rho V^2)$$

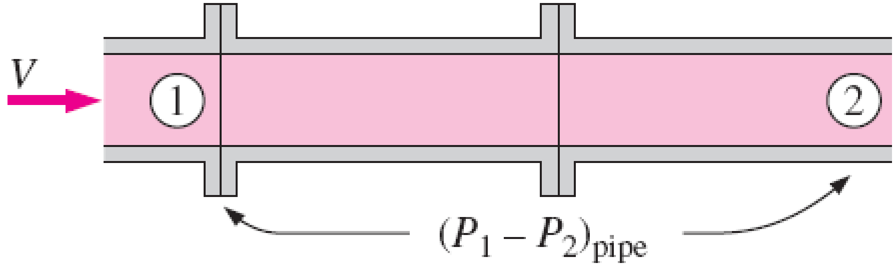
$$\boxed{h_L = K_L \frac{V^2}{2g}}$$

- K_L is the loss coefficient (also called the **resistance coefficient**).
- Is different for each component.
- Is assumed to be independent of Re (Since Re is very large).
- Typically provided by manufacturer or generic table (e.g., Table 8-4 in text).

Pipe section with valve:



Pipe section without valve:



$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$

Minor Losses

- The minor loss occurs *locally* across the minor loss component, but keep in mind that the component influences the flow for several pipe diameters downstream.
- This is the reason why most flow meter manufacturers recommend installing their flow meter at least 10 to 20 pipe diameters downstream of any elbows or valves.
- Minor losses are also expressed in terms of the **equivalent length** L_{equiv}, defined as

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{V^2}{2g} \rightarrow L_{\text{equiv}} = \frac{D}{f} K_L$$

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in the components)

$$h_L = h_{L,major} + h_{L,minor}$$

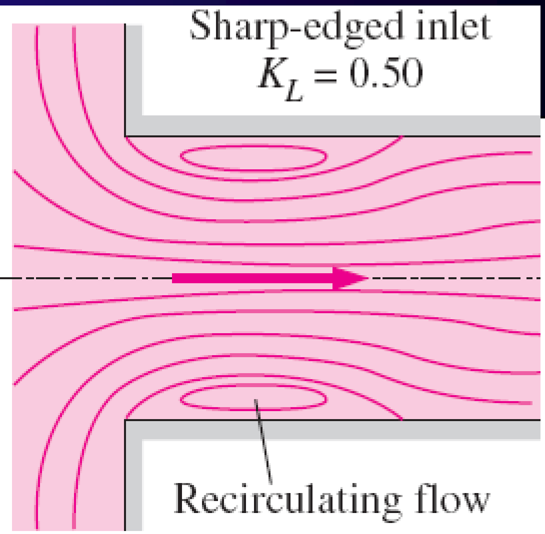
$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}}_{\text{i pipe sections}} + \underbrace{\sum_j K_{L,j} \frac{V_j^2}{2g}}_{\text{j components}}$$

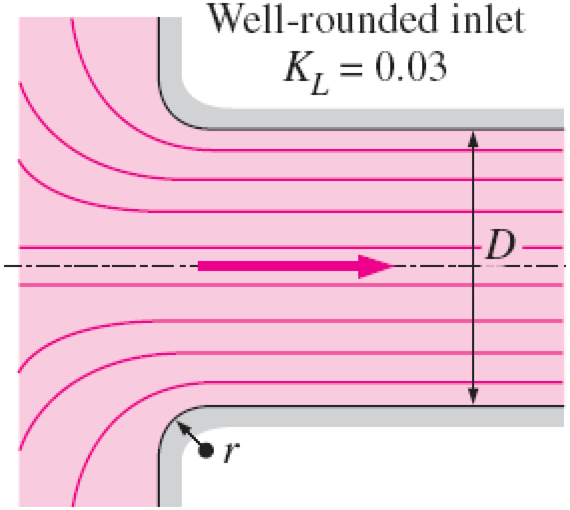
- If the piping system has constant diameter

$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

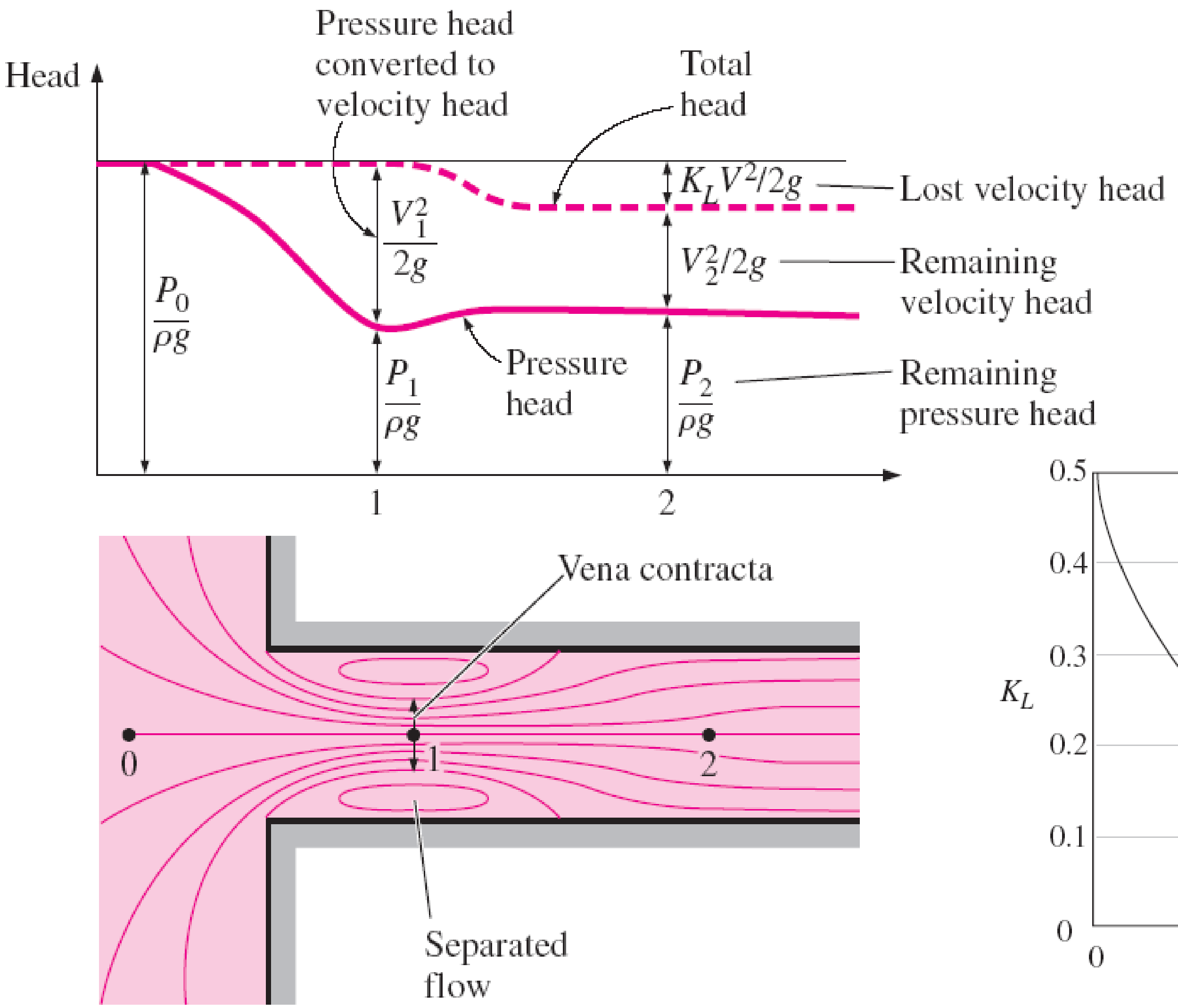
Head loss at the inlet of a pipe

- The head loss at the inlet of a pipe is a strong function of geometry. It is almost negligible for well-rounded inlets (K_L = 0.03 for r/D = 0.2), but increases to about 0.50 for sharp-edged inlets (because the fluid cannot make sharp 90° turns easily, especially at high velocities; therefore, the flow separates at the corners).
- The flow is constricted into the **vena contracta** region formed in the midsection of the pipe.





Head loss at the inlet of a pipe



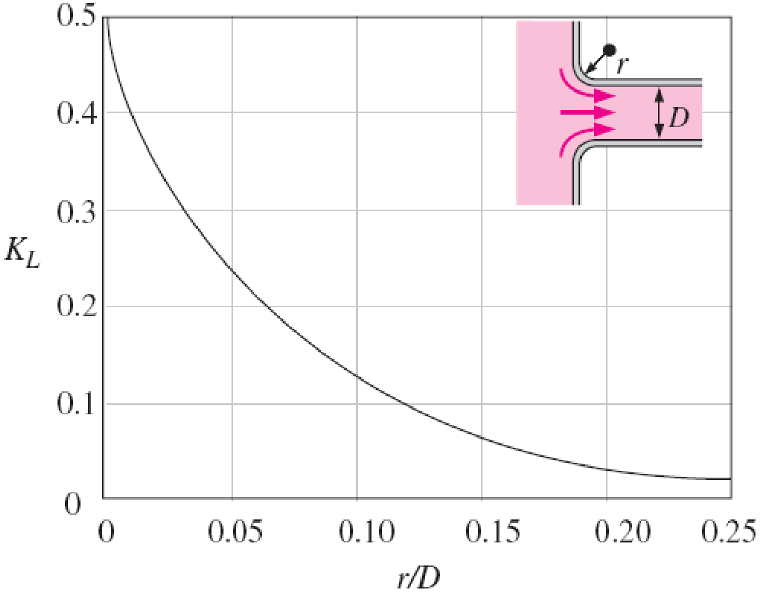
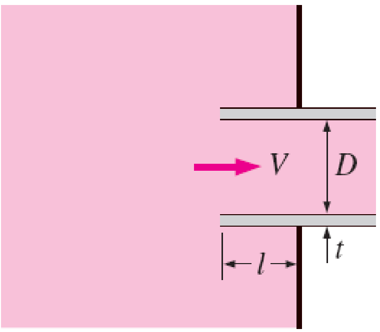


TABLE 8–4

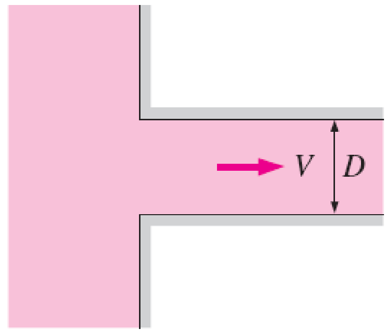
Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where V is the average velocity in the pipe that contains the component)*

Pipe Inlet

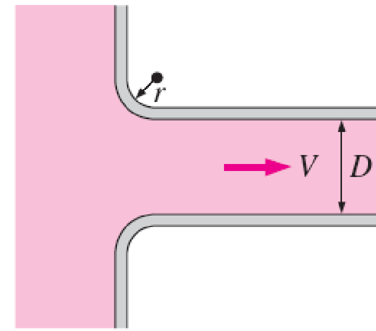
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)



Sharp-edged: $K_L = 0.50$

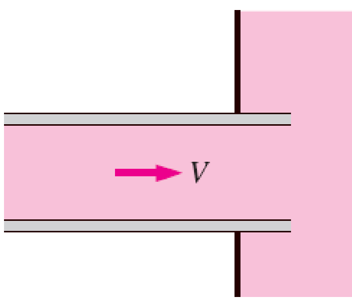


Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)

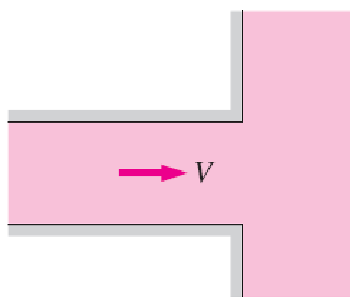


Pipe Exit

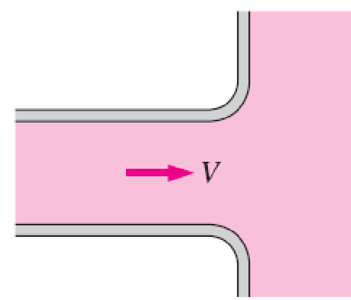
Reentrant: $K_L = \alpha$



Sharp-edged: $K_L = \alpha$



Rounded: $K_L = \alpha$

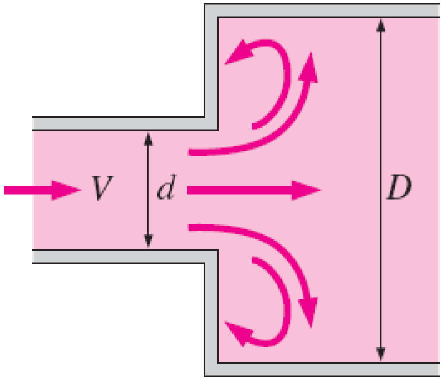


Whether laminar or turbulent, the fluid leaving the pipe loses *all* of its kinetic energy as it mixes with the reservoir fluid and eventually comes to rest

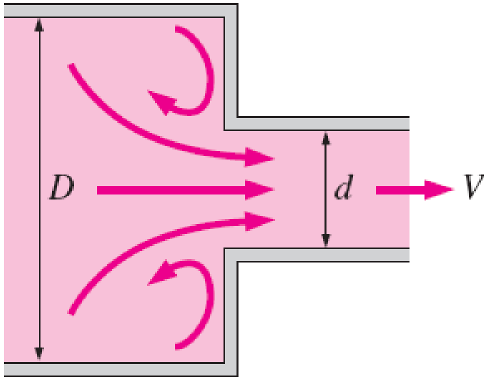
Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1$ for fully developed turbulent flow.

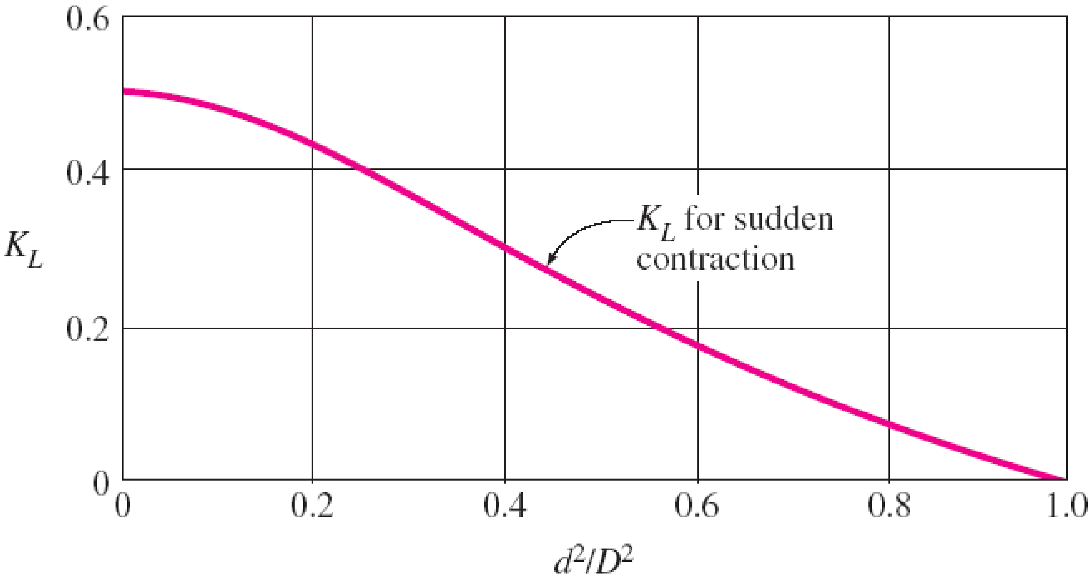
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$

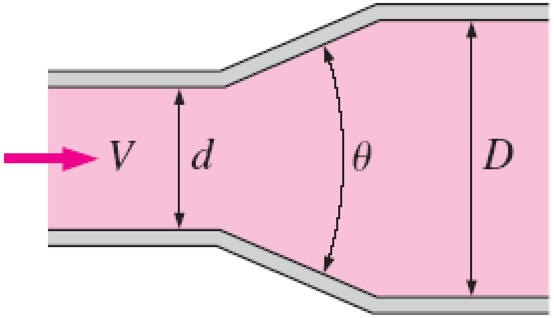


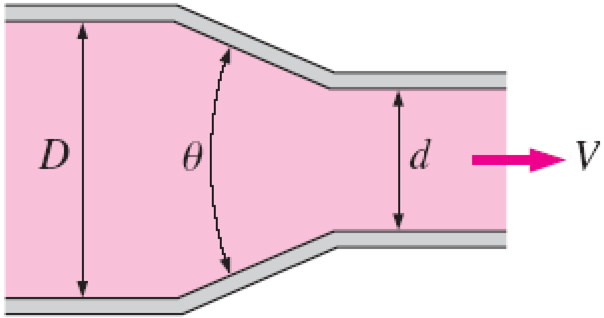
Sudden contraction: See chart.





*Gradual Expansion and Contraction (based on **the velocity in the smaller-diameter pipe**)*





Expansion (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

Contraction

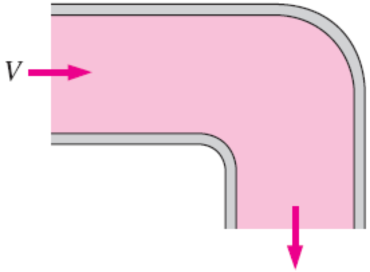
$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

Typos in the text

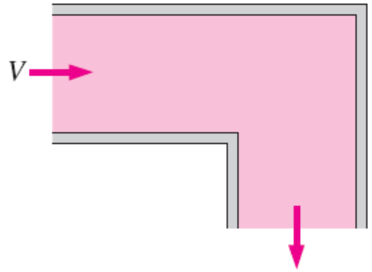
TABLE 8–4 (CONCLUDED)

Bends and Branches

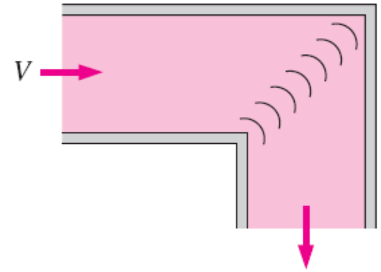
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



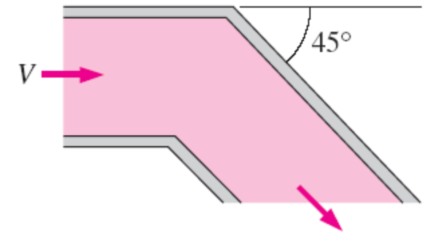
90° miter bend
(without vanes): $K_L = 1.1$



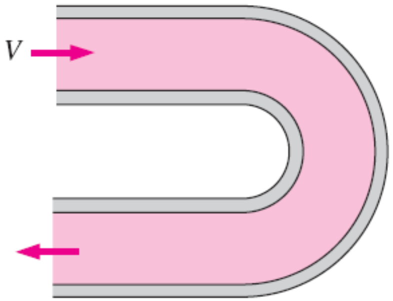
90° miter bend
(with vanes): $K_L = 0.2$



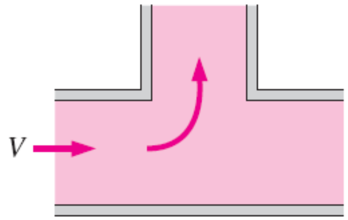
45° threaded elbow:
$K_L = 0.4$



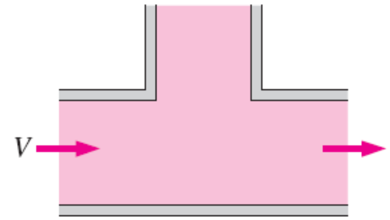
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



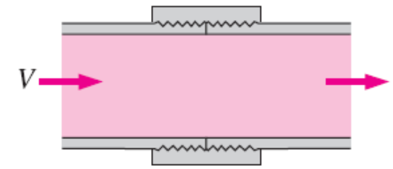
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



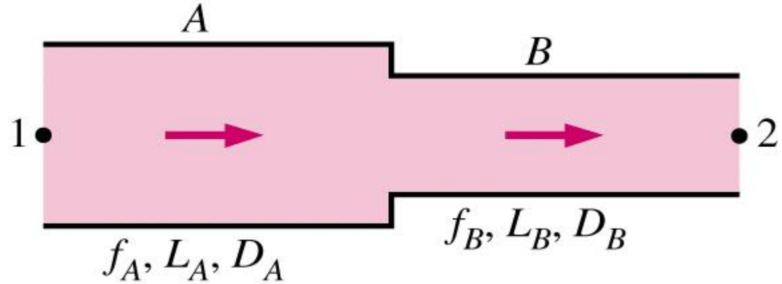
Valves

Globe valve, fully open: $K_L = 10$	*Gate valve,* fully open:	$K_L = 0.2$
Angle valve, fully open: $K_L = 5$	$\frac{1}{4}$ closed:	$K_L = 0.3$
Ball valve, fully open: $K_L = 0.05$	$\frac{1}{2}$ closed:	$K_L = 2.1$
Swing check valve: $K_L = 2$	$\frac{3}{4}$ closed:	$K_L = 17$

* These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

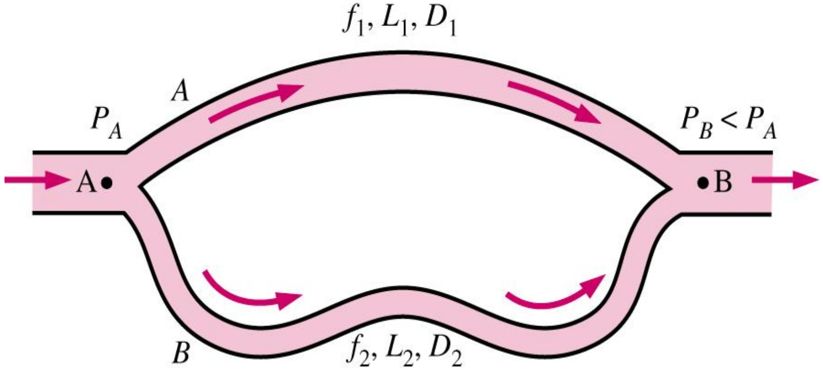
Piping Networks and Pump Selection

- Two general types of networks
  - Pipes in series
    - Volume flow rate is constant
    - Head loss is the summation of parts
  - Pipes in parallel
    - Volume flow rate is the sum of the components
    - Pressure loss across all branches is the same



$$\dot{V}_A = \dot{V}_B$$

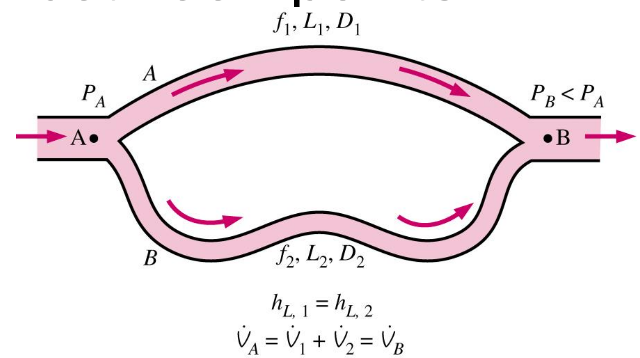
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$



$$h_{L,\,1} = h_{L,\,2}$$

$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

- For parallel pipes, perform CV analysis between points A and B



$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \cancel{\alpha_1 \frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\rho g} + \cancel{\alpha_2 \frac{V_B^2}{2g}} + \cancel{z_B} + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- Since Δp is the same for all branches, head loss in all branches is the same

$$h_{L,1} = h_{L,2} \quad \Longrightarrow \quad f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Piping Networks and Pump Selection

- Head loss relationship between branches allows the following ratios to be developed

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}} \qquad \frac{\dot{\mathcal{V}}_1}{\dot{\mathcal{V}}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}}$$

- Real pipe systems result in a system of non-linear equations.
- Note: the analogy with electrical circuits should be obvious
  - Flow flow rate (V_A) : current (I)
  - Pressure gradient (Δp) : electrical potential (V)
  - Head loss (h_L): resistance (R), however h_L is very nonlinear

Piping Networks and Pump Selection

- The analysis of piping networks, no matter how complex they are, is based on two simple principles:
  - *Conservation of mass throughout the system must be satisfied.*
  - *Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions.*

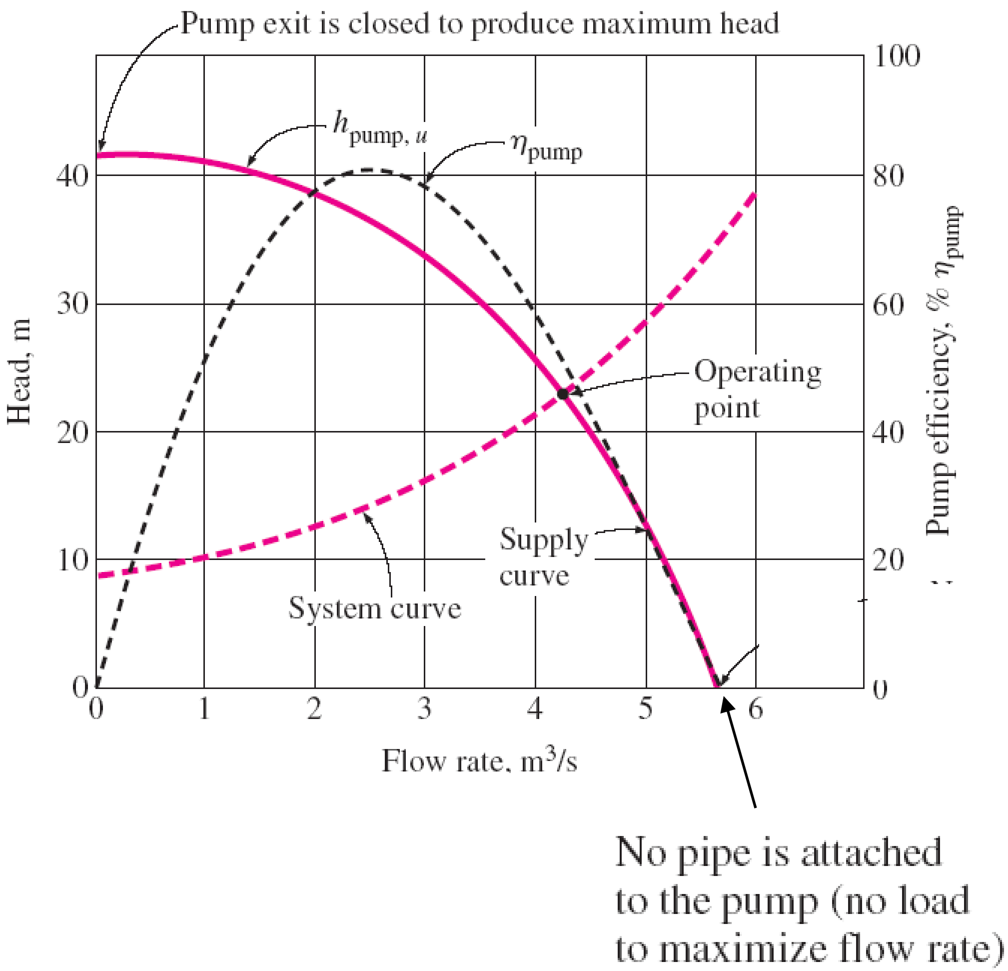
Piping Networks and Pump Selection

- When a piping system involves pumps and/or turbines, pump and turbine head must be included in the energy equation

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \boxed{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \boxed{h_{turbine,e}} + h_L$$

- The useful head of the pump ($h_{pump,u}$) or the head extracted by the turbine ($h_{turbine,e}$), are functions of volume flow rate, i.e., they are not constants.
- Operating point of system is where the system is in balance, e.g., where pump head is equal to the head losses.

Pump and systems curves

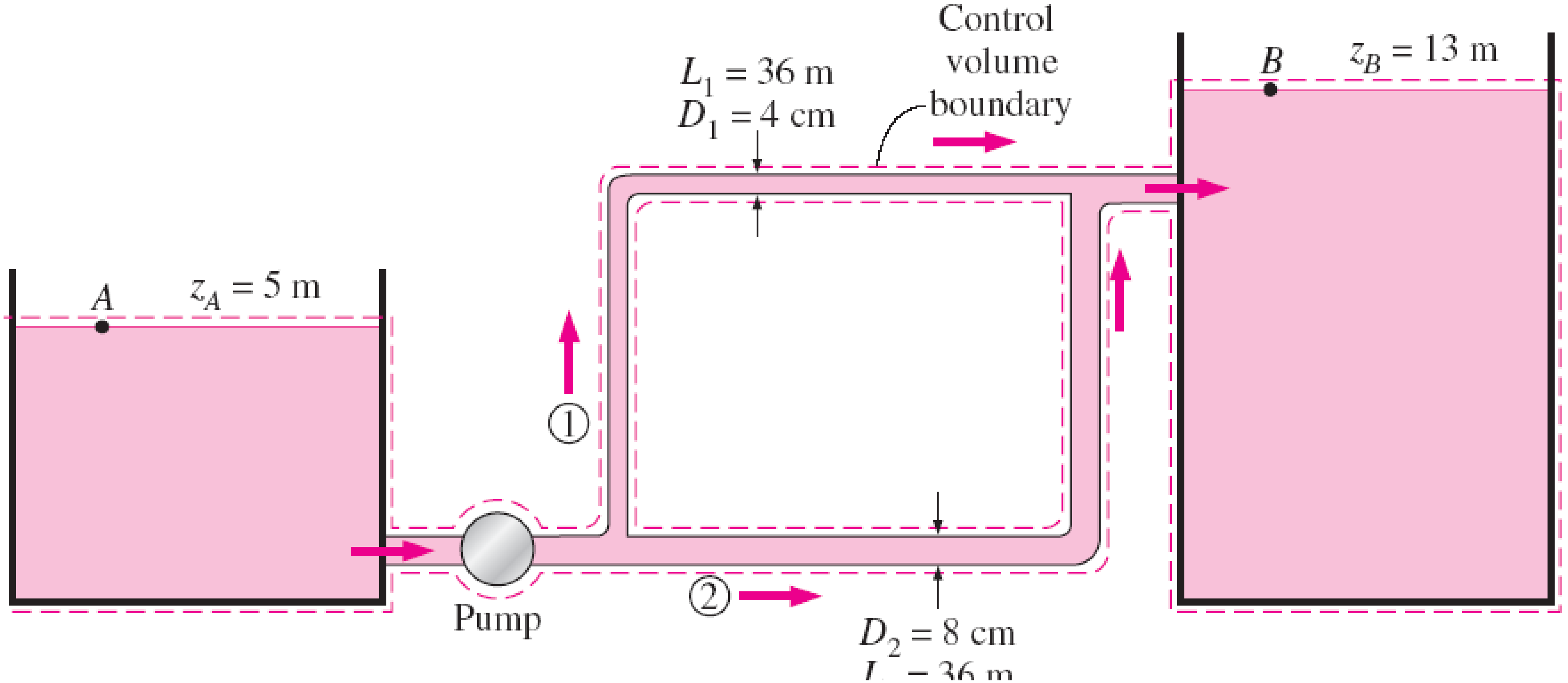


- Supply curve (or characteristic or performance curves) for $h_{pump,u}$: determine experimentally by manufacturer.
- System (or demand) curve determined from analysis of fluid dynamics equations
- Operating point is the intersection of supply and demand curves
- If peak efficiency is far from operating point, pump is wrong for that application.

Examples on pages from 358 to 364
In the text

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

- Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel.



EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

- Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

- Solution:
- Assumptions:

✓ **1** The flow is steady and incompressible.

✓ **2** The entrance effects are negligible, and the flow is fully developed.

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

- Solution:

✓ **3** The elevations of the reservoirs remain constant.

✓ **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible.

✓ **5** Flows through both pipes are turbulent (to be verified).

- The useful head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump},\, u}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 8000\ \text{W} = \frac{(998\ \text{kg/m}^3)\dot{V}(9.81\ \text{m/s}^2)h_{\text{pump},\, u}}{0.70}$$

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

- The energy equation for a control volume between these two points simplifies to

$$\frac{\cancel{P_A}}{\rho g} + \alpha_A \frac{V_A^2}{2g}\nearrow^0 + z_A + h_{\text{pump}, u} = \frac{\cancel{P_B}}{\rho g} + \alpha_B \frac{V_B^2}{2g}\nearrow^0 + z_B + h_L \rightarrow$$

$$h_{\text{pump}, u} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump}, u} = (13 - 5) + h_L$$

Where

$$h_L = h_{L,1} = h_{L,2}$$

- We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. The average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are expressed as

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\ \text{m})^2/4} \tag{5}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\ \text{m})^2/4} \tag{6}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\ \text{kg/m}^3)V_1(0.04\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} \tag{7}$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\ \text{kg/m}^3)V_2(0.08\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} \tag{8}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \tag{9}$$

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \tag{10}$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{36\text{ m}}{0.04\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \tag{11}$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{36\text{ m}}{0.08\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \tag{12}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \tag{13}$$

- This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

$$\dot{V} = \mathbf{0.0300\ m^3/s}, \quad \dot{V}_1 = \mathbf{0.00415\ m^3/s}, \quad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$$

$$V_1 = 3.30\ \text{m/s}, \quad V_2 = 5.15\ \text{m/s}, \quad h_L = h_{L,1} = h_{L,2} = 11.1\ \text{m}, \quad h_{\text{pump}} = 19.1\ \text{m}$$

$$\text{Re}_1 = 131{,}600, \quad \text{Re}_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$$

- Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.
- ***Discussion*** The two parallel pipes are identical, except the diameter of the first pipe is half the diameter of the second one. But only 14 percent of the water flows through the first pipe. This shows the strong dependence of the flow rate (and the head loss) on diameter.

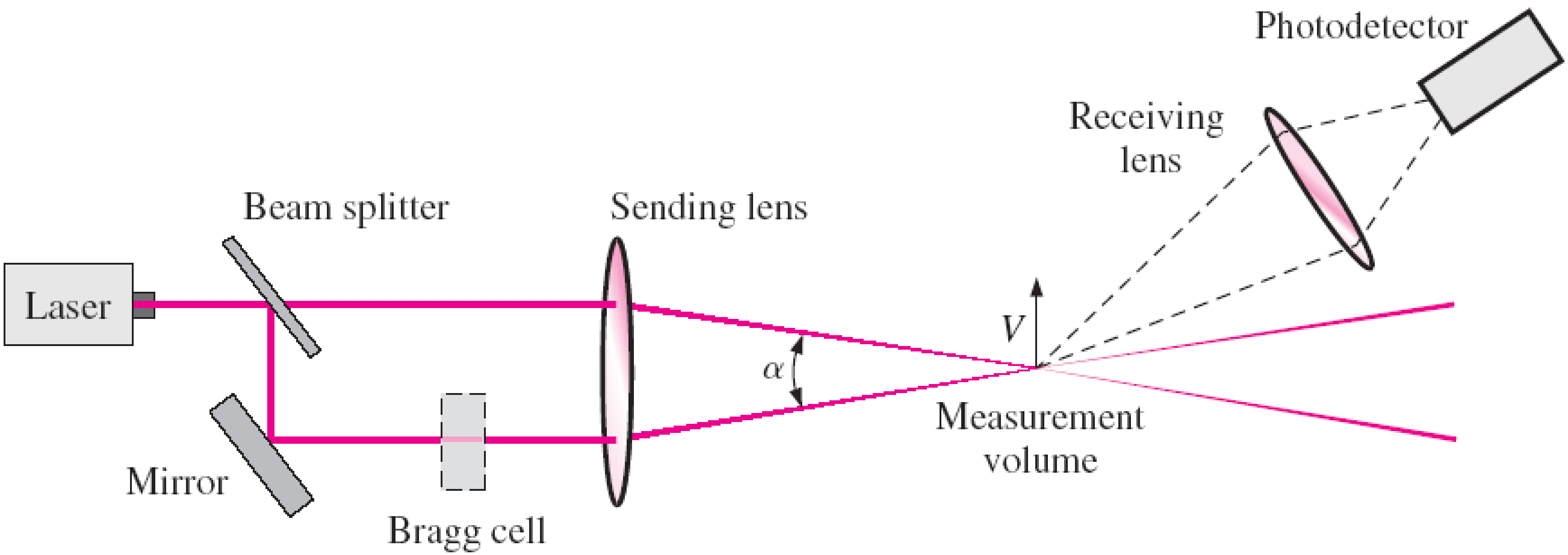
FLOW RATE AND VELOCITY MEASUREMENT

- Please see section 8-8 in the text for the detail. There are various devices to measure flow rates.

- Two optical methods used to measure velocity fields will be introduced:
  - **Laser Doppler Velocimetry (LDV)**
  - **Particle Image Velocimetry (PIV)**

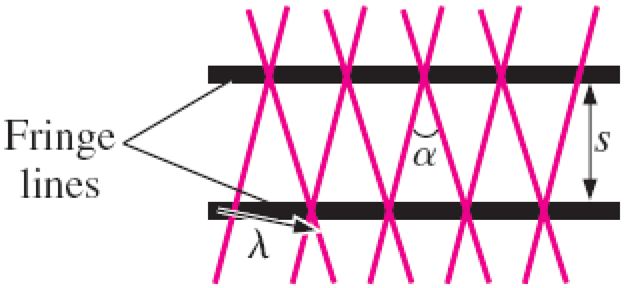
Laser Doppler Velocimetry (LDV)

- **LDV** is an optical technique to measure flow velocity at any desired point without disturbing the flow.

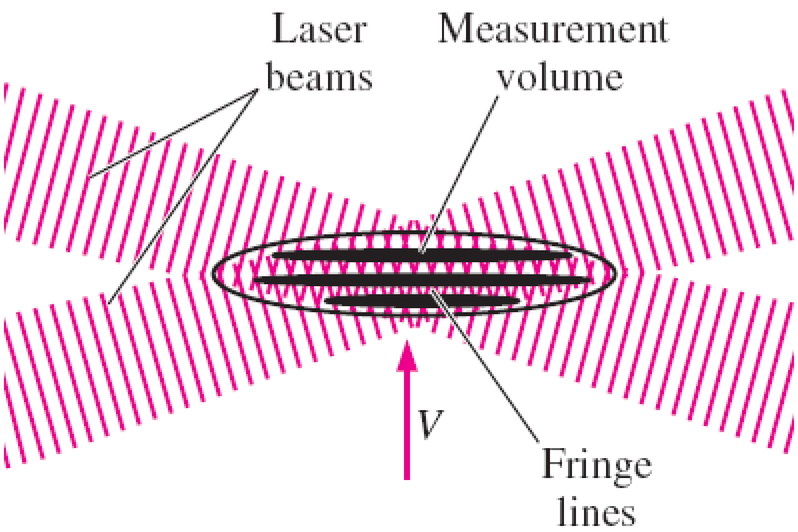


Laser Doppler Velocimetry (LDV)

- When a particle traverses these fringe lines at velocity *V*, the frequency of the scattered fringe lines is.



$$f = \frac{V}{s} = \frac{2V \sin(\alpha/2)}{\lambda}$$



- Particles with a diameter of 1 μm
- The measurement volume resembles an ellipsoid, typically of 0.1 mm diameter and 0.5 mm in length.

Particle Image Velocimetry (PIV)

- PIV provides velocity values simultaneously throughout an entire cross section, and thus it is a whole-field technique.