Fluid Mechanics Homework #12

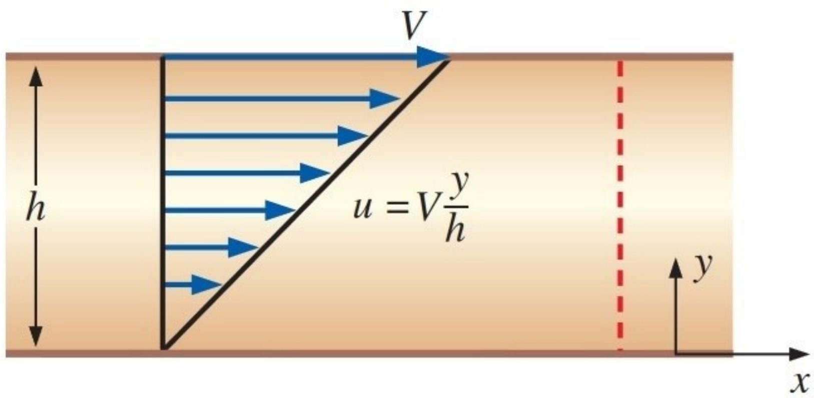
繳交期限：2019/12/25(三) 09:10

共五題，題號為：**9-18,44,58,69,73**

題號的對照書本是 **Yunus A. Cengel and John M. Cimbala “Fluid Mechanics: Fundamentals and Applications 3/e (SI Units) ”**

9-18 The *product rule* can be applied to the divergence of scalar f times vector $\vec{G}$ as: $\vec{\nabla}\cdot(f\vec{G}) = \vec{G}\cdot\vec{\nabla}f + f\vec{\nabla}\cdot\vec{G}$. Expand both sides of this equation in Cartesian coordinates and verify that it is correct.

9-44 Consider fully developed *Couette flow*—flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate stationary as illustrated in Fig. P9–46. The flow is steady, incompressible, and two-dimensional in the xy-plane. The velocity field is given by $\vec{V} = (u, v) = (Vy/h)\vec{i} + 0\vec{j}$. Generate an expression for stream function ψ along the vertical dashed line in Fig. P9–46. For convenience, let $\psi = 0$ along the bottom wall of the channel. What is the value of ψ along the top wall?



Assumption:

1. The flow is steady.

2. The flow is incompressible.
3. The flow is two-dimensional in the xy-plane.
4. The flow is fully developed.

9-58 A steady, two-dimensional, incompressible flow field in the xy-plane has a stream function given by $\psi = ax^2 - by^2 + cx + dxy$, where a, b, c, and d are constants. (a) Obtain expressions for velocity components u and v. (b) Verify that the flow field satisfies the incompressible continuity equation.

Assumption:

1. The flow is steady.
2. The flow is incompressible.
3. The flow is two-dimensional in the xy-plane, implying that w = 0 and neither u nor v depend on z.

9-69 In Example 9–2, we provide expressions for u, v, and ρ for flow through a compressible converging duct. Generate an expression for the compressible stream function ψ_ρ that describes this flow field. For consistency, set $\psi_\rho = 0$ along the x-axis.

$u = u_1 + C_u x$

$v = \frac{-(\rho_1 C_u + u_1 C_\rho) y - 2 C_u C_\rho x y}{\rho}$

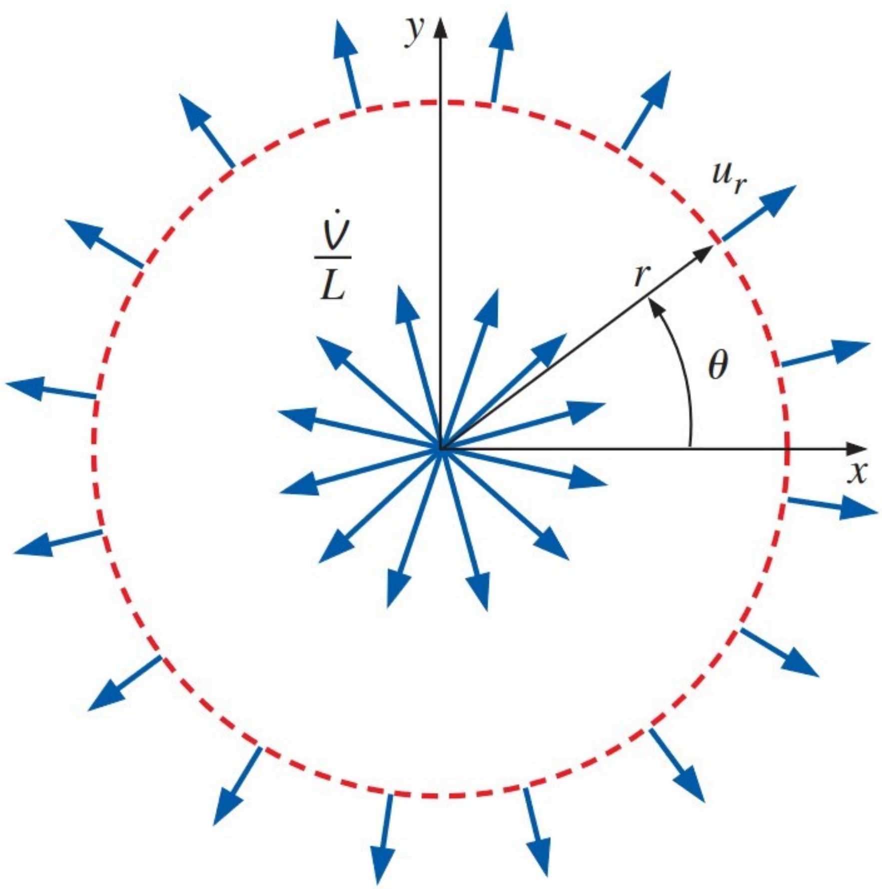
$\rho = \rho_1 + C_\rho x$

$\psi_\rho = f(x, y, u_1, \rho_1, C_u, C_\rho)$

Assumption:

1. The flow is steady.
2. The flow is two-dimensional in the xy-plane.

9-73 Consider steady, incompressible, two-dimensional flow due to a *line source* at the origin (Fig. P9–76). Fluid is created at the origin and spreads out radially in all directions in the *xy*-plane. The net volume flow rate of created fluid per unit width is $\dot{V}/L$ (into the page of Fig. P9–76), where L is the width of the line source into the page in Fig. P9–76. Since mass must be conserved everywhere except at the origin (a singular point), the volume flow rate per unit width through a circle of any radius r must also be $\dot{V}/L$. If we (arbitrarily) specify stream function ψ to be zero along the positive x-axis ($\theta = 0$), what is the value of ψ along the positive y-axis ($\theta = 90°$)? What is the value of ψ along the negative x-axis ($\theta = 180°$)?



Assumption:

1. The flow is steady.
2. The flow is incompressible.