Contents

5 Newton's Laws of Motion

Physics is a study of what can cause an object to accelerate. That cause is a *force*, which is loosely speaking, a push or pull on the object.

5.1 Newtonian Mechanics

The relation between a force and the acceleration it causes was first understood by Isaac Newton (1642-1727). The study of that relation, as Newton presented, is called Newtonian Mechanics.

Newtonian mechanics does not apply to all situations. If the speed of the interacting bodies are very large—an appreciable fraction of the speed of light—we must replace Newtonian mechanics with Einstein's special theory of relativity, which holds at any speed, including those near the speed of light. If the interacting bodies are on the scale of atomic structure, we must replace Newtonian mechanics with quantum mechanics. Physicists now view Newtonian mechanics as a special case of these two comprehensive theories.

5.2 Newton's First Law

Proposition 1 *Newton's First Law: If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.*

In other words, if the body is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude and same direction).

5.3 Force

We know that a force can cause the acceleration of a body. Thus we may define the unit of force in terms of the acceleration that a force gives to a standard reference body. A force is measured by the acceleration it produces. However acceleration is a vector quantity, with both magnitude and direction. Is force also a vector quantity? We can easily assign a direction to a force as the direction of the acceleration. By experiment, forces are indeed vector quantities; they have magnitudes and directions, and they combine according the vector rules. This means when two or more forces act on a body, we can find the net force, or resultant force, by adding the individual forces vectorially. A single force that has the magnitude and direction of the net force has the same effect on the body as all the individual forces together. This fact is called the **principle of superposition for forces**.

In this lecture force are most often represented with a vector symbol such as $\vec{F}$, and a net force is represented with the vector symbol $\vec{F}_{net}$. There may be multiple forces acting on a body, but if their net force is zero the body cannot accelerate.

5.3.1 Inertial Reference Frames

Proposition 2 *An inertial reference frame is one in which Newton's laws hold.*

5.4 Mass

Suppose we exert a force on a standard body whose mass is m_0, and suppose the body accelerates with acceleration a_0. We next apply the same force to a second body, body X, whose mass m_X is not known. We know that a less massive baseball receives a greater acceleration than a more massive bowling ball when the same force is applied to both of them. Let us then make the following conjecture: The ratio of the masses of the tow bodies is equal to the inverse of the ratio of their accelerations when the same force is applied to both. This means that

$$\frac{m_X}{m_0} = \frac{a_0}{a_X}$$

5.5 Newton's Second Law

Proposition 3 *Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.*

$$\vec{F}_{net} = m\vec{a} \tag{1}$$

$\vec{F}_{net}$ is the vector sum of all the forces that act on the body.
(1) is a vector equation, and equivalent to three component equations:

$$F_{net,x} = ma_x, F_{net,y} = ma_y, F_{net,z} = ma_z$$

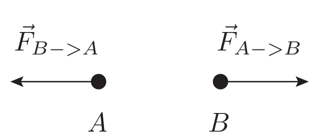
For SI units, (1) tells us that

$$1\ N = (1\ kg)\left(1\ m/s^2\right) = 1\ kg \cdot m/s^2$$

5.6 Newton's Third Law

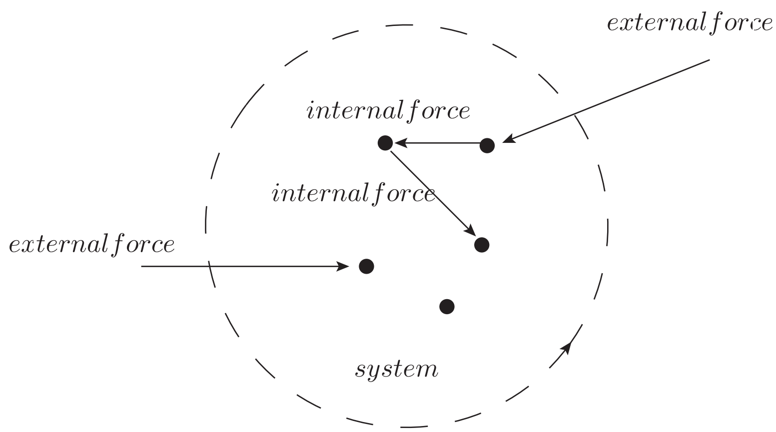
Proposition 4 *Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.*

In the following figure, $\vec{F}_{A->B}$ is the force exerted on B by A and $\vec{F}_{B->A}$ is the force on A exerted by B.



Newton's third law says that $\vec{F}_{A->B} + \vec{F}_{B->A} = 0$. Note that in the above figure, A and B are point particles and the force $\vec{F}_{A->B}$ or $\vec{F}_{B->A}$ are parallel or anti-parallel to $\overrightarrow{AB}$. If this condition is met, the Newton's third law is considered to be enforced in *strong* form. Otherwise, it is valid in *weak* form. An example for weak third law is shown below:

A system consists of one or more bodies, and any force on the bodies inside the system from bodies outside the system is called an **external force**. Forces between two or more bodies inside the system are called **internal forces**.



The net force on the system $\vec{F}_{net}$ is the vector sum of all forces. By Newton's third law, the internal forces cancel in pairs and the vector sum of internal forces vanish. Thus $\vec{F}_{net}$ is equal to the vector sum of external forces. Just as for a single body, we can relate the net force on a system to its acceleration with Newton's second law, $\vec{F}_{net} = m\vec{a}$ where m is the total mass of the system. Note there is ambiguity in defining $\vec{a}$ for a system if not all its constituents are experiencing the same acceleration. This will be discuss later when we define a center of mass for a system of particles.

5.7 Some Particular Forces

5.7.1 The Gravitational Force

Suppose a body of mass m is in free fall with the free-fall acceleration of magnitude g. If we neglect the effects of the air, the only force acting on the body is the gravitational force $\vec{F}_g$. We can relate this downward force and downward acceleration $\vec{g}$ with Newton's second law:

$$\vec{F}_g = m\vec{g}$$

Let us place a vertical z axis along the body's path, with the positive direction upward. For this axis, we have

$$\vec{g} = -g\hat{k}$$

and

$$\vec{F}_g = -F_g\hat{k}$$

Newton's second law $\vec{F}_g = m\vec{g}$ can be written in the form

$$F_g = mg$$

In other words, the magnitude of the gravitational force is equal to the product mg.

5.7.2 Weight

The weight of a body is the magnitude of the net force required to prevent the body from falling freely, as measured by someone on the ground. Consider a body that has a zero acceleration relative to the ground, which we assume to be an inertial frame. Two forces act on the body: a downward gravitational force $\vec{F}_g$ and a balancing upward force $\vec{W}$ of magnitude W. According to Newton's second law,

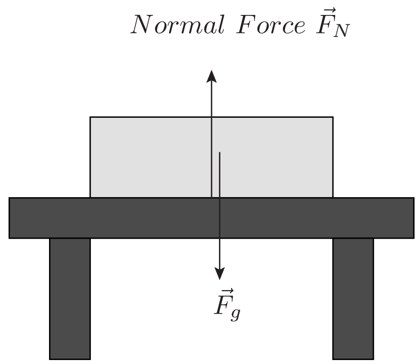
$$\vec{F}_g + \vec{W} = m * 0 = 0$$

Thus $\vec{W} = -\vec{F}_g$ and

$$W = F_g = mg$$

The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

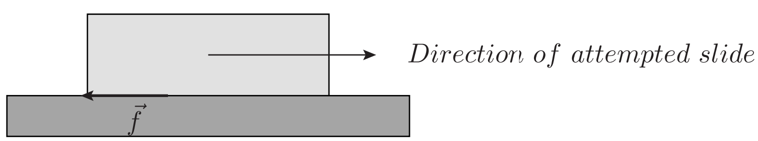
5.7.3 The Normal Force



When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a **normal force** $\vec{F}_N$ that is perpendicular to the surface.

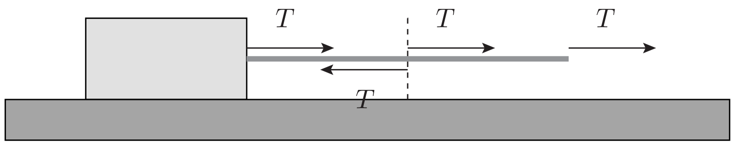
5.7.4 Friction

If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. The resistance is considered to be a single force $\vec{f}$, called either the friction force or simply friction. The force is directed along the surface, opposite the direction of the intended motion.



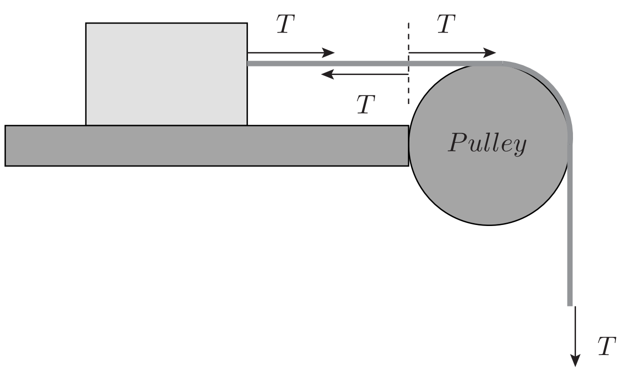
5.7.5 Tension

When a cord (or a rope, cable, or other such object) is attached to a body and pulled taut, the cord pulls on the body with a force $\vec{T}$ directed away from the body and along the cord.



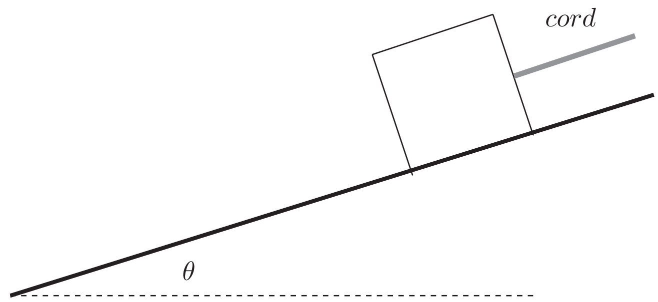
The force is often called a *tension force* because the cord is said to be in a state of tension. The tension in the cord is the magnitude T of the force on the body.

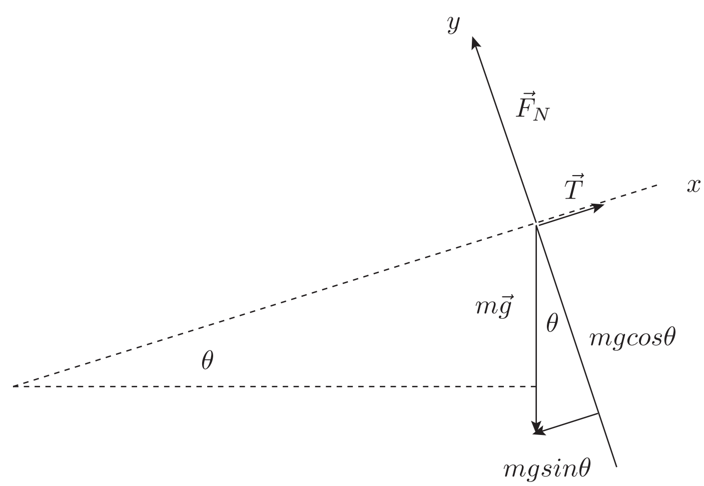
A cord is often *massless* (meaning that its mass is negligible compared to the body's mass) and *unstretchable*. The cord then exists only as a connection between two bodies. It pulls on both bodies with the same force magnitude T, even if the bodies and the cord are accelerating and even if the cord runs around a *massless, frictionless pulley*.



5.8 Sample Problems

5.8.1 (i)





$$m\vec{g} = -mg\sin\theta\hat{\imath} - mg\cos\theta\hat{\jmath}$$

The body is motionless along the y-axis.

$$0 = \vec{F}_N - mg\cos\theta\hat{\jmath}$$

$$F_N = mg\cos\theta$$

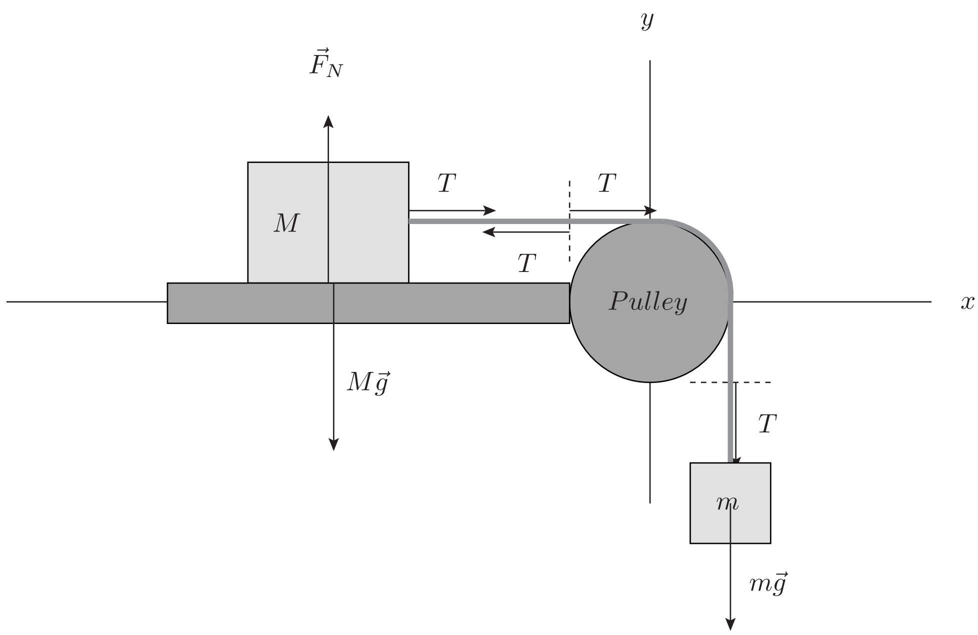
For the motion along the x-axis,

$$\vec{T} - mg\sin\theta\hat{\imath} = m\vec{a}$$

$$\vec{T} = T\hat{\imath}, \vec{a} = a\hat{\imath}$$

$$T - mg\sin\theta = ma$$

5.8.2 (ii)



The normal force $\vec{F}_N$ cancels the gravitation force $M\vec{g}$ on M.

$$\vec{F}_N + M\vec{g} = 0$$

where $\vec{g}$ is downward and

$$\vec{g} = -g\hat{\jmath}$$

The horizontal tension force acting on M is $T\hat{\imath}$.

$$T\hat{\imath} = M\vec{a}_M$$

$$\vec{a}_M = \frac{T}{M}\hat{\imath} = a\hat{\imath}$$

The cord is not stretchable. So the acceleration of m is

$$\vec{a}_m = -a\hat{\jmath} = -\frac{T}{M}\hat{\jmath}$$

The net force on m is vertical and equals to $m\vec{g} + T\hat{\jmath}$. Thus

$$m\vec{g} + T\hat{\jmath} = -ma\hat{\jmath}$$

or

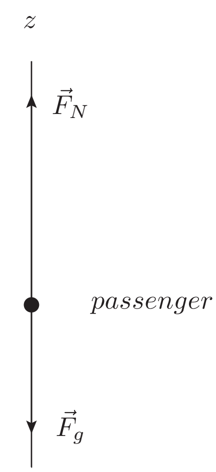
$$-mg + T = -ma = -\frac{m}{M}T$$

We thus have

$$T = \frac{m}{1+\frac{m}{M}}g, a = \frac{T}{M} = \frac{m}{M+m}g$$

5.8.3 (*iii*)

A passenger of mass m stands on a platform scale in an elevator cab.



The upward direction is in the positive z-axis. $\vec{F}_N$ is the normal force exerted by the scale on the passenger.

$$\vec{F}_N = F_N\hat{k}$$

F_N is also the reading on the scale. The gravitational force is

$$\vec{F}_g = m\vec{g} = -mg\hat{k}$$

Let

$$\vec{a} = a\hat{k}$$

Newton's second law the yields

$$\vec{F}_N + \vec{F}_g = m\vec{a}$$

or

$$F_N = mg + ma$$

In a free fall, $\vec{a} = \vec{g}$, $a = -g$, $F_N = 0$.