HW1 ANSWER

9.

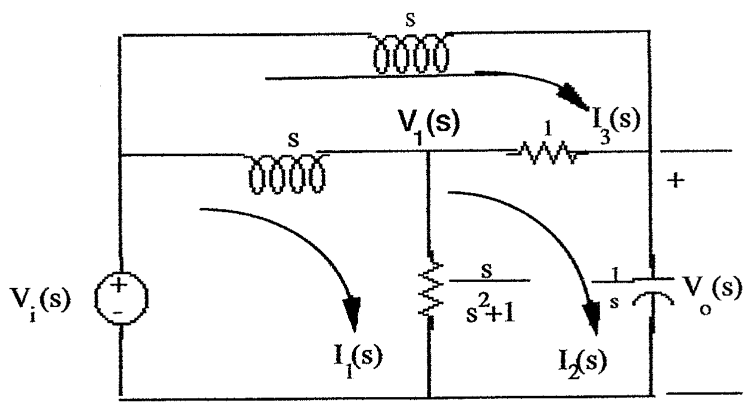
The transfer function is $\dfrac{C(s)}{R(s)} = \dfrac{s^5 + 2s^4 + 4s^3 + s^2 + 4}{s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5}$.

Cross multiplying, $(s^6+7s^5+3s^4+2s^3+s^2+5)C(s) = (s^5+2s^4+4s^3+s^2+4)R(s)$.

Taking the inverse Laplace transform assuming zero initial conditions,

$$\frac{d^6c}{dt^6} + 7\frac{d^5c}{dt^5} + 3\frac{d^4c}{dt^4} + 2\frac{d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5c = \frac{d^5r}{dt^5} + 2\frac{d^4r}{dt^4} + 4\frac{d^3r}{dt^3} + \frac{d^2r}{dt^2} + 4r.$$

15b. Transforming the network yields,



Writing the loop equations,

$$(s + \frac{s}{s^2+1})I_1(s) - \frac{s}{s^2+1}I_2(s) - sI_3(s) = V_i(s)$$

$$-\frac{s}{s^2+1}I_1(s) + (\frac{s}{s^2+1} + 1 + \frac{1}{s})I_2(s) - I_3(s) = 0$$

$$-sI_1(s) - I_2(s) + (2s+1)I_3(s) = 0$$

Solving for $I_2(s)$,

$$I_2(s) = \frac{s(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}V_i(s)$$

But, $V_o(s) = \dfrac{I_2(s)}{s} = \dfrac{(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}V_i(s)$. Therefore,

$$\frac{V_o(s)}{V_i(s)} = \frac{s^2+2s+2}{s^4+2s^3+3s^2+3s+2}$$

18a.

$$Z_1(s) = 4x10^5 + \frac{1}{4x10^{-6}s}$$

$$Z_2(s) = 1.1x10^5 + \frac{1}{4x10^{-6}s}$$

Therefore,

$$G(s) = \frac{Z_1(s)+Z_2(s)}{Z_1(s)} = 1.275\frac{(s+0.98)}{(s+0.625)}$$

21.

The system has two independent translational displacements, so we can write the following two equations:

$$X_1: \quad (s^2+2s+7)X_1(s) - (s+5)X_2(s) = 0$$
$$X_2: \quad -(s+5)X_1(s) + (2s^2+3s+5)X_2(s) = F(s)$$

Solving we get:

$$X_2(s) = \frac{\begin{vmatrix} s^2+2s+7 & 0 \\ -(s+5) & F(s) \end{vmatrix}}{\begin{vmatrix} s^2+2s+7 & -(s+5) \\ -(s+5) & 2s^2+3s+5 \end{vmatrix}} = \frac{(s^2+2s+7)F(s)}{(s^2+2s+7)(2s^2+3s+5)-(s+5)^2}$$

$$= \frac{(s^2+2s+7)F(s)}{2s^4+7s^3+24s^2+21s+10}$$

The resulting transfer function can be written as $\frac{X_2(s)}{F(s)} = \frac{1}{2}\frac{s^2+2s+7}{s^4+3.5s^3+12s^2+10.5s+5}$.

35.

The parameters are:

$$\frac{K_t}{R_a} = \frac{T_s}{E_a} = \frac{5}{5} = 1; \quad K_b = \frac{E_a}{\omega} = \frac{5}{\frac{600}{\pi}2\pi\frac{1}{60}} = \frac{1}{4};$$

$$J_m = 18\left(\frac{1}{4}\right)^2 + 4\left(\frac{1}{2}\right)^2 + 1 = 3.125; \quad D_m = 36\left(\frac{1}{4}\right)^2 = 2.25$$

Thus,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{1}{3.125}}{s(s+\frac{1}{3.125}(2.25+(1)(\frac{1}{4})))} = \frac{0.32}{s(s+0.8)}$$

Since: $\theta_2(s) = \frac{1}{4}\theta_m(s)$; then:

$$\frac{\theta_2(s)}{E_a(s)} = \frac{0.08}{s(s+0.8)}$$