機率與統計 Exercise1 Chapter1　B06505047 陳銘杰

Problem 1.1.3

a. $\mathrm{S} = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$
b. $Z_F = \{aaf, aff, faf, fff\}$
   $X_A = \{aaa, aaf, afa, aff\}$
c. No
d. No
e. $\mathrm{C} = \{aaa, aaf, afa, faa\}$
   $\mathrm{D} = \{aff, faf, ffa, fff\}$
f. Yes
g. Yes

Problem 1.2.13

Define the set of events $\{A_i | i = 1,2,3 \dots\}$ $\mathrm{i} = 1,2 \dots \mathrm{m}, A_i = B_i$, $\mathrm{i} > \mathrm{m}, A_i = \emptyset$
$\mathrm{P}[\cup_{i=1}^{m} B_i] = P[\cup_{i=1}^{\infty} A_i] = \sum_{i=1}^{\infty} P[A_i] = \sum_{i=1}^{m} P[A_i]$ $(i > m, A_i = \emptyset, P[\emptyset] = 0)$
$\mathrm{P}[\cup_{i=1}^{m} B_i] = \sum_{i=1}^{m} P[A_i] = \sum_{i=1}^{m} P[B_i]$ $(i = 1,2 \dots \mathrm{m}, A_i = B_i)$
$=> \mathrm{P}[\cup_{i=1}^{m} B_i] = \sum_{i=1}^{m} P[B_i]$

Problem 1.3.8

a. $\mathrm{P}[L] = 0.16$、$\mathrm{P}[H] = 0.1$、$\mathrm{P}[LH] = 0.1 * P[L \cup H]$
   $\mathrm{P}[L \cup H] = P[L] + P[H] - P[LH]$
   $10 * \mathrm{P}[LH] = 0.16 + 0.1 - P[LH]$
   $11 * \mathrm{P}[LH] = 0.26$
   $\mathrm{P}[LH] = \frac{0.26}{11} = 0.024$
b. $\mathrm{P}[H|L] = \frac{P[LH]}{P[L]} = \frac{0.024}{0.16} = 0.148$

Problem 1.4.3

a.

	H_0	H_1	H_2
F	P_0	P_1	P_2
V	P_3	P_4	P_5

$P_0 + P_3 = P_1 + P_4 = P_2 + P_5 = \frac{1}{3}$

$P_0 + P_1 + P_2 = \frac{5}{12} \Rightarrow P_3 + P_4 + P_5 = \frac{7}{12}$

1.

	H_0	H_1	H_2
F	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{6}$
V	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$

2.

	H_0	H_1	H_2
F	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$
V	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$

3.

	H_0	H_1	H_2
F	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$
V	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

b. $P_0 = \frac{1}{4}$ 、$P_4 = \frac{1}{6}$

$\Rightarrow P_3 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ 、$P_1 = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

$\Rightarrow P_2 = \frac{5}{12} - \frac{1}{4} - \frac{1}{6} = 0$ 、$P_5 = \frac{7}{12} - \frac{1}{12} - \frac{1}{6} = \frac{1}{3}$

	H_0	H_1	H_2
F	$\frac{1}{4}$	$\frac{1}{6}$	0
V	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$

Problem 1.5.11

a. $\mathrm{P}[A \cap B^C]$
$= P[A] - P[A \cap B]$
$= P[A] - P[A]P[B]$
$= P[A](1 - P[B])$
$= P[A]P[B^C]$

b. $\mathrm{P}[A^C \cap B]$
$= P[B] - P[A \cap B]$
$= P[B] - P[A]P[B]$
$= P[B](1 - P[A])$
$= P[B]P[A^C]$
$= \mathrm{P}[A^C]\mathrm{P}[B]$

c. $\mathrm{P}[A^C \cap B^C]$
$= 1 - \mathrm{P}[A \cup B]$
$= 1 - \mathrm{P}[A] - P[B] + P[A \cap B]$
$= 1 - \mathrm{P}[\mathrm{A}] - \mathrm{P}[\mathrm{B}] + \mathrm{P}[\mathrm{A}]\mathrm{P}[B]$
$= 1 - \mathrm{P}[A] - P[B](1 - P[A])$
$= (1 - P[A])(1 - P[B])$
$= \mathrm{P}[A^C]P[B^C]$