B06505047 陳銘杰 HW-Ch1

15. (a) $y_2[n] = x_2[n-2] + \frac{1}{2} x_2[n-3]$

$= \left(2x_1[n-2] + 4x_1[n-3]\right) + \frac{1}{2}\left(2x_1[n-3] + 4x_1[n-4]\right)$

$= 2x_1[n-2] + 4x_1[n-3] + x_1[n-3] + 2x_1[n-4]$

$= 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]$

$\Rightarrow y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$ #

(b) $y_1[n] = 2x_1[n] + 4x_1[n-1]$

$= 2\left(x_2[n-2] + \frac{1}{2} x_2[n-3]\right) + 4\left(x_2[n-3] + \frac{1}{2} x_2[n-4]\right)$

$= 2x_2[n-2] + x_2[n-3] + 4x_2[n-3] + 2x_2[n-4]$

$= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4]$

$\Rightarrow$ Input - output relationship DOESN'T change #

38. (a) $\delta_\Delta(2t) = \begin{cases} 0, & 2t < 0 \\ \frac{1}{\Delta}, & 0 < 2t < \Delta \\ 0, & \Delta < 2t \end{cases}$

$= \begin{cases} 0, & t < 0 \\ \frac{1}{\Delta}, & 0 < t < \frac{\Delta}{2} \\ 0, & \frac{\Delta}{2} < t \end{cases}$

$= \frac{1}{2}\begin{cases} 0, & t < 0 \\ \frac{1}{\frac{\Delta}{2}}, & 0 < t < \frac{\Delta}{2} \\ 0, & \frac{\Delta}{2} < t \end{cases}$

$= \frac{1}{2} \; \delta_{\frac{\Delta}{2}}(t)$

$\Rightarrow \delta_\Delta(2t) = \frac{1}{2}\delta_{\frac{\Delta}{2}}(t)$

$\lim_{\Delta\to 0} \delta_\Delta(2t) = \lim_{\Delta\to 0} \frac{1}{2}\delta_{\frac{\Delta}{2}}(t)$

$\delta(2t) = \frac{1}{2}\delta(t)$

(b) ①

$u^1_\Delta(t) = \begin{cases} 0, & t < -\frac{\Delta}{2} \\ \int_{-\frac{\Delta}{2}}^{t} \frac{1}{\Delta}\, d\tau, & -\frac{\Delta}{2} < t < \frac{\Delta}{2} \\ \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta}\, d\tau, & \frac{\Delta}{2} < t \end{cases}$

$= \begin{cases} 0, & t < -\frac{\Delta}{2} \\ \frac{t}{\Delta} + \frac{1}{2}, & -\frac{\Delta}{2} < t < \frac{\Delta}{2} \\ 1, & \frac{\Delta}{2} < t \end{cases}$

$\Delta \to 0, \; u^1_\Delta(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$

②

$u^2_\Delta(t) = \begin{cases} 0, & t < \Delta \\ \int_{\Delta}^{t} \frac{1}{\Delta}\, d\tau, & \Delta < t < 2\Delta \\ \int_{\Delta}^{2\Delta} \frac{1}{\Delta}\, d\tau, & 2\Delta < t \end{cases}$

$= \begin{cases} 0, & t < \Delta \\ \frac{t-\Delta}{\Delta}, & \Delta < t < 2\Delta \\ 1, & t > 2\Delta \end{cases}$

$\Delta \to 0, \; u^2_\Delta(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$

38. (b)

③

$$u^3_\Delta(t) = \begin{cases} 0, & t < -\Delta \\ \int_{-\Delta}^{t} r^3_\Delta(\tau)\,d\tau, & -\Delta < t < \Delta \\ \int_{-\Delta}^{\Delta} r^3_\Delta(\tau)\,d\tau, & t > \Delta \end{cases}$$

$$\int_{-\Delta}^{t} r^3_\Delta(\tau)\,d\tau = \begin{cases} \int_{-\Delta}^{t} \frac{1}{\Delta^2}(\tau+\Delta)\,d\tau, & t < 0 \\ \int_{-\Delta}^{0} \frac{1}{\Delta^2}(\tau+\Delta)\,d\tau + \int_{0}^{t} \frac{1}{\Delta^2}(\Delta-\tau)\,d\tau, & t > 0 \end{cases}$$

$$= \begin{cases} \frac{(t+\Delta)^2}{2\Delta^2}, & t < 0 \\ \frac{1}{2} - \frac{(t-\Delta)^2}{2\Delta^2} + \frac{1}{2}, & t > 0 \end{cases}$$

$$= \begin{cases} \frac{(t+\Delta)^2}{2\Delta^2}, & t < 0 \\ \frac{1}{2} - \frac{t^2-2\Delta t}{2\Delta^2}, & t > 0 \end{cases}$$

$$\Rightarrow u^3_\Delta(t) = \begin{cases} 0, & t < -\Delta \\ \frac{(t+\Delta)^2}{2\Delta^2}, & -\Delta < t < 0 \\ \frac{1}{2} - \frac{t^2-2\Delta t}{2\Delta^2}, & 0 < t < \Delta \\ 1, & t > \Delta \end{cases}$$

$$\Delta \to 0, \quad u^3_\Delta(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$$

④

$$u^4_\Delta(t) = \begin{cases} 0, & t < -\Delta \\ \int_{-\Delta}^{t} \left(-\frac{\tau}{\Delta^2}\right) d\tau, & -\Delta < t < 0 \\ \frac{1}{2} + \int_{0}^{t} \frac{\tau}{\Delta^2}\,d\tau, & 0 < t < \Delta \\ 1, & t > \Delta \end{cases} = \begin{cases} 0, & t < -\Delta \\ \frac{1}{2} - \frac{t^2}{2\Delta^2}, & -\Delta < t < 0 \\ \frac{1}{2} + \frac{t^2}{2\Delta^2}, & 0 < t < \Delta \\ 1, & t > \Delta \end{cases}$$

$$\Delta \to 0, \quad u^4_\Delta(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$$

⑤

$$u^5_\Delta(t) = \begin{cases} 0, & t < -\Delta \\ \int_{-\Delta}^{t} \left(-\frac{1}{\Delta}\right) d\tau, & -\Delta < t < 0 \\ \int_{-\Delta}^{0} -\frac{1}{\Delta}\,d\tau + \int_{0}^{t} \frac{2}{\Delta}\,d\tau, & 0 < t < \Delta \\ \int_{-\Delta}^{0} -\frac{1}{\Delta}\,d\tau + \int_{0}^{\Delta} \frac{2}{\Delta}\,d\tau, & t > \Delta \end{cases} = \begin{cases} 0, & t < -\Delta \\ -1 - \frac{t}{\Delta}, & -\Delta < t < 0 \\ -1 + \frac{2t}{\Delta}, & 0 < t < \Delta \\ 1, & t > \Delta \end{cases}$$

$$\Delta \to 0, \quad u^5_\Delta(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$$

38 (b)

6

$$u_\Delta^6(t) = \begin{cases} \int_{-\infty}^{t} \frac{1}{2\Delta} e^{\tau/\Delta}\, d\tau, & t < 0 \\ \int_{-\infty}^{0} \frac{1}{2\Delta} e^{\tau/\Delta}\, d\tau + \int_{0}^{t} \frac{1}{2\Delta} e^{-\tau/\Delta}\, d\tau, & t > 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2} e^{\frac{t}{\Delta}}, & t < 0 \\ \frac{1}{2} + \left(-\frac{1}{2} e^{\frac{-t}{\Delta}} + \frac{1}{2} \right), & t > 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2} e^{\frac{t}{\Delta}}, & t < 0 \\ 1 - \frac{1}{2} e^{\frac{-t}{\Delta}}, & t > 0 \end{cases}$$

$$\Delta \to 0, \quad u_\Delta^6(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} = u(t)$$

46.

$$y[n] = e[n-1]$$

$$e[n] = x[n] - y[n]$$

$$\Rightarrow y[n] = x[n-1] - y[n-1], \quad y[n] = 0 \text{ for } n < 0$$

(a) $$x[n] = \delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{other} \end{cases}$$

$$y[0] = \delta[-1] - y[-1] = 0 - 0 = 0$$

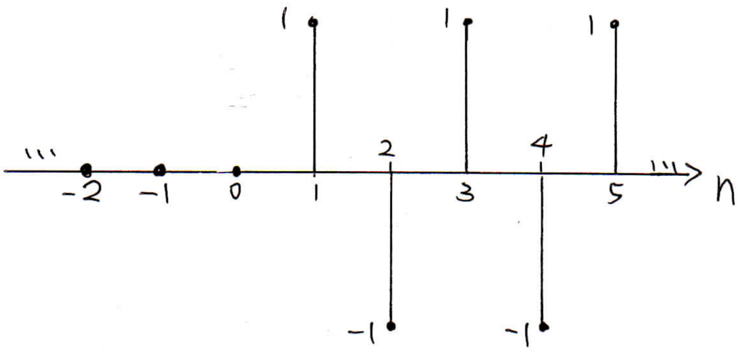
$$y[1] = \delta[0] - y[0] = 1 - 0 = 1$$

$$y[2] = \delta[1] - y[1] = 0 - 1 = -1$$

$$y[3] = \delta[2] - y[2] = 0 - (-1) = 1$$

$\vdots$

$$y[n] = \begin{cases} 0, & n = 0 \\ (-1)^{n-1}, & n \in \mathbb{N} \end{cases}$$



46 (b)

$$u[n] = \begin{cases} 1, & n = 0 \text{ or } n \in \mathbb{N} \\ 0, & \text{other} \end{cases}$$

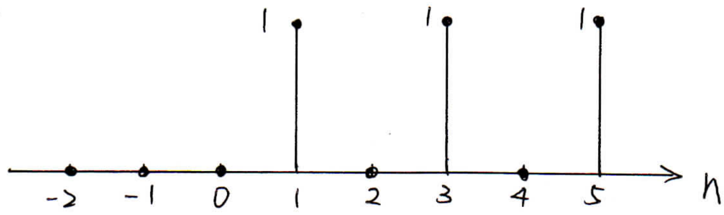
$y[0] = u[-1] - y[-1] = 0 - 0 = 0$

$y[1] = u[0] - y[0] = 1 - 0 = 1$

$y[2] = u[1] - y[1] = 1 - 1 = 0$

$y[3] = u[2] - y[2] = 1 - 0 = 1$

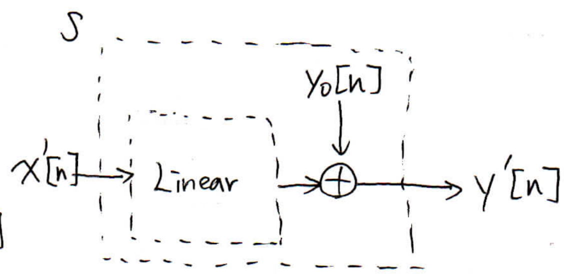
$\vdots$



$$y[n] = \begin{cases} 0, & n \le 0 \\ \dfrac{1 + (-1)^{n-1}}{2}, & n \in \mathbb{N} \end{cases}$$

47.

(a) $y_1[n] = S\{x_1[n]\} = L\{x_1[n]\} + y_0[n]$



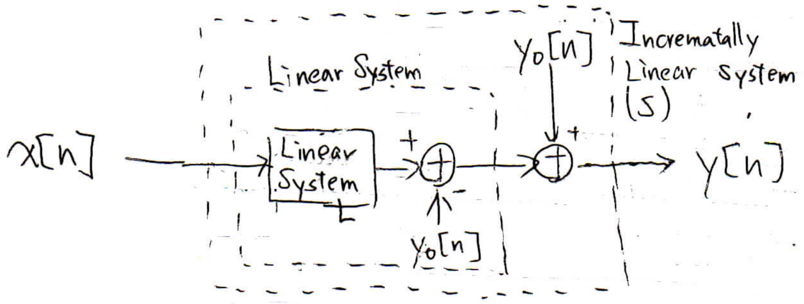
$y[n] = S\{x[n] + x_1[n]\} - y_1[n]$

$= L\{x[n] + x_1[n]\} + y_0[n] - y_1[n]$

$= L\{x[n]\} + L\{x_1[n]\} + y_0[n] - y_1[n]$

$= L\{x[n]\}$ $\Rightarrow$ linear, relationship between $x[n]$ and $y[n]$ doesn't depend on $x_1[n]$ #

(b)



(c)

①

$y_1[n] = n + x_1[n] + 2x_1[n+4]$

$y_2[n] = n + x_2[n] + 2x_2[n+4]$

$y_1[n] - y_2[n] = x_1[n] - x_2[n] + 2x_1[n+4]$
$\qquad - 2x_2[n+4]$

$x[n] \xrightarrow{L} g[n] = x[n] + 2x[n+4]$

$y_0[n] = n$

$\Rightarrow$ incrementally linear

47. (c)

② $y_1[n] = \begin{cases} n/2 & n \text{ even} \\ \frac{n-1}{2} + \sum_{k=-\infty}^{(n-1)/2} x_1[k] & n \text{ odd} \end{cases}$

$y_2[n] = \begin{cases} n/2 & n \text{ even} \\ \frac{n-1}{2} + \sum_{k=-\infty}^{(n-1)/2} x_2[k] & n \text{ odd} \end{cases}$

even n:

$y_1[n] - y_2[n] = \frac{n}{2} - \frac{n}{2} = 0$

odd n:

$y_1[n] - y_2[n] = \left(\frac{n-1}{2} + \sum_{k=-\infty}^{\frac{n-1}{2}} x_1[k]\right) - \left(\frac{n-1}{2} + \sum_{k=-\infty}^{\frac{n-1}{2}} x_2[k]\right)$

$= \sum_{k=-\infty}^{\frac{n-1}{2}} \left(x_1[k] - x_2[k]\right)$

$\Rightarrow \quad x[n] \xrightarrow{L} g[n] = \begin{cases} 0, & n \text{ even} \\ \sum_{k=-\infty}^{\frac{n-1}{2}} x[k], & n \text{ odd} \end{cases}$

$y_0[n] = \begin{cases} \frac{n}{2} & n \text{ even} \\ \frac{n-1}{2} & n \text{ odd} \end{cases}$ $\Rightarrow$ incrementally linear #

③ $y_1[n] = \begin{cases} x_1[n] - x_1[n-1] + 3, & x_1[0] \geq 0 \\ x_1[n] - x_1[n-1] - 3, & x_1[0] < 0 \end{cases}$

$y_2[n] = \begin{cases} x_2[n] - x_2[n-2] + 3 \\ x_2[n] - x_2[n-2] - 3 \end{cases}$

$y_1[n] - y_2[n] = \begin{cases} (x_1[n] - x_1[n-1] + 3) - (x_2[n] - x_2[n-1] + 3), & x_1[0] \geq 0, x_2[0] \geq 0 \\ (x_1[n] - x_1[n-1] + 3) - (x_2[n] - x_2[n-1] - 3), & x_1[0] \geq 0, x_2[0] < 0 \\ (x_1[n] - x_1[n-1] - 3) - (x_2[n] - x_2[n-1] + 3), & x_1[0] < 0, x_2[0] \geq 0 \\ (x_1[n] - x_1[n-1] - 3) - (x_2[n] - x_2[n-1] - 3), & x_1[0] < 0, x_2[0] < 0 \end{cases}$

$= \begin{cases} x_1[n] - x_1[n-1] - x_2[n] + x_2[n-1], & x_1[0] \geq 0, x_2[0] \geq 0 \\ x_1[n] - x_1[n-1] - x_2[n] + x_2[n-1] + 6, & x_1[0] \geq 0, x_2[0] < 0 \\ x_1[n] - x_1[n-1] - x_2[n] + x_2[n-1] - 6, & x_1[0] < 0, x_2[0] \geq 0 \\ x_1[n] - x_1[n-1] - x_2[n] + x_2[n-1], & x_1[0] < 0, x_2[0] < 0 \end{cases}$

$\Rightarrow$ not incrementally linear

47. (c)

④
$$y_1(t) = \frac{d\, t x_1(t)}{dt} = x_1(t) + t\frac{dx_1(t)}{dt}$$
$$y_2(t) = x_2(t) + t\frac{dx_2(t)}{dt}$$

$$y_1(t) - y_2(t) = x_1(t) - x_2(t) + t\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right)$$

$$x(t) \xrightarrow{L} x(t) + t\frac{dx(t)}{dt} - c$$

$y_0(t) = c$, c = const , $\Rightarrow$ incrementally linear

⑤
$$y[n] = [x[n] + \cos(\pi n)]^2 - x^2[n]$$
$$= x^2[n] + 2x[n]\cos(\pi n) + \cos^2(\pi n) - x^2[n]$$
$$= 2x[n]\cos(\pi n) + \cos^2(\pi n)$$

$$y_1[n] = 2x_1[n]\cos(\pi n) + \cos^2(\pi n)$$
$$y_2[n] = 2x_2[n]\cos(\pi n) + \cos^2(\pi n)$$
$$y_1[n] - y_2[n] = 2\cos(\pi n)[x_1[n] - x_2[n]]$$

$$\Rightarrow \quad x[n] \xrightarrow{L} g[n] = 2\cos(\pi n)\,x[n]$$

$y_0[n] = \cos^2(\pi n)$ $\Rightarrow$ incrementally linear

(d)

$$x'[n] \xrightarrow{L} g[n]$$
$$y'[n] = g[n] + y_0[n]$$

output shift $\longrightarrow$ $y'[n-n_0] = g[n-n_0] + y_0[n-n_0]$

input shift $\longrightarrow$ $x'[n-n_0] \xrightarrow{L} g[n-n_0]$

$$y''[n] = g[n-n_0] + y_0[n]$$

if S is time-invariant , $y'[n-n_0] = y''[n]$

$\Rightarrow$ L is time-invariant , $y_0[n]$ = const