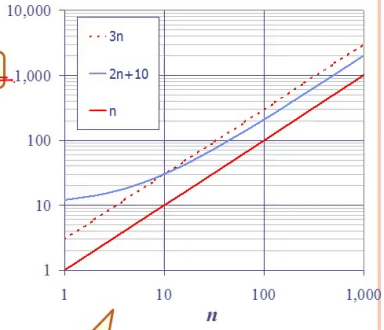
Big-Oh Notation

- Given functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if there are positive constants c and n_0 such that $f(n) \le cg(n)$ for $n \ge n_0$
- Example: $2n + 10$ is $O(n)$
  - $2n + 10 \le cn$
  - $(c - 2)\, n \ge 10$
  - $n \ge 10/(c - 2)$
  - Pick $c = 3$ and $n_0 = 10$

1, n, n^2, n lg n, n^3, 2^n ....

g(n): basic functions

Quiz!



Log scale

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Definition

- That even if f(n) increases, it will always be bound by cg(n)

More Big-Oh Examples

- 7n-2 is O(n)
  - Need c > 0 and $n_0 \ge 1$ such that $7n-2 \le c \bullet n$ for $n \ge n_0$
  - This is true for c = 7 and $n_0 = 1$
- $3n^3 + 20n^2 + 5$ is $O(n^3)$
  - Need c > 0 and $n_0 \ge 1$ such that $3n^3 + 20n^2 + 5 \le c \bullet n^3$ for $n \ge n_0$
  - This is true for c = 28 and $n_0 = 1$
- 3 log n + 5 is O(log n)
  - Need c > 0 and $n_0 \ge 1$ such that $3 \log n + 5 \le c \bullet \log n$ for $n \ge n_0$
  - This is true for c = 8 and $n_0 = 2$

Quiz!

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Examples & Quiz

How to Prove Statements

$\begin{cases} (a \cup b)' = a' \cap b' \\ (a \cap b)' = a' \cup b' \end{cases}$

$P \Rightarrow q \equiv \sim q \Rightarrow \sim P$

- By giving counter example
  - Roger claims that every number of the form 2^i-1 is a prime, where i is an integer greater than 1. Prove he is wrong.
- By contrapositive: Switching the hypothesis and conclusion of a conditional statement and negating both.
  - If p is true, then q is true ↔ If q is not true, then p is not true
  - If a*b is even, then a is even or b is even. ↔ If a is odd and b is odd, then a*b is odd.
  - Be aware of "DeMorgan's Law"
- By math induction
  - Example: Prove that $F(n) < 2^n$, where F(n) is the Fibonacci function with F(n+2)=F(n+1)+F(n), and F(1)=1, F(2)=2.

$\sim Q \Rightarrow \sim P$

$a = 2p+1$, $b = 2q+1$, $ab = 2(\ ) + 1$

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Proving Statements

- 數歸證明（期中考）
- base case
- inductive hypothesis
- inductive step