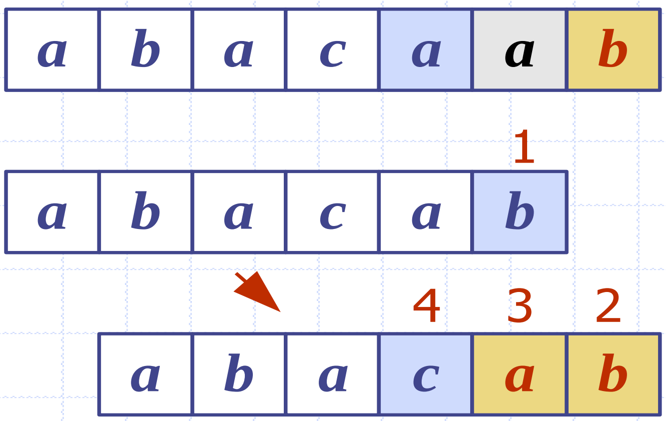
Pattern Matching



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Strings

- A string is a sequence of characters
- Examples of strings:
  - Java program
  - HTML document
  - DNA sequence
  - Digitized image
- An alphabet Σ is the set of possible characters for a family of strings
- Example of alphabets:
  - ASCII
  - Unicode
  - {0, 1}
  - {A, C, G, T}

- Let P be a string of size m
  - A substring $P[i \,..\, j]$ of P is the subsequence of P consisting of the characters with ranks between i and j
  - A prefix of P is a substring of the type $P[0 \,..\, i]$
  - A suffix of P is a substring of the type $P[i \,..m - 1]$
- Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- Applications:
  - Text editors
  - Search engines
  - Biological research

Brute-Force Pattern Matching

- The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T, until either
  - a match is found, or
  - all placements of the pattern have been tried
- Brute-force pattern matching runs in time $O(nm)$
- Example of worst case:
  - $T = aaa \ldots ah$
  - $P = aaah$
  - may occur in images and DNA sequences
  - unlikely in English text

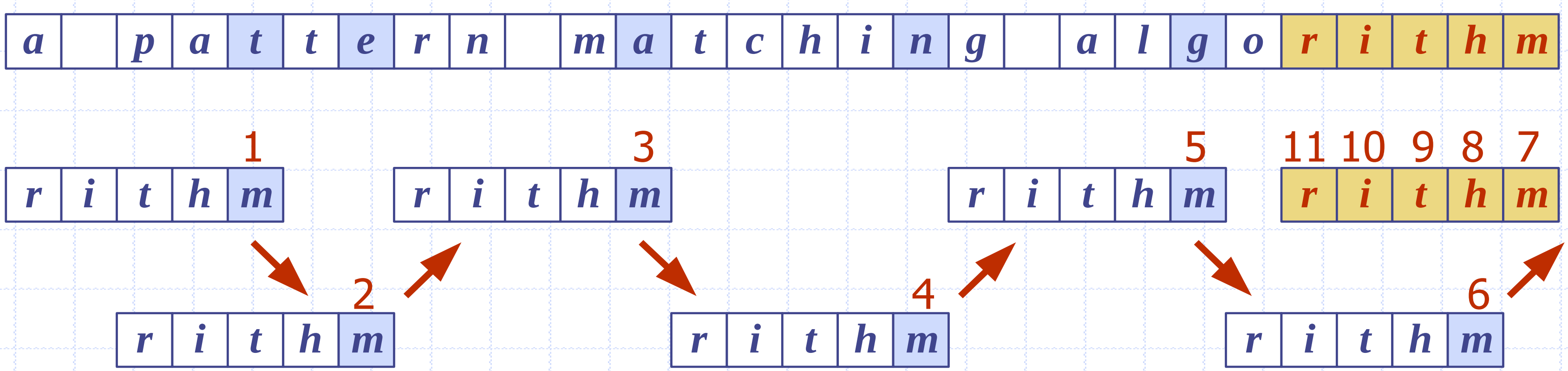
Algorithm *BruteForceMatch*(T, P)
    Input text T of size n and pattern P of size m
    Output starting index of a substring of T equal to P or -1 if no such substring exists
    for $i \leftarrow 0$ **to** $n - m$
        { test shift i of the pattern }
        $j \leftarrow 0$
        while $j < m \wedge T[i + j] = P[j]$
            $j \leftarrow j + 1$
        if $j = m$
            return i {match at i}
        else
            break while loop {mismatch}
    return **-1** {no match anywhere}

Boyer-Moore Heuristics

- The Boyer-Moore's pattern matching algorithm is based on two heuristics

  Looking-glass heuristic: Compare P with a subsequence of T moving backwards

  Character-jump heuristic: When a mismatch occurs at $T[i] = c$
  - If P contains c, shift P to align the last occurrence of c in P with $T[i]$
  - Else, shift P to align $P[0]$ with $T[i+1]$
- Example



Last-Occurrence Function

- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet Σ to build the last-occurrence function L mapping Σ to integers, where $L(c)$ is defined as
  - the largest index i such that $P[i] = c$ or
  - -1 if no such index exists
- Example:
  - $\Sigma = \{a, b, c, d\}$
  - $P = abacab$

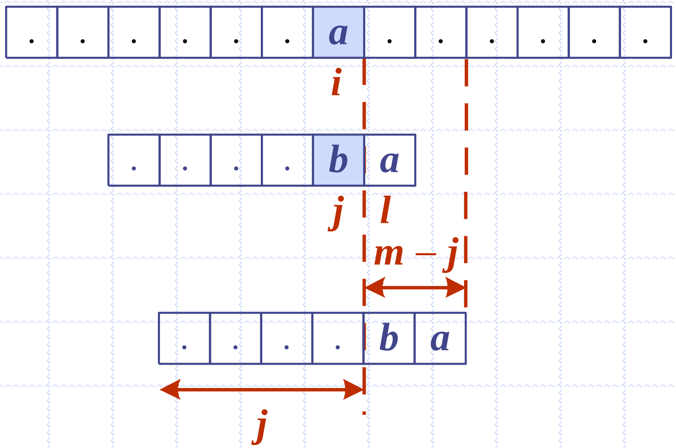
c	a	b	c	d
$L(c)$	4	5	3	-1

- The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- The last-occurrence function can be computed in time $O(m + s)$, where m is the size of P and s is the size of Σ

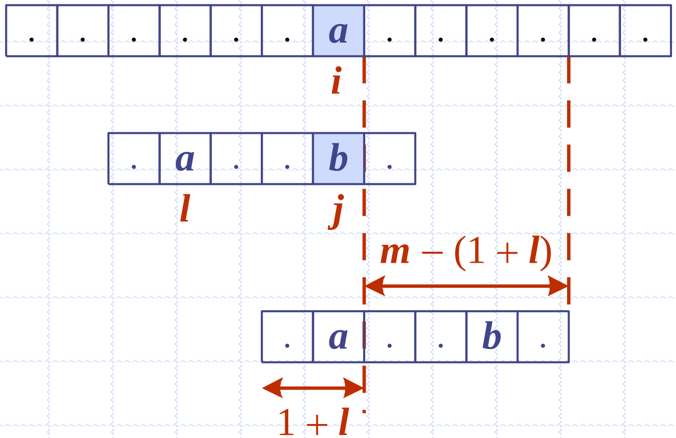
The Boyer-Moore Algorithm

```
Algorithm BoyerMooreMatch(T, P, Σ)
    L ← lastOccurenceFunction(P, Σ )
    i ← m − 1
    j ← m − 1
    repeat
        if T[i] = P[j]
            if  j = 0
                return  i  { match at i }
            else
                i ← i − 1
                j ← j − 1
        else
            { character-jump }
            l ← L[T[i]]
            i ← i + m – min(j, 1 + l)
            j ← m − 1
    until  i > n − 1
    return  −1 { no match }
```

Case 1: $j \leq 1 + l$



Case 2: $1 + l \leq j$



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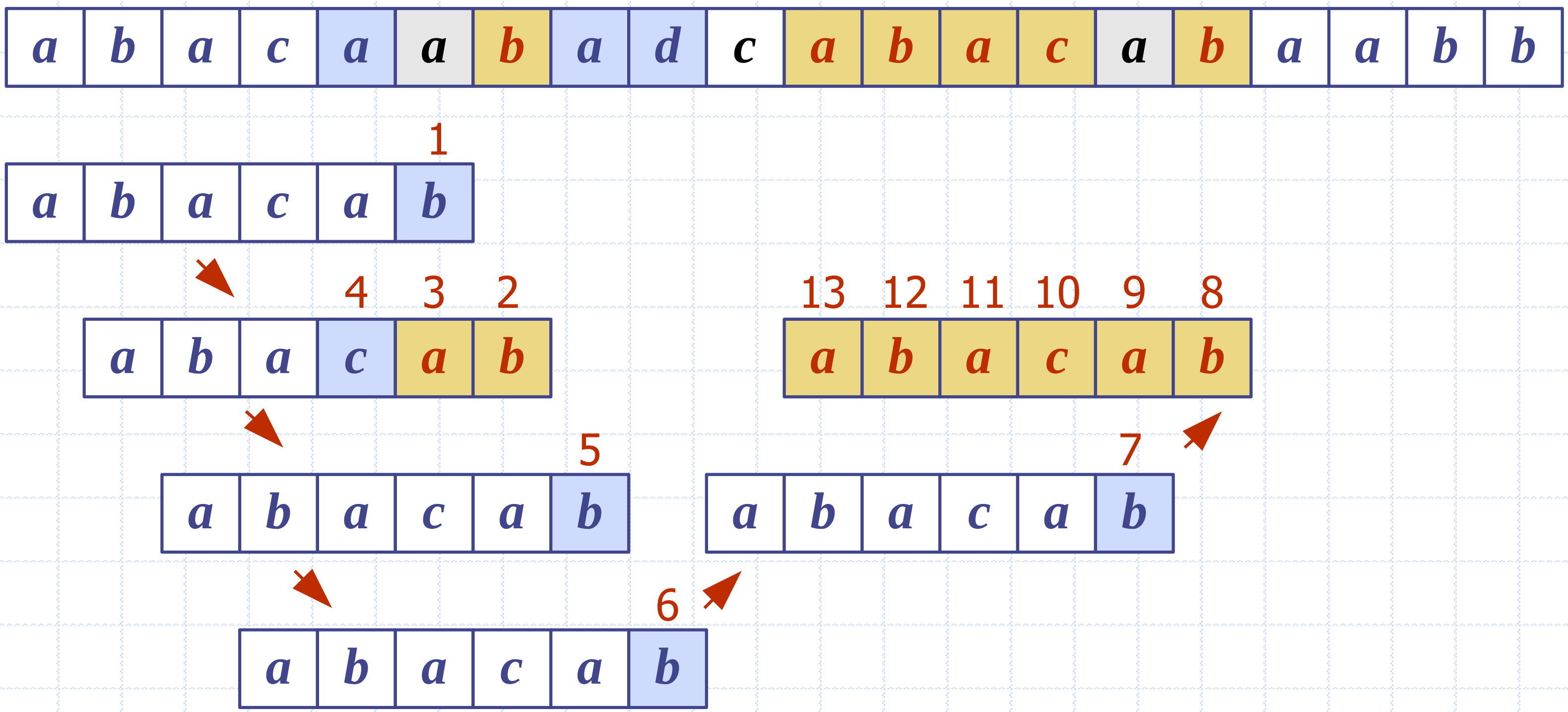
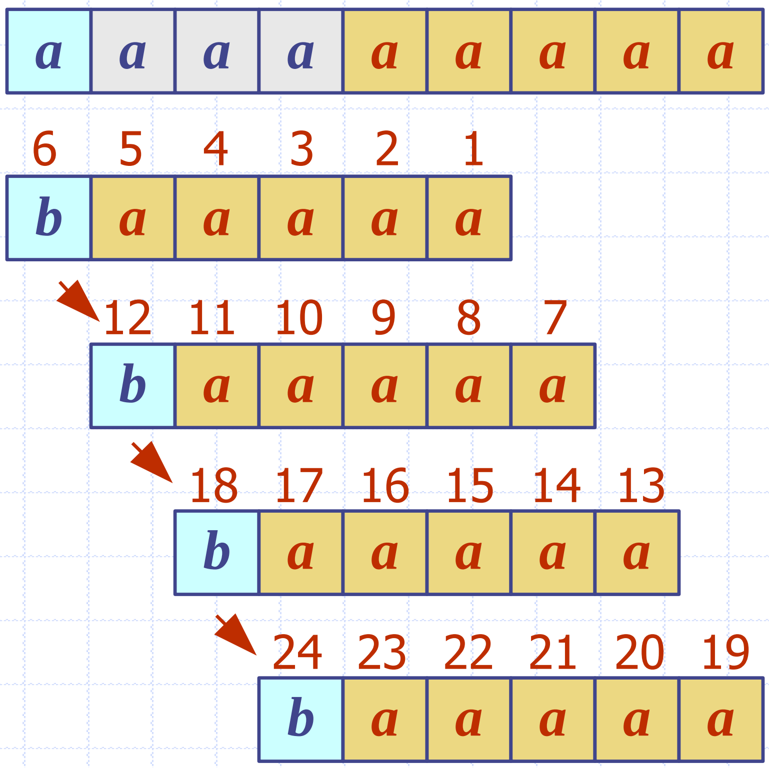
Example


a b a c a a b a d c a b a c a b a a b b
1
a b a c a b
4 3 2
a b a c a b
13 12 11 10 9 8
a b a c a b
5
a b a c a b
7
a b a c a b
6
a b a c a b

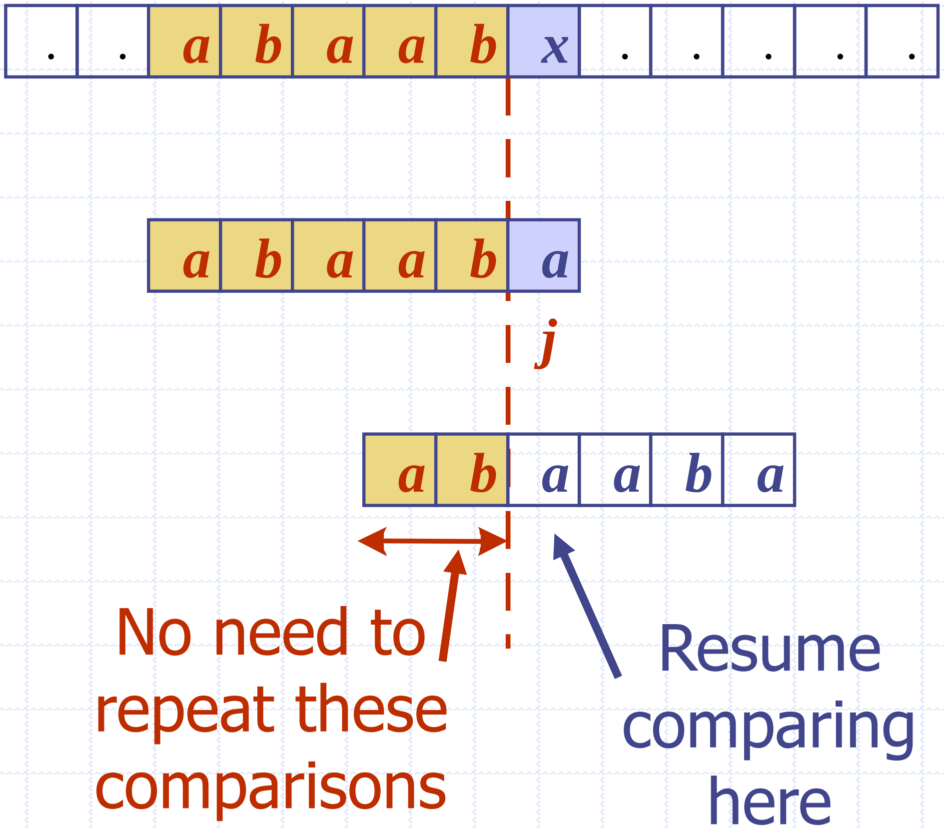
Analysis

- Boyer-Moore's algorithm runs in time $O(nm + s)$
- Example of worst case:
  - $T = aaa \ldots a$
  - $P = baaa$
- The worst case may occur in images and DNA sequences but is unlikely in English text
- Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text



The KMP Algorithm

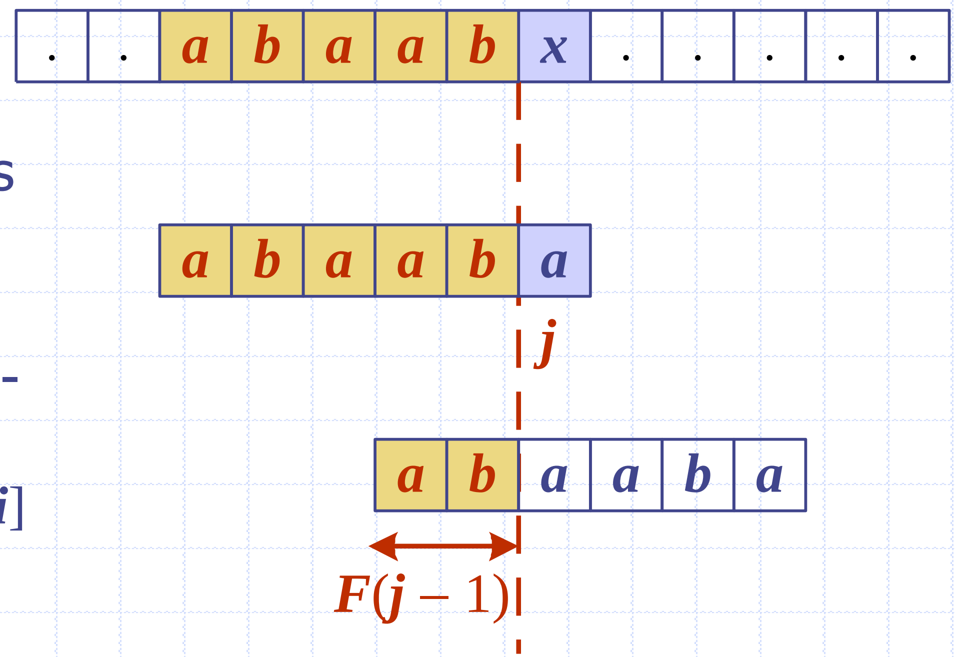
- Knuth-Morris-Pratt's (KMP) algorithm compares the pattern to the text in left-to-right, but shifts the pattern more intelligently than the brute-force algorithm.
- When a mismatch occurs, what is the **most** we can shift the pattern so as to avoid redundant comparisons?
- Answer: the largest prefix of $P[0..j]$ that is a suffix of $P[1..j]$



KMP Failure Function

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The failure function $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j-1)$

j	0	1	2	3	4	5
$P[j]$	a	b	a	a	b	a
$F(j)$	0	0	1	1	2	3



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More About Failure Functions

- Also known as
  - LPS: Longest proper prefix which is also suffix
- Examples

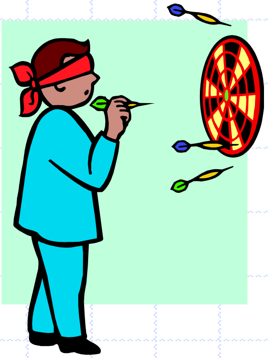
Quiz!

從字串S的位置 i 往前延伸，
最多可以往前幾位，
使得往前的這個位數是S的**真前綴**

abcdabeabf 0000120120	ababaabab 001231234
abcdeabfabc 00000120123	aaaabaacd 012301200

ababaabab

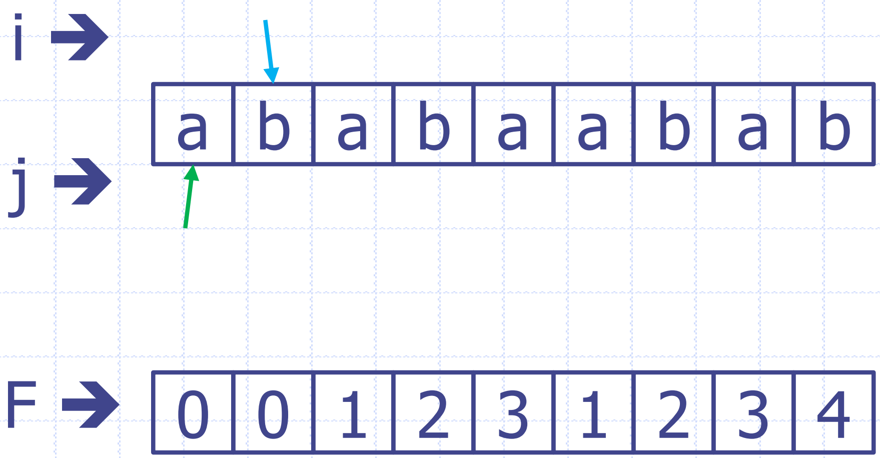
Computing the Failure Function



- The failure function can be represented by an array and can be computed in $O(m)$ time
- The construction is similar to the KMP algorithm itself
- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2m$ iterations of the while-loop

```
Algorithm failureFunction(P)
    F[0] ← 0
    i ← 1
    j ← 0
    while i < m
        if P[i] = P[j]
            {we have matched j + 1 chars}
            F[i] ← j + 1
            i ← i + 1
            j ← j + 1
        else if j > 0 then
            {use failure function to shift P}
            j ← F[j − 1]
        else
            F[i] ← 0 { no match }
            i ← i + 1
```

Computing Failure Functions



Algorithm *failureFunction*(*P*)

$F[0] \leftarrow 0$
$i \leftarrow 1$
$j \leftarrow 0$
while $i < m$
  if $P[i] = P[j]$
    {we have matched $j + 1$ chars}
    $F[i] \leftarrow j + 1$
    $i \leftarrow i + 1$
    $j \leftarrow j + 1$
  else if $j > 0$ **then**
    {use failure function to shift P}
    $j \leftarrow F[j - 1]$
  else
    $F[i] \leftarrow 0$ { no match }
    $i \leftarrow i + 1$

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The KMP Algorithm

- The failure function can be represented by an array and can be computed in $O(m)$ time
- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j - 1) < j$)
- Hence, there are no more than $2n$ iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time $O(m + n)$

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j − 1]
            else
                i ← i + 1
    return −1 { no match }
```

KMP Algorithm in Action (1/2)

There are two matches!

Quiz!

a	b	a	c	a	a	b	a	c	c	a	b	a	c	a	b	a	c	a	b

a	b	a	c	a	b

a	b	a	c	a	b
0	0	1	0	1	2

Failure function

KMP Algorithm in Action (2/2)

x	a	b	c	d	a	a	b	c	d	a	b	c	d	a	b	c	y	b	a

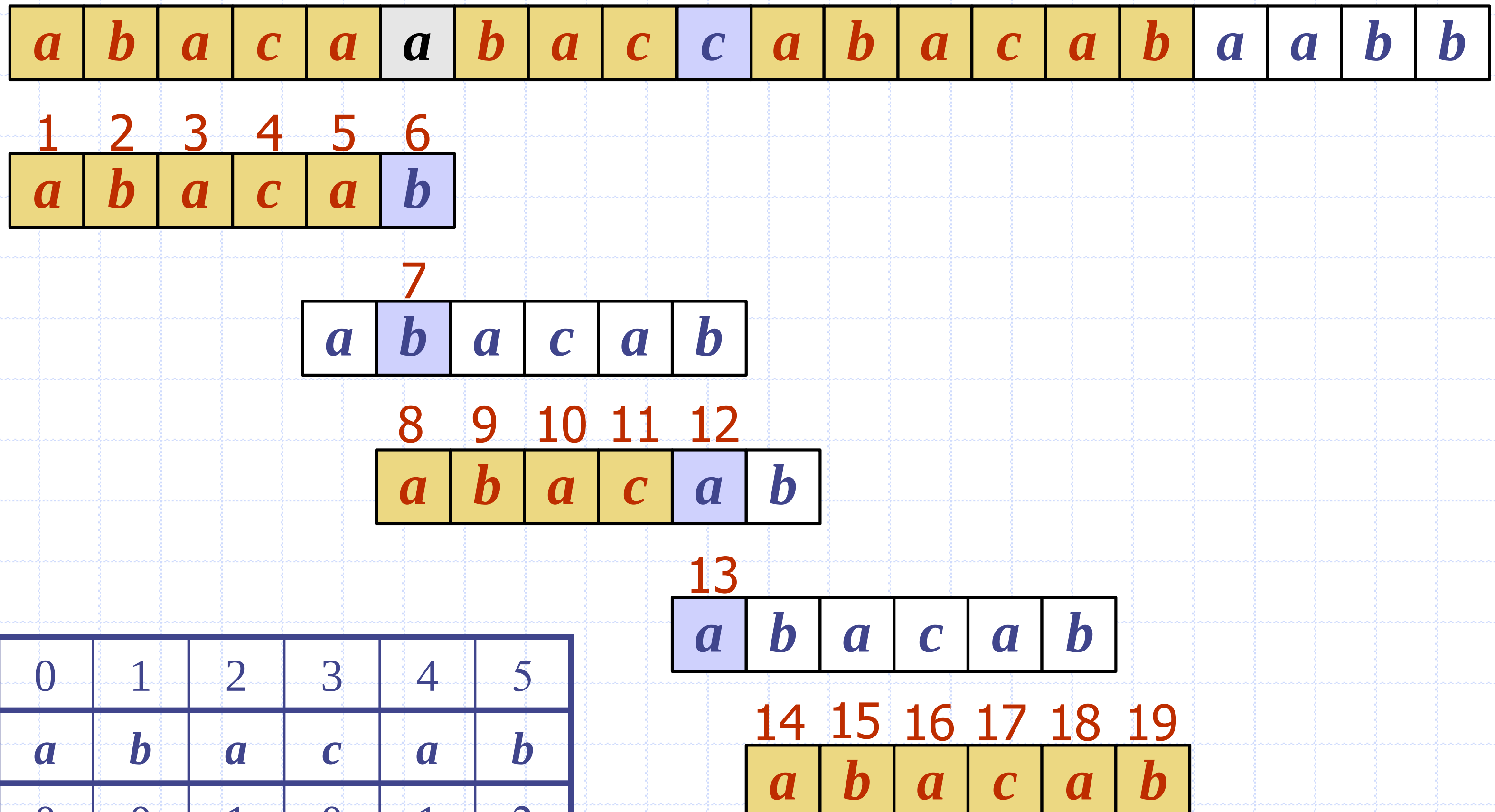
a	b	c	d	a	b	c	y

a	b	c	d	a	b	c	y
0	0	0	0	1	2	3	0

Failure function

Comparison Pattern

Quiz!



j	0	1	2	3	4	5
$P[j]$	a	b	a	c	a	b
$F(j)$	0	0	1	0	1	2

References

- Youtube tutorials
  - Boyer-Moore algorithm by Ben Langmead
  - KMP algorithm by Abdul Bari
  - KMP algorithm by GeeksforGeeks
- Comparisons