PS5
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Part I

1)

We can sort the given list of job into ascending order. Then the algorithm will return the optimal result.

$t_0 + (t_0 + t_{0+1}) + \dots + (t_0 + t_{0+1} + \dots t_n)/n$
where $t_{n-1} < t_n$

We can use known sorting method that is efficient such as quicksort which is divide and conquer algorithm. And the running time is bounded to O (n log n)

2)

We can use greedy algorithm, compare the unhappy score for each person,
For I to N person P
Compare U_{iA} and U_{iB}
If $U_{iA} > U_{iB}$, send P_i to team A else send to team B

The whole process will take O(n),

3)

"messiness" is the cube of the number of spaces left at the end of a line between the final word of the line and W.

The goal here is to minimize the messiness

Each word w have length l + 1(including the separating space), except the last word of the line.
W is the total length of a line, thus $(l_1 + 1) + (l_2 + 1) + \dots (l_{p-1} + 1) + l_p < W$, where lp is the last word of the first line and so on

First we sort $w = \{w_1, w_2, \dots , w_n\}$ into a descending order. And insert w_1 to first line.
For i = w
Int counter k = 0;
$k += (l_i + 1)$

If k < W
Remove w_i from w
i++

if k > W,
skip to w_i and proceed to w_{i+1} until w_n

endfor

if w !isEmtpy()
run for loop again with w

messiness of each line will be optimal, function is taking $O(n^2)$

Part II
1.

```
      7
    6
  5
4
  3
    2
      1
```

2.

```
      4
  2       6
1   3   5   7
```

3.

```
1   2           4
    |          /  |
    3          6   5
               |
               7
```

4.
1 2 3 4 5 6 7

Part III

1. **NO!** There has no prove that NP problem have no polynomial algorithm to solve, especially breaking the cryptography requires factoring and factoring isn't even a NP-complete
2. **YES!** solving 3-SAT hence we can also solve 2-SAT only prove that 2-SAT problem is at least as hard as 3-SAT. In other word, 2-SAT problem can be easier. 2-SAT cannot be proved as NP-Complete
3. **YES!** The solution can be check in polynomial time so this is an NP problem this problem is reducible from NP problem such as Hamilton path problem in a polynomial time. Therefore, this problem is surely NP-complete problem.

Part IV

Optimally solution (Brute force) has running bounded to $O(2^N)$. Therefore slightly larger input will take enormous amount of time to compute.