Hiring Through Referrals

Manolis Galenianos*

Pennsylvania State University

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Abstract

An equilibrium search model of the labor market is combined with a social network. The key features are that the workers' network transmits information about jobs and that wages and entry of firms are determined in equilibrium. When workers are homogeneous referrals mitigate search frictions. When workers are heterogeneous referrals also facilitate the hiring of better workers. Two predictions about the matching efficiency are supported by the data: a higher prevalence of referrals is associated with higher matching efficiency and the efficiency of the aggregate matching function is pro-cyclical.

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1 Introduction

Social networks are an important feature of labor markets (Granovetter, 1995). Approximately half of all American workers report learning about their job through their social network (friends, acquaintances, relatives etc.) and a similar proportion of employers report using the social networks of their current employees when hiring (the evidence is summarized in Section 2 and is surveyed in Ioannides and Loury, 2006, and Topa, 2010).

Surprisingly, however, social networks are typically not included in the equilibrium models that are used to study labor markets. For instance, in their extensive survey of search-theoretic models of the labor market, Rogerson, Shimer and Wright (2005) do not cite any papers that include social networks or referrals. On the other hand, a large literature uses graph theory to study social networks (Jackson, 2008). When applied to labor markets, however, these models usually restrict attention to partial equilibrium analyses where, for instance, wages or labor demand are exogenous (e.g. Calvo-Armengol and Jackson, 2004).[1]

The present paper proposes to bridge this gap by combining an equilibrium search model with a network structure that is simple enough to preserve tractability but also rich enough to deliver a large number of predictions that can be confronted with the data.

In the baseline model, workers are homogeneous in terms of their productivity and network. Each worker is linked with a measure of other workers and the network is exogenous. Vacancies are created both through the free entry of new firms and through the expansion of producing firms.[2] A firm and a worker meet either through search in the frictional market or through a referral, which occurs when a producing firm expands and asks its current employee to refer a link. The flow surplus of a worker-firm match is equal to output plus the value of the referrals and the wage is determined by Nash bargaining.

In equilibrium, referrals affect the labor market in two ways. First, they mitigate search

[1]Calvo-Armengol and Zenou (2005) and Fontaine (2008) are exceptions and are discussed below.

[2]However, the distribution of firm sizes is degenerate: each firm hires one worker and vacancies created through an expansion are immediately sold off.

frictions which reduces unemployment. Second, they discourage the entry of new firms which reduces labor market tightness. This equilibrium outcome generates the model's first prediction: a labor market with higher prevalence of referrals exhibits higher job finding rate for workers despite lower labor market tightness; in other words, higher matching efficiency.

To evaluate this prediction, a matching function without referrals separately for every major industry. Consistent with the prediction, the estimates for matching efficiency are positively and significantly correlated with the prevalence of referrals across industries.[3] A 10 percentage point increase in the prevalence of referrals is associated with a 28% increase in the matching efficiency of the average industry. Furthermore, one quarter of the inter-industry difference in matching efficiency can be accounted for by variation in the prevalence of referrals. This result is particularly relevant given the finding of Davis, Faberman and Haltiwanger (2010) that variation in job filling rates across industries cannot be quantitatively accounted for by the observed variation in labor market tightness.

Another feature of the model is that an increase in the unemployment rate reduces the flow of referrals, in addition to increasing congestion in the unemployment pool. This leads to the model's second prediction: in equilibrium, the job finding rate is a decreasing function of the unemployment rate *conditional on labor market tightness* or, in other words, the efficiency of the aggregate matching function is pro-cyclical. This prediction is consistent with several recent papers that examine the cyclical properties of matching efficiency. Barnichon and Figura (2011) find that the residual of the matching function moves pro-cyclically and Cheremukhin and Restrepo (2011) find that matching efficiency falls in the aftermath of recessions. On the firm side, Davis, Faberman and Haltiwanger (2010) find that a vacancy's hire yield is lower in weak labor markets.

The model is then extended to allow for worker heterogeneity. There are two worker types representing heterogeneity beyond observable characteristics. A worker's type (high or

[3]Data from the National Longitudinal Survey of Youth and the Job Openings and Turnover Survey is used. See Section 3.3 for the full data description.

low) determines his productivity and network. Conditional on type, every worker has the same measure and composition of links and, in accordance with evidence from the sociology literature, the network exhibits *homophily*: a worker has more links with workers of his own type. Firms act similarly to the baseline model. In the context of worker heterogeneity, referrals facilitate the hiring of high type workers in addition to mitigating search frictions.

In equilibrium, a referred worker is more likely to be of a high type than a non-referred worker. The reason is that high productivity workers are more likely to be employed and therefore more likely to act as referrers; the recipients of referrals are therefore more likely to also be high types due to the network's homophily.[4] This prediction is consistent with the following empirical findings: conditional on observable worker characteristics and firm fixed effects, referred candidates are more likely to be hired (Fernandez and Weinberg, 1997; Castilla, 2005; Brown, Setren and Topa, 2011), to receive higher wages (Simon and Warner, 1992; Bayer Ross and Topa, 2008; Brown, Setren and Topa, 2011) and to be more productive on the job (Castilla, 2005; Pinkston, 2011).[5]

Calvo-Armengol and Zenou (2005) and Fontaine (2008) incorporate a finite network in an equilibrium search model. They restrict attention to homogeneous workers who search regardless of employment status and, if employed, forward job information to an unemployed member of their network. Neither paper focuses on the efficiency of the aggregate matching function which is an important feature of the present study. In both papers, however, networks induce persistence in unemployment similar to this paper's second prediction.

[4]This prediction is driven by selection and is similar in spirit to Montgomery (1991). He considers a two-period model of the labor market with heterogeneous workers and a homophilous network among them. That model has no implications about employment rates and does not address the possibility of using referrals when there is no informational advantage concerning the worker's productivity.

[5]See Section 2 for a discussion of papers that have found no wage premium for referrals and Section 4.3 for this model's accommodation of that finding.

2 Evidence About Social Networks and Labor Markets

Numerous studies in economics and sociology have documented the following five salient facts about the interaction between social networks and labor markets.[6]

First, both workers and firms use referrals extensively when searching for a job or trying to fill a vacancy, respectively. More than 85% of worker use informal contacts when searching for a job according to the National Longitudinal Survey of Youth (NLSY) (Holzer, 1988). In terms of outcomes, more than 50% of all workers found their job through their social network according to data from the Panel Study of Income Dynamics (Corcoran, Datcher and Duncan, 1980) while the 24 studies surveyed by Bewley (1999) put that figure between 30% and 60%. In most European countries 25-45% of workers report finding their jobs through referrals according to data from the European Community Household Panel (Pellizzari, 2010).[7]

On the firm side, between 37% and 53% of employers use the social networks of their current employees to advertise jobs according to data from the National Organizations Survey (Mardsen, 2001) and the Employment Opportunity Pilot Project (EOPP) (Holzer, 1987). According to the EOPP 36% of firms filled their last opening through a referral (Holzer, 1987).

Second, increasing access to referrals increases a worker's job finding rate. Using census data Bayer, Ross and Topa (2008) find that when a male individual's access to social networks improves by one standard deviation (say, by moving to a city block where more people have children of the same age) his labor force participation is raised by 3.3 percentage points and his hours worked by 1.8 hours after controlling for other sources of selection. A higher employment rate for the individuals in the network also increases access to referrals. Topa (2001) finds strong evidence of local spillovers in employment rates across different census tracks in the Chicago area. Weinberg, Reagan and Yankow (2004) find that an increase of

[6] See Section 3.3 for the data that concerns the estimation of the matching function parameters.

[7] Denmark, Finland and the Netherlands are the only countries, out of a sample of 14, where that ratio falls below 25%.

one standard deviation in a neighborhood's social characteristics increases annual hours by 6.1% using confidential NLSY data. Using data from the British Household Panel Survey Cappellari and Tatsiramos (2010) show that an additional employed friend is associated with a increase in the probability of finding a job of 3.7 percentage points and a 5% increase in wages.

Third, referred applicants are statistically different from non-referred ones after controlling for observable characteristics. In their firm-level studies, Fernandez and Weinberg (1997), Castilla (2005) and Brown, Setren and Topa (2011) examine all the job applicants (successful and unsuccessful) and find that referred applicants are more likely to be hired after controlling for their observable characteristics. This is consistent with the finding of Holzer (1987) and Blau and Robins (1990) that referrals have a greater "hire yield" for firms than searching in the market, using EOPP data. Controlling for worker observables, Brown, Setren and Topa (2011) find that referred workers have higher wages and lower separation rates and Castilla (2005) and Pinkston (2011) find that referred workers are more productive (Castilla has direct measures of output by worker and Pinkston has subjective measure as reported by the employer). Simon and Warner (1992) and Bayer, Ross and Topa (2008) find that referred workers' wages increase significantly with a referral. Using data from the German Social Security Records Dustmann, Glitz and Schoenberg (2010) find that referred candidates receive higher wages and have lower separation rates after controlling for worker observables and firm fixed effects.

Pistaferri (1999), Pellizzari (2010) and Bentolila, Michelacci and Suarez (2010) report that using the job-finding method as one of the explanatory variables in a wage regression may lead to an insignificant or even negative coefficient of referrals on wages. These studies do not control for firm fixed effects, unlike the studies cited in the previous paragraph which suggests that selection is important on the firm side. The firms' choice of which channel to use for their search is examined in a companion paper, Galenianos (2011).

Fourth, the social ties that are most useful for transmitting information about job opportunities are the more numerous "weak" ties, e.g. acquaintances, as opposed to the "strong" ties, such as close friends (Granovetter, 1973; 1995). *Fifth*, social interactions tend to feature homophily: individuals who socialize together are more likely to share many characteristics, such as race and religion but also educational and professional characteristics (for an exhaustive survey see McPherson, Smith-Lovin and Cook, 2001).

Section 3 introduces the homogeneous worker model which provides predictions consistent with the first and second facts. Section 4 introduces worker heterogeneity and delivers predictions that are consistent with the third fact. Facts four and five will inform the modeling choices throughout the paper.

3 The Labor Market with Homogeneous Workers

This Section adds referrals to a standard equilibrium search model of the labor market.

3.1 The Model

Time runs continuously, the horizon is infinite and the labor market is in steady state. There is free entry of firms and each firm hires one worker, is risk-neutral and maximizes expected discounted profits using discount rate $r > 0$. A firm is either filled and producing or vacant and searching and the flow profit when vacant is 0.

There is a unit measure of workers who are homogeneous, risk-neutral, maximize expected discounted utility and discount the future at the same rate r. A worker is either employed or unemployed and the flow utility of unemployment is b. Every worker is linked with a *measure* ν of other workers, where $\nu \leq 1$.

Modeling a worker's network as a continuum of links is consistent with the (spirit of the) sociology literature's finding that it is a person's more numerous weak ties that help most with finding a job (Granovetter, 1973; 1995) and is crucial for the model's tractability.

A worker's employment opportunities will in general depend on how many of his links are employed which necessitates keeping track of each link's time-varying employment status. Having a continuum of links means that the aggregate (un)employment rate of a worker's social contacts does not change over time due to the law of large numbers, thereby greatly simplifying the analysis.[8]

It is certainly true that some of the richness in the predictions that graph-theoretic models of networks can generate, especially with respect to network architecture, is lost by the assumption of a continuum of links.[9] However, many of these additional predictions would be difficult to empirically verify or refute given the labor market data that is currently available.

Vacancy creation occurs in two ways, both of which cost K: a new firm enters the market or an existing firm expands which occurs at exogenous rate ρ. The position that is created by the expansion is immediately sold off which keeps firms' employment at one. A firm and a worker meet either through search in the market or through a referral, which occurs when a firm expands and asks its current employee to refer a link. The rate of meeting through the market is determined by a matching function. The rate of meeting through referrals is determined by the rate at which firms expand.

An expansion can be interpreted in (at least) a couple of different ways. At rate ρ, the firm meets an entrepreneur who wants to enter the market at which point the firm expands and sells him the new position (that entrepreneur would otherwise create a new firm through free entry). Alternatively, at rate ρ the firm identifies a business opportunity and expands to take advantage of it. However, it is subject to decreasing returns and finds it profitable to sell the new position to some new entrepreneur. For this paper's purposes it makes little

[8]In finite models, additional assumptions are needed to preserve tractability. In Calvo-Armengol and Zenou (2005) each worker is assumed to draw a new network every period so as to avoid keeping track of transitions in the network's employment rate. In Fontaine (2008) each worker belongs to one of a large number of disjoint finite networks and the focus is on the steady state distribution of employment rates across the population of networks.

[9]Note, though, that heterogeneity is not one of them: Section 4 introduces network heterogeneity.

difference which interpretation is adopted.

When an expansion occurs, one of the links of the incumbent worker is contacted at random. If the link is employed then the referral opportunity is lost and search in the market begins; if the link is unemployed then he is hired by the firm. In other words, creating a vacancy through an expansion bears the same cost as creating a new firm but could lead to an immediate hire while a new firm's entry is necessarily followed by time-consuming search in the market. This is consistent with the findings in Holzer (1987) and Blau and Robins (1990) that using referrals as a method of searching for workers exhibits a greater "hire yield" than the alternatives.

The assumption that the referrer contacts one of his links at random regardless of that link's employment status captures the frictions which are present when the referral channel is used. One interpretation is that, consistent with the weak ties view of the network, the referring employee does not know which of his links is currently looking for a job and starts contacting them at random to find out if they are interested in the job. Because this is costly, he will only try a finite number of times and with positive probability will fail to find someone interested in the job. In this paper for simplicity it is assumed that the referring worker stops after a single try but allowing for further tries can be accommodated by appropriately modifying the referral function below.

The flow value of a match is given by the worker's productivity, y, and the value of the referrals that he generates. The worker and the firm split the surplus according to the Nash bargaining solution where the worker's bargaining power is denoted by $\beta \in (0,1)$ and all payoff-relevant information, including the worker's network, are assumed to be common knowledge within the match. Matches are exogenously destroyed at rate δ , where $\delta > \rho$.[10] There is no on the job search.[11] Finally, to avoid trivial outcomes, it is assumed that $y >$

[10]This assumption guarantees that entry of new firms is necessary for steady state: absent entry of new firms, the stock of producing firms will decline.

[11]Since firms are homogeneous and the wage is determined by Nash bargaining there is no incentive to

$b + (r + \delta)K$.

Denote the expected surplus generated during an expansion by E. When a firm expands, it pays K and creates a vacancy, whose value is denoted by V. The incumbent worker contacts one of his links and a match is created if that worker is unemployed, the probability of which is denoted by u. Letting the firm's value of a match be J yields

$$E = -K + V + u(J - V). \tag{1}$$

The new position is immediately sold off and the incumbent firm receives share $\gamma \in [0, 1]$ of that surplus (the remaining $(1 - \gamma)E$ is captured by the buyer). Therefore a match's flow value is given by $y + \rho\gamma E$.[12]

Consider worker j who is linked with ν^j workers, each of whom is in turn linked with ν workers. The number of employed links of worker j is equal to $(1 - u)\nu^j$. The employer of each link expands at rate ρ in which case one of the incumbent employee's ν links receives the referral at random. Therefore, the rate at which worker j is referred to a job is $\alpha_R^j = \rho\nu^j(1 - u)/\nu$. The network's homogeneity ($\nu^j = \nu,\ \forall j$) implies:

$$\alpha_R \quad = \quad \rho(1 - u).$$

Note that the network's size does not affect the equilibrium.[13]

Consider the rate of meeting in the market and let v denote the number of vacancies. The

search while employed, at least under the common assumption that the value of unemployment is the worker's outside option during bargaining (e.g. Pissarides, 1994).

[12] The implicit assumption is that referrals will be used whenever the opportunity arises: a producing firm *will* expand at rate ρ and it *will* ask for a referral from its current employee. Alternatively, one can model the decision of whether to expand and/or ask the current employee for a referral as a decision to be jointly taken by the firm and the worker. Since the use of referrals increases flow surplus by $\rho\gamma E$ and this gain is shared by the worker and the firm they will endogenously choose to do so.

[13] With this specification, the size of one's network affects plays a role in the case of heterogeneity: imagine that half the workers are "insiders" with $\nu_I = 1$ (i.e. they are linked to everyone) while the other half are "outsides" with $\nu_O = 1/2$ (and, for consistency, they are only linked to insiders). It is clear that $\alpha_{RI} > \alpha_{RO}$ and insiders have better job prospects. It is certainly interesting to develop this further, and it is left for future work.

flow of meetings in the market between a vacancy and a worker is given by a Cobb-Douglas function

$$m(v,u) = \mu v^{\eta} u^{1-\eta},$$

where $\mu > 0$ and $\eta \in (0,1)$.

The rate at which a firm meets with a worker through the market is

$$\alpha_F = \frac{m(v,u)}{v} = \mu(\frac{u}{v})^{1-\eta}$$

and the rate at which a worker meets a firm through the market is

$$\alpha_M = \frac{m(v,u)}{u} = \mu(\frac{v}{u})^{\eta}.$$

The aggregate matching function, which includes both meetings through referrals and meetings through the market, is given by

$$\mathcal{M}(v,u) = \mu v^{\eta} u^{1-\eta} + \rho u(1-u) \tag{2}$$

The second term is derived by noting that when the number of producing firms is $1-u$, the rate of vacancy creation through expansion is equal to $\rho(1-u)$ and each referral leads to a new match with probability u.

The steady state condition is that the flows in and out of unemployment are equal:

$$u(\alpha_M + \alpha_R) = (1-u)\delta. \tag{3}$$

The agents' value functions are now described. When vacant, a firm searches in the market and meets with a worker at rate α_F. When producing, the firm's flow payoffs are

$y + \rho\gamma E - w$ where w denotes the wage. The match is destroyed at rate δ. The firm's value of a vacancy (V) and production (J) are given by:

$$\begin{aligned} rV &= \alpha_F (J - V), \\ rJ &= y + \rho\gamma E - w - \delta J. \end{aligned}$$

When unemployed, a worker's flow utility is b and job opportunities appear at rate $\alpha_M + \alpha_R$. When employed, the worker's flow utility is equal to the wage and the match is destroyed at rate δ. The worker's value of unemployment (U) and employment (W) are given by:

$$\begin{aligned} rU &= b + (\alpha_M + \alpha_R)(W - U), \\ rW &= w + \delta(U - W). \end{aligned}$$

The wage solves the Nash bargaining problem

$$w = \text{argmax}_w (W - U)^{\beta} (J - V)^{1-\beta}. \tag{4}$$

The Equilibrium is now defined.

Definition 3.1 *An Equilibrium is the steady state level of unemployment u and the number of vacancies v such that:*

- *The labor market is in steady state as described in* (3)*.*
- *The surplus is split according to* (4)*.*
- *There is free entry of firms: $V = K$.*

3.2 Labor Market Equilibrium

The characterization of equilibrium is fairly standard.

The condition that describes the steady state can be rewritten as follows:

$$\begin{aligned} & u[\mu(\tfrac{v}{u})^\eta + \rho(1-u)] = (1-u)\delta \\ \Rightarrow \qquad & v = [\tfrac{1-u}{\mu}(\tfrac{\delta}{u^{1-\eta}} - \rho u^\eta)]^{1/\eta}. \end{aligned} \tag{5}$$

Equation (5) shows that the steady state rate of unemployment is uniquely determined given v and it is strictly decreasing in v.[14] As a result, in steady state α_M and α_R are strictly increasing in v while α_F is strictly decreasing in v.

The surplus of a match is given by $S = W + J - U - V$. Nash bargaining implies

$$\begin{aligned} W - U &= \beta S, \\ J - V &= (1-\beta)S. \end{aligned}$$

The value functions can be rearranged to yield

$$(r+\delta)S = y + \rho\gamma E - b - (\alpha_M + \alpha_R)\beta S - (r+\delta)V. \tag{6}$$

Combining equation (6) with the definition of E from equation (1) and the free entry condition ($V = K$) and going through some algebra yields an expression that only depends on the number of vacancies in the market (recall that u is a function of v):

$$S = \frac{y - b - (r+\delta)K}{r + \delta + (\alpha_M + \alpha_R)\beta - \rho\gamma u(1-\beta)}.$$

The denominator of the right-hand side is strictly increasing in v which means that $dS/dv < 0$ when the steady state and free entry conditions hold.

[14]It is more convenient mathematically to write v as a function of u, although conceptually the measure of unemployed workers is the dependent variable (determined through the steady state condition for a given v) and the measure of vacancies is the independent variable (to be eventually determined through free entry).

The value function of a vacancy is

$$rV = \alpha_F(1-\beta)S. \tag{7}$$

Since α_F and S are both strictly decreasing in v, there is a unique measure of vacancies such that the value of creating a vacancy is equal to K.

The proposition summarizes the previous statements:

Proposition 3.1 *An equilibrium exists and it is unique.*

3.3 Testable Predictions and Evidence

The model's testable predictions are developed and compared with the data.

3.3.1 Cross-Sectional Properties

A comparative statics exercise is first performed with respect to the rate at which referrals are generated, ρ, to derive some of the model's cross-sectional properties which are then tested on cross-industry data.

Proposition 3.2 *An increase in the rate of generating referrals (ρ) leads to:*

1. *An increase in the proportion of jobs that are found through a referral.*
2. *An increase in the workers' job finding rate*
3. *A decrease in labor market tightness if $K \leq \bar{K}$, where $\bar{K}$ is a function of the model's parameters.*

Proof. See the Appendix. ■

The first part of Proposition 3.2 means that a high prevalence of referrals in the data corresponds to a high ρ in the model.[15] The second and third parts imply that if a matching function *without* referrals is estimated using data that are generated by the model *with* referrals, then a higher ρ will lead to a higher estimate for matching efficiency. This happens because a higher ρ leads to faster job finding *and* lower tightness which a matching function without referrals can only interpret through higher matching efficiency.[16]

It is worth remarking that the model's equilibrium nature, and in particular the fact that the number of vacancies is endogenously determined, is crucial for these results. If, for instance, the number of vacancies were exogenous then a higher referral rate would lead to an *increase* rather than a decrease in labor market tightness which would not generate a clean prediction.

These observations are summarized in the first Prediction:

Prediction 1: The model predicts that the prevalence of referrals and the estimate of matching efficiency from a matching function without referrals are positively correlated across sectors of the economy.

Cross-industry variation in referral prevalence is used to evaluate the first prediction. The prevalence of referrals is calculated from the 1994 wave of the NLSY. The interviewees were asked which method of search led to being offered their current job and the response "contacted friends and relatives" is interpreted as evidence that a referral took place.[17] For each major industry, the proportion of interviewees who report finding their job through referral is calculated.[18]

[15]This is intuitive but not tautological: the unemployment, vacancy and job finding rates depend on ρ.

[16] A sufficient condition for Part (3) is $u \leq \beta/(\beta+\gamma(1-\beta))$ which holds so long as K is not too high. For instance, if $\beta \geq 0.2$ and K is low enough for $u \leq 0.2$ then the condition is satisfied.

[17]The other options are "contacted employer directly," "contacted public employment agency," "contacted private employment agency," "contacted school/university career center," "sent out resumes/filled applications," "placed or answered ads."

[18]The definition of major industries changed in 2000. The numbers here are consisted with the new definition.

The matching function is estimated using monthly data on industry-specific vacancies and hires from JOLTS,[19] and monthly data on industry-specific unemployed from the Current Population Survey (CPS).[20] The data covers all major industries except for agriculture from January 2001 to June 2011.

The following matching function is estimated for each industry using OLS:

$$\ln(\frac{m_{it}}{u_{it}}) \quad = \quad \ln(\hat{\mu}_i) + \hat{\eta}_i \ln(\frac{v_{it}}{u_{it}}) + \zeta_1 t + \zeta_2 t^2$$

where m_{it}, v_{it} and u_{it} are the number of hires, vacancies and unemployed workers, respectively, in industry i at time t, $\hat{\eta}_i$ and $\hat{\mu}_i$ are the industry-specific parameters of the matching function and there is a quadratic time trend (note that the data series are not de-trended).

The following table summarizes the results, with standard errors and sample size in parentheses for the estimates and proportion of referrals, respectively:[21]

[19] See Davis, Faberman and Haliwanger (2010) for a detailed description of JOLTS. Note that an establishment is counted by JOLTS as having a vacancy when it is making 'word of mouth' announcements in order to hire (Davis, Faberman and Haliwanger, 2010, p.6). Therefore there is no systematic undermeasurement of vacancies that end up being filled through a referral, at least in principle.

[20] The CPS assigns unemployed workers to the industry where they were last employed. Sahin, Song, Topa and Violante (2011) estimate the transition matrix across industries and use it to adjust where each worker searches. They find that their estimates of matching function parameters do not change appreciably.

[21] The estimates for matching efficiency are very highly correlated (correlation of 0.89) with those of Sahin, Song, Topa and Violante (2011) even though they assume that the matching function of every industry has the same elasticity. They are also very highly correlated (0.84) with the estimates of the vacancy yield in Davis, Faberman and Haltiwanger (2010).

Industry	$\hat{\eta}_i$ (std. err.)	$\log(\hat{\mu}_i)$ (std. err.)	$\hat{\mu}_i$	% referrals (N)
Mining	0.65 (0.04)	0.323 (0.109)	1.38	40.0 (25)
Construction	0.52 (0.04)	0.408 (0.089)	1.50	39.6 (338)
Manufacturing, Durables	0.59 (0.03)	-0.405 (0.069)	0.67	35.3 (534)
Manufacturing, Non-durables	0.66 (0.04)	0.062 (0.089)	1.06	36.9 (474)
Transportation and Utilities	0.45 (0.04)	-0.244 (0.075)	0.78	33.7 (300)
Wholesale Trade	0.52 (0.04)	-0.232 (0.076)	0.79	32.6 (138)
Retail Trade	0.71 (0.04)	0.400 (0.064)	1.49	27.6 (675)
FIRE	0.59 (0.05)	-0.278 (0.065)	0.76	27.3 (334)
Information	0.55 (0.03)	-0.381 (0.077)	0.68	26.5 (83)
Professional and Business Services	0.67 (0.04)	0.242 (0.049)	1.27	27.4 (343)
Leisure and Hospitality	0.62 (0.04)	0.354 (0.051)	1.42	37.9 (145)
Health	0.65 (0.05)	-0.332 (0.047)	0.72	23.8 (638)
Education	0.36 (0.06)	-0.232 (0.98)	0.79	22.6 (349)
Other services	0.42 (0.06)	-0.178 (0.079)	0.84	33.3 (285)
Government	0.63 (0.09)	-0.378 (0.078)	0.69	24.4 (357)

Table 1: Estimates for industry-specific matching functions.

Consistent with the model's prediction, the estimated efficiency parameters of the matching function are positively and significantly correlated with the proportion of referrals as can be seen in Figure 1 and from the outcome of the following regression (standard errors in parentheses):[22]

$$\begin{array}{ccc} \hat{\mu}_i = & \tau_0 + & \tau_1 PR_i \\ & 0.099(0.416) & 2.845(1.31) \end{array}$$

[22]Each industry is equally weighted. Weighting each industry by its employment share changes the estimate of τ_1 to 2.73.

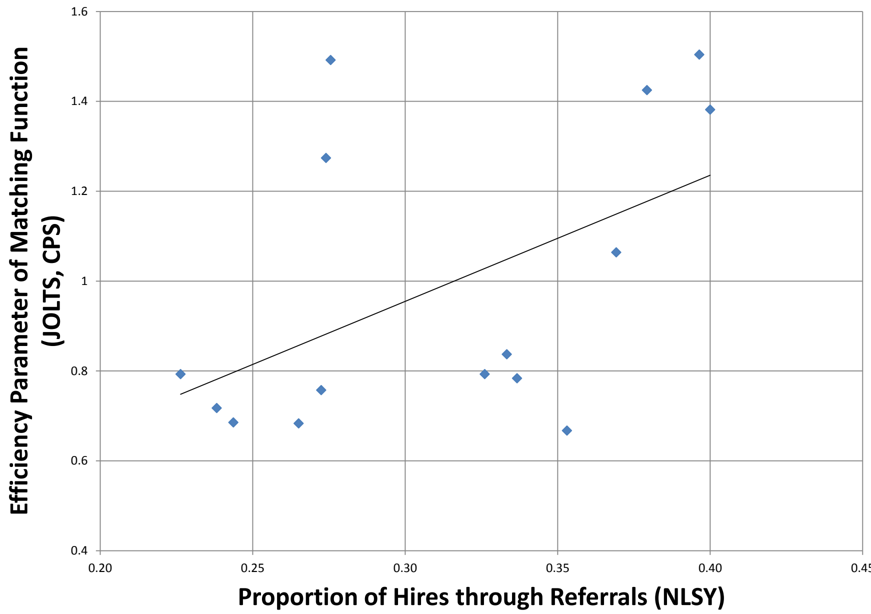

Figure 1: Estimates of matching efficiency and prevalence of referrals

where $\hat{\mu}_i$ is the estimate of matching efficiency for industry i and PR_i is the proportion of jobs in industry i that are found through a referral (the prevalence of referrals).

These estimates suggest that a significant proportion of the variation in matching efficiency across industries is associated with variation in the prevalence of referrals. A ten percentage point increase in the prevalence of referrals is associated with a increase in the efficiency parameter of 0.28 which corresponds to a 28% increase in the efficiency parameter of the average industry (the unweighted average of the $\hat{\mu}_i$'s is 0.99). Furthermore, if the referral rate in every industry is set to the overall average in the NLSY sample, 30%, and the matching efficiency is adjusted according to τ_1, then the variance of matching efficiencies across industries is reduced by a quarter.

Additionally, the estimates are informative regarding the type of modeling choices that are necessary to endogenize the prevalence of referrals. Specifically, they suggest that the use of referrals is not the response to low efficiency in the market matching function: such an assumption (a low market μ leading to a high ρ) would lead to a *negative* correlation between the aggregate matching efficiency and referral prevalence.

This exercise should nevertheless be seen as a first step for (at least) two reasons. First, the "correct" matching function (according to the model) has not been estimated. The reason is that some of the assumptions that have been made for theoretical convenience should be modified to give a good empirical fit (e.g. the assumption of instantaneous matching when a referral is used) which introduces complications that are beyond the present paper's scope. Second, if the prevalence of referrals is correlated with factors affecting other aspects of labor market matching, then the estimates are biased. Since the prevalence of referrals is the outcome of decisions by firms and workers, such correlation is likely to be present.[23] Of course, as mentioned above, the estimates still inform the modeling decisions of a model where referral prevalence is endogenous.

3.3.2 Cyclical Properties

To examine the model's cyclical properties, a comparative statics exercise is performed with respect to the unemployment rate and compared with recent evidence.

Proposition 3.3 *Conditional on labor market tightness, the number of matches is decreasing in the unemployment rate.*

Proof. See the Appendix. ■

To see the intuition for Proposition 3.3, consider how an unemployed worker's job finding rate is affected by a proportional increase in the measure of unemployed workers and vacancies. Such a change does not affect his meeting rate through the market, since labor market tightness is unchanged; however, it reduces his meeting rate through referrals, since the number of employed links that act as the source of a referral is lower. In other words,

[23] A model where referral prevalence is an endogenous outcome is developed in Galenianos (2012).

an increase in the unemployment rate reduces the effectiveness of the referral channel which corresponds to a decrease in the efficiency of the aggregate matching function. Identifying the (inverse of the) unemployment rate with the business cycle, leads to the second prediction.

Prediction 2: The efficiency of the aggregate matching function is pro-cyclical.

This prediction is consistent with the evidence presented in several recent papers that have examined the cyclical properties of matching efficiency. Barnichon and Figura (2011) estimate a matching function and calculate its residual (essentially, the efficiency of the matching function). They find that the residual exhibits cyclical regularities, increasing in the late parts of expansions and falling in the aftermath of recessions. Cheremukhin and Restrepo (2011) perform a business cycle accounting exercise of the standard search and matching model to decompose the labor wedge into several components one of which is variations in matching efficiency. They find that matching efficiency drops significantly in the aftermath of recessions. On the firm side, Davis, Faberman and Haltiwanger (2010) find that the vacancy filling rate is lower than would be predicted by a matching function with constant efficiency in weak labor markets.

These findings are consistent with Prediction 2 which provides a natural interpretation that is complementary to the explanations that have already been proposed.[24] Assessing the relative importance of referrals and the alternatives is a quantitative issue which is left for future work.

3.3.3 Further Properties

The next two predictions relate to the effects of network heterogeneity on wages and employment rates.

[24]Barnichon and Figura (2011) attribute a large part of the variation to the varying composition of the unemployment pool. Davis, Faberman and Haltiwanger (2010) propose that the variation is due to lower recruiting intensity by employers.

Prediction 3: Ceteris paribus, increasing the size of a worker's network leads to a drop in the probability that he is unemployed and an increase in his wage.

Prediction 4: Ceteris paribus, increasing the employment rate of a worker's network leads to a drop in the probability that he is unemployed and an increase in his wage.

The logic is straightforward: increasing the size of a worker's network or the employment rate of his links raise his job finding rate which reduces his unemployment probability and raises his wage by increasing his value of unemployment. These prediction are consistent with the finding of Bayer, Ross and Topa (2008) about network size and Topa (2001), Weinberg, Reagan and Yankow (2004) and Cappellari and Tatsiramos (2010) about the network's employment rate.

Predictions 3 and 4 are essentially partial equilibrium predictions due to the ceteris paribus statement, however the results hold in a fully specified model with network heterogeneity. Igarashi (2011) analyzes a similar model where workers have homogeneous productivity but heterogeneous network sizes and finds that workers who have greater access to networks enjoy higher wages and lower unemployment.

4 The Labor Market with Heterogeneous Workers

This Section introduces worker heterogeneity.

4.1 The Model

Firms are identical to Section 3. Workers are heterogeneous and each worker belongs to a high or a low type (H or L). The measure of each type is equal to one and the two types differ in terms of their productivity and their network. The different types represent heterogeneity beyond the workers' observable characteristics. The relevant modeling assumption is that a firm cannot post a type-specific vacancy and both types search for jobs in the same market.[25]

Conditional on his type, every worker has the same network. The network of a worker of type $i \in \{H, L\}$ is fully described by the measure of other workers that he is linked with, ν_i, and the proportion of these links that are with workers of his own type, ϕ_i. Consistency requires that the measure of links that high type workers have with low types is equal to the measure of links that low type workers have with high types: $\nu_H(1-\phi_H) = \nu_L(1-\phi_L)$.

The following assumption will be maintained about the network structure: a worker has more links with workers of the same type (homophily) and this is weakly more prevalent for high type workers:[26]

Assumption A: $\phi_H \geq \phi_L \geq \frac{1}{2}$.

Assumption A is not necessary to prove existence but it is sufficient to prove the uniqueness of equilibrium and facilitate its characterization.

As earlier, workers and firms meet either through referrals or through the market. When a worker and a firm meet, the match-specific productivity is drawn from a distribution that depends on the worker's type and remains constant for the duration of the match. All payoff-relevant variables (match-specific productivity, worker's type and network) become common knowledge and the pair decides whether to form the match.

More precisely, with probability p_i a worker of type i is productive and flow output is y_i; with probability $1 - p_i$ he is unproductive and the match is not formed.[27] It is assumed that $p_H > p_L$ and $y_H > y_L$ so that high type workers draw from a productivity distribution that first order stochastically dominates that of the low type workers.[28] It is also assumed that

[25]However, there is no private information: once a worker and firm meet, the worker's type becomes common knowledge.

[26]If $\phi_H = \phi_L = 1/2$, then workers are only heterogeneous in terms of their productivity but have the same networks.

[27]Alternatively, and equivalently, the worker's flow output is a large negative number when unproductive.

[28]A high-type worker is more likely to be hired when meeting a firm and has higher productivity conditional on being employed. An earlier version of this paper delivered the same qualitative results with a distribution of match-specific productivities that was continuous and log-concave and had a higher mean for the high type workers. That specification complicated the analysis without adding further insights.

$y_L > b + (r+\delta)K$ which guarantees that low type workers are hired when productive.[29]

As in Section 3, a referral occurs when some firm expands and it is sent at random to one of the incumbent worker's links. When a firm that employs a type-i worker expands, it meets a type-i worker with probability $\phi_i u_i$ and a type-k ($\neq i$) worker with probability $(1-\phi_i)u_k$, where u_i is the unemployment rate of a type-i worker. Therefore, in addition to the possibility of instantaneous matching, a referred worker is also drawn from a different pool than a random draw of unemployed workers.

Denoting the value of employing a type-i worker by J_i, the value of expanding when employing a type i worker is equal to:

$$E_i = -K + V + \phi_i u_i p_i (J_i - V) + (1-\phi_i) u_k p_k (J_k - V). \tag{8}$$

The flow value to a match between a firm and a type-i worker is $y_i + \rho\gamma E_i$.

A worker of type i is referred to a firm when the employer of one of his links expands and this worker is chosen among the referrer's links. A type i worker has $\nu_i \phi_i$ links of type i and $\nu_i(1-\phi_i)$ links of type k. Each link of type i is employed with probability $1-u_i$ and gets the opportunity to refer at rate ρ. A referrer of type i has ν_i links and each of them is equally likely to receive the referral. Therefore, our worker is referred to a job at rate

$$\begin{aligned}\alpha_{Ri} &= \frac{\rho\nu_i\phi_i(1-u_i)}{\nu_i} + \frac{\rho\nu_i(1-\phi_i)(1-u_k)}{\nu_k} \\ &= \rho\phi_i(1-u_i) + \rho(1-\phi_k)(1-u_k),\end{aligned}$$

where the consistency condition $\nu_H(1-\phi_H) = \nu_L(1-\phi_L)$ was substituted in the second term.

Three types of agents search in the same market: measure v vacancies, measure u_H high-type unemployed workers and measure u_L low-type unemployed workers.[30] The flow

[29] Introducing a probability $1-p$ that the worker is unproductive to the baseline model of Section 3 yields results that are identical to a rescaling of the parameters that determine the speed of matching to $\tilde{\mu} = \mu * p$ and $\tilde{\rho} = \rho * p$.

[30] Since there is a unit measure of each type, u_i denotes both the proportion and the measure of type-i

of meetings in the market between a vacancy and a worker of either type is given by a Cobb-Douglas function

$$m(v, u_H, u_L) = \mu v^{\eta}(u_H + u_L)^{1-\eta},$$

where $\mu > 0$ and $\eta \in (0, 1)$.

When a firm meets a worker, the worker is drawn at random from the unemployed population. The rate at which a firm meets with a type i worker through the market is

$$\alpha_{Fi} = \frac{m(v, u_H, u_L)}{v} \frac{u_i}{u_H + u_L} = \mu(\frac{u_H + u_L}{v})^{1-\eta} \frac{u_i}{u_H + u_L}.$$

The rate at which a type i worker meets a firm through the market is

$$\alpha_{Mi} = \frac{m(v, u_H, u_L)}{u_H + u_L} = \mu(\frac{v}{u_H + u_L})^{\eta}.$$

Since this rate does not depend on the worker's type, the i subscript is henceforth dropped.

The steady state conditions are that each type's flows in and out of unemployment are equal:

$$u_H(\alpha_M + \alpha_{RH})p_H = (1 - u_H)\delta, \tag{9}$$

$$u_L(\alpha_M + \alpha_{RL})p_L = (1 - u_L)\delta. \tag{10}$$

The agents' value functions are now described. Consider a firm. When vacant, it searches in the market and meets with a type-i worker at rate α_{Fi}. With probability p_i the worker is productive and the match is formed. When producing, the firm's flow payoffs are $y_i + \rho\gamma E_i - w_i$ where w_i denotes the wage. The match is destroyed at rate δ. The firm's value of a vacancy

unemployed.

(V) and production with a type-i worker (J_i) are given by:

$$\begin{aligned} rV &= \alpha_{FH} p_H (J_H - V) + \alpha_{FL} p_L (J_L - V). \\ rJ_i &= y_i + \rho\gamma E_i - w_i - \delta J_i. \end{aligned}$$

Consider a worker of type i. When unemployed his flow utility is b. Job opportunities appear at rate $\alpha_M + \alpha_{Ri}$ and a match is formed with probability p_i. When employed, the worker's flow utility is equal to the wage and the match is destroyed at rate δ. The worker's value of unemployment (U_i) and employment (W_i) are given by:

$$\begin{aligned} rU_i &= b + (\alpha_M + \alpha_{Ri}) p_i (W_i - U_i). \\ rW_i &= w_i + \delta(U_i - W_i). \end{aligned}$$

The wage solves the Nash bargaining problem

$$w_i = \text{argmax}_w (W_i - U_i)^\beta (J_i - V)^{1-\beta}. \tag{11}$$

The Equilibrium is defined as follows.

Definition 4.1 *An Equilibrium is the steady state unemployment levels* $\{u_H, u_L\}$ *and the number of vacancies* v *such that:*

- *The labor market is in steady state as described in* (9) *and* (10).
- *The surplus is split according to* (11).
- *There is free entry of firms:* $V = K$.

4.2 Labor Market Equilibrium

This Section's analysis mirrors Section 3.2.

The following lemmata characterize the steady state labor market flows. Although these results are conceptually straightforward, they are non-trivial to prove as shown in the Appendix. The source of the complication is that the unemployment rates for the two worker types are implicitly defined by equations (9) and (10) and one type's unemployment rate affects the other's meeting rate through both the referral and the market channel. Therefore a change in v affects u_H both directly, through the steady state condition of high-type workers, and indirectly, through its effect on u_L.

Lemma 4.1 *In steady state, the unemployment rates for the two worker types $\{u_H, u_L\}$ are uniquely determined given any number of vacancies, v. Furthermore, the unemployment rate of both types is monotonically decreasing in v.*

The unemployment rate for the two types is characterized as follows:

Lemma 4.2 *The high productivity workers have lower unemployment rates than the low types in a steady state ($u_H < u_L$) under Assumption A.*

The rate at which a firm contacts workers is characterized as follows:

Lemma 4.3 *If $p_H \leq 3/4$, $1 - \eta - \eta^2 \geq 0$ and Assumption A holds then in steady state the rate at which a firm meets with a type i worker (α_{Fi}) is decreasing in v.*

Lemma 4.3 requires a restriction on η because a change in v affects both the rate at which a vacancy meets some unemployed worker, $\mu(\frac{u_H+u_L}{v})^{1-\eta}$, and the proportion of type-i workers in the unemployment pool, $\frac{u_i}{u_H+u_L}$. When η is high, the level of v affects the flow of matches less and, therefore, the change in the proportion plays a more important role. In the extreme, when $\eta = 1$ the arrival rate of a certain type only depends on that type's proportion in the unemployed population and, consequently, if α_{Fi} is decreasing in v then α_{Fk} must be increasing in v.

As a result, η needs to be bounded away from 1 for the (reasonable) requirement that the worker-meeting rate is declining in v. The bound derived in Lemma 4.3 is equivalent to $\eta \leq 0.62$. Empirically, the coefficient on vacancies has been estimated to be 0.28 by Shimer (2005) while Petrongolo and Pissarides (2001) report values between 0.3-0.5 in their survey of the matching function which suggests that the upper bound is not very restrictive.[31]

The surplus of a match between a firm and a type-i worker is given by $S_i = W_i - U_i + J_i - V$. Nash bargaining implies that

$$\begin{aligned} W_i - U_i &= \beta S_i, \\ J_i - V &= (1-\beta) S_i, \end{aligned}$$

and the value functions can be rearranged to yield

$$(r+\delta)S_i = y_i + \rho\gamma E_i - b - (\alpha_M + \alpha_{Ri})\beta S_i - (r+\delta)V.$$

Combine the above with equation (8) and the free entry condition to arrive at:

$$S_i = \frac{y_i - b - (r+\delta)K + \rho\gamma(1-\beta)(1-\phi_i)u_k p_k \ S_k}{r+\delta+(\alpha_M+\alpha_{Ri})p_i\beta - \rho\gamma(1-\beta)\phi_i u_i p_i}. \tag{12}$$

Equation (12) illustrates that the dependence between S_i and S_k is due to the fact that a type-i worker may refer a type-k in the case of an expansion. If $\phi_i = 1$ then i types only refer workers of the same type and the term multiplying S_k drops out.

The value of a vacancy is given by

$$rV = \alpha_{FH} p_H (1-\beta) S_H + \alpha_{FL} p_L (1-\beta) S_L \tag{13}$$

[31] Note, however, that the estimates of η using JOLTS are significantly higher than what the previous literature has found.

The following proposition states the main result.

Proposition 4.1 *An equilibrium exists. The equilibrium is unique if* $p_H \leq 3/4$, $1-\eta-\eta^2 \geq 0$ *and Assumption A holds.*

Proof. See the Appendix. ■

4.3 Testable Predictions and Evidence

The extended model's predictions are presented and compared with empirical evidence reported in the literature.

In the model with worker heterogeneity, referrals help firms find high type workers, in addition to mitigating search frictions. The intuition is quite straightforward. High productivity workers are more likely to be employed at any point in time and therefore they are more likely to refer a member of their network. The assumption of homophily ($\phi_H \geq \frac{1}{2}$) implies that the recipients of these referrals are more likely to be other high-type workers. Formally:

Proposition 4.2 *When a firm and a worker meet, it is more likely that the worker is of high type if the meeting is through a referral rather than through the market if Assumption A holds.*

Proof. See the Appendix. ■

Proposition 4.2 leads to the following predictions:

Prediction 5: When a worker and a firm meet, the match is more likely to be formed if they meet through a referral.

Prediction 6: When a worker and a firm meet, the match is more productive in expectation if they meet through a referral.

Prediction 7: When a worker and a firm meet, the wage is higher in expectation if they meet through a referral.

There is ample evidence supporting the above predictions as detailed in Section 2. In their firm-level studies Fernandez and Weinberg (1997), Castilla (2005) and Brown, Setren and Topa (2011) examine all job applicants and find evidence that supports prediction 4. Regarding prediction 5, Castilla (2005) and Pinkston (2011) find that, conditional on being hired, referred candidates have higher productivity while Bayer, Ross and Topa (2011) and Brown, Setren and Topa (2011) find that referred candidates receive higher wages, after controlling for worker observables and firm fixed effects.

In Section 3, where workers are homogeneous, every worker receives the same wage. In Section 4, where they are heterogeneous, referred workers receive a higher wage on average. Therefore the relation between a worker's wage and the channel through which he found his job crucially depends on whether heterogeneity is an important feature or not which leads to the final Prediction.

Prediction 8: A referred worker receives a higher wage only in markets where worker heterogeneity beyond observables is an important feature.

A corollary is that including workers from both types of markets in a wage regression might lead to a zero effect on wages for finding a job through a referral even if this is positive for some workers (of course, a negative estimate can never be had from the present model). One could try to distinguish between the two cases by, for instance, using a measure of the job's complexity as a proxy for the importance of heterogeneity and having a dummy on the *interaction* between a referral and that proxy.

5 Conclusions

The aim of this paper is to combine social networks, which have long been recognized as an important feature of labor markets, with the equilibrium models that are used to study labor markets. This is achieved in a tractable theoretical framework which, despite its simplicity, is consistent with a large number of empirical findings and yields novel testable predictions.

Of particular interest is the prediction that variation in the prevalence of referrals is a source of variation in the speed of matching. This is relevant in light of the finding in Davis, Faberman and Haltiwanger (2010) that there is significant variation in the speed with which vacancies are filled across different industries and, especially, that this variation cannot be explained by differences in the tightness across industries. Section 3.3.1 provides empirical support for the model's prediction and suggests that an important component of the frictions that the aggregate matching function represents are associated with referrals. An exploration of the source of cross-industry variation in the prevalence of referrals is left for future work.

A related question is studied in a companion paper (Galenianos, 2012): what type of firm chooses to hire through the market or a referral. In that paper, referrals alleviate a learning friction by facilitating the hiring of workers with better match quality. An important implication of that model is that larger firms use referrals to a lower extent because they have alternative methods for alleviating that friction (e.g. better human resource departments) which is consistent with the evidence in Dustmann, Glitz and Schoenberg (2011) and many other papers.

A further avenue for future work is to introduce social networks in the study of individuals' migration decisions. There is ample evidence to suggest that social networks affect these decisions. For instance, Munshi (2003) finds that Mexican migrants are more likely to move to locations with more people from their region of origin and this helps them with finding employment while Belot and Ermisch (2009) show that an individual is less likely to move if he has more friends at his current location.[32] Therefore, it seems natural to combine the decision to migrate with an explicit model of how the social network helps a worker to find a job.

Finally, this paper's focus is on the positive implications of combining social networks and labor market models. Having provided a theoretical framework, one can move towards asking normative questions. A first step is taken in Igarashi (2011) who studies the effect of banning referrals in a market where some workers have no access to networks. Surprisingly, he finds that non-networked workers might become worse off even though they have no direct access to referrals.

[32]Belot and Ermisch (2009) focus on the number of close friends and they interpret their findings to reflect the intrinsic value of friendship. To the extent that the number of one's close friends in some location reflects the overall ties to that location, close friends can be used as a proxy for the overall number of one's contacts.

6 Appendix

Proposition 3.2: An increase in the rate of generating referrals (ρ) leads to:

1. An increase in the proportion of jobs that are found through a referral.

2. An increase in the job finding rate

3. A decrease in labor market tightness if $K \leq \bar{K}$, where $\bar{K}$ is a function of the model's parameters.

Proof. We start with the effect of ρ on the equilibrium level of unemployment. The steady state condition implies that

$$\frac{v}{u} = \frac{1}{\mu^{1/\eta}}[(1-u)(\frac{\delta}{u}-\rho)]^{1/\eta}$$

The free entry condition can therefore be rearranged as follows:

$$\frac{(y-b-(r+\delta)K)(1-\beta)}{rK} = \frac{1}{\mu^{1/\eta}}[(1-u)(\frac{\delta}{u}-\rho)]^{\frac{1-\eta}{\eta}}(r+\delta+\frac{(1-u)\delta\beta}{u}-\rho\gamma(1-\beta)u).$$

In this expression u is the only endogenous variable and the steady state condition determines a unique number of vacancies for each u. In equilibrium:

$$\begin{aligned} Q(\rho,u) &= C, \quad \text{where} \\ Q(\rho,u) &\equiv [(1-u)(\frac{\delta}{u}-\rho)]^{\frac{1-\eta}{\eta}}[r+(1-\beta)\delta+\frac{\delta\beta}{u}-\rho\gamma(1-\beta)u] \\ C &\equiv \frac{[y-b-(r+\delta)K](1-\beta)\mu^{1/\eta}}{rK}. \end{aligned}$$

It is easy to verify that Q is decreasing both in u and in ρ and the implicit function theorem implies:

$$\frac{du}{d\rho} = -\frac{\partial Q/\partial\rho}{\partial Q/\partial u} < 0$$

To sustain a lower level of unemployment, the steady state condition implies that the flow of matches is higher, which proves (2).

The proportion of matches that occur through referrals is given by:

$$P_R \equiv \frac{\alpha_R}{\alpha_R + \alpha_M} = \frac{(1-u)\rho}{(1-u)\delta/u} = \frac{u\rho}{\delta}$$

where the steady state condition was used. Therefore:

$$\frac{dP_R}{d\rho} = \frac{1}{\delta}(\frac{du}{d\rho}\rho + u) = \frac{1}{\delta \partial Q/\partial u}(-\frac{\partial Q}{\partial \rho}\rho + \frac{\partial Q}{\partial u}u)$$

which is positive if

$$\begin{aligned}
\frac{\partial Q}{\partial \rho}\rho &> \frac{\partial Q}{\partial u}u \\
(\frac{1-\eta}{\eta}\frac{\hat{Q}}{\delta/u-\rho} + \gamma(1-\beta)u)\rho &< [\frac{1-\eta}{\eta}\frac{\hat{Q}}{1-u} + \frac{1-\eta}{\eta}\frac{\hat{Q}}{\delta/u-\rho}\frac{\delta}{u^2} + \frac{\delta}{u^2} + \rho\gamma(1-\beta)]u \\
0 &< \frac{1-\eta}{\eta}\frac{\hat{Q}}{1-u} + \frac{1-\eta}{\eta}\frac{u\hat{Q}}{\delta/u-\rho}(\frac{\delta}{u} - \rho) + \frac{\delta}{u^2} \\
\text{where } \hat{Q} &= r + (1-\beta)\delta + \frac{\beta\delta}{u} - \rho\gamma(1-\beta)u > 0
\end{aligned}$$

which proves (1).

To find how a change in ρ affects v/u start with:

$$\frac{d(v/u)}{d\rho} = \frac{[(1-u)(\delta/u-\rho)]^{1/\eta-1}}{\eta\mu^{1/\eta}}[-\frac{du}{d\rho}(\frac{\delta}{u^2} - \rho) - (1-u)],$$

which implies

$$\frac{d(v/u)}{d\rho} < 0 \Leftrightarrow -\frac{du}{d\rho} < \frac{1-u}{\delta/u^2 - \rho}. \tag{14}$$

It is straightforward (though tedious) to use the implicit function theorem and arrive at:

$$\frac{du}{d\rho} = -\frac{1+\gamma(1-\beta)u(\delta/u-\rho)/\Xi}{(\delta/u^2-\rho)/(1-u)+(\rho\gamma(1-\beta)+\beta\delta/u^2)(\delta/u-\rho)/\Xi} \tag{15}$$

where $\Xi = \frac{1-\eta}{\eta}[r+\delta(1-\beta)-\rho\gamma(1-\beta)u+\frac{\beta\delta}{u}]$.

Combining (15) with (14) and going through the algebra yields:

$$\frac{d(v/u)}{d\rho} < 0 \Leftrightarrow$$

$$\rho\gamma(1-\beta)+\frac{\delta}{u^2}[\beta-u(\beta+\gamma(1-\beta))] > 0$$

A sufficient condition for this inequality to hold is $u \leq \hat{u} \equiv \beta/[\beta+(1-\beta)\gamma]$ which is mentioned in footnote 16. A necessary and sufficient condition is:

$$G(u) = u^2\rho\gamma(1-\beta) - u\delta(\beta+\gamma(1-\beta)) + \delta\beta > 0$$

Note that $G(u) > 0$ and $G(1) < 0$ which means that there is a unique $\bar{u}$ such that $u \leq \bar{u} \Rightarrow G(u) \geq 0$. Furthermore:

$$\bar{u} = \frac{\delta}{2\rho}\left(\frac{\beta}{\gamma(1-\beta)}+1-\sqrt{\left(\frac{\beta}{\gamma(1-\beta)}+1\right)^2-\frac{4\beta\rho}{\delta\gamma(1-\beta)}}\right)$$

Recall that the equilibrium is determined by $Q(\rho,u)=C$ with $\lim_{u\to 0} Q(\rho,0) = +\infty > C$, $Q(\rho,1) = 0 < C$ and $\partial Q/\partial u < 0$. Therefore:

$$u \leq \bar{u} \Leftrightarrow Q(\rho,\bar{u}) \leq C \Leftrightarrow K \leq \bar{K}$$

$$\text{where} \quad \bar{K} = \frac{y-b}{rQ(\rho,\bar{u})+(r+\delta)(1-\beta)\mu^{1/\eta}}$$

Therefore (3) holds if $K \leq \bar{K}$. ■

Proposition 3.3: The number of matches is decreasing in the unemployment rate conditional on labor market tightness.

Proof. Consider the effect an increase in the measure of unemployed workers and vacancies by a factor $\xi > 1$ on the aggregate matching function (equation (2)):

$$\begin{aligned}\mathcal{M}(\xi v, \xi u) &= \mu(\xi v)^{\eta}(\xi u)^{1-\eta} + \rho(\xi u)(1-(\xi u)) \\ &= \xi[\mu v^{\eta} u^{1-\eta} + \rho u(1-u)] + \xi(1-\xi)\rho u^2\end{aligned}$$

The last term is strictly decreasing in u which proves the result. ■

Lemma 4.1: In steady state, the unemployment rates for the two worker types $\{u_H, u_L\}$ are uniquely determined given any number of vacancies, v. Furthermore, the unemployment rate of both types is monotonically decreasing in v.

Proof. Define

$$\begin{aligned}H(v, u_H, u_L) &\equiv u_H \mu(\frac{v}{u_H + u_L})^{\eta} + u_H \rho(\phi_H(1-u_H) + (1-\phi_L)(1-u_L)) - \frac{\delta}{p_H}(1-u_H) \\ L(v, u_H, u_L) &\equiv u_L \mu(\frac{v}{u_H + u_L})^{\eta} + u_L \rho(\phi_L(1-u_L) + (1-\phi_H)(1-u_H)) - \frac{\delta}{p_L}(1-u_L)\end{aligned}$$

and note that in a steady state $H(v, u_H, u_L) = L(v, u_H, u_L) = 0$ holds. From now on, let $H_x(v, u_H, u_L) \equiv \partial H(v, u_H, u_L)/\partial x$ where $x \in \{v, u_H, u_L\}$, and similarly for $L(v, u_H, u_L)$. Define $h^H(v, u_L)$ and $h^L(v, u_L)$ to be the set of $\{u_H\}$ that satisfy $H(v, u_H, u_L) = 0$ and $L(v, u_H, u_L) = 0$, respectively, for every $v > 0$ and $u_L \in [0, 1]$.

The proof proceeds by showing that (1) $h^H(v, u_L)$ and $h^L(v, u_L)$ include at most one point for any given (v, u_L) (i.e. they are functions); (2) they are strictly increasing in u_L and strictly decreasing in v; (3) for every $v > 0$ there is a unique $u_L(v) \in (0, 1)$ such that $h^H(v, u_L(v)) = h^L(v, u_L(v)) \equiv h(v, u_L(v))$ and $h(v, u_L(v)) \in (0, 1)$; (4) $h(v, u_L(v))$ and $u_L(v)$ are decreasing in v. The steady state unemployment levels for high and low type workers are then given by $h(v, u_L(v))$ and $u_L(v)$, respectively.

Observe that

$$\begin{aligned}
H(v,0,u_L) &= -\frac{\delta}{p_H} < 0 \\
H(v,1,u_L) &= \mu(\frac{v}{1+u_L})^\eta + \rho(1-\phi_L)(1-u_L) > 0 \\
H_{u_H}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^\eta(1-\frac{\eta u_H}{u_H+u_L}) + \rho(\phi_H(1-u_H)+(1-\phi_L)(1-u_L)) \\
&\quad +\frac{\delta}{p_H} - u_H\rho\phi_H > 0
\end{aligned}$$

The above equations imply that $h^H(v, u_L)$ is uniquely defined and belongs to $(0, 1)$ given any $v > 0$ and $u_L \in [0, 1]$. Furthermore,

$$\begin{aligned}
H_{u_L}(v,u_H,u_L) &= -\frac{\eta u_H}{u_H+u_L}\mu(\frac{v}{u_H+u_L})^\eta - u_H\rho(1-\phi_L) < 0 \\
H_v(v,u_H,u_L) &= \frac{\eta u_H}{v}\mu(\frac{v}{u_H+u_L})^\eta > 0
\end{aligned}$$

Therefore $h^H(v, u_L)$ is strictly increasing in u_L and strictly decreasing in v.

Turning to $h^L(v, u_L)$, note that

$$\begin{aligned}
L(v,u_H,1) &= \mu(\frac{v}{u_H+1})^\eta + \rho(1-\phi_H)(1-u_H) > 0, \quad \forall\ u_H \in [0,1] \\
L(v,u_H,0) &= -\frac{\delta}{p_L} < 0, \quad \forall\ u_H \in [0,1] \\
L_{u_L}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^\eta\frac{u_H+(1-\eta)u_L}{u_H+u_L} + \rho(\phi_L(1-u_L)+(1-\phi_H)(1-u_H)) \\
&\quad +\frac{\delta}{p_L} - u_L\rho\phi_L > 0
\end{aligned}$$

The first equation shows that $L(v, u_H, u_L) = 0$ has no solution for u_L "close enough" to 1. The second equation shows that $L(v, u_H, u_L) = 0$ has no solution for u_L "close enough" to 0. The third equation implies that a solution to $L(v, u_H, u_L) = 0$ with $u_H \in [0, 1]$ only exists if $u_L \in [\underline{u}_L(v), \overline{u}_L(v)]$ where $\underline{u}_L(v) > 0$ and $\overline{u}_L(v) < 1$.

Furthermore,

$$L_{u_H}(v,u_H,u_L) \;=\; -\frac{\eta u_L}{u_H+u_L}\mu(\frac{v}{u_H+u_L})^{\eta} - u_L\rho(1-\phi_H) < 0$$

implies $h^L(v,\underline{u}_L(v)) = 0$, $h^L(v,\overline{u}_L(v)) = 1$ and $0 < \underline{u}_L(v) < \overline{u}_L(v) < 1$.

To complete the analysis of $h^L(v,u_L)$, note that $L_{u_L}(v,u_H,u_L) > 0 > L_{u_H}(v,u_H,u_L)$ and

$$L_v(v,u_H,u_L) \;=\; \frac{\eta u_L}{v}\mu(\frac{v}{u_H+u_L})^{\eta} > 0$$

imply that given any $v > 0$ and $u_L \in [\underline{u}_L(v),\overline{u}_L(v)]$, $h^L(v,u_L)$ is uniquely defined and is strictly decreasing in v and strictly increasing in u_L.

The next step is to examine the intersection of $h^H(v,u_L)$ and $h^L(v,u_L)$. Observing that $h^L(v,\underline{u}_L(v)) = 0 < h^H(\underline{u}_L(v))$ and $h^L(v,\overline{u}_L(v)) = 1 > h^H(\overline{u}_L)$ implies that there is some $u_L(v) \in (0,1)$ such that $h^H(v,u_L(v)) = h^L(v,u_L(v))$. To show that the intersection is unique it suffices to show

$$\begin{aligned}
\frac{\partial h^H(v,u_L)}{\partial u_L} &< \frac{\partial h^L(v,u_L)}{\partial u_L} \\
&\Leftrightarrow \\
-\frac{H_{u_L}(v,u_H,u_L)}{H_{u_H}(v,u_H,u_L)} &< -\frac{L_{u_L}(v,u_H,u_L)}{L_{u_H}(v,u_H,u_L)}
\end{aligned}$$

Noting that

$$\begin{aligned}
L_{u_L}(v,u_H,u_L) + H_{u_L}(v,u_H,u_L) \;=\;& \mu(\frac{v}{u_H+u_L})^{\eta}(\frac{(1-\eta)(u_H+u_L)}{u_H+u_L}) + \rho\phi_L(1-u_L) \\
&+\rho(1-\phi_H)(1-u_H) + \frac{\delta}{p_L} - \rho(u_L\phi_L + (1-\phi_L)u_H) > 0
\end{aligned}$$

and

$$
\begin{aligned}
H_{u_H}(v,u_H,u_L)+L_{u_H}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^{\eta}(\frac{(1-\eta)(u_H+u_L)}{u_H+u_L})+\rho\phi_H(1-u_H)\\
&\quad +\rho(1-\phi_L)(1-u_L)+\frac{\delta}{p_H}-\rho(u_H\phi_H+(1-\phi_H)u_L)>0
\end{aligned}
$$

proves that the intersection is unique.

Finally, $H_v(v,u_H,u_L)>0$ and $L_v(v,u_H,u_L)>0$ imply that the steady state u_H and u_L decrease in v. ■

Lemma 4.2: If $\phi_H \geq \phi_L$ then the high productivity workers have lower unemployment rates than the low types in a steady state ($u_H < u_L$).

Proof. The aim is to prove that $u_L(v) > h(v,u_L(v))$. Define f^H and f^L by $h^H(v,f^H)=f^H$ and $h^H(v,f^L)=f^L$ (of course, f^H and f^L depend on v but since v will be kept constant throughout this proof this is omitted for notational brevity). Let $T^H(v,u)\equiv H(v,u,u)$ and $T^L \equiv L(v,u,u)$ and note that $T^i(v,u)=0 \Leftrightarrow u=f^i$. The proof's steps are to prove that (1) f^H and f^L are uniquely defined; (2) $f^H<f^L \Leftrightarrow h(v,u_L(v))<u_L(v)$; (3) $\phi_H\geq\phi_L\geq 1/2$ suffices for $f^H<f^L$.

The following proves that f^i exists and is unique:

$$
\begin{aligned}
T^i(v,u) &= \mu u^{1-\eta}(\frac{v}{2})^{\eta}+u(1-u)\rho(\phi_i+1-\phi_k)-\frac{\delta}{p_i}(1-u)\\
T^i(v,0) &= -\frac{\delta}{p_i}<0\\
T^i(v,1) &= \mu(\frac{v}{2})^{\eta}>0\\
\frac{\partial T^i(v,u)}{\partial u} &= (1-\eta)\mu(\frac{v}{2u})^{\eta}-u\rho(\phi_i+1-\phi_k)+\frac{\delta}{p_i}>0
\end{aligned}
$$

Define $\overline{f}=\max\{f^H,f^L\}$ and $\underline{f}=\min\{f^H,f^L\}$. Recall that $h^H(v,0)>0$ and therefore $u_L<f^H \Leftrightarrow h^H(v,u_L)>u_L$. Similarly, $h^L(v,\underline{u}_L(v))=0<\underline{u}_L(v)$ implies $u_L<f^L \Leftrightarrow h^L(v,u_L)<u_L$.

In steady state $u_L(v) \in [\underline{f}, \overline{f}]$ necessarily holds because $u_L < \underline{f} \Rightarrow h^H(v, u_L) > h^L(v, u_L)$ and $u_L > \overline{f} \Rightarrow h^H(v, u_L) < h^L(v, u_L)$. If $f^L < f^H$ then the intersection between $h^H(u_L)$ and $h^L(u_L)$ occurs above the 45 degree which implies that $h(v, u_L) > u_L$; and the opposite happens if $f^L > f^H$. It has been shown that $f^H < f^L \Leftrightarrow u_H < u_L$.

Perform the following monotonic transformation: $\tilde{T}^i(u) = \frac{T^i(u)}{1-u}$ which preserves $T^i(u) = 0 \Leftrightarrow \tilde{T}^i(u) = 0$ and therefore $\tilde{T}^i(f^i) = 0$. Note that $0 > \tilde{T}^H(0) = -\delta/p_H > -\delta/p_L = \tilde{T}^L(0)$. For $f^H < f^L$ it is necessary that $\tilde{T}^L$ "overtakes" $\tilde{T}^H$ before the latter reaches zero. To examine whether this happens define

$$\begin{aligned} \Delta T(u) &\equiv \tilde{T}^H(u) - \tilde{T}^L(u) \\ &= 2u\rho(\phi_H - \phi_L) - \frac{\delta}{p_H} + \frac{\delta}{p_L} \end{aligned}$$

and note that $\phi_H \geq \phi_L$ implies that $f^H < f^L$. ■

Lemma 4.3: If $\phi_H \geq \phi_L \geq 1/2$, $p_H \leq 3/4$ and $1 - \eta - \eta^2 \geq 0$ then in steady state the rate at which a firm meets with a type i worker (α_{Fi}) is decreasing in v.

Proof. To prove that the rate at which firms meet workers of type i decreases in v recall that:

$$\begin{aligned} \alpha_{Fi} &= u_i(u_H + u_L)^{-\eta} v^{-1+\eta} \\ \frac{d\alpha_{Fi}}{dv} &= v^{-1+\eta}(u_H + u_L)^{-\eta-1}[(1-\eta)u_i(\frac{du_i}{dv} - \frac{u_H + u_L}{v}) + \frac{du_i}{dv}u_k - \eta\frac{du_k}{dv}u_i] \end{aligned} \tag{16}$$

In steady state u_H and u_L are defined by $H(v, u_H, u_L) = 0$ and $L(v, u_H, u_L) = 0$ which defines an implicit system of two equations and two unknowns. Using the implicit function theorem yields

$$\begin{aligned}
\frac{du_H}{dv} &= -\frac{L_{u_L}H_v - H_{u_L}L_v}{Det} \\
\frac{du_L}{dv} &= -\frac{H_{u_H}L_v - L_{u_H}H_v}{Det} \\
Det &= H_{u_H}L_{u_L} - H_{u_L}L_{u_H}
\end{aligned}$$

Equation (16) for α_{FL} can be rewritten as:

$$\begin{aligned}
\frac{d\alpha_{FL}}{dv} &= -\kappa\Bigg[(1-\eta)u_L\{\eta\alpha_M u_L H_{u_H} - \eta\alpha_M u_H L_{u_H} + (u_H+u_L)(H_{u_H}L_{u_L} - H_{u_L}L_{u_H})\} \\
&\quad + \eta\alpha_M(u_H u_L H_{u_H} - u_H^2 L_{u_H}) - \eta^2\alpha_M(u_L u_H L_{u_L} - u_L^2 H_{u_L})\Bigg] \qquad (17)
\end{aligned}$$

where

$$\kappa = \frac{v^{-1+\eta}(u_H+u_L)^{-\eta-1}}{v\ Det}$$

To prove that equation (17) is negative, it suffices to show that the term in the square brackets is positive. Label that term B_L.

Expanding the term $(u_H+u_L)(H_{u_H}L_{u_L} - H_{u_L}L_{u_H})$, B_L can be written as:

$$\begin{aligned}
B_L &= (1-\eta)u_L\{\eta\alpha_M u_L H_{u_H} - \eta\alpha_M u_H L_{u_H} + \alpha_M u_H L_{u_L} - \alpha_M\eta u_H L_{u_L} + \alpha_M u_L L_{u_L} \\
&\quad + \alpha_M(u_H + u_L(1-\eta))(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H) + (u_H+u_L)(\frac{\delta}{p_H} + \alpha_{RH} - u_H\rho\phi_H)(\frac{\delta}{p_L} \\
&\quad + \alpha_{RL} - \rho\phi_L u_L) + \alpha_M\eta u_H L_{u_H} - \alpha_M\eta u_H\rho u_L(1-\phi_L) - (u_H+u_L)\rho u_H(1-\phi_L)\rho u_L(1-\phi_H)\} \\
&\quad + \eta\alpha_M(u_H u_L H_{u_H} - u_H^2 L_{u_H}) - \alpha_M\eta^2 u_L u_H L_{u_L} + \alpha_M\eta^2 u_L^2 H_{u_L}
\end{aligned}$$

It will prove helpful to group terms as follows:

$$\begin{aligned}
B_{L1} &= (1-\eta)u_L\{\eta\alpha_M u_L H_{u_H} + \alpha_M(u_H + u_L(1-\eta))(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H)\} + \eta\alpha_M u_H u_L H_{u_H} \\
&= \alpha_M u_L\{H_{u_H}\eta((1-\eta)u_L + u_H) + (1-\eta)(u_H + u_L(1-\eta))(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H)\} \\
&= \alpha_M u_L(u_H + (1-\eta)u_L)\{\frac{\eta\alpha_M(u_H(1-\eta)+u_L)}{u_H+u_L} + (\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H)\} \\
B_{L2} &= (1-\eta)\alpha_M u_L L_{u_L}(u_L - \eta u_H) > 0 \\
B_{L3} &= -(1-\eta)\eta u_L^2\alpha_M\rho u_H(1-\phi_L) + \eta\alpha_M(-u_H^2 L_{u_H} + \eta u_L^2 H_{u_L}) \\
&= \alpha_M\eta u_H^2 u_L[\frac{\alpha_M\eta}{u_H+u_L} + \rho(1-\phi_H)] - \alpha_M\eta u_L^2 u_H[\frac{\alpha_M\eta^2}{u_H+u_L} + \rho(1-\phi_L)] \\
B_{L4} &= (1-\eta)u_L\{-\eta\alpha_M u_H L_{u_H} + L_{u_H}\eta u_H\alpha_M\} = 0 \\
B_{L5} &= (1-\eta)u_L(u_H+u_L)\{(\alpha_{RH} + \frac{\delta}{p_H} - \rho\phi_H u_H)(\frac{\delta}{p_L} + \alpha_{RL} - \rho\phi_L u_L) \\
&\quad -\rho u_L(1-\phi_H)\rho u_H(1-\phi_L)\} \\
B_{L6} &= \alpha_M u_H u_L L_{u_L}(1-\eta) - \alpha_M u_H u_L L_{u_L}\alpha_M\eta^2 = (1-\eta-\eta^2)\alpha_M u_H u_L L_{u_L} \geq 0
\end{aligned}$$

where $B_L = B_{L1} + B_{L2} + B_{L3} + B_{L4} + B_{L5} + B_{L6}$. Note that $B_{L5} > 0$ since $\alpha_{RH} + \frac{\delta}{p_H} - u_H\rho\phi_H - \rho(1-\phi_H)u_H = \alpha_{RH} + \frac{\delta}{p_H} - \rho u_H > 0$ (and similarly for the L-terms). Furthermore $B_{L1} > 0$, $B_{L2} > 0$, $B_{L6} \geq 0$ and $B_{L4} = 0$.

Combine B_{L1} with B_{L3} so that $B_{La} + B_{Lb} = B_{L1} + B_{L3}$:

$$\begin{aligned}
B_{La} &= \frac{\alpha_M^2\eta u_L}{u_H+u_L}[(u_H + u_L(1-\eta))(u_H(1-\eta)+u_L) + u_H^2\eta - \eta^2 u_H u_L] > 0 \\
B_{Lb} &= \alpha_M u_L[(u_H + (1-\eta)u_L)(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H) + \eta u_H^2\rho(1-\phi_H) - \eta u_L u_H\rho(1-\phi_L)] \\
&= \alpha_M u_L[(1-\eta)u_L(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H) + \eta u_H^2\rho(1-\phi_H) \\
&\quad + u_H(\frac{\delta}{p_H} + \alpha_{RH} - \rho\phi_H u_H - \eta u_L\rho(1-\phi_L))]
\end{aligned}$$

A sufficient condition for $B_{Lb} > 0$ is $\delta \geq \rho(\phi_H + \eta(1-\phi_L))p_H$. Note that $\rho(\phi_H+\eta(1-\phi_L))p_H > \rho\frac{4}{3}p_H$. Therefore $\phi_H \geq \phi_L \geq 1/2$, $p_H \leq \frac{3}{4}$ and $1-\eta-\eta^2 \geq 0$ suffice for $\frac{d\alpha_{FL}}{dv} < 0$.

The grouping of terms and resulting calculations for α_{FH} yield a weaker condition, are very similar and are omitted (but are available upon request). ■

Proposition 4.1: An equilibrium exists. The equilibrium is unique if $\phi_H \geq \phi_L \geq 1/2$, $p_H \leq 3/4$ and $1 - \eta - \eta^2 \geq 0$.

Proof. The S_i's can be expressed as follows:

$$S_H = \frac{D_{H2} + D_{L2}D_{H3}/D_{L1}}{D_{H1} - D_{H3}D_{L3}/D_{L1}},$$

$$S_L = \frac{D_{L2} + D_{H2}D_{L3}/D_{H1}}{D_{L1} - D_{L3}D_{H3}/D_{H1}},$$

where

$$\begin{aligned} D_{i1} &= r + \delta + (\alpha_M + \alpha_{Ri})p_i\beta - \rho\gamma(1-\beta)\phi_i u_i p_i, \\ D_{i2} &= y_i - b - (r+\delta)K, \\ D_{i3} &= \rho\gamma(1-\beta)(1-\phi_i)u_k p_k. \end{aligned}$$

Note that an increase in v leads to an increase D_{i1} and a fall in D_{i3} and so S_i is decreasing in v. The value of a vacancy is given by:

$$rV = \alpha_{FH}(1-\beta)S_H + \alpha_{FL}(1-\beta)S_L \tag{18}$$

The steady state equations imply that

$$\begin{aligned} v \to 0 &\Rightarrow (u_H, u_L) \to (1,1) \Rightarrow \alpha_{Fi} \to \infty, \\ v \to \infty &\Rightarrow (u_H, u_L) \to (0,0) \Rightarrow \alpha_{Fi} \to 0. \end{aligned}$$

These observations, together with the fact that S_i is strictly decreasing in v, means that a vacancy's value is above K for v near zero and below K for v very large and, therefore, an equilibrium exists.

If $1 - \eta - \eta^2 \geq 0$ and $\phi_H \geq \phi_L \geq 1/2$ then α_{Fi} is monotonically decreasing in v and therefore the right-hand side of equation (18) is strictly decreasing in v. As a result, in that case, the equilibrium is unique. ■

Proposition 4.2: When a firm and a worker meet, it is more likely that the worker is of high type if the meeting is through a referral rather than through the market if $\phi_H \geq \phi_L \geq 1/2$.

Proof. In a meeting through the market the probability that the worker is of high type is given by

$$P[H|\text{market}] = \frac{u_H}{u_H + u_L}$$

In a meeting through referrals the probability that the worker is of high type is given by:

$$\begin{aligned} P[H|\text{referral}] &= \frac{P[\text{ref. from } H] * P[H|\text{ref. from } H] + P[\text{ref. from } L] * P[H|\text{ref. from } L]}{P[\text{referral from } H] + P[\text{referral from } L]} \\ &= \frac{[(1-u_H)\phi_H + (1-u_L)(1-\phi_L)]u_H}{[(1-u_H)\phi_H + (1-u_L)(1-\phi_L)]u_H + [(1-u_H)(1-\phi_H) + (1-u_L)\phi_L]u_L} \end{aligned}$$

Noting that

$$(1-u_H)\phi_H + (1-u_L)(1-\phi_L) \geq (1-u_H)(1-\phi_H) + (1-u_L)\phi_L$$

completes the proof. ■

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