University of Pittsburgh

ECE 1150: Computer Networks

Noise, Errors and Channel Capacity

Mai Abdelhakim, PhD

ECE Department

Swanson School of Engineering

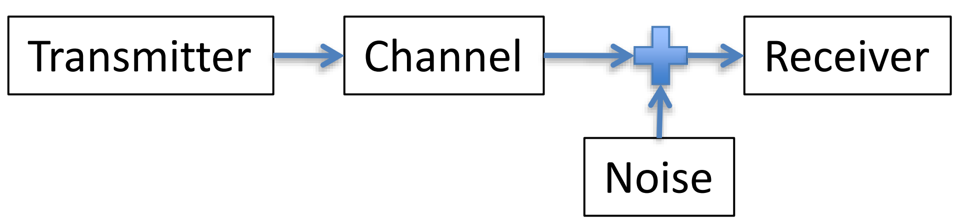
University of Pittsburgh

Objectives of This Unit

- Describe what is noise and its impact
- What is the signal to noise ratio
- Quantify and measure channel errors
- Analyze the impact of noise on channel capacity

Communications Impairments

- Various impairments that affect the signal
  - Attenuation Loss (covered)
  - Noise



Noise

- Noise is undesirable signal that impacts the quality of a desired signal
  - Analogy: Snow in analog video transmission

- In telecommunication networks noise comes from
  - Electromagnetic interference (EMI)
  - Heat in cables
  - System processing (inside the devices)

- Results in **errors** in transmissions

Question

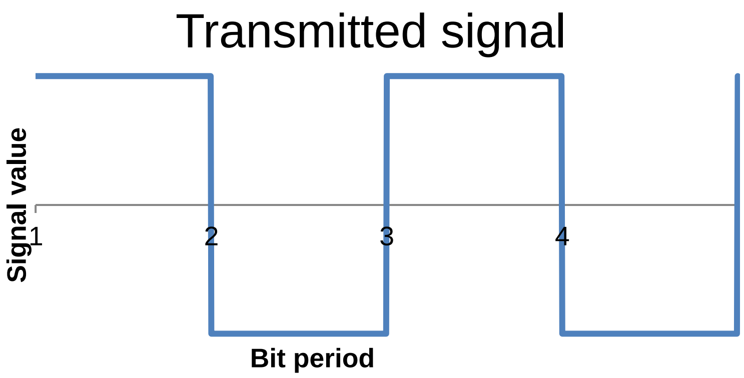
Is noise random or deterministic?

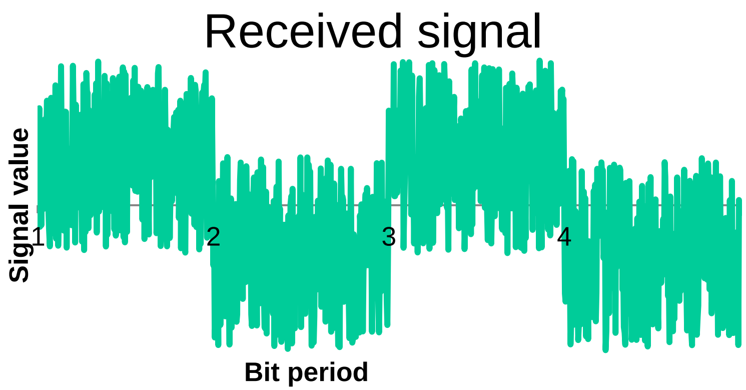
Types of Noise: AWGN

Many types, most common one is Additive White Gaussian Noise (AWGN)

- **White** means: all frequency components are affected the same way
- **Additive** means: noise adds to the signal
- Ideal case
  - There will always be AWGN, but hopefully that is all there is

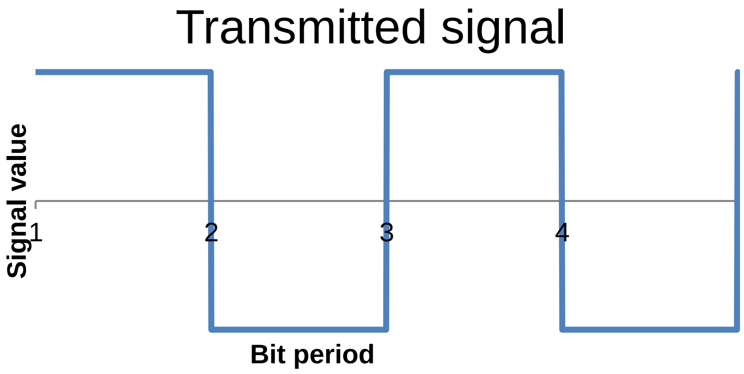
Impact of Noise on Digital Signals

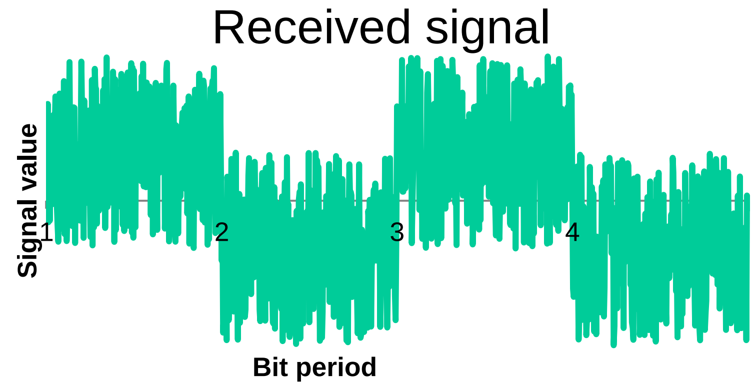




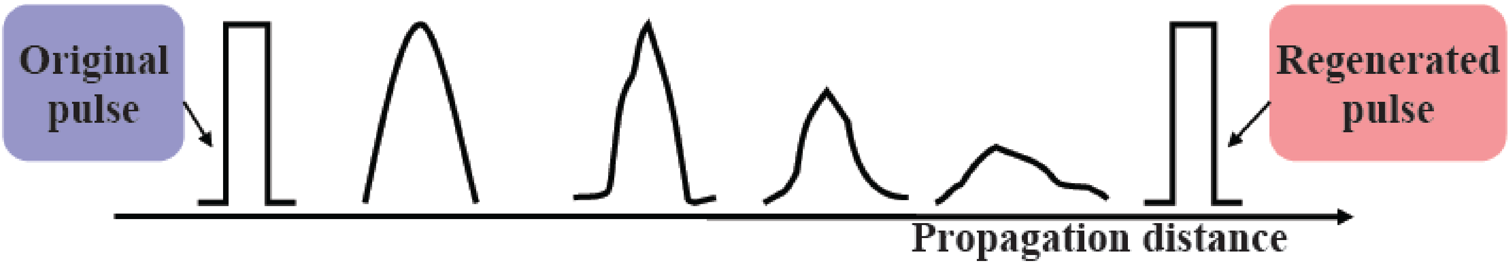
- Received signal get affected by noise
- For reliable data transmission, receiver should interpret the received signal accurately
- Why digital systems preferred over analog?

Impact of Noise on Digital Signals





- Digital systems are **more immune to noise**.
  - Efficient **repeaters** (devices) can regenerate attenuated and noisy signals



Measuring Noise

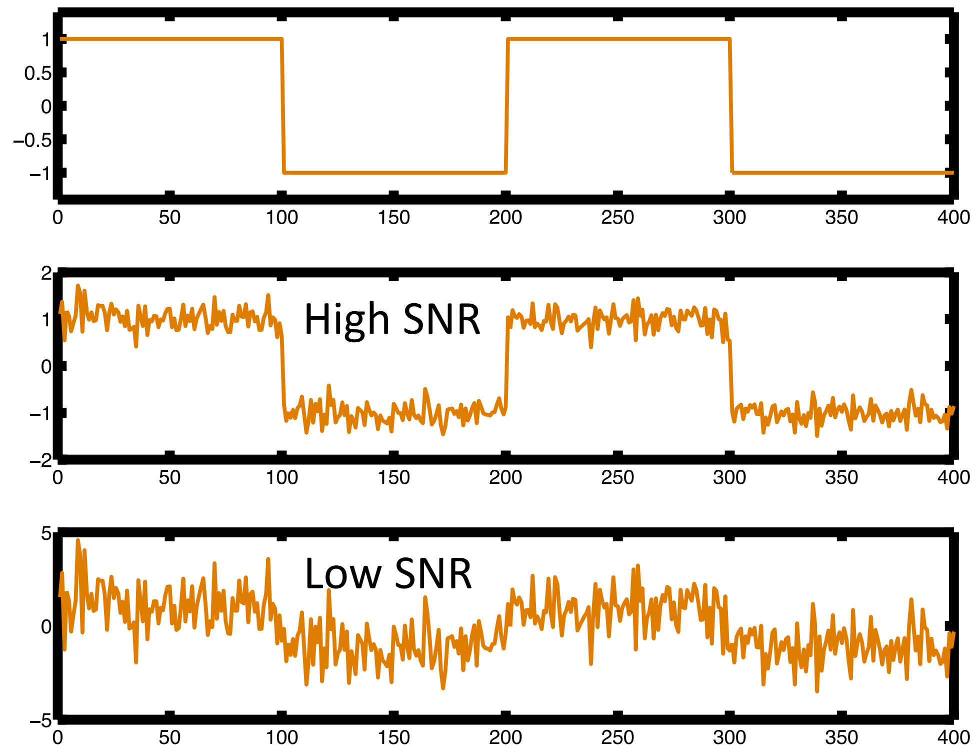
- Measuring noise
  - Signal-to-Noise Ratio (SNR): (S/N)
    - Ratio between the received signal power (S) to noise power (N) at a receiver
  - $SNR_{dB} = 10 \log_{10} (S/N)$
  - Measured in *decibels*

Signal-to-Noise Ratio

- **Measure of "quality" of the received signal**
  - **Ratio of the signal power (S) to noise power (N) at a receiver:**

SNR = *S*/*N*, where *S* and *N* in watts (W) or milliwatts (mW)

$SNR_{dB} = 10 \log_{10} (S/N)$ in decibels (dB)



[Demo: https://demonstrations.wolfram.com/FrequencySpectrumOfANoisySignal/](https://demonstrations.wolfram.com/FrequencySpectrumOfANoisySignal/)

Measuring Power Ratios

- Example: A cell phone receives a signal with average power of 4W and noise power of 0.5W what is the SNR in dB?

  – SNR = 4/0.5=8

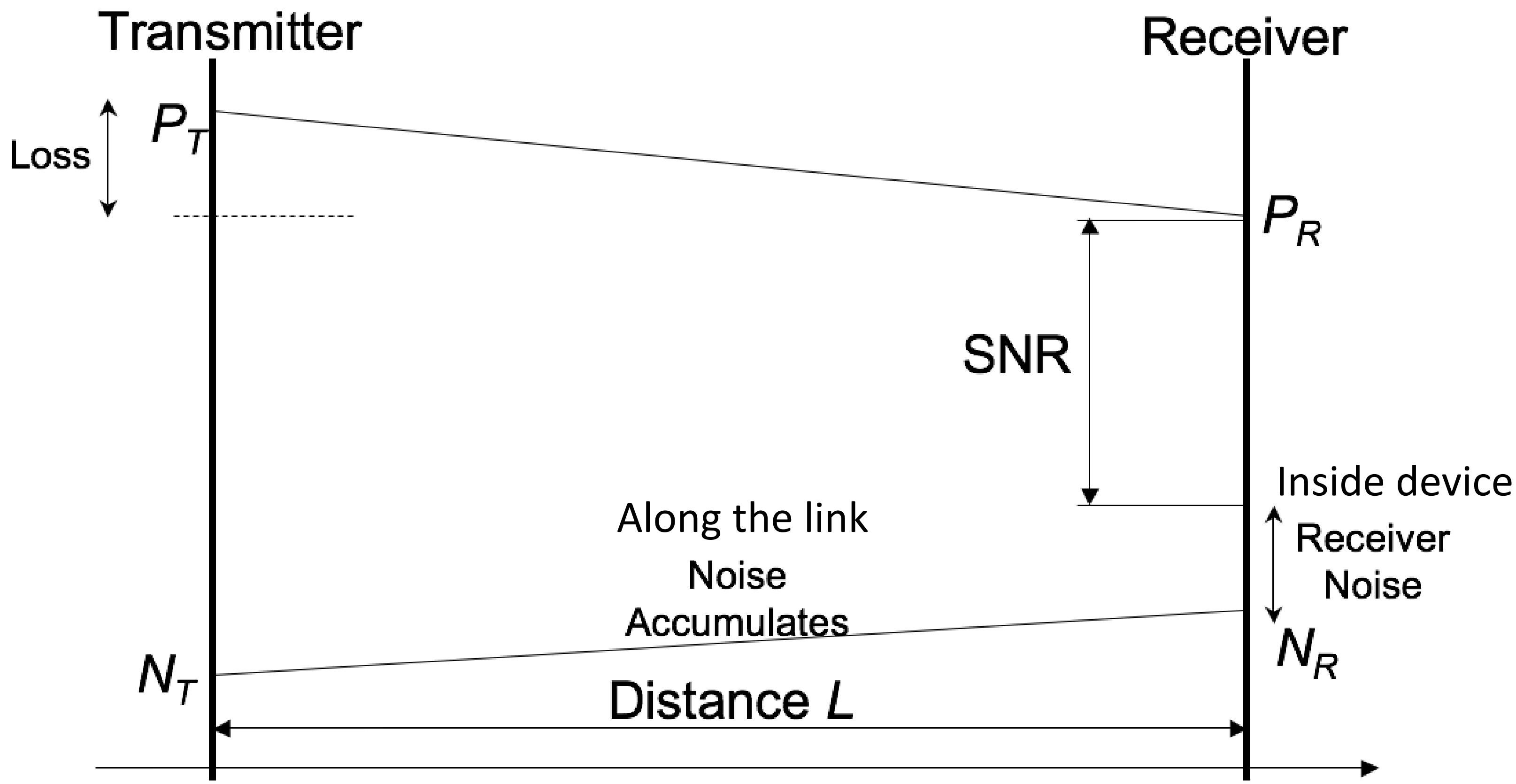
  – SNR (dB) = 10 $\log_{10}$ (S/N) = 10 $\log_{10}$ (4/0.5)

  = 9.03 dB

Link Power Budget

- Balance sheet of gains and losses in a system
- Allows you to determine
  - **Coverage** of your communication link
    - How far can the signals go?
  - Transmit **power**
  - Receiver **capability** needed
    - **Receiver sensitivity** is the minimum power that receiver can detect
      - Received power should be greater than or equal to the receiver sensitivity for the receiver to be able to detect the signal

Basic Concept of Link Budget



Impact of Impairments

- Channel impairments result in errors
- For digital communications, we measure performance by **bit error rates**
  - On average, how many bits you received in error
- Depends on **attenuation**, **SNR**, and other factors

Bit Error Rate or Probability of Bit Error

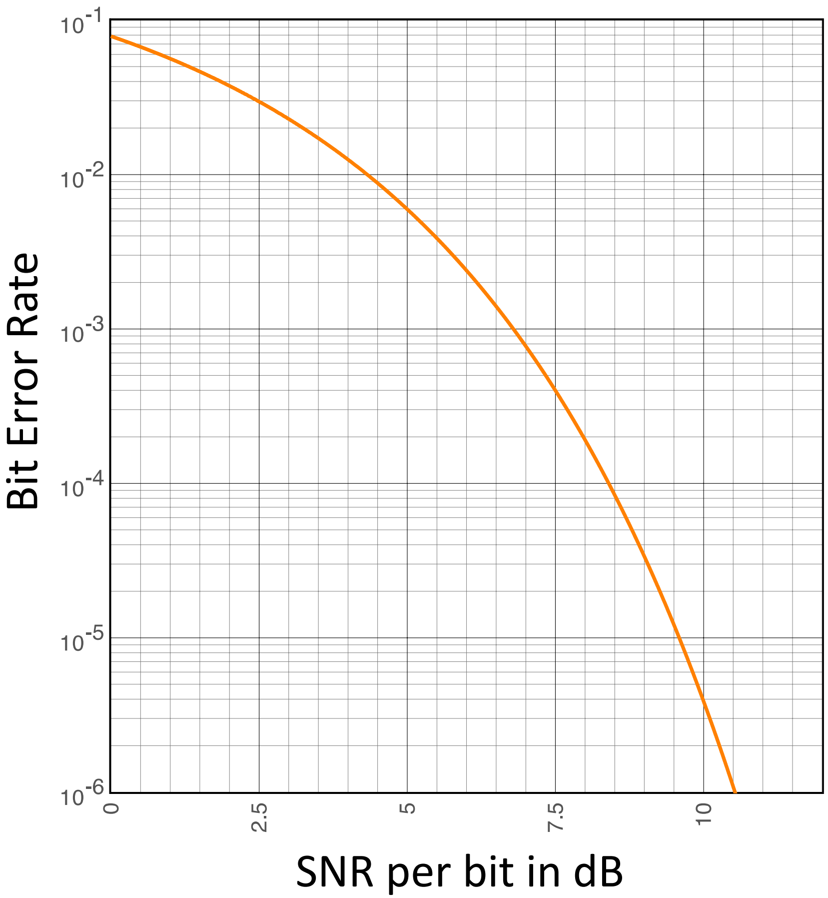
- Oversimplified view
  - Say we have 10 transmissions, each of 1000 bits
  - In the 10 transmissions, the receiver gets 999, 998, 1000, 995, 1000, 1000, 1000, 1000, 1000, 998 bits correctly, respectively
  - Total number of errors in the 10 transmissions, are (1 + 2 + 0 + 5 + 0 + 0 + 0 + 0 + 0 + 2)=10
  - Bit error rate = number of bit errors / total number of bits transmitted = 10/10,000
  - The bit error rate is 1/1000 or 10^{-3}
    - On average 1 bit error for each 1000 bits transmitted

Tophat : BER vs SNR

Noise and Bit Error Rate

- Higher noise (lower SNR) => results in more errors

- Bit error rates are very **low** in optical **fiber** links, **higher** on **copper**, and much **higher** on **radio links**
  - Due to their noise susceptibility
    - More susceptible to noise ➔ lower SNR ➔ higher error

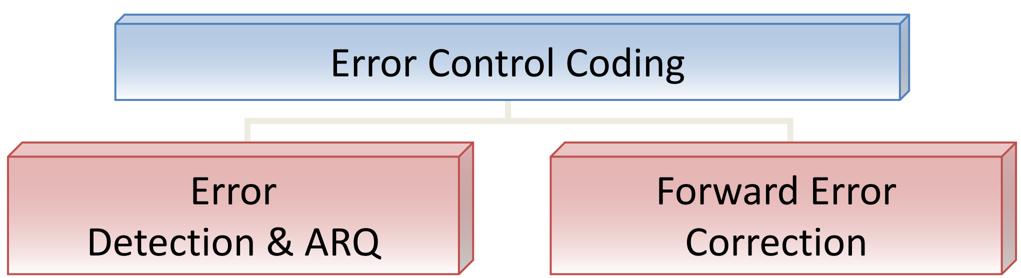


Eb/No vs SNR

- Noise power spectral density (one sided) = No watts/Hz
- Energy per bit = Eb
  Power = Eb /time = Eb x bit rate
- SNR = Eb x bit rate / (No x Bandwidth)

Error Control Coding

- Systematically add redundant bits for error detection or correction
- Part of link layer, but sometimes part of physical layer
- Approaches to error control
  - Error Detection + ARQ (retransmission) → Link layer
  - Error Correction (FEC) → Physical layer



Channel Capacity

- Losses **impact the rate** at which information can be sent over a link
- **Capacity** (C) = **Maximum rate at which data is communicated reliably over a channel**
  - In bits per second
  - Reliably means with low errors
  - Capacity is function of the received signal power to noise power ratio (S/N) and the bandwidth

Channel Capacity

Shannon-Hartley Law: the capacity in an AWGN channel is

$$\mathbf{C = B \log_2 (1+S/N)}$$

- **B is bandwidth**
- **S/N is the received signal power to noise power ratio** (magnitude value <u>not dB</u>)
  - Noise limits capacity
    - » Noisy channels ➔ less information rate

- How to increase capacity?

Channel Capacity - Question

- A twisted pair telephone line has bandwidth of 4000 Hz and a SNR of 100. What is the channel capacity?

Final Answer: 26,635 bps

Channel Capacity - Solution

- A twisted pair telephone line has bandwidth of 4000 Hz and a SNR of 100. What is the channel capacity?

$$C = B \log_2 (1 + SNR) \ = 4000 \ [\log_{10}(101)/\log_{10}(2)]$$

$$= \ 4000[2.004/0.301] = 26{,}635 \text{ bps}$$

Channel Capacity - Example

- If the channel capacity is 20 Kbps, and bandwidth is 4 KHz, what is SNR of the channel?

Final Answer: SNR = 31

Channel Capacity - Example

- If the channel capacity is 20 Kbps, and bandwidth is 4 KHz, what is SNR of the channel?

$C = B \log_2 (1+SNR)$ - > $SNR = 2^{C/B} – 1= 2^{20/4} – 1$

$SNR = 31$

Exercise

- Bandwidth of:
  - TV channel: 6 MHz (Mega is 10^6)
  - Single model fiber: 20 GHz (Giga is 10^9)

If the signal to noise ratio in all channels is 2^{10}-1.

What is the maximum bit rate (capacity) of each case?

Key Takeaway

- **Signal to noise ratio** measures the noise in a channel
- Noise could result in **errors** in communications
- Noise limits the **capacity** of the channel
  - Capacity is the maximum rate that can be transmitted over the channel with low errors
- Shannon theorem is used to obtain capacity
  - The channel capacity increases by increasing the signal to noise ratio or the bandwidth