Basics of Sensor Fusion D - HW1

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Question

Assume that instead of distances to the corners as in Figure 2, we measure azimuth angles and heights with respect to each of the corners. Write the resulting model in form

$$y = g(\mathrm{x}) + r$$

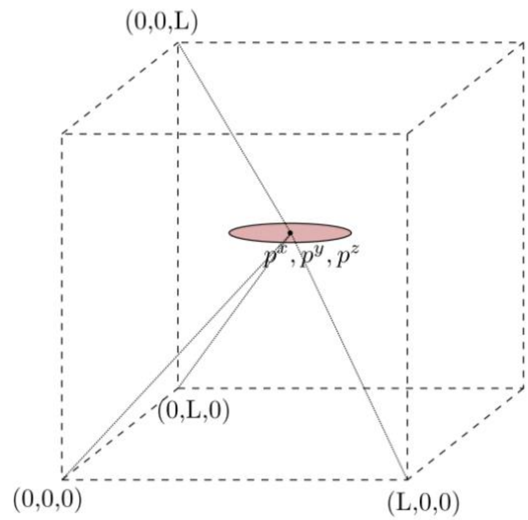

Figure 2: Drone positioning problem II.

Solution

The position P of the corners is

$$P_1 = (0,\ 0,\ 0)$$
$$P_2 = (L,\ 0,\ 0)$$
$$P_3 = (0,\ L,\ 0)$$
$$P_4 = (L,\ L,\ 0)$$
$$P_5 = (0,\ 0,\ L)$$
$$P_6 = (L,\ 0,\ L)$$
$$P_7 = (0,\ L,\ L)$$
$$P_8 = (L,\ L,\ L)$$

$+y$ is determined as the north.
The height h with respect to each of the corners is

$$h_1 = p^z + r_1$$
$$h_2 = p^z + r_2$$
$$h_3 = p^z + r_3$$
$$h_4 = p^z + r_4$$
$$h_5 = p^z - L + r_5$$
$$h_6 = p^z - L + r_6$$
$$h_7 = p^z - L + r_7$$
$$h_8 = p^z - L + r_8$$

Counting clockwise from the north, the azimuth angles θ with respect to each corner is

$$\theta_1 = \tan^{-1}\left(\frac{p^x}{p^y}\right) + r_9$$
$$\theta_2 = \tan^{-1}\left(\frac{p^y}{L - p^x}\right) + \frac{3}{2}\pi + r_{10}$$
$$\theta_3 = \tan^{-1}\left(\frac{L - p^y}{p^x}\right) + \frac{1}{2}\pi + r_{11}$$
$$\theta_4 = \tan^{-1}\left(\frac{L - p^x}{L - p^y}\right) + \pi + r_{12}$$
$$\theta_5 = \tan^{-1}\left(\frac{p^x}{p^y}\right) + r_{13}$$
$$\theta_6 = \tan^{-1}\left(\frac{p^y}{L - p^x}\right) + \frac{3}{2}\pi + r_{14}$$
$$\theta_7 = \tan^{-1}\left(\frac{L - p^y}{p^x}\right) + \frac{1}{2}\pi + r_{15}$$
$$\theta_8 = \tan^{-1}\left(\frac{L - p^x}{L - p^y}\right) + \pi + r_{16}$$

$$y = \begin{bmatrix} y_1 & y_2 & \cdots & y_{16} \end{bmatrix}^\top = \begin{bmatrix} H_1 & H_2 & \cdots & H_8 & \theta_1 & \theta_2 & \cdots & \theta_8 \end{bmatrix}^\top$$
$$x = \begin{bmatrix} p^x & p^y & p^z \end{bmatrix}^\top = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top$$
$$r = \begin{bmatrix} r_1 & r_2 & \cdots & r_{16} \end{bmatrix}^\top$$

Write in a vector notation with the following form

$$\boldsymbol{y} = \boldsymbol{g}(\mathbf{x}) + \boldsymbol{r}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \end{bmatrix} = \begin{bmatrix} p^z \\ p^z \\ p^z \\ p^z \\ p^z - L \\ p^z - L \\ p^z - L \\ p^z - L \\ \tan^{-1}\left(\dfrac{p^x}{p^y}\right) \\ \tan^{-1}\left(\dfrac{p^y}{L - p^x}\right) + \dfrac{3}{2}\pi \\ \tan^{-1}\left(\dfrac{L - p^y}{p^x}\right) + \dfrac{1}{2}\pi \\ \tan^{-1}\left(\dfrac{L - p^x}{L - p^y}\right) + \pi \\ \tan^{-1}\left(\dfrac{p^x}{p^y}\right) \\ \tan^{-1}\left(\dfrac{p^y}{L - p^x}\right) + \dfrac{3}{2}\pi \\ \tan^{-1}\left(\dfrac{L - p^y}{p^x}\right) + \dfrac{1}{2}\pi \\ \tan^{-1}\left(\dfrac{L - p^x}{L - p^y}\right) + \pi \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \\ r_{10} \\ r_{11} \\ r_{12} \\ r_{13} \\ r_{14} \\ r_{15} \\ r_{16} \end{bmatrix}$$