

CSE 417T

# Introduction to Machine Learning

Lecture 21

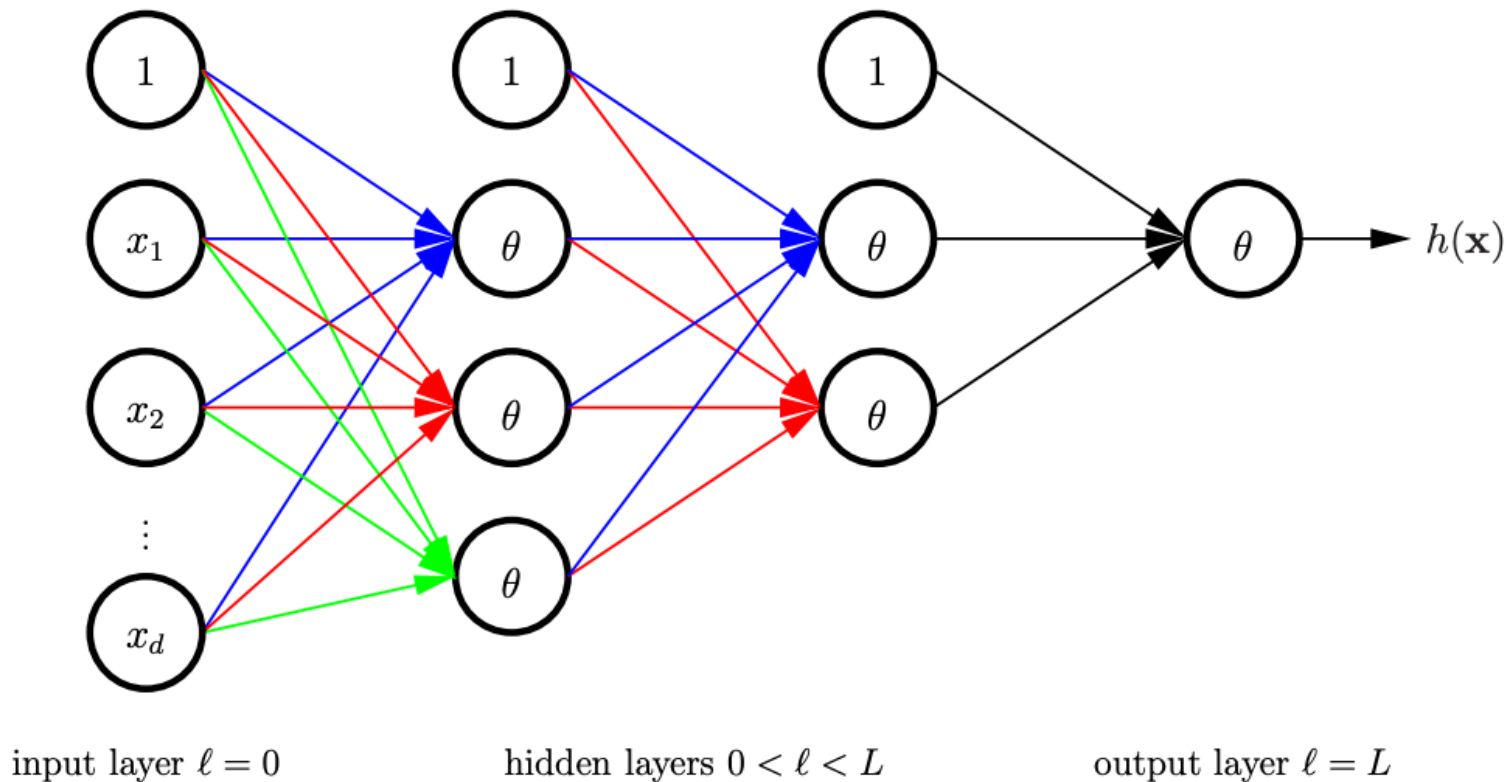
Instructor: Chien-Ju (CJ) Ho

# Logistics

- Homework 5 is due Dec 2 (Friday)
- Exam 2 will be on Dec 8 (Thursday)
  - Will focus on the topics in the second half of the semester
    - Note though knowledge is cumulative, so we still assume you know the concepts earlier
  - Format / logistics will be similar with what we have in Exam 1
    - Timed exam (75 min) during lecture time in the classroom
    - Closed-book exam with 2 letter-size cheat sheets allowed (4 pages in total)
      - No format limitations (it can be typed, written, or a combination)
- Dec 6 (Tuesday) will be a review lecture

Recap

# Neural Networks



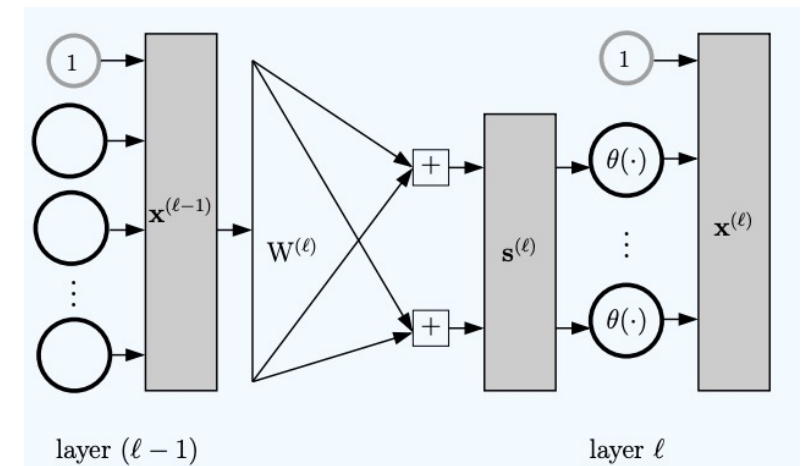
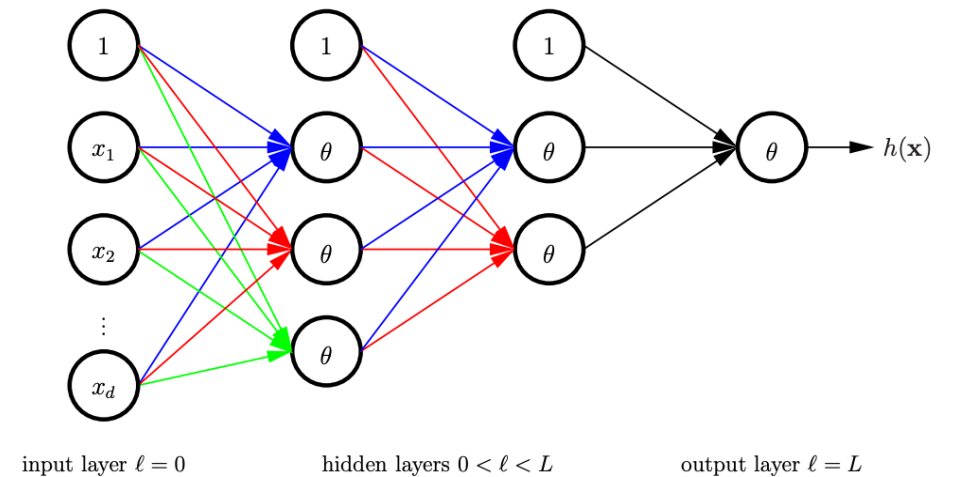
$\theta$ : **activation function**  
(Specify the “activation” of the neuron)



We mostly focus on **feed-forward** network structure

# Notations of Neural Networks (NN)

- Notations:
  - $\ell = 0$  to  $L$ : layer
  - $d^{(\ell)}$ : dimension of layer  $\ell$
  - $\vec{x}^{(\ell)}$ : the nodes in layer  $\ell$
  - $w_{i,j}^{(\ell)}$ : weights; characterize hypothesis in NN
  - $s_j^{(\ell)} = \sum_{i=0}^{d^{(\ell-1)}} w_{i,j}^{(\ell)} x_i^{(\ell-1)}$ : linear signals
  - $\theta$ : activation function
    - $x_j^{(\ell)} = \theta(s_j^{(\ell)})$



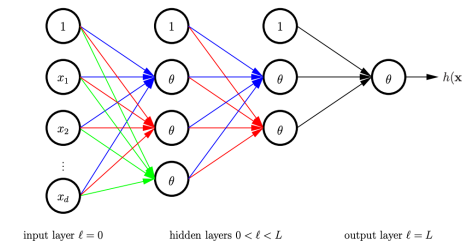
# Evaluate $h(\vec{x})$ - Forward Propagation

- A NN hypothesis  $h$  is characterized by  $\{w_{i,j}^{(\ell)}\}$
- How to evaluate  $h(\vec{x})$ ?

$$\mathbf{x} = \mathbf{x}^{(0)} \xrightarrow{W^{(1)}} \mathbf{s}^{(1)} \xrightarrow{\theta} \mathbf{x}^{(1)} \xrightarrow{W^{(2)}} \mathbf{s}^{(2)} \xrightarrow{\theta} \mathbf{x}^{(2)} \dots \xrightarrow{W^{(L)}} \mathbf{s}^{(L)} \xrightarrow{\theta} \mathbf{x}^{(L)} = h(\mathbf{x}).$$

Forward propagation to compute  $h(\mathbf{x})$ :

```
1:  $\mathbf{x}^{(0)} \leftarrow \mathbf{x}$                                 [Initialization]
2: for  $\ell = 1$  to  $L$  do                                [Forward Propagation]
3:    $\mathbf{s}^{(\ell)} \leftarrow (W^{(\ell)})^T \mathbf{x}^{(\ell-1)}$ 
4:    $\mathbf{x}^{(\ell)} \leftarrow \begin{bmatrix} 1 \\ \theta(\mathbf{s}^{(\ell)}) \end{bmatrix}$ 
5: end for
6:  $h(\mathbf{x}) = \mathbf{x}^{(L)}$                                 [Output]
```



Given weights  $w_{i,j}^{(\ell)}$  and  $\vec{x}^{(0)} = \vec{x}$ , we can calculate all  $\vec{x}^{(\ell)}$  and  $\vec{s}^{(\ell)}$  through forward propagation.

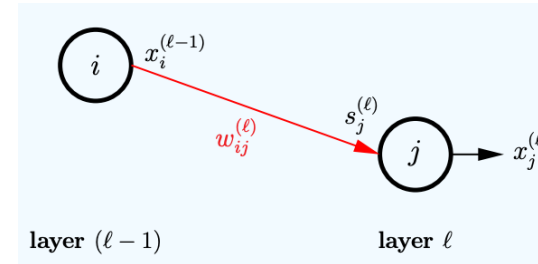
# How to Learn NN From Data?

- Given  $D$ , how to learn the weights  $W = \{w_{i,j}^{(\ell)}\}$ ?
- Intuition: Minimize  $E_{in}(W) = \frac{1}{N} \sum_{n=1}^N e_n(W)$
- How?
  - Gradient descent:  $W(t+1) \leftarrow W(t) - \eta \nabla_W E_{in}(W)$
  - Stochastic gradient descent  $W(t+1) \leftarrow W(t) - \eta \nabla_W e_n(W)$
- Key step: we need to be able to evaluate the gradient...
  - Not trivial given the network structure
  - **Backpropagation** is an algorithmic procedure to calculate the gradient

# Compute the Gradient $\nabla_W e_n(W)$

- Applying chain rule

$$\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}} \frac{\partial s_j^{(\ell)}}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$$



- Calculating  $\delta_j^{(\ell)}$  (Using dynamic programming idea)

- Boundary conditions

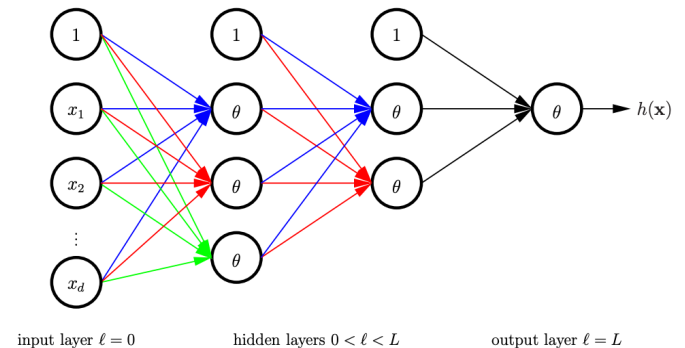
- The output layer (assume regression)

$$\delta_1^{(L)} = 2 \left( s_1^{(L)} - y_n \right) \text{ (generalizable to other differentiable error)}$$

- Backward recursive formulation

$$\delta_j^{(\ell)} = \sum_{k=1}^{d^{(\ell+1)}} \frac{\partial e_n(W)}{\partial s_k^{(\ell+1)}} \frac{\partial s_k^{(\ell+1)}}{\partial x_j^{(\ell)}} \frac{\partial x_j^{(\ell)}}{\partial s_j^{(\ell)}} = \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)} \theta' \left( s_j^{(\ell)} \right)$$

- Backward propagation





# Backpropagation Algorithm

- Recall that  $\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$
- Backpropagation Algorithm
  - Initialize  $w_{i,j}^{(\ell)}$  randomly
  - For  $t = 1$  to  $T$ 
    - Randomly pick a point from  $D$  (for stochastic gradient descent)
    - Forward propagation: Calculate all  $x_i^{(\ell)}$  and  $s_i^{(\ell)}$
    - Backward propagation: Calculate all  $\delta_j^{(\ell)}$
    - Update the weights  $w_{i,j}^{(\ell)} \leftarrow w_{i,j}^{(\ell)} - \eta \delta_j^{(\ell)} x_i^{(\ell-1)}$
- Return the weights

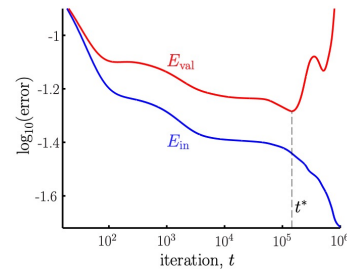
# Discussion

- Backpropagation is gradient descent with efficient gradient computation
- Note that the  $E_{in}$  is **not convex** in weights
- Gradient descent doesn't guarantee to converge to global optimal
- Potential approaches:
  - Run it many times
  - Choose better initializations (the choice of initialization matters)

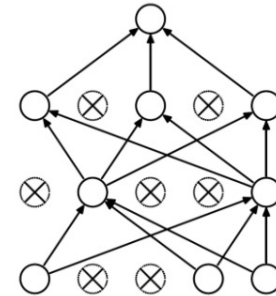
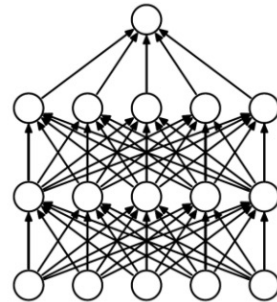
# Regularizations in Neural Networks

- Weight-based regularization

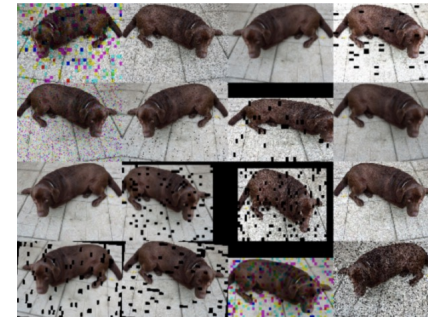
- Early stopping



- Dropout



- Adding noises  
(Data augmentation)



# Today's Lecture

The notes are not intended to be comprehensive. They should be accompanied by lectures and/or textbook.  
Let me know if you spot errors.

# Deep Learning

# ImageNet Challenge 2012

## Task 1: Classification



Car

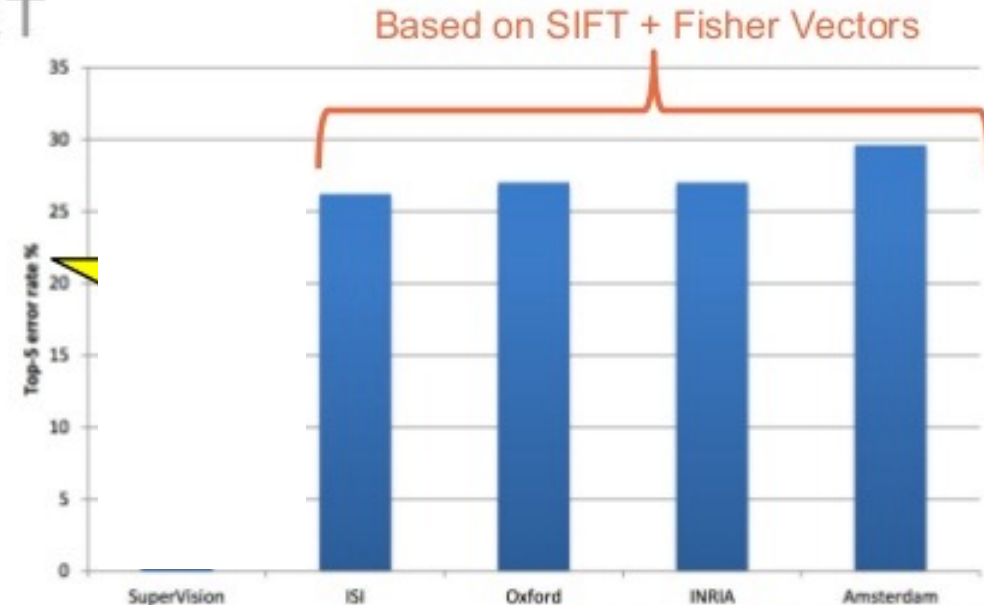
- Predict a class label
- 5 predictions / image
- 1000 classes
- 1,200 images per class for training
- Bounding boxes for 50% of training.

## ImageNet Challenge

Image Classification 2012

IMAGENET

Slide credit:  
[Rob Fergus](#) (NYU)



Russakovsky, O., Deng, J., Su, H., Krause, J., Satheesh, S., Ma, S., ... & Fei-Fei, L. (2014). [Imagenet large scale visual recognition challenge](#). *arXiv preprint arXiv:1409.0575*. [\[wsh\]](#)

# ImageNet Challenge 2012

## Task 1: Classification



Car

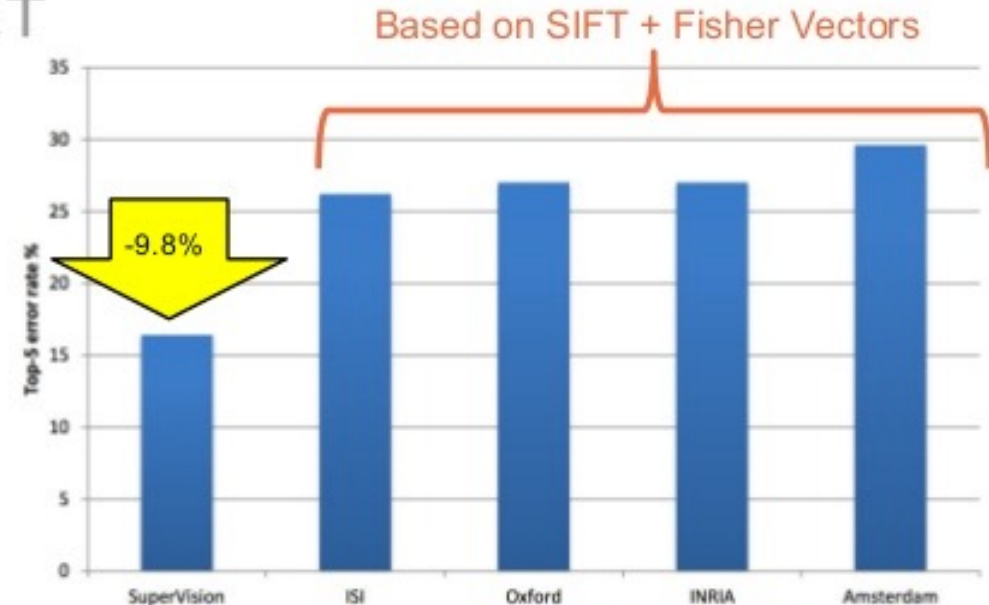
- Predict a class label
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## ImageNet Challenge

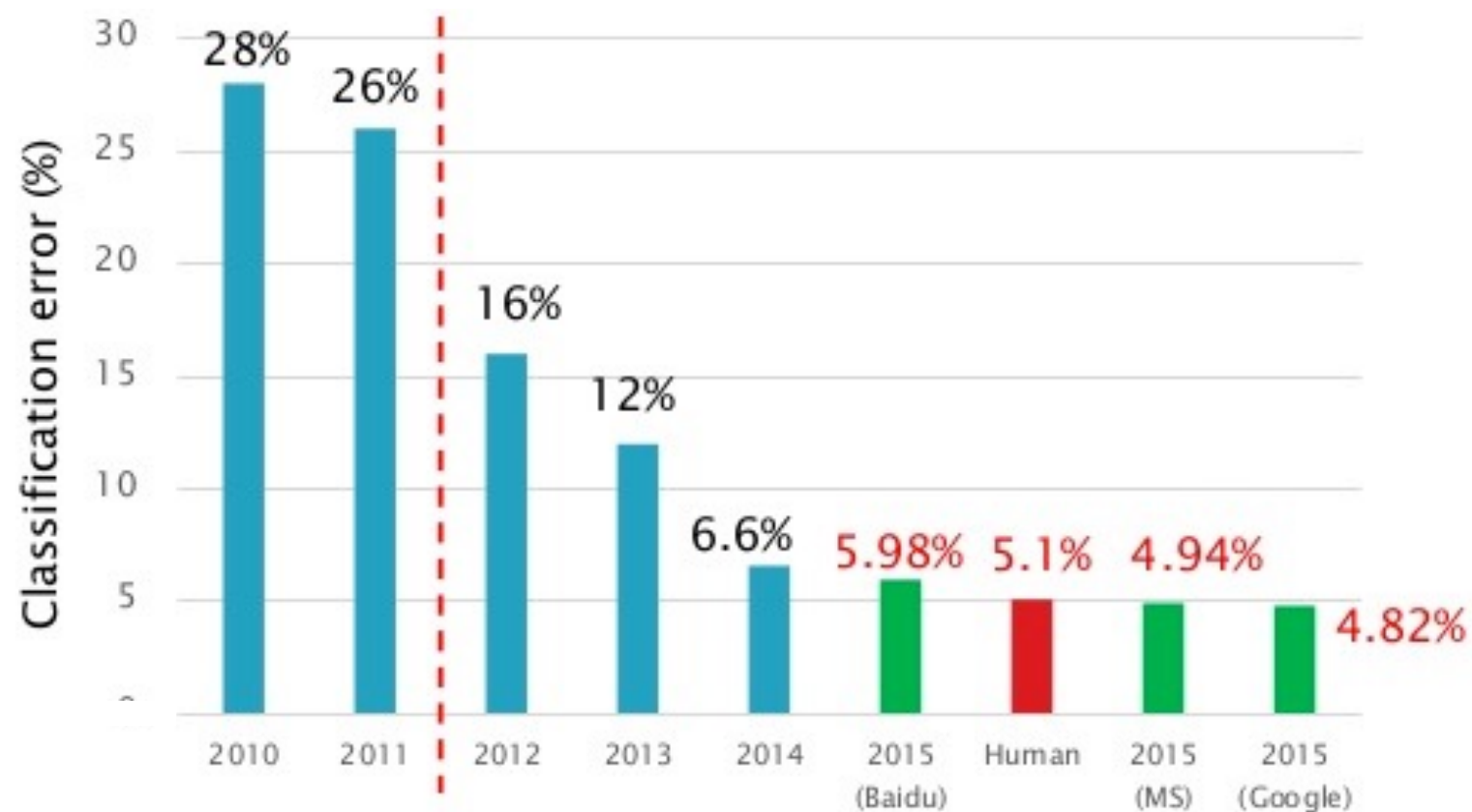
Image Classification 2012

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He et al., "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", arXiv, 2015.

Ioffe et al., "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", arXiv, 2015.

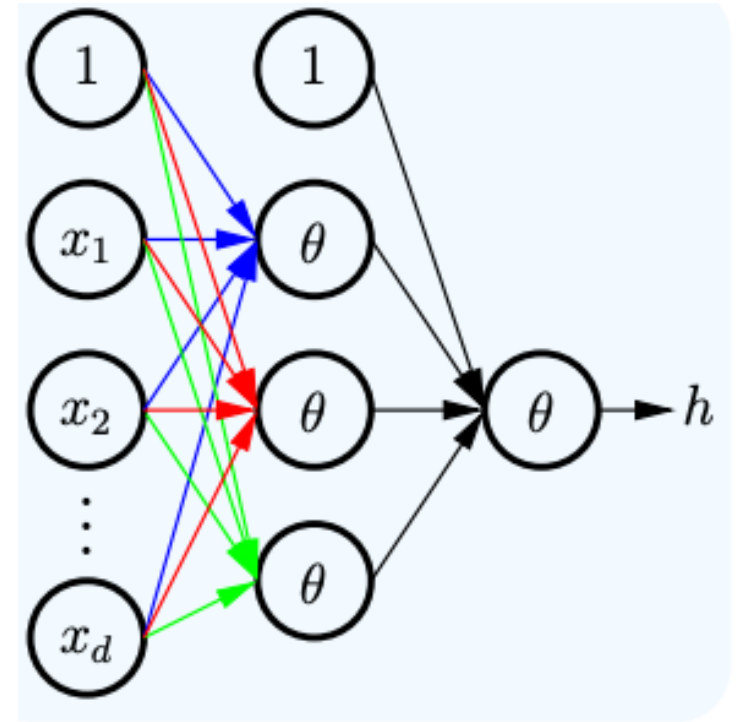


# What is “Deep” Learning

Neural networks with many layers

# Single Hidden-Layer Neural Network

- How do we write a hypothesis in a single-hidden layer NN mathematically?



# Single Hidden-Layer Neural Network

- How do we write a hypothesis in a single-hidden layer NN mathematically?

- $$h(\vec{x}) = \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} x_j^{(1)} \right)$$
$$= \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} \theta \left( \sum_{i=0}^{d^{(0)}} w_{i,j}^{(1)} x_i^{(0)} \right) \right)$$

- How do we write a linear model with nonlinear transform

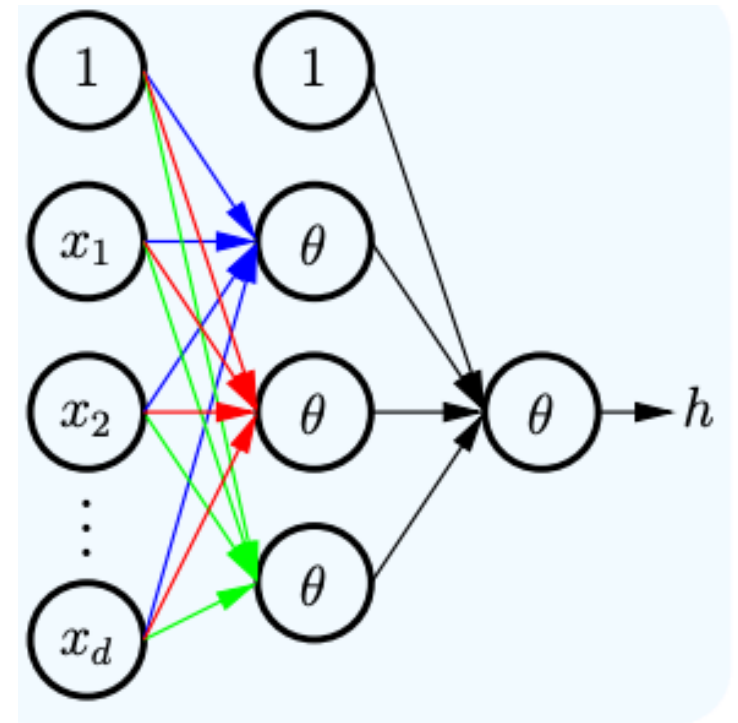
- $$h(\vec{x}) = \theta(w_0 + \sum w_i \phi_i(\vec{x}))$$

- How do we write a Kernel SVM hypothesis

- $$g(\vec{x}) = \theta \left( b^* + \sum_{\alpha_n^* > 0} \alpha_n^* y_n K(\vec{x}_n, \vec{x}) \right)$$

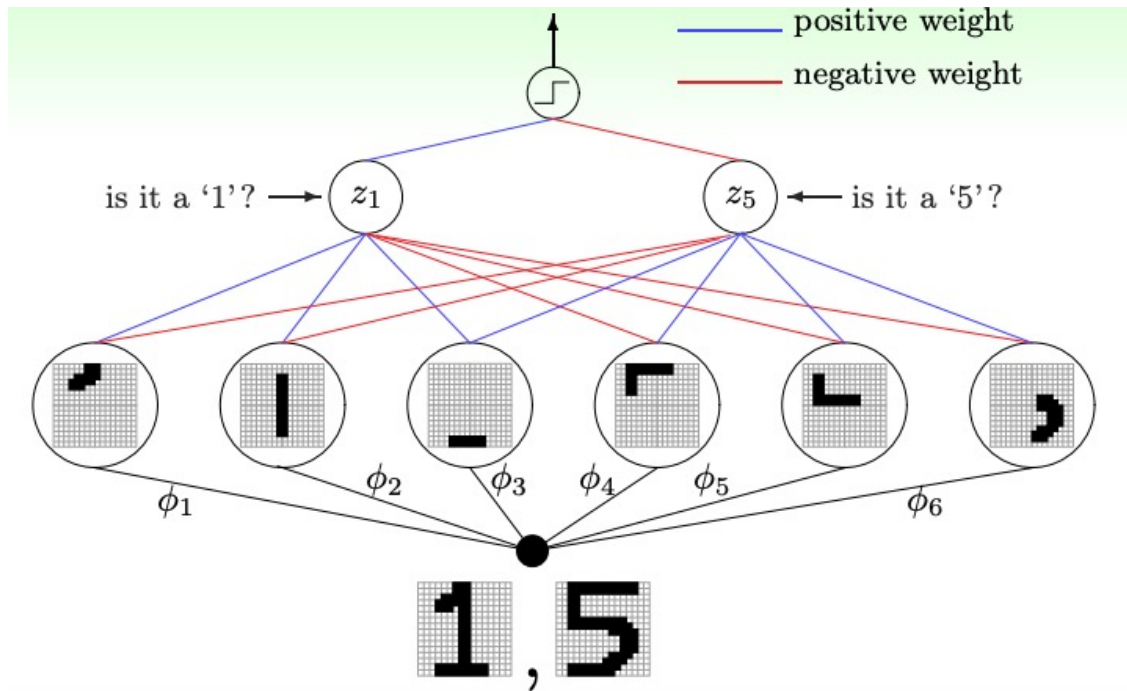
- Interpretation:

- The hidden layer is like **feature transform**
  - Shallow learning vs. deep learning



# Deep Neural Network

- “Shallow” neural network is powerful (universal approximation theorem holds with a single hidden layer). Why “deep” neural networks?



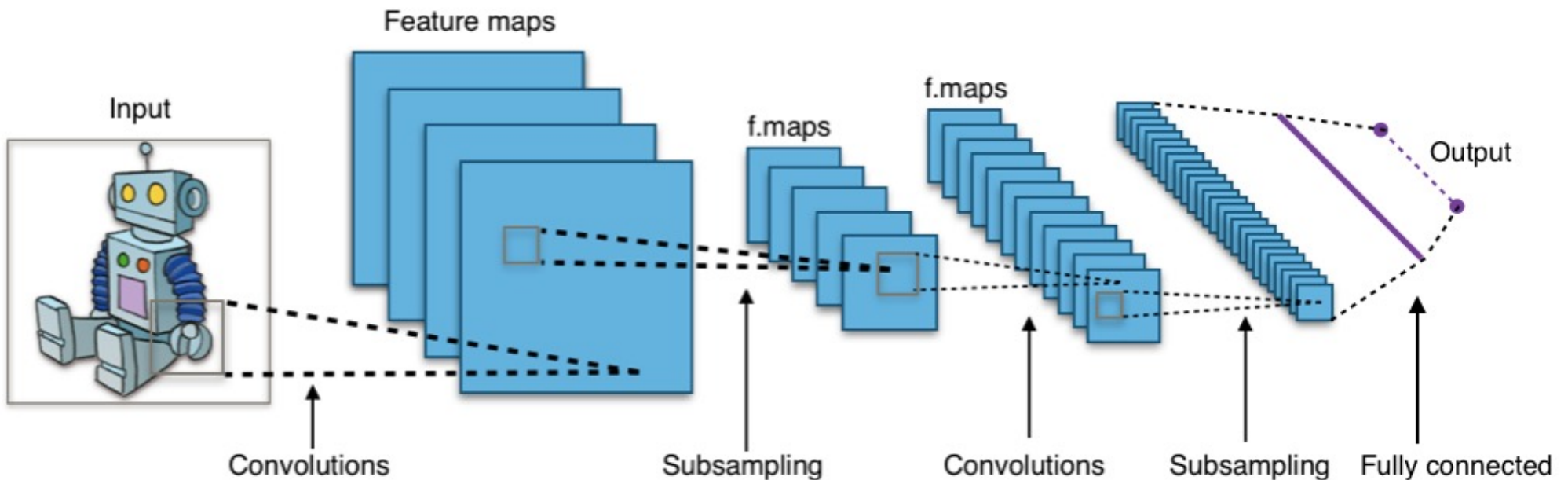
Each layer captures **features** of the previous layers.

We can use “raw data” (e.g., pixels of an image) as input. The hidden layer are extracting the **features**.

Design different **network architectures** to incorporate domain knowledge.

# Convolutional Neural Networks (CNN)

- Captures the localized properties of features hierarchically



# Convolutional Filters

- A convolutional filter is like a matrix version of a dot product.

0	0	0	0	0	0
0	1	2	2	1	0
0	2	4	4	2	0
0	1	3	3	1	0
0	1	2	3	1	0
0	0	1	1	0	0

\*

0	1	0
1	-4	1
0	1	0

# Convolutional Filters

- A convolutional filter is like a matrix version of a dot product.

0	0	0	0	0	0
0	1	2	2	1	0
0	2	4	4	2	0
0	1	3	3	1	0
0	1	2	3	1	0
0	0	1	1	0	0

\*

0	1	0
1	-4	1
0	1	0

=

0			

$$\begin{aligned} & (0 * 0) + (0 * 1) + (0 * 0) + \\ & (0 * 1) + (1 * -4) + (2 * 1) + \\ & (0 * 0) + (2 * 1) + (4 * 0) \\ & = 0 \end{aligned}$$

# Convolutional Filters

- A convolutional filter is like a matrix version of a dot product.

0	0	0	0	0	0
0	1	2	2	1	0
0	2	4	4	2	0
0	1	3	3	1	0
0	1	2	3	1	0
0	0	1	1	0	0

\*

0	1	0
1	-4	1
0	1	0

=

0			



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0	0	0	0	0	0
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0	0	1	1	0	0

\*

0	1	0
1	-4	1
0	1	0

=

0	-1		

$$\begin{aligned} & (0 * 0) + (0 * 1) + (0 * 0) + \\ & (1 * 1) + (2 * -4) + (2 * 1) + \\ & (2 * 0) + (4 * 1) + (4 * 0) \\ & = -1 \end{aligned}$$

# Convolutional Filters

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0	0	0	0	0	0
0	1	2	2	1	0
0	2	4	4	2	0
0	1	3	3	1	0
0	1	2	3	1	0
0	0	1	1	0	0

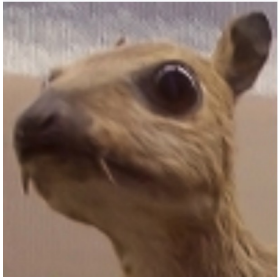
\*

0	1	0
1	-4	1
0	1	0

=

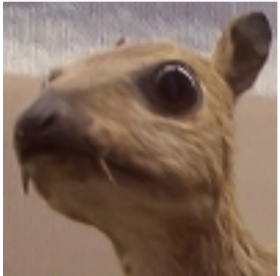
0	-1	-1	0
-2	-5	-5	-2
2	-2	-1	3
-1	0	-5	0

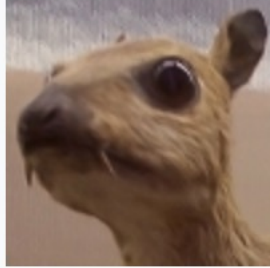
# Convolutional Filters



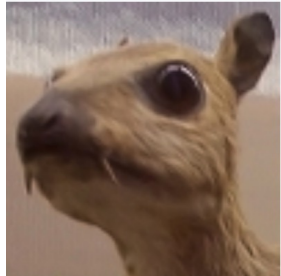
Operation	Kernel $\omega$	Image result $g(x,y)$
	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

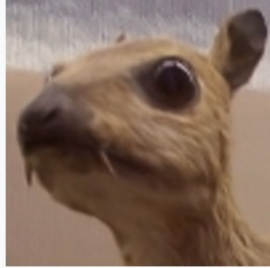
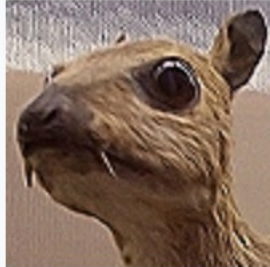
# Convolutional Filters



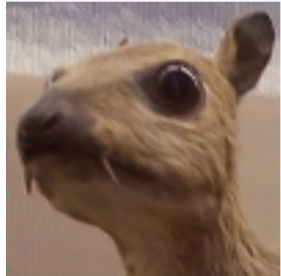
Operation	Kernel $\omega$	Image result $g(x,y)$
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

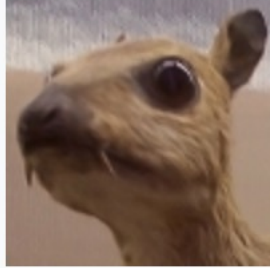
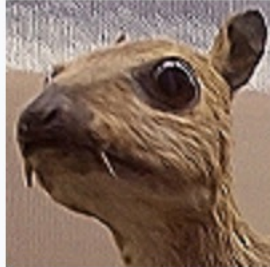
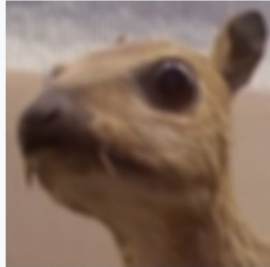
# Convolutional Filters



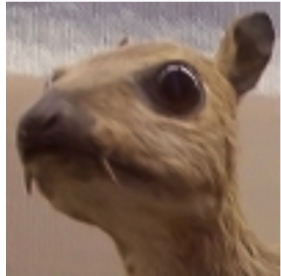
Operation	Kernel $\omega$	Image result $g(x,y)$
<b>Identity</b>	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
<b>Sharpen</b>	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

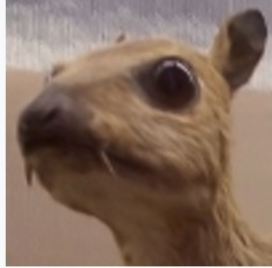
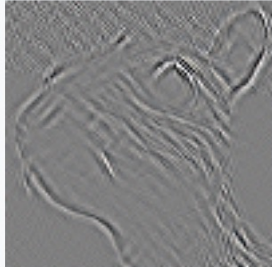
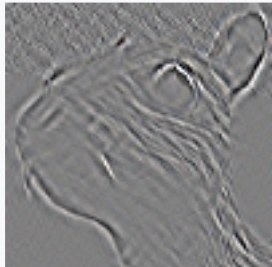
# Convolutional Filters



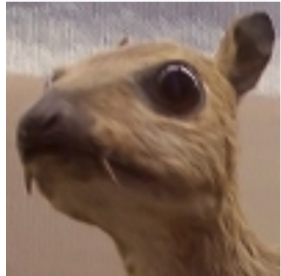
Operation	Kernel $\omega$	Image result $g(x,y)$
<b>Identity</b>	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
<b>Sharpen</b>	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
<b>Box blur</b> (normalized)	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

# Convolutional Filters



Operation	Kernel $\omega$	Image result $g(x,y)$
<b>Identity</b>	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
<b>Ridge detection</b>	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	

# Convolutional Filters



Operation	Kernel $\omega$	Image result $g(x,y)$
<b>Box blur</b> (normalized)	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	
<b>Gaussian blur 3 × 3</b> (approximation)	$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	
<b>Gaussian blur 5 × 5</b> (approximation)	$\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$	



# Connection to Neural Networks

- Convolutions can be represented by a network structure
  - Nodes in the previous layer are only connected to “adjacent” nodes in the next layer.
  - Many of the weights have the same value.

0	0	0	0	0	0
0	1	2	2	1	0
0	2	4	4	2	0
0	1	3	3	1	0
0	1	2	3	1	0
0	0	1	1	0	0

\*

0	1	0
1	-4	1
0	1	0

=

0	-1	-1	0
-2	-5	-5	-2
2	-2	-1	3
-1	0	-5	0

# Pooling Layers

- Commonly used in convolutional neural networks.

A subsampling / down-sampling process:

- Combines multiple adjacent nodes into a single node

0	-1	-1	0
-2	-5	-5	-2
2	-2	2	3
-1	1	-5	0

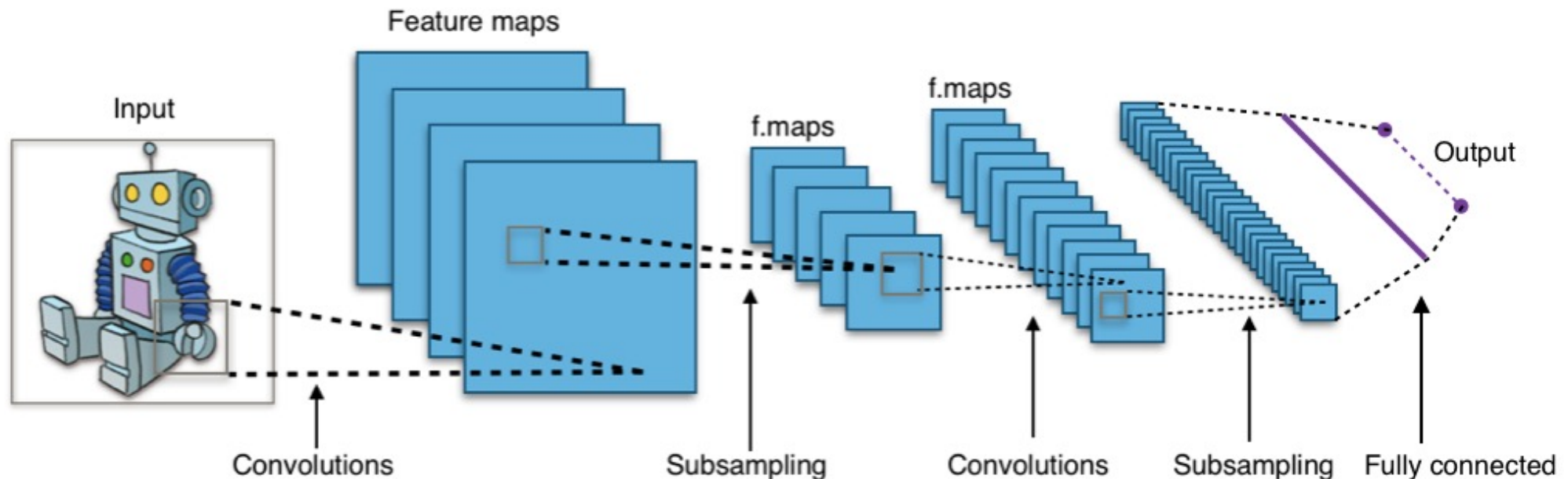
*average  
pooling*

-2	-2
0	0

- Reduce the dimensionality of input. More robust to noise.

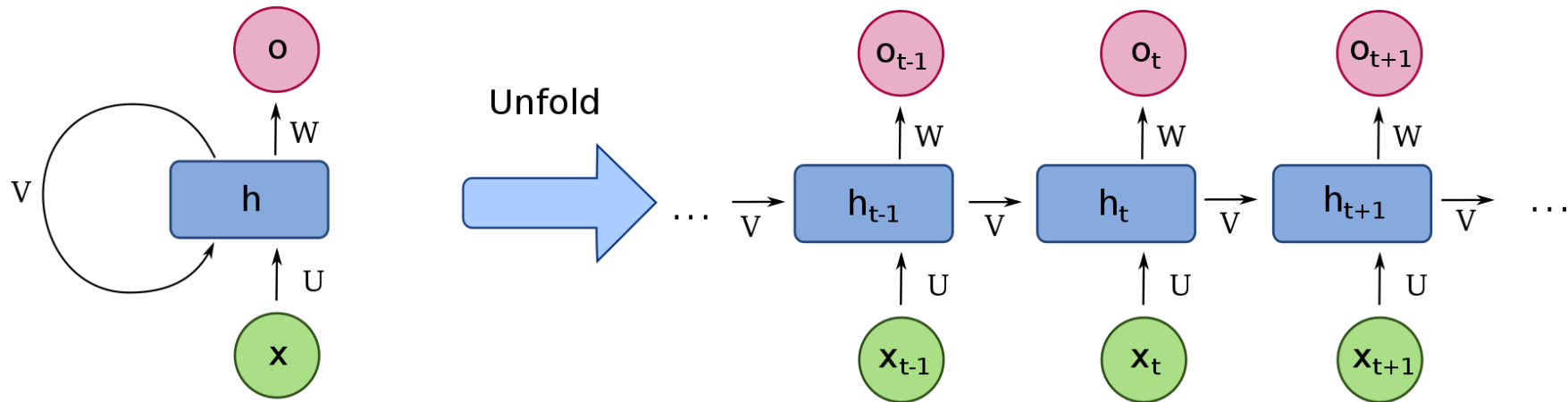
# Convolutional Neural Networks (CNN)

- Captures the localized properties of features
  - Particularly suitable for computer vision (images)
  - Go (AlphaGo) is another famous application of CNN



# Another Example Network Structure [Safe to Skip for the Exam]

- Recurrent Neural Network (RNN)
  - Aim to deal with time-series data, such as natural language processing
  - Using hidden layers to store temporal information
  - Allow previous outputs to be used as inputs and keep hidden states



# Some Techniques in Improving Deep Learning

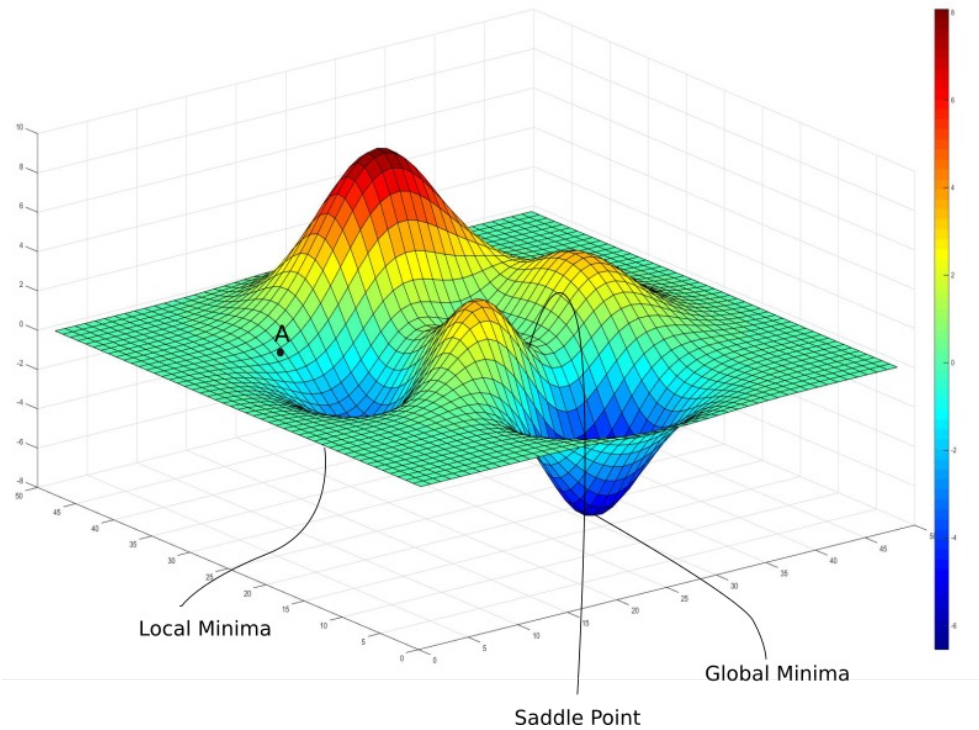
- Regularization to mitigate overfitting
  - Weight-based, early stopping, dropout, etc
- Incorporating domain knowledges
  - Network architectures (e.g., Convolutional Neural Nets)
- Improving computation with huge amount of data
  - Hardware architecture to improve parallel computation
- Improving gradient-based optimization
  - See more in LFD 7.5 (Steepest descent, conjugate gradient, higher-order optimization)
  - Choosing better **initialization** points

# Initialization

Why initialization matters in deep learning

- Error is nonconvex in NN
- Vanishing/exploding gradient problem

# Error is Nonconvex in Neural Networks



- We mostly adopt gradient-descent-style algorithms for optimization
- No guarantee to converge to global optimal
- Could run it many times
- Initialization matters

# Vanishing Gradient Problem

- Backpropagation

- $\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$

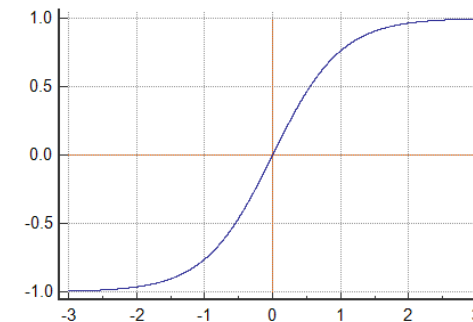
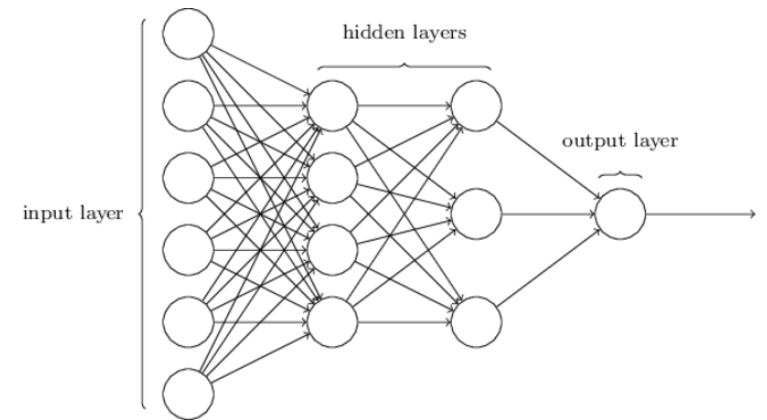
- $\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$

- If we use activation function  $\theta(s) = \tanh(s)$

- $\theta'(s) = 1 - \theta(s)^2 < 1$

- In deep learning with a lot of layers,

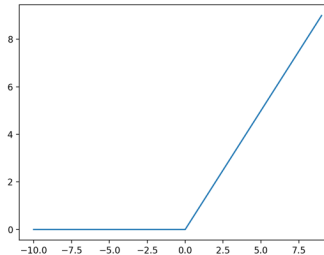
- the gradient might vanish
  - hard to update the early layers





# Vanishing Gradient Problem

- $\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$
- There is also a corresponding “exploding gradient problem”
- What can we do
  - Choose different activation functions
    - One common choice is Rectified Linear Unit (ReLU) and its variant
      - $\theta(s) = \max(0, s)$
  - Choose better **initialization**
    - Many approaches



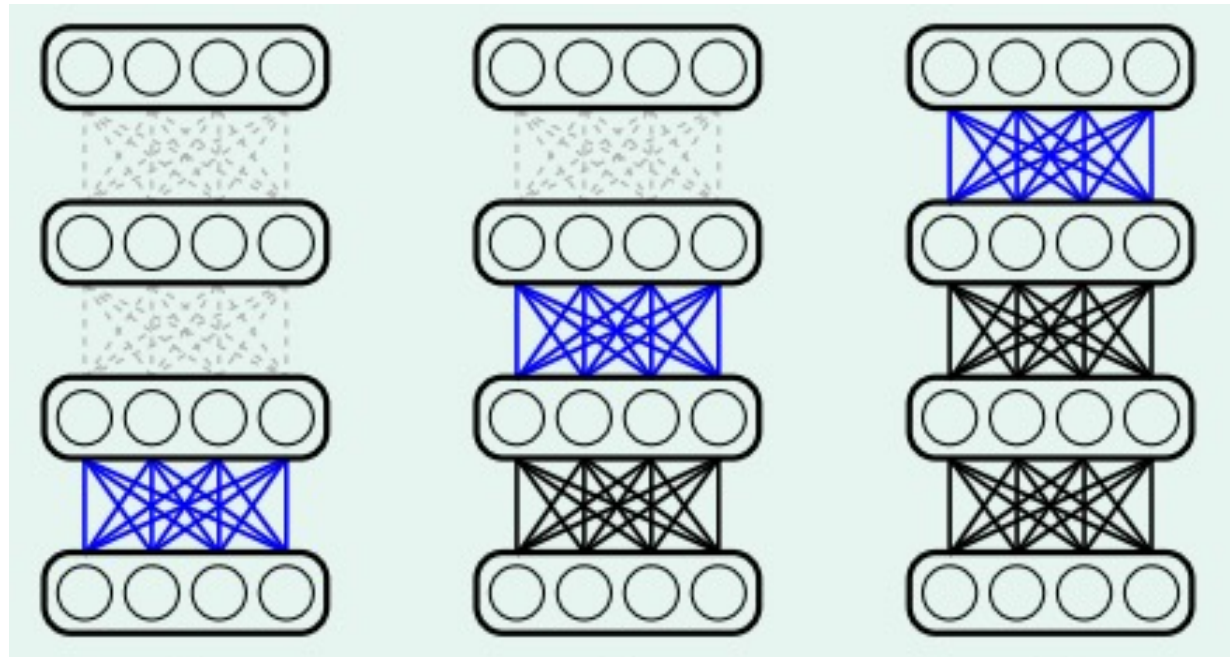
# Weight Initialization

- Initializing weights to all 0 is a bad idea
  - Q6b of HW1
  - Hint: Look at the backpropagation formulation
- Randomly Initializing weights to regions so that vanishing/exploding gradients are less likely to happen
  - Activation-function dependent
    - e.g., Xavier initialization for tanh
- Learning the initialization that might be closer to the optimal
  - E.g., using autoencoder

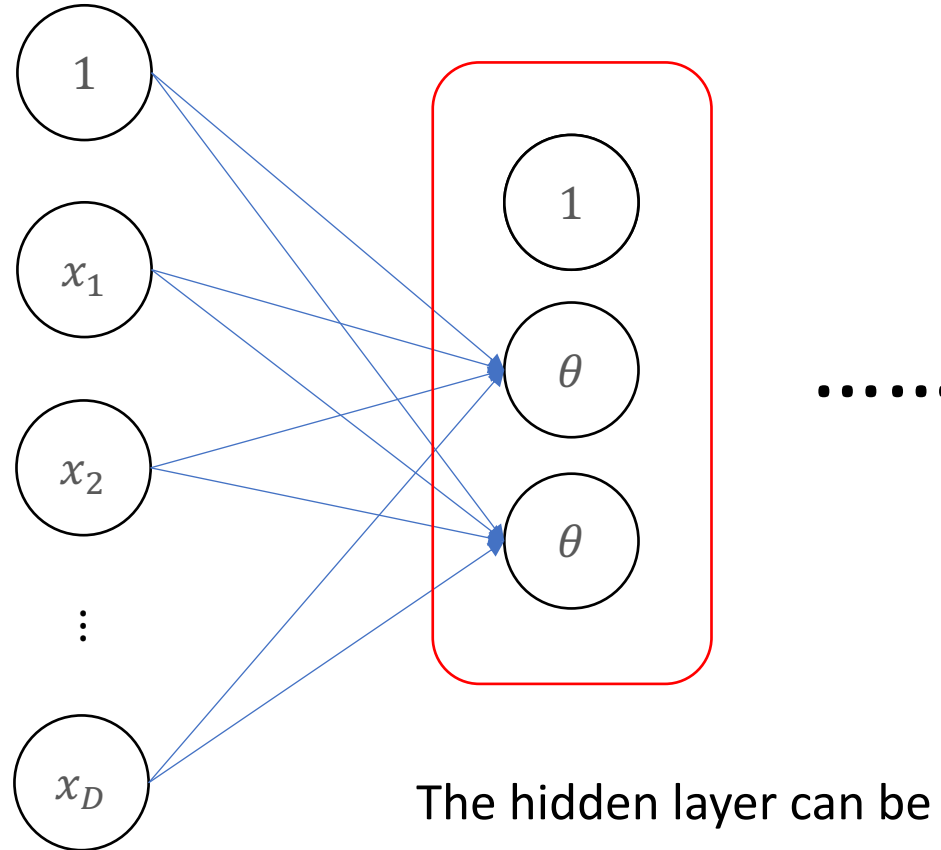
$$\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$$

# Initialization

- Hard to initialize the entire network well.
- Intuition: Initialize the weights **layer by layer** such that each layer **preserves** the properties of the previous layer.



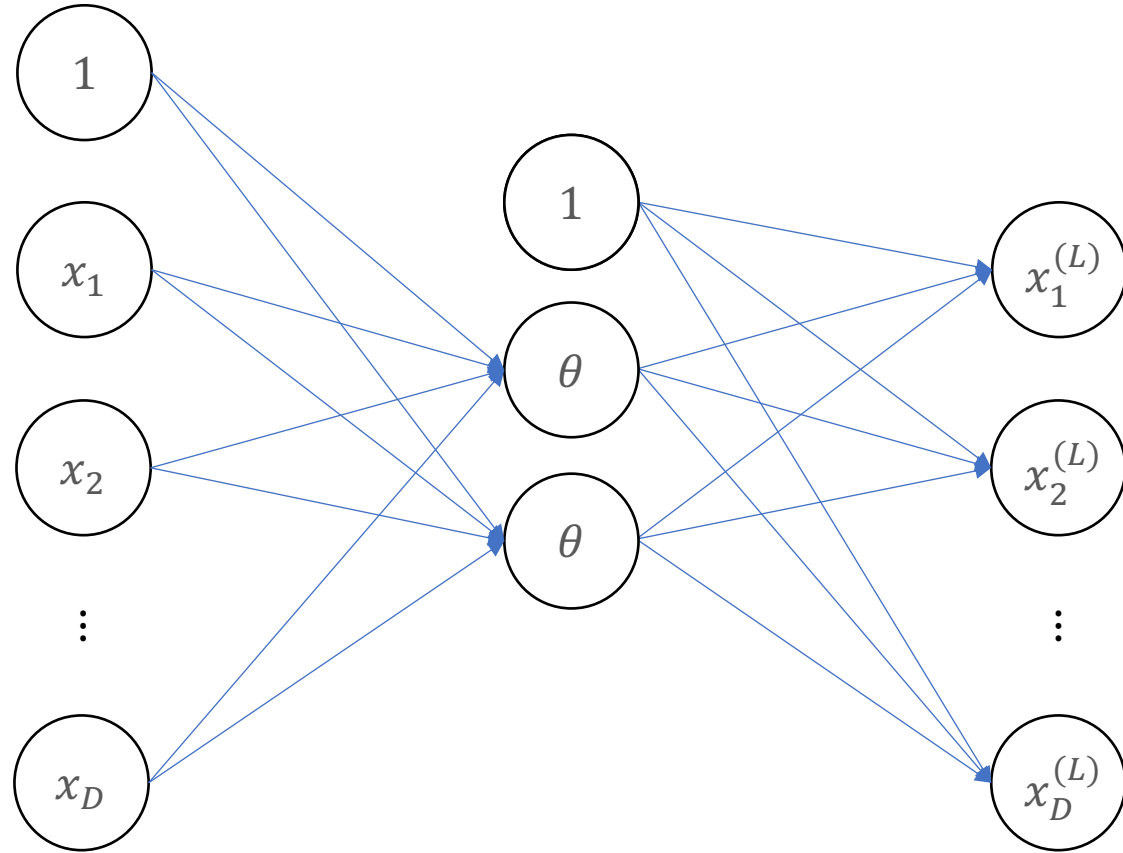
# Autoencoders



The hidden layer can be considered as a feature transform.

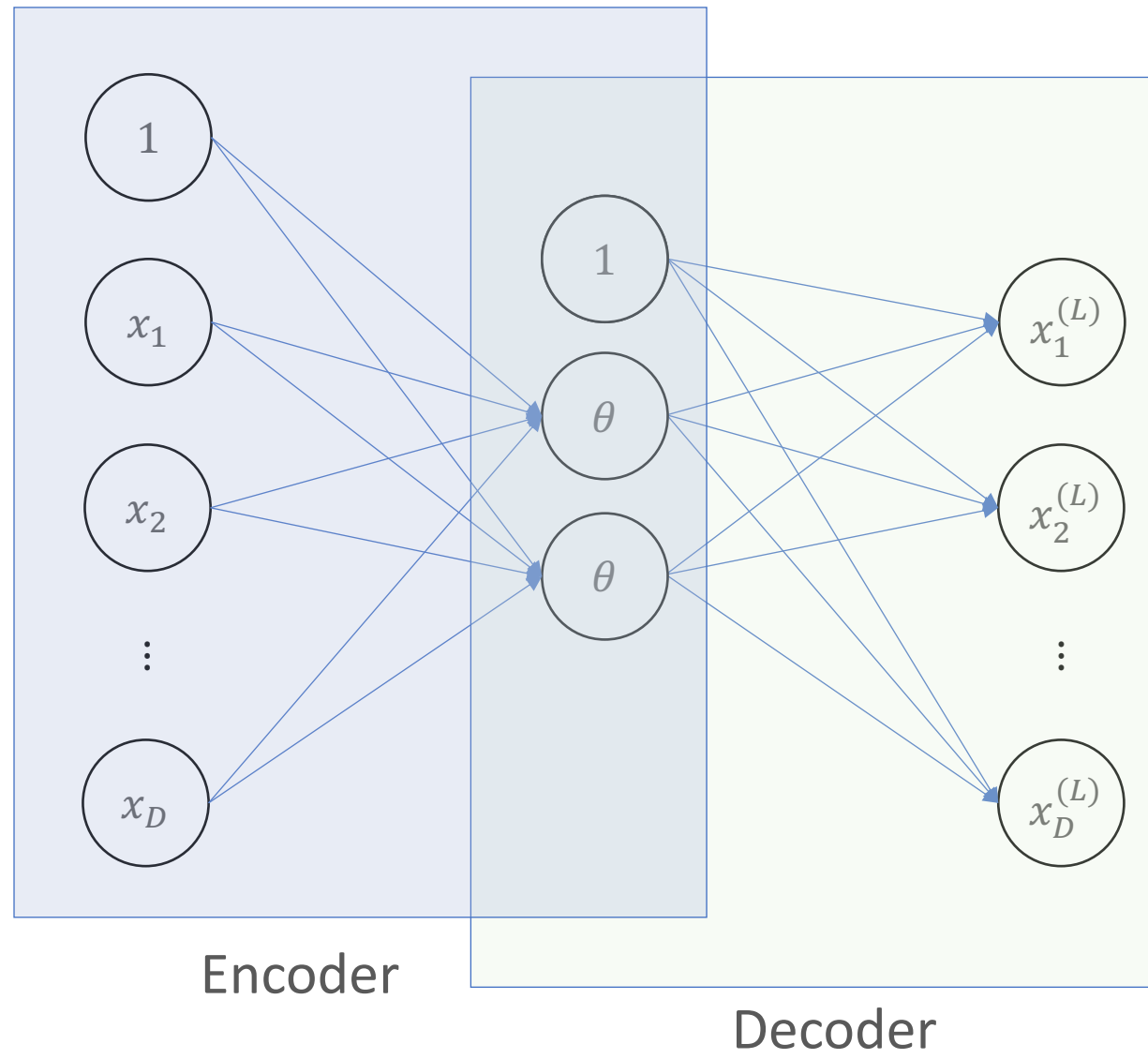
Can we ensure this transformation contain as much information about the original input as possible?

# Autoencoders

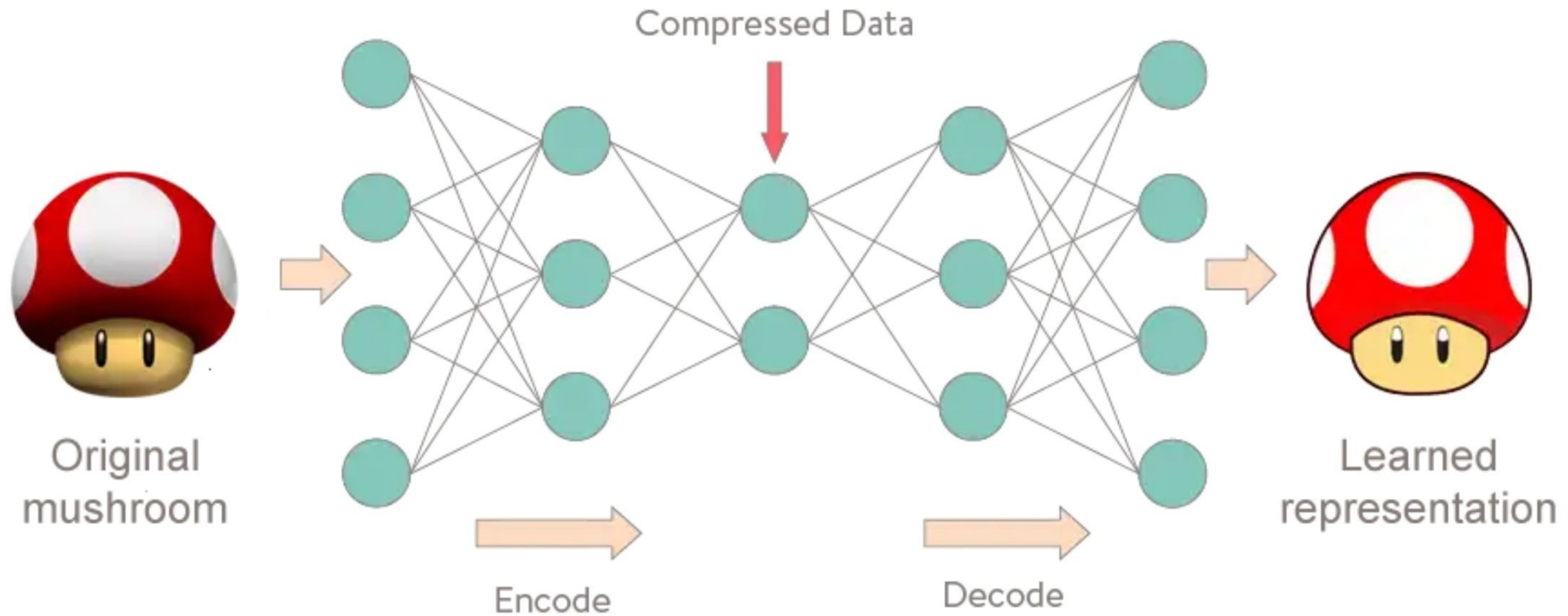


Minimize error of  $\left\| \vec{x} - \vec{x}^L \right\|$

# Autoencoders



# Autoencoders



Unsupervised learning!

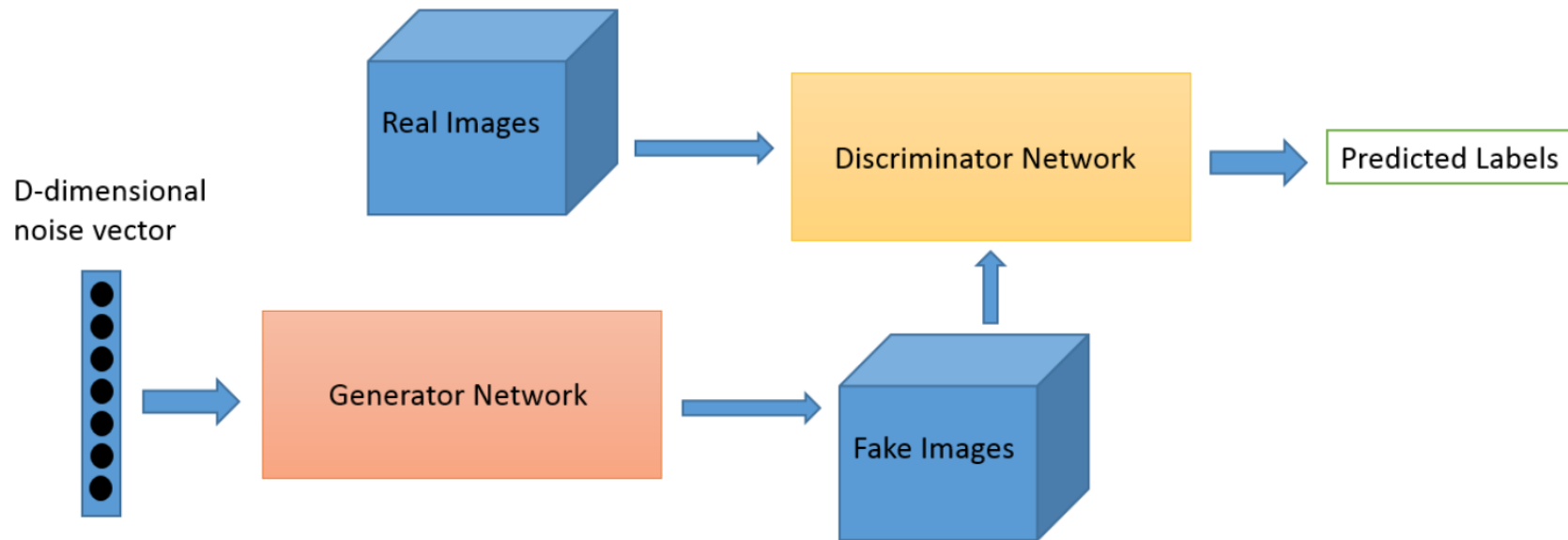
# Cool Stuffs for Deep Learning

[Safe to Skip for the exam]



# Generative Adversarial Nets (GAN)

- A Competition: Generator vs Discriminator
  - Discriminator wants to correctly classify the images (true images or not)
  - Generator wants to generate images that discriminator can't classify

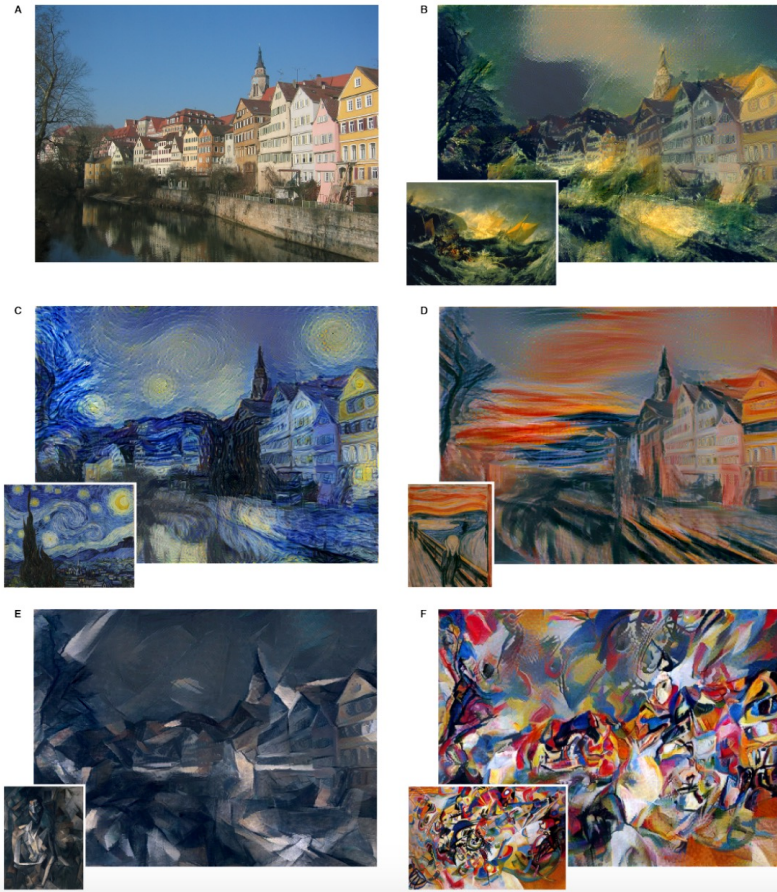


[Safe to Skip for the Exam]



<https://thisPersonDoesNotExist.com/>

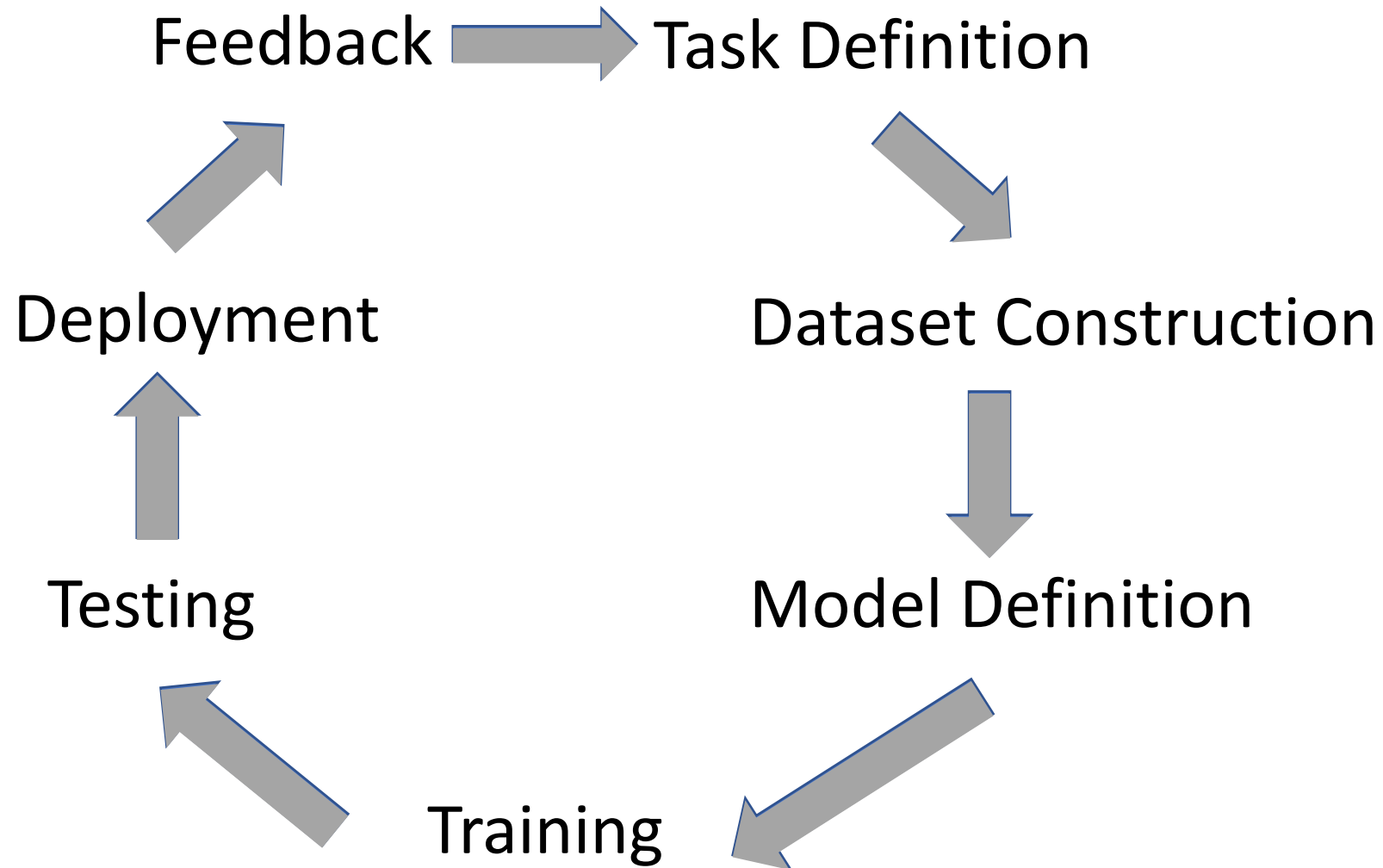
# Style Transfer



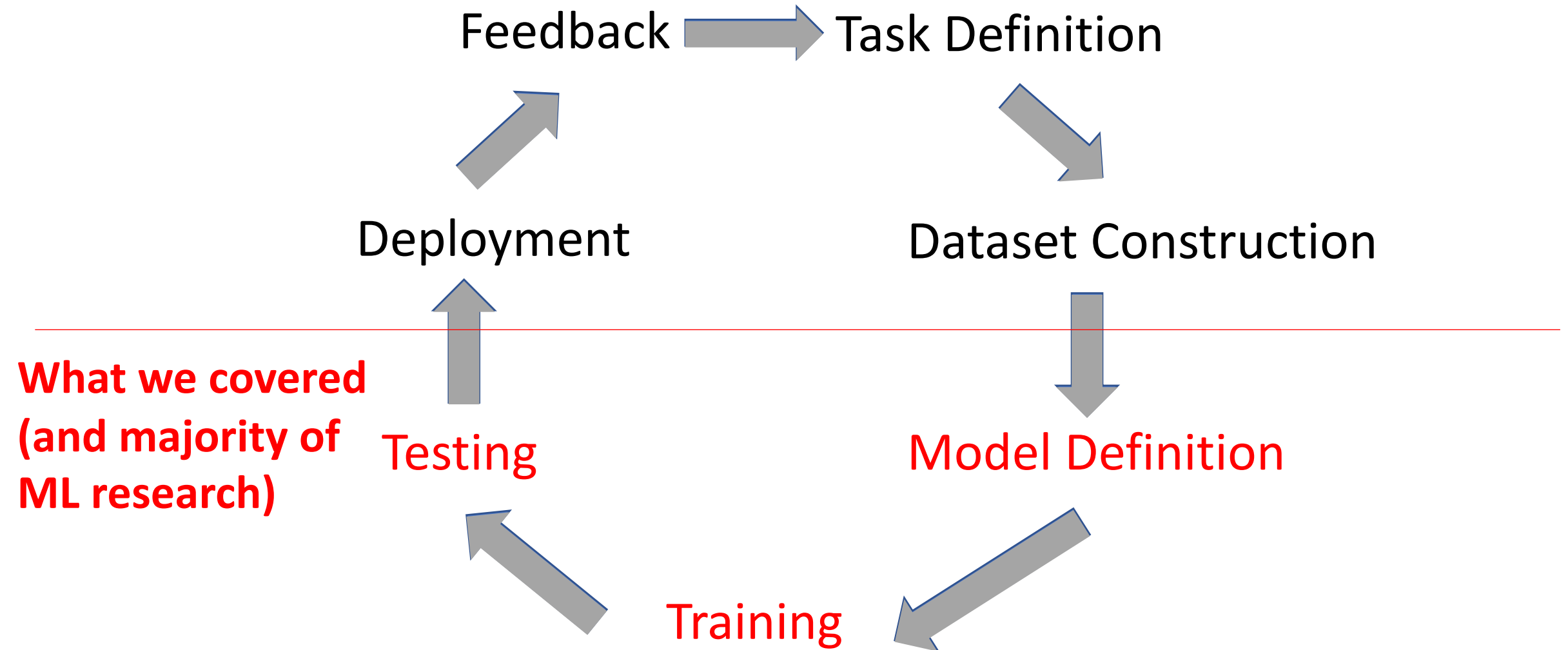
- Informal intuitions:
  - Recall that we can treat hidden layers as **feature transforms**
  - Deep learning is learning **representation** of data
  - How to achieve style transfer:
    - Learn a **content representation** for an image using hidden layers
    - Learn a **style representation** for an image using hidden layers
    - Compute an image that jointly minimizes the distance from the content image's content representation and the style image's style representation
    - <https://arxiv.org/pdf/1508.06576.pdf>

# Machine Learning Life Cycle

# Machine Learning Lifecycle



# Machine Learning Lifecycle



# Machine Learning Lifecycle

To have “positive” impacts,  
we need to be careful in  
every stage

Feedback → Task Definition

Deployment

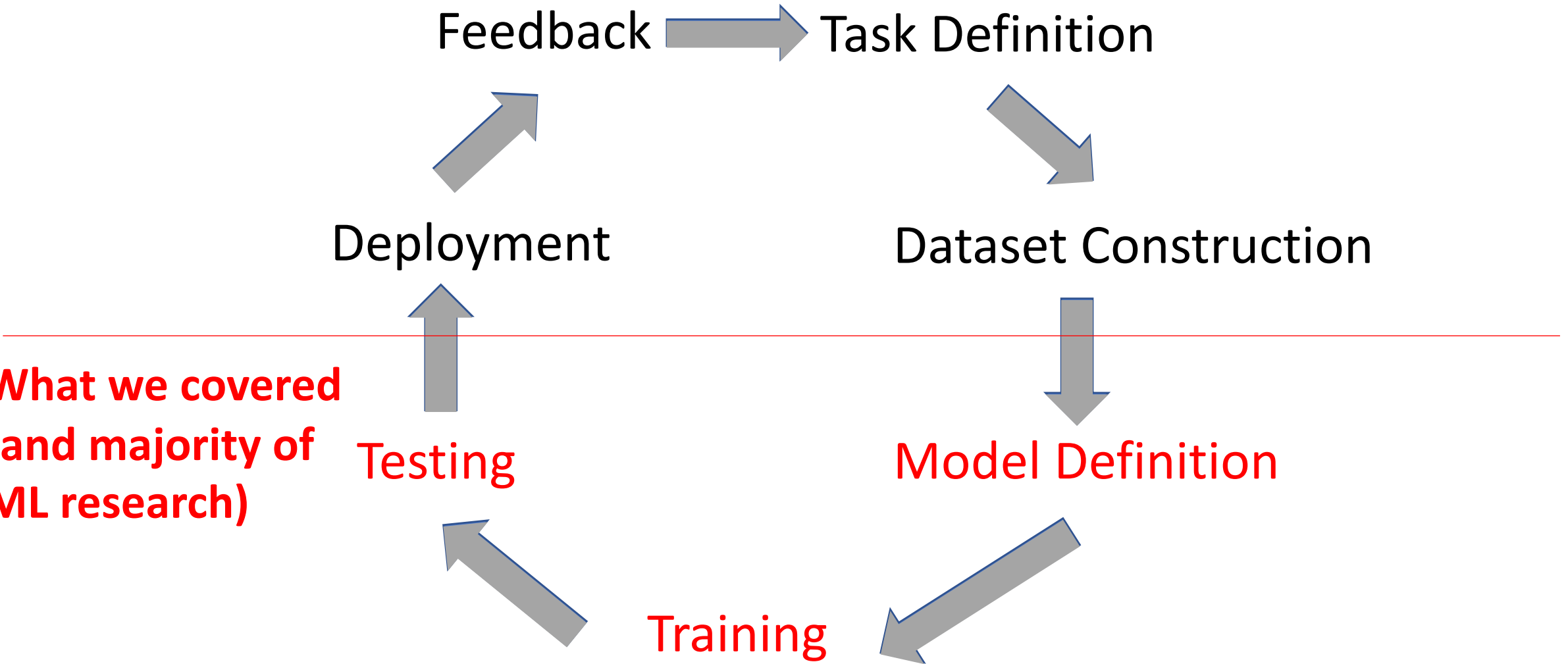
Dataset Construction

Testing

Model Definition

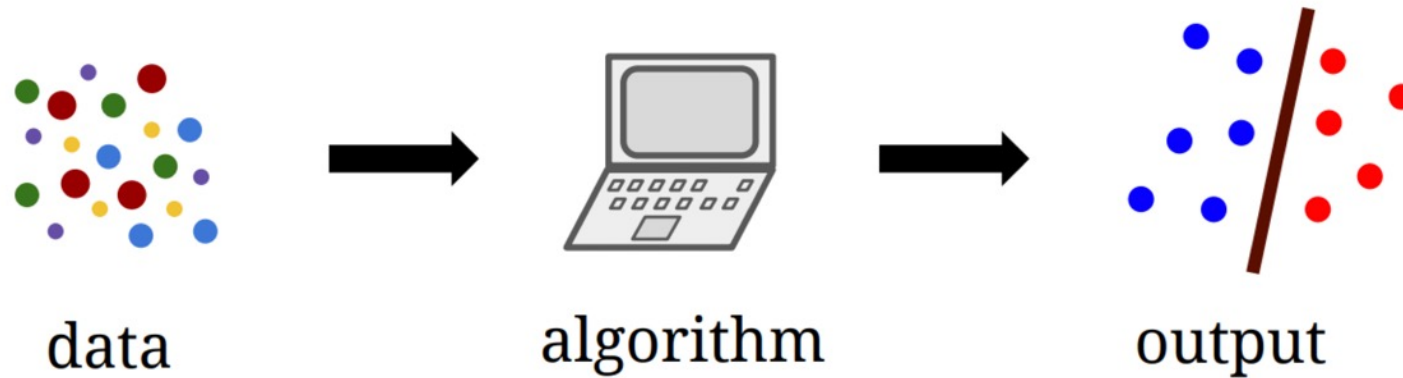
Training

What we covered  
(and majority of  
ML research)



# Supervised Learning

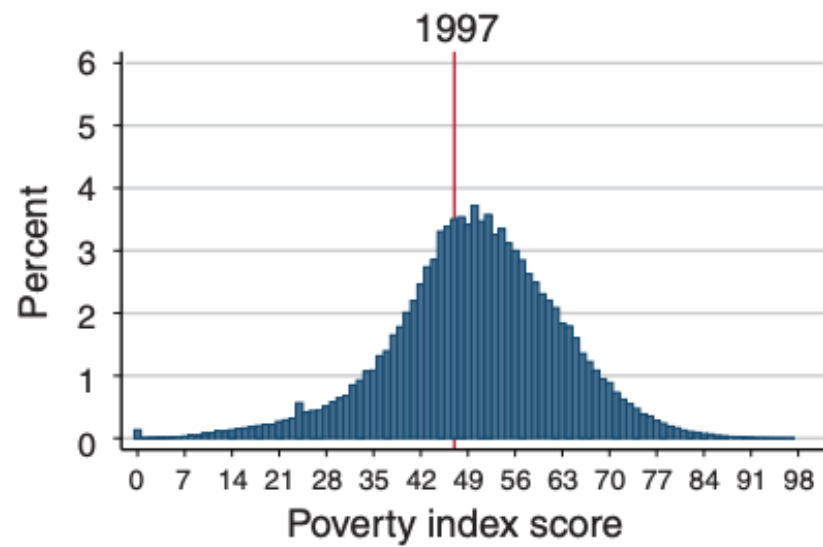
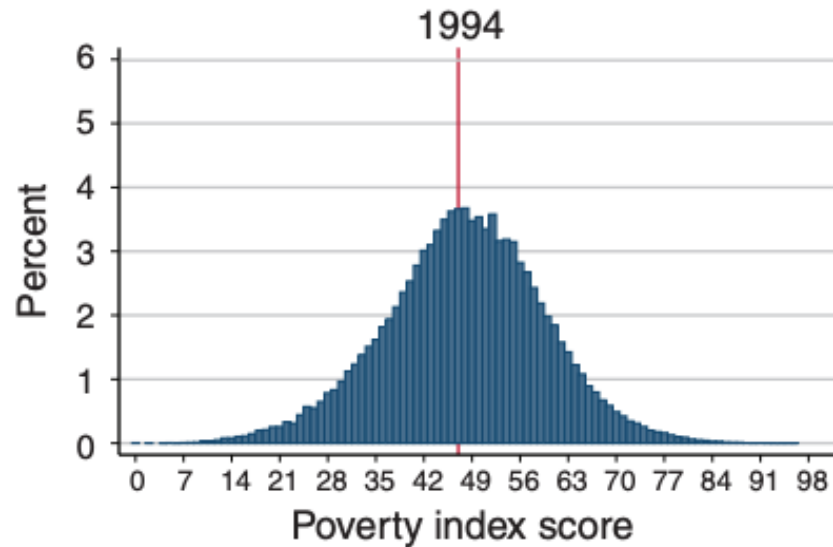
- Standard setup of (supervised) machine learning



- Finding patterns from the given training datasets
  - Use the pattern to make predictions on new testing data
- 
- Fundamental assumption:
    - Training and testing data points are i.i.d. drawn from the same distribution



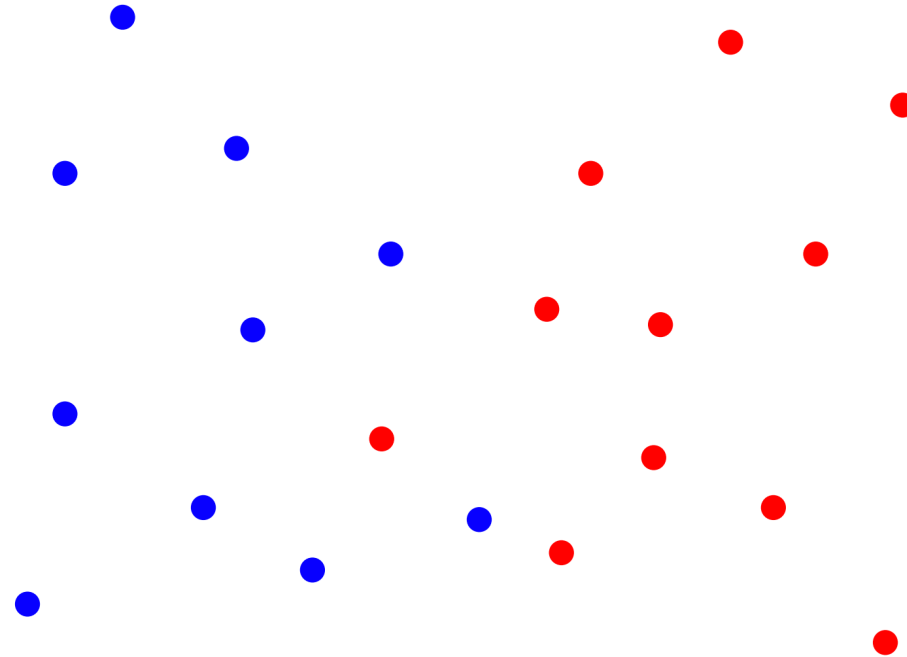
# Social Program Eligibility [Camacho and Conover, 2012]



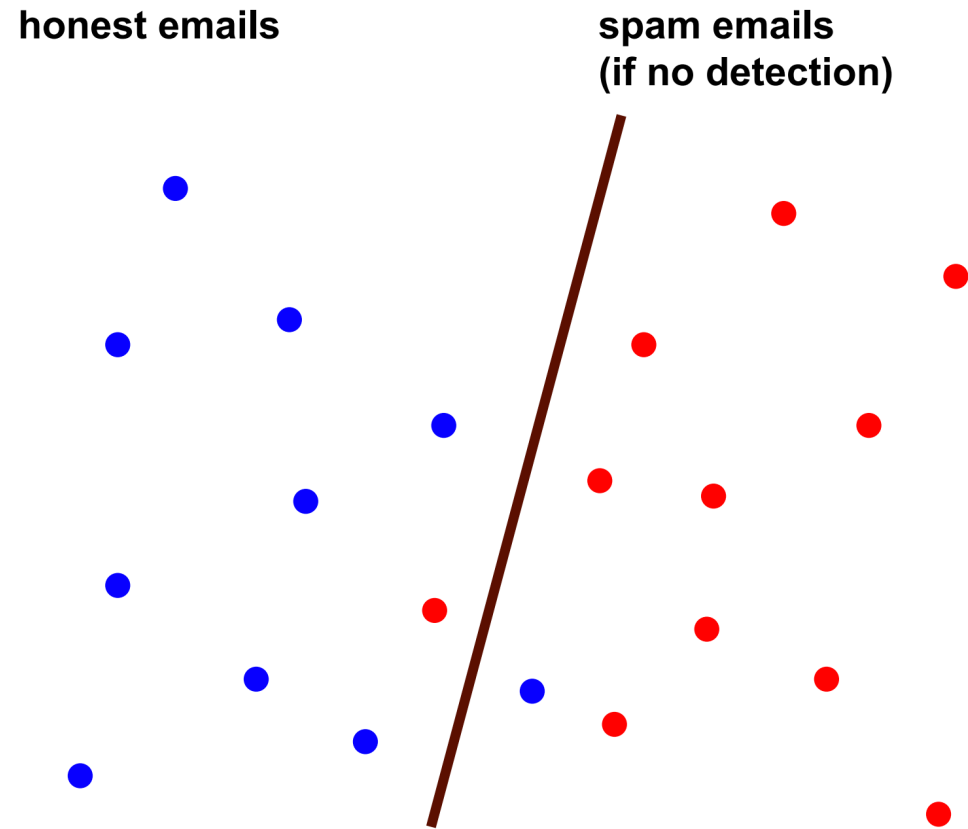
# A More ML Example: Spam Filter

**honest emails**

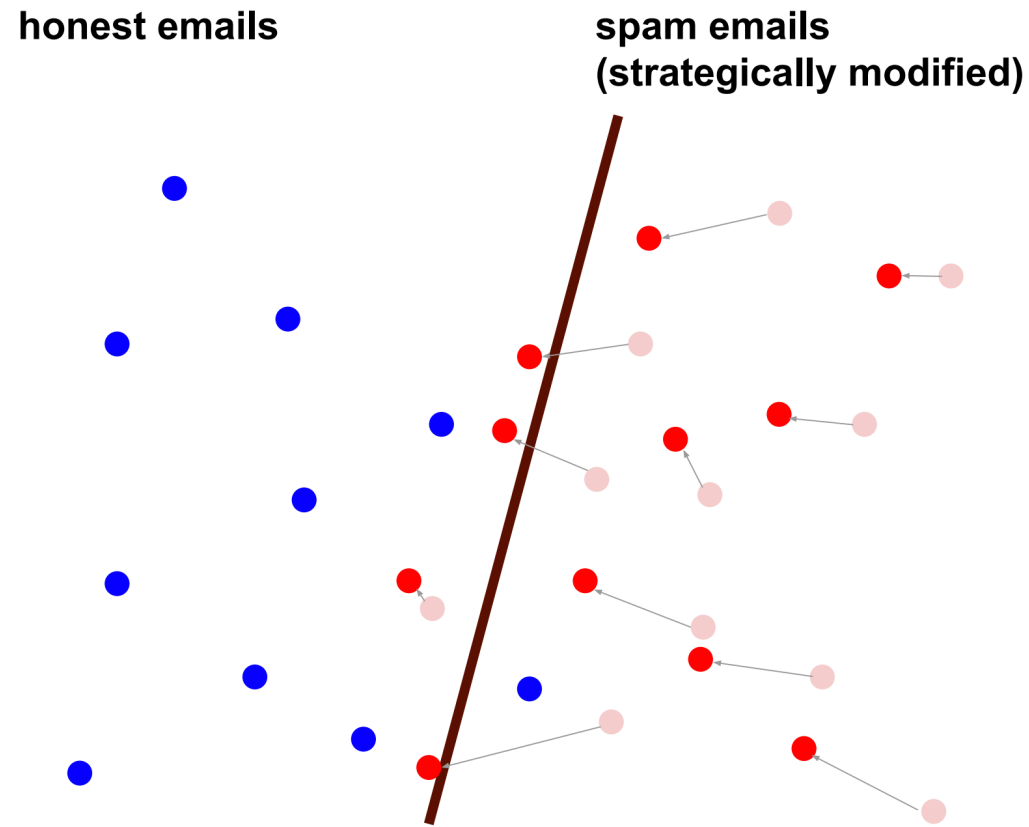
**spam emails  
(if no detection)**



# A More ML Example: Spam Filter



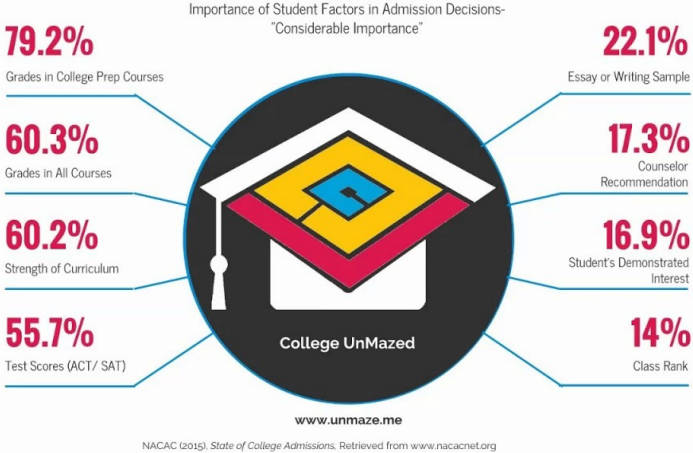
# A More ML Example: Spam Filter



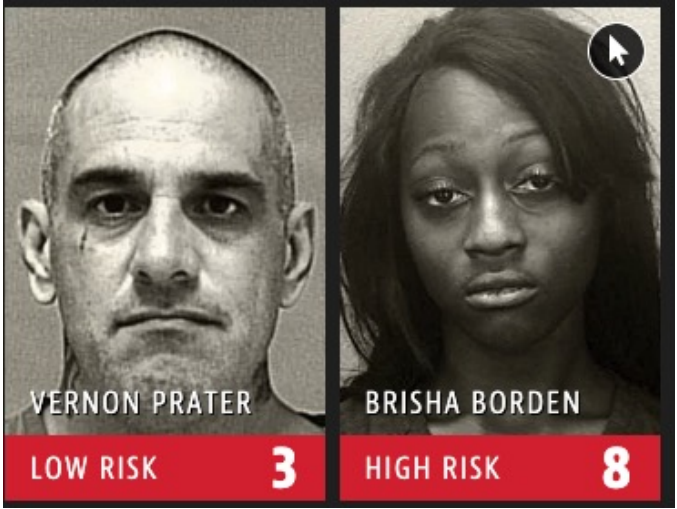
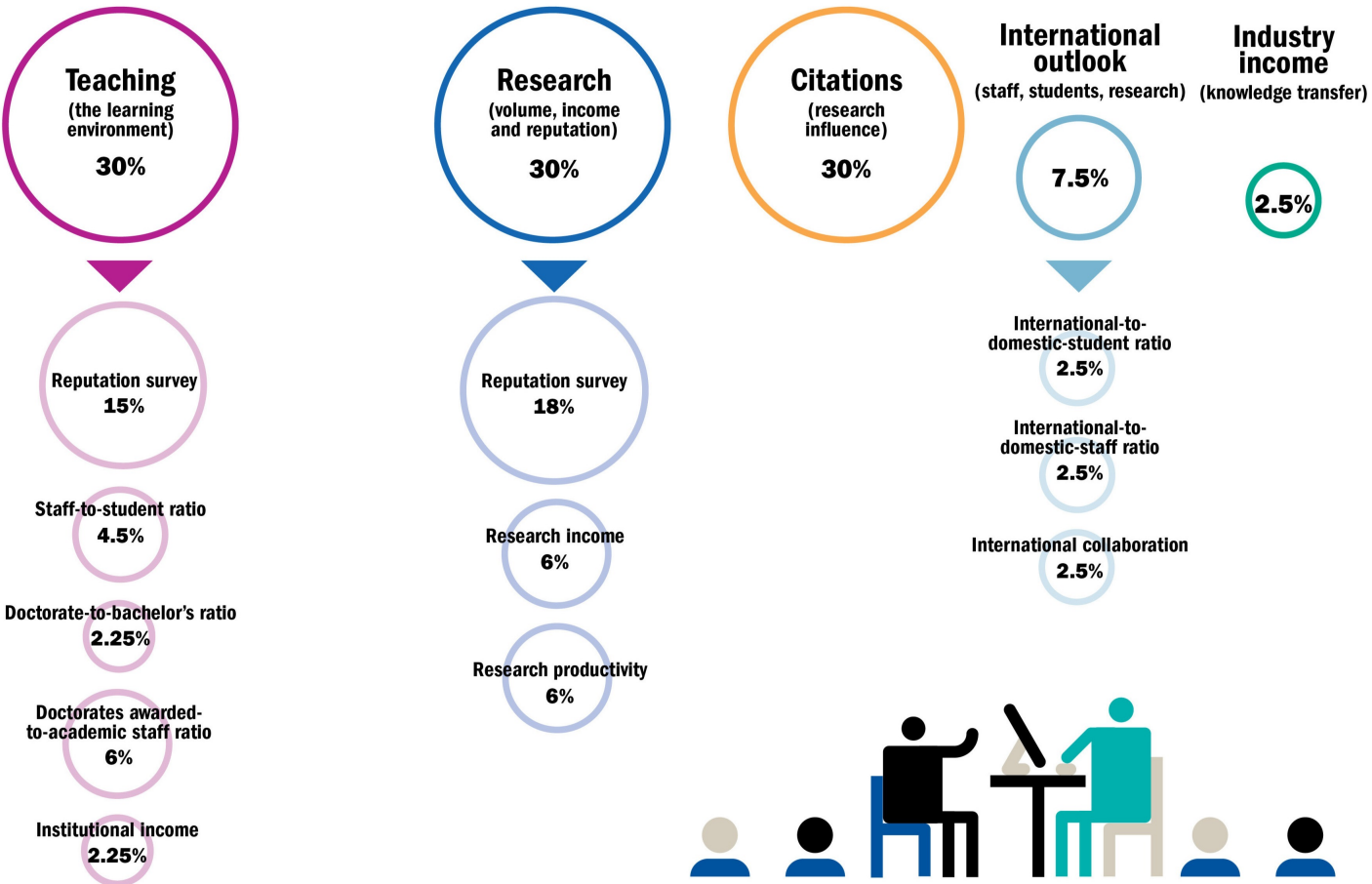
Goodhart's law:

“If a measure becomes the public's goal,  
it is no longer a good measure.”

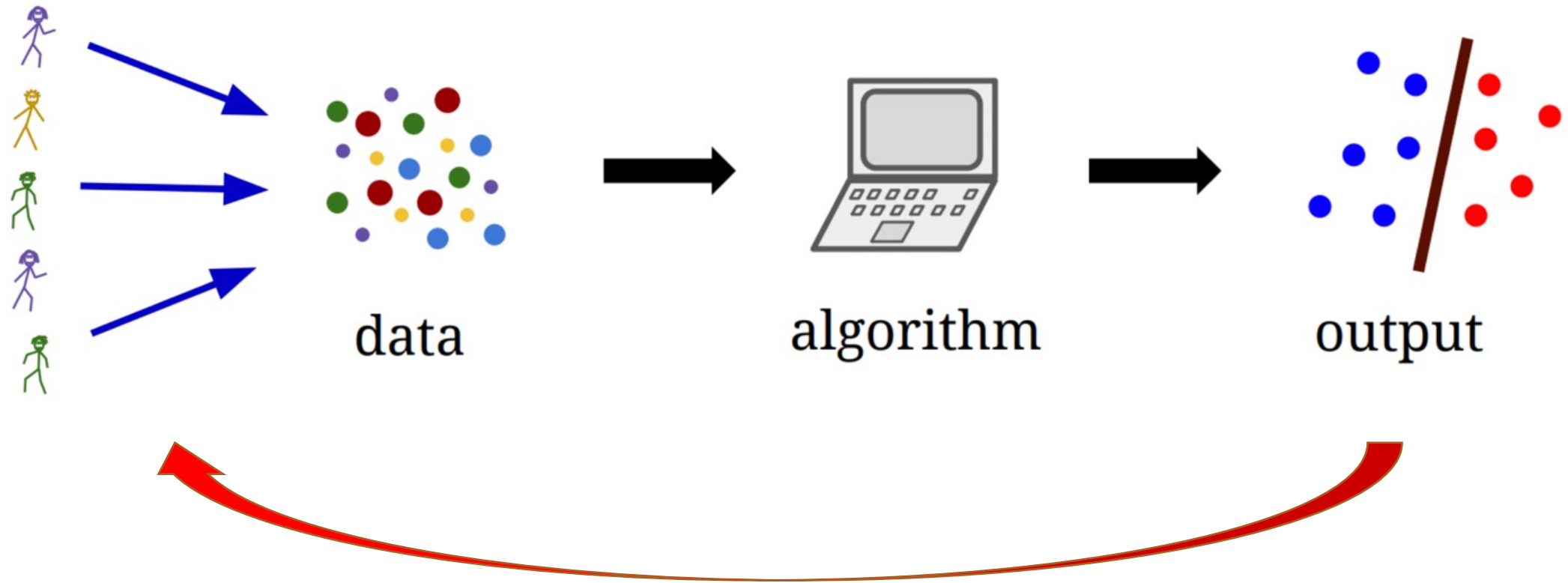
# COLLEGE ADMISSIONS



# Methodology



# Strategic Classification



# Machine Learning Lifecycle

