# CSE 417T Introduction to Machine Learning

Lecture 20

Instructor: Chien-Ju (CJ) Ho

#### Logistics

Homework 5 is due December 2 (Friday)

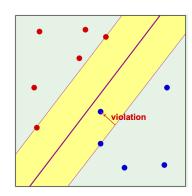
- Exam 2 will be on December 8 (Thursday)
  - Will focus on the topics in the second half of the semester
  - Format / logistics will be similar to Exam 1
  - More details to come

# Recap

#### Support Vector Machines

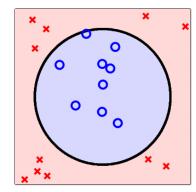
• Soft-margin SVM (approximates hard-margin SVM with  $C \to \infty$  )

minimize 
$$\overrightarrow{w},b,\overrightarrow{\xi}$$
  $\frac{1}{2}\overrightarrow{w}^T\overrightarrow{w} + C\sum_{n=1}^N \xi_n$  subject to  $y_n(\overrightarrow{w}^T\overrightarrow{x}_n + b) \ge 1 - \xi_n, \forall n$   $\xi_n \ge 0, \forall n$ 



• Kernel version of the soft-margin SVM (with Kernel  $K_{\Phi}$ )

maximize 
$$\vec{\alpha} \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_n \alpha_m y_n y_m K_{\Phi}(\vec{x}_n, \vec{x}_m)$$
 subject to  $\sum_{n=1}^{N} \alpha_n y_n = 0$   $0 \le \alpha_n \le C$ ,  $\forall n$ 



• Solve for  $\vec{\alpha}^*$  in the kernel SVM using QP

$$g(\vec{x}) = sign(\vec{w}^{*T}\Phi(\vec{x}) + b^{*})$$

$$= sign(\sum_{\alpha_{n}^{*}>0} \alpha_{n}^{*} y_{n} K_{\Phi}(\vec{x}_{n}, \vec{x}) + b^{*}),$$
where  $b^{*} = y_{m} - \sum_{\alpha_{n}^{*}>0} \alpha_{n}^{*} y_{n} K_{\Phi}(\vec{x}_{n}, \vec{x}_{m})$  for some  $\alpha_{m}^{*} > 0$ 

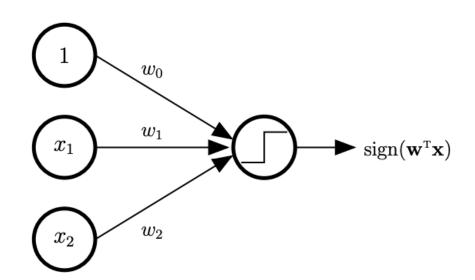
## Neural Networks

#### Perceptron

A hypothesis in Perceptron

$$h(\vec{x}) = sign(\vec{w}^T \vec{x})$$

Graphical representation of Perceptron



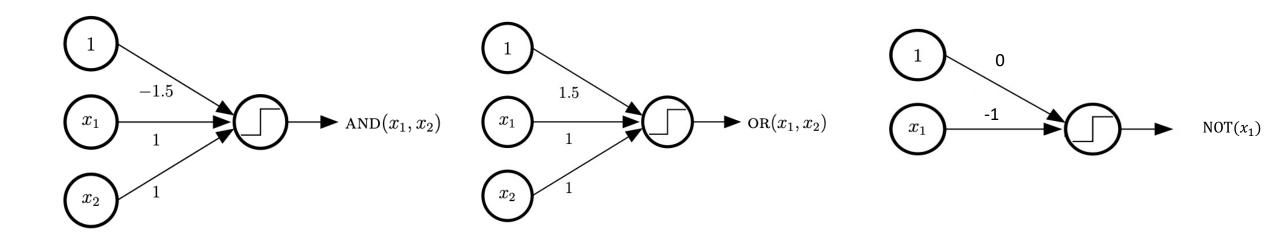
- Notations
  - $\vec{x} = (x_0, x_1, ..., x_d)$
  - $\overrightarrow{w} = (w_0, w_1, \dots, w_d)$
  - Linear separator

$$h(\vec{x}) = sign(\vec{w}^T \vec{x})$$

#### Inspired by neurons:

The output signal is triggered when the weighted combination of the inputs is larger than some threshold

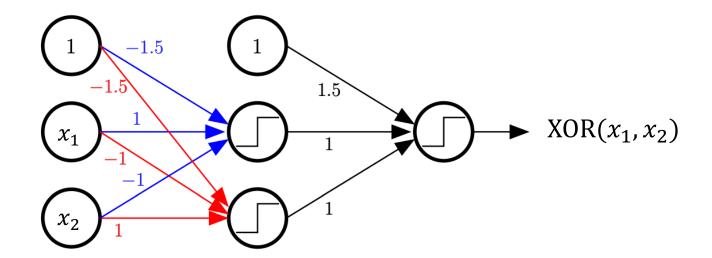
#### Implementing Logic Gates with Perceptron



Impossible to implement XOR using a single perceptron

#### Multi-Layer Perceptron

•  $XOR(x_1, x_2) \to x_1 \bar{x}_2 + \bar{x}_1 x_2$ 



- Note: you are asked to create a neural network with one hidden layer that implements XOR(AND  $(x_1, x_2), x_3$ ) in HW5
  - Hint: Try to operate the Boolean algebra first
  - Using sign as the activation function would make sense

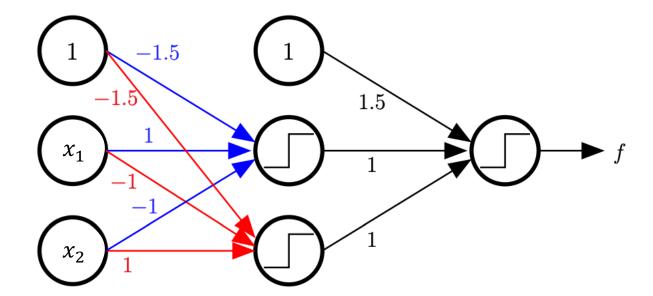
#### Universal Approximation Theorem

- A feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of  $\mathbb{R}^n$ , under mild assumptions on the activation function.
- Single-hidden-layer MLP can approximate ANY continuous target function!

- What about overfitting?
  - We'll discuss regularization methods later

#### Learn MLP From Data?

• Given D and the network structure, how to learn the "weights" (i.e., the weight vectors of every Perceptron)?



• Computationally challenging due to the "sign" function  $(\Box)$ 

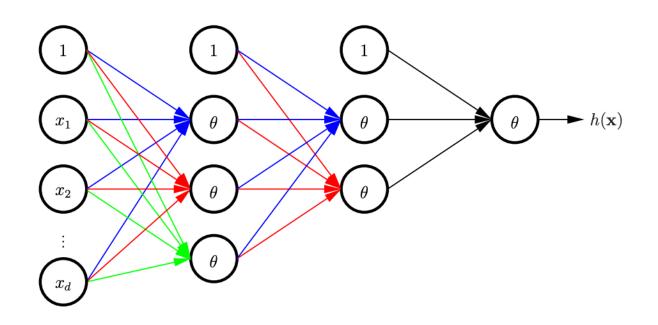


#### Neural Networks

input layer  $\ell = 0$ 

A softened version of multi-layer Perceptron (MLP)

output layer  $\ell=L$ 



hidden layers  $0 < \ell < L$ 

 $\theta$ : activation function

(Specify the "activation" of the neuron)

(The activation function in the output layer is often separately considered)

#### **Activation Function**

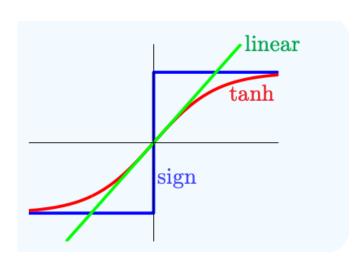
- Activation functions in Neural Networks
  - sign function: hard to optimize
  - linear function: the entire neural network is linear
  - tanh: a softened version of sign

• 
$$tanh(s) = \frac{e^s - e^{-s}}{e^s + e^{-s}}$$

• Examine tanh(s)

• 
$$tanh(s) = \begin{cases} 1 & \text{when } s \to \infty \\ 0 & \text{when } s = 0 \\ -1 & \text{when } s \to \infty \end{cases}$$

• For  $\theta(s) = \tanh(s)$ ,  $\theta'(s) = 1 - \theta(s)^2$ 



#### **Activation Function**

- There are other activation functions with different benefits. However, it doesn't impact our discussions, and we'll focus on tanh() as the activation function
- A few more examples

ArcTan	$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)	$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \ge 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \ge 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) <sup>[2]</sup>	$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \ge 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \ge 0 \end{cases}$
Exponential Linear Unit (ELU) <sup>[3]</sup>	$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \ge 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \ge 0 \end{cases}$
SoftPlus	$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$

https://towardsdatascience.com/activation-functions-neural-networks-1cbd9f8d91d6

# Today's Lecture

The notes are not intended to be comprehensive. They should be accompanied by lectures and/or textbook. Let me know if you spot errors.

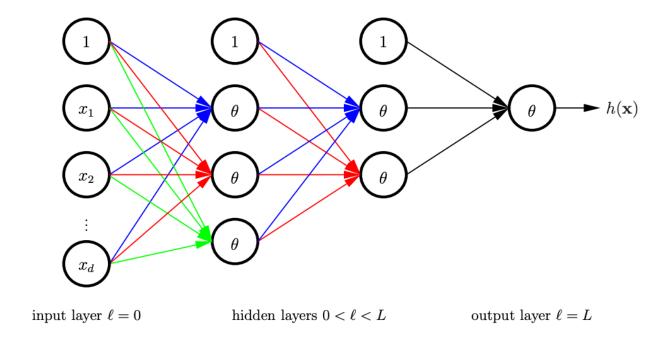
## Goal of Today

Formally characterize Neural Networks (introduce notations)

• Given a Neural Network hypothesis h, how do we make prediction  $h(\vec{x})$ 

• Given D, how do we learn a Neural Network hypothesis

- Layers  $\ell = 0$  to L
  - Layer 0: input layer
  - Layer 1 to L-1: hidden layers
  - Layer *L*: output layer
- $d^{(\ell)}$ : dimension of layer  $\ell$ 
  - # nodes (excluding 1s) in the layer
- $\vec{x}^{(\ell)}$ : the nodes in layer  $\ell$ 
  - $\vec{x}^{(0)}$  is the input feature  $\vec{x}$
  - $x_i^{(\ell)}$  is the *i*-th node in layer  $\ell$



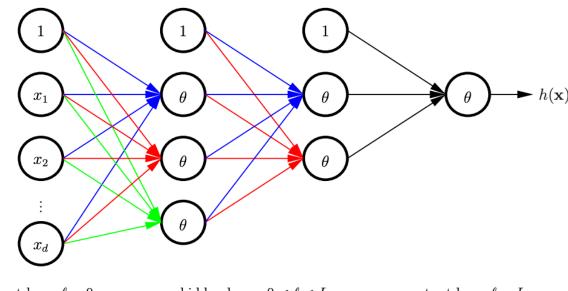
- A hypothesis in linear model is specified by the weights  $\{w_i\}$
- Similarly, a hypothesis in NN is characterized by the weights  $\{w_{i,j}^{(\ell)}\}$ 
  - $1 \le \ell \le L$

- layers
- $0 \le i \le d^{(\ell-1)}$

inputs

•  $1 \le j \le d^{(\ell)}$ 

outputs

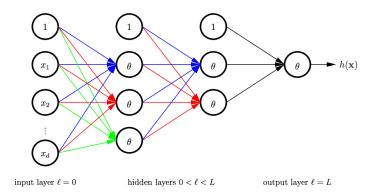


input layer  $\ell = 0$ 

hidden layers  $0 < \ell < L$ 

output layer  $\ell=L$ 

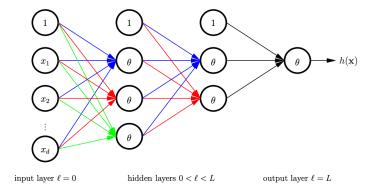
- Notations so far:
  - $d^{(\ell)}$ : dimension of layer  $\ell$
  - $\vec{x}^{(\ell)}$ : the nodes in layer  $\ell$
  - $w_{i,j}^{(\ell)}$ : weights; characterize hypothesis in NN



- Lastly, linear signal  $s_j^{(\ell)} = \sum_{i=0}^{d^{(\ell-1)}} w_{i,j}^{(\ell)} x_i^{(\ell-1)}$ 
  - By definition:  $x_j^{(\ell)} = \theta(s_j^{\ell})$

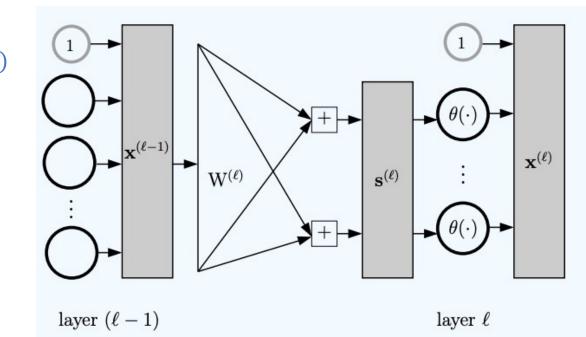
$$\mathbf{s}^{(\ell)} \xrightarrow{\theta} \mathbf{x}^{(\ell)}$$

- Notations so far:
  - $d^{(\ell)}$ : dimension of layer  $\ell$
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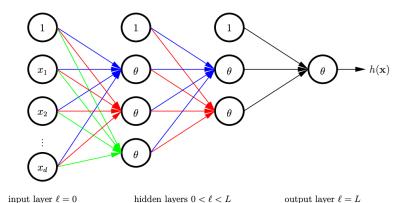
$$\mathbf{s}^{(\ell)} \stackrel{ heta}{-\!\!\!-\!\!\!\!-\!\!\!\!-} \mathbf{x}^{(\ell)}$$



# Short Break and Q&A

#### Practice:

For a neural network with L=2,  $d^{(0)}=3$ ,  $d^{(1)}=2$ ,  $d^{(2)}=1$ , what is the total # weights?



#### Notations so far:

 $d^{(\ell)}$ : dimension of layer  $\ell$ 

 $\vec{x}^{(\ell)}$ : the nodes in layer  $\ell$ 

 $w_{i,j}^{(\ell)}$ : weights; characterize hypothesis in NN

$$s_j^{(\ell)} = \sum_{i=0}^{d^{(\ell-1)}} w_{i,j}^{(\ell)} x_i^{(\ell-1)}$$
: linear signal

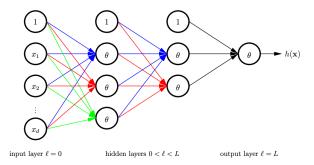
# Forward Propagation

Given a NN hypothesis and a point  $\vec{x}$ , how do we make predictions

# Backpropagation

Learn a Neural Network hypothesis from data

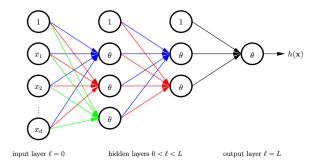
#### Forward Propagation



- A Neural network hypothesis h is characterized by  $\left\{w_{i,j}^{(\ell)}\right\}$
- How to evaluate  $h(\vec{x})$ ?

$$\mathbf{x} = \mathbf{x}^{(0)} \xrightarrow{\mathrm{W}^{(1)}} \mathbf{s}^{(1)} \xrightarrow{\theta} \mathbf{x}^{(1)} \xrightarrow{\mathrm{W}^{(2)}} \mathbf{s}^{(2)} \xrightarrow{\theta} \mathbf{x}^{(2)} \cdots \xrightarrow{\mathrm{W}^{(L)}} \mathbf{s}^{(L)} \xrightarrow{\theta} \mathbf{x}^{(L)} = h(\mathbf{x}).$$

#### Forward Propagation



- A Neural network hypothesis h is characterized by  $\left\{w_{i,j}^{(\ell)}\right\}$
- How to evaluate  $h(\vec{x})$ ?

$$\mathbf{x} = \mathbf{x}^{(0)} \xrightarrow{\mathbf{w}^{(1)}} \mathbf{s}^{(1)} \xrightarrow{\theta} \mathbf{x}^{(1)} \xrightarrow{\mathbf{w}^{(2)}} \mathbf{s}^{(2)} \xrightarrow{\theta} \mathbf{x}^{(2)} \cdots \xrightarrow{\mathbf{w}^{(L)}} \mathbf{s}^{(L)} \xrightarrow{\theta} \mathbf{x}^{(L)} = h(\mathbf{x}).$$

```
Forward propagation to compute h(\mathbf{x}):

\mathbf{x}^{(0)} \leftarrow \mathbf{x} \qquad \qquad \text{[Initialization]}
\mathbf{for} \ \ell = 1 \text{ to } L \text{ do} \qquad \qquad \text{[Forward Propagation]}
\mathbf{s}^{(\ell)} \leftarrow (\mathbf{W}^{(\ell)})^{\mathrm{T}} \mathbf{x}^{(\ell-1)}
\mathbf{x}^{(\ell)} \leftarrow \begin{bmatrix} 1 \\ \theta(\mathbf{s}^{(\ell)}) \end{bmatrix}
\mathbf{s} \text{ end for}
\mathbf{s} \ h(\mathbf{x}) = \mathbf{x}^{(L)} \qquad \qquad \text{[Output]}
```

Given weights  $w_{i,j}^{(\ell)}$  and  $\vec{x}^{(0)} = \vec{x}$ , we can calculate all  $\vec{x}^{(\ell)}$  and  $\vec{s}^{(\ell)}$  through forward propagation.

#### How to Learn NN From Data?

- Given D, how to learn the weights  $W = \{w_{i,j}^{(\ell)}\}$ ?
- Intuition: Minimize  $E_{in}(W) = \frac{1}{N} \sum_{n=1}^{N} e_n(W)$
- How?
  - Gradient descent:  $W(t+1) \leftarrow W(t) \eta \nabla_W E_{in}(W)$
  - Stochastic gradient descent  $W(t+1) \leftarrow W(t) \eta \nabla_W e_n(W)$

- Key step: we need to be able to evaluate the gradient...
  - Not trivial to do given the network structure
  - Backpropagation is an algorithmic procedure to calculate the gradient

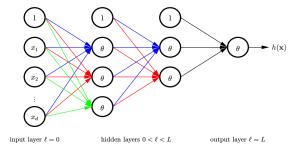
# Backpropagation

Use dynamic programming to evaluate the gradient

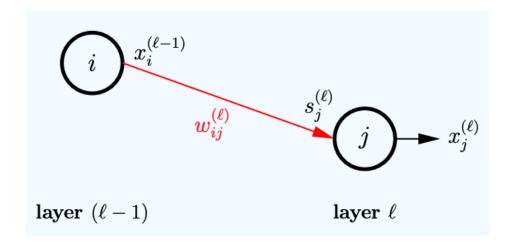
## Quick Reminders on Dynamic Programming

- Example: Fibonacci number
  - $F_n = F_{n-1} + F_{n-2}$  for  $n \ge 2$
  - $F_0 = 0, F_1 = 1$
  - To evaluate  $F_N$ 
    - Recursively apply the definition
      - Wasted computation
    - Dynamic programming: evaluate and store  $F_0$ ,  $F_1$ , ...,  $F_N$ 
      - Use space to exchange for time
- Key step in backpropagation
  - Find a recursive definition of some key quantities
  - Solve the boundary conditions
  - Adopt dynamic programming

# Compute the Gradient $\nabla_W e_n(W)$



- To evaluate  $\nabla_W e_n(W)$ , we need to calculate  $\frac{\partial e_n(W)}{\partial w_{i,i}^{(\ell)}}$  for all  $(i,j,\ell)$
- Zoom in on the region around  $w_{i,i}^{(\ell)}$



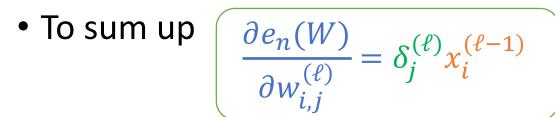
• Apply chain rule
$$\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}} \frac{\partial s_j^{(\ell)}}{\partial w_{i,j}^{(\ell)}}$$

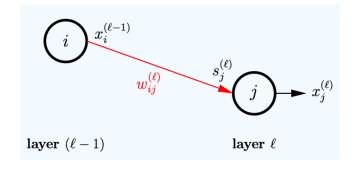
## Compute the Gradient $V_W e_n(W)$

Apply chain rule

$$\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}} \frac{\partial s_j^{(\ell)}}{\partial w_{i,j}^{(\ell)}}$$

- Let's look at the second term first
  - Remember  $s_i^{(\ell)} = \sum_{i=0}^{d^{(\ell-1)}} w_{i,i}^{(\ell)} x_i^{(\ell-1)}$
  - Therefore,  $\frac{\partial s_j^{(\ell)}}{\partial w_{i,j}^{(\ell)}} = x_i^{(\ell-1)}$

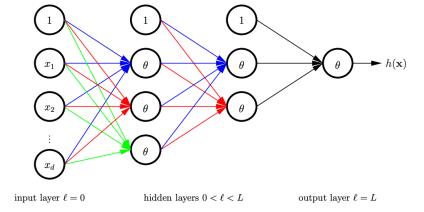




- What about the first term?
  - Let's define  $\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_i^{(\ell)}}$
  - We'll apply dynamic programming style algorithm to deal with this term

Compute 
$$\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}}$$

- Using dynamic programming style approach
  - Check boundary case (what is the boundary case?)
  - Write the recursive formulation



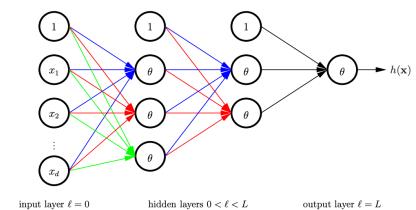
- Check boundary case (when  $\ell = L$ )
  - Output layer
  - For simplicity, assume we are doing regression and the error is squared error

• 
$$e_n(W) = \left(s_1^{(L)} - y_n\right)^2$$
 (Usually only one node in the output layer)

- $\delta_1^{(L)} = 2(s_1^{(L)} y_n)$  (similar discussion applies for other differentiable error function)
- So the boundary condition at L is checked.
- Next we will derive the backward recursive formulation (hence, backpropagation)

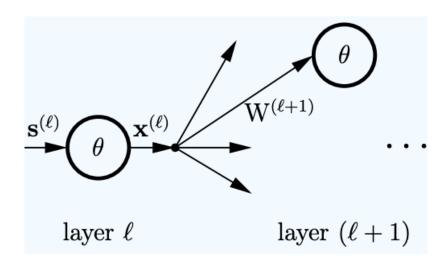
Compute 
$$\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}}$$

Zoom in to see the chain of dependencies

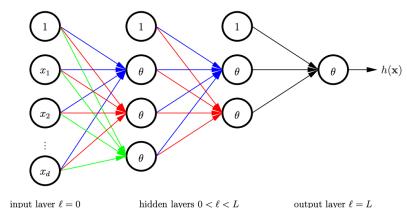


Compute 
$$\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}}$$

Zoom in to see the chain of dependencies

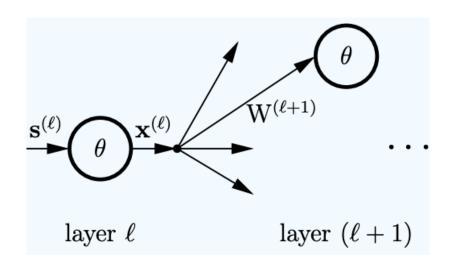


$$\mathbf{s}^{(\ell)} \longrightarrow \mathbf{x}^{(\ell)} \longrightarrow \mathbf{s}^{(\ell+1)}$$

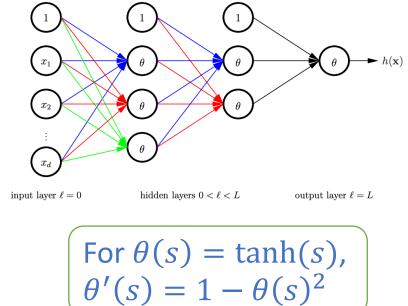


Compute 
$$\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}}$$

Zoom in to see the chain of dependencies



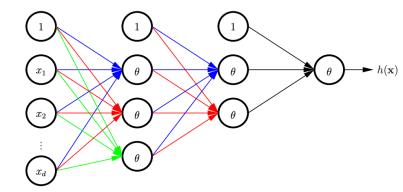
$$\mathbf{s}^{(\ell)} \longrightarrow \ \mathbf{x}^{(\ell)} \ \longrightarrow \ \mathbf{s}^{(\ell+1)}$$



$$\hat{S}_{j}^{(\ell)} = \frac{\partial e_{n}(W)}{\partial s_{j}^{(\ell)}} = \frac{\partial e_{n}(W)}{\partial s_{j}^{(\ell+1)}} \frac{\partial e_{n}(W)}{\partial s_{k}^{(\ell+1)}} \frac{\partial s_{k}^{(\ell+1)}}{\partial s_{j}^{(\ell)}} \frac{\partial x_{j}^{(\ell)}}{\partial s_{j}^{(\ell)}} = \sum_{k=1}^{d(\ell+1)} \frac{\delta_{k}^{(\ell+1)}}{\delta_{k}^{(\ell+1)}} \frac{\partial s_{k}^{(\ell+1)}}{\partial s_{j}^{(\ell)}} \frac{\partial x_{j}^{(\ell)}}{\partial s_{j}^{(\ell)}} = \sum_{k=1}^{d(\ell+1)} \frac{\delta_{k}^{(\ell+1)}}{\delta_{k}^{(\ell+1)}} w_{j,k}^{(\ell+1)} \theta' \left(s_{j}^{(\ell)}\right)$$

We have the backward recurve definition!

Compute 
$$\delta_j^{(\ell)} = \frac{\partial e_n(W)}{\partial s_j^{(\ell)}}$$



- We can calculate  $\delta_j^{(\ell)}$  in a dynamic programming manner:
- Boundary condition:  $\delta_1^{(L)} = 2(s_1^{(L)} y_n)$
- Recursive formulation:  $\delta_j^{(\ell)} = \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)} \theta' \left(s_j^{(\ell)}\right)$
- Calculate  $\delta_i^{(\ell)}$  for  $\ell < L$  in a backward manner

#### Backpropagation Algorithm

- Recall that  $\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$
- Backpropagation Algorithm
  - Initialize  $w_{i,j}^{(\ell)}$  randomly [You will discuss the impacts of initialization in HW5]
  - For t = 1 to T
    - Randomly pick a point from D (for stochastic gradient descent)
    - Forward propagation: Calculate all  $x_i^{(\ell)}$  and  $s_i^{(\ell)}$
    - Backward propagation: Calculate all  $\delta_i^{(\ell)}$
    - Update the weights  $w_{i,j}^{(\ell)} \leftarrow w_{i,j}^{(\ell)} \eta \delta_j^{(\ell)} x_i^{(\ell-1)}$
  - Return the weights

#### Discussion

- Backpropagation is gradient descent with efficient gradient computation
- Note that the  $E_{in}$  is not convex in weights
- Gradient descent doesn't guarantee to converge to global optimal

- Common approaches:
  - Run it many times
  - Each with a different initialization (the choice of initialization matters)
    - Initialization matters (more discussion next lecture)
    - Initializing at 0 is not a good choice (Q6b of HW5)
    - Initializing at larger weights is not a good idea for tanh as activation function (Q6a of HW5)

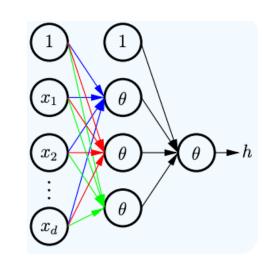
## Single Hidden-Layer Neural Network

How do we write a hypothesis in single-hidden layer mathematically?

• 
$$h(\vec{x}) = \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} x_j^{(1)} \right)$$
  
 $= \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} \theta \left( \sum_{i=0}^{d^{(0)}} w_{i,j}^{(1)} x_i \right) \right)$ 

 How do we write a Kernel SVM hypothesis (linear model with nonlinear transformation)

• 
$$g(\vec{x}) = \theta \left( b^* + \sum_{\alpha_n^* > 0} \alpha_n^* y_n K(\vec{x}_n, \vec{x}) \right)$$



- Interpretation:
  - The hidden layer is like "feature transform"
  - Shallow learning vs. deep learning
  - More discussion on neural networks and deep learning next lecture

### Neural Network is Expressive

- Universal approximation theorem:
  - A feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of  $\mathbb{R}^n$ , under mild assumptions on the activation function.
  - A single-hidden-layer NN can approximate ANY continuous target function!

• We also seem to only discuss how to minimize  $E_{in}$ 

What about overfitting?

# Regularization in Neural Networks

### Weight-Based Regularization

Weight decay

$$E_{aug}(W) = E_{in}(W) + \frac{\lambda}{N} \sum_{i,j,\ell} \left( w_{i,j}^{(\ell)} \right)^2$$

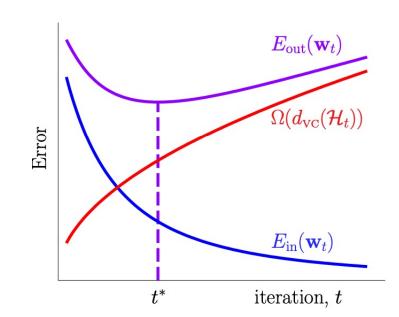
Weight elimination

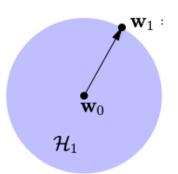
$$E_{aug}(W) = E_{in}(W) + \frac{\lambda}{N} \sum_{i,j,\ell} \frac{\left(w_{i,j}^{(\ell)}\right)^2}{1 + \left(w_{i,j}^{(\ell)}\right)^2}$$

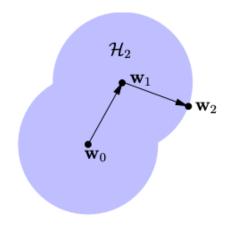
- When  $w_{i,j}^{(\ell)}$  is small, approximates weight decay
- When  $w_{i,j}^{(\ell)}$  is large, approximates adding a constant (no impacts to gradient)
- "Decaying" more on smaller weights (i.e., eliminating small weights)

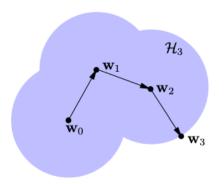
## Early Stopping

- Consider gradient descent (GD)
  - $H_1$ : the set of hypothesis GD can reach at t=1
  - $H_2$ : the set of hypothesis GD can reach at t=2
  - •
  - $H_1 \subseteq H_2 \subseteq H_3 \subseteq \cdots$



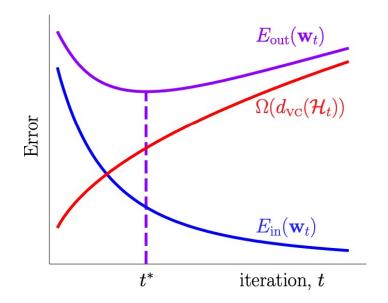


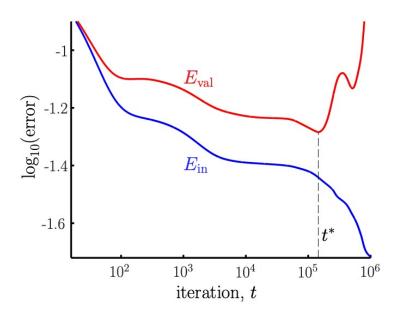




## Early Stopping

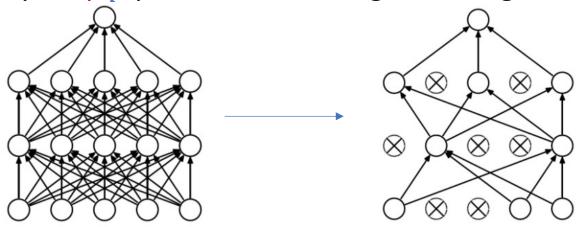
- Stopping gradient descent early is a regularization method
  - Constrain the hypothesis set
- How to find the optimal stopping point t\*?
  - Using validation is a common approach





### Dropout

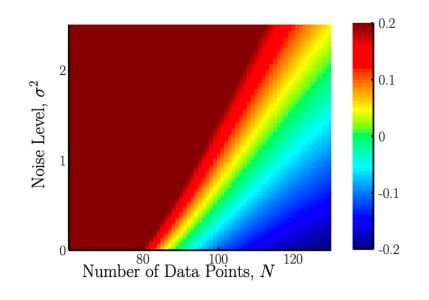
- Neural networks is very expressive (low bias, potentially high variance)
- Dropout
  - Randomly drop p portion of the weights during training

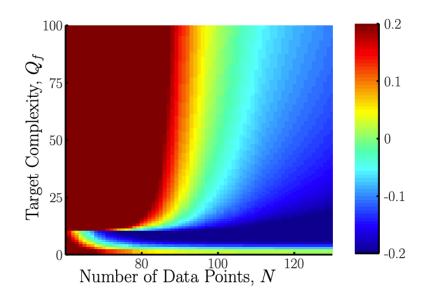


- Learn many models with dropout
- Average them during prediction (reduce weights by a ratio of p)

### A Nontraditional Method to Avoid Overfitting

What's the cause of overfitting?

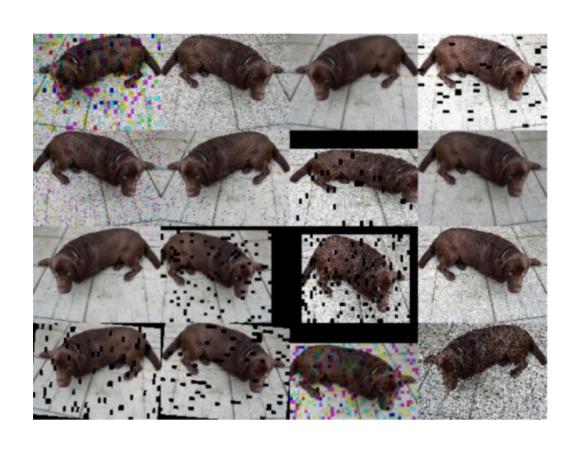




- Fitting the noise instead of the target
- Regularization: Constrain H so it's not that powerful to fit noise
- How about adding noises to data?

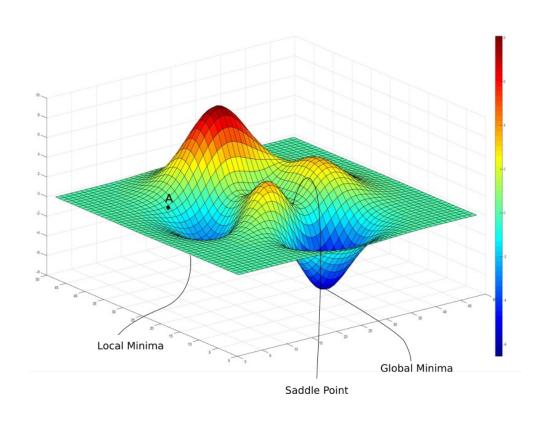
# Adding Noises as Regularization





# Initialization

### Error is Nonconvex in Neural Networks



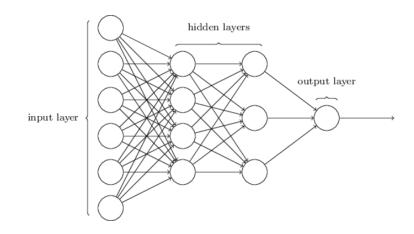
- We mostly adopt gradient-descent-style algorithms for optimization.
- No guarantee to converge to global optimal.
- Need to run it many times.
- Initialization matters!

### Vanishing Gradient Problem

Backpropagation

• 
$$\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$$

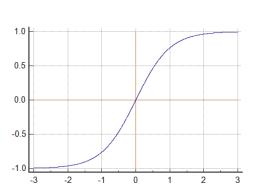
• 
$$\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$$



• If we use activation function  $\theta(s) = \tanh(s)$ 

• 
$$\theta'(s) = 1 - \theta(s)^2 < 1$$

- In deep learning with a lot of layers,
  - the gradient might vanish
  - hard to update the early layers



### Vanishing Gradient Problem

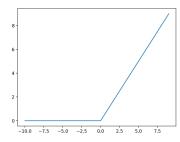
• 
$$\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$$

There is also a corresponding "exploding gradient problem"

- What can we do
  - Choose more suitable activation functions
    - One common choice is Rectified Linear Unit (ReLU) and its variant

• 
$$\theta(s) = \max(0, s)$$

- Choose better initialization
  - Many approaches

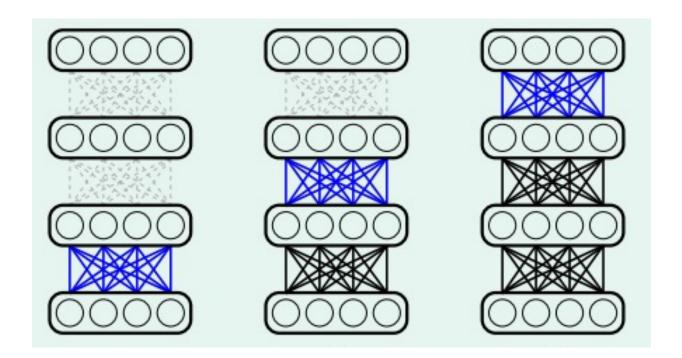


### Weight Initialization

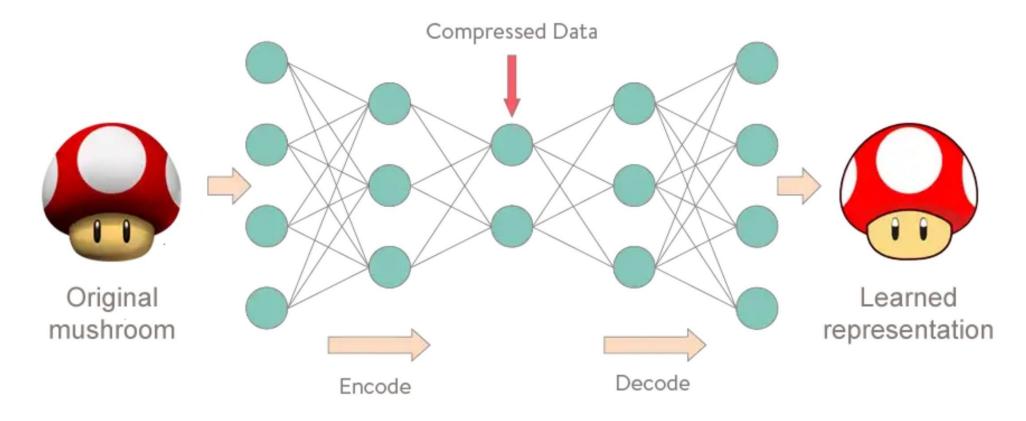
- Initializing weights to 0 is a bad idea
  - Q5 of HW1
- Randomly Initializing weights to regions so that vanishing/exploding gradients are less likely to happen
  - Activation-function dependent
    - e.g., Xavier initialization for tanh
- Learning the initialization
  - E.g., autoencoder

#### Initialization

- Hard to initialize the entire network well.
- Intuition: Initialize the weights layer by layer such that each layer preserves the properties of the previous layer.



#### Autoencoder



## Unsupervised learning!