

# Lecture 8:

## Modeling Humans: Incentive Design

Instructor: Chien-Ju (CJ) Ho

# Logistics: Project Proposal

- Due this Friday
- Topics: Focus on “humans” in computation
  - I might suggest/require changes after the proposal
  - You may check the list of example/past projects
- Logistics
  - **Include all teammates using Gradescope interface**
  - See the link on the website for the requirements
    - <http://chienjuho.com/courses/cse518a/project-proposal.pdf>

# Logistics: Assignment 3

- Assignment 3 has been posted on the website
  - 3 long-ish math questions that extend the lecture today
- Due: Oct 12 (Wed)
  - A bit less than 2 weeks after the Assignment 2 deadline
  - To avoid clashing with the deadline of project milestone 1 (Oct 14)

# Logistics: Presentation

- For presenters:
  - Give a **55~60 min** presentation based on the **required reading** and at least **two optional reading** (3 optional readings for 3-person groups) of a lecture
    - The papers are the “backbone” of the presentation
  - Prepare **2 reading questions** for the required reading
  - Prepare **at least 2 discussion sessions**
  - Lead the discussion for the discussion sessions
  - Template format (if you are not sure what to do):
    - Explain the required reading (5~10 min)
    - Discussion session (5~10 min)
    - Discussion on the optional readings (15~20 min)
    - Another discussion session (5~10 min)
    - Discussion on the optional readings (15~20 min)
    - A short summary (3~5 min)
    - Feel free to be creative and include materials outside of the papers

# Logistics: Presentation

- For presenters:
  - Talk to me **one week before your presentation.**
    - Default time: talk to me after class
  - You need to be ready for the following before meeting with me
    - Finish reading the papers
    - A structure of your presentation
    - Two reading questions for the required reading
    - Topics for one or two discussion sessions
- Send me the slides **before the noon of the presentation day**

# Today's Lecture: Modeling Incentives

# Warm-Up Discussion

- What are the **incentives** that motivate you/people to do things?
- Can you try to **model** the incentives? For example, how do you mathematically specify the relationships of the “strength” of the incentives and how that impacts your actions?
- **Mechanism design**: Given your model of how humans respond to incentives, how can you design mechanisms (set of rules) that encourage people to do what you want them to do? (You can try to think about these in some specific applications).

# Today's Lecture: Modeling Incentives

- Game theory basics
  - Utility, games, equilibrium
  - Example usage in crowdsourcing
    - Contract design (Principal agent model)
- Proper scoring rules (eliciting truthful probability estimates)
  - Example usage in crowdsourcing
    - Prediction markets
- Peer prediction
  - Example usage in crowdsourcing
    - Peer grading in MOOCs



# Game Theory

- Mathematical study of interactions between **rational** and **self-interested** agents.
- Agents are often assumed to be rational and choose actions to maximize their **expected utility**.

# Utility

- A way to quantify agents' preferences over the state of the world.
- Example

$$\Omega = \{\text{Sunny}, \text{Cloudy}, \text{Rainy}\}$$

$$\text{Sunny} \succ \text{Cloudy} \succ \text{Rainy}$$

(Sunny is preferred over Cloudy and Rainy, and Cloudy is preferred over Rainy)

- Using von Neumann–Morgenstern utility

$$u(\text{Sunny}) = 10, u(\text{Cloudy}) = 5, u(\text{Rainy}) = 3$$

A natural way to use numerical value to represent preference. It also satisfies nice properties:

- *Completeness*
- *Transitivity*
- *Continuity*
- *Independence*

# Expected Utility Theory

- Agents take actions to maximize their expected utility

$$\sum_{\omega \in \Omega} p(\omega)u(\omega)$$

- Game theory deals with situations in which  $p(\omega)$  and  $u(\omega)$  are influenced by **agents' joint actions**

# Example 1: Prisoner's Dilemma



B

Normal-form game



A

	Stay Silent	Confess
Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
Confess	A: free B: 10 years	A: 5 years B: 5 years

Solution Concept

What should the prisoners do?

“Confess” is a **dominant strategy** – it maximizes the prisoner’s utility no matter what action the other player chooses.

# Normal-Form Game

- Players take actions simultaneously
- The elements of a normal-form game
  - **Players:** (prisoner A, prisoner B)
  - **Strategies:** (stay silent, confess)
  - **Payoffs:** (sentences for all strategy combinations)

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

# Normal-Form Game (More formally)

- A finite, n-player normal-form game is a tuple  $(N, A, \vec{u})$ 
  - $N$  is a finite set of  $n$  agents, indexed by  $i \in \{1, \dots, n\}$
  - $A = A_1 \times A_2 \cdots \times A_n$ , where  $A_i$  is a finite set of actions available to agent  $i$
  - $\vec{u} = \{u_1, u_2, \dots, u_n\}$ , where  $u_i: A \rightarrow \mathbb{R}$  is a real-valued utility function for agent  $i$

# Example 2: Coordination Game

- Two friends A and B are deciding what to do on Friday night
  - A prefers to go to the movie
  - B prefers to go to the bar
  - Both prefer to do something together than doing something separately

		B	
		Movie	Bar
A	Movie	(2, 1)	(0, 0)
	Bar	(0, 0)	(1, 2)

“Nash equilibrium”  
of this game

What is the “solution” of the game: what should A and B do?

(Movie, Movie) and (Bar, Bar) seem to be two ***stable*** outcomes

# Example 3: Rock, Paper, Scissors

- A zero-sum game

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

Is there a dominant strategy?

Are there stable pairs of actions?

Need a more general solution concept.



# Definitions

- Mixed strategy
  - Instead of deterministic actions, think about **randomized** actions
  - Let  $A_i$  be the set of actions agent  $i$  can take
  - Let  $S_i$  be the set of all probability distributions over  $A_i$
  - Each  $s_i \in S_i$  is a ***mixed strategy***, where
    - $s_i(a_i)$  denotes the probability for agent  $i$  choosing action  $a_i$
- Payoffs can be calculated using expected utility

# Definitions

- Best response
  - $\vec{s}$  : a strategy profile, i.e., the set of strategies for all agents
  - $\vec{s}_{-i}$  : the strategies of all agents except agent  $i$
  - A strategy  $s_i^* \in S_i$  is a **best response** to  $\vec{s}_{-i}$  if
$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \text{ for all } s_i \in S_i$$
  - Given what others do, a best response leads to highest utility

# Definitions

- Nash equilibrium
  - A strategy profile  $\vec{s}$  is a **Nash equilibrium** if, for all agent  $i$ ,  $s_i$  is a best response to  $\vec{s}_{-i}$
- Intuitive interpretations:
  - If all agents except  $i$  follow the strategies in the Nash equilibrium, agent  $i$  would maximize her payoff by following the strategy in Nash.
  - No incentive to deviate if everyone else follows the strategy in Nash

# Let's look at the examples again

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

- (Confess, Confess) is the dominant strategy equilibrium
- Strongest solution concept

# Let's look at the examples again

	Movie	Bar
Movie	(2, 1)	(0, 0)
Bar	(0, 0)	(1, 2)

- (Movie, Movie) and (Bar, Bar) are pure strategy Nash equilibria

# Let's look at the examples again

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

- Both players play each action with  $1/3$  probability is the mixed strategy Nash equilibrium
- It is the unique equilibrium

## **Theorem (Nash, 51):**

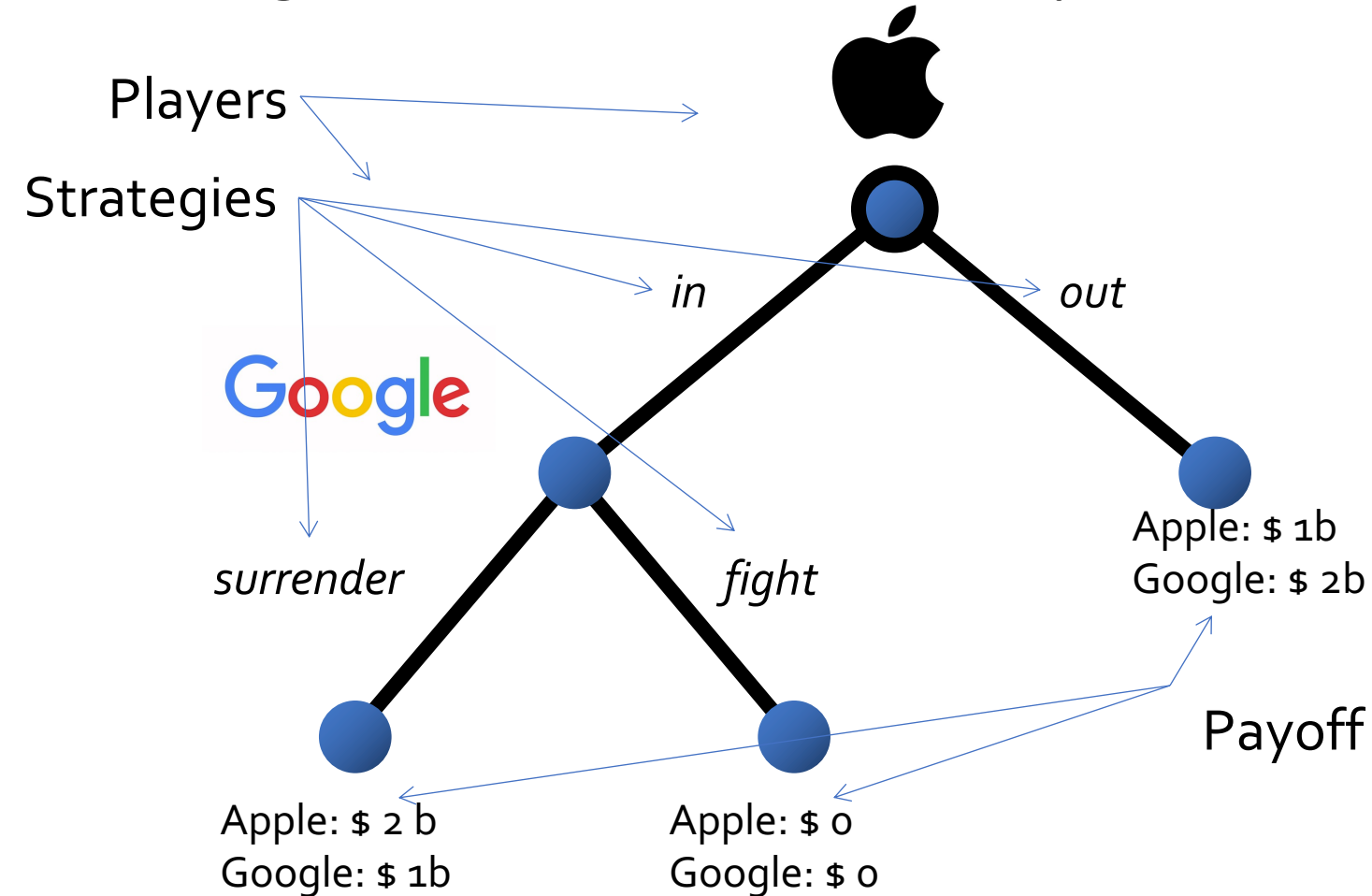
Every game with a finite number of players and actions has at least one equilibrium.

### Notes:

- This is the “existence” proof. Computing the equilibrium could be hard (computationally expensive).
- If it’s hard to calculate the equilibrium, can we really expect humans to follow the equilibrium?
  - There have been recent studies developing new solution concepts which assume humans are “learning” to adapt to the game.

# Extensive-Form Game

- Agents take decisions in a sequential manner

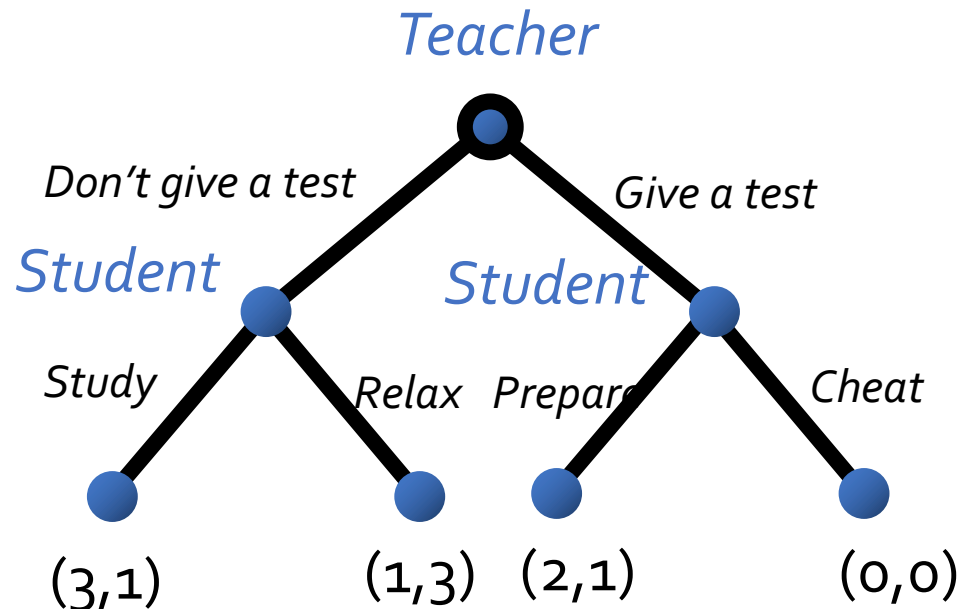


	Surrender	Fight
In	(2, 1)	(0, 0)
Out	(1, 2)	(1, 2)



# Solution Concepts in Extensive Games

- There are some weird cases by directly extending Nash equilibrium.



- Nash equilibrium:

- Teacher chooses "Don't give a test"
- Student chooses (Relax, Cheat)

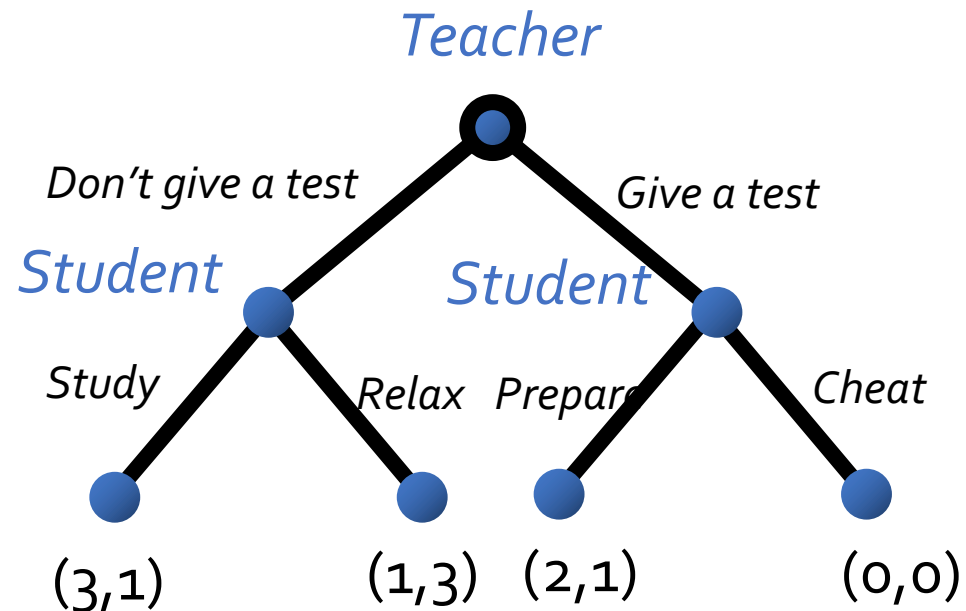
Is it stable?

Why is this the weird case?

- Student gives a "non-credible threat"
- "I'll choose to cheat if you give a test"

# Solution Concepts in Extensive Games

- Subgame Perfect Equilibrium (SPE)
  - Play in each subgame is a Nash equilibrium.
  - Rule out the “non-credible threat”

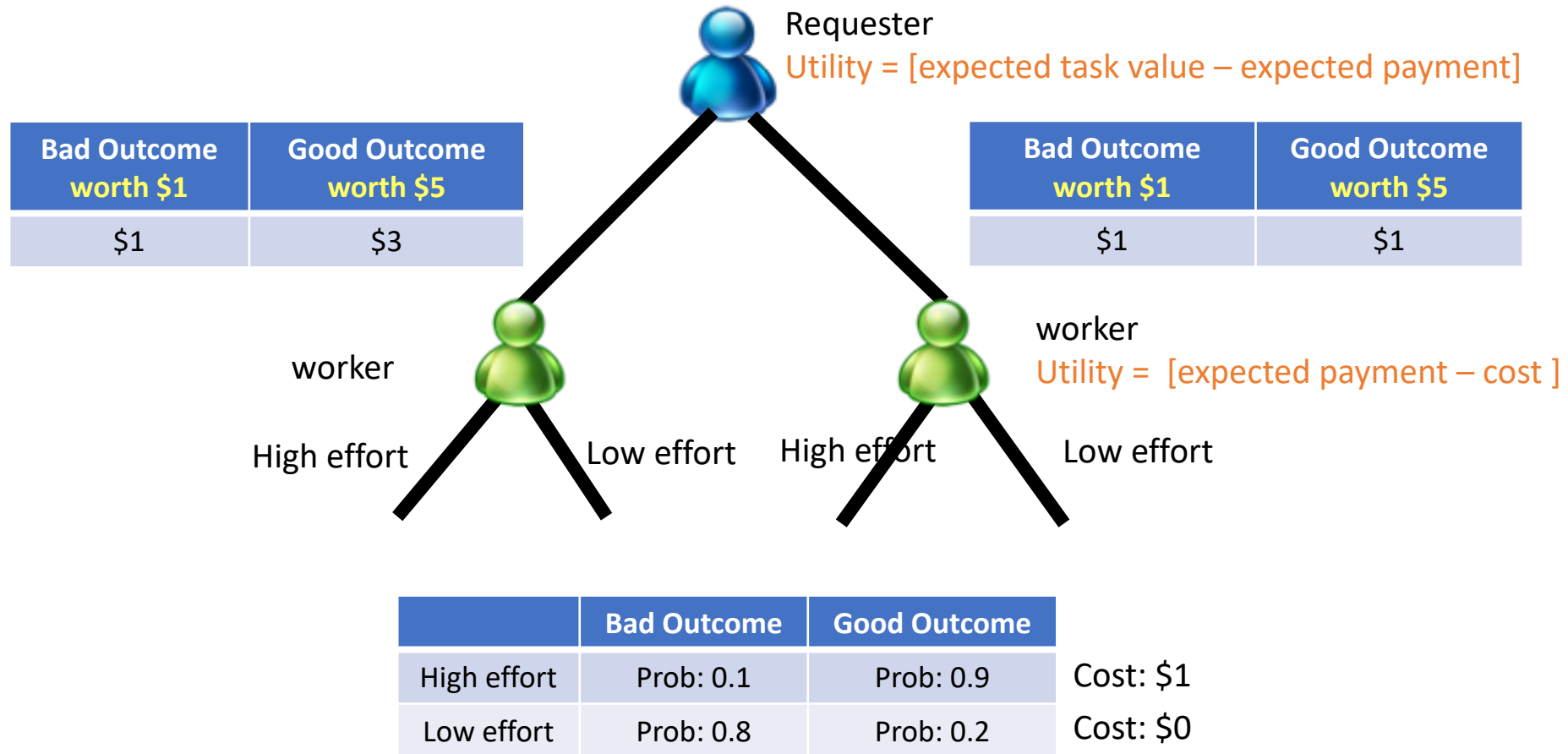


- SPE
  - Teacher chooses “Give a test”
  - Student chooses (Relax, Prepare)
- Can usually be calculated using backward induction.

# Applying Game Theory in this Course

- Formulate users' incentives
- Describe the game structure
  - Sometime there are no interactions between workers.  
It becomes a simpler optimization problem.
- Analyze the equilibrium as the prediction of the outcome
- Mechanism design
  - Specify the desired outcome
  - Design incentive and game structures such that the outcome is the equilibrium

# Practice Example – Contract Design



Which contract should the requester choose?

Can be generalized to any real-valued contracts (Principal Agent problem)

# Discussion

- The main assumption we usually make in incentive design is that users are **rational**. While it approximates human behavior fine in some cases, this assumption has often been criticized.
- Questions
  - What are the examples of the “non-rational” human behavior you can think of?
  - Can you still try to model the non-rational behavior?

# Example of Human Bias

- Present bias:
  - Humans value immediate payoffs heavier than future payoffs.
  - What will happen if we don't require project milestone reports?
- Herding bias:
  - Humans tend to follow what others do/say.
  - Will you eat at a restaurant with a lot of people lining up or another with no one inside.
- Prospect theory:
  - Nobel-winning theory in explaining biases in decision-making with uncertainty events.
- And more...
- A growing research direction to account for human biased behavior in computational systems.
  - [First workshop on Behavioral EC](#)

Sep 27    Incentive Design: Financial Incentives  
**[Presentation Slot #1]**

Are workers really rational in financial incentives?

Sep 29    Incentive Design: Badges and Attention  
**[Presentation Slot #2]**

Can we formulate non-financial incentives?

Oct 4    Application: Darpa Network Challenge  
**[Presentation Slot #3]**

Incentivizing users through social networks

Oct 6    Application: Prediction Markets  
**[Presentation Slot #4]**

Using market mechanisms for eliciting users' beliefs

# Proper Scoring Rules



# Incentivizing Truthful Reports About Probabilities

- Example scenarios:
  - Ask a weather forecaster: will it rain tomorrow?
  - Ask a political researcher: will Trump win 2020 election?
  - Ask a Microsoft employer: will the new version of Office be shipped on time?
- Want to obtain forecasts about future events
- How do we make sure we obtain **truthful** reports?

# Incentivizing Truthful Reports

- Setting
  - Consider a rational agent with linear utility for cash
  - Suppose there are  $n$  mutually exclusive and exhaustive states of the world  $\Omega = \{w_1, w_2, \dots, w_n\}$  (e.g., Sun, Rain, Snow)
  - $p_i$  is the subjective belief of the agent that state  $w_i$  will occur
- Question
  - How do we motivate this agent to tell us her beliefs about the likelihood of each state?

# Scoring Rules

- A scoring rule rewards an agent  $S(\vec{r}, w)$  when her reported distribution is  $\vec{r}$  and the realized outcome is  $w$

# Scoring Rules

- Let's consider a linear scoring rule

$$S(\vec{r}, w_i) = r_i$$

- If a risk-neutral agent believes the probability for Rain and Sun are  $\vec{p} = (0.7, 0.3)$

What report should the agent provide?

# Scoring Rules

- A scoring rule rewards an agent  $S(\vec{r}, w)$  when her reported distribution is  $\vec{r}$  and the realized outcome is  $w$
- A scoring rule is called **proper** if the agent maximizes her expected utility by providing truthful report

$$\vec{p} = \operatorname{argmax}_{\vec{r}} \sum_{i=1}^n p_i S(\vec{r}, w_i)$$

- A scoring rule is **strictly proper** if honestly reporting is the **unique** maximizer.

# Examples of Strictly Proper Scoring Rules

- Quadratic scoring rule (Brier score):

$$S(\vec{r}, w_i) = r_i - \frac{1}{2} \sum_j r_j^2$$

We can verify this by taking the gradient of the expected payoff

- Affine transformation of the proper scoring rule is still proper.

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## MONTHLY WEATHER REVIEW

EDITOR, JAMES E. CASKEY, JR.

Volume 78  
Number 1

JANUARY 1950

Closed March 5, 1950  
Issued April 15, 1950

VERIFICATION OF FORECASTS EXPRESSED IN TERMS OF PROBABILITY

GLENN W. BRIER

U. S. Weather Bureau, Washington, D. C.  
[Manuscript received February 10, 1950]

# Examples of Strictly Proper Scoring Rules

- Logarithmic scoring rule:

$$S(\vec{r}, w_i) = \log r_i$$

We can verify this by taking a gradient of the expected payoff

- In logarithmic scoring rule, the score for outcome  $w_i$  only depends on the report  $r_i$  and not  $r_j$  for  $j \neq i$

# More examples?

- How do we construct a strictly proper scoring rule?
- How many strictly proper scoring rules are there?



# Characterization of Proper Scoring Rules

- Connections between convex functions and proper scoring rules.
- A scoring rule  $S(\vec{r}, w_i)$  is (strictly) proper **if and only** if

$$S(\vec{r}, w_i) = G(\vec{r}) - \sum_{j \neq i} G'_j(\vec{r})p_j + G'_i(\vec{r})$$

where  $G(\vec{r})$  is a (strictly) convex function,  $G'(\vec{r})$  is a subgradient of  $G$  at  $\vec{r}$ , and  $G'_i(\vec{r})$  is its  $i$ -th component.

# Connection to Prediction Market

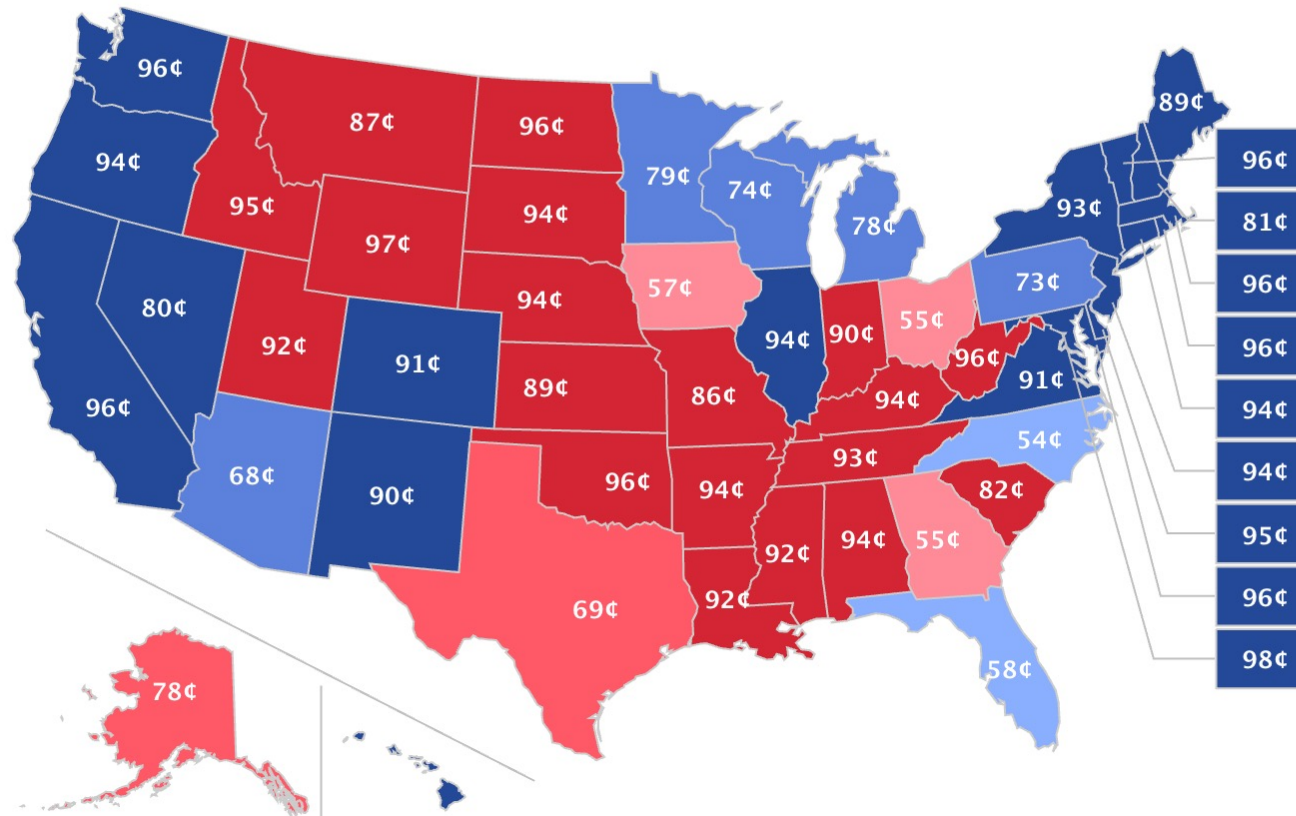
Which party will win the Electoral College?

Democratic 335

203 Republican

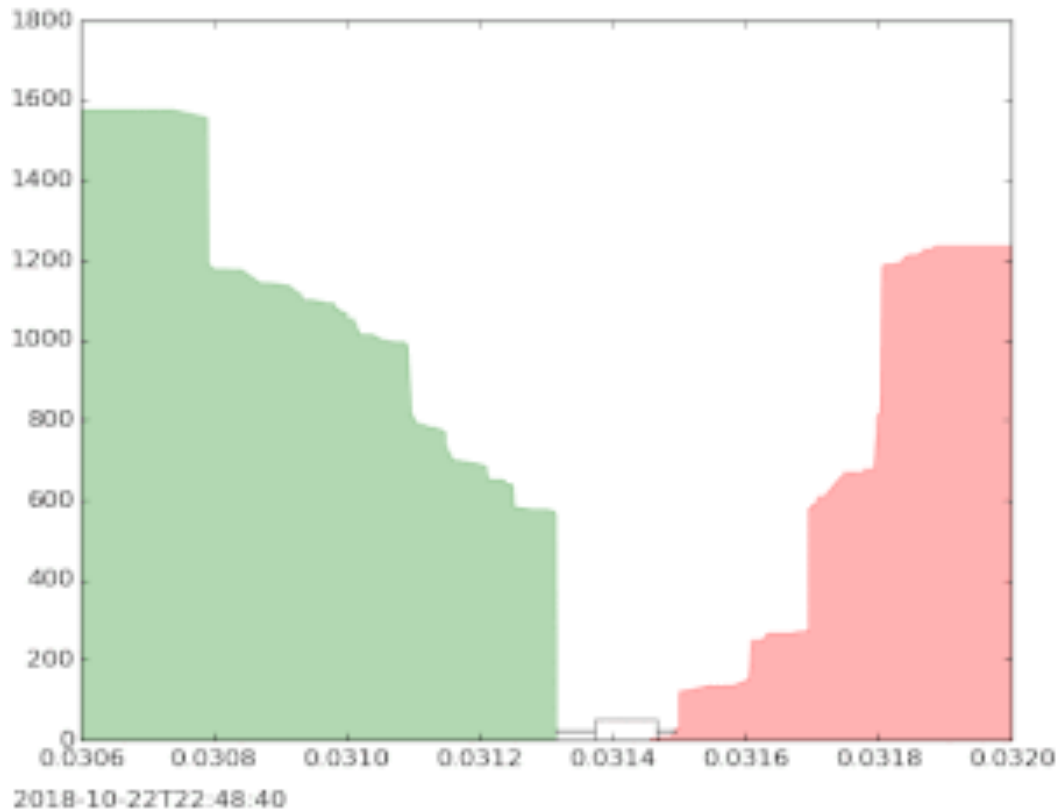


Majority



# Designing Automatic Market Makers

- Traditional market mechanisms might not work when the market is **thin**



# Market Scoring Rules

- See Hanson's papers in the optional readings of the Prediction Market lecture
- Intuitions: a “sequentially shared scoring rule”
  - An automatic market maker
  - Market maintains a vector of predictions  $\vec{r}^{(t)}$
  - If a trader changes the vector from  $\vec{r}^{(t)}$  to  $\vec{r}^{(t+1)}$  and the outcome is  $w_i$ , the trader obtains reward

$$S(\vec{r}^{(t+1)}, w_i) - S(\vec{r}^{(t)}, w_i)$$

- Under some conditions:
  - Agents truthfully report their beliefs
  - The prediction will converge

# Market Scoring Rules

- The connection to convex optimization opens up an interesting line of research in the design of efficient market maker...

**Required**

[The Promise of Prediction Markets](#), K.J. Arrow et. al., Science. 2008  
[Results from a Dozen Years of Election Futures Markets Research](#). Berg et al. 2001.

Please read both. The first one is a two-page non-technical paper.

**Optional**

Empirical reports:

[Using Prediction Markets to Track Information Flows: Evidence from Google](#). Cowgill, Wolfers, and Zitwewitz. 2008.

[Using prediction markets to estimate the reproducibility of scientific research](#). Dreber et al. PNAS 2015.

[Prediction Without Markets](#). Goel et al. EC 2010.

[Prediction Markets](#). Wolfers and Zitzewitz, The Journal of Economic Perspectives 2004.

Automatic market makers

[Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation](#). Hanson. Journal of Prediction Markets 2007.

[Combinatorial Information Market Design](#). Hanson. Information Systems Frontier 2003.

A [blogpost](#) by David Pennock that discusses how to implement market scoring rules as a market maker.

- We won't cover too much on prediction markets. In case you are interested, below are a few more papers to follow up:
  - [A New Understanding of Prediction Markets Via No-Regret Learning](#). Chen and Vaughan. EC 2010.
  - [An Optimization-Based Framework for Automated Market-Making](#). Abernethy, Chen, and Vaughan. EC 2011.
  - and more (the papers by these authors)

# Very Brief Intro of Peer Prediction

See more discussion in Assignment 3

# Eliciting Truthful Reports

- Scoring rule relies on the “truth” to be revealed in the future
- What if there is no ground truth (or the ground truth is hard to obtain)
  - Do you like this movie?
  - Peer grading in MOOCs
- Output agreement:
  - Randomly pick two persons
  - If their reports match, reward them 1, otherwise reward 0
  - Truthful reporting is not an equilibrium (you are encouraged to report the majority’s opinion)



# Peer Prediction

- How to fix the issue?
  - Assume knowledge about the report distribution, re-weighting the rewards to make sure truthful reporting is a equilibrium
- Drawbacks:
  - Require knowledge of the prior
  - There are usually multiple equilibrium (including naïve bad ones...)
- Still an ongoing research area
  - Some nice theoretical results, however there is little practical success so far

# Related Course

- The course on peer grading at Northwestern by Jason Hartine.
  - <https://sites.northwestern.edu/hartline/eecs-497-peer-grading/>

# Assignment 3

# Cooperation and Repeated Prisoner's Dilemma

- Prisoner's dilemma predicts that people are not going to cooperate in the game setup, but in practice, people sometimes do.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

- Will look at this using repeated versions of prisoner's dilemma

# Peer Grading and Peer Prediction

- Can we design “incentives” for peer grading?
  - Ground truth (goodness of assignment) is hard to obtain
- Randomly pick two students to grade the same assignment
  - Rewarding “the same score” is probably not a good idea
    - Every grader can just give high score for every assignment
  - How should we do it?

# Information Design with Bayesian Persuasion

- A company wants to hire interns from our class and asks me for recommendation letters
- Assumption
  - 30% of students are “good” -> meet their requirement
  - They don’t know who are good but I know
- How do I write letters to maximize the number of students getting hired?