Lecture 5
Label Aggregation: Matrix-Based Methods

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Logistics: Late Days

- Reminders of the late-day policy (copied from Lecture 1)

- Late day policy
  - Assignments
    - 4 late days in total. No 2 late days per assignment.
  - Reviews
    - No late submissions. But you can skip 2 of them without penalty.
  - Project-related reports
    - No late submissions.

Logistics: Assignment 1

- Due this Friday

- If you use Figure Eight and can’t find enough tasks
  - Try to satisfy $0.25 requirement (we’ll relax the 3 tasks requirements)
  - If even the $0.25 requirement is not possible (e.g., not enough copies of tasks), take screenshots of available tasks, and submit whatever you have.
    - Include discussion on why you think this happens, and how to solve this issue.

- If you cannot get any accounts set up, discuss with me after class.

- No additional extension will be given if you only bring up the issue tomorrow/Friday.

Logistics: Project Proposal

- Due: September 20 (next Friday)
- Example/past projects are posted on the course website

- Requirements:
  - Title, team members
  - 1~2 paragraphs describing what you want to do
  - At least one relevant paper

- Submission:
  - Submit on Gradescope.
  - One submission per group.
  - Need to include all teammates using the Gradescope interface.

Logistics: Bidding for Presentations

- Check out the course schedule for the presentation slots:

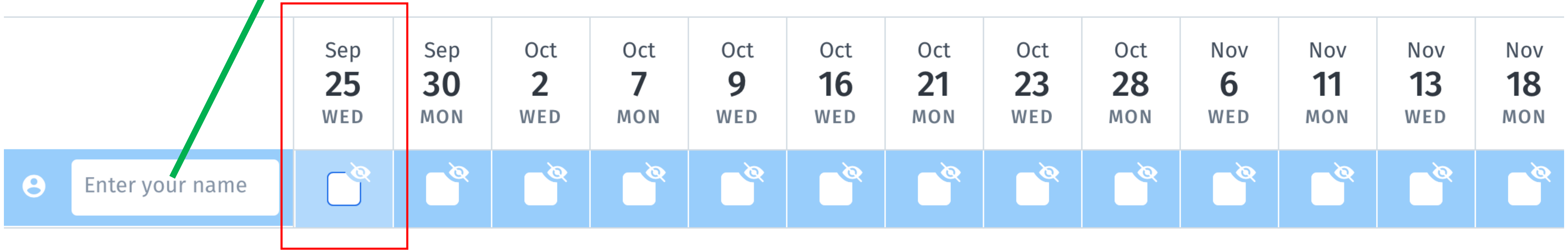
Sep 25 Incentive Design: Financial Incentives

[Student Presentation]

- Provide around 3~5 bids by the end of today (hard deadline).
  - https://doodle.com/poll/yycvun8fx8z8bde2
  - You might want to glance over the papers of your bidding.
  - You can bid more than 5 bids.
    - It might help in decreasing the chance you get assigned to slots outside of your bids.

Logistics: Bidding for Presentations

- Bidding interface
  - Enter the **names of all members** in your group



Make sure you can make it before bidding this one.

There might be small changes on the exact list of papers for later topics.

- I'll announce the assignment by this Thursday.
  - Manually solve the max-cover problem.
  - I'll try to accommodate your interests, but no guarantee on that.
  - Random assignments will be used if there is no feasible solutions.
  - I'll fill in the slots if there are fewer groups than slots.

Logistics: Bid for Presentations

Sep 25 WED	Sep 30 MON	Oct 2 WED	Oct 7 MON	Oct 9 WED	Oct 16 WED	Oct 21 MON	Oct 23 WED	Oct 28 MON	Nov 6 WED	Nov 11 MON	Nov 13 WED	Nov 18 MON
✓0	✓0	✓0	✓2	✓1	✓2	✓2	✓0	✓1	✓4	✓1	✓1	✓1
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
			✓			✓			✓			
					✓			✓	✓			✓
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			✓			✓			✓			

Quick Recap

EM-Based Approach

- Notations
  - $D = \{d_1, \ldots, d_n\}$: Observations
  - θ: latent variables
- Concepts
  - Likelihood: $\Pr(D|\theta)$
  - Posterior: $\Pr(\theta|D)$
- Steps for MLE approach
  - Define label generation model $\Pr(d_i|\theta)$
    - θ contains the true labels and other latent factors in your models
  - Optimization: Find $\theta^* = argmax_\theta \sum_{i=1}^{n} \log \Pr(d_i|\theta)$
    - In last lecture, there are only two possible values for θ. So we brute-force find it.

EM-Based Approach

- Connection to supervised learning
  - Model of the labeling process: Hypothesis set / Loss Function
  - EM: an algorithm to find a hypothesis within the set that minimizes the error

EM-Based Approach: Pros and Cons

- Pros
  - **Empirically performs well**
  - A generic framework
    - There is a HUGE amount of papers along this line, with different models of label generation
- Cons
  - EM only attempts to find the local optimal of the objective function
  - **Lack of theoretical guarantees** on the final performance
    - Are we just getting lucky?

Today’s Lecture:

Matrix-Based Approach

Matrix Representation of Workers' Answers

	Task 1	Task 2	Task 3	Task 4	…
Worker 1	1	-1	1	1	
Worker 2	1	-1	-1	-1	
Worker 3	-1	1	-1	1	
Worker 4	1	-1	1	1	
…					
	?	?	?	?	

Goal: Infer the true label of each task

Let’s Look at Another Problem First

- Movie recommendation

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6
Alice	5	4		1		
Bob	4			2	5	
Charlie	1		4		2	
David		3	2			4
…						

Warmup Discussion:

- Which movie will you recommend to Alice? Why?

Collaborative Filtering

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6
Alice	5	4		1		
Bob	4			2	5	
Charlie	1		4		2	
David		3	2			4
…						

Bob is probably most similar to Alice

- User-based collaborative filtering
  - Examine users' rating *vector*
    - Alice and Bob seem to have similar tastes
    - Bob likes Movie 5
    - Alice probably also likes Movie 5
  - We can also calculate similarities among users, and weight their opinions accordingly

Collaborative Filtering

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6
Alice	5	4		1		
Bob	4			2	5	
Charlie	1		4		2	
David		3	2			4
…						

- Item-based collaborative filtering
  - Examine items' rating *vectors*
    - People who like/hate Movie 1 seem to like/hate Movie 5 as well
    - Since Alice likes Movie 1, she might also like Movie 5

Intuitions

- Pros and Cons
  - Simple and interpretable
  - Cold-start and data sparsity problem (won’t discuss much in this lecture)

- Key intuitions for collaborative filtering to work
  - A big number of ratings are controlled by a small number of parameters
  - You probably can see why this is related to crowdsourcing already

- Low rank matrix approximation
  - A principled method to utilize the above intuition

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6
Alice	5	4		1		
Bob	4			2	5	
Charlie	1		4		2	
David		3	2			4
…						

A very short intro to

Low rank matrix approximation

Rank of a Matrix

- Matrix Rank
  - # linearly independent row (or column) vectors in a matrix

- Example

$$\begin{bmatrix} 1 & 2 & 4 & 4 \\ 3 & 4 & 8 & 0 \end{bmatrix}$$

Rank: 2

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{bmatrix}$$

Rank: 2

- What does low rank matrix imply?

Singular Value Decomposition (SVD)

$$A_{[m \times n]} \approx U_{[m \times r]} \Sigma_{[r \times r]} \left(V_{[n \times r]}\right)^T$$

- A: Input matrix
  - $m \times n$ matrix (m users, n movies; m workers, n tasks)

- U: Left singular matrix
  - $m \times r$ matrix (m users, r latent concepts)
- Σ: Singular values
  - $r \times r$ diagonal matrix (strength of each latent concept)
- V: Right singular matrix
  - $n \times r$ matrix (n movies, r latent concepts)

(Technically, the SVD definition here is slightly different from standard definition, but we can get this one with some discussion on matrix ranks.
(See the lecture notes by Tim Roughgarden for more details.)

Singular Value Decomposition (SVD)

- It is always possible to make such decomposition exactly equal (if we don't put any restrictions on the **rank** r)

$$A_{[m\times n]} = U_{[m\times r]}\Sigma_{[r\times r]}\left(V_{[n\times r]}\right)^T$$

- Low rank matrix decomposition
  - Can we approximate A with this decomposition with **small rank r**

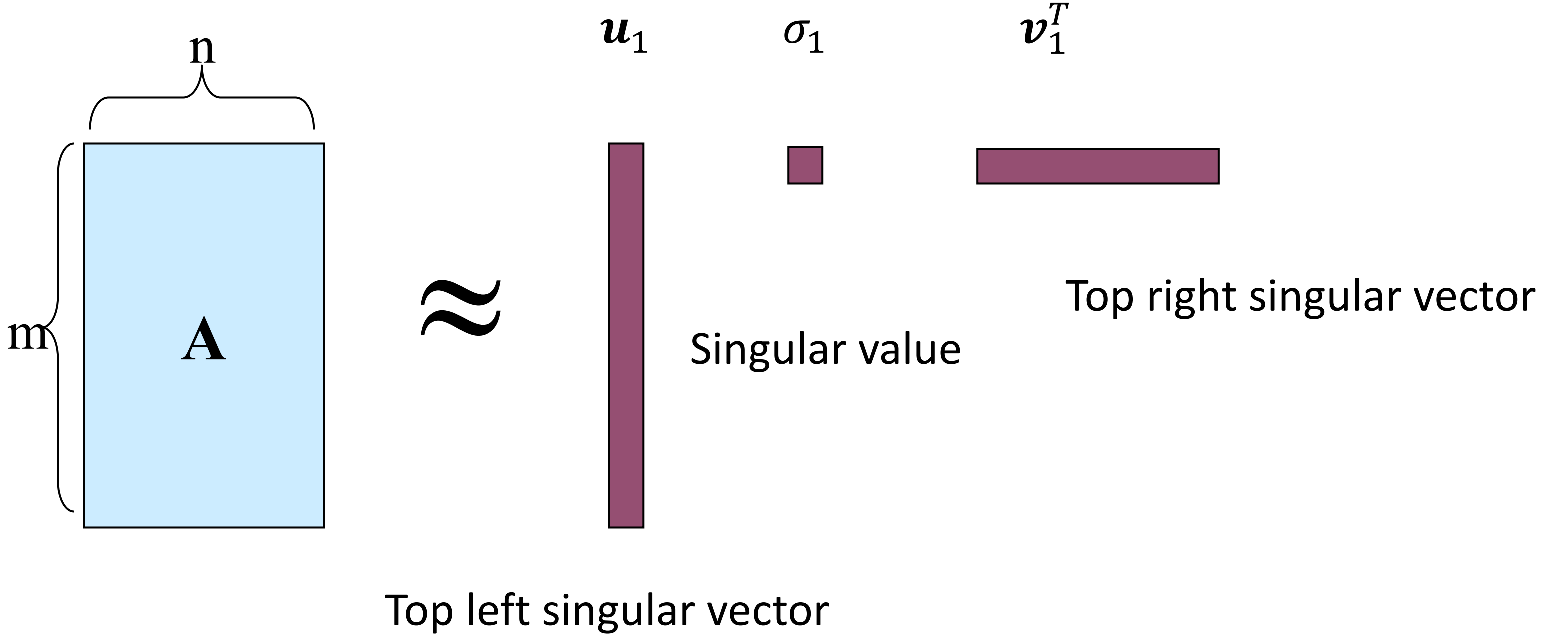
$$A_{[m\times n]} \approx U_{[m\times r]}\Sigma_{[r\times r]}\left(V_{[n\times r]}\right)^T$$

A dimension reduction technique;
Reduce the number of parameters (and therefore requires less data to learn)

Rank 1 Approximation

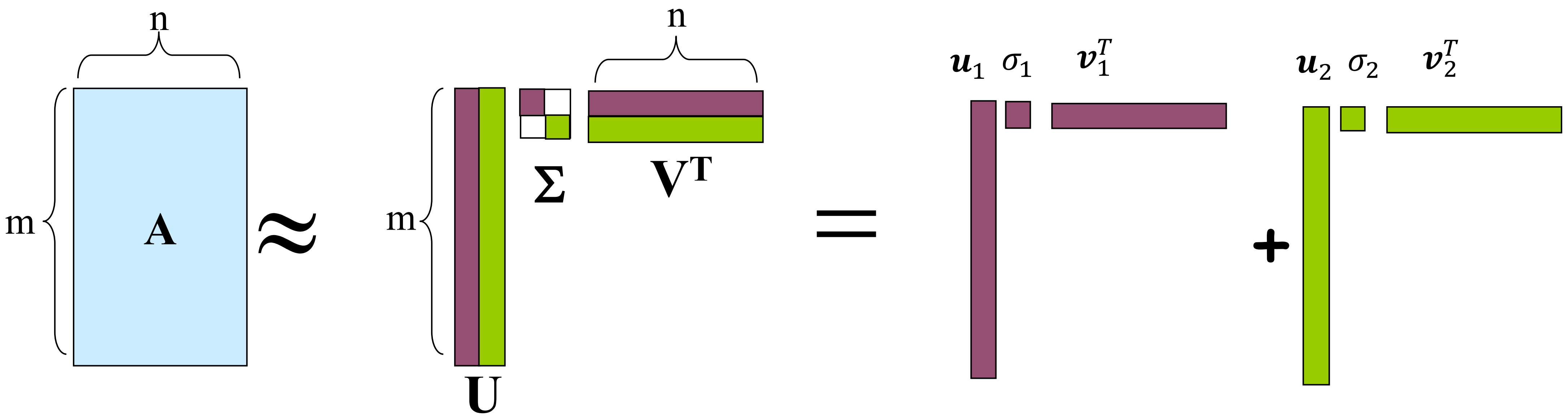
$$A_{[m\times n]} \approx U_{[m\times 1]}\Sigma_{[1\times 1]}\left(V_{[n\times 1]}\right)^T$$

$$= \boldsymbol{u}_1\sigma_1\boldsymbol{v}_1^T$$



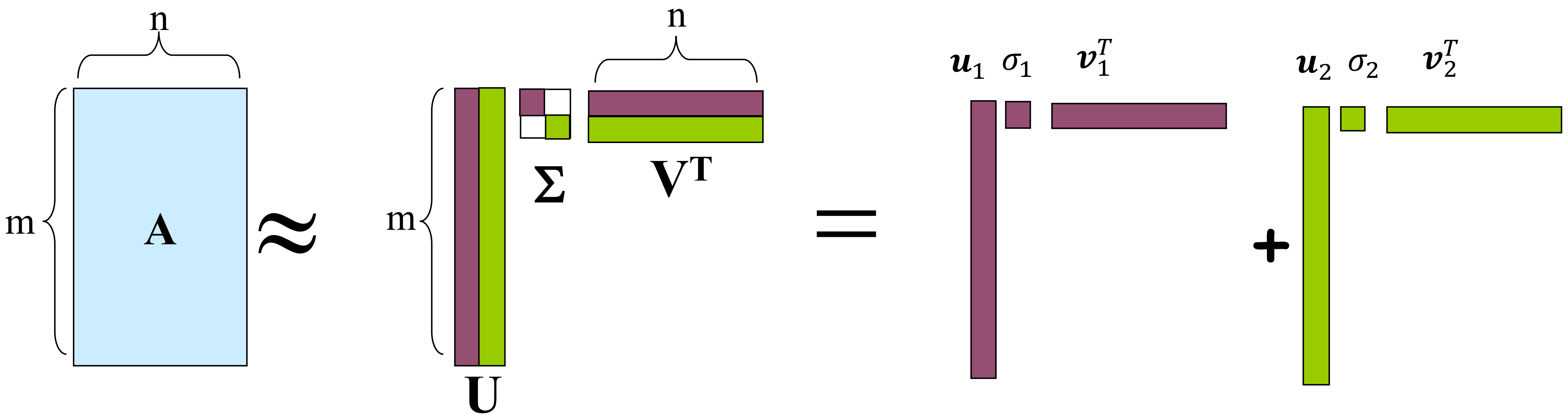
Rank k Approximation

$$A_{[m \times n]} \approx U_{[m \times k]} \Sigma_{[k \times k]} \left(V_{[n \times k]}\right)^T$$



Rank k Approximation

$$A_{[m\times n]} \approx U_{[m\times k]}\Sigma_{[k\times k]}\left(V_{[n\times k]}\right)^T = \sum_i \boldsymbol{u_i}\sigma_i\boldsymbol{v}_i^T$$



Movie Recommendation Example

$$A_{[m \times n]} \approx U_{[m \times r]} \Sigma_{[r \times r]} \left(V_{[n \times r]}\right)^T$$

$$
\begin{array}{c}
\begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \end{array} \\
\begin{bmatrix}
\mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 \\
\mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & 0 \\
\mathbf{4} & \mathbf{4} & \mathbf{4} & 0 & 0 \\
\mathbf{5} & \mathbf{5} & \mathbf{5} & 0 & 0 \\
0 & \mathbf{2} & 0 & \mathbf{4} & \mathbf{4} \\
0 & 0 & 0 & \mathbf{5} & \mathbf{5} \\
0 & \mathbf{1} & 0 & \mathbf{2} & \mathbf{2}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{0.13} & 0.02 & -0.01 \\
\mathbf{0.41} & 0.07 & -0.03 \\
\mathbf{0.55} & 0.09 & -0.04 \\
\mathbf{0.68} & 0.11 & -0.05 \\
0.15 & \mathbf{-0.59} & \mathbf{0.65} \\
0.07 & \mathbf{-0.73} & \mathbf{-0.67} \\
0.07 & \mathbf{-0.29} & \mathbf{0.32}
\end{bmatrix}
\times
\begin{bmatrix}
\mathbf{12.4} & 0 & 0 \\
0 & \mathbf{9.5} & 0 \\
0 & 0 & \mathbf{1.3}
\end{bmatrix}
\times
\begin{bmatrix}
\mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\
0.12 & -0.02 & 0.12 & \mathbf{-0.69} & \mathbf{-0.69} \\
0.40 & \mathbf{-0.80} & 0.40 & 0.09 & 0.09
\end{bmatrix}
$$

Movie Recommendation Example

$$A_{[m \times n]} \approx U_{[m \times r]} \Sigma_{[r \times r]} \left(V_{[n \times r]}\right)^T$$

SciFi-concept

Romance-concept

“strength” of the SciFi-concept

$$\begin{array}{c} \begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \end{array} \\ \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 \\ \mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & 0 \\ \mathbf{4} & \mathbf{4} & \mathbf{4} & 0 & 0 \\ \mathbf{5} & \mathbf{5} & \mathbf{5} & 0 & 0 \\ 0 & \mathbf{2} & 0 & \mathbf{4} & \mathbf{4} \\ 0 & 0 & 0 & \mathbf{5} & \mathbf{5} \\ 0 & \mathbf{1} & 0 & \mathbf{2} & \mathbf{2} \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{0.13} & 0.02 & -0.01 \\ \mathbf{0.41} & 0.07 & -0.03 \\ \mathbf{0.55} & 0.09 & -0.04 \\ \mathbf{0.68} & 0.11 & -0.05 \\ 0.15 & \mathbf{-0.59} & \mathbf{0.65} \\ 0.07 & \mathbf{-0.73} & \mathbf{-0.67} \\ 0.07 & \mathbf{-0.29} & \mathbf{0.32} \end{bmatrix} \times \begin{bmatrix} \mathbf{12.4} & 0 & 0 \\ 0 & \mathbf{9.5} & 0 \\ 0 & 0 & \mathbf{1.3} \end{bmatrix} \times$$

$$\begin{array}{r} \text{SciFi-concept} \rightarrow \\ \text{Romance-concept} \rightarrow \\ \\ \end{array} \begin{bmatrix} \mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & \mathbf{-0.69} & \mathbf{-0.69} \\ 0.40 & \mathbf{-0.80} & 0.40 & 0.09 & 0.09 \end{bmatrix}$$

Netflix Challenge

- $1 million award for people beating their algorithm by 10%

- Simply implementing SVD already beats the algorithm Netflix was using…
- The winning team uses an ensemble of many methods
  - SVD is one major component

How to Perform SVD

- Should be covered in linear algebra class….

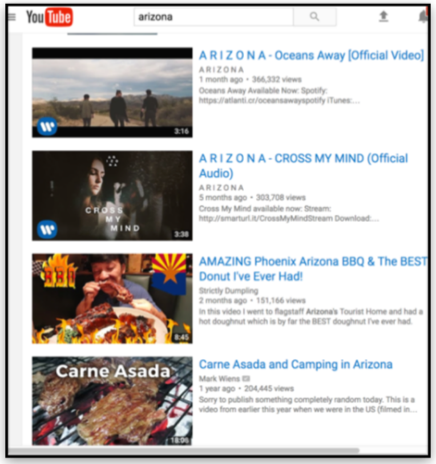
- Most likely, you will just call an existing library

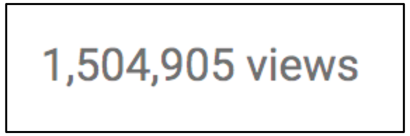
Let’s (finally) get back to label aggregation

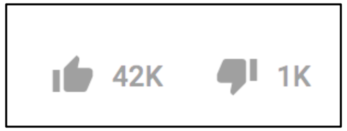
Who moderates the moderators? Crowdsourcing abuse detection in user-generated content. Ghosh, Kale, and McAfee. EC 2011.

User Generated Content

- Common practice to use user ratings to determine whether a content is good or not







- When a content receives a bad rating
  - is the content bad, or
  - is the rating bad?

- Given users ratings, how to decide content quality.

- This is a crowdsourcing label aggregation problem. With each rating being a label provided by a worker.

Model

NOTE:
1. The notations here are different from the previous slides about SVD.
2. The matrix also uses a different representation compared with the movie rating examples. In movie rating, each row is a user's ratings. Here, each column is a rater's ratings.

- Basic components
  - n raters, $i = 1, \dots, n$
  - T contributions, $t = 1, \dots, T$
  - $u_{t,i} \in \{-1,1\}$ is the rating rater i gives to contribution t

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	1	-1	-1	1	
Contribution 2	-1	1	1	-1	
Contribution 3	1	1	-1	1	
…					

U

- Label generation process
  - Each contribution t has a true quality $q_t \in \{-1,1\}$
  - Each rater i gives *correct* rating with probability ψ_i
- Goal: Infer q_t for all t from rating matrix U (both q_t and ψ_i are unknown)

The Goal Seems Different from Movie Recommendation

- In movie recommendation
  - Goal: Fill in the empty ratings (and recommend the movie with highest one)
- In this paper
  - Assumption: all ratings are given
  - Goal: Infer the latent variable (true quality)

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	1	-1	-1	1	
Contribution 2	-1	1	1	-1	
Contribution 3	1	1	-1	1	
…					

- Connection: Low rank approximation of the rating matrix

Look at the "Expected" Rating Matrix $E[U]$

$U =$

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	1	-1	-1	1	
Contribution 2	-1	1	1	-1	
Contribution 3	1	1	-1	1	
…					

$E[U] =$

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	$q_1(2\psi_1 - 1)$	$q_1(2\psi_2 - 1)$	$q_1(2\psi_3 - 1)$	$q_1(2\psi_4 - 1)$	
Contribution 2	$q_2(2\psi_1 - 1)$	$q_2(2\psi_2 - 1)$	$q_2(2\psi_3 - 1)$	$q_2(2\psi_4 - 1)$	
Contribution 3	$q_3(2\psi_1 - 1)$	$q_3(2\psi_2 - 1)$	$q_3(2\psi_3 - 1)$	$q_3(2\psi_4 - 1)$	
…					

q_t: true label of contribution t
ψ_i: prob of rater i to give correct rating

Look at the "Expected" Rating Matrix $E[U]$

$E[U] =$

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	$q_1(2\psi_1 - 1)$	$q_1(2\psi_2 - 1)$	$q_1(2\psi_3 - 1)$	$q_1(2\psi_4 - 1)$	
Contribution 2	$q_2(2\psi_1 - 1)$	$q_2(2\psi_2 - 1)$	$q_2(2\psi_3 - 1)$	$q_2(2\psi_4 - 1)$	
Contribution 3	$q_3(2\psi_1 - 1)$	$q_3(2\psi_2 - 1)$	$q_3(2\psi_3 - 1)$	$q_3(2\psi_4 - 1)$	
…					

$$= \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dots \end{bmatrix} [(2\psi_1 - 1) \ (2\psi_2 - 1) \ (2\psi_3 - 1) \ \dots]$$

$$= q \, [1] \, (2\psi - \mathbf{1})^T$$

This is the exact singular value decomposition with **rank 1**

Look at the “Expected” Rating Matrix $E[U]$

$$E[U] = q\ [1]\ (2\psi - \mathbf{1})^T$$

- The decomposition is not unique:
  - Multiply all elements in q by -1 and all elements in $(2\psi - \mathbf{1})$ by -1
  - What does this mean intuitively?

- Additional assumption:
  - $\psi_1 > 0.5$
  - Use this assumption to determine the sign

From $E[U]$ to U

- Exact rank 1 matrix decomposition

$$E[U] = q\,[1]\,(2\psi - \mathbf{1})^T$$

- Rank 1 matrix approximation

$$U \approx q'\,[1]\,(2\psi' - \mathbf{1})^T$$

Take the sign of q' as the prediction of q

More Details

- They don’t directly do singular value decomposition
- The top left singular vector of U = Top eigenvector of UU^T
- In the algorithm, they perform eigenvalue decomposition of UU^T

- Proposed algorithm: Spectral-Rating
  - Calculate the top eigenvector v of UU^T
  - Let $s = sign(v)$
  - Correct sign
    - If the majority of the sign is the same as the prediction of user 1, do nothing
    - Else, $s \leftarrow -s$
  - s is the final prediction

Theoretical Guarantee

THEOREM 3.1. *There is a constant c such that if $T > \frac{2}{\gamma^2}\log(4/\eta)$ and $\frac{n}{\log(n)} > \frac{128}{c\bar{\kappa}^2}$, then for any $\eta \in (0,1)$, with probability at least $1-\eta$, we have*

$$\frac{1}{T}|\{t : q'_t \neq q_t\}| \leq \frac{8}{\bar{\kappa}}\sqrt{\frac{\log(n)}{cnT}\log(\frac{4}{\eta})}.$$

n: # raters
T: # contributions

- Utilizing the matrix form of Hoeffding's inequality
- Average prediction error $= O\left(\frac{1}{2\,\bar{\psi}-1}\sqrt{\frac{\log n}{nT}}\right)$
- Focus on parameters you care about
  - How does error change as $\bar{\psi}$ changes
  - How does error changes as n changes
  - How does error change as T changes

How to interpret a bound like this?

Extensions

- Not every rater rates every contribution

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	1		-1	1	
Contribution 2	-1	1			
Contribution 3			-1	1	
…					

Extensions

- Not every rater rates every contribution

	Rater 1	Rater 2	Rater 3	Rater 4	…
Contribution 1	1	**0**	-1	1	
Contribution 2	-1	1	**0**	**0**	
Contribution 3	**0**	**0**	-1	1	
…					

- An updated label generation process
  - Each rater i has a probability p_i to rate a contribution

$$u_{ti} = \begin{cases} q_t & \text{w.p. } p_i \psi_i \\ -q_t & \text{w.p. } p_i(1-\psi_i) \\ 0 & \text{w.p. } 1 - p_i. \end{cases}$$

Is this a reasonable model?
Why do you think we need this model?

Extensions

- Computation is expensive for large datasets
  - UU^T is a T by T matrix
  - T (# contributions) is often huge in practice

- Online algorithm
  - For a small subset of contributions, solve for their quality
  - Used this subset to infer rater's skills ψ
  - Use weighted majority voting for new contributions (as in our lecture 3)

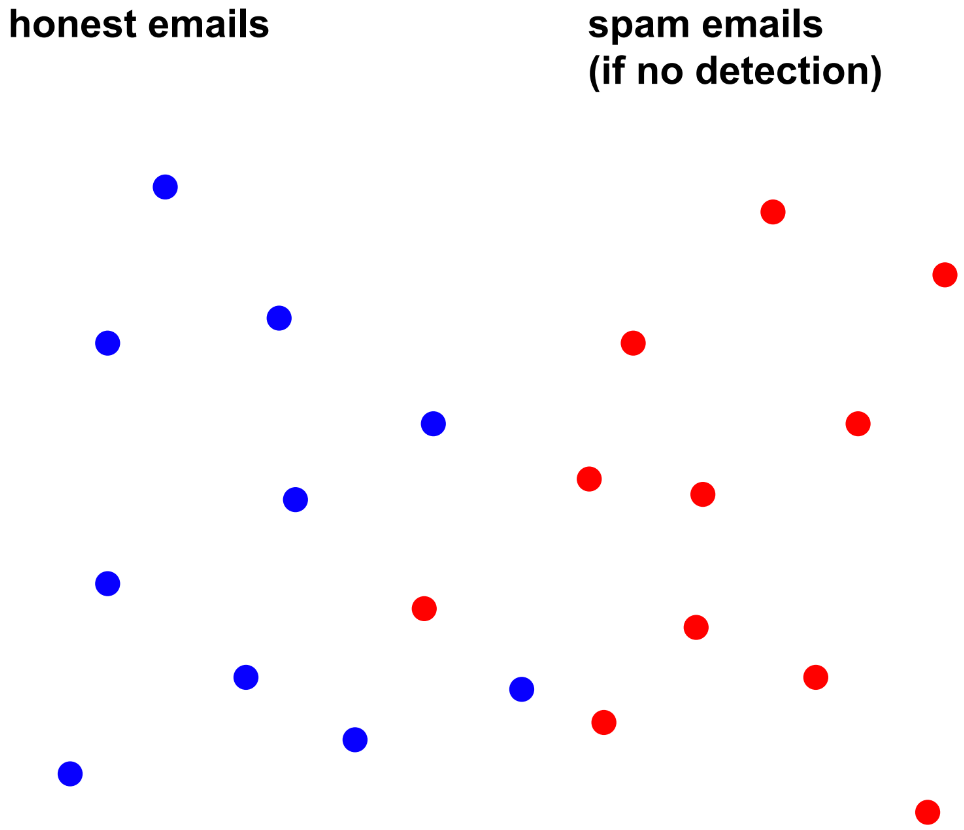
$$q_t = sign\left(\sum_i ln \frac{\psi_i}{1-\psi_i} u_{t,i}\right)$$

Extensions

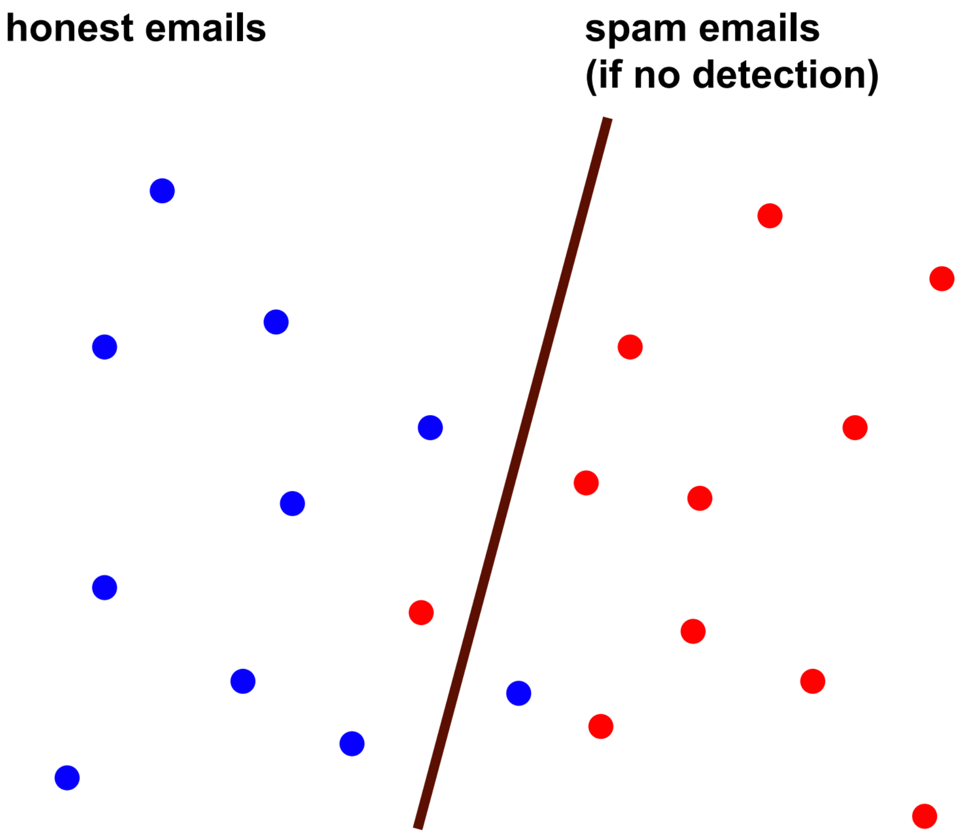
- Algorithm robustness against rater manipulations
  - Changing their “skills” to influence the algorithm outcome
  - Changing their labeling strategy (no randomized) to influence the algorithm outcome
  - Collude with other raters to influence the outcome for a particular contribution
- The results are “robust” if the ratio of manipulations is not large
- What if manipulations are prevalent
  - Learning with the presence of strategic behavior
    - We will discuss more on this in the lecture of Nov 13

Learning with the Presence of Strategic Behavior

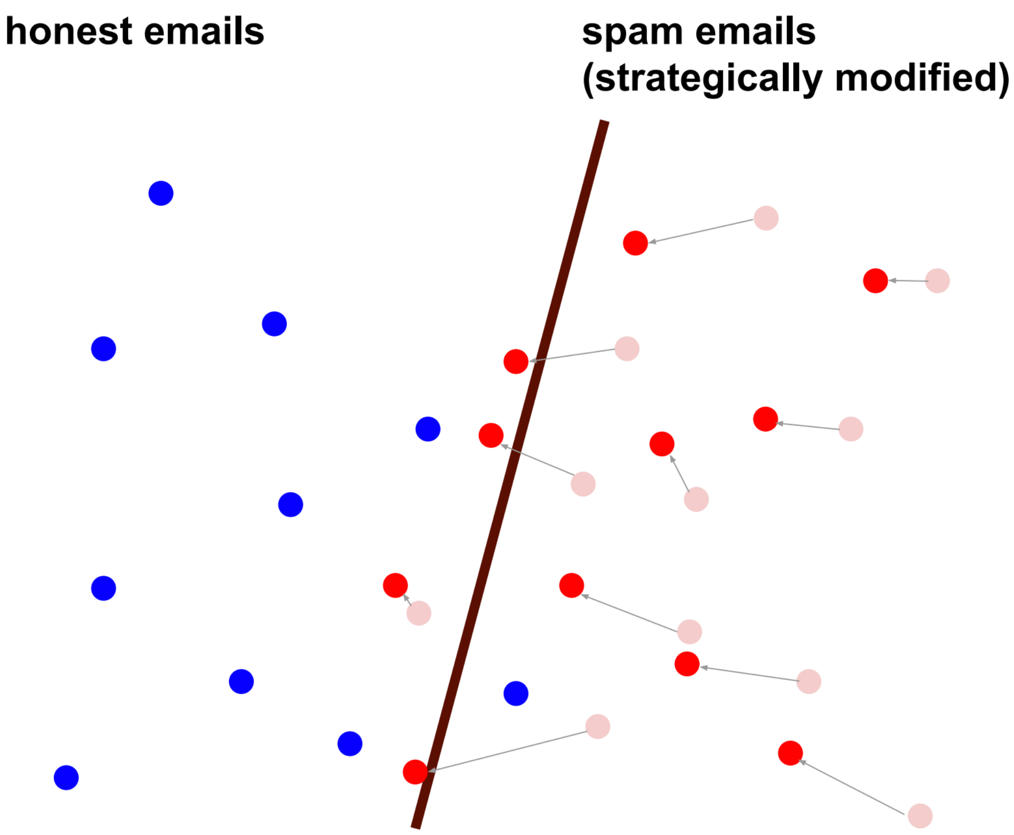
Example: Spam Classification



Example: Spam Classification



Example: Spam Classification



Goodhart’s law:
“If a measure becomes the public’s goal, it is no longer a good measure.”

What We Learned So Far

- EM-based methods
  - Empirically performs well
  - Relatively computationally efficient
  - No theoretical guarantee

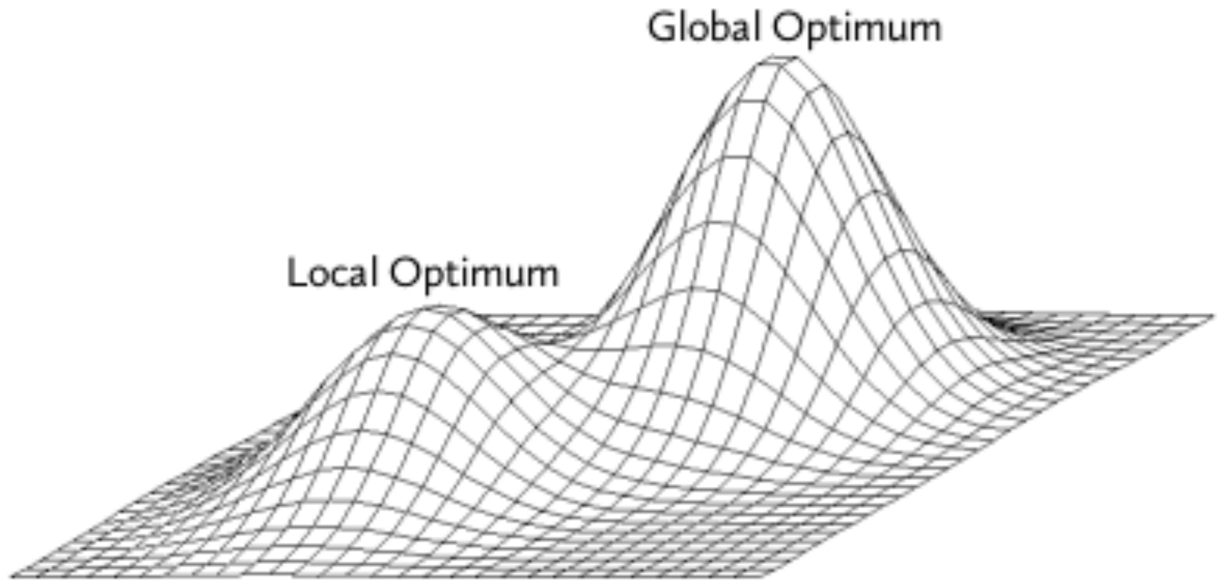
- Matrix-based methods
  - Comes with theoretical guarantee
  - Computationally expensive

- Can we achieve the best of both worlds?

Spectral Methods Meet EM

- Spectral Methods Meet EM: A Provably Optimal Algorithm for Crowdsourcing. Zhang et al. JMLR 2016.

- The main issue for EM: Might converge to local optimum



Spectral Methods Meet EM

- Key idea:
  - Estimate the “confusion matrix” from data
  - Using the estimation as the initial point for running the EM algorithm

- Key results
  - Given this fine-tuned starting point, with high probability, EM can achieve global optimal

Reading Next Monday

- Our last “label aggregation” lecture

- One of the well-cited papers in label aggregation
  - One of the early non-EM-based papers
  - The algorithm is very simple and intuitive
  - Solid theoretical guarantees
  - One of the first to formally address the task assignment question

Discussion

- General thoughts about this work.
- What are your thoughts on the manipulation issue? Any way to fight again it?
- What kind of research do you like? Why?
  - Empirically oriented: Design new algorithms and examine them on some datasets. No theoretical guarantees.
  - Theoretically oriented: Make assumptions on the target problems. Design algorithms and prove theoretical properties of the algorithms.