

CSE 417T

# Introduction to Machine Learning

Lecture 22

Instructor: Chien-Ju (CJ) Ho

# Logistics

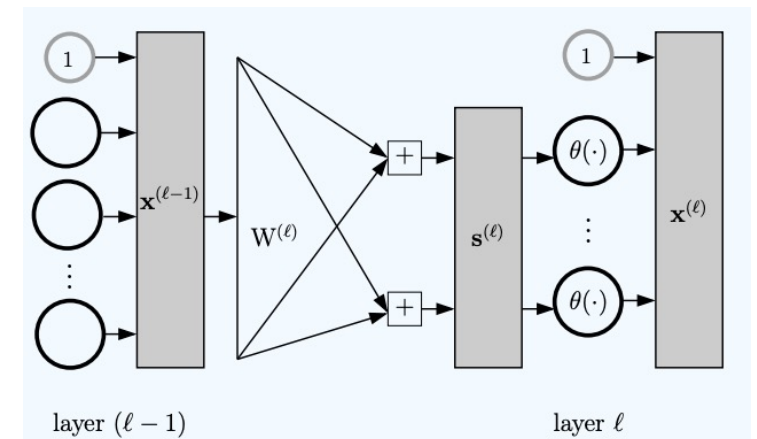
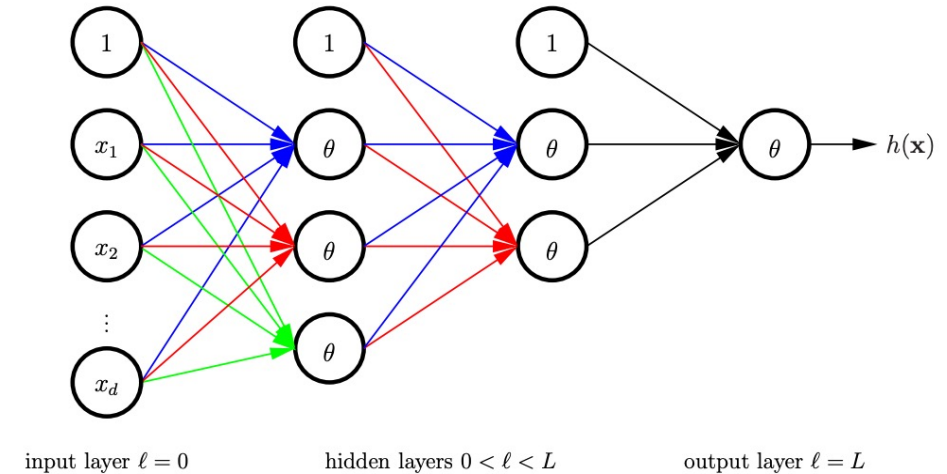
- Homework 5 is due Apr 19 (Tuesday)
- Exam 2 will be on April 28 (Thursday)
  - Will focus on the topics in the second half of the semester
  - Format / logistics will be similar with what we have in Exam 1
    - Timed exam (75 min) during lecture time in the classroom
    - Closed-book exam with 2 letter-size cheat sheets allowed (4 pages in total)
      - No format limitations (it can be typed, written, or a combination)
  - April 26 (Tuesday) will be a review lecture

Recap

# Neural Networks (NN)

- Notations:
  - $\ell = 0$  to  $L$ : layer
  - $d^{(\ell)}$ : dimension of layer  $\ell$
  - $\vec{x}^{(\ell)}$ : the nodes in layer  $\ell$
  - $w_{i,j}^{(\ell)}$ : weights; characterize hypothesis in NN
  - $s_j^{(\ell)} = \sum_{i=0}^{d^{(\ell-1)}} w_{i,j}^{(\ell)} x_i^{(\ell-1)}$ : linear signals
  - $\theta$ : activation function
    - $x_j^{(\ell)} = \theta(s_j^{(\ell)})$

Feed-forward network

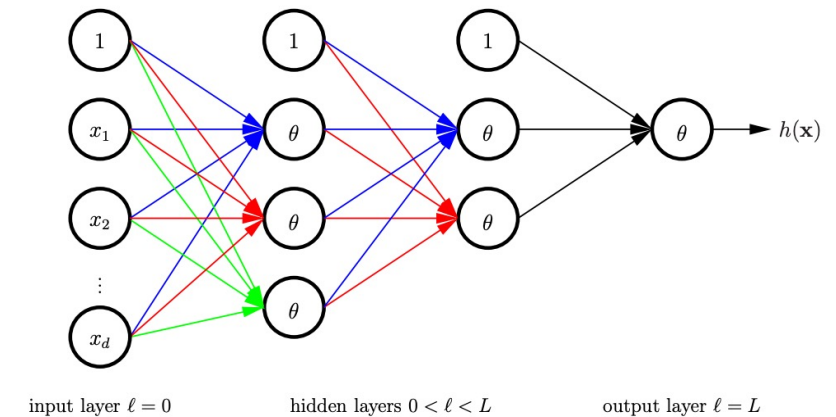


# Forward Propagation and Backpropagation

- Evaluate  $h(\vec{x})$  given  $h$  (characterized by  $\{w_{i,j}^{(\ell)}\}$ )
  - Forward propagation

$$\mathbf{x} = \mathbf{x}^{(0)} \xrightarrow{w^{(1)}} \mathbf{s}^{(1)} \xrightarrow{\theta} \mathbf{x}^{(1)} \xrightarrow{w^{(2)}} \mathbf{s}^{(2)} \xrightarrow{\theta} \mathbf{x}^{(2)} \dots \xrightarrow{w^{(L)}} \mathbf{s}^{(L)} \xrightarrow{\theta} \mathbf{x}^{(L)} = h(\mathbf{x}).$$

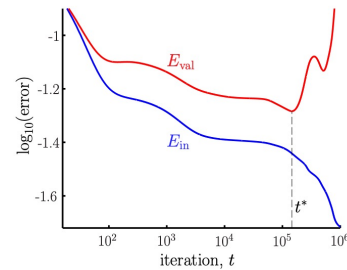
- Given  $D$ , learn the weights  $W = \{w_{i,j}^{(\ell)}\}$ 
  - Backpropagation
  - Initialize  $w_{i,j}^{(\ell)}$  randomly
  - For  $t = 1$  to  $T$ 
    - Randomly pick a point from  $D$  (for stochastic gradient descent)
    - **Forward propagation:** Calculate all  $x_i^{(\ell)}$  and  $s_i^{(\ell)}$
    - **Backward propagation:** Calculate all  $\delta_j^{(\ell)}$
    - Update the weights  $w_{i,j}^{(\ell)} \leftarrow w_{i,j}^{(\ell)} - \eta \delta_j^{(\ell)} x_i^{(\ell-1)}$
  - Return the weights



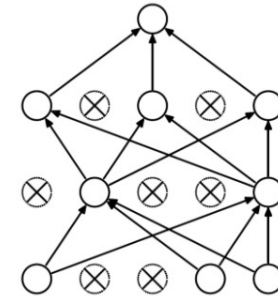
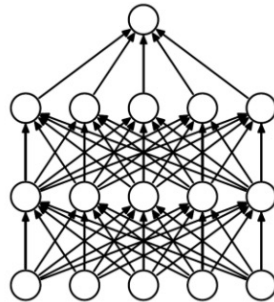
# Regularizations in Neural Networks

- Weight-based regularization

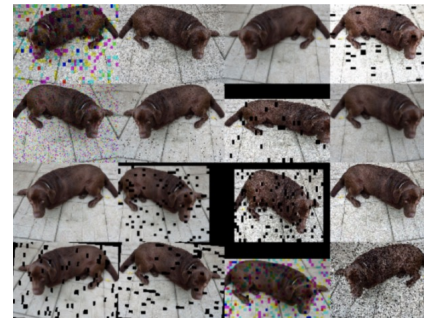
- Early stopping



- Dropout



- Adding noises



# Today's Lecture

The notes are not intended to be comprehensive. They should be accompanied by lectures and/or textbook.  
Let me know if you spot errors.

# Deep Learning

Neural networks with many layers



# Single Hidden-Layer Neural Network

- How do we write a hypothesis in a single-hidden layer NN mathematically?

- $$h(\vec{x}) = \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} x_j^{(1)} \right)$$
$$= \theta \left( w_{0,1}^{(2)} + \sum_{j=1}^{d^{(1)}} w_{j,1}^{(2)} \theta \left( \sum_{i=0}^{d^{(0)}} w_{i,j}^{(1)} x_i^{(0)} \right) \right)$$

- How do we write a linear model with nonlinear transform

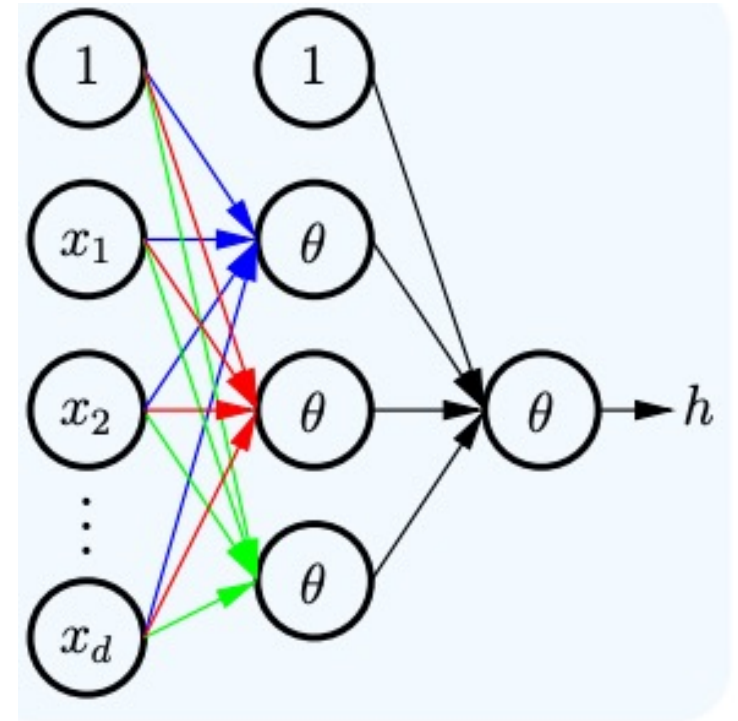
- $$h(\vec{x}) = \theta(w_0 + \sum w_i \phi_i(\vec{x}))$$

- How do we write a Kernel SVM hypothesis

- $$g(\vec{x}) = \theta \left( b^* + \sum_{\alpha_n^* > 0} \alpha_n^* y_n K(\vec{x}_n, \vec{x}) \right)$$

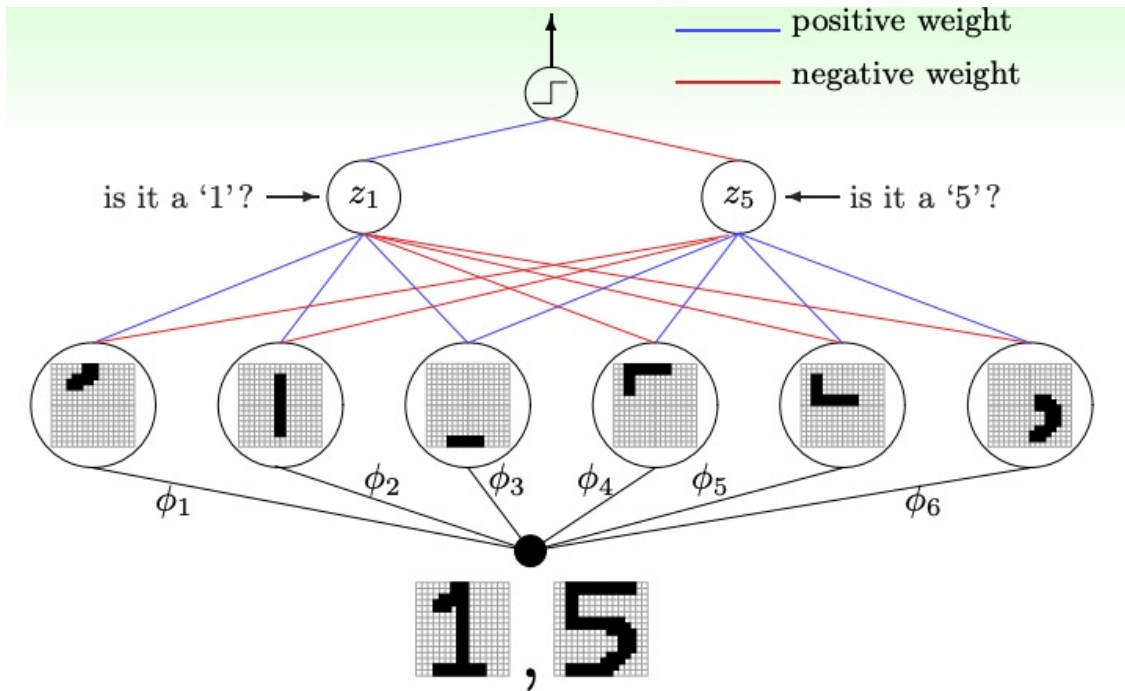
- Interpretation:

- The hidden layer is like **feature transform**
  - Shallow learning vs. deep learning



# Deep Neural Network

- “Shallow” neural network is powerful (universal approximation theorem holds with a single hidden layer). Why “deep” neural networks?



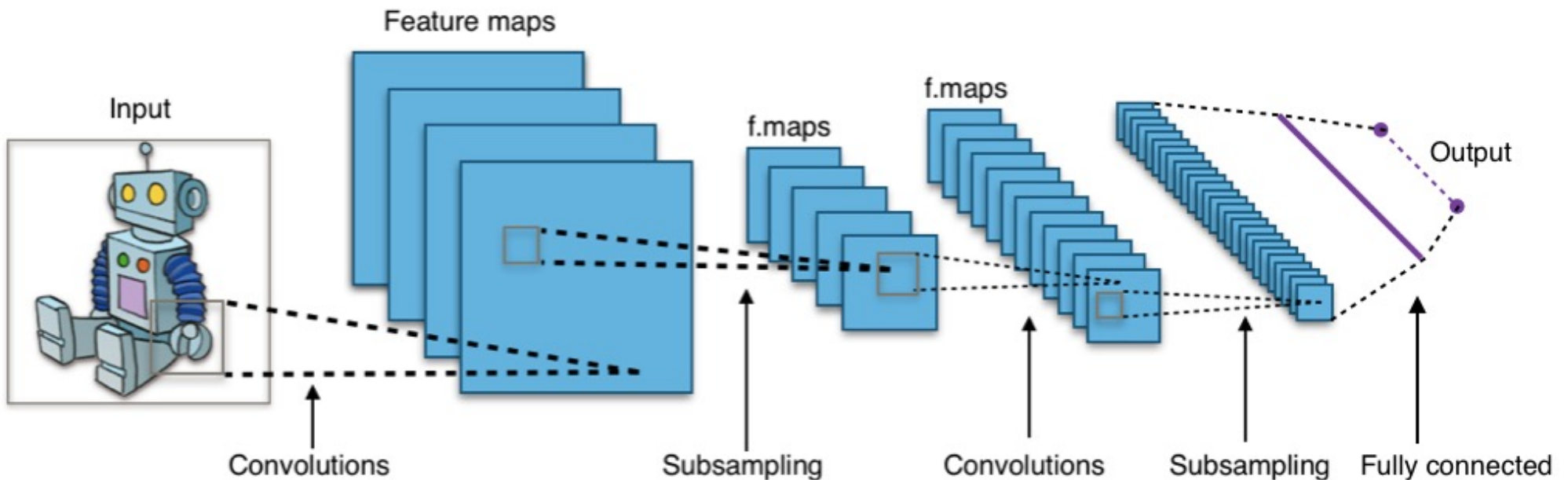
Each layer captures **features** of the previous layers.

We can use “raw data” (e.g., pixels of an image) as input. The hidden layer are extracting the **features**.

Design different **network architectures** to incorporate domain knowledge.

# Convolutional Neural Networks (CNN)

- Captures the localized properties of features hierarchically



# Convolutional Filters

- A convolutional filter is like a matrix version of a dot product.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 1 | 0 |
| 0 | 2 | 4 | 4 | 2 | 0 |
| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

\*

|   |    |   |
|---|----|---|
| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

# Convolutional Filters

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| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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|   |    |   |
|---|----|---|
| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

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|---|--|--|--|
| 0 |  |  |  |
|   |  |  |  |
|   |  |  |  |
|   |  |  |  |

$$\begin{aligned} & (0 * 0) + (0 * 1) + (0 * 0) + \\ & (0 * 1) + (1 * -4) + (2 * 1) + \\ & (0 * 0) + (2 * 1) + (4 * 0) \\ & = 0 \end{aligned}$$

# Convolutional Filters

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|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 1 | 0 |
| 0 | 2 | 4 | 4 | 2 | 0 |
| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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|   |    |   |
|---|----|---|
| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

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| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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|   |    |   |
|---|----|---|
| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

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|   |    |  |  |
|---|----|--|--|
| 0 | -1 |  |  |
|   |    |  |  |
|   |    |  |  |
|   |    |  |  |

$$\begin{aligned} & (0 * 0) + (0 * 1) + (0 * 0) + \\ & (1 * 1) + (2 * -4) + (2 * 1) + \\ & (2 * 0) + (4 * 1) + (4 * 0) \\ & = -1 \end{aligned}$$

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|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 1 | 0 |
| 0 | 2 | 4 | 4 | 2 | 0 |
| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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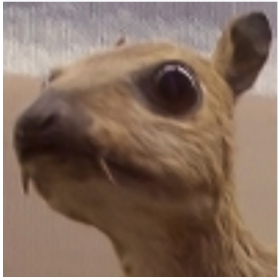
|   |    |   |
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| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

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|----|----|----|----|
| 0  | -1 | -1 | 0  |
| -2 | -5 | -5 | -2 |
| 2  | -2 | -1 | 3  |
| -1 | 0  | -5 | 0  |

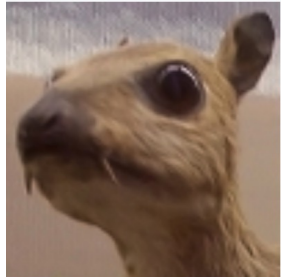


# Convolutional Filters



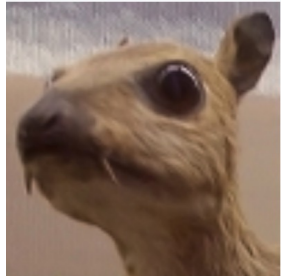
| Operation | Kernel $\omega$   | Image result $g(x,y)$ |
|-----------|---|-----------------------|
|           | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$         |                       |
|           | $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$       |                       |
|           | $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$        |                       |
|           | $\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ |                       |

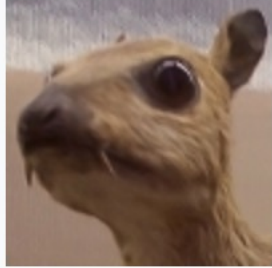
# Convolutional Filters



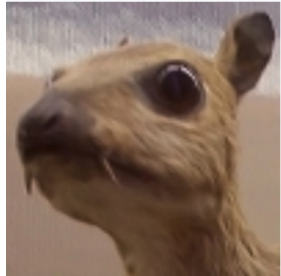
| Operation       | Kernel $\omega$   | Image result $g(x,y)$ |
|-----------------|---|-----------------------|
| Identity        | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$         |                       |
| Ridge detection | $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$       |                       |
|                 | $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$        |                       |
|                 | $\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ |                       |

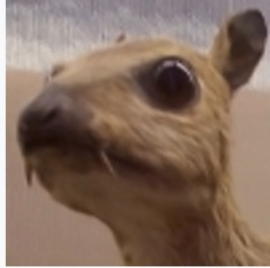
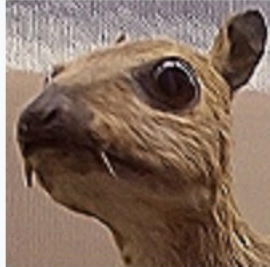
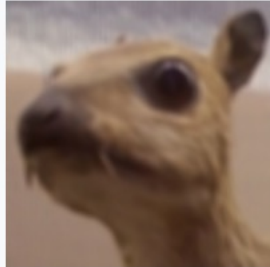
# Convolutional Filters



| Operation | Kernel $\omega$   | Image result $g(x,y)$   |
|-----------|---|---|
| Identity  | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$             |  |
|           | $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$         |   |
|           | $\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ |   |

# Convolutional Filters



| Operation                       | Kernel $\omega$   | Image result $g(x,y)$  |
|---------------------------------|---|--|
| <b>Identity</b>                 | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$             |   |
| <b>Sharpen</b>                  | $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$         |   |
| <b>Box blur</b><br>(normalized) | $\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ |  |

# Connection to Neural Networks

- Convolutions can be represented by a network structure
  - Nodes in the previous layer are only connected to “adjacent” nodes in the next layer.
  - Many of the weights have the same value.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 1 | 0 |
| 0 | 2 | 4 | 4 | 2 | 0 |
| 0 | 1 | 3 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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|   |    |   |
|---|----|---|
| 0 | 1  | 0 |
| 1 | -4 | 1 |
| 0 | 1  | 0 |

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|    |    |    |    |
|----|----|----|----|
| 0  | -1 | -1 | 0  |
| -2 | -5 | -5 | -2 |
| 2  | -2 | -1 | 3  |
| -1 | 0  | -5 | 0  |

# Pooling Layers

- Commonly used in convolutional neural networks.

A subsampling / down-sampling process:

- Combines multiple adjacent nodes into a single node

|    |    |    |    |
|----|----|----|----|
| 0  | -1 | -1 | 0  |
| -2 | -5 | -5 | -2 |
| 2  | -2 | 2  | 3  |
| -1 | 1  | -5 | 0  |

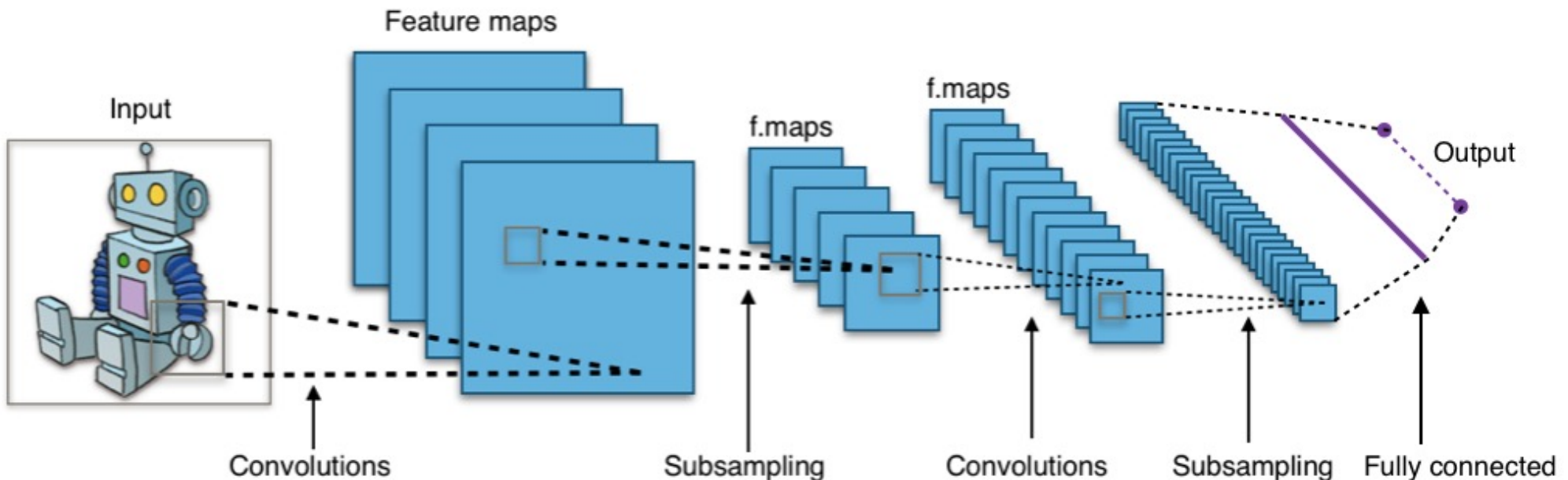
*average  
pooling*

|    |    |
|----|----|
| -2 | -2 |
| 0  | 0  |

- Reduce the dimensionality of input. More robust to noise.

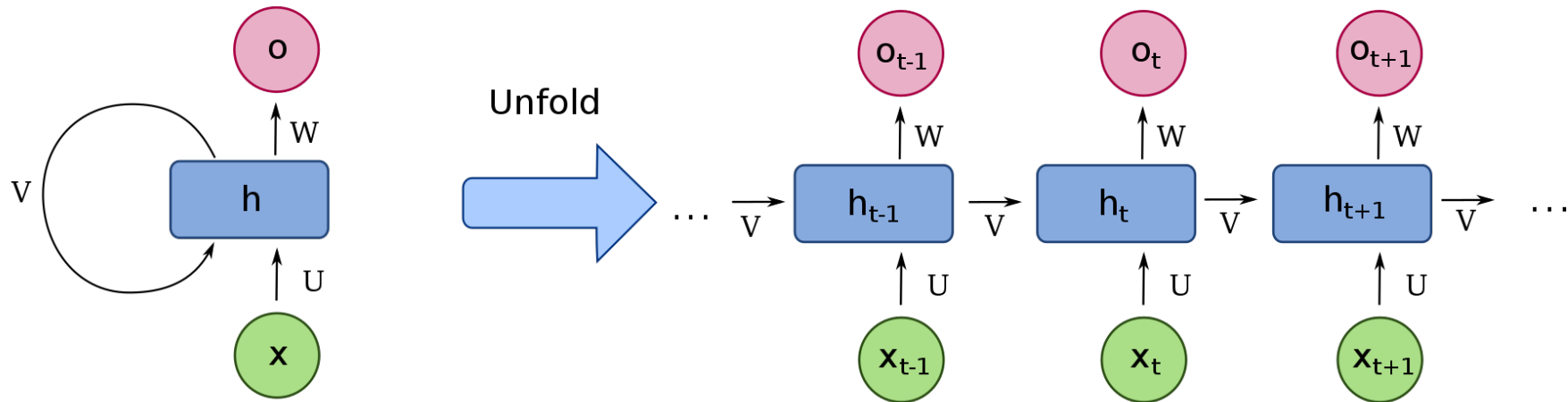
# Convolutional Neural Networks (CNN)

- Captures the localized properties of features
  - Particularly suitable for computer vision (images)
  - Go (AlphaGo) is another famous application of CNN



# Another Example Network Structure [Safe to Skip for the Exam]

- Recurrent Neural Network (RNN)
  - Aim to deal with time-series data, such as natural language processing
  - Using hidden layers to store temporal information
  - Allow previous outputs to be used as inputs and keep hidden states





# Some Techniques in Improving Deep Learning

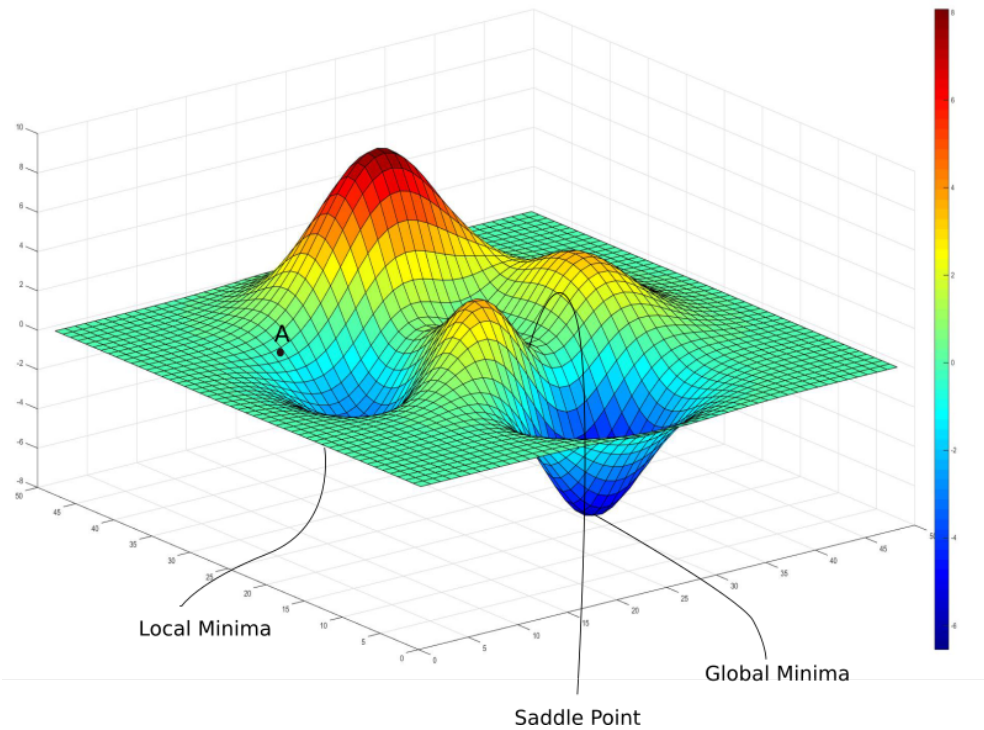
- Regularization to mitigate overfitting
  - Weight-based, early stopping, dropout, etc
- Incorporating domain knowledges
  - Network architectures (e.g., Convolutional Neural Nets)
- Improving computation with huge amount of data
  - Hardware architecture to improve parallel computation
- Improving gradient-based optimization
  - Choosing better **initialization** points

# Initialization

Why initialization matters in deep learning

- Error is nonconvex in NN
- Vanishing/exploding gradient problem

# Error is Nonconvex in Neural Networks



- We mostly adopt gradient-descent-style algorithms for optimization.
- No guarantee to converge to global optimal.
- Need to run it many times.
- Initialization matters.

# Vanishing Gradient Problem

- Backpropagation

- $\frac{\partial e_n(W)}{\partial w_{i,j}^{(\ell)}} = \delta_j^{(\ell)} x_i^{(\ell-1)}$

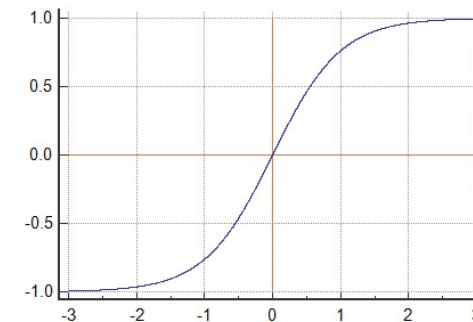
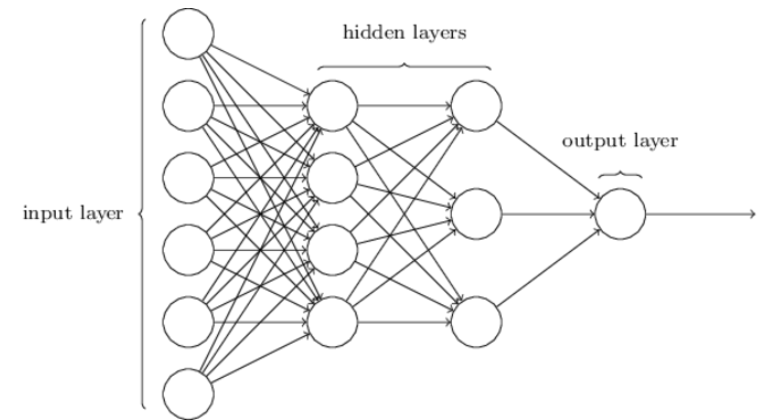
- $\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$

- If we use activation function  $\theta(s) = \tanh(s)$

- $\theta'(s) = 1 - \theta(s)^2 < 1$

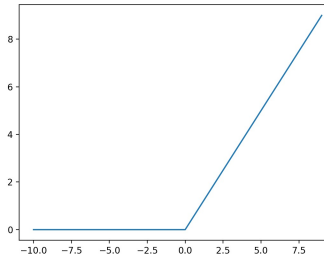
- In deep learning with a lot of layers,

- the gradient might vanish
  - hard to update the early layers



# Vanishing Gradient Problem

- $\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$
- There is also a corresponding “exploding gradient problem”
- What can we do
  - Choose different activation functions
    - One common choice is Rectified Linear Unit (ReLU) and its variant
      - $\theta(s) = \max(0, s)$
  - Choose better **initialization**
    - Many approaches



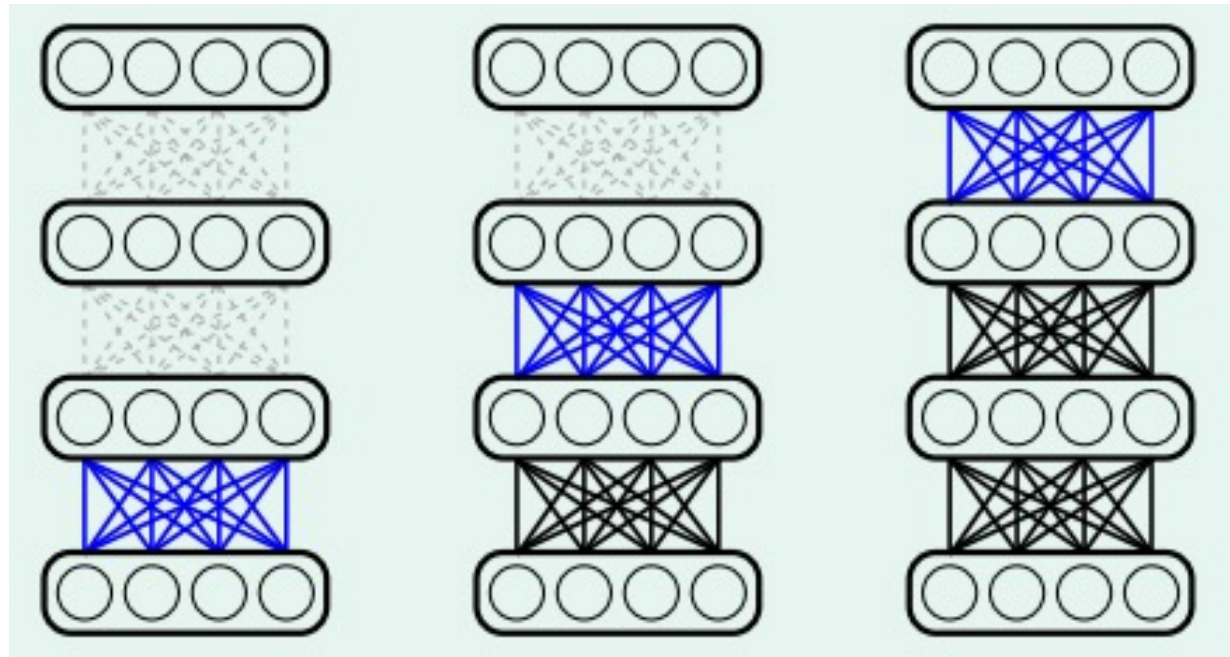
# Weight Initialization

- Initializing weights to all 0 is a bad idea
  - Q6b of HW1
  - Hint: Look at the backpropagation formulation
- Randomly Initializing weights to regions so that vanishing/exploding gradients are less likely to happen
  - Activation-function dependent
    - e.g., Xavier initialization for tanh
- Learning the initialization that might be closer to the optimal
  - E.g., using autoencoder

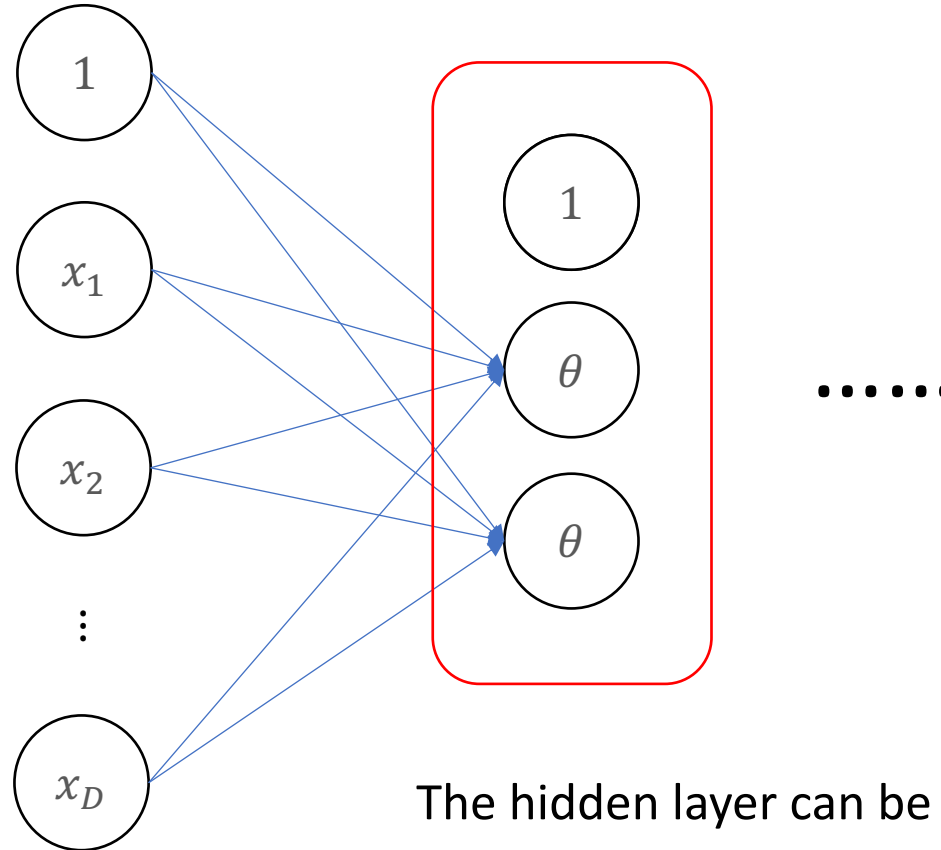
$$\delta_j^{(\ell)} = \theta' \left( s_j^{(\ell)} \right) \sum_{k=1}^{d^{(\ell+1)}} \delta_k^{(\ell+1)} w_{j,k}^{(\ell+1)}$$

# Initialization

- Hard to initialize the entire network well.
- Intuition: Initialize the weights **layer by layer** such that each layer **preserves** the properties of the previous layer.



# Autoencoders

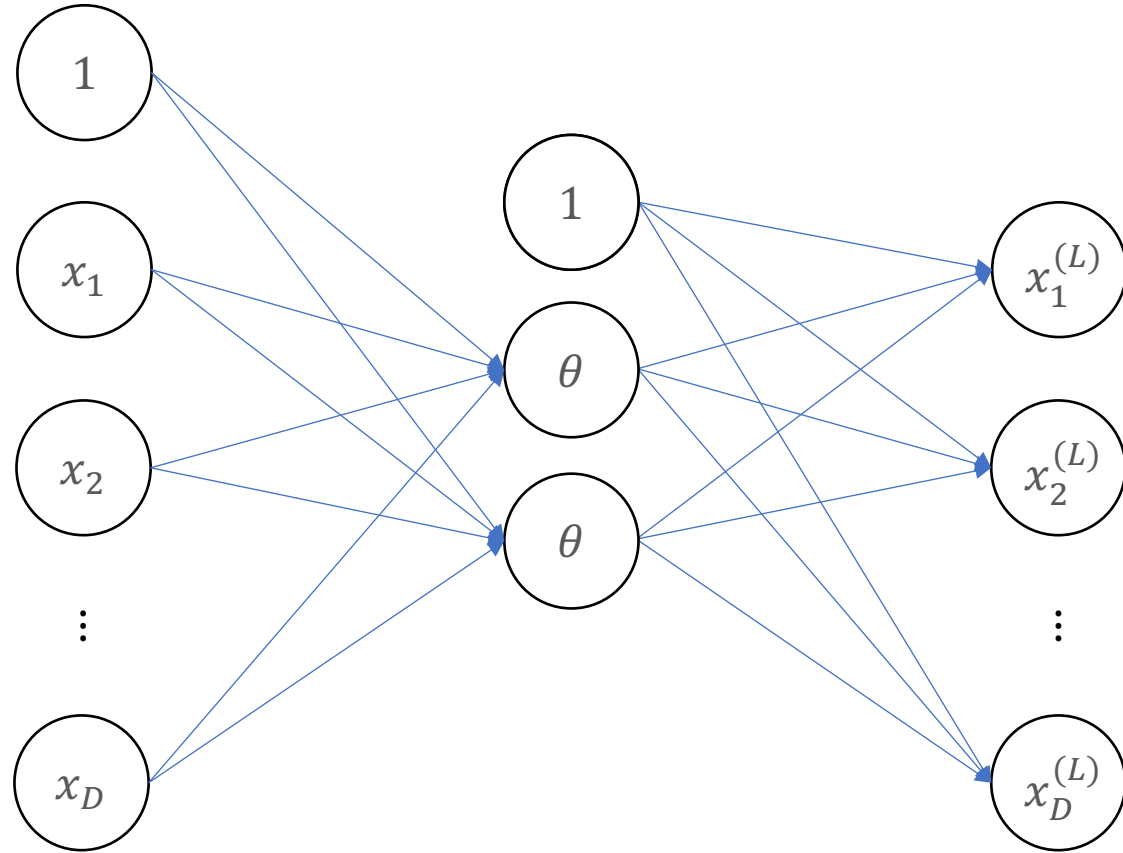


The hidden layer can be considered as a feature transform.

Can we ensure this transformation contain as much information about the original input as possible?

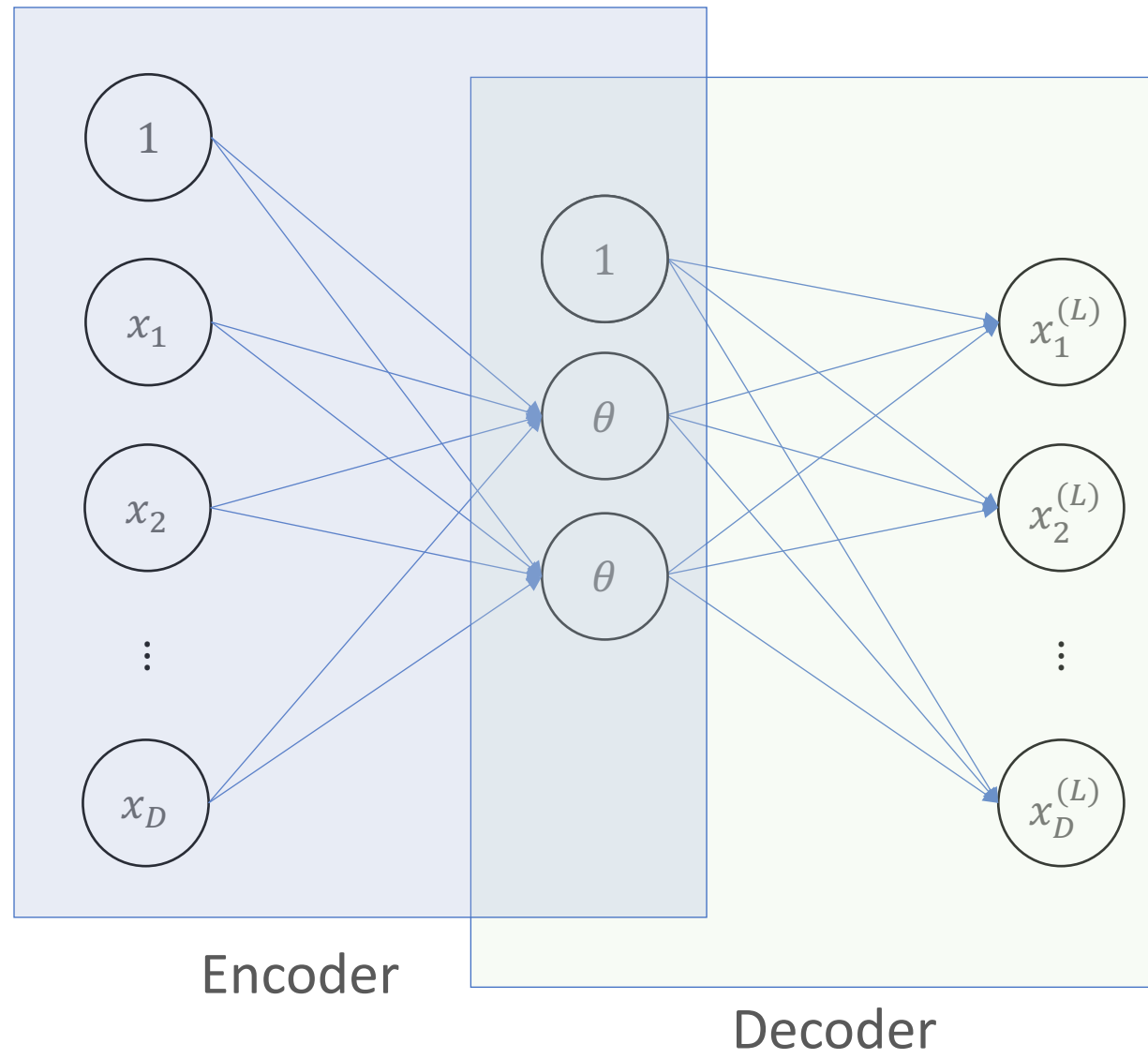


# Autoencoders

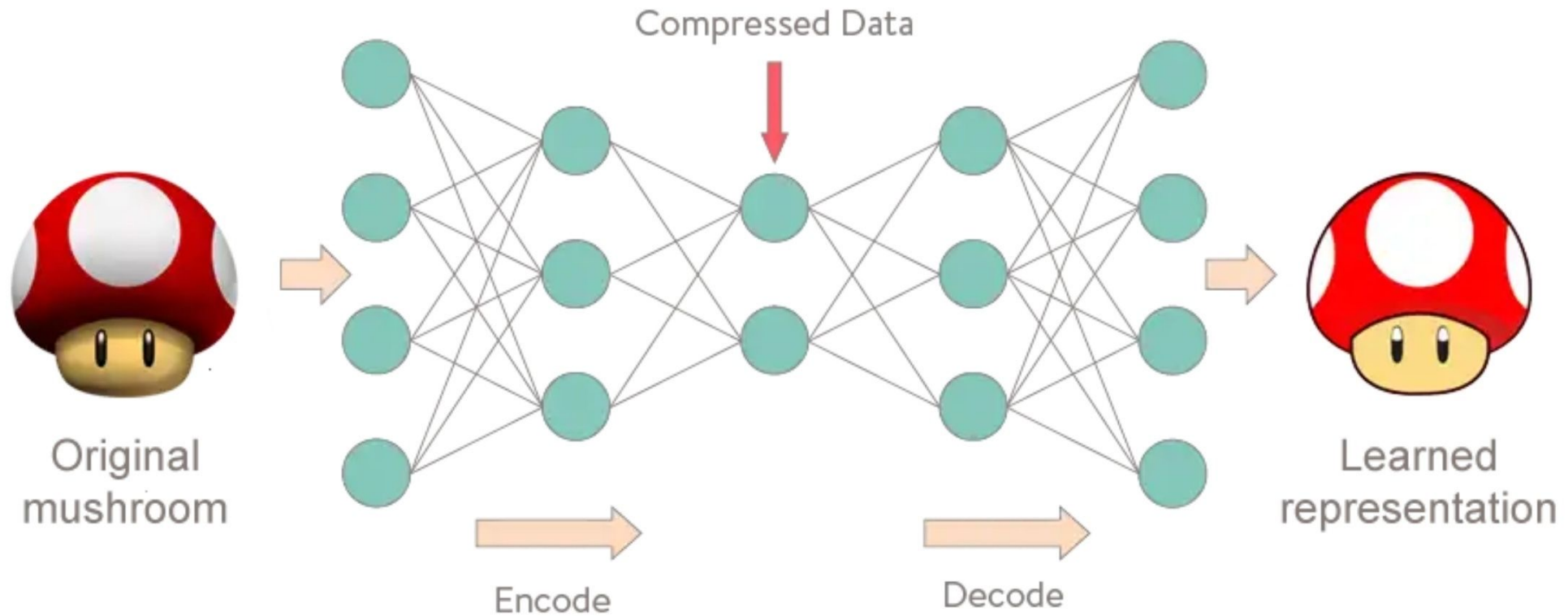


Minimize error of  $\left\| \vec{x} - \vec{x}^L \right\|$

# Autoencoders



# Autoencoder



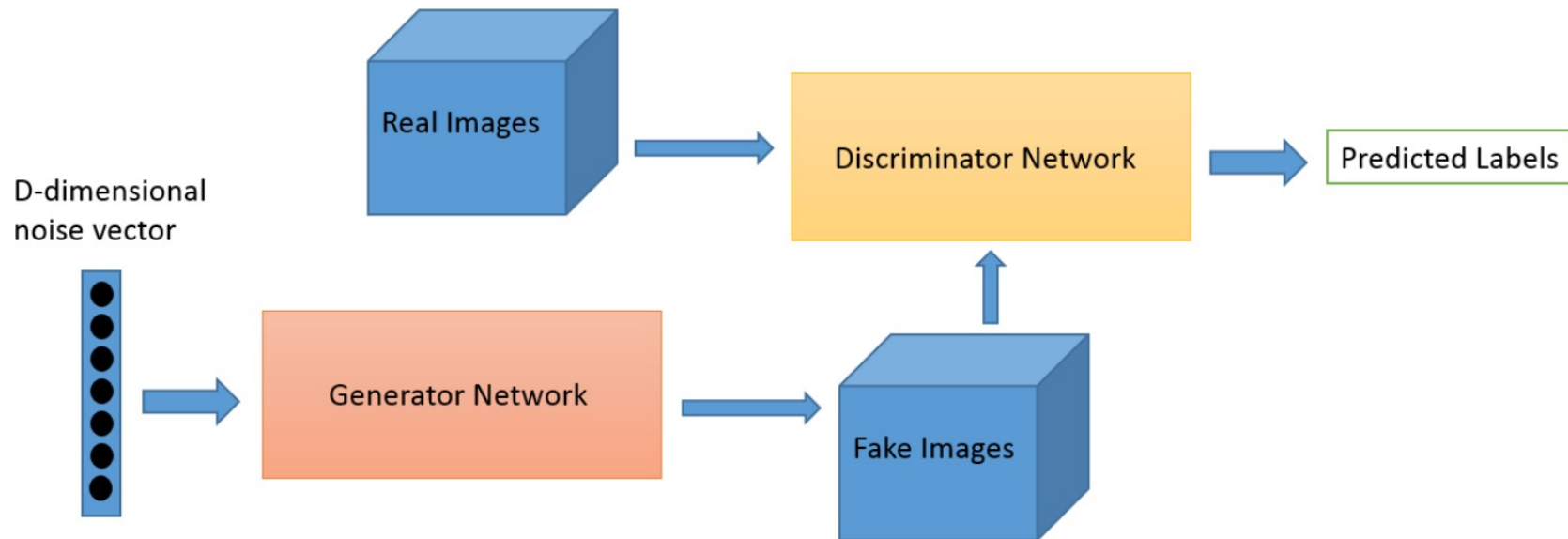
Unsupervised learning!

# Cool Stuffs for Deep Learning

[Safe to Skip for the exam]

# Generative Adversarial Nets (GAN)

- A Competition: Generator vs Discriminator
  - Discriminator wants to correctly classify the images (true images or not)
  - Generator wants to generate images that discriminator can't classify



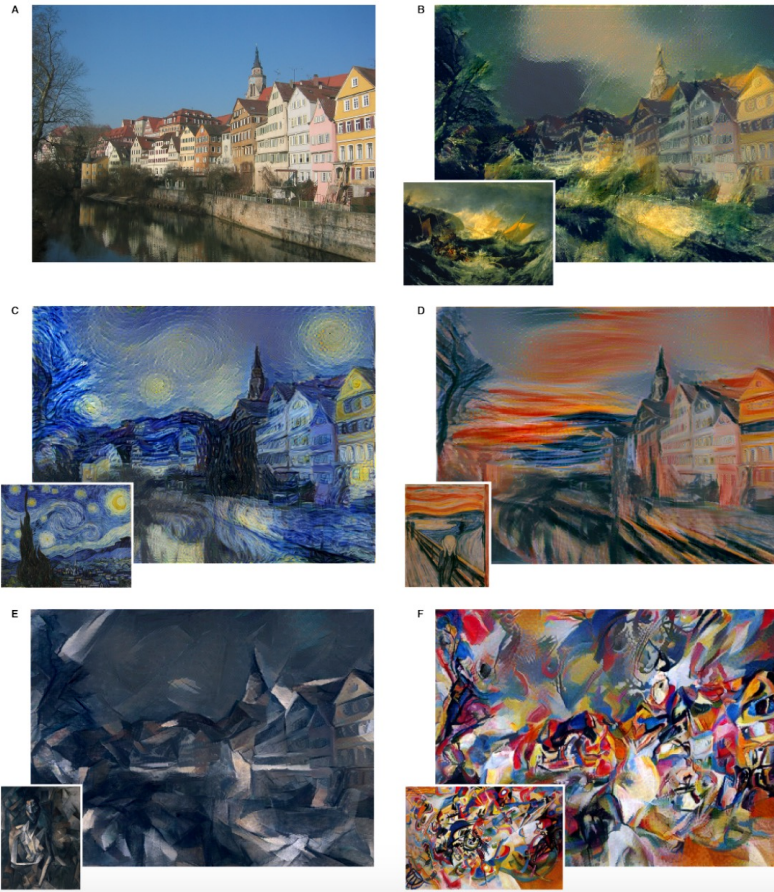
[Safe to Skip for the Exam]



<https://thisPersonDoesNotExist.com/>



# Style Transfer



- Informal intuitions:
  - Recall that we can treat hidden layers as **feature transforms**
  - Deep learning is learning **representation** of data
  - How to achieve style transfer:
    - Learn a **content representation** for an image using hidden layers
    - Learn a **style representation** for an image using hidden layers
    - Compute an image that jointly minimizes the distance from the content image's content representation and the style image's style representation
    - <https://arxiv.org/pdf/1508.06576.pdf>

A Quick Flashback to RBF



# RBF Function

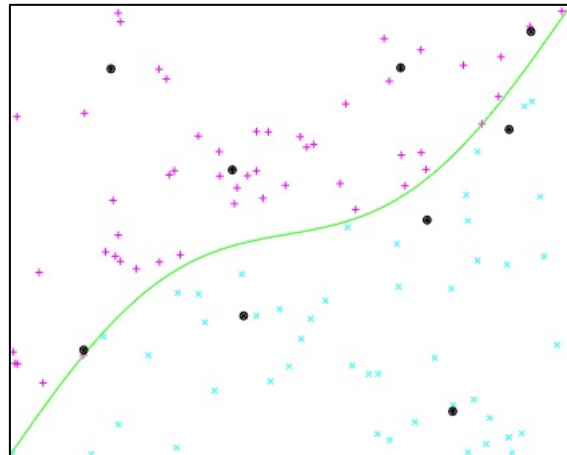
- $$h(\vec{x}) = \sum_{k=1}^K w_k \phi \left( \frac{\|\vec{x} - \vec{\mu}_k\|}{r} \right)$$

- Connection to linear models
  - Parametric RBF is essentially linear model with nonlinear transformation
- Connection to nearest neighbor
  - Radial Basis Function is defined by “similarity”
  - A prediction for a point is based on the “similarity” of the points to be predicted and other points

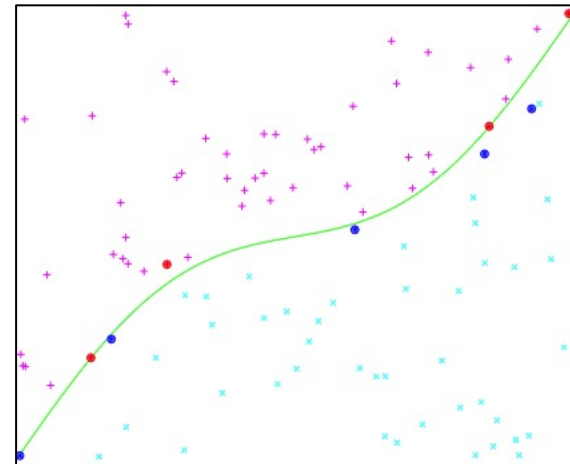
# More Discussion on RBF

- $h(\vec{x}) = \sum_{k=1}^K w_k \phi\left(\frac{\|\vec{x} - \vec{\mu}_k\|}{r}\right)$
- Connection to SVM
  - Gaussian RBF Kernel:  $g(\vec{x}) = \text{sign}\left(\sum_{\alpha_n^* > 0} \alpha_n^* y_n K(\vec{x}_n, \vec{x}) + b^*\right)$
  - Use **cluster centers** or **support vectors** to characterize a hypothesis

RBF

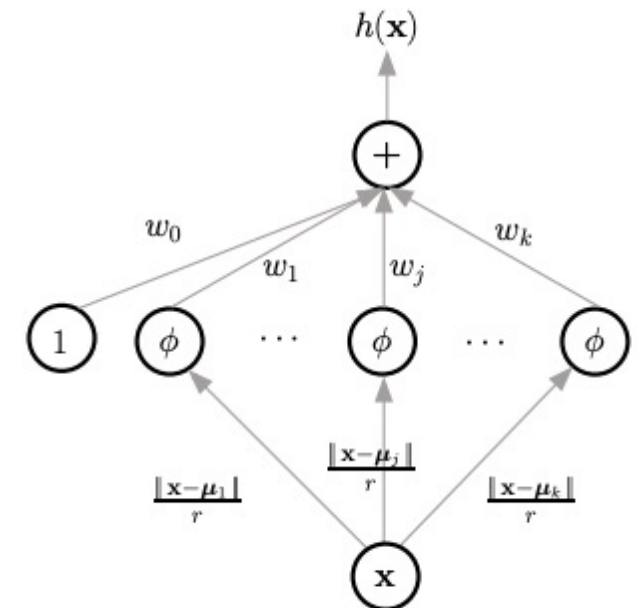


SVM



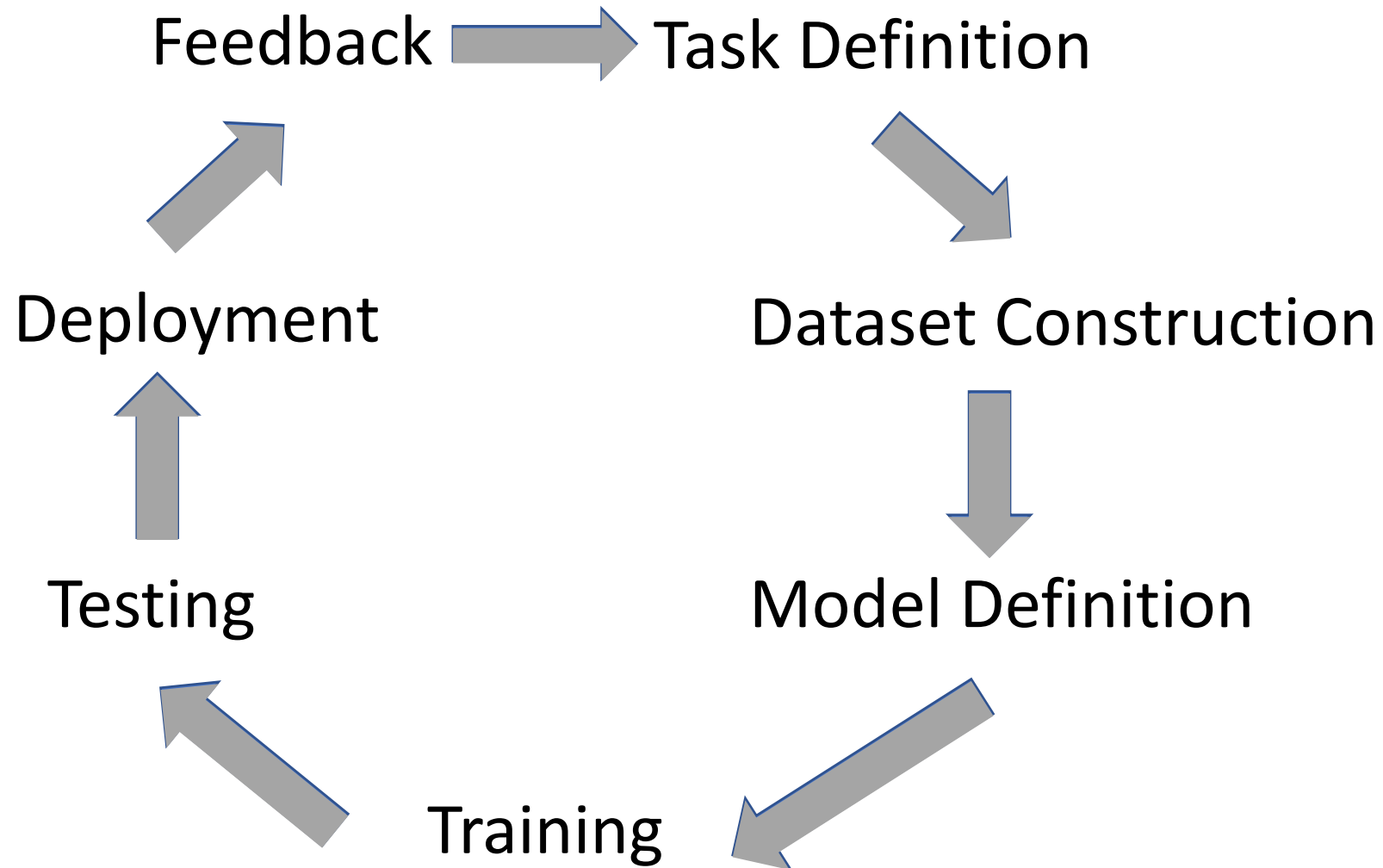
# More Discussion on RBF

- $h(\vec{x}) = \sum_{k=1}^K w_k \phi\left(\frac{\|\vec{x} - \vec{\mu}_k\|}{r}\right)$
- Connection to Neural Network
  - RBF can be graphically presented:
  - (Similar to SVM, it's a single-hidden layer NN)

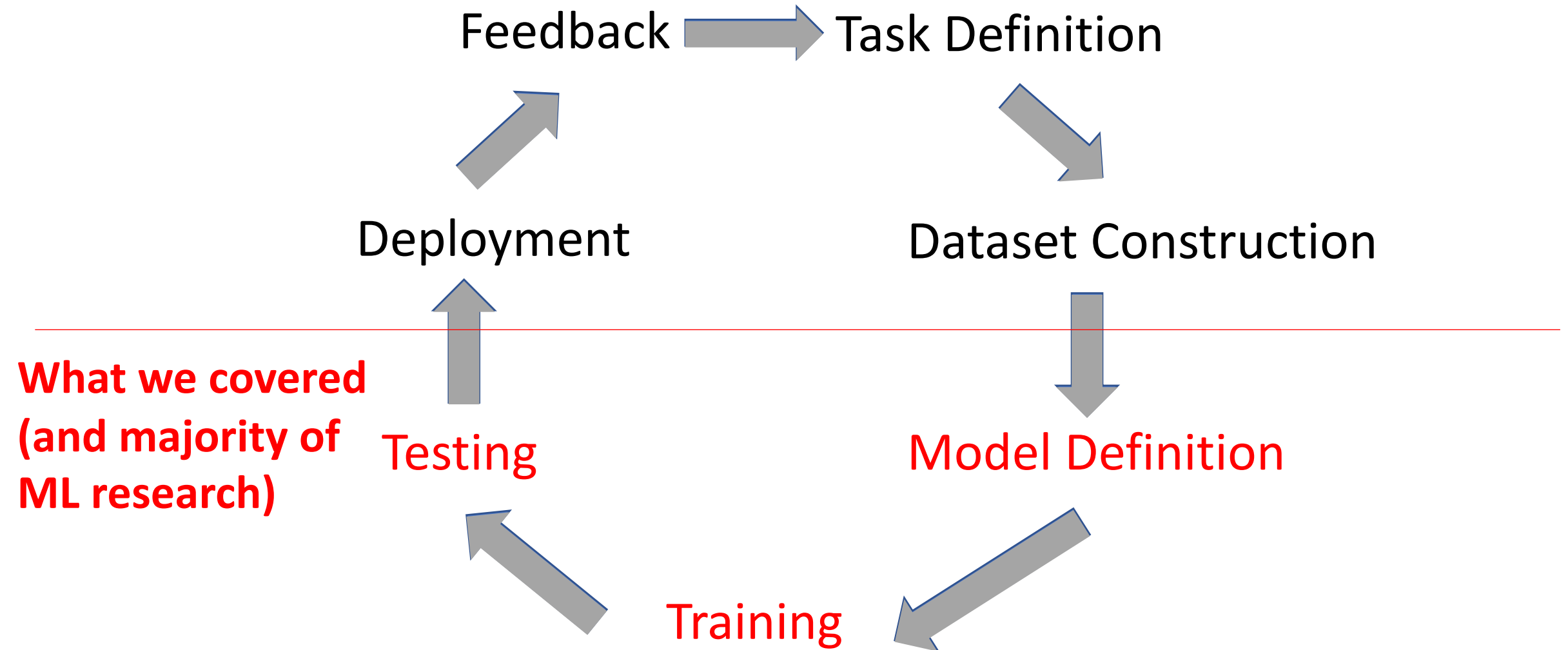


# Machine Learning Life Cycle

# Machine Learning Lifecycle



# Machine Learning Lifecycle



# Machine Learning Lifecycle

To have “positive” impacts,  
we need to be careful in  
every stage

Feedback → Task Definition

Deployment

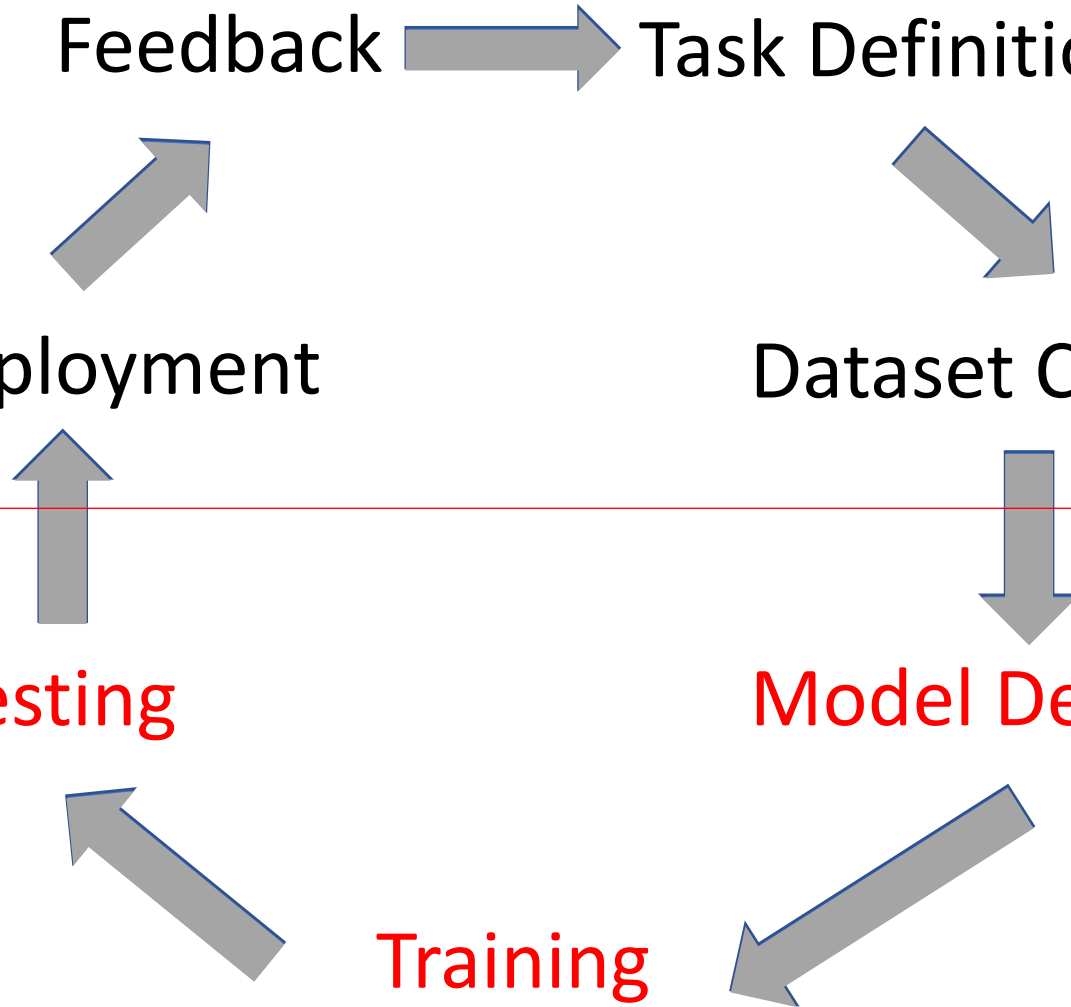
Dataset Construction

Testing

Model Definition

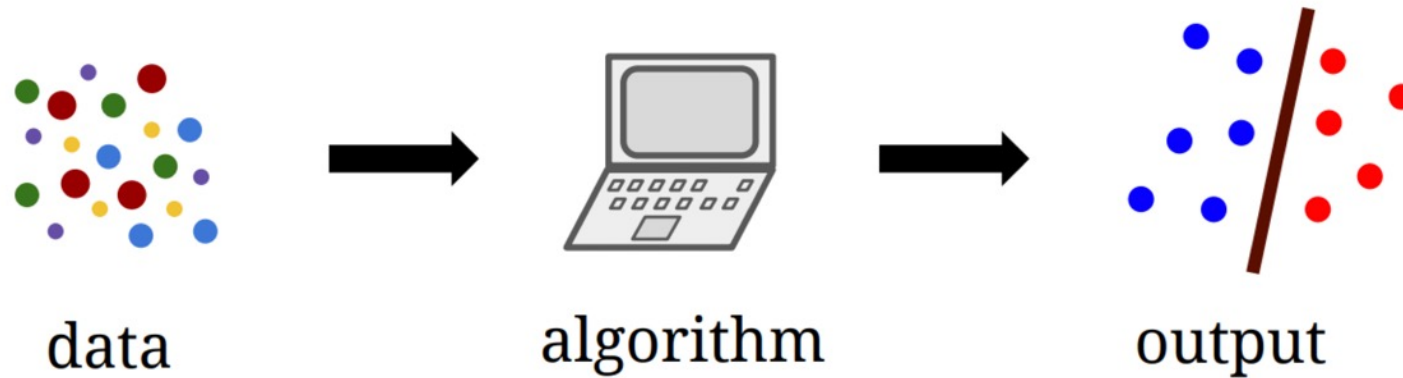
Training

What we covered  
(and majority of  
ML research)



# Supervised Learning

- Standard setup of (supervised) machine learning



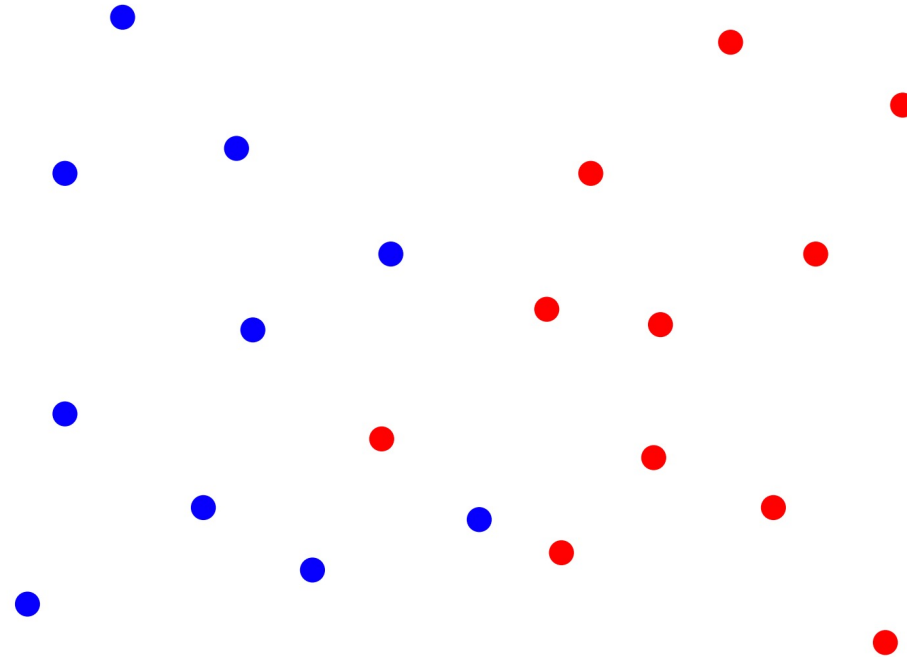
- Finding patterns from the given training datasets
  - Use the pattern to make predictions on new testing data
- 
- Fundamental assumption:
    - Training and testing data points are i.i.d. drawn from the same distribution



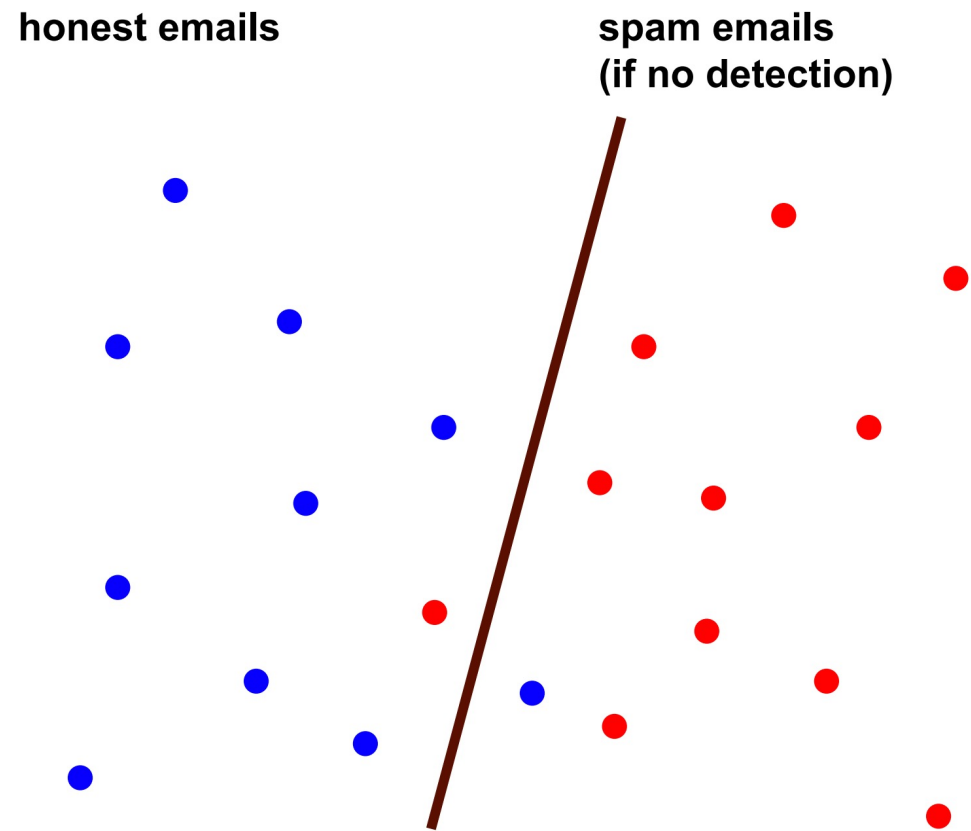
# Example: Spam Filter

**honest emails**

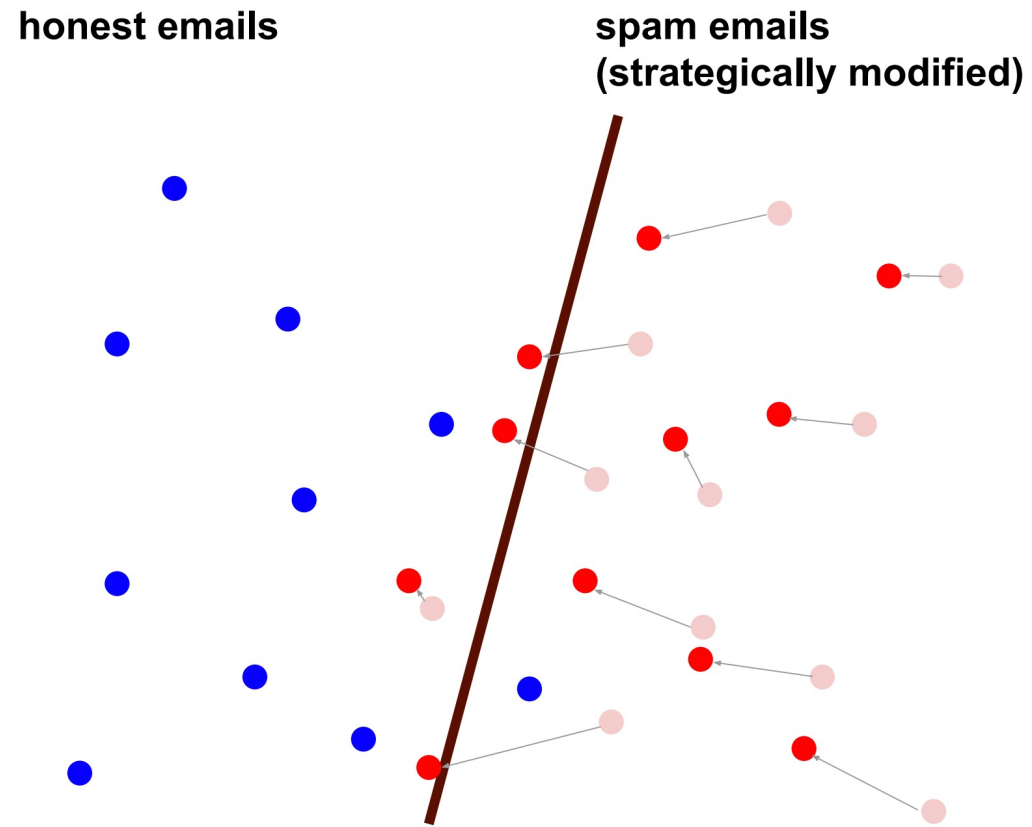
**spam emails  
(if no detection)**



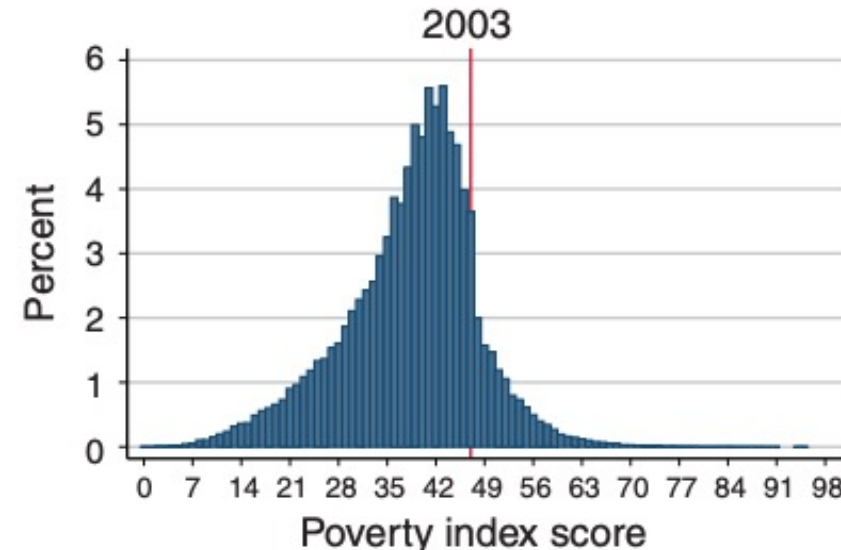
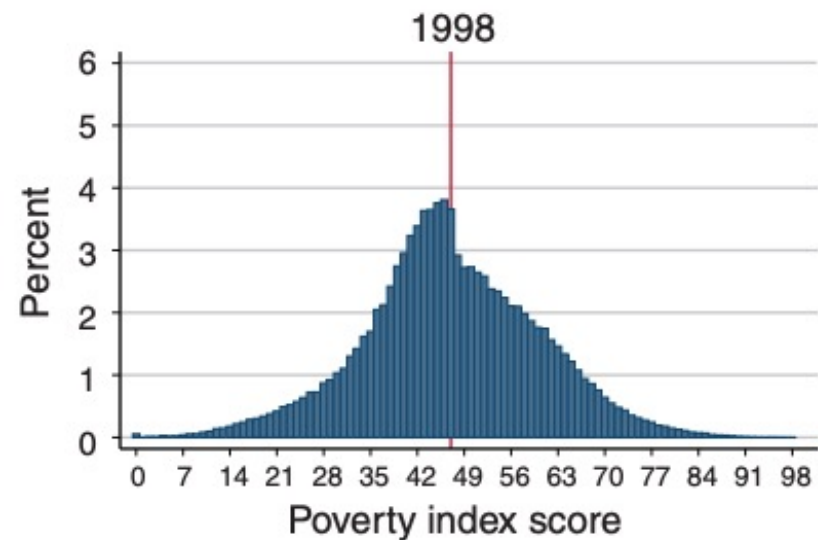
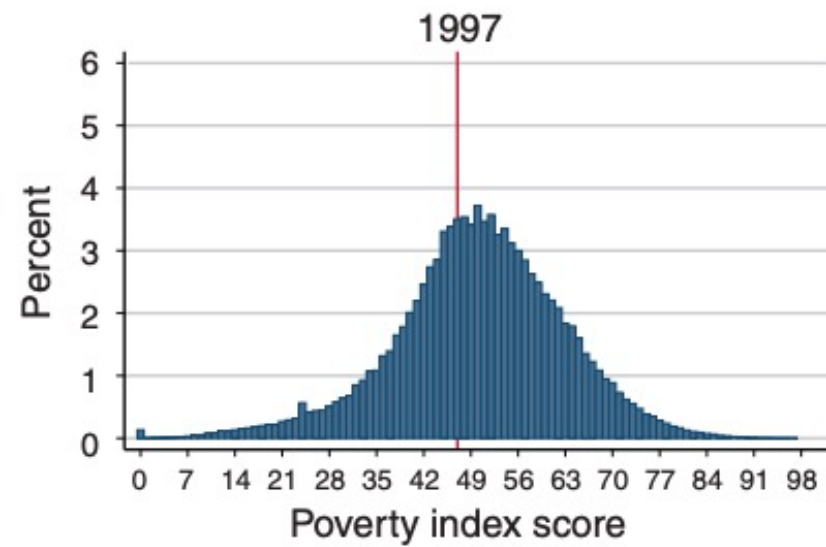
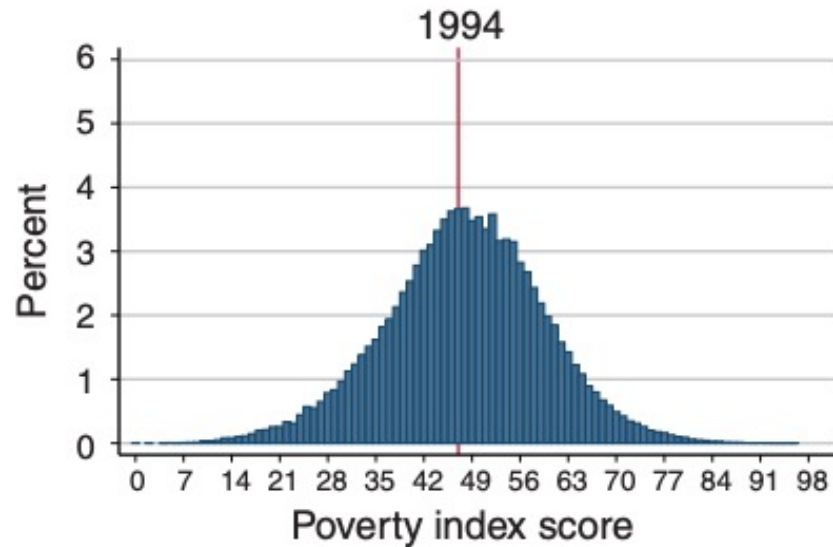
# Example: Spam Filter



# Example: Spam Filter



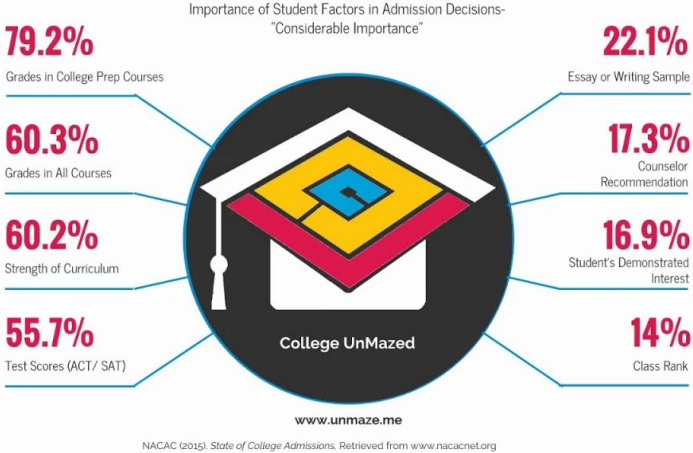
# Social Program Eligibility [Camacho and Conover, 2012]



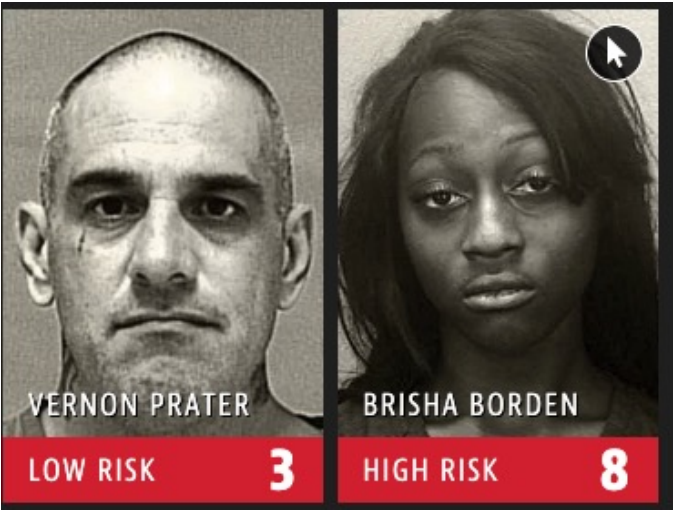
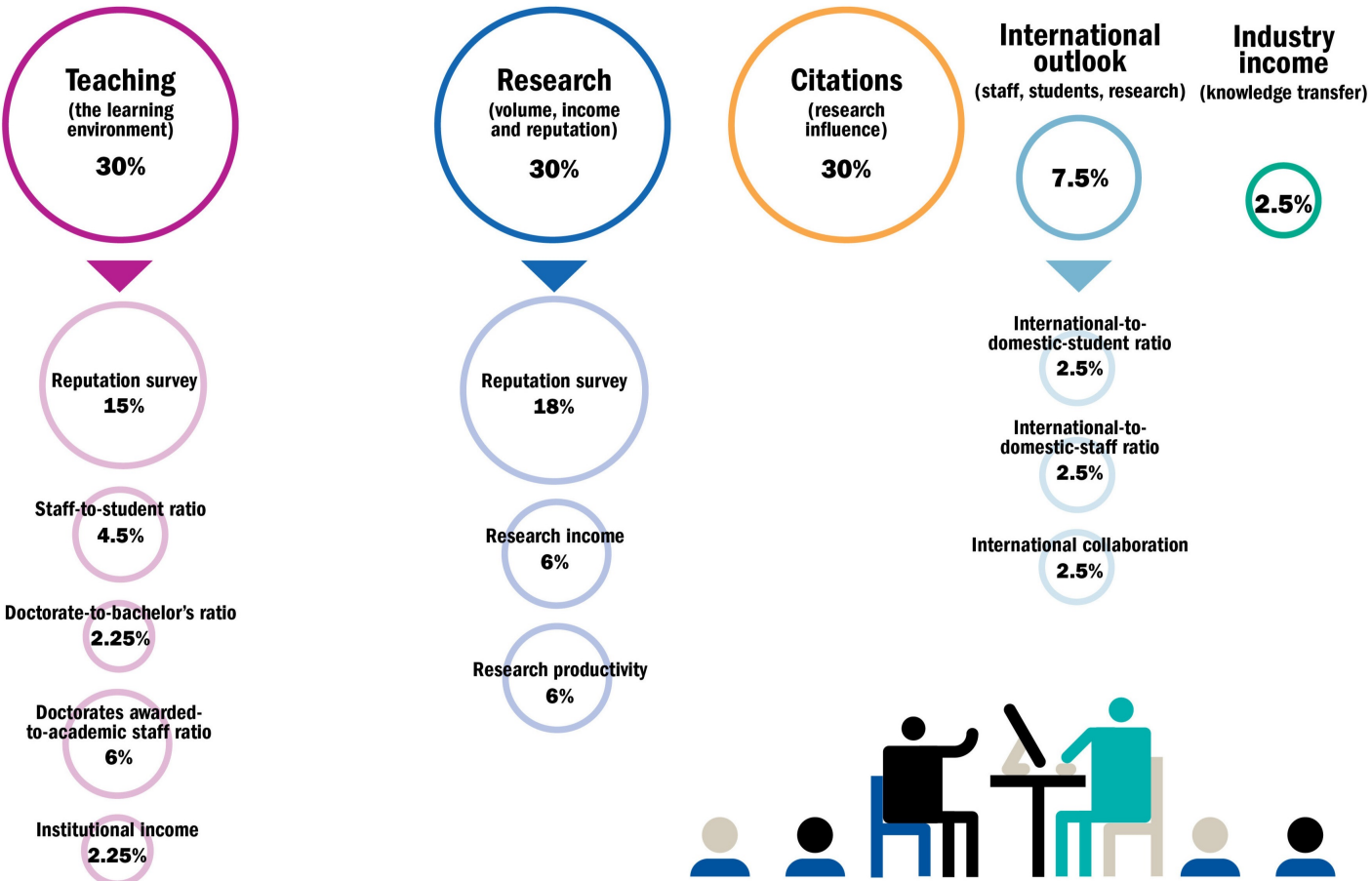
Goodhart's law:

“If a measure becomes the public's goal,  
it is no longer a good measure.”

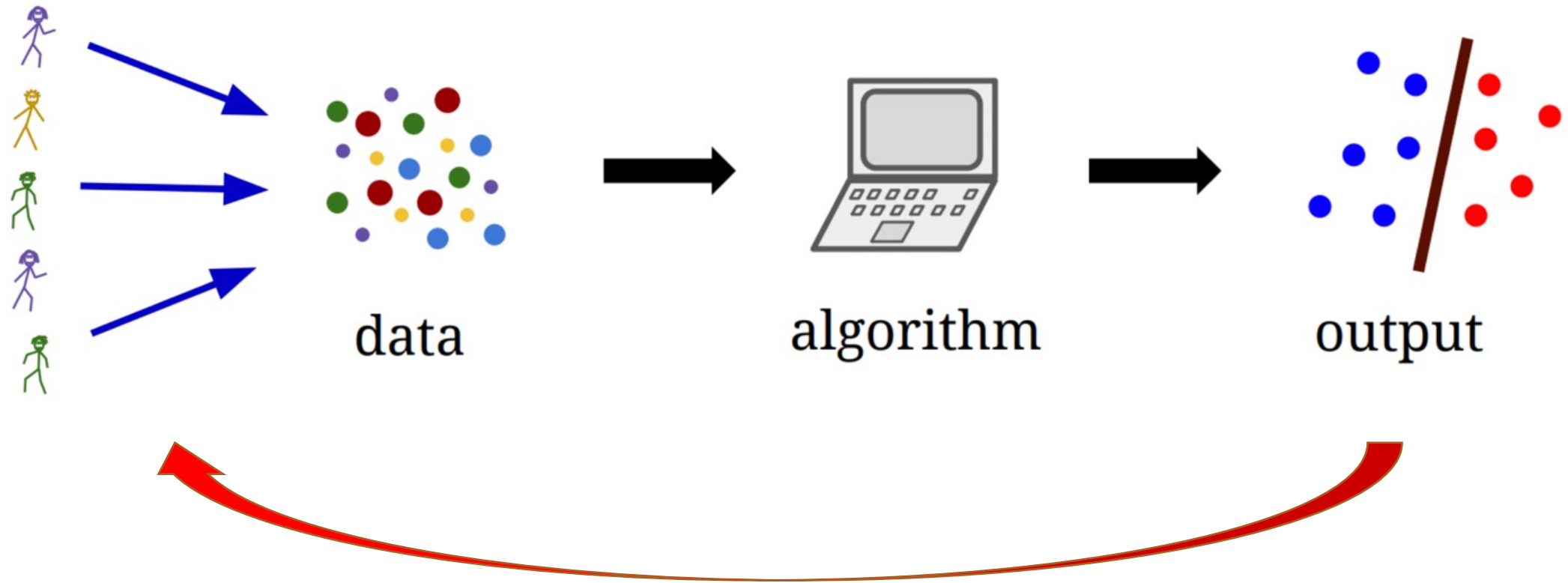
# COLLEGE ADMISSIONS



# Methodology



# Strategic Classification



# Machine Learning Lifecycle

