Logistics: Assignment 3 and Project Milestone 1

- No class next Thursday, October 13
- Assignment 3: Due Oct 17 (Monday)
- Project milestone 1: Due Oct 14 (Fri)
  - Initial literature survey (know what other works are out there)
  - A plan on what you want to do for the remaining of the semester
    - Formalize your research question and approaches, e.g.,
      - Theory/simulation project: formalize your models
      - Data-analysis project: figure out where and how to get data and what you plan to do with it
      - Experiment/application project: have a prototype design and an evaluation plan
    - Include a timeline (weekly or biweekly) on what you plan to do
      - Nov 1: Midterm Project Pitch and Discussion
      - Nov 4: Milestone 2

Logistics: Peer Review Forms

- The link to the form is posted on the course website

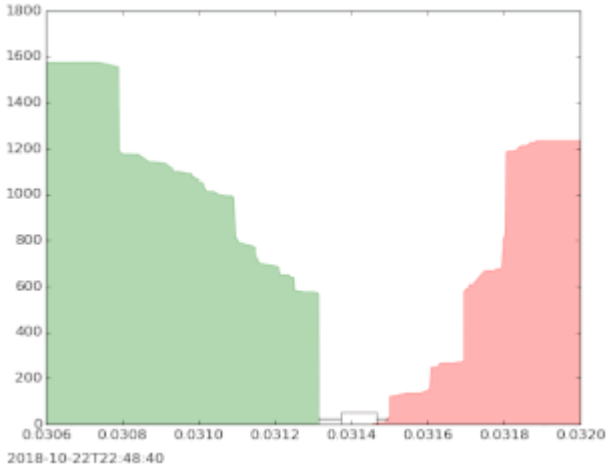
- Please try to submit the form before 6pm
- It might make sense to keep it open and write comments as the presentation goes.

Lecture 12
Additional Discussion for Prediction Markets: Automated Market Maker

Instructor: Chien-Ju (CJ) Ho

How Stock Market Works?

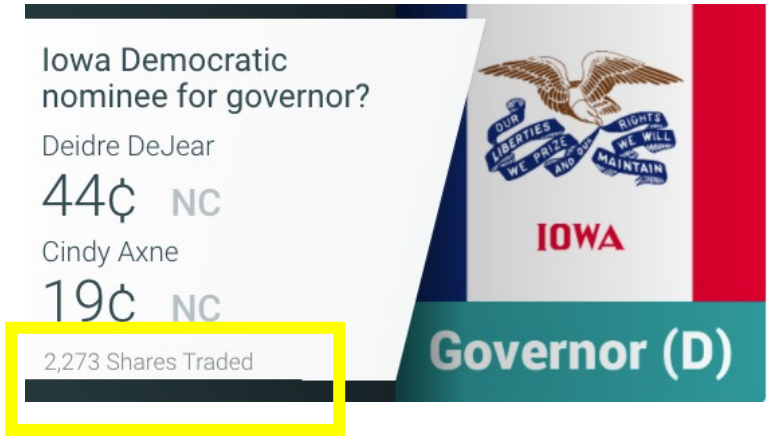
- Common mechanism: Continuous double-auction
  - Buyers bid prices and quantities
  - Sellers ask prices and quantities
  - Keep bids/asks in order books
  - Decide price accordingly



- When the market is **thick** (i.e., there are many buyers/sellers), double auction is a mechanism with many good properties
- What if the market is **thin**?
  - Think about prediction market on events with less attention

Automated Market Maker

- Thick vs thin markets



- Enable trade even without a counter-party to sell/buy.
  - How to determine the price?

Other Usage of Automated Market Maker

- The ability to deal with thin markets:
  - Well suited for distributed markets
- It has attracted attention in blockchain
  - No need to maintain a central order book
  - AI can execute the transactions with pre-defined formulas
  - Possible to formally ensure some nice properties

Proper Scoring Rules

Incentivizing Truthful Reports About Probabilities

- Example scenarios:
  - Ask a weather forecaster: will it rain tomorrow?
  - Ask a political researcher: will Trump be nominated for presidential election?
  - Ask a Microsoft employer: will the new version of Office be shipped on time?

- Want to obtain forecasts about future events
- How do we make sure we obtain **truthful** reports?

Incentivizing Truthful Reports

- Setting
  - Consider a rational agent with linear utility for cash
  - Suppose there are n mutually exclusive and exhaustive states of the world $\Omega = \{w_1, w_2, \dots, w_n\}$ (e.g., Sun, Rain, Snow)
  - p_i is the subjective belief of the agent that state w_i will occur

- Question
  - How do we motivate this agent to tell us her beliefs about the likelihood of each state?

Scoring Rules

- A scoring rule rewards an agent $S(\vec{r}, w)$ when her reported distribution is $\vec{r}$ and the realized outcome is w

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- A scoring rule rewards an agent $S(\vec{r}, w)$ when her reported distribution is $\vec{r}$ and the realized outcome is w

  - $\vec{r}$: (rain: 20%, sun: 80%)
  - w: it rains

- A scoring rule is called ***proper*** if the agent maximizes her utility by providing truthful report

$$\vec{p} = \text{argmax}_{\vec{r}} \sum_{i=1}^{n} p_i S(\vec{r}, w_i)$$

- A scoring rule is ***strictly proper*** if honestly reporting is the **unique** maximizer.

Example of Non-Proper Scoring Rule

- Consider a linear scoring rule $S(\vec{r}, w_i) = r_i$
  - Assume your report $\vec{r} = (\text{rain: } 70\%, \text{sun: } 30\%)$
    - If it rains, you get a reward of 0.7
    - If it is sunny, you get a reward of 0.3

- If you believes the probability for Rain and Sun $\vec{p} = (0.7, 0.3)$

  Expected payoff for truthful reporting, i.e., $\vec{r} = \vec{p}$

  $$0.7 * 0.7 + 0.3 * 0.3 = 0.58$$

  Expected payoff for non-truthful reporting, e.g., $\vec{r} = (\text{rain: } 1, \text{sun: } 0)$

  $$0.7 * 1 + 0.3 * 0 = 0.7$$

Examples of Strictly Proper Scoring Rules

- Quadratic scoring rule (Brier score):

$$S(\vec{r}, w_i) = r_i - \frac{1}{2}\sum_j r_j^2$$

- Logarithmic scoring rule:

$$S(\vec{r}, w_i) = \log r_i$$

- We can verify this by taking the gradient of the expected payoff
- Affine transformation of the proper scoring rule is still proper.

Back to Automated Market Maker

Goal:

Incentivize ***multiple*** agents to share their beliefs, and find a way to ***aggregate*** these beliefs into a unified prediction

1. Could use one scoring rule per agent, but not clear how to aggregate

2. Market itself is an aggregation mechanism (use final price as the prediction). However, standard stock-market-style trading might encounter issues for less popular predictions (market is too *thin*).

Market Scoring Rules [Hanson. 2007]

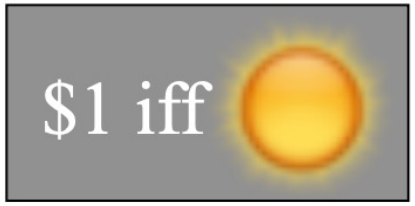
- Intuition: a "sequentially shared scoring rule"
  - Market maintains a vector of predictions $\vec{r}^{(t)}$
  - If a trader changes the vector from $\vec{r}^{(t)}$ to $\vec{r}^{(t+1)}$ and the outcome is w_i, the trader obtains reward

$$S\left(\vec{r}^{(t+1)}, w_i\right) - S\left(\vec{r}^{(t)}, w_i\right)$$

- Properties
  - Traders will truthfully report their beliefs as long as S is proper
  - Bounded loss from the market maker (assume w_i is realized)
    - $\sum_{t=1}^{T}\left(S\left(\vec{r}^{(t+1)}, w_i\right) - S\left(\vec{r}^{(t)}, w_i\right)\right) = S\left(\vec{r}^{(T+1)}, w_i\right) - S\left(\vec{r}^{(1)}, w_i\right)$

Cost-Function Based Market Maker

- Consider a complete prediction market (covers all possible outcomes)



- Determines a price via a cost function C
  - Let q_i be the current number of purchased shares of the security for state i
  - The cost for purchasing a bundle $\vec{r} = (r_1, \dots)$ of shares is

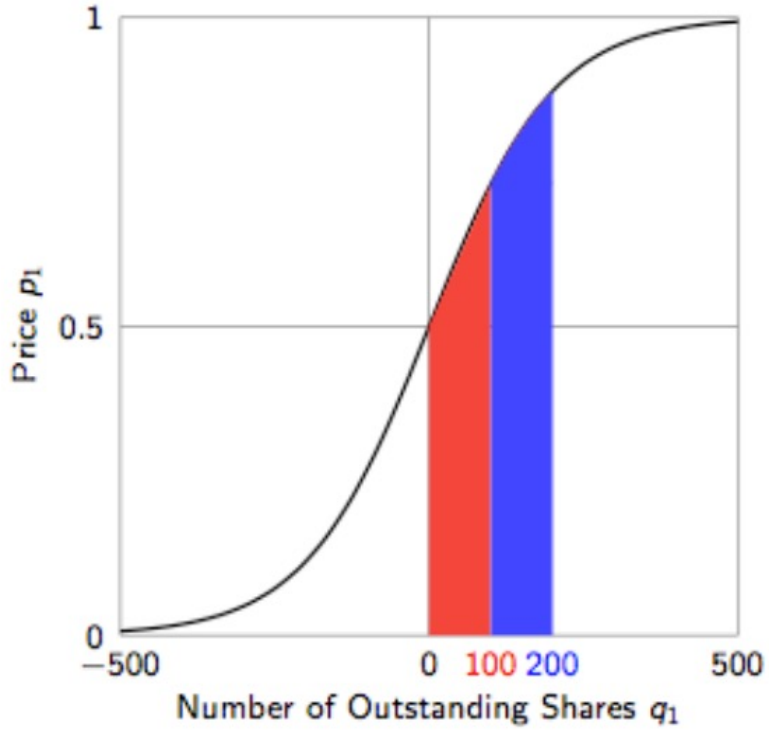
$$C(\vec{q} + \vec{r}) - C(\vec{q})$$

- The price of security $p_i = \partial C / \partial q_i$

Prediction

Cost-Function Based Market Maker

- Another way to look at the connection between cost and price



- Price is the derivative of cost
- Cost is the integral of price

- There is an equivalency to market scoring rules

Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation. Hanson. Journal of Prediction Markets 2007.

Cost-Function Based Market Maker

- How to determine the cost function (or the price function)?

- Want the market maker to have ”nice” properties:
  - **Existence of instantaneous prices**.
    C must be continuous and differentiable
  - **No arbitrage**:
    Can’t buy a bundle r with a guaranteed positive profit regardless of outcome
  - **Expressiveness**:
    A trader must always be able to set the market prices to reflect his beliefs
  - **Information incorporation:**
    Purchase of a bundle should not decrease its price
  - And others…

Connection to Convex Optimization

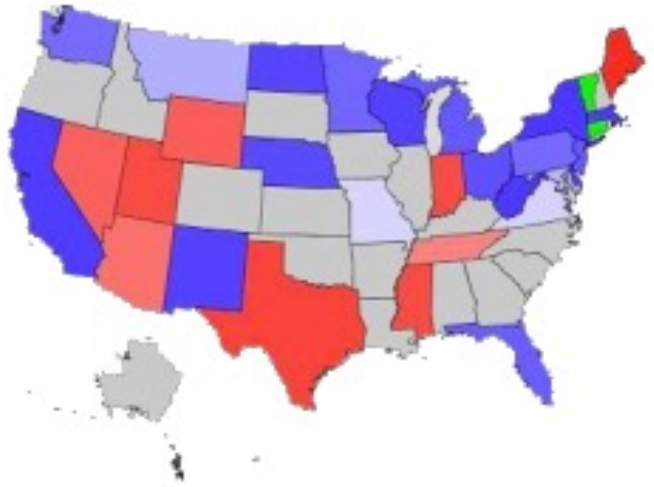
- You can formulate the design of C as a design of convex functions

- It enables an interesting line of work
  - A New Understanding of Prediction Markets Via No-Regret Learning. Chen and Vaughan. EC 2010.
  - An Optimization-Based Framework for Automated Market-Making. Abernethy, Chen, and Vaughan. EC 2011.
  - and more (the papers by these authors)

Other Interesting Questions and Explorations

- Combinatorial markets

$n!$



2^n

Other Interesting Questions and Explorations

- Incentives/Manipulations in Prediction Markets
  - Information aggregation
    - Assume you have private information others don't know
    - Should you report as soon as possible or wait till later?
  - Bluffing:
    - People are influence by the market price
    - Can I **bluff** to mislead others for future gains?
  - Outside incentives and manipulation
    - Assume the price has impacts to the real outcome (e.g., election)
    - People might want to manipulate the market
  - …

Next 3 Lectures

Oct 13	No Class
Oct 18	Practical Issues: Complex Tasks Workflow Design Presenter: CJ
Oct 20	Practical Issues: Complex Tasks Expert Crowdsourcing and Teams Presenter: Danielle and Kaushik
Oct 25	Practical Issues: Non-Independent Work and Argumentation Presenter: Cenhao, Ruiwei, and Yang