Lecture 5: Introduction to Techniques

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Logistics

- Presentations
  - The presentation schedule is announced on the website
  - **Required**:
    - Discuss with me one week before your presentation (e.g., after class)
      - Finish (required and 1~2 optional) readings by then
      - Think about reading questions
    - Content of presentations
      - You can choose what exactly to present (constrained by the theme of the lecture)
      - If/When presenting multiple papers, try to put them into context instead of separately presenting them.

~~Jan 29: Deciding the team members~~ (Submit this with your proposal)

2 Weeks

Feb 12: Project proposal

- Brief description of the proposed project (1~2 paragraph)
- Citing at least one paper that's relevant to your proposal

3 Weeks

Mar 5: Milestone 1

- A brief literature review and the description of your plan (one page)
- Last chance to change the topic of the project

5 Weeks

Apr 9: Milestone 2

- Summary of your current progress (2 pages)
- Last chance to convert the research project to a (more extensive) literature review

2 Weeks

Apr 23/25: In-class project presentations

Apr 28: Project report due (up to 6 pages)

Logistics

- Assignment 1 is due tonight (maximum 2 out of 3 late days can be used)
- Heads up of Assignment 2 and 3
  - Assignment 2 will be a math assignment with questions on game theory, scoring rules, and possibly applications to crowdsourcing.
    - Expected to be out this weekend. Due in another 2 weeks.

  - Assignment 3 is a programming assignment. You will be asked to implement label aggregation algorithms on a dataset labeled using crowdsourcing.
    - You can use any programming language you like. No code submissions are needed. Just submit a reports of your results.
    - Will be announced before the due of Assignment 2. You will have 2~3 weeks after the due of Assignment 2 to work on it.

Today’s Lecture

- Game theory basics
  - Utility, Games, Equilibrium
  - Example usage in crowdsourcing
    - Contract design (Principal Agent Model)

- Proper scoring rules (Eliciting Truthful Probability Estimates)
  - Example usage in crowdsourcing
    - Prediction markets

- Peer prediction
  - Example usage in crowdsourcing
    - Peer grading in MOOCs

Game Theory

- Mathematical study of interactions between **rational** and **self-interested** agents.

- Agents are often assumed to be rational and choose actions to maximize their **expected utility.**

Utility

- A way to quantify agents' preferences over the state of the world.
- Example

$$\Omega = \{\text{Sunny}, \text{Cloudy}, \text{Rainy}\}$$

$$\text{Sunny} \succ \text{Cloudy} \succ \text{Rainy}$$

(Sunny is preferred over Cloudy and Rainy, and Cloudy is preferred over Rainy)

- Using von Neumann–Morgenstern utility

u(Sunny) = 10, u(Cloudy) = 5, u(Rainy) = 3

Expected Utility Theory

- Agents take actions to maximize their expected utility

$$\sum_{\omega \in \Omega} p(\omega) u(\omega)$$

- There have been discussions/extensions on risk attitudes, irrational behavior, etc.

- Game theory deals with situations in which $p(\omega)$ and $u(\omega)$ are influenced by agents' joint actions

Example 1: Prisoner’s Dilemma

Normal-form game

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

Solution Concept

What should the prisoners do?

“Confess” is a **dominant strategy** – it maximizes the prisoner’s utility no matter what action the other player chooses.

Normal-Form Game

- Players take actions simultaneously
- The elements of a normal-form game
  - Players: (prisoner A, prisoner B)
  - Strategies: (stay silent, confess)
  - Payoff: (sentences for all strategy combinations)

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

Normal-Form Game (More formally)

- A finite, n-player normal-form game is a tuple $(N, A, \vec{u}\ )$
  - N is a finite set of n players, indexed by $i \in \{1, \ldots, n\}$
  - $A = A_1 \times A_2 \cdots \times A_n$, where A_i is a finite set of actions available to agent *i*
  - $\vec{u} = \{u_1, u_2, \ldots, u_n\}$, where $u_i \colon A \rightarrow \mathbb{R}$ is a real-valued utility function for agent *i*

Example 2: Coordination Game

- Two friends A and B are deciding what to do on Friday night
  - A prefers to go to the movie
  - B prefers to go to the bar
  - Both prefer to do something together than doing something separately

B

A	Movie	Bar
Movie	(2, 1)	(0, 0)
Bar	(0, 0)	(1, 2)

"Nash equilibrium" of this game

What should A and B do?

(Movie, Movie) and (Bar, Bar) seem to be two ***stable*** outcomes

Example 3: Rock, Paper, Scissors

- A zero-sum game

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

Is there a dominant strategy?

Are there stable pairs of actions?

Need a more general solution concept.

Definitions

- Mixed strategy
  - Let S_i be the set of all probability distributions over A_i
  - Each $s_i \in S_i$ is a ***mixed strategy***, where
    - $s_i(a_i)$ denotes the probability for agent i choosing action a_i

- Payoffs can be calculated using expected utility

Definitions

- Best response
  - $\vec{s}$: a strategy profile, i.e., the set of strategies for all agents
  - $\vec{s}_{-i}$: the strategies of all agents except i

  - A strategy $s_i^* \in S_i$ is a ***best response*** to $\vec{s}_{-i}$ if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \text{ for all } s_i \in S_i$$

  - There could be multiple best responses.
  - If a best response put positive probability on 2 or more actions, the agent must be indifferent between these actions. (Think about why)

Definitions

- Nash equilibrium
  - A strategy profile $\vec{s}$ is a ***Nash equilibrium*** if, for all agent i, $\mathrm{s_i}$ is a best response to $\vec{s}_{-i}$
- Intuitive interpretations:
  - If all agents except i follow the strategies in the Nash equilibrium, agent i would maximize her payoff by following the strategy in Nash.
  - No incentive to deviate if everyone else follows the strategy in Nash

Let’s look at the examples again

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

- (Confess, Confess) is the dominant strategy equilibrium
- Strongest solution concept

Let’s look at the examples again

	Movie	Bar
Movie	(2, 1)	(0, 0)
Bar	(0, 0)	(1, 2)

- (Movie, Movie) and (Bar, Bar) are pure strategy Nash equilibria
- Are there other equilibria?
  - Using the indifference property to derive a mixed-strategy equilibria

Let's look at the examples again

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

- Both players play each action with 1/3 probability is the mixed strategy Nash equilibrium
- It is the unique equilibrium

Theorem (Nash, 51):

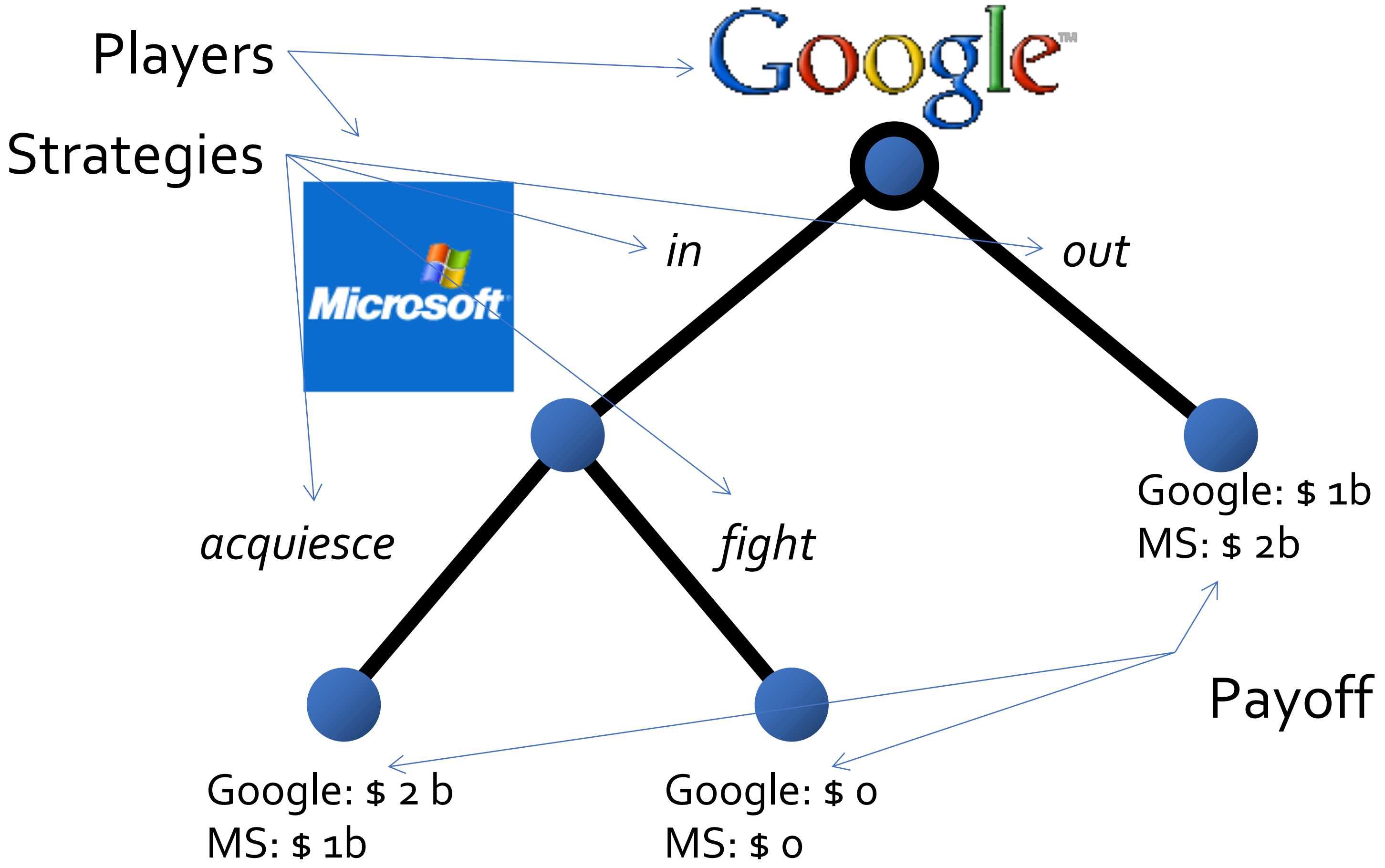
Every game with a finite number of players and actions has at least one equilibrium.

Notes:

- This is the “existence” proof. Computing the equilibrium could be hard (computationally expensive).
- If it’s hard to calculate the equilibrium, can we really expect humans to follow the equilibrium?
  - There have recent studies developing new solution concepts which assume humans are “learning” to adapt to the game.

Extensive-Form Game

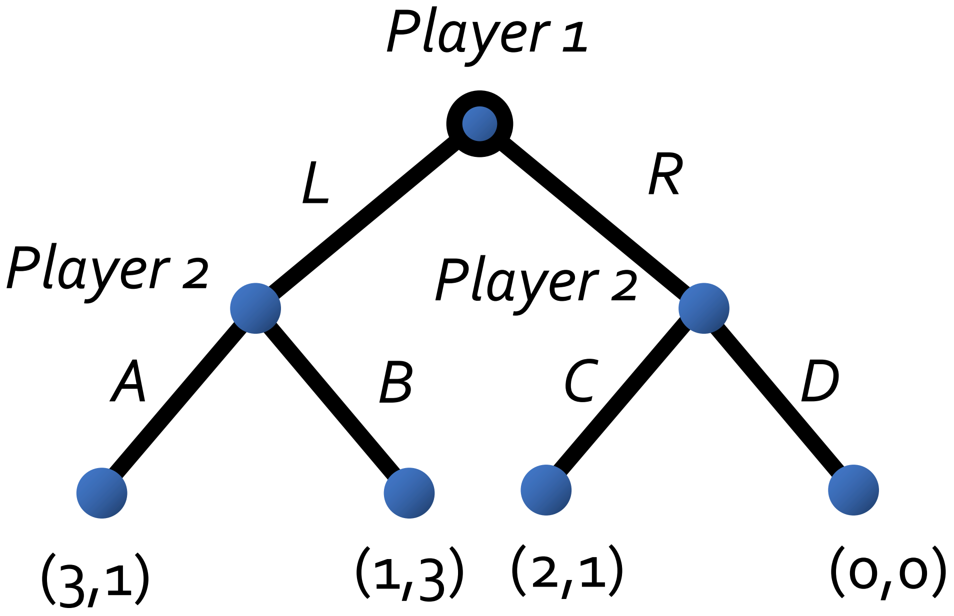
- Agents take decisions in a sequential manner



	Acquiesce	Fight
In	(2, 1)	(0, 0)
Out	(1, 2)	(1, 2)

Solution Concepts in Extensive Games

- There are some weird cases by directly extending Nash equilibrium.



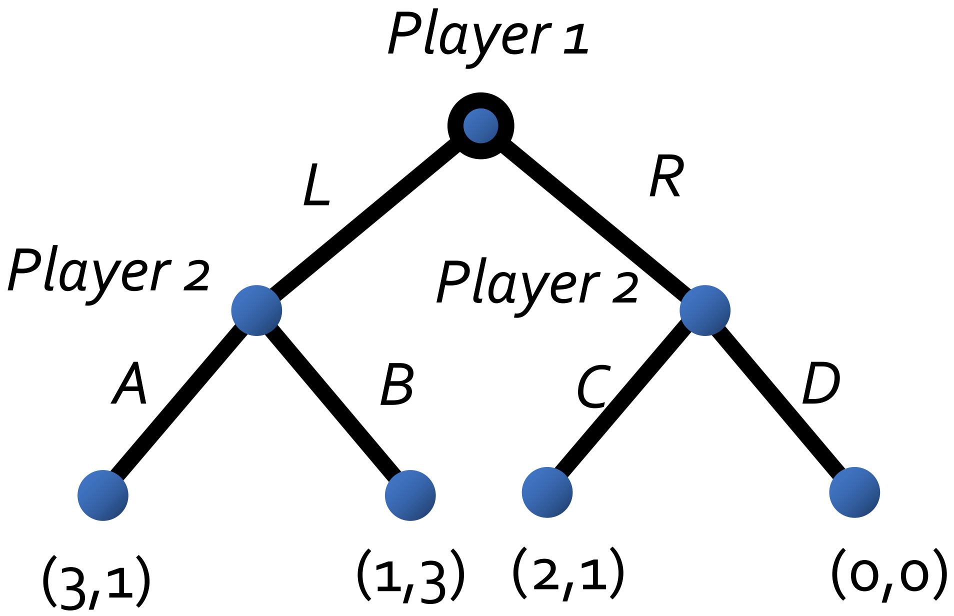
- Nash equilibrium:
  - Player 1 chooses L
  - Player 2 chooses (B, D)

Is it stable?

Why is this the weird case?
- Player 2 gives a “non-credible threat”
- I’ll choose D if you choose R

Solution Concepts in Extensive Games

- Subgame Perfect Equilibrium (SPE)
  - Play in each subgame is a Nash equilibrium.
  - Rule out the "non-credible threat"



- SPE
  - Player 1 chooses R
  - Player 2 chooses (B, C)

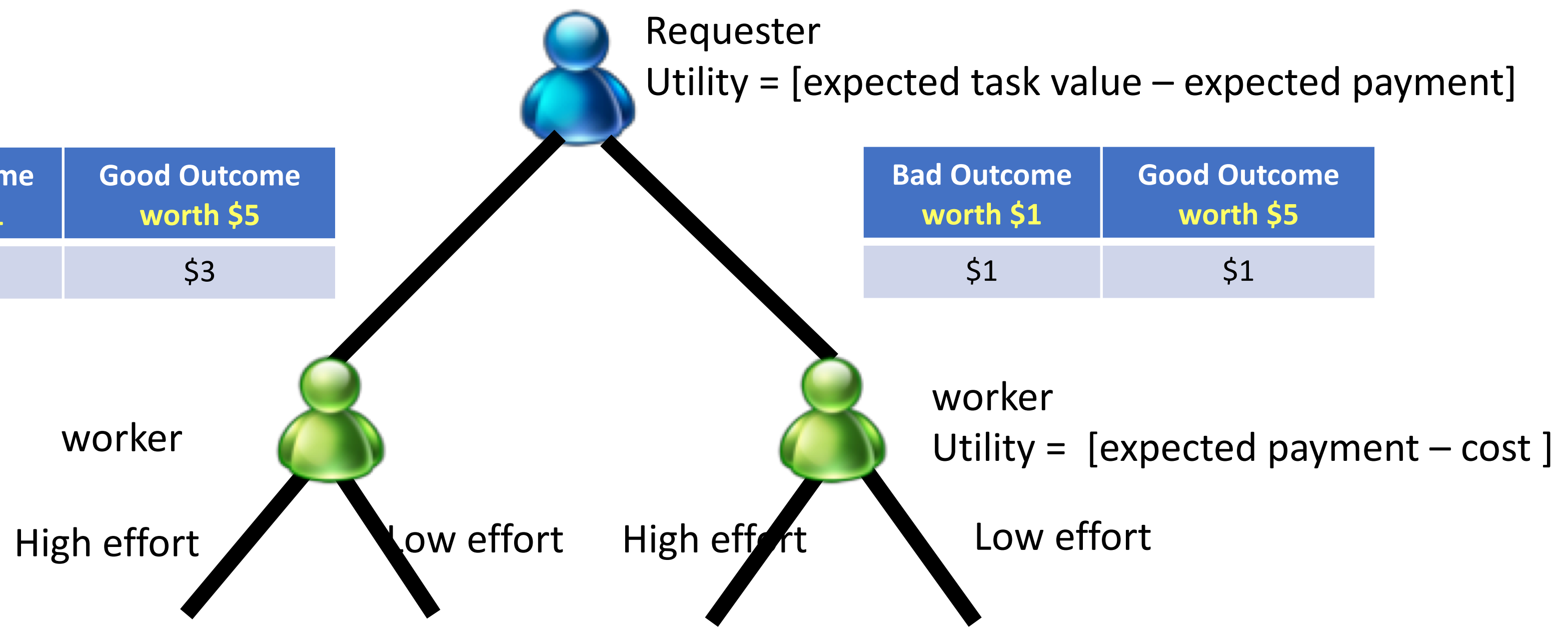
- Can usually be calculated using backwards induction.

Apply Game Theory to Crowdsourcing

- Formulate users’ incentives.
- Describe the game structure
  - Sometime there are no interactions between workers. It becomes a simpler optimization problem.
- Analyze the equilibrium as the prediction of the outcome

- Mechanism design
  - Specify the desired outcome
  - Design incentive and game structures such that the outcome is the equilibrium

Practice Example – Contract Design



Bad Outcome worth $1	Good Outcome worth $5
$1	$3

Bad Outcome worth $1	Good Outcome worth $5
$1	$1

	Bad Outcome	Good Outcome	
High effort	Prob: 0.1	Prob: 0.9	Cost: $1
Low effort	Prob: 0.8	Prob: 0.2	Cost: $0

Which contract should the requester choose?

Can be generalized to any real-valued contracts (Principal Agent problem)

Feb 7	Incentive Design: Financial Incentives Presenter: Michael and Rajagopal	**Required** Incentivizing High Quality Crowdwork. Ho et al. WWW 2015. **Optional** Financial Incentives and the "Performance of Crowds". Mason and Watts. HCOMP 2009. The Effects of Performance-Contingent Financial Incentives in Online Labor Markets. Yin et al. AAAI 2013. The Effects of Pay-to-Quit Incentives on Crowdworker Task Quality. Harris. CSCW 2015. Adaptive Contract Design for Crowdsourcing Markets: Bandit Algorithms for Repeated Principal-Agent Problems. Ho, Slivkins, and Vaughan. JAIR 2016.	Are workers really rational?
Feb 12	Incentive Design: Badges Presenter: Andrew, Kateryna, and Michelle	**Required** Steering User Behavior with Badges. Anderson et al. WWW 2013. **Optional** Incentives, Gamification, and Game Theory: An Economic Approach to Badge Design. Easley and Ghosh. EC 2013. Social Status and Badge Design. Immorlica, Stoddard, and Syrgkanis. WWW 2015.	Formulating users' utility on badges.
Feb 14	Incentive Design: Attention Presenter: Baihao, Taylor, and Xin	**Required** Incentivizing High-Quality User-Generated Content. Ghosh and McAfee. WWW 2011. **Optional** A Game-Theoretic Analysis of Rank-Order Mechanisms for User-Generated Content. Ghosh and Hummel. EC 2011. Learning and Incentives in User-Generated Content: Multi-Armed Bandits with Endogenous Arms. Ghosh and Hummel. ITCS 2013.	Formulating users' utility on attention.
Feb 19	Application: Darpa Network Challenge Presenter: Feiran, Jason, and Wei	**Required** Time Critical Social Mobilization. Pickard et al. Science 2011. Here is the long version of the article. **Optional** Mechanisms for Multi Level Marketing. Emek et al. EC 2011. Task Routing for Prediction Tasks. Zhang et al. AAMAS 2012.	Incentivizing users through social networks
Feb 21	Application: Prediction Markets Presenter: Owen and Sam	**Required** Results from a Dozen Years of Election Futures Markets Research. Berg. 2001. **Optional** Using Prediction Markets to Track Information Flows: Evidence from Google. Cowgill, Wolfers, and Zitwewitz. 2008. Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation. Hanson. Journal of Prediction Markets. 2007 Combinatorial Information Market Design. Hanson. Information Systems Frontier. 2003. A blogpost by David Pennock that discuss how to implement market scoring rules as a market maker.	

Proper Scoring Rules

Incentivizing Truthful Reports

- Example scenarios:
  - Ask a weather forecaster: will it rain tomorrow?
  - Ask a political researcher: will Trump win 2020 election?
  - Ask a Microsoft employer: will the new version of Office be shipped on time?

- Want to obtain forecasts about future events
- How do we make sure we obtain “truthful” reports?

Incentivizing Truthful Reports

- Setting
  - Consider a rational agent with linear utility for cash
  - Suppose there are n mutually exclusive and exhaustive states of the world $\Omega = \{w_1, w_2, \ldots, w_n\}$ (e.g., Sun, Rain, Snow)
  - p_i is the subjective belief of the agent that state w_i will occur
- Question
  - How do we motivate this agent to tell us her beliefs about the likelihood of each state?

Scoring Rules

- A scoring rule rewards an agent $S(\vec{r}, w)$ when his reported distribution is $\vec{r}$ and the realized outcome is w

- A scoring rule is called ***proper*** if the agent maximizes her utility by providing truthful report

$$\vec{p} = \mathrm{argmax}_{\vec{r}} \sum_{i=1}^{n} p_i S(\vec{r}, w_i)$$

- A scoring rule is ***strictly proper*** if honestly reporting is the **unique** maximizer.

Scoring Rules

- Let's consider a linear scoring rule

$$S(\vec{r}, w_i) = r_i$$

- If a risk-neutral agent believes the probability for Rain, Sun, and Snow are

$$\vec{p} = (0.7, 0.2, 0.1)$$

What report should the agent provide?

Examples of Strictly Proper Scoring Rules

- Quadratic scoring rule (Brier score):

$$S(\vec{r}, w_i) = r_i - \frac{1}{2}\sum_j r_j^2$$

We can verify this by taking the gradient of the expected payoff

- Affine transformation of the proper scoring rule is still proper.

Examples of Strictly Proper Scoring Rules

- Logarithmic scoring rule:
$$S(\vec{r}, w_i) = \log r_i$$

  We can verify this by taking a gradient of the expected payoff

- In logarithmic scoring rule, the score for outcome w_i only depends on the report r_i and not r_j for $j \neq i$

More examples?

- How many strictly proper scoring rules are there?

- How do we construct a strictly proper scoring rule?

Characterization of Proper Scoring Rules

- Connections between convex functions and proper scoring rules.

- A scoring rule $S(\vec{r}, w_i)$ is (strictly) proper if and only if

$$S(\vec{r}, w_i) = G(\vec{r}) - \sum_{j \neq i} G_j'(\vec{r}) p_j + G_i'(\vec{r})$$

where $G(\vec{r})$ is a (strictly) convex function, $G'(\vec{r})$ is a subgradient of G at $\vec{r}$, and $G_i'(\vec{r})$ is its i-th component.

Section 3.1 of Strictly Proper Scoring Rules, Prediction, and Estimation by Gneiting and Raftery.

How does this connect to prediction markets?

Goal:

Incentivize ***multiple*** agents to share their beliefs, and find a way to ***aggregate*** these beliefs into an unified prediction

1. Could use one scoring rule per agent, but not clear how to aggregate
2. Could use standard stock-market-style trading. However, the market might be *thin* for less popular predictions

Market Scoring Rules

- See Hanson's papers in the optional readings on Feb 21
- Intuitions: a "sequentially shared scoring rule"
  - An automatic market maker
  - Market maintains a vector of predictions $\vec{r}^{(t)}$
  - If a trader changes the vector from $\vec{r}^{(t)}$ to $\vec{r}^{(t+1)}$ and the outcome is w_i, the trader obtains reward

$$S\left(\vec{r}^{(t+1)}, w_i\right) - S\left(\vec{r}^{(t)}, w_i\right)$$

- Under some conditions:
  - Agents truthfully report their beliefs
  - The prediction will converge

Market Scoring Rules

- The connection to convex optimization opens up an interesting line of research in the design of efficient market maker…

Feb 21	Application: Prediction Markets Presenter: Owen and Sam	**Required** Results from a Dozen Years of Election Futures Markets Research. Berg. 2001. **Optional** Using Prediction Markets to Track Information Flows: Evidence from Google. Cowgill, Wolfers, and Zitwewitz. 2008. Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation. Hanson. Journal of Prediction Markets. 2007 Combinatorial Information Market Design. Hanson. Information Systems Frontier. 2003. A blogpost by David Pennock that discuss how to implement market scoring rules as a market maker.

- We won't cover too much on prediction markets. In case you are interested, below are a few more papers to follow up:
  - A New Understanding of Prediction Markets Via No-Regret Learning. Chen and Vaughan. EC 2010.
  - An Optimization-Based Framework for Automated Market-Making. Abernethy, Chen, and Vaughan. EC 2011.
  - and more (the papers authored on these authors)

Very Brief Intro of Peer Prediction

Eliciting Truthful Reports

- Scoring rule relies on the “truth” to be revealed in the future
- What if there is no ground truth (or the ground truth is hard to obtain)
  - Do you like this movie?
  - Peer grading in MOOCs

- Output agreements:
  - Randomly pick two persons
  - If their reports match, reward them 1, otherwise reward 0
  - Truthful reporting is not an equilibrium (you are encouraged to report the majority’s opinion)

Peer Prediction

- How to fix the issue?
  - Assume knowledge about the report distribution, re-weighting the rewards to make sure truthful reporting is a equilibrium

- Drawbacks:
  - Require knowledge of the prior
  - Usually there are multiple equilibrium (including naïve bad ones…)

- Still an ongoing research area
  - Some nice theoretical results, however there is little practical success so far

Eliciting Informative Feedback: The Peer-Prediction Method. Miller et al. 2004

Next Lecture

- Will discuss techniques from probability and machine learning that are relate to label aggregation
  - Concentration bounds
  - Weighted majority voting aggregation
  - Maximum likelihood estimation
  - (maybe) brief explanation of the EM framework

- There will be heavy math involved in the next lecture.