CSE 417T
Introduction to Machine Learning

Lecture 14

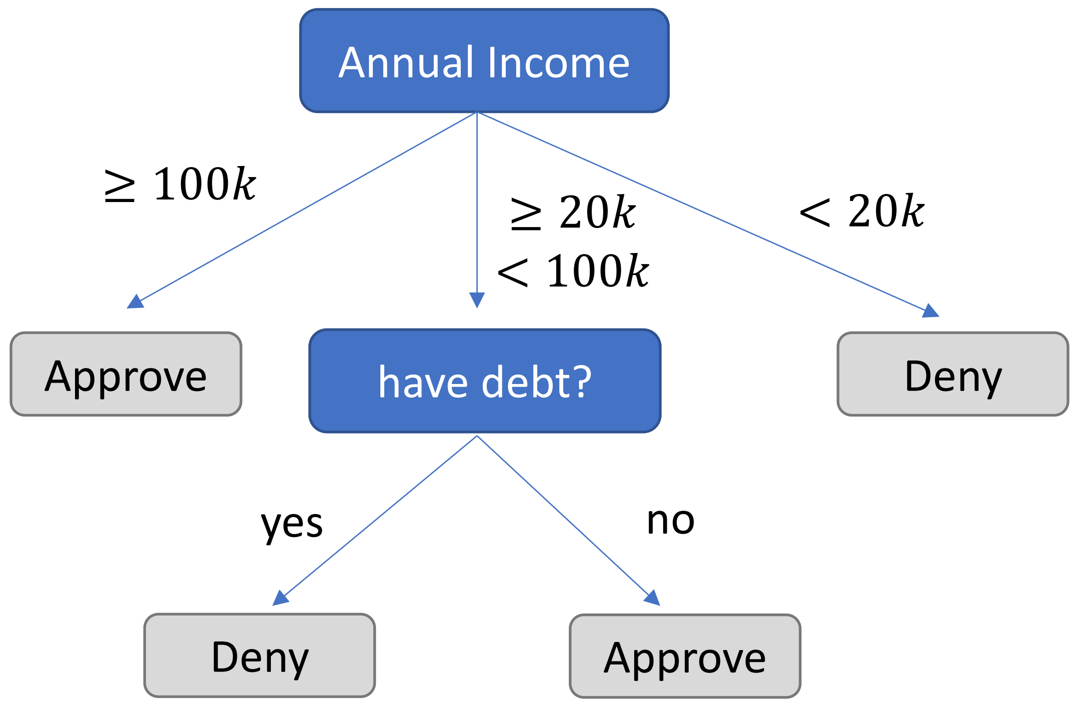
Instructor: Chien-Ju (CJ) Ho

Logistics

- Homework 3: due yesterday (Wednesday)
  - If you still have late days, the final due is tomorrow
  - Track your own late-day usages
    - Assignments over-using late days won't be graded
- Exam 1: **October 27 (Thursday)**
  - Topics: LFD Chapters 1 to 5
  - Timed exam (75 min) during lecture time
  - Location TBD
  - Closed-book exam with 2 letter-size cheat sheets allowed (4 pages in total)
    - No format limitations (it can be typed, written, or a combination)
- October 25 (Tuesday) will be a review session
  - Practice questions are posted on Piazza (will be discussed during the review session)

Recap

Decision Tree Hypothesis



Credit Card Approval Example

- Pros
  - Easy to interpret (interpretability is getting attention and is important in some domains)
  - Can handle multi-type data (Numerical, categorical. …)
  - Easy to implement (Bunch of if-else rules)
- Cons
  - Generally speaking, bad generalization
  - VC dimension is infinity
  - High variance (small change of data leads to very different hypothesis)
  - Easily overfit
- Why we care?
  - One of the classical model
  - Building block for other models (e.g., random forest)

ID3: Using Information Gain as Selection Criteria

- Information gain of choosing feature A to split
  - $Gain(D, A) = H(D) - \sum_i \frac{|D_i|}{|D|} H(D_i)$ [The amount of decrease in entropy]
- ID3: Choose the split that maximize $Gain(D, A)$

Notations:
$H(D)$: Entropy of D
$|D|$ is the number of points in D

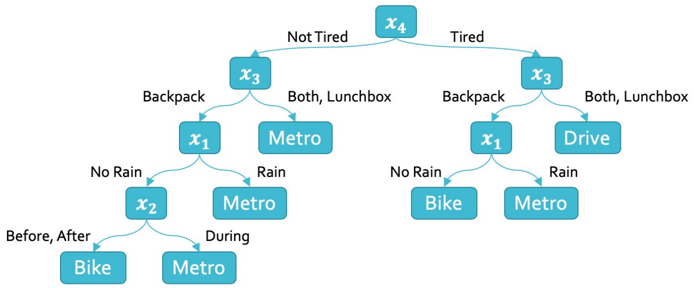
DecisionTreeLearn(D)
    Create a root node r
    If termination conditions are met
        return a single node tree with leaf prediction based on
    Else: Greedily find a feature A to split according to split criteria
    For each possible value v_i of A
        Let D_i be the dataset containing data with value v_i for feature A
        Create a subtree DecisionTreeLearn(D_i) that being the child of root r

- ID3 termination conditions
  - If all labels are the same
  - If all features are the same
  - If dataset is empty
- ID3 leaf predictions
  - Most common labels (majority voting)
- ID3 split criteria
  - Information gain

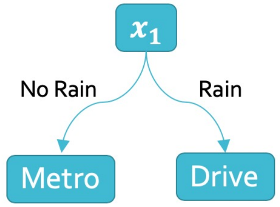
Ensemble Learning

- Goal: Utilize a set of weak learners to obtain a strong learner.

- One common way to construct weak learners is via decision trees
  - Fully grown decision trees
    - High variance
    - Low bias



  - Decision stump (One-depth decision trees, split on only one attribute)
    - Low variance
    - High bias



Ensemble Learning

- Goal: Utilize a set of weak learners to obtain a strong learner.

- Format of ensemble learning
  - Construct many diverse weak learners
  - Aggregate the weak learners

Bagging
- Construct diverse weak learners
  - (Simultaneously) bootstrap datasets
  - Train weak learners on them

- Aggregate the weak learners
  - Uniform aggregation

Bagging - Bootstrapped Aggregating

- Bootstrap M datasets $\{\widetilde{D}^{\{m\}}\}$ (Sample with replacement from D)
- Learn a hypothesis from each of them

$$
H \Longrightarrow
\begin{array}{ccc}
\widetilde{D}^{\{1\}} & \longrightarrow & g_1 \\
\widetilde{D}^{\{2\}} & \longrightarrow & g_2 \\
\dots & & \\
\widetilde{D}^{\{M\}} & \longrightarrow & g_M
\end{array}
$$

- Aggregate the learned hypothesis (assume we are doing classification)

$$G(\vec{x}) = \bar{g}(\vec{x}) = sign\left(\frac{1}{M}\sum_{m=1}^{M} g_m(\vec{x})\right)$$

Out-Of-Bag (OOB) Error

	$\widetilde{D}^{(1)}$	$\widetilde{D}^{(2)}$	$\widetilde{D}^{(3)}$	$\widetilde{D}^{(4)}$	…
$(\vec{x}_1, y_1)$	Yes	Yes	No	No	…
$(\vec{x}_2, y_2)$	Yes	No	No	No	…
…	…	…	…	…	…
$(\vec{x}_N, y_N)$	No	Yes	Yes	Yes	…

Whether a point is in a bootstrapped dataset

- G_n^- : the aggregation of hypothesis that $\vec{x}_n$ is OOB of
  - $G_1^- = \text{aggregate}(g_3, g_4, \dots)$
  - $G_2^- = \text{aggregate}(g_2, g_3, g_4, \dots)$
  - $G_N^- = \text{aggregate}(g_1, , \dots)$

Aggregate:
Majority voting for classification
Average for regression

- OOB Error
  - $E_{OOB}(G) = \frac{1}{N}\sum_{n=1}^{N} \text{error}(G_n^-(\vec{x}_n), y_n)$

Error:
Binary error for classification
Squared error for regression

Out-Of-Bag (OOB) Error

$$E_{OOB}(G) = \frac{1}{N}\sum_{n=1}^{N} \text{error}(G_n^-(\vec{x}_n), y_n)$$

- Bagging provided an intrinsic mechanism for us to perform validation
- Practical issues (you might face this in HW4)
  - What if some $\vec{x}_n$ appears in all bootstrapped datasets?
    - The probability of this happening is small when the number of bags M is large
  - Let S be the set of points that is out of bag for at least one bootstrapped dataset
    - $E_{OOB}(G) = \frac{1}{|S|}\sum_{(\vec{x}_n, y_n)\in S} \text{error}(G_n^-(\vec{x}_n), y_n)$

Random Forest

- Construct many random trees
  - Bootstrapping datasets and learn a max-depth tree for each of them
  - Other randomizations (not required in HW4)
    - When choosing split features, choose from a random subset (instead of all features)
    - Randomly project features (similar to non-linear transformation) for each tree

- Aggregate the random trees
  - Classification: Majority vote $\bar{g}(\vec{x}) = sign\left(\frac{1}{M}\sum_{m=1}^{M} g_m(\vec{x})\right)$
  - Regression: Average $\bar{g}(\vec{x}) = \frac{1}{M}\sum_{m=1}^{M} g_m(\vec{x})$

Today’s Lecture

The notes are not intended to be comprehensive. They should be accompanied by lectures and/or textbook.

Let me know if you spot errors.

Boosting

Ensemble Learning

- Goal: Utilize a set of weak learners to obtain a strong learner.

- Format of ensemble learning
  - Construct many diverse weak learners
  - Aggregate the weak learners

<u>Bagging:</u>
- Construct diverse weak learners
  - (Simultaneously) bootstrapping datasets
  - Train weak learners on them
- Aggregate the weak learners
  - Uniform aggregation

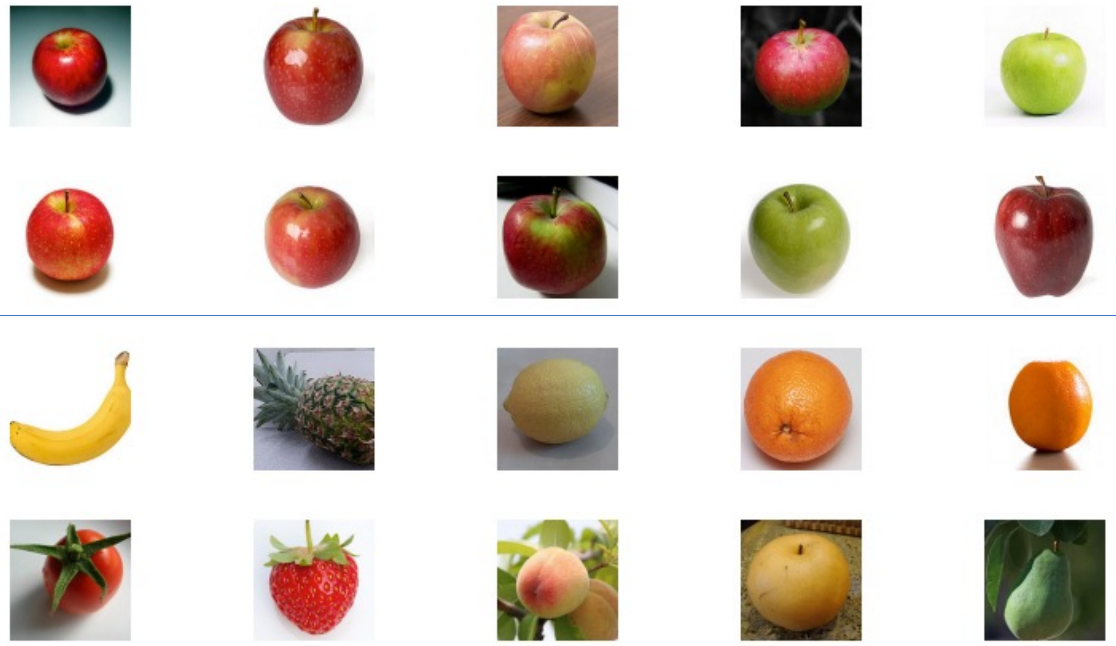
<u>Boosting</u>
- Construct diverse weak learners
  - Adaptively generating datasets
  - Train weak learners on them
- Aggregate the weak learners
  - Weighted aggregation

Informal Intuitions about Boosting

- Example : Teach a class of kids to identify apples from data

Apples

Not Apples



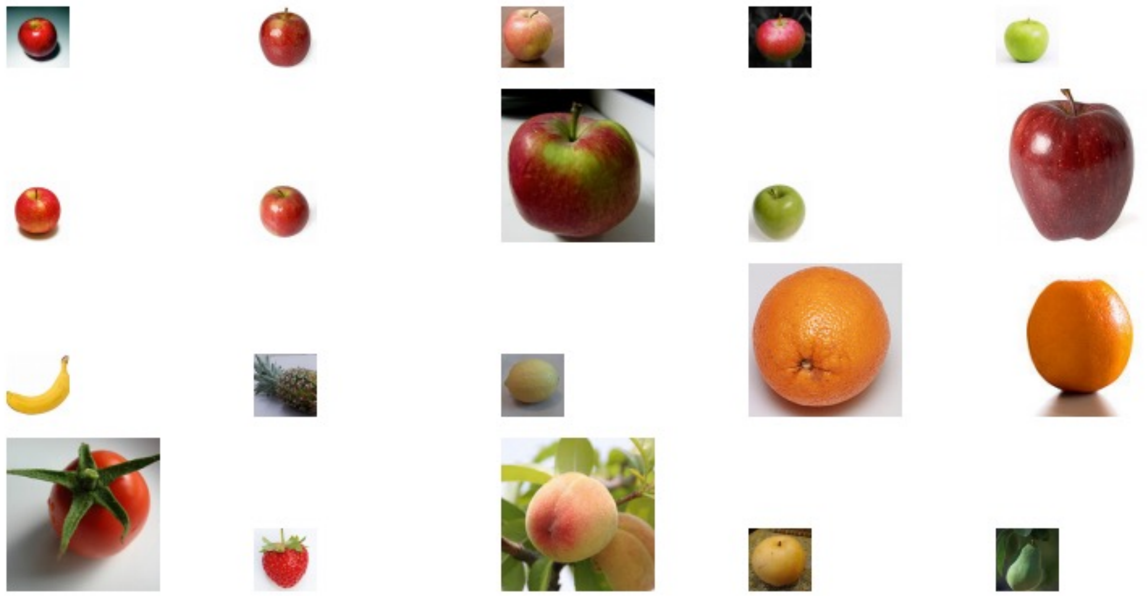
- Alice: Apples are circular
- Teacher:
  Circular is a good feature, but using this feature might make some mistakes

  Let me highlight the mistakes.
  - Make correct images smaller
  - Make incorrect images larger

Example created by Hsuan-Tien Lin

Informal Intuitions about Boosting

- Example : Teach a class of kids to identify apples from data

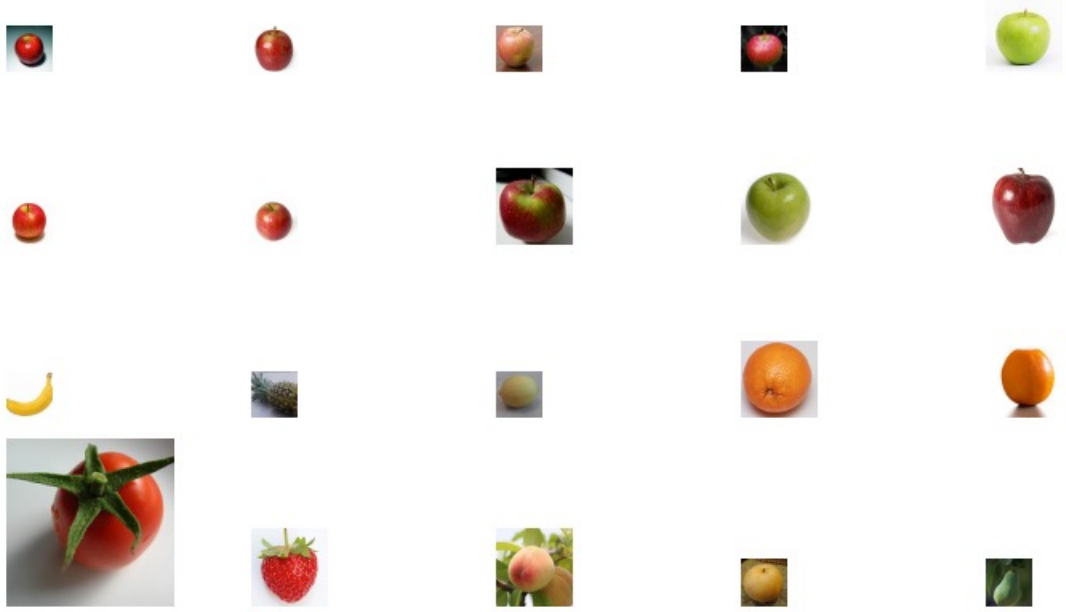


- Alice: Apples are circular
- Bob: Apples are red

Example created by Hsuan-Tien Lin

Informal Intuitions about Boosting

- Example : Teach a class of kids to identify apples from data

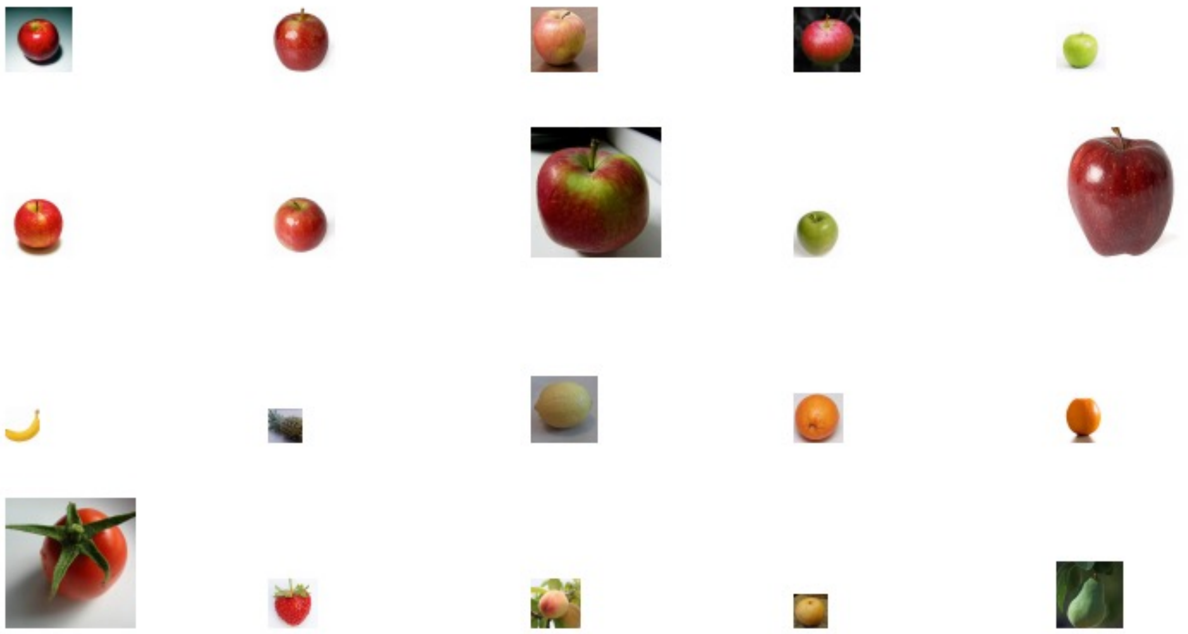


- Alice: Apples are circular
- Bob: Apples are red
- Charlie: Apples could be green

Example created by Hsuan-Tien Lin

Informal Intuitions about Boosting

- Example : Teach a class of kids to identify apples from data



Example created by Hsuan-Tien Lin

- Alice: Apples are circular
- Bob: Apples are red
- Charlie: Apples could be green
- David: Apples have stems at the top
- Class: Apples are somewhat circular, somewhat red, possibly green, and may have stems at the top

Informal Intuitions about Boosting

- Example : Teach a class of kids to identify apples from data

Key steps of this process:

- Learn a simple hypothesis for each dataset
- Iteratively update the dataset to focus on what we got wrong (i.e., create diversity)
- Aggregate the learned simple hypothesis

Example created by Hsuan-Tien Lin

- Alice: Apples are circular
- Bob: Apples are red
- Charlie: Apples could be green
- David: Apples have stems at the top
- Class: Apples are somewhat circular, somewhat red, possibly green, and may have stems at the top

Outline of a Boosting Algorithm

- Initialize D_1 (usually the same as the initial dataset D)
- For $t = 1$ to T
  - Learn g_t from D_t
  - Reweight the distribution and obtain D_{t+1} based on g_t and D_t
- Output weighted-aggregate$(g_1, \dots, g_T)$
  - Classification: $G(\vec{x}) = \bar{g}(\vec{x}) = sign\left(\frac{1}{T}\sum_{t=1}^{M} \alpha_t g_t(\vec{x})\right)$

Questions

How to learn g_t from D_t

How to reweight the distribution and obtain D_{t+1}

How to perform weighted aggregation

Discussion on Re-weighted D_t (What does re-weighting mean?)

- Original Dataset $D = \{(\vec{x}_1, y_1), \dots, (\vec{x}_N, y_N)\}$
- Notation of D_t
  - $D_t(n)$ is the weight/probability of data point $(\vec{x}_n, y_n)$ in D_t
  - $\sum_{n=1}^{N} D_t(n) = 1$

- What is $E_{in}(h)$ on D_t? (Expressed as $E_{in}^{(D_t)}(h)$)
  - Re-sample dataset (noisier)
    - Re-sample the dataset from D according to distribution D_t
    - Calculate E_{in} on the re-sampled dataset as usual

  - Calculate weighted error
    - $E_{in}^{(D_t)}(h) = \sum_{n=1}^{N} D_t(n)\, \mathrm{error}(h(\vec{x}_n), y_n)$

When $D_t(n) = 1/N$. This reduces to standard definition of E_{in}.

AdaBoost – Adaptive Boosting

How to learn g_t from D_t
How to reweight the distribution and obtain D_{t+1}
How to perform weighted aggregation

[AdaBoost focuses on **classification** problem]

Boosting Background

- A theoretical question asked by Kearns and Valiant
  - Whether a “weak” learning algorithm which performs just slightly better than random guessing in the PAC model can be “boosted” into an arbitrarily accurate “strong” learning algorithm

- AdaBoost
  - The first adaptive boosting algorithm that
    - has nice theoretical guarantees
    - successfully deployed in real-world applications

What Does AdaBoost Do?

Outline of a Boosting Algorithm

Initialize D_1 (usually the same as the initial dataset D)

For $t = 1$ to T

Learn g_t from D_t

Reweight the distribution and obtain D_{t+1} based on g_t and D_t

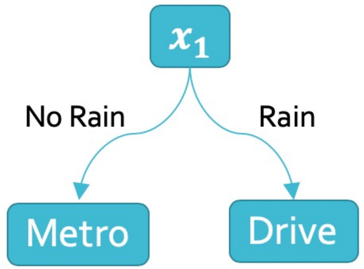
Output weighted-aggregate$(g_1, \ldots, g_T)$

Classification: $G(\vec{x}) = \bar{g}(\vec{x}) = sign\left(\frac{1}{T}\sum_{t=1}^{M} \alpha_t g_t(\vec{x})\right)$

- Will discuss the following for AdaBoost
  1. How to learn g_t from D_t
  2. How to reweight the distribution and obtain D_{t+1}
  3. How to perform weighted aggregation

1. Learn a Weak Learner g_t from D_t

- AdaBoost uses *simple* weak learners
  - low variance, high bias
  - Decision stump (one-level decision tree) is one good option



- How to learn g_t from D_t
  - Find the decision stump that
    - Minimizes $E_{in}^{(D_t)}$ (recall the definition on this ***weighted in-sample error*** earlier)
    - Approximately: Maximize (weighted) information gain (you can call decision tree library directly)

2. How to Reweight D_{t+1} (based on g_t and D_t)

- We want to make g_{t+1} (learned from D_{t+1}) to be **diverse** from g_t
  - Increase the weights of points that g_t makes wrong predictions
  - Decrease the weights of points that g_t makes correct predictions

- Define a parameter $\gamma > 1$
  - If g_t makes wrong predictions on $\vec{x}_n$
    - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \cdot \gamma$ (increase the weight)
  - If g_t makes correct predictions on $\vec{x}_n$
    - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) / \gamma$ (decrease the weight)

Z_t: normalization constant
We need to ensure $\sum_{n=1}^{N} D_{t+1}(n) = 1$

- Goal:
  - Choose γ such that $E_{in}^{(D_{t+1})}(g_t) = 0.5$
  - Since g_{t+1} minimizes $E_{in}^{(D_{t+1})}$ => g_t and g_{t+1} are **diverse**

Choose γ such that $E_{in}^{(D_{t+1})}(g_t) = 0.5$

Math derivations in the next few slides

- Define $\epsilon_t = E_{in}^{(D_t)}(g_t) = \sum_{n=1}^{N} D_t(n)\mathbb{I}[g_t(\vec{x}_n) \neq y_n]$
  - Weighted in-sample error of g_t on D_t
  - $\epsilon_t < 0.5$ (requirement of weak learners)

We consider the case weak learners are better than random guessing: $\epsilon_t < 0.5$

- Goal: Want to make $E_{in}^{(D_{t+1})}(g_t) = 0.5$

$$E_{in}^{(D_t)}(h) = \sum_{n=1}^{N} D_t(n)\ \mathrm{error}(h(\vec{x}_n), y_n)$$

- If g_t makes wrong predictions on $\vec{x}_n$
  - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \cdot \gamma$ (increase the weight)
- If g_t makes correct predictions on $\vec{x}_n$
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- Define $\epsilon_t = E_{in}^{(D_t)}(g_t) = \sum_{n=1}^{N} D_t(n)\mathbb{I}[g_t(\vec{x}_n) \neq y_n]$
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We consider the case weak learners are better than random guessing: $\epsilon_t < 0.5$

- Goal: Want to make $E_{in}^{(D_{t+1})}(g_t) = 0.5$

$$E_{in}^{(D_{t+1})}(g_t) = \sum_{n=1}^{N} D_{t+1}(n)\mathbb{I}[g_t(\vec{x}_n) \neq y_n]$$
$$= \sum_{n=1}^{N} \frac{1}{Z_t} D_t(n)\gamma\, \mathbb{I}[g_t(\vec{x}_n) \neq y_n]$$
$$= \frac{\gamma}{Z_t} \sum_{n=1}^{N} D_t(n)\, \mathbb{I}[g_t(\vec{x}_n) \neq y_n] = \frac{\gamma}{Z_t}\epsilon_t$$

$$E_{in}^{(D_t)}(h) = \sum_{n=1}^{N} D_t(n)\, \mathrm{error}(h(\vec{x}_n), y_n)$$

- If g_t makes wrong predictions on $\vec{x}_n$
  - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \cdot \gamma$ (increase the weight)
- If g_t makes correct predictions on $\vec{x}_n$
  - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) / \gamma$ (decrease the weight)

- Remember Z_t is the normalization constant

$$Z_t = \sum_{n=1}^{N} D_t(n)\gamma\, \mathbb{I}[g_t(\vec{x}_n) \neq y_n] + \sum_{n=1}^{N} D_t(n)\frac{1}{\gamma}\, \mathbb{I}[g_t(\vec{x}_n) = y_n]$$
$$= \gamma\epsilon_t + \frac{1}{\gamma}(1 - \epsilon_t)$$

- Want to make $E_{in}^{(D_{t+1})}(g_t) = 0.5$
  - $E_{in}^{(D_{t+1})}(g_t) = \frac{\gamma}{Z_t}\epsilon_t$
  - $Z_t = \gamma\epsilon_t + \frac{1}{\gamma}(1 - \epsilon_t)$

- Want to make $E_{in}^{(D_{t+1})}(g_t) = 0.5$
  - $E_{in}^{(D_{t+1})}(g_t) = \frac{\gamma}{Z_t}\epsilon_t$
  - $Z_t = \gamma\epsilon_t + \frac{1}{\gamma}(1 - \epsilon_t)$
- $\frac{\gamma\epsilon_t}{\gamma\epsilon_t + (1-\epsilon_t)/\gamma} = 0.5$ => $\frac{1-\epsilon_t}{\gamma} = \gamma\epsilon_t$ => $\gamma = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$

Both $g_t(\vec{x}_n)$ and y_n are either +1 or -1

If $g_t(\vec{x}_n) \neq y_n$, $g_t(\vec{x}_n)y_n = -1$

If $g_t(\vec{x}_n) = y_n$, $g_t(\vec{x}_n)y_n = 1$

- The rule for reweighting
  - If $g_t(\vec{x}_n) \neq y_n$, then $D_{t+1}(n) = \frac{1}{Z_t}D_t(n)\left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)$
  - If $g_t(\vec{x}_n) = y_n$, then $D_{t+1}(n) = \frac{1}{Z_t}D_t(n)\left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-1}$

- Want to make $E_{in}^{(D_{t+1})}(g_t) = 0.5$
  - $E_{in}^{(D_{t+1})}(g_t) = \frac{\gamma}{Z_t}\epsilon_t$
  - $Z_t = \gamma\epsilon_t + \frac{1}{\gamma}(1 - \epsilon_t)$
- $\frac{\gamma\epsilon_t}{\gamma\epsilon_t + (1-\epsilon_t)/\gamma} = 0.5$ => $\frac{1-\epsilon_t}{\gamma} = \gamma\epsilon_t$ => $\gamma = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$

- The rule for reweighting
  - If $g_t(\vec{x}_n) \neq y_n$, then $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right) = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-g_t(\vec{x}_n)y_n}$
  - If $g_t(\vec{x}_n) = y_n$, then $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-1} = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-g_t(\vec{x}_n)y_n}$

- Reweight rule: $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-g_t(\vec{x}_n)y_n}$

2. How to Reweight D_{t+1}: Summary

- Reweight rule:
  - $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) \left(\sqrt{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{-g_t(\vec{x}_n)y_n}$
- A bit more manipulations (the reason will be clear later)
  - Define $\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
  - $e^{\alpha_t} = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$
- Final reweight rule: $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) e^{-\alpha_t g_t(\vec{x}_n)y_n}$

3. How to Aggregate Weak Learners

- Intuition:
  - We want to put more weights on better weak learners
  - $\epsilon_t = E_{in}^{(D_t)}(g_t)$ is a proxy on how well g_t performs (smaller ϵ_t => better g_t)

  - Recall that $\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
    - Better g_t, smaller ϵ_t, higher α_t
    - When $\epsilon_t = 0.5, \alpha_t = 0$ (random guessing leads to 0 weights)
    - When $\epsilon_t = 0, \alpha_t = \infty$ (if a feature perfectly classifies the data, use it as our final hypothesis)

- Aggregation rule
  - $G(\vec{x}) = sign(\sum_{t=1}^{T} \alpha_t g_t(\vec{x}))$

AdaBoost Algorithm

- Given $D = \{(\vec{x}_1, y_1), \dots, (\vec{x}_N, y_N)\}$
- Initialize $D_1(n) = 1/N$ for all $n = 1, \dots, N$
- For $t = 1, \dots, T$
    - Learn g_t from D_t (using decision stumps)
    - Calculate $\epsilon_t = E_{in}^{(D_t)}(g_t)$
    - Set $\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
    - Update $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) e^{-\alpha_t y_n g_t(\vec{x}_n)}$
- Output $G(\vec{x}) = sign(\sum_{t=1}^{T} \alpha_t g_t(\vec{x}))$

Theoretical Properties of AdaBoost

- See Freund & Schapire's Tutorial for more discussion

- The training error of AdaBoost converges fast
  - Let $\gamma_t = \frac{1}{2} - \epsilon_t$ (how good each weak learner is better than random guessing)
  - $E_{in} \leq e^{-2\sum_{t=1}^{T} \gamma_t^2}$

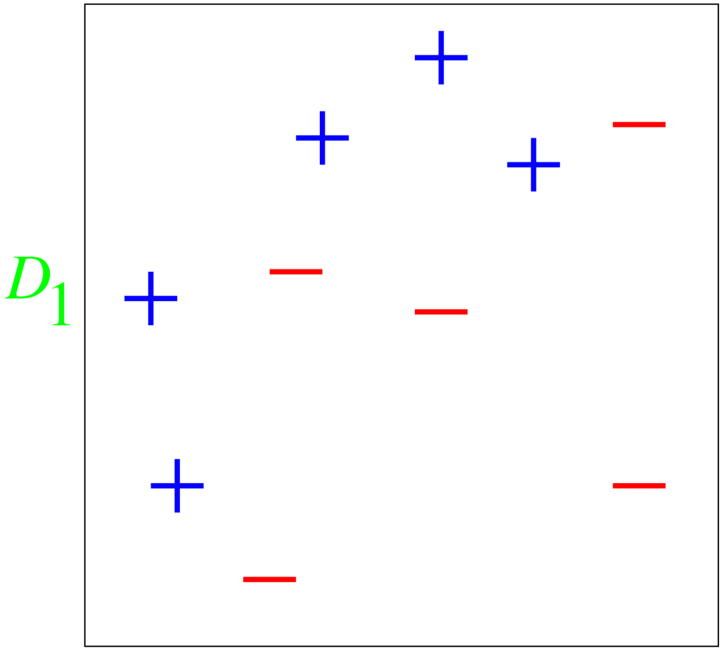
d_{vc} is the VC dimension of the weak learner

- Generalization error
  - VC analysis gives us $E_{out} \leq E_{in} + \tilde{O}\left(\sqrt{\frac{T d_{vc}}{m}}\right)$
  - It seems as T goes large, overfitting could happen
  - Empirically, AdaBoost is relatively robust to overfitting
  - There are some more delicate analysis using the idea of **margins** to explain why

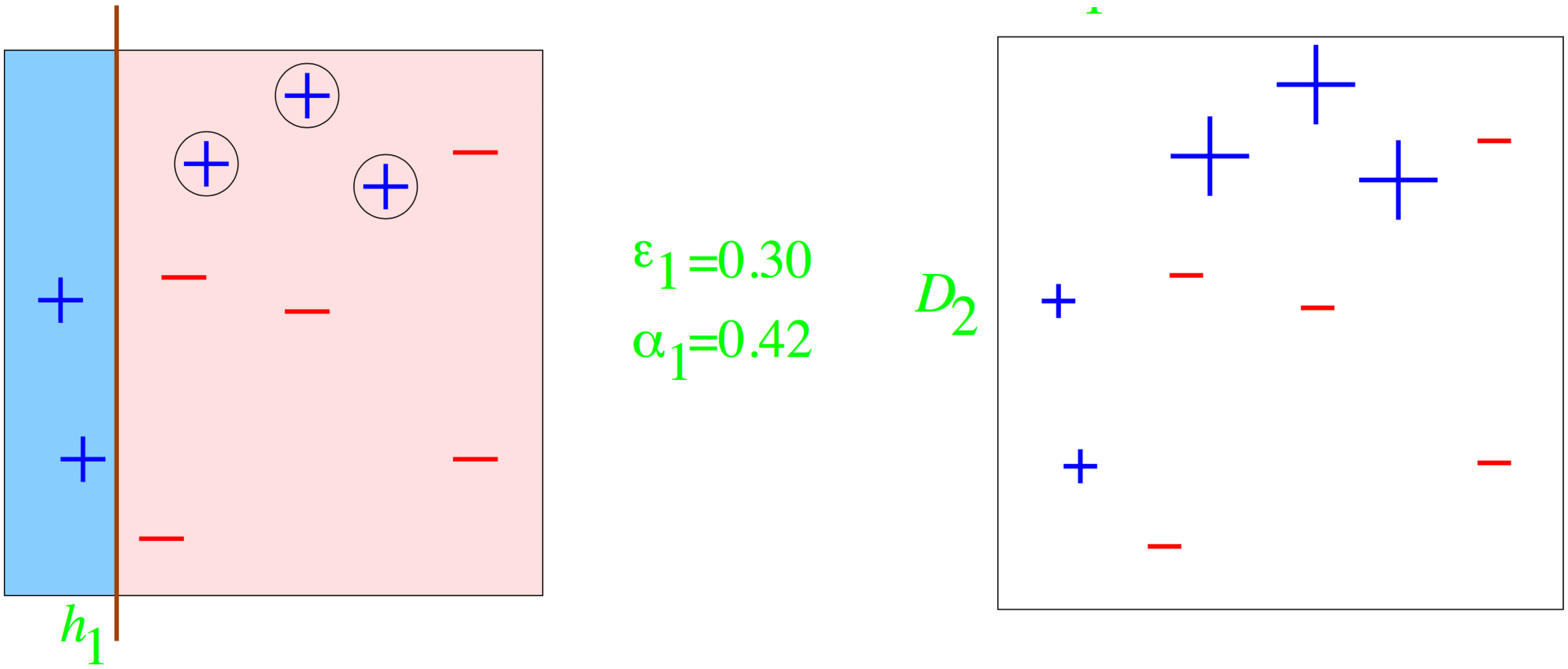
AdaBoost in Action

AdaBoost in Action

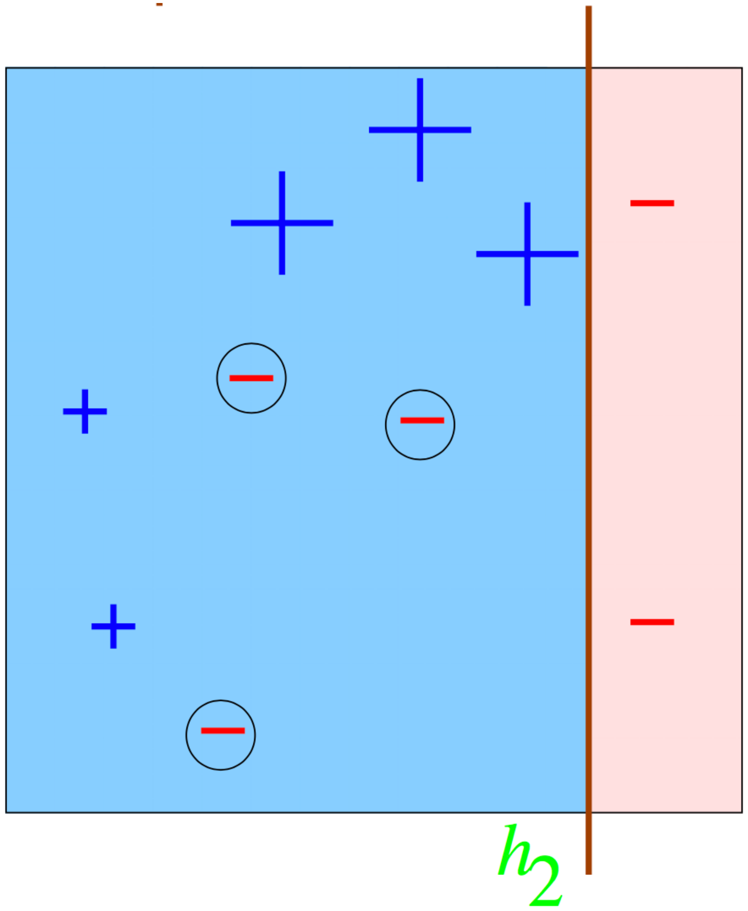
- A toy example (by Yoav Freund Rob Schapire)
- Weak learner: decision stump (one-level decision tree)



Round 1

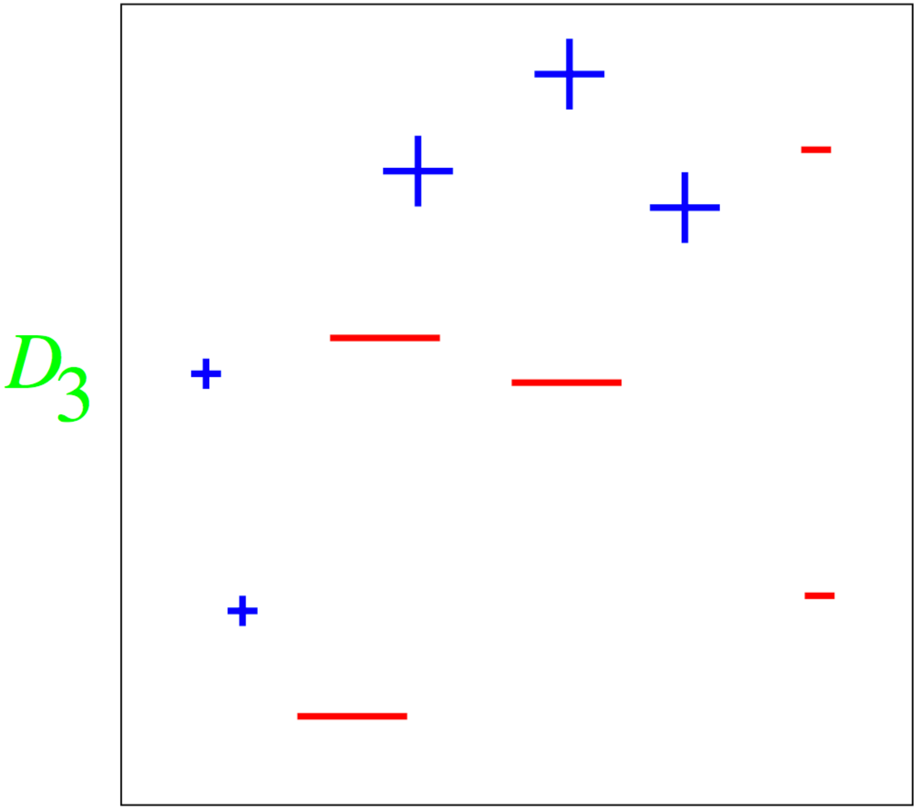


Round 2

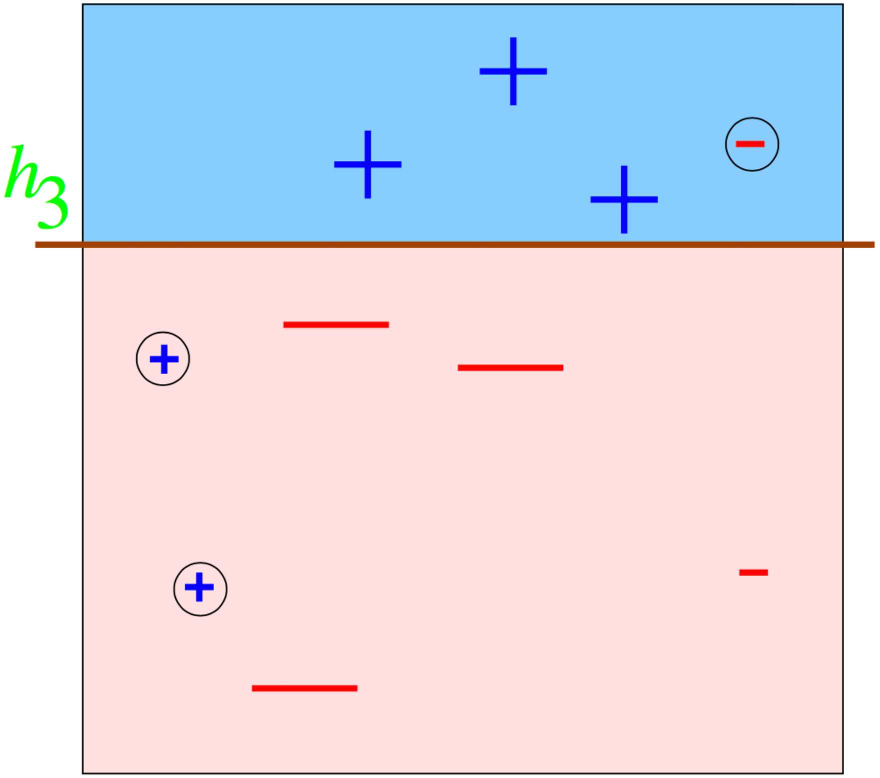


$\varepsilon_2 = 0.21$

$\alpha_2 = 0.65$

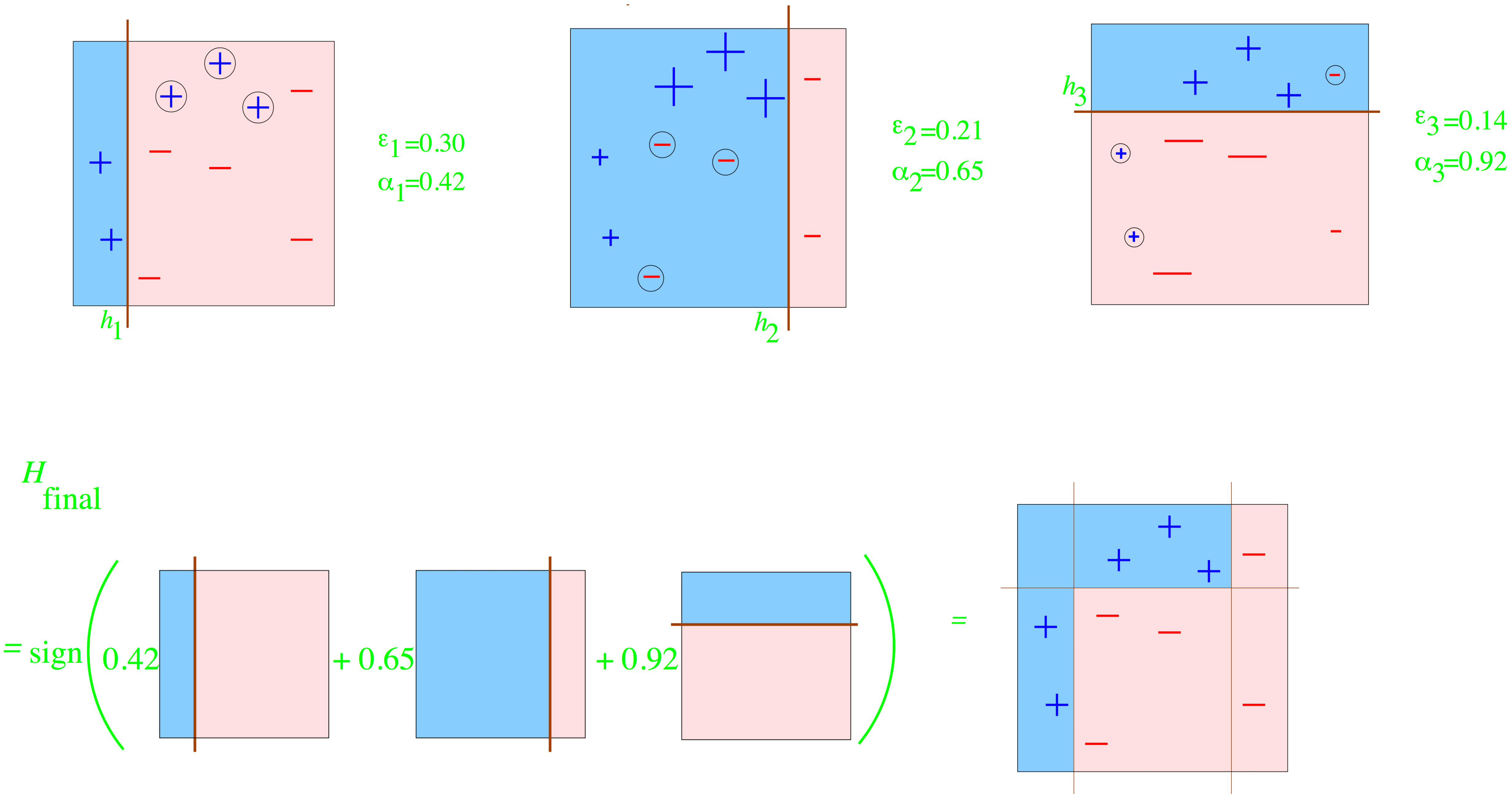


Round 3



$\varepsilon_3=0.14$

$\alpha_3=0.92$

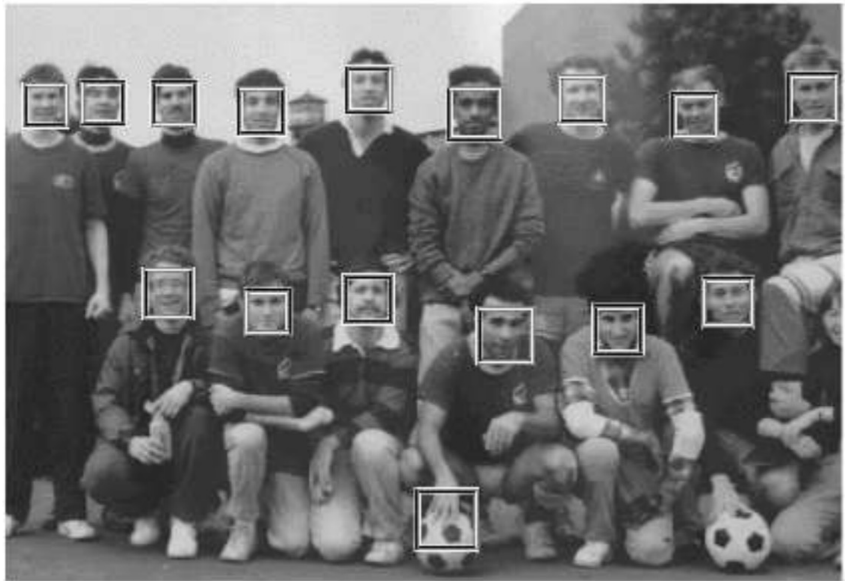


h_1
$\varepsilon_1=0.30$
$\alpha_1=0.42$
h_2
$\varepsilon_2=0.21$
$\alpha_2=0.65$
h_3
$\varepsilon_3=0.14$
$\alpha_3=0.92$
H_{final}
= sign(0.42 + 0.65 + 0.92) =

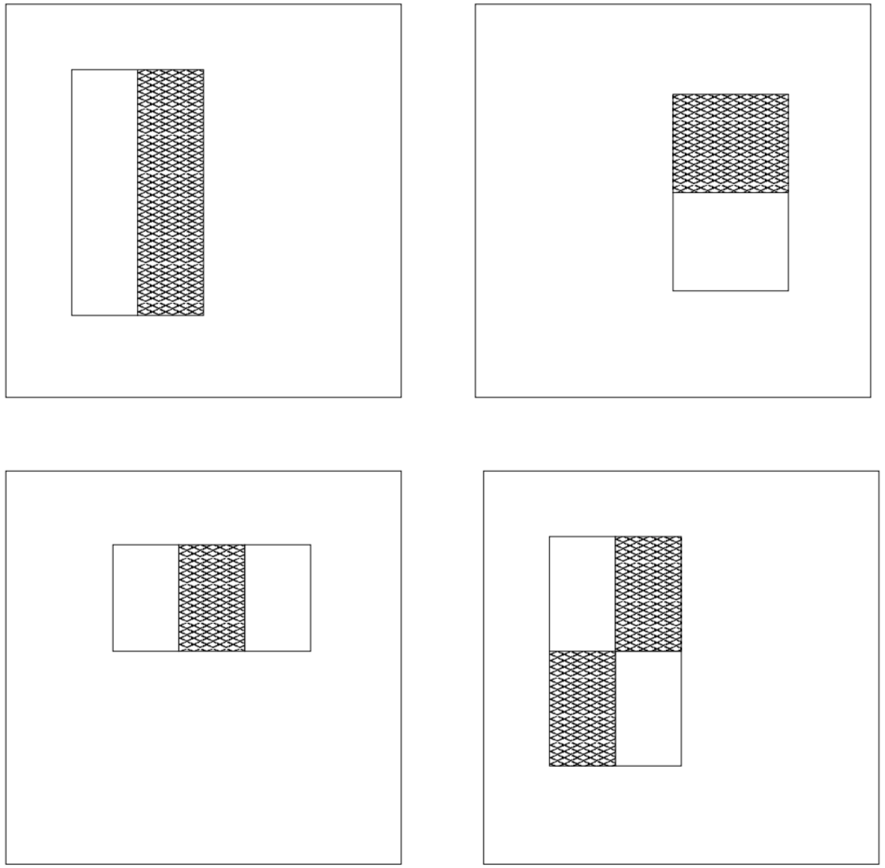
Practical Success of AdaBoost

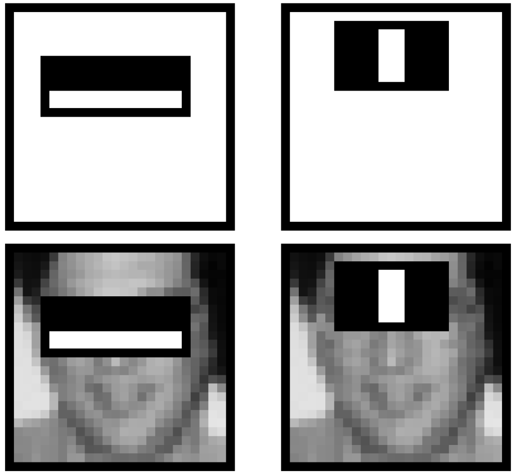
Viola-Jones Face Detection (2001)

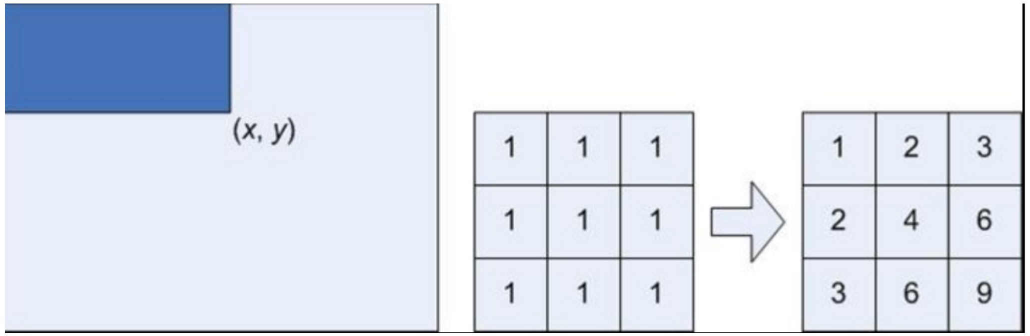
- First real-time object detection framework
- Paul Viola and Michael Jones. Rapid object detection using a boosted cascade of simple features. CVPR 2001.



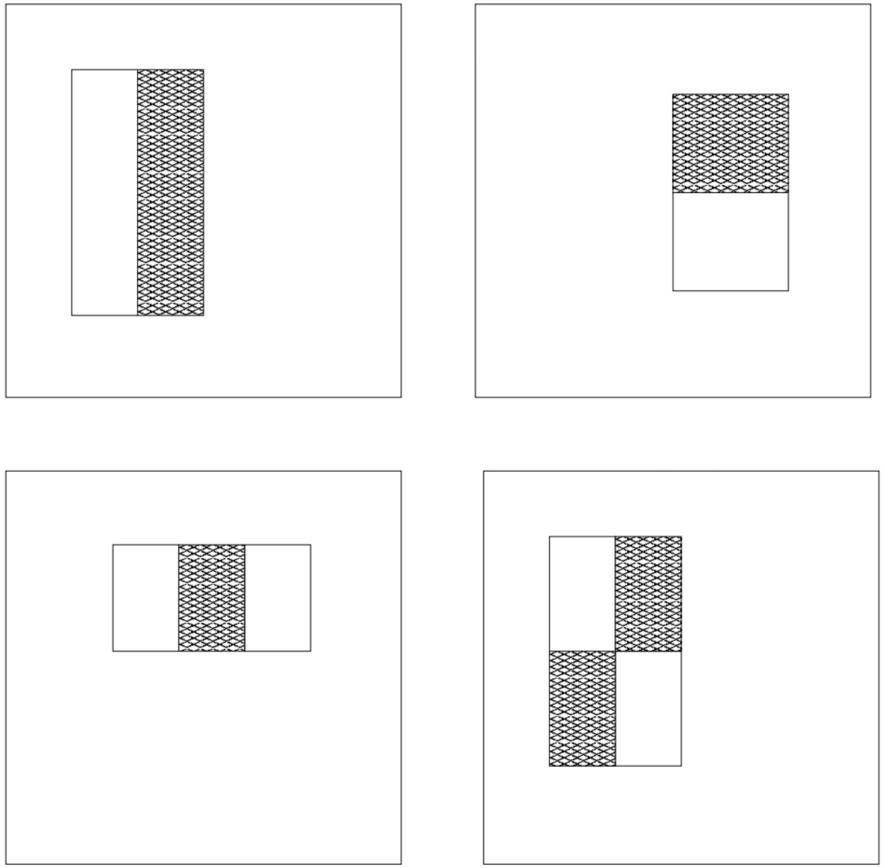
Weak Learners (Haar wavelet features)







Weak Learners (Haar wavelet features)



- Each hypothesis is very weak.
- There are many possible features.
  - For a 24x24 detection region, more then 160,000 features

- AdaBoost!
  - Training is slow
  - Testing is fast
    - (inherent feature selection)

Brief Discussion on Gradient Boosting

Gradient boosting is **safe to skip** for Exam 2

Look at the AdaBoost Algorithm Again

Given $D = \{(\vec{x}_1, y_1), \dots, (\vec{x}_N, y_N)\}$
Initialize $D_1(n) = 1/N$ for all $n = 1, \dots, N$
For $t = 1, \dots, T$
Learn g_t from D_t (using decision stumps)
Calculate $\epsilon_t = E_{in}^{(D_t)}(g_t)$
Set $\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
Update $D_{t+1}(n) = \frac{1}{Z_t} D_t(n) e^{-\alpha_t y_n g_t(\vec{x}_n)}$
Output $G(\vec{x}) = sign(\sum_{t=1}^{T} \alpha_t g_t(\vec{x}))$

→

Initialize $G(\vec{x}) = 0$
For $t = 1, \dots, T$
$G(\vec{x}) \leftarrow G(\vec{x}) + \alpha_t g_T(\vec{x})$
Output $sign(G(\vec{x}))$

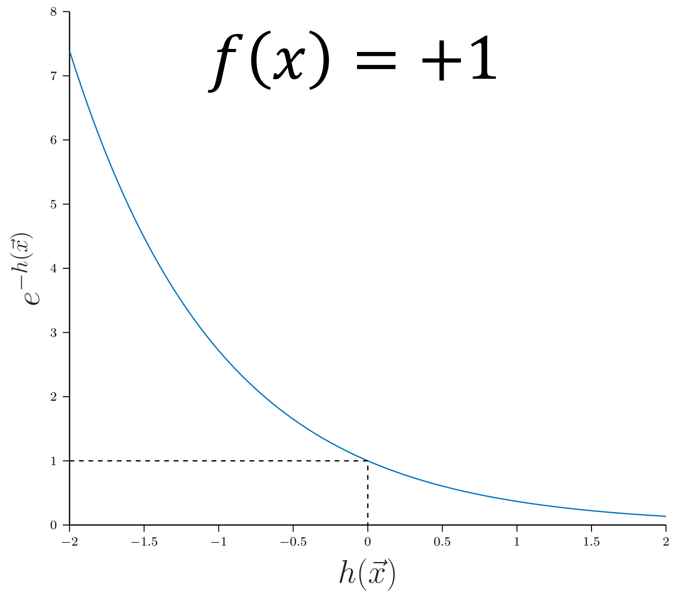
- The format is similar to gradient descent!
  - If we consider the space of the weak learners (i.e., $g_t(\vec{x})$) as the space of "weights"
  - This observation leads to a general class of boosting algorithms: gradient boosting
  - XGBoost is one implementation of gradient boosting that is popular in practice
  - See CASI 17.4 and the reference in CASI P.350 for more discussion

[Safe to Skip]

Gradient Boosting

Initialize $G(\vec{x}) = 0$
For $t = 1, \dots, T$
$G(\vec{x}) \leftarrow G(\vec{x}) + \alpha_t g_T(\vec{x})$
Output $sign(G(\vec{x}))$

- AdaBoost is a special case of Gradient Boosting
  - minimizing the exponential loss ($\mathrm{e}_{\mathrm{exp}}(h(\vec{x}), y) = e^{-yh(\vec{x})}$)
  - using decision stump as the weak learners



- e_{exp} is a surrogate loss function of the binary classification error we care about
  - Minimizing an alternative error (loss function) is a common trick in ML, especially when the target loss function is hard to optimize.
  - There are some theoretical discussions on when doing this makes sense ("calibration": whether minimizing the surrogate is consistent with minimizing the original loss).

[Safe to Skip]