Lecture 7:
Overview on Game Theory and Incentive Design

Instructor: Chien-Ju (CJ) Ho

Logistics: Assignment 1

- Due: Feb 9 (Friday)

- For majority voting, we mean simple majority voting
  - Break ties randomly when there are equal votes for each label
    - Equivalent (in expectation) to set the error to 0.5 in this case

- EM (only assuming unknown worker skills):
  - How to choose the weights during the step of weighted majority voting?
  - The MLE weight $\log(p/(1-p))$ is very sensitive to noise when p approaches 0 or 1
  - You can do either one of the following:
    - Crop the probability using ideas in Section 4 of "Who Moderates the Moderators"
    - Use the $(2p-1)$ weight for workers with correct probability p
      - technically not leading to MLE, but it give the smallest theoretical error bounds (see lecture 3 and 4)
- SVD
  - As a sanity check, you should get an error 0 when you simulate a complete matrix
    - Let every worker label every task according to ground truth and worker skills

Logistics: Assignment 2 and Project Proposal

- Assignment 2
  - Due: Feb 28 (Wed)
  - 3 long-ish math questions that extend the lecture today

- Project Proposal
  - Tentative due: Mar 1 (Friday)
  - Requirement
    - Project topic
    - 1~2 paragraph description of the project
    - Identify at least one research paper on the topic

Today’s Lecture:
Modeling Incentives

Warm-Up Discussion

- What are the incentives that motivate you/people to do things?
- Can you try to model the incentives? For example, how do you mathematically specify the relationships of the "strength" of the incentives and how that impacts your actions?
- Mechanism design: Given your model of how humans respond to incentives, how can you design mechanisms (set of rules) that encourage people to do what you want them to do? (You can try to think about these in some specific applications).

Today’s Lecture: Modeling Incentives

- Game theory basics
  - Utility, games, equilibrium
  - Example usage in crowdsourcing
    - Contract design (Principal agent model)

- Proper scoring rules (eliciting truthful probability estimates)
  - Example usage in crowdsourcing
    - Prediction markets

- Peer prediction
  - Example usage in crowdsourcing
    - Peer grading in MOOCs

Game Theory

- Mathematical study of interactions between **rational** and **self-interested** agents.

- Agents are often assumed to be rational and choose actions to maximize their **expected utility.**

Utility

- A way to quantify agents' preferences over the state of the world.
- Example

$$\Omega = \{\text{Sunny}, \text{Cloudy}, \text{Rainy}\}$$

$$\text{Sunny} \succ \text{Cloudy} \succ \text{Rainy}$$

(Sunny is preferred over Cloudy and Rainy, and Cloudy is preferred over Rainy)

- Using von Neumann–Morgenstern utility

u(Sunny) = 10, u(Cloudy) = 5, u(Rainy) = 3

A natural way to use numerical value to represent preference. It also satisfies nice properties:

- *Completeness*
- *Transitivity*
- *Continuity*
- *Independence*.

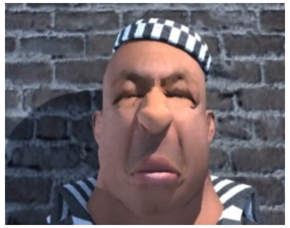
Expected Utility Theory

- Agents take actions to maximize their expected utility

$$\sum_{\omega \in \Omega} p(\omega) u(\omega)$$

- Game theory deals with situations in which $p(\omega)$ and $u(\omega)$ are influenced by **agents' joint actions**

Example 1: Prisoner’s Dilemma



B

Normal-form game

A

	Stay Silent	Confess
Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
Confess	A: free B: 10 years	A: 5 years B: 5 years

Solution Concept

What should the prisoners do?

“Confess” is a **dominant strategy** – it maximizes the prisoner’s utility no matter what action the other player chooses.

Normal-Form Game

- Players take actions simultaneously
- The elements of a normal-form game
  - Players: (prisoner A, prisoner B)
  - Strategies: (stay silent, confess)
  - Payoffs: (sentences for all strategy combinations)

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

Normal-Form Game (More formally)

- A finite, n-player normal-form game is a tuple $(N, A, \vec{u})$
  - N is a finite set of n agents, indexed by $i \in \{1, \ldots, n\}$
  - $A = A_1 \times A_2 \cdots \times A_n$, where A_i is a finite set of actions available to agent *i*
  - $\vec{u} = \{u_1, u_2, \ldots, u_n\}$, where $u_i: A \rightarrow \mathbb{R}$ is a real-valued utility function for agent *i*

Example 2: Coordination Game

- Two friends A and B are deciding what to do on Friday night
  - A prefers to go to the movie
  - B prefers to go to the bar
  - Both prefer to do something together than doing something separately

B

A

	Movie	Bar
Movie	(2, 1)	(0, 0)
Bar	(0, 0)	(1, 2)

"Nash equilibrium" of this game

What is the "solution" of the game: what should A and B do?

(Movie, Movie) and (Bar, Bar) seem to be two ***stable*** outcomes

Example 3: Rock, Paper, Scissors

- A zero-sum game

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

Is there a dominant strategy?

Are there stable pairs of actions?

Need a more general solution concept.

Definitions

- Mixed strategy
  - Instead of deterministic actions, think about randomized actions

  - Let A_i be the set of actions agent i can take
  - Let S_i be the set of all probability distributions over A_i
  - Each $s_i \in S_i$ is a ***mixed strategy***, where
    - $s_i(a_i)$ denotes the probability for agent i choosing action a_i

- Payoffs can be calculated using expected utility

Definitions

- Best response
  - $\vec{s}$: a strategy profile, i.e., the set of strategies for all agents
  - $\vec{s}_{-i}$: the strategies of all agents except agent i
  - A strategy $s_i^* \in S_i$ is a ***best response*** to $\vec{s}_{-i}$ if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \text{ for all } s_i \in S_i$$

  - Given what others do, a best response leads to highest utility

Definitions

- Nash equilibrium
  - A strategy profile $\vec{s}$ is a ***Nash equilibrium*** if, for all agent i, $\mathrm{s_i}$ is a best response to $\vec{s}_{-i}$

- Intuitive interpretations:
  - If all agents except i follow the strategies in the Nash equilibrium, agent i would maximize her payoff by following the strategy in Nash.
  - No incentive to deviate if everyone else follows the strategy in Nash

Let’s look at the examples again

	B Stay Silent	B Confess
A Stay Silent	A: 6 months B: 6 months	A: 10 years B: free
A Confess	A: free B: 10 years	A: 5 years B: 5 years

- (Confess, Confess) is the dominant strategy equilibrium
- Strongest solution concept

Let’s look at the examples again

	Movie	Bar
Movie	(2, 1)	(0, 0)
Bar	(0, 0)	(1, 2)

- (Movie, Movie) and (Bar, Bar) are pure strategy Nash equilibria

Let’s look at the examples again

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

- Both players play each action with 1/3 probability is the mixed strategy Nash equilibrium
- It is the unique equilibrium

Theorem (Nash, 51):

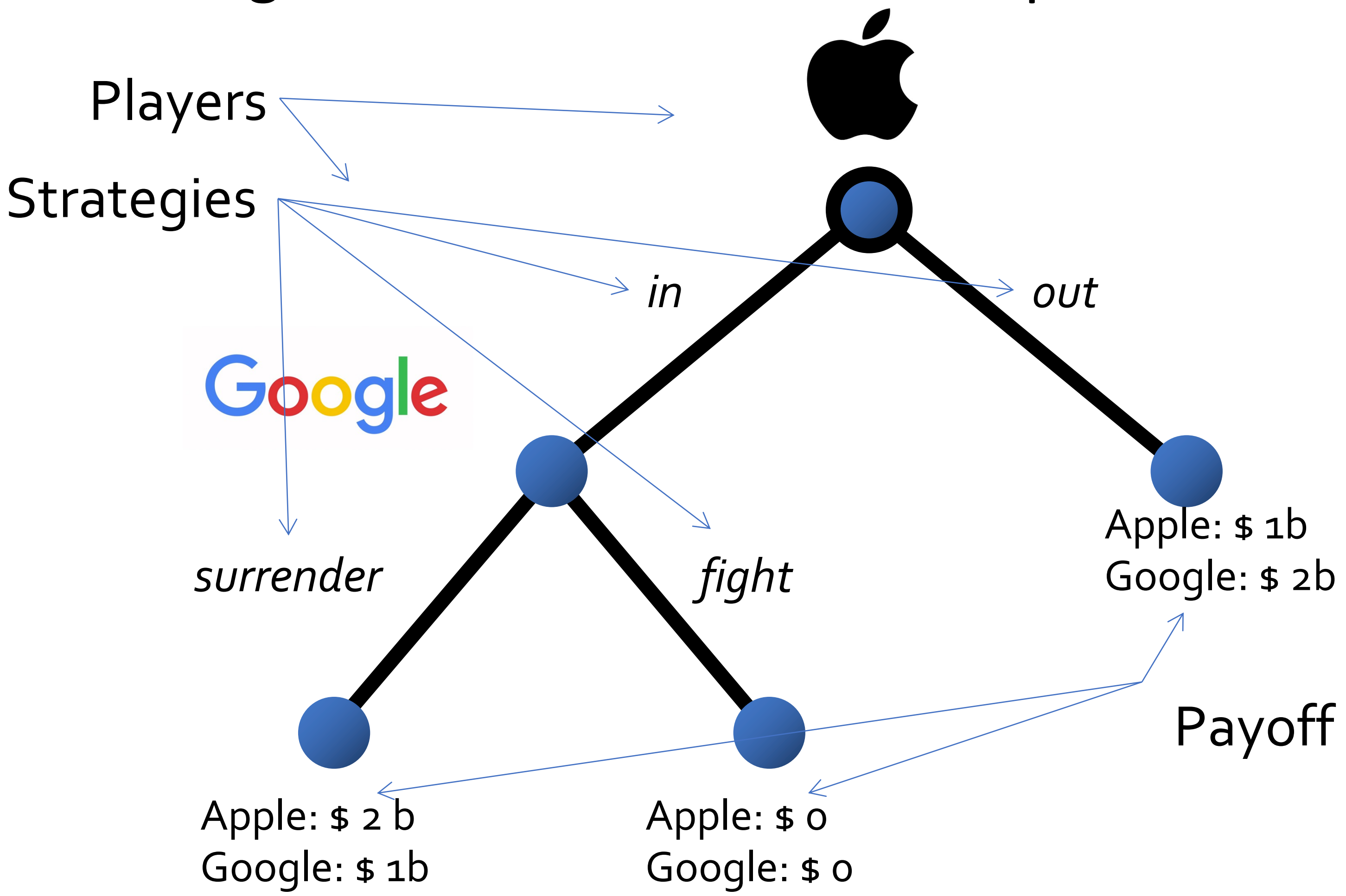
Every game with a finite number of players and actions has at least one equilibrium.

Notes:

- This is the "existence" proof. Computing the equilibrium could be hard (computationally expensive).
- If it's hard to calculate the equilibrium, can we really expect humans to follow the equilibrium?
  - There have been recent studies developing new solution concepts which assume humans are "learning" to adapt to the game.

Extensive-Form Game

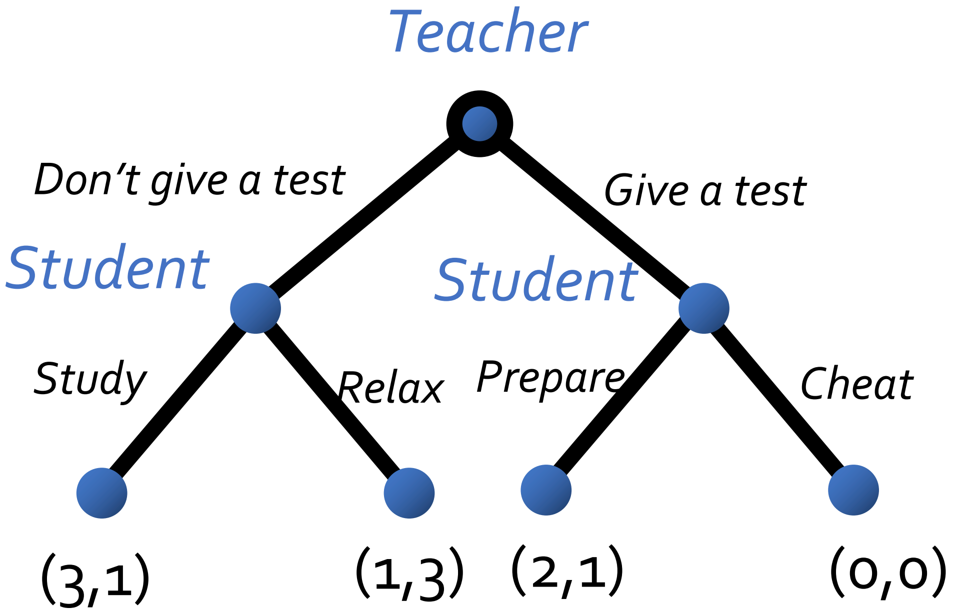
- Agents take decisions in a sequential manner



	Surrender	Fight
In	(2, 1)	(0, 0)
Out	(1, 2)	(1, 2)

Solution Concepts in Extensive Games

- There are some weird cases by directly extending Nash equilibrium.



- Is this a Nash equilibrium:
  - Teacher chooses "Don't give a test"
  - Student chooses (Relax, Cheat)

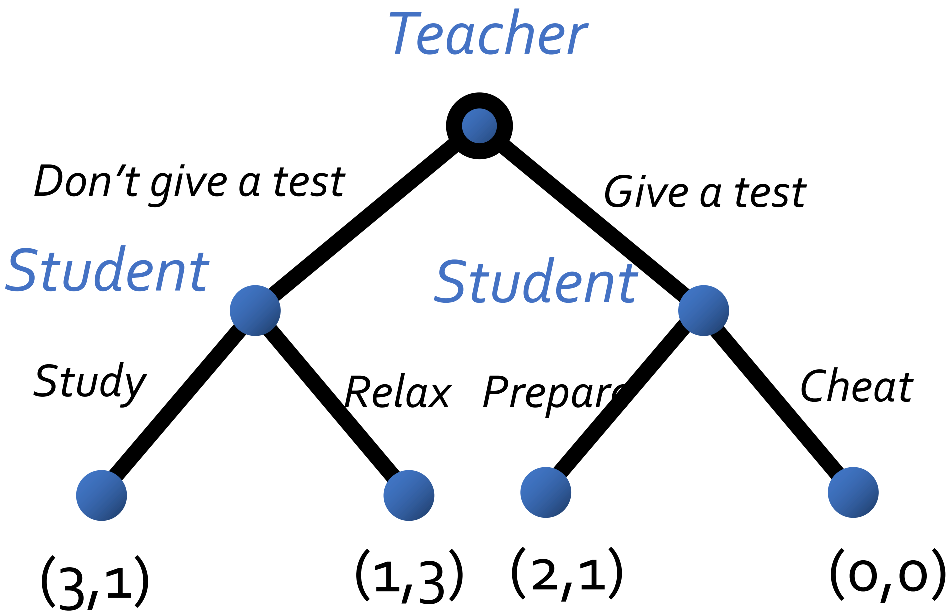
Is it stable?

Why is this a weird case?

- Student gives a "non-credible threat"
- "I'll choose to cheat if you give a test"

Solution Concepts in Extensive Games

- Subgame Perfect Equilibrium (SPE)
  - Play in each subgame is a Nash equilibrium.
  - Rule out the "non-credible threat"



- SPE
  - Teacher chooses "Give a test"
  - Student chooses (Relax, Prepare)

- Can usually be calculated using backward induction.

Applying Game Theory in this Course

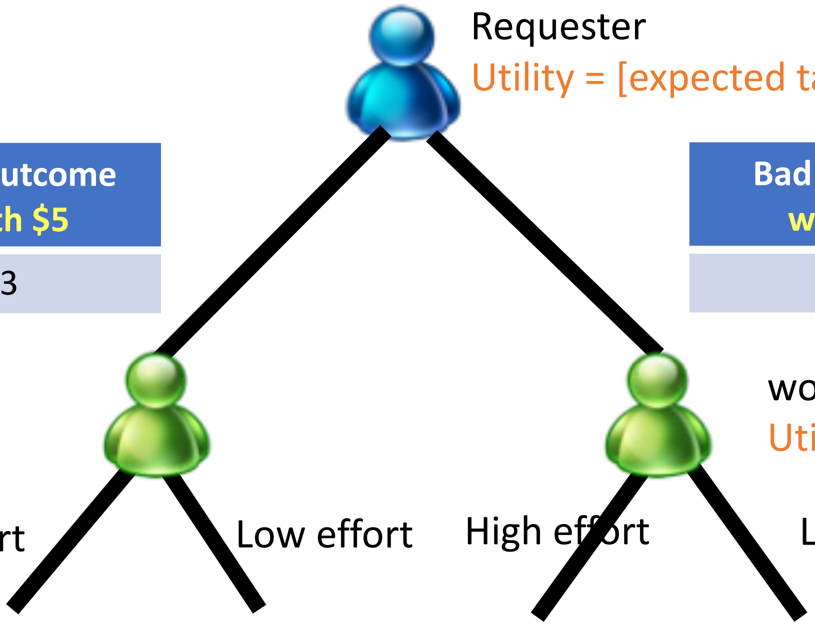
- Formulate users' incentives
- Describe the game structure
  - Sometime there are no interactions between workers.
    It becomes a simpler optimization problem.
- Analyze the equilibrium as the prediction of the outcome

- Mechanism design
  - Specify the desired outcome
  - Design incentive and game structures such that the outcome is the equilibrium

Practice Example – Contract Design

Bad Outcome worth $1	Good Outcome worth $5
$1	$3

Bad Outcome worth $1	Good Outcome worth $5
$1	$1



	Bad Outcome	Good Outcome	
High effort	Prob: 0.1	Prob: 0.9	Cost: $1
Low effort	Prob: 0.8	Prob: 0.2	Cost: $0

Which contract should the requester choose?

Can be generalized to any real-valued contracts (Principal Agent problem)

Discussion

- The main assumption we usually make in incentive design is that users are rational. While it approximates human behavior fine in some cases, this assumption has often been criticized.

- Questions
  - What are the examples of the “non-rational” human behavior you can think of?
  - Can you still try to model the non-rational behavior?

Example of Human Bias

- Present bias:
  - Humans value immediate payoffs heavier than future payoffs.
  - What will happen if we don't require project milestone reports?
- Herding bias:
  - Humans tend to follow what others do/say.
  - Will you eat at a restaurant with a lot of people lining up or another with no one inside.
- Prospect theory:
  - Nobel-winning theory in explaining biases in decision-making with uncertainty events.
- And more…
- A growing research direction to account for human biased behavior in computational systems.
  - First workshop on Behavioral EC

Proper Scoring Rules

Incentivizing Truthful Reports About Probabilities

- Example scenarios:
  - Ask a weather forecaster: will it rain tomorrow?
  - Ask a political researcher: will Trump be the presidential candidate in 2024?
  - Ask an Apple employer: will the new version of iOS be shipped on time?

- Want to obtain forecasts about future events
- How do we make sure we obtain **truthful** reports?

Incentivizing Truthful Reports

- Setting
  - Consider a rational agent with linear utility for cash
  - Suppose there are n mutually exclusive and exhaustive states of the world $\Omega = \{w_1, w_2, \dots, w_n\}$ (e.g., Sun, Rain, Snow)
  - p_i is the subjective belief of the agent that state w_i will occur
- Question
  - How do we motivate this agent to tell us her beliefs about the likelihood of each state?

Scoring Rules

- A scoring rule rewards an agent $S(\vec{r}, w)$ when her reported distribution is $\vec{r}$ and the realized outcome is w

Scoring Rules

- Let's consider a linear scoring rule

$$S(\vec{r}, w_i) = r_i$$

- If a risk-neutral agent believes the probability for Rain and Sun are

$$\vec{p} = (0.7, 0.3)$$

What report should the agent provide?

Scoring Rules

- A scoring rule rewards an agent $S(\vec{r}, w)$ when her reported distribution is $\vec{r}$ and the realized outcome is w

- A scoring rule is called ***proper*** if the agent maximizes her utility by providing truthful report

$$\vec{p} = \mathrm{argmax}_{\vec{r}} \sum_{i=1}^{n} p_i S(\vec{r}, w_i)$$

- A scoring rule is ***strictly proper*** if honestly reporting is the **unique** maximizer.

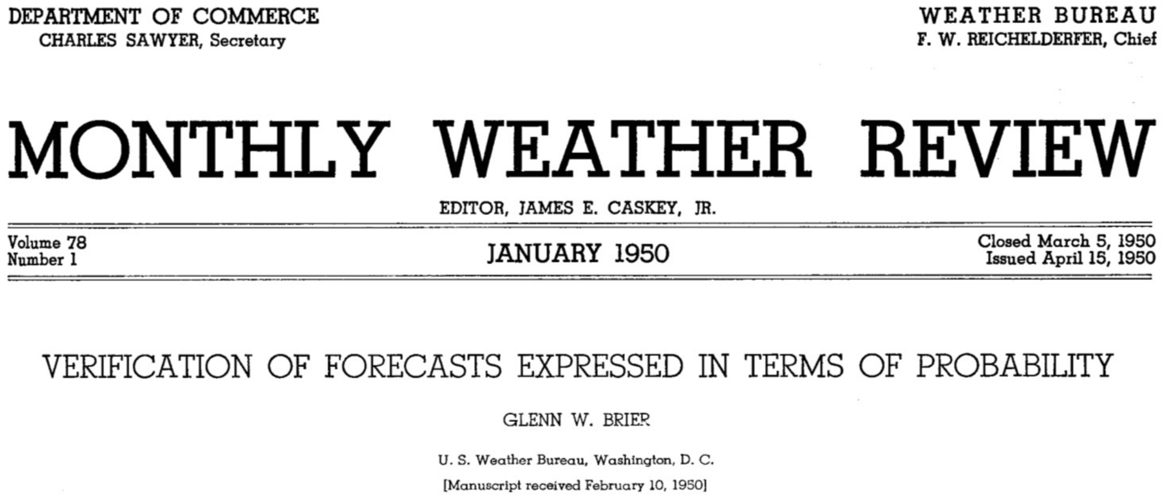
Examples of Strictly Proper Scoring Rules

- Quadratic scoring rule (Brier score):

$$S(\vec{r}, w_i) = r_i - \frac{1}{2}\sum_j r_j^2$$

We can verify this by taking the gradient of the expected payoff

- Affine transformation of the proper scoring rule is still proper.



DEPARTMENT OF COMMERCE
CHARLES SAWYER, Secretary

WEATHER BUREAU
F. W. REICHELDERFER, Chief

MONTHLY WEATHER REVIEW

EDITOR, JAMES E. CASKEY, JR.

Volume 78
Number 1

JANUARY 1950

Closed March 5, 1950
Issued April 15, 1950

VERIFICATION OF FORECASTS EXPRESSED IN TERMS OF PROBABILITY

GLENN W. BRIER

U. S. Weather Bureau, Washington, D. C.

[Manuscript received February 10, 1950]

Examples of Strictly Proper Scoring Rules

- Logarithmic scoring rule:

$$S(\vec{r}, w_i) = \log r_i$$

  We can verify this by taking a gradient of the expected payoff

- In logarithmic scoring rule, the score for outcome w_i only depends on the report r_i and not r_j for $j \neq i$

More examples?

- How do we construct a strictly proper scoring rule?

- How many strictly proper scoring rules are there?

Characterization of Proper Scoring Rules

- Connections between convex functions and proper scoring rules.

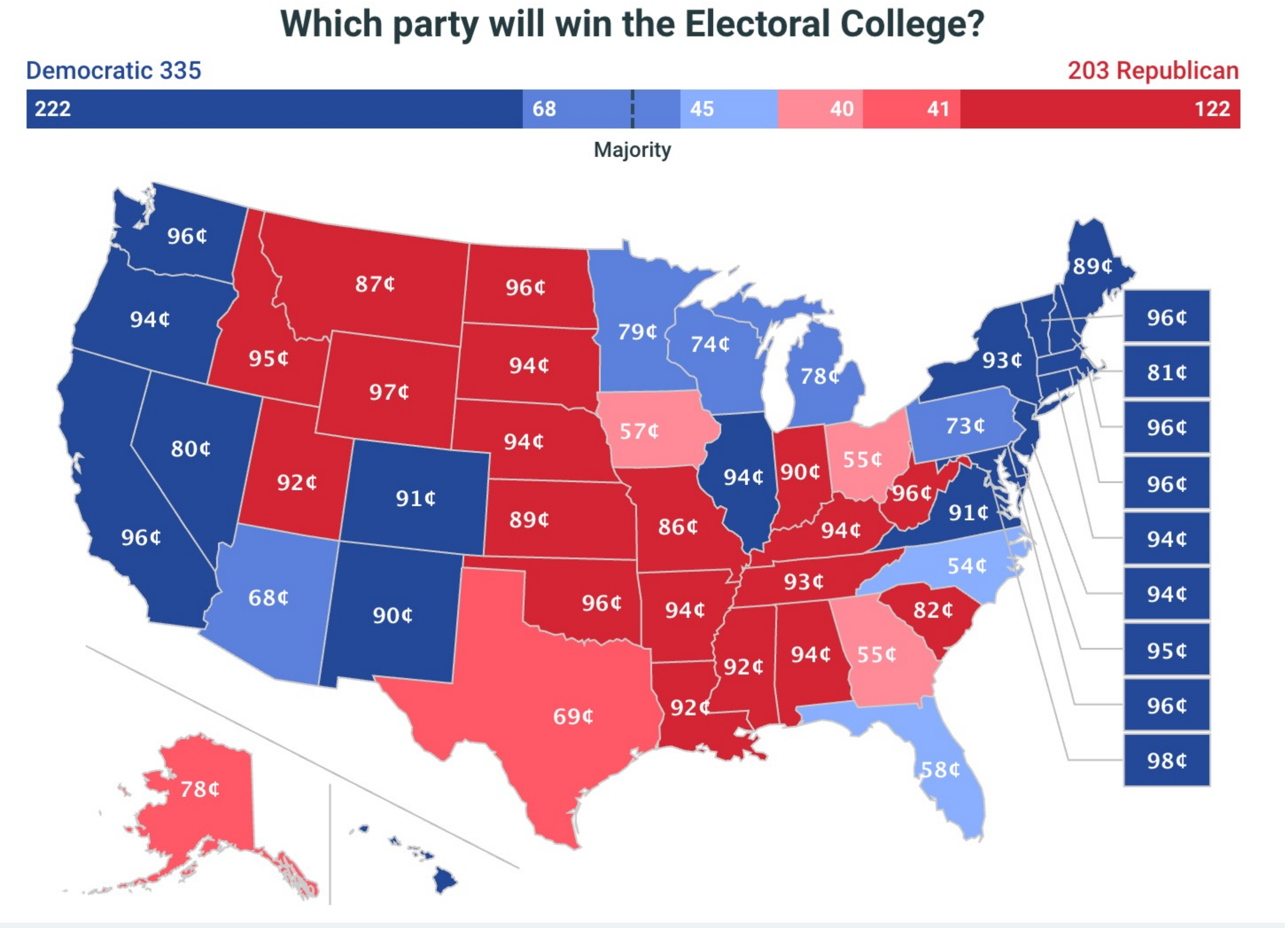
- A scoring rule $S(\vec{r}, w_i)$ is (strictly) proper if and only if

$$S(\vec{r}, w_i) = G(\vec{r}) - \sum_{j \neq i} G_j'(\vec{r}) p_j + G_i'(\vec{r})$$

where $G(\vec{r})$ is a (strictly) convex function, $G'(\vec{r})$ is a subgradient of G at $\vec{r}$, and $G_i'(\vec{r})$ is its i-th component.

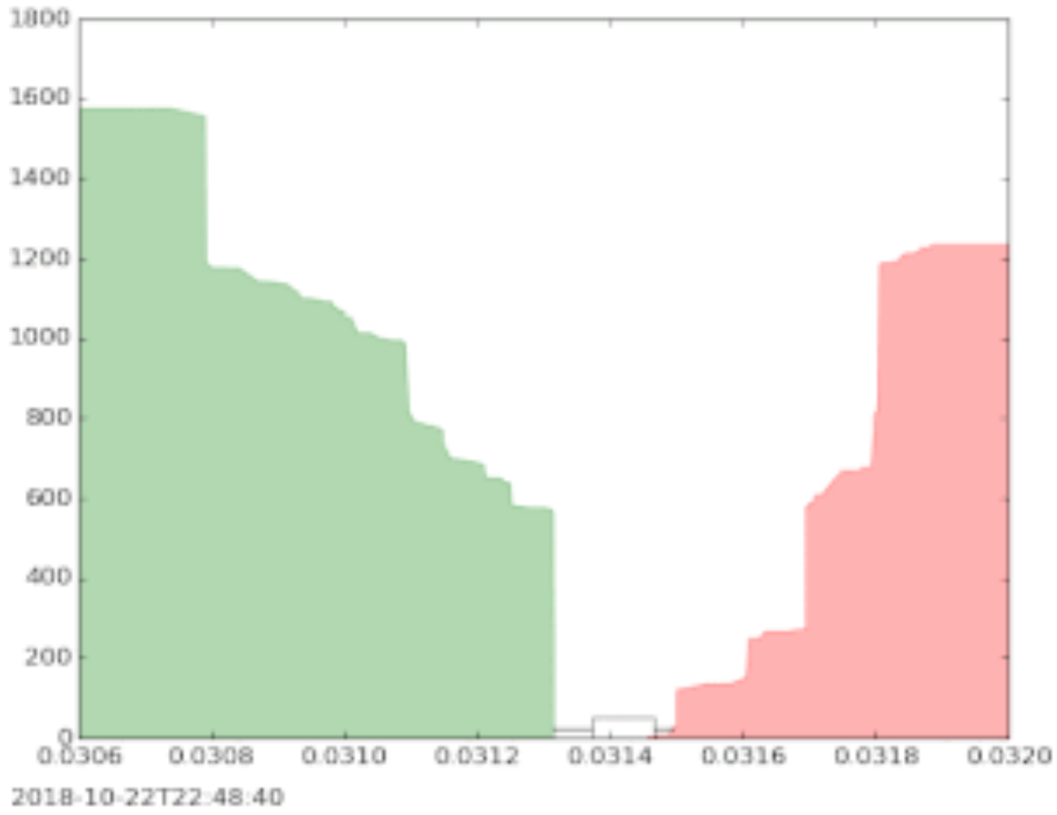
Section 3.1 of Strictly Proper Scoring Rules, Prediction, and Estimation by Gneiting and Raftery.

Connection to Prediction Market



Designing Automatic Market Makers

- Traditional market mechanisms might not work when the market is thin



Market Scoring Rules

- See Hanson's papers in the optional readings of the Prediction Market lecture
- Intuitions: a "sequentially shared scoring rule"
  - An automatic market maker
  - Market maintains a vector of predictions $\vec{r}^{(t)}$
  - If a trader changes the vector from $\vec{r}^{(t)}$ to $\vec{r}^{(t+1)}$ and the outcome is w_i, the trader obtains reward

$$S\left(\vec{r}^{(t+1)}, w_i\right) - S\left(\vec{r}^{(t)}, w_i\right)$$

- Under some conditions:
  - Agents truthfully report their beliefs
  - The prediction will converge

Market Scoring Rules

- The connection to convex optimization opens up an interesting line of research in the design of efficient market maker…

Very Brief Intro of Peer Prediction

See more discussion in Assignment 3

Eliciting Truthful Reports

- Scoring rule relies on the “truth” to be revealed in the future
- What if there is no ground truth (or the ground truth is hard to obtain)
  - Do you like this movie?
  - Peer grading in MOOCs

- Output agreement:
  - Randomly pick two persons
  - If their reports match, reward them 1, otherwise reward 0
  - Truthful reporting is not an equilibrium (you are encouraged to report the majority’s opinion)

Peer Prediction

- How to fix the issue?
  - Assume knowledge about the report distribution, re-weighting the rewards to make sure truthful reporting is a equilibrium

- Drawbacks:
  - Require knowledge of the prior
  - There are usually multiple equilibrium (including naïve bad ones…)

- Still an ongoing research area
  - Some nice theoretical results, however there is little practical success so far

Eliciting Informative Feedback: The Peer-Prediction Method. Miller et al. 2004

Related Course

- The course on peer grading at Northwestern by Jason Hartine.
  - https://sites.northwestern.edu/hartline/eecs-497-peer-grading/

Assignment 2

Cooperation and Repeated Prisoner's Dilemma

- Prisoner's dilemma predicts that people are not going to cooperate in the game setup, but in practice, people sometimes do cooperate.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

- Will look at this using repeated versions of prisoner's dilemma

Peer Grading and Peer Prediction

- Can we design ”incentives” for peer grading?
  - Ground truth (goodness of assignment) is hard to obtain
- Randomly pick two students to grade the same assignment
  - Rewarding or the same report is probably not a good idea
  - How should we do it?

- Setting:
  - Assignment quality {Good or Bad}: Prior: $\Pr(\text{“}Good\text{”}) = 0.8$
  - Assessment signal {G, B}

	Signal	
	G	B
Good	80%	20%
Bad	40%	60%

Information Design with Bayesian Persuasion

- A company wants to hire interns from our class and asks me for recommendation letters

- Assumption
  - 30% of students are “good” -> meet their requirement
  - They don’t know which students are good, but I know

- How do I write letters to maximize the number of students getting hired?