UMEM 1.0.0

# Unified Memristor Model

Technical Manual

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# 1. Introduction

UMEM is a unified compact model for memristive devices [1]. Currently, it includes 3 devices: RRAM, FERAM/FTJ, and STT-MRAM.

UMEM has three inout nodes. n1 and n2 are the electrical nodes for electrodes. s is the state node which should not be connected to anything. It is used to calculate and probe the state. It can also be used to assign initial condition. In RRAM (devmod=0), s node represents the tunneling length in nanometer. In FERAM/FTJ (devmod=1), it represents the polarization in C/m$^2$. In MRAM, it is the magnetic moment.

```
module umem_va(n1, n2, s);
```

# 2. Model Framework

The unified switching model is based on (2.1) and (2.2) to model memristor state dynamics. (2.1) can be solved by the equivalent circuit in Fig. 2.1 using Verilog-A. $F_+$ is the switching function for forward switching. $F_-$ is the switching function for reverse switching. $F_+$ and $F_-$ can be expressed as (2.2). $s_+$ and $s_-$ are the final state for forward and reverse switching. $\tau_+$ and $\tau_-$ are the effective time constants. In forward switching, $\tau_-$ becomes $\infty$, and similar for $\tau_-$ in reverse switching.

$$\frac{ds}{dt} = F_+ + F_- \quad (2.1)$$

$$F_+ = \frac{s_+ - s}{\tau_+}, F_- = \frac{s_- - s}{\tau_-} \quad (2.2)$$

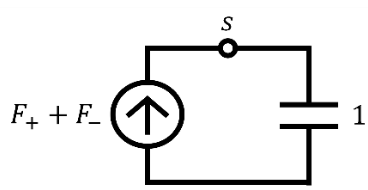



Fig. 2.1. The equivalent circuit to solve (2.1).

The unified resistance model is shown as (2.3) and (2.4). $va$ is the applied voltage, $\rho rhotfinal$ is the tunneling resistivity, $x$ is the tunneling thickness, $area$ is the device area, $x_t$ is the thickness dependence factor, $v0fianl$ is the voltage dependence factor, $rsfianl$ is the series resistance, $vs0$ is the voltage factor for series resistance, $areanom$ is the nominal area. and $ratio$ is the resistance change due to tunnel magnetoresistance (TMR) effect. The DC current is calculated by (2.4). The electrical equivalent circuit of UMEM is shown as Fig. 2.2.

$$rm = ratio \times \frac{rhotfinal \cdot x}{area} \frac{exp\left(\frac{x}{x_t}\right)}{1 + (va/v0final)^2} + \frac{rsfinal \cdot areanom}{(1 + (va/vs0)^2)area} \quad (2.3)$$

$$im = V/rm \quad (2.4)$$

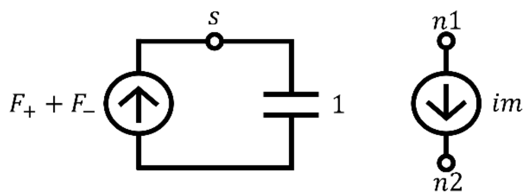



Fig. 2.2. The electrical equivalent circuit of UMEM.

# 3. Physical Contant

Common physical constants in UMEM.

$$tnomK = tnom + 273.15 \quad (4.1)$$

$$devtemp = \$temperature + \Delta T + dtemp \quad (4.2)$$

$$vt = 8.617087 \times 10^{-5} \times devtemp \quad (4.3)$$

$$vt = 8.8542 \times 10^{-12} \times epar \,/\, tm * area \quad (4.4)$$

$$va1 = va + voff \quad (4.5)$$

# 4. RRAM Switching Model

In RRAM, state $s$ is the tunneling thickness in nanometer. The filament growth in RRAM changes $x$ to change resistance. The state dynamics of RRAM is modeled by the following equations.

$$x = 10^{-9} V(s) \quad (4.1)$$

$$x = smoothmin(x, 0, 5 \times 10^{-12}) \quad (4.2)$$

$$x = smoothmin(x, tm, 5 \times 10^{-12}) \quad (4.3)$$

$$efield = va1/tm \quad (4.4)$$

$$t0 = x/tm \quad (4.5)$$

$$t1 = 1 - t0 \quad (4.6)$$

$$ea\_f = ea0 \times (a0 \times t1^{b0} + 1) \; if \; a0 > 0 \quad (4.7)$$

$$ea\_f = ea0 \times (1 - a0 \times t1^{b0})/(1 - a0) \; if \; a0 < 0 \quad (4.8)$$

$$ea\_r = ea0\_r \times (a0\_r \times t1^{b0\_r} + 1) \; if \; a0\_r > 0 \quad (4.9)$$

$$ea\_r = ea0\_r \times (1 - a0\_r \times t1^{b0\_r})/(1 - a0\_r) \; if \; a0\_r < 0 \quad (4.10)$$

$$SF = smoothstep(efield, dedc) \quad (4.11)$$

$$F\_f = \frac{(0 - 10^{-9}\, V(s))}{tau0} \times exp(\frac{10^{-9} am \times efield - ea\_f}{vt}) \times SF \quad (4.12)$$

$$F\_r = \frac{\left(tm - 10^{-9}\, V(s)\right)}{tau0} \times exp(\frac{-10^{-9} am\_r \times efield - ea\_r}{vt}) \times (1 - SF) \quad (4.13)$$

$$F = 10^9(F\_f + F\_r) \quad (4.14)$$

# 5. FE Switching Model

In ferroelectrics (FE), state $s$ is the polarization. The state dynamics of FE is modeled by the following equations.

$$pr\_t = pr \times \exp\left(-tpr(devtemp - tnomK)\right) \quad (5.1)$$

$$efield = va1/tm \quad (5.2)$$

$$ef\_f = smoothmin(efield, 0, 10^6) \quad (5.3)$$

$$ef\_r = smoothmax(efield, 0, 10^6) \quad (5.4)$$

$$t0 = V(s)/pr\_t \quad (5.5)$$

$$t0 = smoothmin(t0, -0.999,\ 0.01) \quad (5.6)$$

$$t0 = smoothmax(t0, 0.999,\ 0.01) \quad (5.7)$$

The minor loop states are tracked by the following differential equations.

$$\frac{dV(s1)}{dt} = \frac{V(s) - V(s1)}{10^{-12}} \times smoothstep(-efield - eth, defd) \quad (5.8)$$

$$\frac{dV(s2)}{dt} = \frac{V(s) - V(s2)}{10^{-12}} \times smoothstep(efield - eth, defd) \quad (5.9)$$

$$t1 = V(s1)/pr\_t \quad (5.10)$$

$$t2 = 1 + t1 \quad (5.11)$$

$$t2 = smoothmin(t2, 0, 0.01) \quad (5.12)$$

$$mf = 1 + md1(1 - \exp(-md2 \times t2)) \quad (5.13)$$

$$t1 = V(s2)/pr\_t \quad (5.14)$$

$$t2 = 1 - t1 \quad (5.15)$$

$$t2 = smoothmin(t2, 0, 0.01) \quad (5.16)$$

$$mr = 1 + md1\_r(1 - \exp(-md2\_r \times t2)) \quad (5.17)$$

In this model, two types of $E_A$ distribution are provided: Gaussian (pdfmod=0) and log-normal (pdfmod=1).

pdfmod=0:

$$ea\_f = smoothmin(10^8 \frac{ea0}{mf}\left(1 + 0.5mf \times a0 \ln\left(\frac{1+t0}{1-t0}\right)\right), 0, 10^5) \quad (5.18)$$

$$ea\_r = smoothmin(10^8 \frac{ea0\_r}{mr}\left(1 + 0.5mr \times a0\_r\ln\left(\frac{1-t0}{1+t0}\right)\right), 0, 10^5) \quad (5.19)$$

pdfmod=1:

$$ea\_f = 10^8 \frac{ea0}{mf}\left(1 + 0.5mf \times a0\ln\left(\frac{1+t0}{1-t0}\right)\right) \quad (5.20)$$

$$ea\_r = 10^8 \frac{ea0\_r}{mr}\left(1 + 0.5mr \times a0\_r\ln\left(\frac{1-t0}{1+t0}\right)\right) \quad (5.21)$$

The final switching equations are as follows.

$$SF = smoothstep(efield + efb \times t0, dedc) \quad (5.22)$$

$$F\_f = \frac{(pr - V(s))}{tau0} \times \left(\frac{tnomK}{devtemp}\right)^{ttau0} exp\left(-\left(\frac{tnomK}{devtemp}\right)^{c0} \left(\frac{ea\_f}{ef\_f}\right)^{am}\right) \times b0\left(-\ln\left(\frac{1-t0}{2}\right)\right)^{\frac{b0-1}{b0}} \quad (5.23)$$

$$F\_r = \frac{(-pr - V(s))}{tau0} \times \left(\frac{tnomK}{devtemp}\right)^{ttau0} exp\left(-\left(\frac{tnomK}{devtemp}\right)^{c0} \left(\frac{ea\_r}{ef\_r}\right)^{am\_r}\right) \times b0\_r\left(-\ln\left(\frac{1+t0}{2}\right)\right)^{\frac{b0\_r-1}{b0\_r}} \quad (5.24)$$

$$F = F\_f \times SF + F\_r \times (1 - SF) \quad (5.25)$$

# 6. MRAM Switching Model

In STT-MRAM, state $s$ is the magnetization. The state dynamics of STT-MRAM is modeled by the following equations.

$$ip = smoothmin(im, 0, 10^{-10}) - 5 \times 10^{-11} \quad (6.1)$$

$$in = im - ip \quad (6.2)$$

$$vf = tf \times area \quad (6.3)$$

$$vfnom = tfnom \times areanom \quad (6.3)$$

$$icp = ic0 \times vf/vfnom \quad (6.5)$$

$$icn = ic0\_r \times vf/vfnom \quad (6.6)$$

$$t0 = V(s) \quad (6.7)$$

$$t0 = smoothmax(t0, 1, 10^{-3}) \quad (6.8)$$

$$t0 = smoothmin(t0, -1, 10^{-3}) \quad (6.9)$$

$$t1 = V(s) \quad (6.10)$$

$$t1 = smoothmax(t1, 0.999, 10^{-3}) \quad (6.11)$$

$$t2 = V(s) \quad (6.12)$$

$$t2 = smoothmin(t2, -0.999, 10^{-3}) \quad (6.13)$$

$$F\_f = \frac{(-1 - V(s))}{tau0} \times (-1 + t1) \times (0.5t0 - \frac{ip}{icp}) \quad (6.14)$$

$$F\_r = \frac{(1 - V(s))}{tau0} \times (1 + t2) \times (0.5t0 - \frac{in}{icn}) \quad (6.15)$$

$$F = F\_f + F\_r \quad (6.16)$$

# 7. Resistance Model

Before using (2.3), there are several variables that should be calculated. This model supports asymmetric resistance by smoothly connecting 2 model parameters.

$$SF = smoothstep(va, 0.01) \quad (7.1)$$

$$x0final = x0 \times SF + x0\_r \times (1 - SF) \quad (7.2)$$

$$v0final = v0 \times SF + v0\_r \times (1 - SF) \quad (7.3)$$

$$rhotfinal = rhot \times SF + rhot\_r \times (1 - SF) \quad (7.4)$$

$$rsfinal = rs \times SF + rs\_r \times (1 - SF) \quad (7.5)$$

RRAM (devmod=0):

$$xt = x0final \quad (7.6)$$

FE (devmod=1):

$$xt = x0final(1 + x1 \times t0) \quad (7.7)$$

MRAM (devmod=2):

$$xt = x0final \quad (7.8)$$

$$tmr = \frac{tmr0}{1+(\frac{va}{vr0})^2} \quad (7.9)$$

$$ratio = \frac{2(1+tmr)}{2+(1+t0)tmr} \quad (7.10)$$

The above variables will be plugged into (2.3) to calculate $rm$ and $im$.

# 8. Charge Model

RRAM (devmod=0):

$$Q = cp \times va1 \quad (8.1)$$

FE (devmod=1):

$$Q = cp \times va1 + V(s) \times area \quad (8.2)$$

MRAM (devmod=2):

$$Q = cp \times va1 \quad (8.3)$$

# 9. Self-heating Model

Self-heating is calculated by a thermal subcircuit solving (9.1).

$$rth \times cth \times \frac{d\Delta T}{dt} = -\Delta T + rth \times va \times im \quad (9.1)$$

# 10. Model Testing

This section demonstrates model fitting results with corresponding parameters and quality tests.

## 10.1 Validation with Experimental Data

**RRAM**

.model rram umem_va devmod=0 tm=1.69n area=25e-14 rth=5e5 cth=1e-15 x0=0.15n

+tau0=1e-15 ea0=1.24 am=0.85 a0=1.2 b0=4 am_r=0.35 a0_r=0.15 b0_r=4

+rhot=3e-3 v0=0.45 rs=4000 vs0=1 rhot_r=8e-4 v0_r=0.7 rs=4000 vs0=1 areanom=25e-14

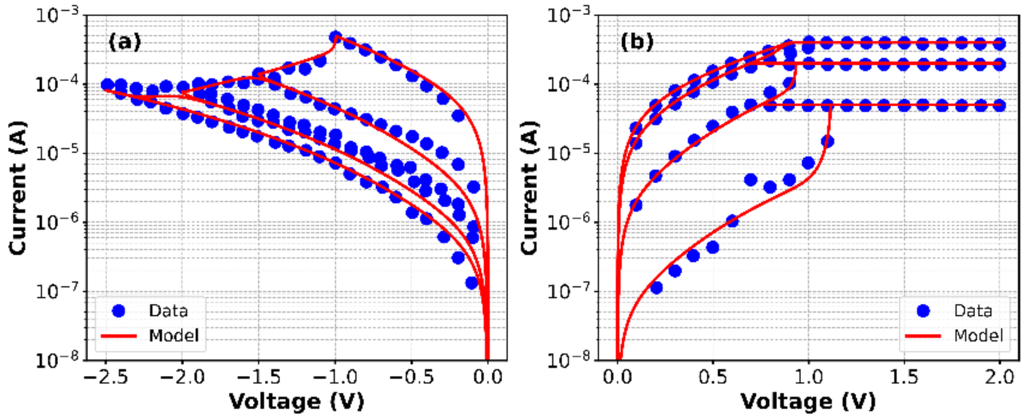



Fig. 10.1. RRAM fitting results versus data from [2].

**FE**

.model fe umem_va devmod=1 pdfmod=1 tm=8.3n pr=pr tau0=400n ea0=2 ea0_r=1.95 a0=0.2 b0=2.0 am=3.4 voff=-0.08 +md1=0.18 md1_r=0.07 md2=5 epar=34 efb=1e7

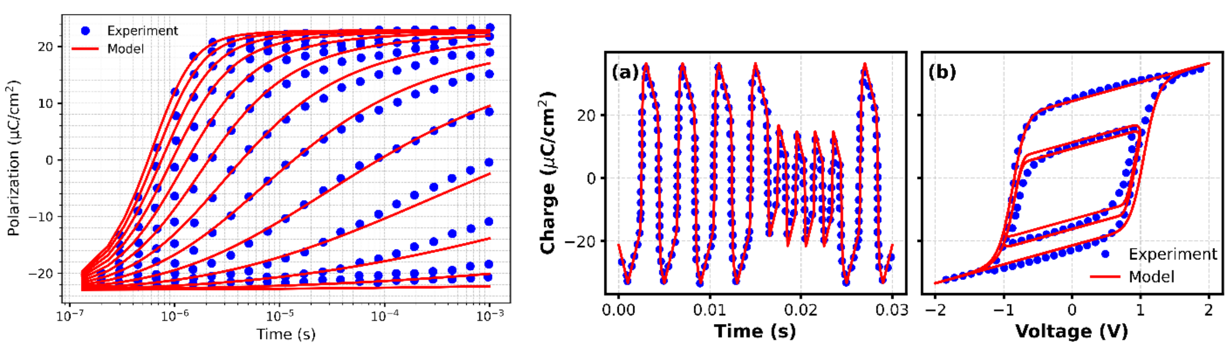



Fig. 10.2. FE fitting results versus data from [3].

**MRAM**

.model mram umem_va devmod=2 tm=1e-9 area=49e-16 areanom=49e-16 rhot=4.6e-3 ic0=3.8e-5 ic0_r=2.6e-5 v0=1.5 v0_r=2 tmr0=1.13 vr0=0.8 tau0=1.8e-8

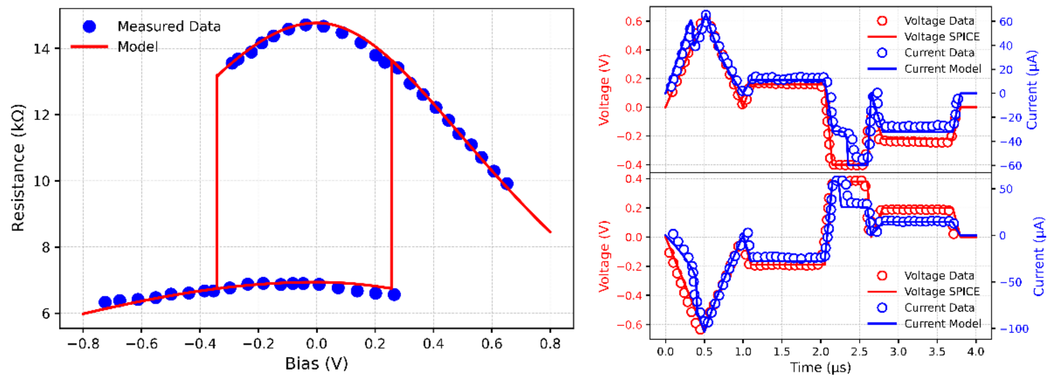



Fig. 10.3. MRAM fitting results versus data from [4].

## 10.2 Quality Test

Model smoothness and solution uniqueness are examined by plotting $F$ and its zero contour for each device. All models show a smooth function surface and a single zero contour.

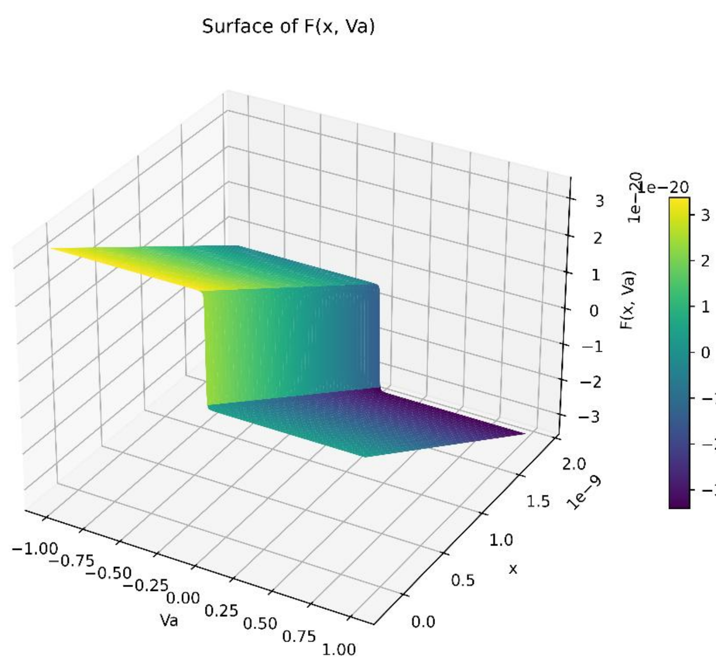



Fig. 10.4. Function surface of $F$ for RRAM.

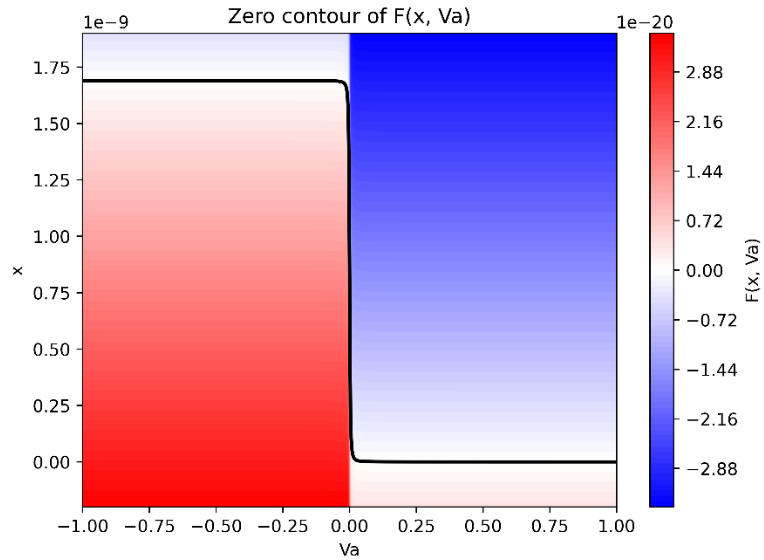



Fig. 10.5. Zero contour of $F$ for RRAM.

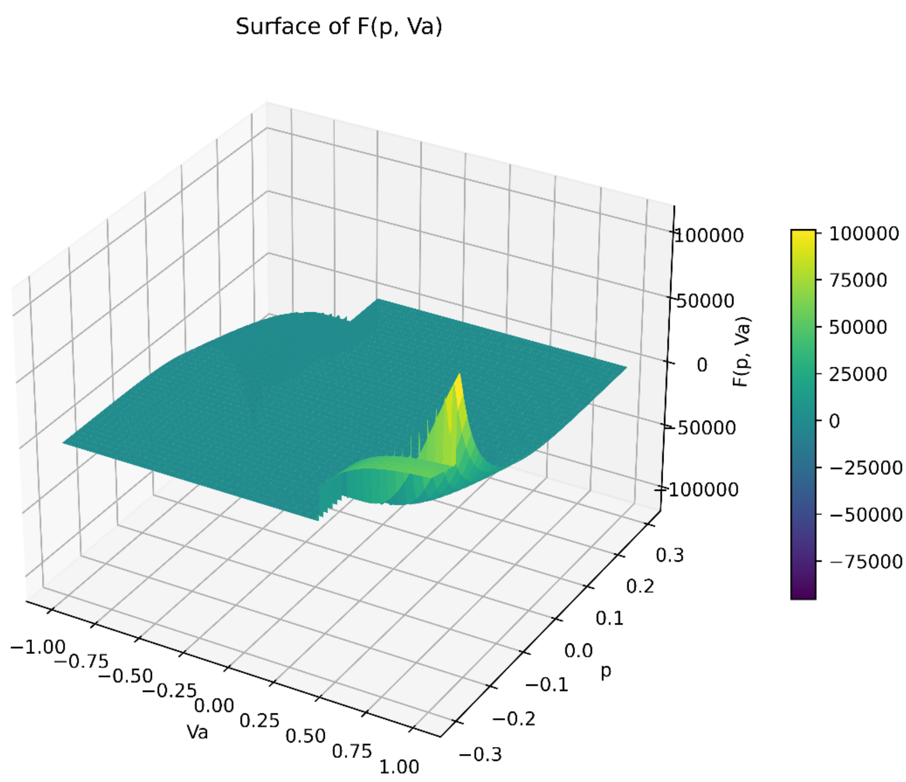



Fig. 10.6. Function surface of $F$ for FE.

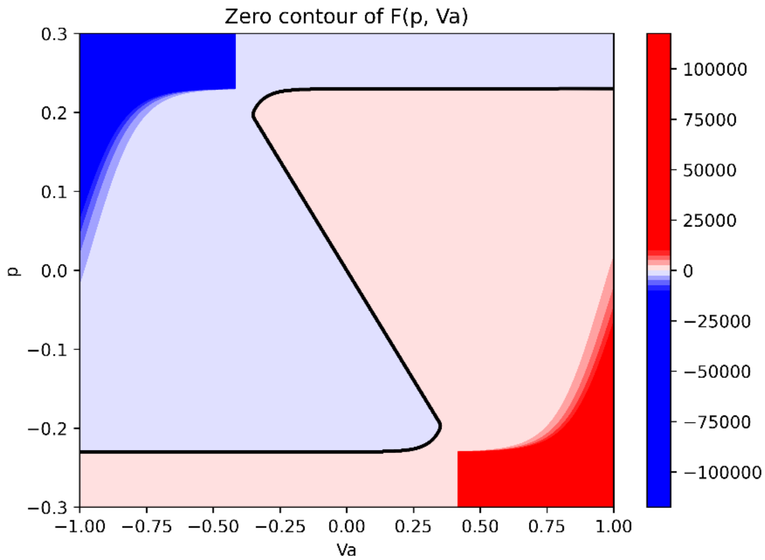



Fig. 10.7. Zero contour of $F$ for FE.

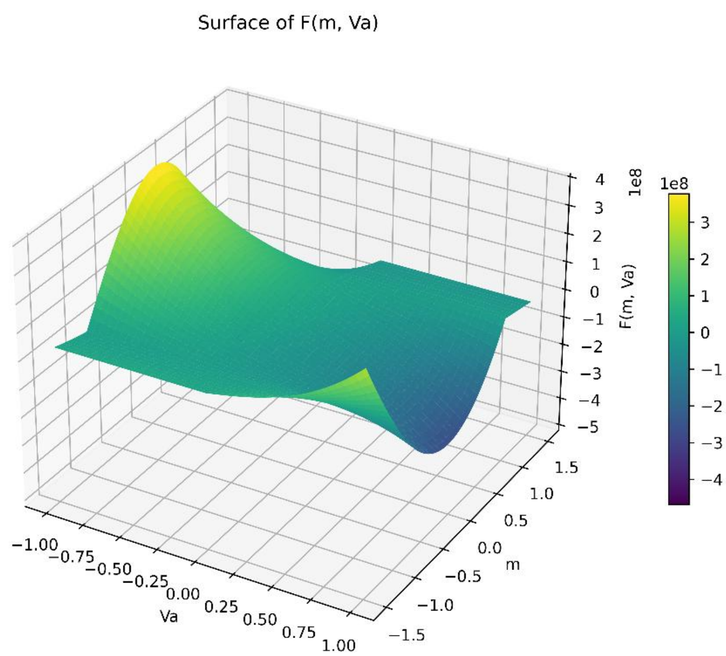



Fig. 10.8. Function surface of $F$ for MRAM.

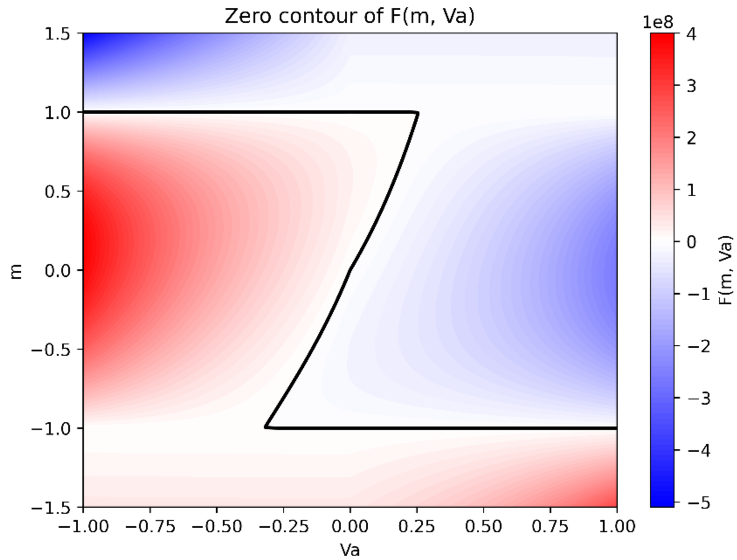



Fig. 10.9. Zero contour of $F$ for MRAM.

# 11. Parameter Extraction

## 11.1 RRAM

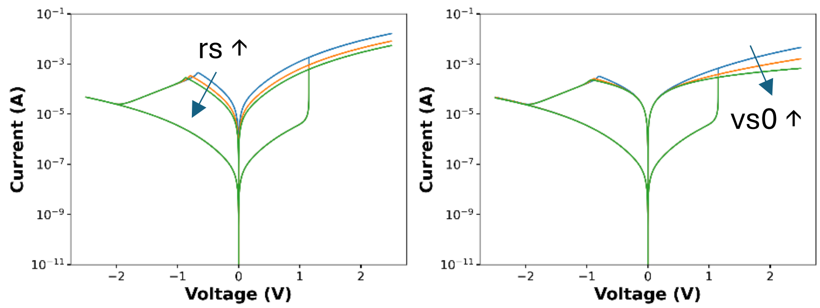
To use RRAM mode, please set devmod=0.

These are the parameters used in the demonstration.

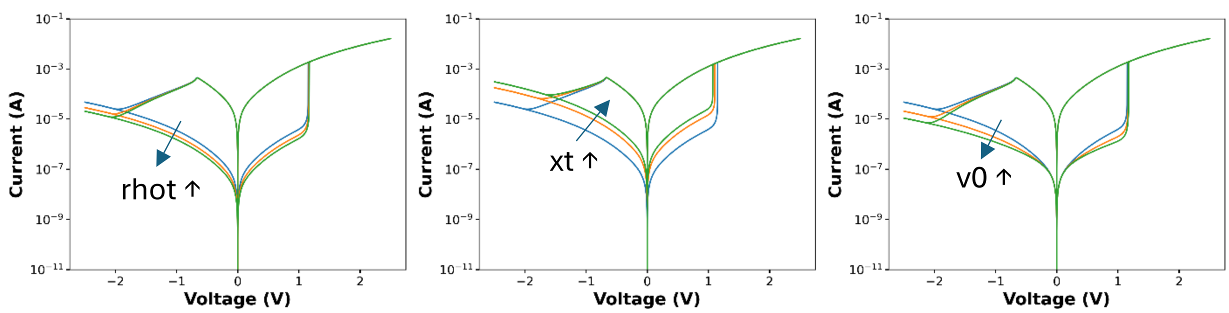
.model rram umem_va devmod=0 tm=1.7n area=25e-14 rth=5e5 cth=1e-15 x0=0.15n tau0=1e-15

+ ea0=1.24 ea0_r=1.24 am=0.8 am_r=0.5 a0=-0.1 b0=3 rhot=3e-3 v0=0.45 rs=4000 vs0=1 areanom=25e-14

First step is extracting the resistance parameters. This step can be applied to different device modes. To fit the on-state current, adjust rs for magnitude and vs0 for slope.



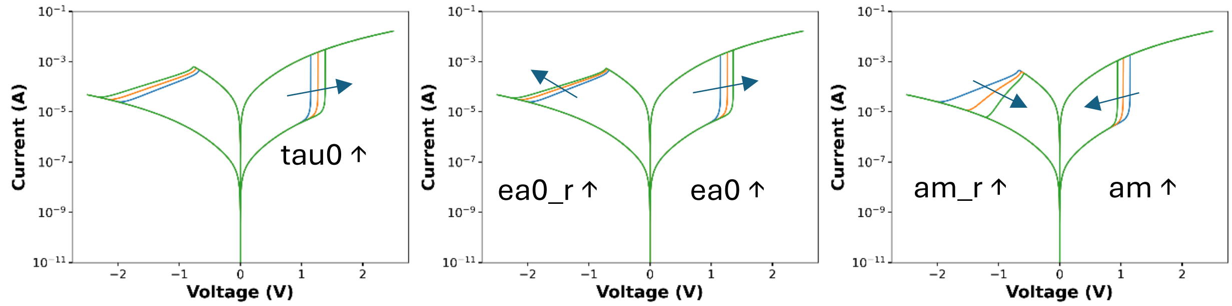

To fit the off-state current, adjust rhot and xt for magnitude and v0 for slope. xt controls the length dependence.



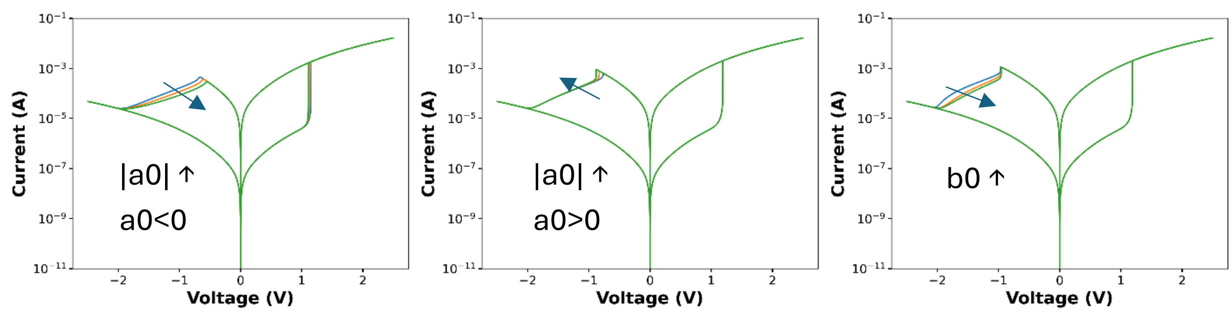

The next step is calibrating the SET and RESET switching.

Increasing tau0, ea0, ea0_r will increase the SET and RESET voltages.

Increasing am and am_r will decrease the SET and RESET voltages and increase the slope.




To change the slope of RESET transition, a0 and b0 can be adjusted. a0>0 and a0<0 will create different RESET behaviors. a0 and b0 will also affect the positive region which alter the state dependence of the SET voltage.



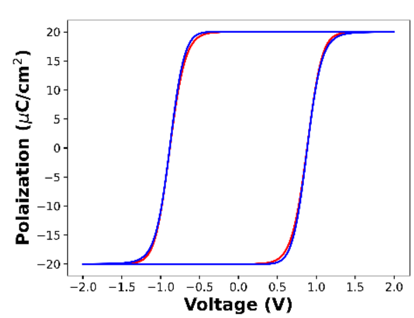

## 11.2 FE

To use FE mode, please set devmod=1.

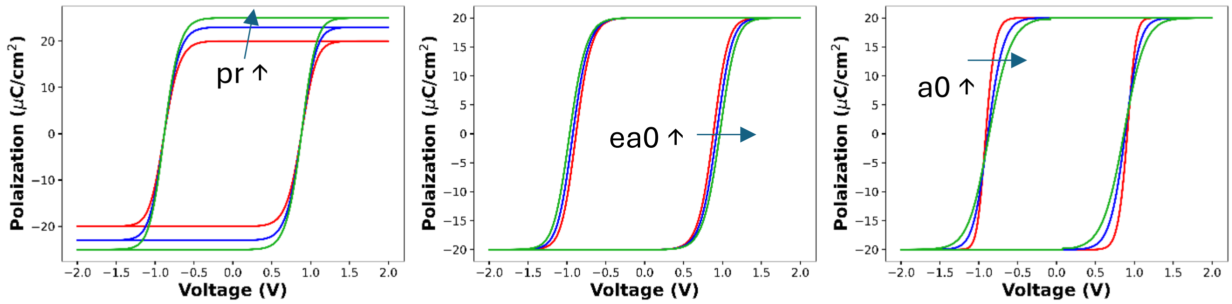
There are 2 distributions to choose from. pdfmod=0 is Gaussian (red). pdfmod=1 is log-normal (blue). For log-normal distribution, the transition region is asymmetric. There are more grains in the high $E_A$ side with longer distribution tail. The following set of parameters are used for demonstrations.

.model fe umem_va devmod=1 pdfmod=0 tm=8n pr=0.2 tau0=400n ea0=2 a0=0.2 b0=2.0 am=3.0

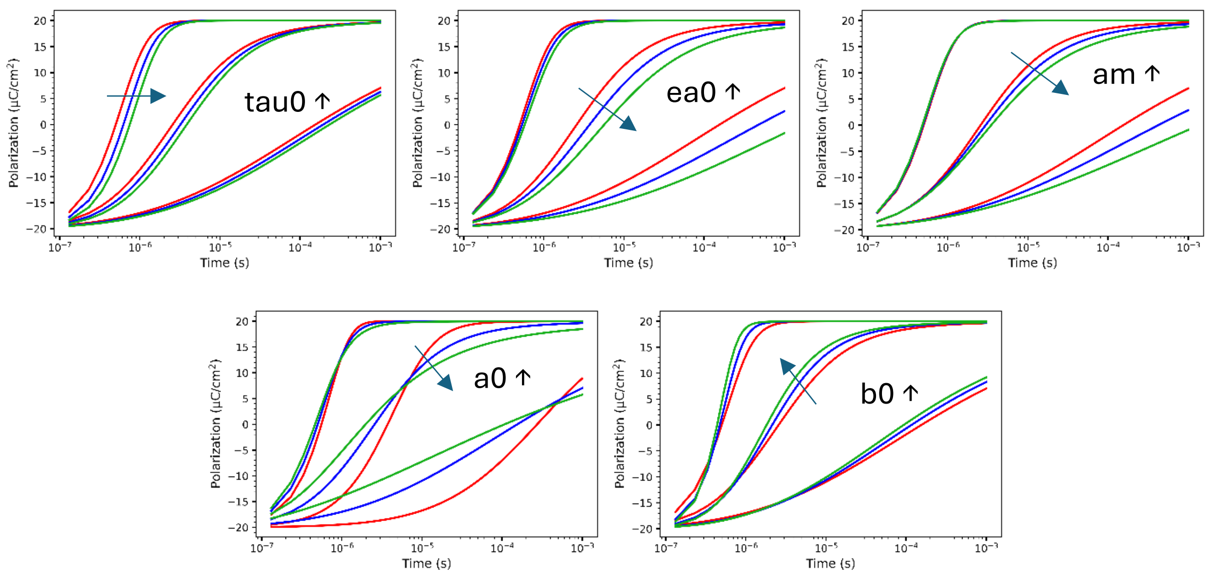
+md1=0.1 md2=5 epar=34 efb=1e7 rhot=1e-5 x1=0.2 v0=0.45



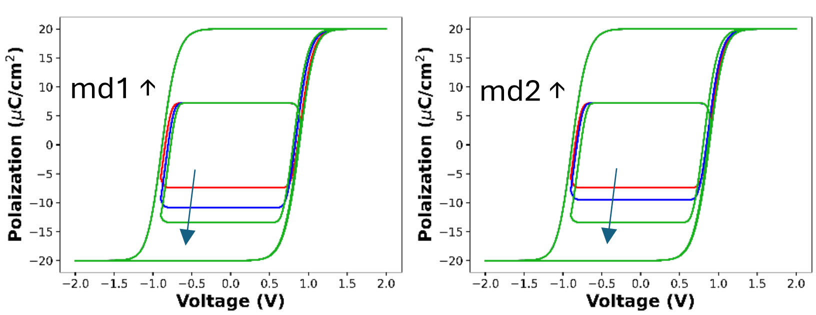
Major PV loop can be adjusted by 3 key parameters: pr, ea0, and a0. a0 controls the slope in the transition region.



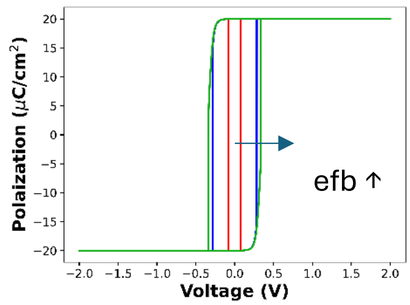

A more detailed extraction should be done by fitting P-t curves. tau0 shifts the delay. ea0 and am determine the voltage dependence of switching. ea0 shifts the entire voltage dependence while am tunes the space between each P-t curve. a0 can adjust the slope and final state the curve reaches. b0 can further increase the slope without changing other aspects of the curve.



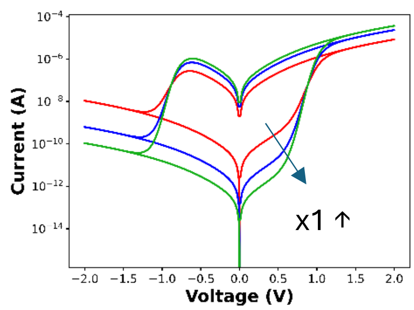

Minor loops can be tuned by md1 and md2.



In DC simulation, the hysteresis window is controlled by dedc. The maximum window size will be restricted by ea0.




For FTJ IV, the on-off current ratio is determined by x1. The other parameters can follow the RRAM case.



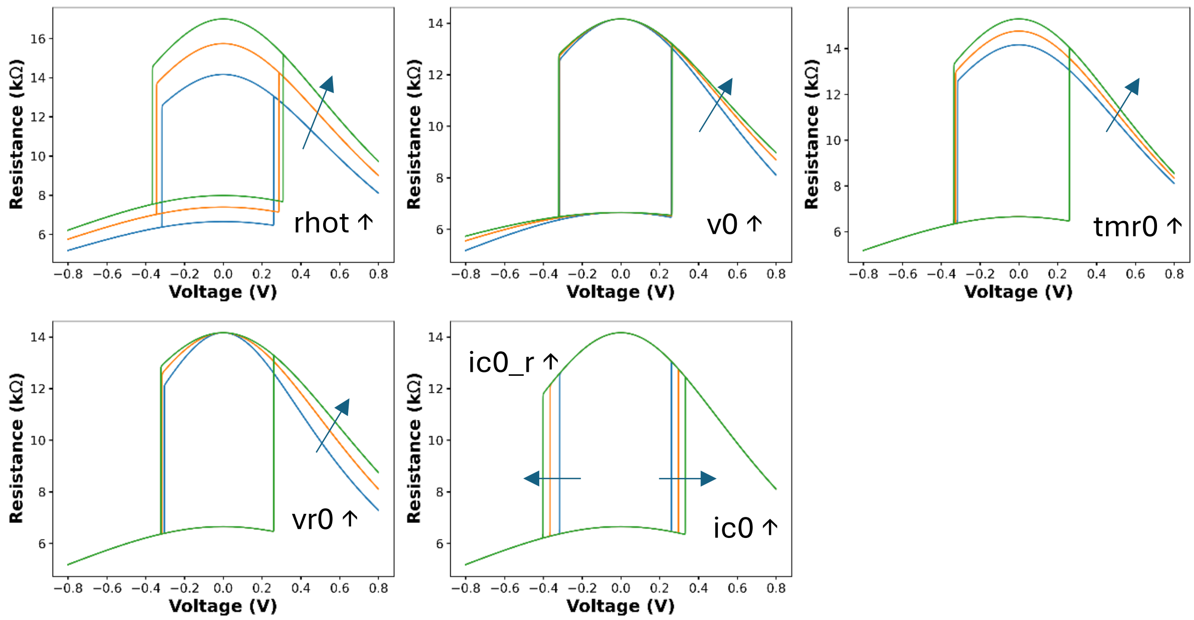

## 11.3 MRAM

To use MRAM mode, please set devmod=2.

This parameter set is used for demonstrations.

.model mram umem_va devmod=2 tm=1e-9 area=5e-15 areanom=5e-15 rhot=4.5e-3 ic0=4e-5 ic0_r=2.5e-5 v0=1.5 tmr0=1.13 vr0=0.8 tau0=2e-8 rs=0

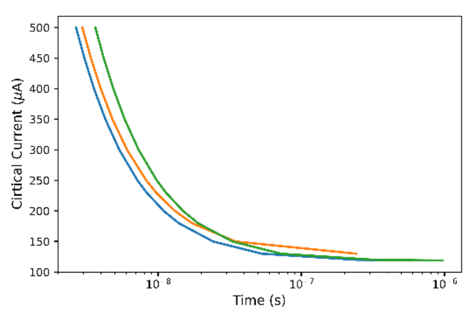
To fit the RV curves, rhot or xt will affect not just the overall resistance level but also the switching voltage since I=V/R. v0 will change both the slope or on and off resistance. tmr0 controls the on-off ratio. vr0 only adjusts the slope of off resistance. ic0 and ic0_r determine the critical currents and switching voltages.




The pulse dependent Ic is adjusted by ic0/ic0_r and tau0. A simple parameter set is used as an example.

.model mram umem_va devmod=2 tau0=2.2e-9 ic0=120e-6

Blue line: nominal. Orange line: increase ic0. Green line: increase tau0. ic0/ic0_r will shift the curve. tau0 tunes the slope.



# 12. Parameter List

| Name | Unit | Default | Min | Max | Description |
|---|---|---|---|---|---|
| tm | m | 5e-9 | 0 | - | Thickness/length of the device |
| area | $m^2$ | 1e-14 | 0 | - | Area |
| tf | m | 5e-9 | 0 | - | Free layer thickness of the device (MRAM only) |
| pr | $C/m^2$ | 0.02 | 0 | - | Remanent polarization at nominal temperature (FE only) |
| epar | - | 0 | 0 | - | Parasitic dielectric constant |
| dtemp | degC | 0.0 | - | - | Variability in device temperature |
| devmod | - | 0 | 0 | 2 | RRAM: 0; FE: 1; MRAM: 2 |
| pdfmod | - | 0 | 0 | 1 | Gaussian: 0; Log-normal: 1 |
| shmod | - | 0 | 0 | 1 | Self-heating Off: 0; On: 1 |
| voff | V | 0 | - | - | Offset voltage |
| tnom | degC | 27.0 | -273.15 | inf | Nominal temperature |
| tfnom | m | 5e-9 | 0 | - | Nominal free layer thickness of the device (MRAM only) |
| areanom | $m^2$ | 1e-14 | 0 | - | Nominal area |
| tau0 | s | 1e-10 | 0 | - | Switching time constant |
| ea0 | eV (RRAM) or MV/cm (FE) | 1.0 | 0 | - | Nominal activation energy (RRAM) or field (FE) |
| am | - | 1 | 0 | - | Field amplification factor (RRAM and FE only) |
| a0 | - | 0 | - | - | Fitting parameter for switching voltage dependence (RRAM and FE only) |

| | | | | | |
|---|---|---|---|---|---|
| b0 | - | 0 | 0 | - | Fitting parameter for switching slope (RRAM and FE only) |
| c0 | - | 1 | 0 | - | FE temperature switching coefficient |
| ttau0 | - | 0 | 0 | - | FE temperature dependence of tau0 |
| tpr | - | 0 | 0 | - | FE temperature coefficient of pr |
| eth | V/m | 5e7 | 0 | - | Threshold field for minor loop detection (FE only) |
| defd | V/m | 1e2 | 0 | - | Smoothing factor for minor loop detection (FE only) |
| md1 | - | 0 | 0 | - | Distribution scaling factor for minor loops (FE only) |
| md2 | - | 0 | 0 | - | Distribution scaling factor for minor loops (FE only) |
| dedc | V/m | 1e4 | 0 | - | Smoothing factor for DC curve (RRAM and FE only) |
| efb | V/m | 1e7 | 0 | - | Feedback field from polarization (FE only) |
| ic0 | A | 1e-5 | 0 | - | Critical current (MRAM only) |
| ea0_r | eV (RRAM) or MV/cm (FE) | ea0 | 0 | - | Nominal activation field (for RRAM and FE only) |
| am_r | - | am | 0 | - | Field amplification factor (for RRAM and FE only) |
| a0_r | - | a0 | - | - | Fitting parameter for switching voltage dependence (for RRAM and FE only) |
| b0_r | - | b0 | 0 | - | Fitting parameter for switching slope |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | (for RRAM and FE only) |
| md1_r | - | md1 | 0 | - | Distribution scaling factor for minor loops (FE only) |
| md2_r | - | md2 | 0 | - | Distribution scaling factor for minor loops (FE only) |
| ic0_r | - | ic0 | 0 | - | Critical current (MRAM only) |
| rhot | Wm | 1e-5 | 0 | - | Resistivity of tunneling resistance |
| x0 | m | 5e-10 | 0 | - | Thickness dependence factor of resistance |
| x1 | - | 0 | - | - | Polarization dependent barrier factor (FE only) |
| v0 | V | 0.5 | 0 | - | Voltage dependence factor of tunnel resistance |
| vs0 | V | 0.5 | 0 | - | Voltage dependence factor of series resistance |
| vr0 | V | 1 | 0 | - | Voltage dependence factor of tmr |
| tmr0 | - | 1 | 0 | - | Nominal tunnel magnetoresistance |
| rs | W | 0.1 | 0 | - | Series resistance |
| rhot_r | Wm | rhot | 0 | - | Resistivity of tunneling resistance |
| x0_r | m | x0 | 0 | - | Thickness dependence factor of resistance |
| v0_r | V | v0 | 0 | - | Voltage dependence factor of tunnel resistance |
| rs_r | W | rs | 0 | - | Series resistance |
| rth | K/W | 0 | 0 | - | Thermal resistance |
| cth | J/K | 0 | 0 | - | Thermal capacitance |

# 12. References

[1] C. -T. Tung, “A Unified Memristor Compact Model for RRAM, FERAM/FTJ, and MRAM,” *IEEE Trans. Electron Devices*, pp. 1–8, 2025, doi: 10.1109/TED.2025.3633652.

[2] Z. Jiang *et al.*, “A Compact Model for Metal–Oxide Resistive Random Access Memory With Experiment Verification,” *IEEE Trans. Electron Devices*, vol. 63, no. 5, pp. 1884–1892, 2016, doi: 10.1109/ted.2016.2545412.

[3] C. Alessandri, P. Pandey, A. Abusleme, and A. Seabaugh, “Monte Carlo Simulation of Switching Dynamics in Polycrystalline Ferroelectric Capacitors,” *IEEE Trans. Electron Devices*, vol. 66, no. 8, pp. 3527–3534, Aug. 2019, doi: 10.1109/ted.2019.2922268.

[4] R. Carboni *et al.*, “A Physics-Based Compact Model of Stochastic Switching in Spin-Transfer Torque Magnetic Memory,” *IEEE Trans. Electron Devices*, vol. 66, no. 10, pp. 4176–4182, 2019, doi: 10.1109/ted.2019.2933315.