Dynamic Economic Modelling
Problem Set 1

David Murakami

3rd May, 2024

Deadline: 27th of May 2024

Instructions: Carefully read and attempt all questions. Be sure that your answers directly address the questions. You are encouraged to use a TEX language processor, or some other type setting program to write your answers. If submitting hand-written answers, then try to write neatly – grades will not be awarded for writing that is ineligible. It is not necessary to submit all mathematical/algebraic working out, but provide enough so that the marker can follow your intuition.

For problems that require numerical computation, then you are welcome to use any software that you like – even Microsoft Excel (but you are encouraged to use julia, MATLAB, Python, R, and so on). There is no requirement on how to present statistics, tables, or plots – so long as they look nice and are easy to understand. Cite your data sources and please also provide a copy of your code, ensuring that it is coherent.

You are also encouraged to work together and collaborate. You may work in **groups of up to four people**.

Submission: Please email a PDF document of your answers to me and Domenico. Please copy all members of your group in the email, and ensure that all your names appear in the documents and code. Your code can be submitted as either a PDF copy of the code, or in a replication kit so that with "one-click" all your tables and figures can be replicated.

1 Consumption with different generations

An economy is composed of identical individuals. Each individual lives for 2 periods (you may imagine them as adulthood and old age). Individuals may work during the first period of their life for a proportion L of the day, for an income equal to wL. In the second period they retire and consume their remaining lifetime savings. Their lifetime utility is given by:

$$U = \ln C_1 + \ln C_2 + \ln(1 - L),$$

where C_i is consumption in period i and L is the fraction of time the individual spends working in period 1.

(a) If the rate of interest on savings is R, write down the individual's budget constraints for both periods, and then combine them in a lifetime (intertemporal) budget constraint.

(b) Solve for optimal consumption each period and the optimal work effort. Comment on what you find.

(c) The government introduces a fixed pension paid to individuals in the second period of their lives, funded by a "lump sum tax" paid by those who work.[1] Re-write the intertemporal budget constraint. What will be the impact of the pension on consumption and labour supply decisions of the young? What about the old when the pension is introduced? Give intuition for your answers.

(d) The pension is now funded by an income tax, i.e., a tax equal to τwL where τ is the tax rate. Will the behaviour of the young change in the case where $R = 0$? Interpret this result.

1. A lump sum tax is one that is independent of income.

2 Stylised facts of the business cycle

A business cycle is made of an expansion (boom) and a contraction (recession). During the expansion all good things (GDP, employment, productivity, and so on) tend to go up, or grow faster than "normal", and bad things (e.g. unemployment) tend to fall. During the contraction good things go down and bad things go up.

Table 1 gives a more complete description of the volatilities and cross correlations of consumption and labour market variables. These results are from the US because most of the theoretical models we shall examine have been constructed with this data in mind.

Variable	SD%	Cross-correlation of output with:								
		$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
GNP	1.72	0.16	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.16
CND	0.86	0.40	0.55	0.68	0.78	0.77	0.64	0.47	0.27	0.06
CD	4.96	0.37	0.49	0.65	0.75	0.78	0.61	0.38	0.11	-0.13
H	1.59	0.09	0.30	0.53	0.74	0.86	0.82	0.69	0.52	0.32
$AveH$	0.63	0.16	0.34	0.48	0.63	0.62	0.52	0.37	0.23	0.09
L	1.14	0.04	0.23	0.46	0.69	0.85	0.86	0.76	0.59	0.40
GNP/L	0.90	0.14	0.20	0.30	0.33	0.41	0.19	0.00	-0.18	-0.25
$AveW$	0.55	0.25	0.21	0.14	0.09	0.03	-0.07	-0.09	-0.09	-0.09

Note: SD% denotes standard deviations, $t-j$ denotes the correlation between GNP at time t and the variable denoted by the first column at time $t-j$. CND stands for non-durable consumption, CD for durable consumption, H for total hours worked, $AveH$ is average hours worked per employee, L is employment, GNP/L is productivity, $AveW$ is average hourly wage based on national accounts. All unemployment data is based on household surveys. Source: "Frontiers of Business Cycle Research" (Cooley and Prescott, 1995).

Table 1: Cyclical Behaviour of the US Economy (1954Q1-1991Q2)

Consider also the study by King and Rebelo (1999), who also verified the findings of Cooley and Prescott:

Variable	SD	Relative SD	ρ	Corr$(\cdot, Y)$
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Note: All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. SD is standard deviation, ρ denotes a variable's first-order autocorrelation, and Corr$(\cdot, Y)$ is a variable's contemporaneous correlation with output. Data sources are described in Stock and Watson (1999), who also created the real rate series. Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity. Source: "Resuscitating Real Business Cycles" (King and Rebelo, 1999).

Table 2: Business Cycle Statistics for the US Economy

(a) Replicate Table 1 and 2 for the US economy from – ideally – 1950 Q1 to the newest data you can find (either 2020 or 2023). You can use real GDP instead of GNP if you want.

(b) Verify whether or not the following business cycle facts from Cooley and Prescott (1995) still hold today:

- Consumption is smoother than output.
- Volatility in GNP is similar in magnitude to volatility in total hours.
- Volatility in employment is greater than volatility in average hours. Therefore most labour market adjustments operate on the extensive rather than intensive margin.
- Productivity is slightly pro-cyclical.
- Wages are less variable than productivity.
- There is no correlation between wages and output (nor with employment for that matter).

(c) Verify whether or not the following business cycle facts from King and Rebelo (1999) still hold today:

- Consumption of non-durables is less volatile than output.
- Consumer durables are more volatile than output.
- Investment is three times more volatile than output.
- Government expenditures are less volatile than output.

- Total hours worked are about the same volatility as output.
- Capital is much less volatile than output.
- Employment is as volatile as output, while hours per worker are much less volatile than output.
- Labour productivity is less volatile than output
- The real wage is much less volatile than output.

References

Cooley, Thomas F., and Edward C. Prescott. 1995. *Frontiers of Business Cycle Research.* Princeton University Press.

King, Robert G., and Sergio T. Rebelo. 1999. "Resuscitating Real Business Cycles." *Handbook of Macroeconomics* 1 (B): 927–1007.

Stock, James H., and Mark W. Watson. 1999. "Business Cycle Fluctuations in US Macroeconomic Time Series." *Handbook of Macroeconomics* 1 (A): 3–64.