Complex Integration: Cauchy's Theorem - Notes

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1 Introduction

- $\int_a^b$ in $\mathbb{R}$ has only way to from a to b but in $\mathbb{C}$, the two points are in a plane. In general, the integral will depend on the path (countour) taken.
- Unlike differentiation which made sense for strictly limited set of analytic functions, integration of non-analytic function is possible.
- We will see under what conditions the value of integral is independent of contour.
- Cauchy's Theorem essentially says that any two integrals from a to b will agree, provided that the mapping is *analytic everywhere in the region lying between the two countours.*

2 The Real Integral

- *Riemann sum* $R \equiv \sum_{i=1}^{n} f(x_i)\Delta_i$. Desired area can be obtained by simultaneously letting n tend to ∞ while each Δ_i shrinks to nothing. There are many ordinary functions whose antiderivative does not exist. That's when we will require Riemann sum.
- Two familiar ways to compute R are trapezoidal rule and Simpson's rule.
- The trapezoid rule can be accurately approximated by Riemann sum using a modest value of n and choosing x_i to be the midpoint of its Δ_i. This is called *Midpoint Riemann Sum* R_M.

3 The Complex Integral

3.1 Complex Riemann Sums