6-4

$E(X_i)=\mu$

$V(X_i)=\sigma^2=E(X_i^2)-\mu^2$

$E(\bar{x})=\mu$

$V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x}^2)-\mu^2$

$$E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{x})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{x}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$$

$$=\frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{x}^2)}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{x}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$$

$$=\sigma^2$$

$\hat{\theta}_2=\frac{\sum_{i=1}^{n}(X_i-\bar{x})^2}{(n-1)}$ 為母体変異数 σ^2 之不偏估計量，

而 $\hat{\theta}_1=\frac{\sum_{i=1}^{n}(X_i-\bar{x})^2}{n}$ 為母体変異数 σ^2 之偏誤估計量。