Week5

9. (1) $S = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$

$= \sqrt{\frac{1284 - 6\times 14.33^2}{5}} = \sqrt{10.38} = 3.22$ ✱

(2) $1-\alpha = 0.9$, $\frac{\alpha}{2} = 0.05$, $n-1 = 5$

$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(5) = 11.07$

$\chi^2_{1-\alpha}(n-1) = \chi^2_{0.95}(5) = 1.15$

$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}}\right) = \left(\sqrt{\frac{5\times 10.38}{11.07}}, \sqrt{\frac{5\times 10.38}{1.15}}\right)$

$= (2.17, 6.12)$ ✱

20. (1) $V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1}{n_1}\right)^2}{(n_1-1)} + \frac{\left(\frac{S_2}{n_2}\right)^2}{(n_2-1)}}$

$n_1 = 9$, $\bar{x} = 7.67$, $S_1 = 9.27$

$n_2 = 9$, $\bar{y} = 6.78$, $S_2 = 21.15$

$V = \frac{\left(\frac{9.27^2}{9} + \frac{21.15^2}{9}\right)}{\frac{\left(\frac{9.27}{9}\right)^2}{8} + \frac{\left(\frac{21.15}{9}\right)^2}{8}} = 10.96 \fallingdotseq 11$

$(\bar{x}-\bar{y}) \pm t_{\frac{\alpha}{2}}(V)\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$= (7.67-6.78) \pm t_{0.025}(11)\sqrt{\frac{9.27^2}{9} + \frac{21.15^2}{9}}$

$= 0.89 \pm 2.201 \times 7.7$

$= 0.89 \pm 16.95$

$\Rightarrow (-16.06, 17.84)$ ✱

(2) $\left(\sqrt{\frac{8\times 9.27^2}{\chi^2_{0.05}(8)}}, \sqrt{\frac{8\times 9.27^2}{\chi^2_{0.95}(8)}}\right)$

$= \left(\sqrt{\frac{687.46}{15.51}}, \sqrt{\frac{687.46}{2.73}}\right)$

$= (6.66, 15.87)$ ✱

(3) $\left(\frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{S_1^2}{S_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)}\right)$

$= \left(\frac{9.27^2}{21.15^2} \times \frac{1}{3.44}, \frac{9.27^2}{21.15^2} \times \frac{1}{0.29}\right)$

$= (0.06, 0.66)$ ✱