

# Sheet 2

## Motion Models and Robot Odometry

Group 4

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### Exercise 1

- c) Group Picture
- d) Graph Visualization
- e) Specify the odometry model

### Exercise 2

- d) Kalman Filter Covariance Ellipse Screenshot

Figure 1 is the screenshot of the covariance ellipse from the original Q matrix visualized by rviz.<sup>1</sup>

Figure 2 shows the screenshot of the estimated two-dimensional trajectory from the given bag file.

- e) Kalman Filter with Higher Noise Screenshots

Figure 3 shows the screenshot of the covariance ellipse from the modified Q matrix, which drifts two times more in the global x-direction.

Figure 4 shows the screenshot of the estimated two-dimensional trajectory from the bag file with modified Q matrix.<sup>2</sup>

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<sup>1</sup>In order to easily compare the difference between the effect of the two different Q matrices, we took both screenshots at the last moment of the given bagfile.

<sup>2</sup>There is no difference of the trajectories between the two different Q matrix, since until now, the Q matrix just adjusts the covariance ellipse but not the state vector, hence the trajectory.

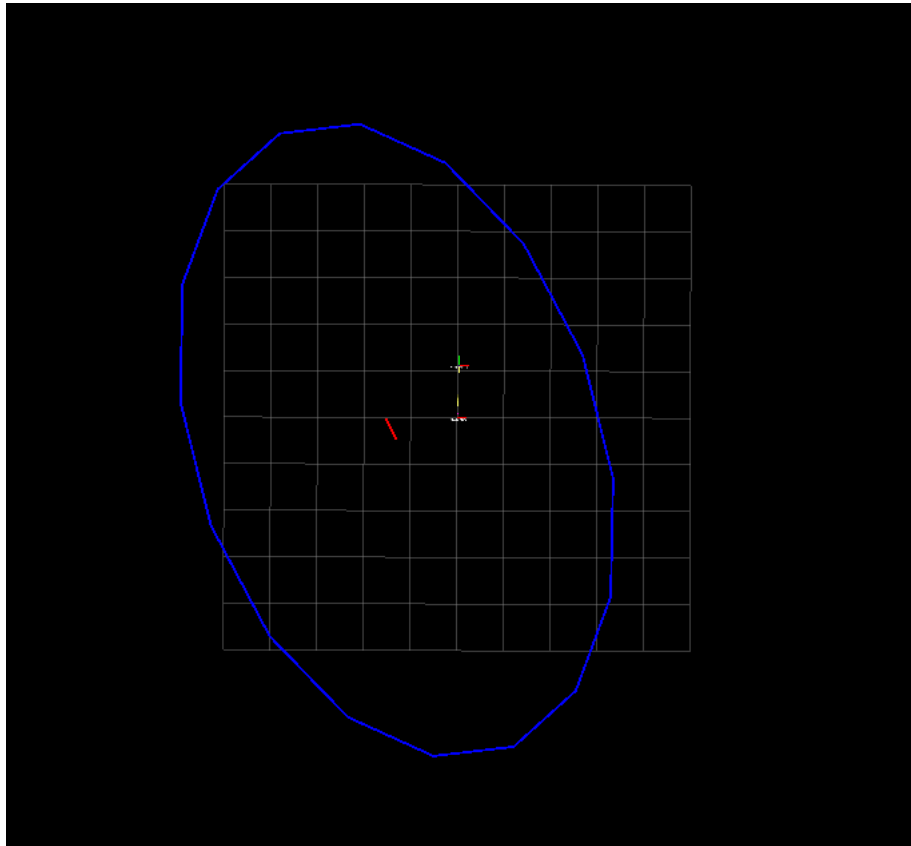


Figure 1: Covariance ellipse with the origin Q matrix

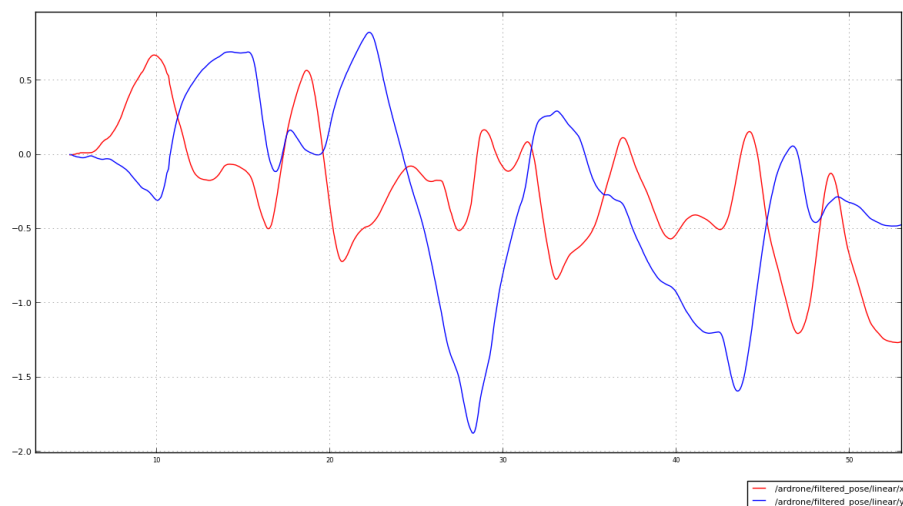


Figure 2: Estimated two-dimensional trajectory from the given bag file

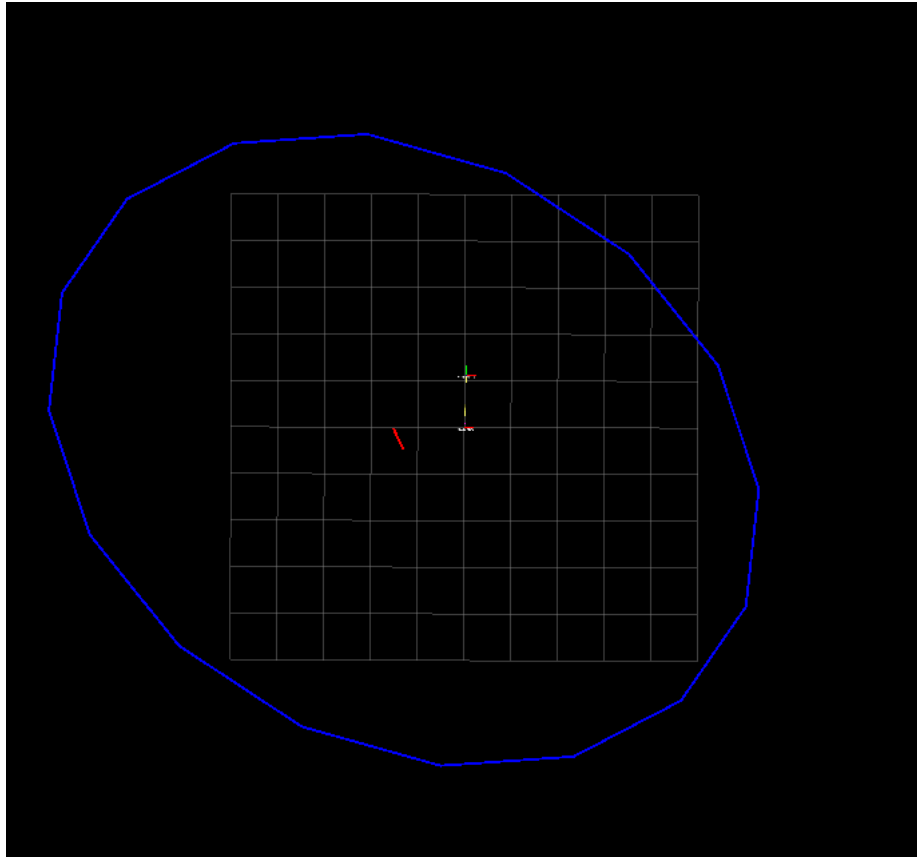


Figure 3: Covariance ellipse with the modified Q matrix

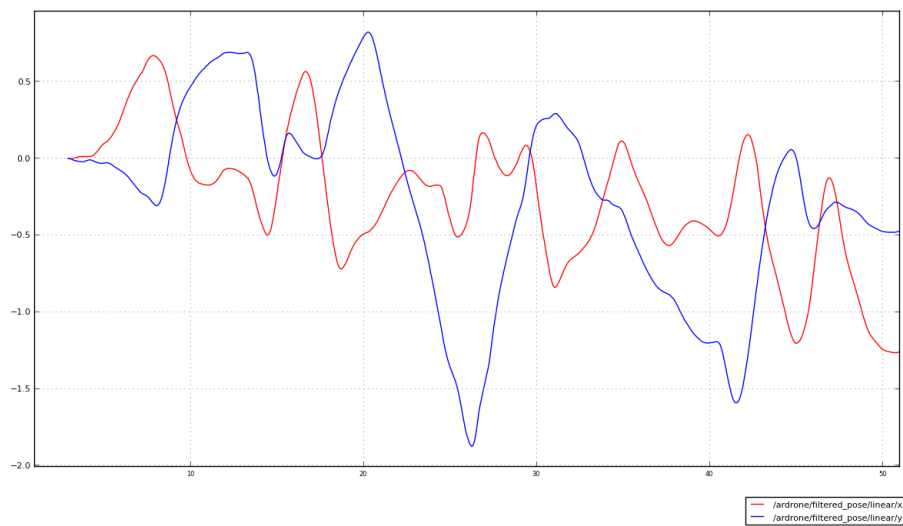


Figure 4: Estimated two-dimensional trajectory from the bag file with modified Q matrix.

## f) Noise Prediction for Experimental Setup

## g) Observation Function and Its Jacobian

### Observation Function

Observed marker pose is calculated with function  $h(x)$  (eq. 1). This  $h(x)$  observation function predicts the marker pose  $z_{pre}$  given  $x$ , estimated robot world state (eq 2), and  $z_g$ , the marker pose in global frame (eq. 3).

1.  $z_{pre} = h(x)$   
 $z_{pre} = (x_{pre} \quad y_{pre} \quad \psi_{pre})^T$
2.  $x = (x_w \quad y_w \quad \psi_w)^T$
3.  $z_g = (x_g \quad y_g \quad \psi_g)^T$

In order to find the observation, we need to transform the global marker pose to local frame.

If  $X$  is homogeneous transformation matrix of  $x$ , robot pose,

$$X = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \psi_w & -\sin \psi_w & x_w \\ \sin \psi_w & \cos \psi_w & y_w \\ 0 & 0 & 1 \end{pmatrix}$$

then we can transform any local frame to global frame as follows;

$$\vec{t}_g = X t_p \vec{re}$$

We want to transform from global to local. In order to do that we should take inverse of  $X$  transformation matrix;

$$X^{-1} = \begin{pmatrix} R^{-1} & -R^{-1}t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \psi_w & \sin \psi_w & -x_w \cos \psi_w - y_w \sin \psi_w \\ -\sin \psi_w & \cos \psi_w & x_w \sin \psi_w - y_w \cos \psi_w \\ 0 & 0 & 1 \end{pmatrix}$$

Now we can compute the local marker position from global marker position;

$$\vec{t}_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix}$$

$$t_{pre}^{\sim} = X^{-1} \vec{t}_g = \begin{pmatrix} (x_g - x_w) \cos \psi_w + (y_g - y_w) \sin \psi_w \\ -(x_g - x_w) \sin \psi_w + (y_g - y_w) \cos \psi_w \\ 1 \end{pmatrix}$$

Since yaw angle is always in the global frame, observed yaw angle is

$$\psi_{pre} = (\psi_w - \psi_g)$$

At the end we get following observation function

$$h(x) = \begin{pmatrix} (x_g - x_w) \cos \psi_w + (y_g - y_w) \sin \psi_w \\ -(x_q - x_w) \sin \psi_w + (y_g - y_w) \cos \psi_w \\ (\psi_w - \psi_g) \end{pmatrix}$$

### Jacobian of Observation Function

Now we can compute the jacobian of observation function as following;

$$H = \frac{\partial h(x)}{\partial x} = \begin{pmatrix} \frac{\partial h(x)}{\partial x_w} & \frac{\partial h(x)}{\partial \psi_w} & \frac{\partial h(x)}{\partial \psi_g} \end{pmatrix}$$

$$= \begin{pmatrix} -\cos \psi_w & -\sin \psi_w & -(x_q - x_w) \sin \psi_w + (y_g - y_w) \cos \psi_w \\ \sin \psi_w & -\cos \psi_w & -(x_q - x_w) \cos \psi_w - (y_g - y_w) \sin \psi_w \\ 0 & 0 & -1 \end{pmatrix}$$

### i) Trajectory

Figure 5 shows the screenshot of the from EKF corrected two-dimensional trajectory from the given bag file.

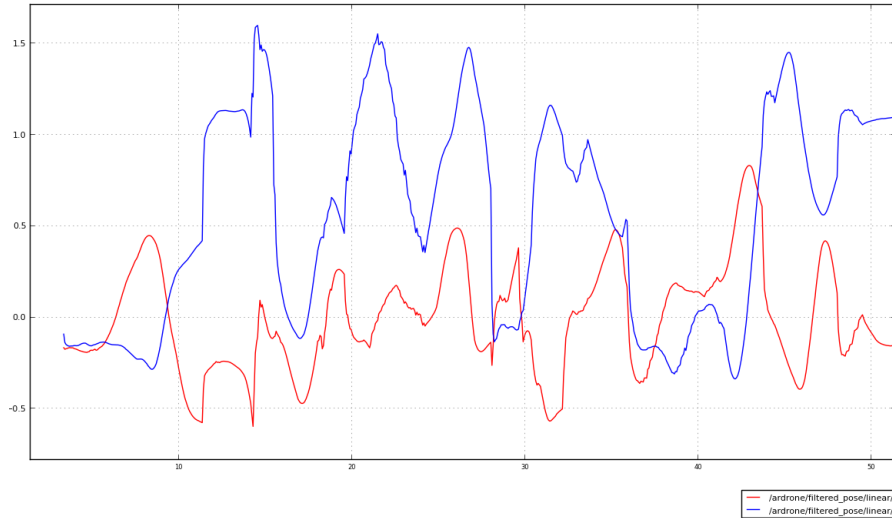


Figure 5: From the EKF corrected two-dimensional trajectory from the given bag file

**j) Drift on Pose Estimation**