IS 609 Assignment 12

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Allowing back ordering is a good strategy if the storage costs are known to exceed the costs of stock-outs. If we calculate storage costs by integrating the area under the curve for one cycle, it is easy to see that shifting the curve downward - the reverse of allowing a buffer stock - will storage costs by decreasing the area. We should assume that costs are assigned to stock-outs, or unsatsified demand, as that is likely to decrease customer loyalty and risks losing business in the short and long-term to competitors.

Model 13.7 assumes the entire inventory is used up in each cycle and that demand is met immediately when items are in stock, but not when they are out-of-stock. The model makes the assumption that demand will be constant over multiple cycles and will not be negatively affected by the stock-outs. We can determine the percentage of time an item is unfilled by taking a cycle t and comparing the interval that $Q \leq 0$ to the interval that $Q > 0$.

We can determine the loss-of-good-will cost, w, of a stock-out by adapting the storage-cost formula. To find the cost of a stock out, we multiply the w by the average daily invetory and the number of days of the stock-out. Where t^* is the day the stock hits zero:

$$loss\ of\ good\ will\ cost = \frac{wq(t - t^*)}{2}$$

$$cost\ per\ cycle = d + \frac{sqt^*}{2} + \frac{wq(t - t^*)}{2}$$

To find the average daily cost, we divide by t. Since the time the stock hits zero is unknown, we will replace the resulting proportions, $\frac{t^*}{t}$ with a and $\frac{t-t^*}{t}$ with $(1 - a)$.

$$c = \frac{d}{t} + at\frac{sq}{2} + (1 - a)t\frac{wq}{2}$$

Sub $q = rt$:

$$c = \frac{d}{t} + a\frac{srt}{2} + (1 - a)\frac{wrt}{2}$$

Differentiate c with respect to t:

$$c' = \frac{-d}{t^2} + a\frac{sr}{2} + (1 - a)\frac{wr}{2} = 0$$

Our critical point, is:

$$T^* = \sqrt{\frac{2d}{asr + (1 - a)wr}}$$

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Since the optimal order quantity is $Q^* = rT^*$,

$$Q^* = r\sqrt{\frac{2d}{asr + (1-a)wr}}$$

As we can see, the optimal quantity depends in part on the proportion of time that the item is in-stock, a versus out-of-stock $a-1$. Before running a system that allows stock-outs, we should know this proportion, as well as the the ratio of storage costs to loss-of-good-costs, $s:w$. Both are needed to determine the feasibility of the operation.

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Fine the local minimum value of the function:

$$f(x,y) = 3x^2 + 6xy + 7y^2 - 2x + 4y$$

$$\frac{df}{dx} = 6x + 6y - 2 = 0$$
$$\frac{df}{dy} = 6x + 14y + 4 = 0$$

Solve for x and y:

$$6x + 6y - 2 = 6x + 14y + 4$$
$$6y - 2 = 14y + 4$$
$$-8y = 6$$
$$y = -\frac{3}{4}$$
$$x = \frac{26}{24}$$

We can confirm that this is a minimum by checking that the second derivatives are positive:

$$\frac{df^2}{dx^2} = 6$$
$$\frac{df^2}{dy^2} = 14$$

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Using the method of Lagrange multipliers, find the hottest point (x, y, z) along the elliptical orbit:

$$4x^2 + y^2 + 4z^2 = 16$$

Where the temperature function is:

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

$$L(x, y, z, \lambda) = 8x^2 + 4yz - 16z + 600 - \lambda(4x^2 + y^2 + 4z^2 - 16)$$

$$\frac{dl}{dx} = 16x - 8x\lambda = 0$$

$$\lambda = 2$$

$$\frac{dl}{dz} = 4y - 16 - 4\lambda = 0$$

$$y = 6$$

$$\frac{dl}{dy} = 4z - 2y\lambda = 0$$

$$4z = 2y\lambda$$

$$z = 6$$

$$\frac{dl}{d\lambda} = 4x^2 + y^2 + 4z - 16 = 0$$

$$4x^2 + 36 + 24 - 16 = 0$$

$$4x^2 = -44$$

$$x = \sqrt{-11}$$

The hottest point is $f(\sqrt{-11}, 6, 6)$. We can check this is a maximum (hottest temperature) by making sure one of the second derivatives is negative.

$$\frac{dl^2}{dy^2} = -2\lambda = -12$$