IS 604 Assignment 6

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6.1

A tool crib has exponential interarrival and service times and servers a very large group of mechanics. The mean time between arrivals is 4 minutes. It takes 3 minutes on the average for a tool-crib attendant to service a mechanic. The attendant is paid $10 per hour and the mechanic is paid $15 per hour. Would it be advisable to have a second tool-crib attendant?

To answer this question, we must compare the costs of each scenario. Hiring the attendants costs $10/hour/server. The costs of the mechanics waiting in line is less straight-forward. With an arrival rate of $\hat{\lambda}$ mechanics per hour, the average cost per hour is: $15 \cdot \hat{\lambda} \hat{w_Q}$.

$$\lambda = 15$$
$$\mu = 20$$
$$\rho = \frac{\lambda}{\mu} = \frac{15}{20}$$

For an $M/M/1$ queue, $\hat{w_Q} = \frac{\rho}{\mu(1-\rho)}$.

$$\hat{w_Q} = \frac{0.75}{20 \cdot 0.25} = 0.15$$

The cost per hour of having one server is:

$$Mechanic\,Cost = 15 \cdot 15 \cdot 0.15 = 33.75$$
$$Attendant\,Cost = 10$$
$$Total\,Cost = 43.75$$

For an $M/M/c$ queue, $\hat{w_Q} = w - \frac{1}{\mu}$, where:

$$w = \frac{L}{\lambda}$$
$$L = c\rho + \frac{\rho P(L(\infty) \geq c)}{1-\rho}$$
$$P(L(\infty) \geq c) = \frac{(cp)^c P_0}{c!(1-\rho)}$$
$$P_0 = \{\Big[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\Big] + \Big[(cp)^c(\frac{1}{c!})\frac{1}{1-\rho}\Big]\}^{-1}$$

For $c = 2$, we can work backwards to get $\hat{w_Q}$:

$$P_0 = \Big[0.75 + (2 \cdot 0.75)^2(\frac{1}{2!})(\frac{1}{1-0.75})\Big]^{-1} = \frac{4}{21}$$

$$P(L(\infty) \geq c) = \frac{(2 \cdot 0.75)^2 \cdot \frac{4}{21}}{2!(1-0.75)} = 0.4285714$$

$$L = 2 \cdot 0.75 + \frac{0.75 \cdot 0.4285714}{1-0.75} = 2.785714$$

$$w = \frac{2.785714}{15} = 0.1857143$$

$$\hat{w_Q} = 0.1857143 - \frac{1}{20} = 0.1357143$$

The cost per hour of having two servers is:

$$Mechanic\,Cost = 15 \cdot 15 \cdot 0.1357143 = 30.54$$

$$Attendant\,Cost = 20$$

$$Total\,Cost = 50.54$$

It appears that the cost of having two servers exceeds the cost of one. It would not be advisable to hire a second tool-crib attendant.

6.2

A two-runway (one runway for landing, one runway for taking off) airport is being designed for propeller-driven aircraft. The time to land an airplane is known to be exponentially distributed, with a mean of 1.5 minutes. If airplane arrivals are assumed to occur at random, what arrival rate can be tolerated if the average wait in the sky is not to exceed 3 minutes?

For this problem, we are only modeling the landing process, so with one runway, we will use the steady-state parameters of the $M/M/1$ queue. The average wait for this queue is:

$$\hat{w_Q} = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$\mu = \frac{1}{1.5} = \frac{2}{3}$$

If the average wait is not to exceed three minutes, then we find the maximum λ at $\hat{w_Q} = 3$.

$$\frac{\lambda}{\frac{2}{3}(\frac{2}{3}-\lambda)} = 3$$

$$\lambda = \frac{4}{9}$$

The maximum arrival rate is $lambda = \frac{4}{9}$, or one arrival every 2.25 minutes.