IS 604 Assignment 7

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November 23, 2015

9.14

Use the Kolmogorov-Smirnov test to discover whether the distribution of location of accidents is uniformly distributed for the month of September.

To test for uniformity, our hypotheses are:

$$H_0 : R_i \sim Uniform[0, 1]$$
$$H_A : R_i \nsim Uniform[0, 1]$$

```
accidents <- c(88.3,91.7,98.8,32.4,20.6,76.6,40.7,67.3,90.1,87.8,
               73.1,73.2,36.3,7.0,17.2,69.8,21.6,27.3,27.3,45.2,
               23.7,62.6,6.0,87.6,36.8,23.3,97.4,99.7,45.3,87.2)
accidents <- accidents/100 # scale to [0,1]
Ri <- sort(accidents) # sort from smallest to largest
idivn <- (1:30)/30
dPlusVec <- idivn - Ri
dMinusVec <-  Ri - ((1:30) - 1)/30
dPlus <- max(dMinusVec)
dMinus <- max(dPlusVec)
d <- max(dMinus,dPlus)
d
```

```
## [1] 0.172
```

We will use table A.8 in the Discrete-Event System Simulation to test the critical value of $D = 0.172$ for the sample size $N = 30$.

For a significance level of $\alpha = 0.10$ at $N = 30$, $D_{0.10} = 0.22$. Since our computed value of $D = 0.172$ is less than the critical value of $D_{0.10} = 0.22$, we do not reject the null hypothesis that the accidents are uniformly distributed.

9.17

The time required for 50 different employees to compute and record the number of hours worked during the week was measured, with the following results in minutes:

```
employee <- 1:50
minutes <- c(1.88,0.54,1.90,0.15,0.02,2.81,1.50,0.53,2.62,2.67,
             3.53,0.53,1.80,0.79,0.21,0.80,0.26,0.63,0.36,2.03,
             1.42,1.28,0.82,2.16,0.05,0.04,1.49,0.66,2.03,1.00,
             0.39,0.34,0.01,0.10,1.10,0.24,0.26,0.45,0.17,4.29,
             0.80,5.50,4.91,0.35,0.36,0.90,1.03,1.73,0.38,0.48)
```

Use the chi-square test to test the hypothesis that these service times are exponentially distributed. Let the number of class intervals be $k = 6$. Use the level of significance $\alpha = 0.05$.

$$H_0 = exponentially\ distributed$$
$$H_A = not\ exponentially\ distributed$$

Each interval will have equal probability $p = 0.1666667$. To find the enpoints of these intervals we must solve for the expression below where a_i is the endpoint of the i th interval.

$$a_i = -\frac{1}{\lambda} ln(1 - ip), i = 0, 1, ...6$$

```r
suppressWarnings(suppressMessages(library(knitr)))
lambda <- 1/mean(minutes)
p <- 1/6
endpts <- c()
for (i in 0:6){
  endpts[i+1] <- log(1-(i*p))/-lambda
}
bins <- cut(minutes, breaks=endpts, labels=c("bin 1","bin 2","bin 3","bin 4"," bin 5"," bin 6"))
Oi <- summary(bins)
Ei <- rep(p*length(minutes),6)
fit <- ((Oi-Ei)^2)/Ei
intervals <- c("[0,0.220)","[0.220,0.489)","[0.489,0.836)",
               "[0.836,1.325)","[1.325,2.161)","[2.161,inf)")
df <- data.frame("Class Intervals"=intervals,
                 "Observed Freq., Oi"=Oi,
                 "Expected Freq., Ei"=Ei,
                 "((Oi-Ei)^2)/Ei"=fit)
kable(df)
```

	Class.Intervals	Observed.Freq...Oi	Expected.Freq...Ei	X..Oi.Ei..2..Ei
bin 1	[0,0.220)	8	8.333333	0.0133333
bin 2	[0.220,0.489)	11	8.333333	0.8533333
bin 3	[0.489,0.836)	9	8.333333	0.0533333
bin 4	[0.836,1.325)	5	8.333333	1.3333333
bin 5	[1.325,2.161)	10	8.333333	0.3333333
bin 6	[2.161,inf)	7	8.333333	0.2133333

Our value for χ_0^2 value is the sum of the right-most column, 2.8. At $\alpha = 0.05$ and $k - 1 = 5$ degrees of freedom, $\chi^2_{0.05,5} = 11.1$. Since $\chi_0^2 < \chi^2_{0.05,5}$, we do not reject the null hypothesis.

10.1

Our statistical test for the simulation model of the job shop with an average number of jobs X and a true mean value μ_0 will be a double-sided test:

$$H_0 : E(X) = 22.5$$
$$H_A : E(X) \neq 22.5$$

The results of the simulation are:

$$18.9, 22.0, 19.4, 22.1, 19.8, 21.9, 20.2$$

$$For\ \alpha = 0.05, n = 7$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = 20.6$$

$$S = \left[\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}\right]^{\frac{1}{2}} 1.36$$

$$t_{\alpha/2,n-1} = t_{0.025,6} = 2.45$$

$$t_0 = \frac{20.6 - 22.5}{1.36/\sqrt{7}} = -3.69$$

For a two-sided test, if $|t_0| > t_{0.025,6}$, we reject H_0. Since $|-3.69| > 2.45$, we reject the null hypothesis. The model hypothesis is not consistent with system behavior.

If two jobs is viewed as critical, we can determint the value of the test using the open characteristic curves for the two-sided t test in table $A.10$ in the $Discrete - Event\ System\ Simulation$ textbook.

$$\hat{\delta} = \frac{E(X) - \mu_0}{S} = \frac{2}{1.36} = 1.47$$

$$for\ n = 7\ and\ \alpha = 0.05, \beta(\hat{\delta}) = \beta(1.47) \approx 0.1$$

The power of the test is $1 - \beta \approx 0.9$

Using the table, we see that the sample size needed to guarantee a power of 0.8 or higher for $\alpha = 0.05$ is 6.