IS 609 Assignment 13

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15.1 #4

Discuss how you might go about validating the nuclear arms race model. What data would you collect? Is it possible to obtain the data?

In order to validate the nuclear arms race model, one would need information on the three factors that define the shape of the curve that defines each countries minimum missile requirements.

The minimum required missiles for Country Y is:

$$y = f(x) = \frac{y_0}{s^{x/y}}$$

The minimum required missiles for Country X is:

$$x = g(y) = \frac{x_0}{s^{y/x}}$$

Taking the variables, for country Y as an example, we would need to know: y_0, the minimum number of missile required to maintain second-strike capabilities; s, the survivability percentage, which is a function of the security of Y's missiles as well as the effectiveness of X's missiles, and the exchange ration $e = x/y$. To validate this model, even historically, would be an incredibly challenge due to secrecy that still remains around these weapons systems (for example: the exact number and location of missiles as well as the peak technology each country developed). During the Cold War, both the US and USSR spent a great deal of resources trying to gain intelligence on each other's stockpile as well as technological advances. The most informative data would be the number of missile each country had over time to see if they accurately represent the shape of $f(x)$ or $g(y)$. I think even with this information, it is unlikely either nation's stockpile would faithfully reflect the shape of the curves. After all, the megatonnage of a nuclear weapon is based on analytical determinations and a country wouldn't have perfect knowledge of even their own weapons' effectiveness unless they actually tested them. The Castle Bravo test, for instance, was expected to be a 6 Mt weapon and ended up being 15 Mt!

15.2 #1

Build a numerical solution to Equations (15.8):

$$Let\, n = \ stage$$

$$x_n = number\, of\, weapons\, possessed\, by\, X\, in\, stage\, n$$

$$y_n = number\, of\, weapons\, possessed\, by\, Y\, in\, stage\, n$$

$$y_{n+1} = 120 + \frac{1}{2}x_n$$

$$x_{n+1} = 60 + \frac{1}{3}y_n$$

$$x_0 = 100$$

$$y_0 = 200$$

a. Graph your results.
b. Is an equilibrium value reached?
c. Try other starting values. Do you think the equilibrium value is stable?
d. Explore other values for the survival coefficients of Countries X and Y . Describe your results.

```
suppressWarnings(suppressMessages(library(knitr)))
suppressWarnings(suppressMessages(library(ggplot2)))
x <- c(100)
y <- c(200)

for (i in 1:20){
  x[i+1] <- 60 + y[i]/3
  y[i+1] <- 120 + x[i]/2
  }

unevenStart <- data.frame(n=c(0:20),x,y)
unevenStart
```

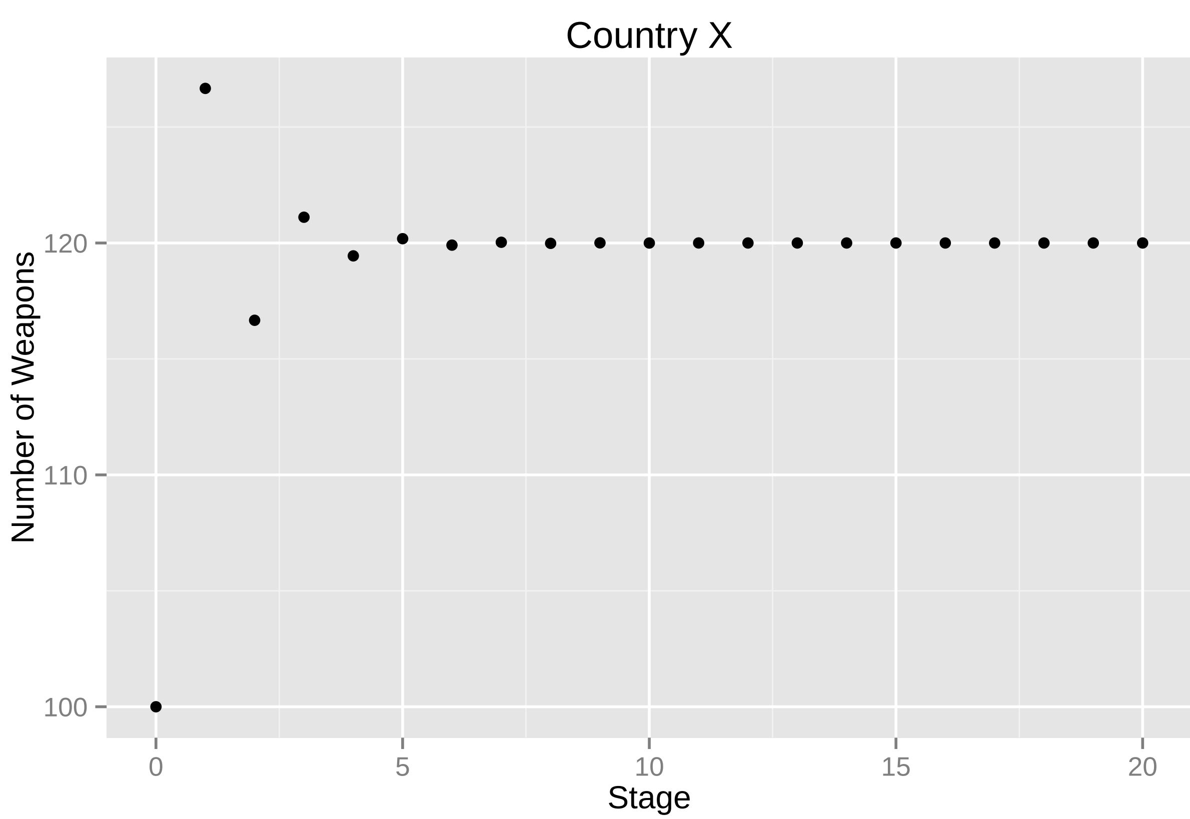
```
##     n        x        y
## 1   0 100.0000 200.0000
## 2   1 126.6667 170.0000
## 3   2 116.6667 183.3333
## 4   3 121.1111 178.3333
## 5   4 119.4444 180.5556
## 6   5 120.1852 179.7222
## 7   6 119.9074 180.0926
## 8   7 120.0309 179.9537
## 9   8 119.9846 180.0154
## 10  9 120.0051 179.9923
## 11 10 119.9974 180.0026
## 12 11 120.0009 179.9987
## 13 12 119.9996 180.0004
## 14 13 120.0001 179.9998
## 15 14 119.9999 180.0001
## 16 15 120.0000 180.0000
## 17 16 120.0000 180.0000
## 18 17 120.0000 180.0000
## 19 18 120.0000 180.0000
## 20 19 120.0000 180.0000
## 21 20 120.0000 180.0000
```

```
kable(unevenStart)
```

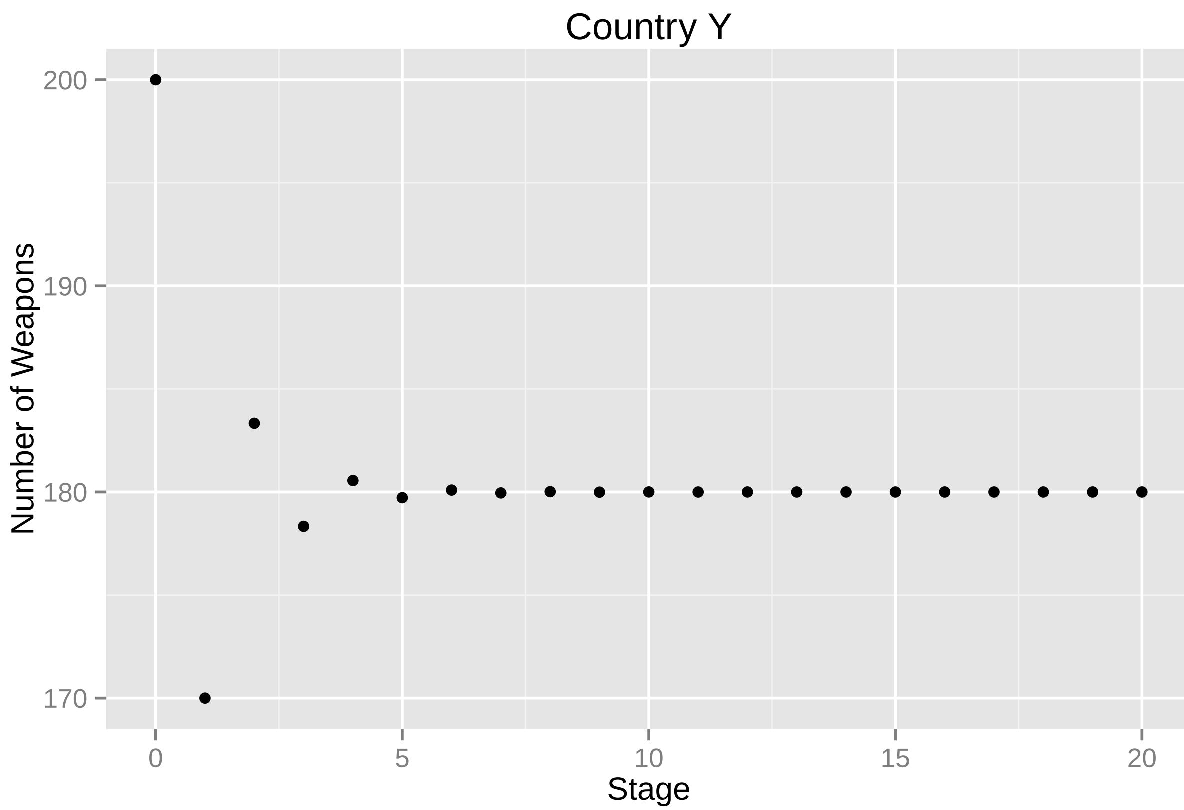
n	x	y
0	100.0000	200.0000
1	126.6667	170.0000
2	116.6667	183.3333
3	121.1111	178.3333
4	119.4444	180.5556
5	120.1852	179.7222
6	119.9074	180.0926
7	120.0309	179.9537

n	x	y
8	119.9846	180.0154
9	120.0051	179.9923
10	119.9974	180.0026
11	120.0009	179.9987
12	119.9996	180.0004
13	120.0001	179.9998
14	119.9999	180.0001
15	120.0000	180.0000
16	120.0000	180.0000
17	120.0000	180.0000
18	120.0000	180.0000
19	120.0000	180.0000
20	120.0000	180.0000

```
ggplot(unevenStart) +
  geom_point(aes(x=n,y=x)) +
  ggtitle("Country X") +
  ylab("Number of Weapons") +
  xlab("Stage")
```




```
ggplot(unevenStart) +
  geom_point(aes(x=n,y=y)) +
  ggtitle("Country Y") +
  ylab("Number of Weapons") +
  xlab("Stage")
```



Here we see that an equilbirum is reached fairly quickly. The number of weapons possessed by Country X approaches 120 and Country Y approaches 180. The equilibrium appears to be very stable. If we swap the number of weapons that each country starts off with, we also reach equilbirum fairly quickly.

```
x <- c(200)
y <- c(100)

for (i in 1:20){
  x[i+1] <- 60 + y[i]/3
  y[i+1] <- 120 + x[i]/2
  }

unevenStart <- data.frame(n=c(0:20),x,y)
kable(unevenStart)
```

n	x	y
0	200.00000	100.0000
1	93.33333	220.0000
2	133.33333	166.6667
3	115.55556	186.6667
4	122.22222	177.7778
5	119.25926	181.1111
6	120.37037	179.6296
7	119.87654	180.1852
8	120.06173	179.9383
9	119.97942	180.0309
10	120.01029	179.9897

n	x	y
11	119.99657	180.0051
12	120.00171	179.9983
13	119.99943	180.0009
14	120.00029	179.9997
15	119.99990	180.0001
16	120.00005	180.0000
17	119.99998	180.0000
18	120.00001	180.0000
19	120.00000	180.0000
20	120.00000	180.0000

The equilbrium does depend greatly on the survival coefficients. Here we will halve the coefficient for Country X and triple it for Country Y. We see that that the system settles on an equilbirum of 157 weapons for Country X and 146 weapons for Country Y.

```
x <- c(100)
y <- c(200)

for (i in 1:10){
  x[i+1] <- 60 + y[i]/1.5
  y[i+1] <- 120 + x[i]/6
  }

unevenStart <- data.frame(n=c(0:10),x,y)
kable(unevenStart)
```

n	x	y
0	100.0000	200.0000
1	193.3333	136.6667
2	151.1111	152.2222
3	161.4815	145.1852
4	156.7901	146.9136
5	157.9424	146.1317
6	157.4211	146.3237
7	157.5492	146.2369
8	157.4912	146.2582
9	157.5055	146.2485
10	157.4990	146.2509

15.3 #4

Verify the result that the marginal revenue of the $q+1st$ unit equals the price of that unit minus the loss in revenue on previous units resulting from price reduction.

To do so, we will take the derivative of the Total Revenue curve. Total Revenue is defined as:

$$TR(q) = P(q) \cdot (q)$$
$$MR(q) = TR'(q) = TR(q+1) = TR(q)$$

$$MR(q) = P(q+1) \cdot (q+1) - P(q) \cdot q$$
$$MR(q) = P(q+1) + P(q+1) \cdot q - P(q) \cdot q$$
$$MR(q) = P(q+1) + q \cdot (P(q+1) - P(q))$$
$$MR(q) = P(q+1) + q \cdot \Delta p$$

This last expression shows us that the the marginal revenue of the $q + 1st$ unit equals the price at $q + 1$ plus the product of the change in price and the new quantity. If the change in price is negative, then marginal revenue will be the new price minus the loss in revenue.