Universidad Autónoma de México

Matemáticas para las Ciencias Aplicadas III

# Tarea I

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**Anton-Bivens-Davis**

**Sección 14.2**

**15-18** Evaluate the double integral in two ways using iterated integrals: (a) viewing $R$ as a type I region, and (b) viewing $R$ as a type II region.

**18.**

$\iint\limits_R y \,\mathrm{d}A$; $R$ is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.

**19-24.** Evaluate the double integral in two ways using iterated integrals:

**22.** $\iint\limits_R x \,\mathrm{d}A$; $R$ is the region enclosed by $y = sin^{-1}x, x = \frac{1}{\sqrt{2}}$, and $y = 0$.

**28.**

(a) By hand or with the help of a graphing utility, make a sketch of the region $R$ enclosed between the curves $y = 4x^3 - x^4$ and $y = 3 - 4x + 4x^2$.

(b) Find the intersections of the curves in part (a).

(c) Find $\iint\limits_R x \,\mathrm{d}A$

**37-38** Use double integration to find the volume of the solid.

**37.**

img1.png

**57.** Try to evaluate the integral with a CAS using the stated order of integration, and then by reversing the order of integration.

(a) $\int\limits_0^4 \int\limits_{\sqrt{x}}^2 \sin \pi y^3 \,\mathrm{d}x\mathrm{d}y$

(b) $\int\limits_0^1 \int\limits_{sin^{-1}y}^{\frac{\pi}{2}} \sec \cos x^2 \,\mathrm{d}x\mathrm{d}y$

**59.** Evaluate $\iint\limits_R xy^2 \,\mathrm{d}A$ over the region R shown in the accompanying figure.

img2.png

**63.** Suppose that the temperature in degrees Celsius at a point $(x, y)$ on a flat metal plate is $T(x, y) = 5xy + x^2$, where $x$ and $y$ are in meters. Find the average temperature of the diamond-shaped portion of the plate for which $|2x + y| \leq 4$ and $|2x - y| \leq 4$.

**Sección 14.5**

**12.** $\iiint\limits_G \cos\frac{z}{y}\,\mathrm{d}V$, where $G$ is the solid defined by the inequalities $\frac{\pi}{6} \le y \le \frac{\pi}{2}, y \le x \le \frac{\pi}{2}, 0 \le z \le xy$.

**37.** Let $G$ be the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, (a > 0, b > 0, c > 0)$$

(a) List six different iterated integrals that represent the volume of $G$.

(b) Evaluate any one of the six to show that the volume of $G$ is $\frac{1}{6}abc$.

**38.** Use a triple integral to derive the formula for the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

**Hughes-Hallet**

**Sección 16.2**
**35.**

$$\int\limits_0^1 \int\limits_{\sqrt{y}}^1 \sqrt{2 + x^3}\,\mathrm{d}x\,\mathrm{d}y$$

**37.**

$$\int\limits_0^1 \int\limits_{e^y}^e \frac{x}{\ln x}\,\mathrm{d}x\,\mathrm{d}y$$

**60.** Show that for a right triangle the average distance from any point in the triangle to one of the legs is one-third the length of the other leg. (The legs of a right triangle are the two sides that are not the hypotenuse.)

**62.** Find the area of the crescent-moon shape with circular arcs as edges and the dimensions shown in Figure 16.22.

**Sección 16.3**

In Problems 14–18, decide whether the integrals are positive,negative, or zero. Let $S$ be the solid sphere $x^2 + y^2 + z^2 \le 1$, and $T$ be the top half of this sphere (with $z \ge 0$), and $B$ be the bottom half (with $z \le 0$), and $R$ be the right half of the sphere (with $x \ge 0$), and $L$ be the left half (with $x \le 0$).

**14.**

$$\int_T e^z\,\mathrm{d}V$$

**15.**

$$\int_B e^z\,\mathrm{d}V$$

**16.**

$$\int_S \sin z \, \mathrm{d}V$$

**17.**

$$\int_T \sin z \, \mathrm{d}V$$

**18.**

$$\int_R \sin z \, \mathrm{d}V$$

**31.** A trough with triangular cross-section lies along the $x$-axis for $0 \le x \le 10$. The slanted sides are given by $z = y$ and $z = -y$ for $0 \le z \le 1$ and the ends by $x = 0$ and $x = 10$, where $x, y, z$ are in meters. The trough contains a sludge whose density at the point $(x, y, z)$ is $\delta = e^{-3x}$ kg per $m^3$.

**a)** Express the total mass of sludge in the trough in terms of triple integrals.

**b)** Express the total mass of sludge in the trough in terms of triple integrals.

**55.** $E$ is the region bounded by $x = 0, y = 0, z = 0, z = 2$, and $2x + 4y + z = 4$.

**57.** Figure 16.28 shows part of a spherical ball of radius 5 cm. Write an iterated triple integral which represents the volume of this region.

**66.** Find the center of mass of the tetrahedron that is bounded by the $xy, yz, xz$ planes and the plane $x + 2y + 3z = 1$. Assume the density is 1 $gm/cm^3$ and $x, y, z$ are in centimeters.

Problems 67–69 concern a rotating solid body and its *moment of inertia* about an axis; this moment relates angular acceleration to torque (an analogue of force). For a body of constant density and mass $m$ occupying a region $W$ of volume $V$ , the moments of inertia about the coordinate axes are

$$I_x = \frac{m}{V} \int_W (y^2 + z^2) \, \mathrm{d}V$$

$$I_y = \frac{m}{V} \int_W (x^2 + z^2) \, \mathrm{d}V$$

$$I_z = \frac{m}{V} \int_W (x^2 + z^2) \, \mathrm{d}V$$

**67.** Find the moment of inertia about the $z$-axis of the rectangular solid of mass $m$ given by $0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3$.

Are the statements in Problems 74–83 true or false? Give reasons for your answer.

**75.** The region of integration of the triple iterated integral $\int_0^1 \int_0^1 \int_0^x f dzdydx$ lies above a square in the $xy$-plane and below a plane.

**78.** The iterated integrals $\int_{-1}^1 \int_0^1 \int_0^{1-x^2} f dzdydx$ and $\int_0^1 \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} f dzdydx$ are equal.

**80.** If $W$ is the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ and $\int_W f \, \mathrm{d}V = 0$, then $f = 0$ everywhere in the unit.