

UNIVERSIDAD AUTÓNOMA DE MÉXICO

MATEMÁTICAS PARA LAS CIENCIAS APLICADAS III

## Tarea I

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## Sección 14.2

**15-18** Evaluate the double integral in two ways using iterated integrals: (a) viewing  $R$  as a type I region, and (b) viewing  $R$  as a type II region.

**18.**

$\iint_R y \, dA$ ;  $R$  is the region in the first quadrant enclosed between the circle  $x^2 + y^2 = 25$  and the line  $x + y = 5$ .

**19-24.** Evaluate the double integral in two ways using iterated integrals:

**22.**  $\iint_R x \, dA$ ;  $R$  is the region enclosed by  $y = \sin^{-1}x$ ,  $x = \frac{1}{\sqrt{2}}$ , and  $y = 0$ .

**28.**

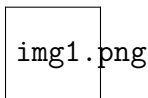
(a) By hand or with the help of a graphing utility, make a sketch of the region  $R$  enclosed between the curves  $y = 4x^3 - x^4$  and  $y = 3 - 4x + 4x^2$ .

(b) Find the intersections of the curves in part (a).

(c) Find  $\iint_R x \, dA$

**37-38** Use double integration to find the volume of the solid.

**37.**

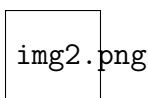


**57.** Try to evaluate the integral with a CAS using the stated order of integration, and then by reversing the order of integration.

(a)  $\int_0^4 \int_{\sqrt{x}}^2 \sin \pi y^3 \, dx dy$

(b)  $\int_0^1 \int_{\sin^{-1}y}^{\frac{\pi}{2}} \sec \cos x^2 \, dx dy$

**59.** Evaluate  $\iint_R xy^2 \, dA$  over the region  $R$  shown in the accompanying figure.



**63.** Suppose that the temperature in degrees Celsius at a point  $(x, y)$  on a flat metal plate is  $T(x, y) = 5xy + x^2$ , where  $x$  and  $y$  are in meters. Find the average temperature of the diamond-shaped portion of the plate for which  $|2x + y| \leq 4$  and  $|2x - y| \leq 4$ .

## Sección 14.5

12.  $\iiint_G \cos \frac{z}{y} \, dV$ , where  $G$  is the solid defined by the inequalities  $\frac{\pi}{6} \leq y \leq \frac{\pi}{2}, y \leq x \leq \frac{\pi}{2}, 0 \leq z \leq xy$ .

37. Let  $G$  be the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, (a > 0, b > 0, c > 0)$$

(a) List six different iterated integrals that represent the volume of  $G$ .

(b) Evaluate any one of the six to show that the volume of  $G$  is  $\frac{1}{6}abc$ .

38. Use a triple integral to derive the formula for the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

## Hughes-Hallet

## Sección 16.2

35.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3} \, dx \, dy$$

37.

$$\int_0^1 \int_{e^y}^e \frac{x}{\ln x} \, dx \, dy$$

60. Show that for a right triangle the average distance from any point in the triangle to one of the legs is one-third the length of the other leg. (The legs of a right triangle are the two sides that are not the hypotenuse.)

62. Find the area of the crescent-moon shape with circular arcs as edges and the dimensions shown in Figure 16.22.

## Sección 16.3

In Problems 14–18, decide whether the integrals are positive, negative, or zero. Let  $S$  be the solid sphere  $x^2 + y^2 + z^2 \leq 1$ , and  $T$  be the top half of this sphere (with  $z \geq 0$ ), and  $B$  be the bottom half (with  $z \leq 0$ ), and  $R$  be the right half of the sphere (with  $x \geq 0$ ), and  $L$  be the left half (with  $x \leq 0$ ).

14.

$$\int_T e^z \, dV$$

15.

$$\int_B e^z \, dV$$

16.

$$\int_S \sin z \, dV$$

17.

$$\int_T \sin z \, dV$$

18.

$$\int_R \sin z \, dV$$

**31.** A trough with triangular cross-section lies along the  $x$ -axis for  $0 \leq x \leq 10$ . The slanted sides are given by  $z = y$  and  $z = -y$  for  $0 \leq z \leq 1$  and the ends by  $x = 0$  and  $x = 10$ , where  $x, y, z$  are in meters. The trough contains a sludge whose density at the point  $(x, y, z)$  is  $\delta = e^{-3x}$  kg per  $m^3$ .

a) Express the total mass of sludge in the trough in terms of triple integrals.

b) Express the total mass of sludge in the trough in terms of triple integrals.

**55.**  $E$  is the region bounded by  $x = 0, y = 0, z = 0, z = 2$ , and  $2x + 4y + z = 4$ .

**57.** Figure 16.28 shows part of a spherical ball of radius 5 cm. Write an iterated triple integral which represents the volume of this region.

**66.** Find the center of mass of the tetrahedron that is bounded by the  $xy, yz, xz$  planes and the plane  $x + 2y + 3z = 1$ . Assume the density is  $1 \text{ gm/cm}^3$  and  $x, y, z$  are in centimeters.

Problems 67–69 concern a rotating solid body and its *moment of inertia* about an axis; this moment relates angular acceleration to torque (an analogue of force). For a body of constant density and mass  $m$  occupying a region  $W$  of volume  $V$ , the moments of inertia about the coordinate axes are

$$I_x = \frac{m}{V} \int_W (y^2 + z^2) \, dV$$

$$I_y = \frac{m}{V} \int_W (x^2 + z^2) \, dV$$

$$I_z = \frac{m}{V} \int_W (x^2 + y^2) \, dV$$

**67.** Find the moment of inertia about the  $z$ -axis of the rectangular solid of mass  $m$  given by  $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$ .

Are the statements in Problems 74–83 true or false? Give reasons for your answer.

**75.** The region of integration of the triple iterated integral  $\int_0^1 \int_0^1 \int_0^x f \, dz \, dy \, dx$  lies above a square in the  $xy$ -plane and below a plane.

**78.** The iterated integrals  $\int_{-1}^1 \int_0^1 \int_0^{1-x^2} f \, dz \, dy \, dx$  and  $\int_0^1 \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} f \, dz \, dy \, dx$  are equal.

**80.** If  $W$  is the unit cube  $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$  and  $\int_W f \, dV = 0$ , then  $f = 0$  everywhere in the unit.