1. We have

$$202202 = 202 \cdot 1001 = (2 \cdot 101)(7 \cdot 11 \cdot 13) = 22(7 \cdot 13 \cdot 101),$$

so the answer is $7 \cdot 13 \cdot 101 = \boxed{9191}$.

2. The triangle is right with legs $3, 4$ so the answer is $\frac{1}{2} \cdot 3 \cdot 4 = \boxed{6}$.

3. We have that

$$43 = x - \frac{x}{2} - \frac{x}{4} = \frac{x}{4},$$

so $x = 4 \cdot 43 = \boxed{172}$.

4. Let E denote the expression. Then order of operations gives

$$\begin{aligned} E &= (1+5) \times [(5 \times 3) - 2^2] - 3^2 \times 2 \\ &= 6 \times [15 - 2^2] - 3^2 \times 2 \\ &= 6 \times [15 - 4] - 9 \times 2 \\ &= 6 \times 11 - 9 \times 2 \\ &= 66 - 18 \\ &= \boxed{48}. \end{aligned}$$

5. He flips heads with probability $\frac{1}{2}$, so the answer is $\left(\frac{1}{2}\right)^5 = \boxed{\frac{1}{32}}$.

6. Noting that there are 25 primes less than 100, we can work backwards to get that the 21st prime is $\boxed{73}$.

7. Let r denote the radius of the sphere. We are given that

$$4\pi r^2 = 36\pi \implies r = 3,$$

so the volume of the sphere is $\frac{4}{3}\pi r^3 = \boxed{36\pi}$.

8. Let $S = \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}} = 4$. Squaring the equation gives

$$16 = x + S = x + 4 \implies x = \boxed{12},$$

as desired.

9. Note that

$$421^2 - 420^2 = 421 + 420 = 841 = 29^2,$$

so $29^2 + 420^2 = 421^2$, and the answer is $\boxed{421}$.

10. Let $S = \frac{1}{7} + \frac{2}{49} + \frac{3}{343} + \ldots$. Then $\frac{S}{7} = \frac{0}{7} + \frac{1}{49} + \frac{2}{343} + \ldots$, so subtracting the equations gives

$$\frac{6S}{7} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343} + \cdots = \frac{\frac{1}{7}}{1 - \frac{1}{7}} = \frac{1}{6},$$

and $S = \boxed{\frac{7}{36}}$.

11. The nth hexagonal number is given by $n(2n-1)$, so the 7th hexagonal number is $7 \cdot 13 = \boxed{91}$.

12. The sum of the interior angles of an $n-$gon is $180(n-2)$, so the answer is

$$180(29-2) = 180 \cdot 27 = \boxed{4860}.$$

13. The hundreds digit of 10! is the units digits of

$$10!/100 = 3 \cdot 4 \cdot 6 \cdot 7 \cdot 8 \cdot 9,$$

which can be found to be $\boxed{8}$.

14. Since hexagon is regular, the triangle is equilateral. Additionally, $AB = BC = 8$, $\angle ABC = 120^\circ$, so from geometry we get $AC = 8\sqrt{3}$. Then the answer is $3 \cdot 8\sqrt{3} = \boxed{24\sqrt{3}}$.

15. This is equal to $641 \cdot 22 + 2 = \boxed{14104}$.

16. The smallest such integer must be one of

$$p_1p_2p_3p_4,\ p_1^3p_2p_3,\ p_1^3p_2^3,\ p_1^7p_2,\ p_1^{15},$$

where $p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7$. Plugging these in, the smallest integer out of these five is $p_1^3p_2p_3 = 2^3 \cdot 3 \cdot 5 = \boxed{120}$.

17. We have

$$\begin{aligned} 2021^2 &= (2000+21)(2000+21) = 2000^2 + 2 \cdot 21 \cdot 2000 + 21^2 \\ &= 4000000 + 84000 + 441 = \boxed{4084441}, \end{aligned}$$

as desired.

18. The total amount Anthony will pay is given by

$$\$100,000(1 + 0.012 \cdot 30) = \$136,000,$$

so his average monthly payment is $\frac{\$136,000}{30 \cdot 12} = \boxed{\$\frac{3400}{9}}$.

19. The area of a regular octagon with side length s is $2s^2(1+\sqrt{2})$, so the answer is

$$2 \cdot 4^2 \cdot (1+\sqrt{2}) = \boxed{32 + 32\sqrt{2}}.$$

20. Every second, the distance between Anthony and the baseball decreases by 125 feet, so Anthony reaches the ball after $375/125 = 3$ seconds. In this time, he runs

$$3 \cdot 25 \text{ feet} = 75 \text{ feet} = 75 \cdot 12 \text{ inches} = \boxed{900} \text{ inches}.$$

21. This factors as $20^{21} = 2^{42} \cdot 5^{21}$, which has $43 \cdot 22 = \boxed{946}$ positive divisors.

22. Note that $3^4 \equiv 1 \pmod{10}$, so

$$1^{337} + 13^{37} + 133^7 \equiv 1 + 3^1 + 3^3 = 31 \equiv \boxed{1} \pmod{10},$$

as desired.

23. At the end of every day, the number of monkeys in the tree triples, so if we let k denote the number of days since the first day, the total number of monkeys in the tree at the end of that day is 3^k. Then 6 days from Friday, the total number of monkeys in the tree is $3^6 = 729$, and on the following day, there would be $3^7 = 2187 > 2021$ monkeys in the tree. This means that the tree collapses 7 days from Friday, which is on a $\boxed{\text{Friday}}$.

24. Every week, Yang Yang spends $1+2+\ldots+7=28$ hours teaching his dog. 420 days is equivalent to $420/7=60$ weeks, so the total number of hours he spends teaching his dog is $28\cdot 60=\boxed{1680}$.

25. Let O be the common center of the circle and the square, $\overline{AB}$ be one of the sides of the octagon, and M be the midpoint of $\overline{AB}$. Since $\overline{AB}$ is the side of an octagon inscribed in a unit square, we have $AB=\frac{1}{\sqrt{2}+1}=\sqrt{2}-1$. Since $\overline{AB}$ is a chord in the circle centered at O, we have $\overline{OM}\perp\overline{AB}$, so $\triangle OMA$ is a right triangle. Then

$$r^2=OA^2=MA^2+MO^2=\left(\frac{\sqrt{2}-1}{2}\right)^2+\left(\frac{1}{2}\right)^2=\boxed{\frac{2-\sqrt{2}}{2}}.$$