Mental Math 2022 Solutions

Chiles Mini Mu

December 2022

1. The first five natural numbers are $1, 2, 3, 4$, and 5 and the first five whole numbers are $0, 1, 2, 3$, and 4. Summing these 10 numbers, we get 25.

2. In 42 seconds, Rachel has crossed 16 roads as she crosses 8 roads ever 21 seconds. Since she gets 13 points for every 4 roads she crosses, she gets $13 * 4 = 52$ points.

3. 18 gallons of gas at 4.50 gives a final price of $18 * 4.50 = \$81$.

4. 33 miles per gallon with 18 gallons of gas gives a total travel distance of $33 * 18 = 594$ miles.

5. $2020 + 2021 + 2022 + 2023 = 2021.5 * 4 = 8086$.

6. Linda and Jessica can write 37 words in a minute, combined, by adding their independent words per minute. With 111 words in total, $\frac{111}{37} = 3$ minutes.

7. The rectangular prism has length 1 cm, width 100 cm, and height 2 cm. The volume is then $1 * 100 * 2 = 200$ cm^3.

8. 4 Aces is equal to $4 * 4 = 16$ Boos, 16 Boos is equal to $16 * \frac{2}{7} = \frac{32}{7}$ Bours, and $\frac{32}{7}$ Bours is equal to $\frac{32}{7} * \frac{7}{4} = 8$ Bevins.

9. The sum of the solutions to a quadratic $ax^2 + bx + c = 0$ is equal to $\frac{-b}{a} = \frac{15}{1} = 15$ and the product of the solutions to the same quadratic is equal to $\frac{c}{a} = \frac{54}{1} = 54$. Adding the two $15 + 54 = 69$.

10. The volume of a sphere is given by $V = \frac{4\pi}{3}r^3$, so the volume of this sphere is $\frac{4\pi}{3} * 1^3 = \frac{4\pi}{3} \approx 4$.

11. The probability that Elise gets 1st and Cathleen get 2nd is $\frac{1}{2}$ and $\frac{9}{10}$, respectively. Since they are independent probabilities, multiply the two to get the probability both occur, so $\frac{1}{2} * \frac{9}{10} = \frac{9}{20}$.

12. There is only one even prime number, 2, and there are 49 even natural numbers between 0 and 100. Thus $1 + 49 = 50$.

13. A cube has 6 faces, 8 vertices, and 12, so $6 + 8 + 12 = 26$.

14. The number of permutations of a 6 letter word with all distinct letters is equal to 6!, but since there are 2 sets of 2 duplicate letters, we must divide by 2! twice. $\frac{6!}{2!2!} = 180$.

15. $3 * 7 = 21 \equiv 4 \pmod{17}$. Thus, $(3 * 7)^2 \equiv 4^2 \equiv 16 \pmod{17}$, so the remainder is 16.

16. 8 divides 2000, so the remainder of dividing the two is equal to the remainder when 23 divided by 8, or 7.

17. There were 31 days in January and 28 days in February. $31 + 28 = 59$ total days.

18. If $f(x) = 0$, then $0 = \sqrt{2x+3} - 7$ and $\sqrt{2x+3} = 7$. Squaring both sides yields $2x + 3 = 49$ and simplifying gives $x = 23$.

19. $x = 2^3 * 5^2 + 3^4 * 4^2 = 2^3(5^2 + 3^4 * 2) = 2^3(25 + 162) = 2^3(187)$. Now dividing $\frac{2^3(187)}{2*3*4*5} = \frac{187}{15}$.

20. There are 600 seconds in 10 minutes, so $\frac{600}{40} = 15$ seconds per question.

21. Dividing by 42 gives $\frac{12}{7}x = y - k$, so the slope is $\frac{12}{7}$.

22. Finding the reciprocal of the original inequality gives $\frac{2023}{22} < \frac{b}{a} < \frac{2022}{21}$. Simplifying the fractions on both sides gives $91 + k_1 < \frac{b}{a} < 96 + k_2$, with $0 \geq k_1, k_2 \geq 1$. Thus the possible even values of $\frac{b}{a}$ are $92, 94$ and 96. Summing gives $92 + 94 + 96 = 282$.

23. When dealing with a problem that involves first moving to a line and then a point from an initial point, if the two points are on the same side of the line, then it is often effective to reflect the final point over the line and find the distance of the original point and this new point. $(10, 9)$ reflected over $x = 1$ gives the point $(-8, 9)$. The distance from $(4, 4)$ to $(-8, 9)$ is given by the distance formula $\sqrt{(-8-4)^2 + (9-4)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$.

24. Dividing the second equation by 17 and substituting x in for y, it becomes $x = 11x + 1$, so $x = -\frac{1}{10} = y$. Thus $x + y = -\frac{1}{5}$.

25. The denominator simplifies to $8! * \sqrt{9} = 8! * 3$. Since the numerator is equal to $\frac{10!}{5}$, then dividing the numerator by the denominator is equal to $\frac{10*9}{3*5} = 6$.

26. This is a geometric series with first time $\frac{1}{11}$ and common ratio $\frac{1}{11}$. Thus its sum is equal to $\frac{\frac{1}{11}}{1-\frac{1}{11}} = \frac{1}{10}$.

27. Since this is a square, it is sufficient to find the distance between two points, which is the length of the side, and square it. The distance between $(1, 3)$ and $(7, 11)$ squared is equal to $(7-1)^2 + (11-3)^2 = 36 + 64 = 100$.

28. $112 = 2^4 * 7$. Thus it has 10 total factors, the powers of 2 from the 0th power to the 4th power and those same powers multiplied by 7.

29. There are 3 ways to pick a Theta student, 4 ways to pick a Alpha student, and $\frac{8!}{2!6!} = \frac{8*7}{2} = 28$ ways to pick a Mu student by picking a combination of 2 from 8 Mu students. Multiplying all three gives $3 * 4 * 28 = 336$ total combinations.

30. $6969 = 101 * 69 = 101 * 3 * 23$, giving it 8 factors. Since it has 3 prime factors and 7 proper factors, the probability that she picks a prime factor is $\frac{3}{7}$.

31. Bruce will multiply 3 by 2 and 6 by 3 and obtain 6 and 18. Jiayi knows that Auska could have told Bruce 2 and 6, 3 and 6, 2 and 9, or 3 and 9 for a total of 4 possible combinations.

32. If Shilpa has two \$10 bills, then she can have either five \$1 bills or one \$5 bill. If she has one \$10 bill, then she can have either zero, one, two, or three \$5 bills and the rest be ones. If she has no \$10 bills, then she can have from zero to five \$5 bills. Summing two, four, and six from the three cases gives a total of 12 combinations.

33. Jose will take $45 * 15 = 675$ seconds while Julia will take $3 * 60 * 3 = 540$ seconds. Jose will take $675 - 540 = 135$ seconds longer.

34. Sabrina and Nikki need to pay \$9.50 each for a total of \$19 for themselves. The students' tickets will cost $\$5 * 40 = \$200$. Thus they will need $\$19 + \$200 = \$219$ in total.

35. Observe that $31 * 41 = 30 * 41 + 1 * 41 = 1230 + 41 = 1271$.

36. Observe that $80 * 70 = (75 - 5)(75 + 5) = 75^2 - 25$, so $75^2 = 80 * 70 + 25 = 5600 + 25 = 5625$.

37. The Fibonacci sequence is defined by $a_{n+2} = a_{n+1} + a_n$. So the sequence is as so: $1, 1, 2, 3, 5, 8, 13, 21, \cdots$. Adding the 3rd and 8th numbers gives $2 + 21 = 23$.

38. By the Pythagorean Theorem, the hypotenuse of a triangle with legs of length 9 and 40 is equal to $\sqrt{9^2 + 40^2} = \sqrt{1681} = 41$.

39. There are seven 1's and four -1's giving a total sum of 3.

40. Well, there is a negative and a positive one and a bunch of zeroes, so the total sum is equal to 0.