Elementary Round 2 Problems and Solutions

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2022-2023

1. Yimo has come up with the idea to replace the Hubble Space Telescope. He has named the new telescope the "James Webb Space Telescope". There will be a meeting held at the Kennedy Space Center at $(10, 2)$. Yimo is currently studying Barnard 68 at the Mauna Kea Observatory at $(-2, -3)$. If each unit of length in the grid is 350 miles, how many miles will Yimo need to travel to reach the Kennedy Space Center from his current location at the Mauna Kea Observatory?

Solution. The distance between the two points is

$$\sqrt{(10-(-2))^2+(2-(-3))^2} = \sqrt{12^2+5^2} = 13 \text{ units},$$

so the answer is $13 \text{ units} \cdot 350 \text{ miles/unit} = \boxed{4550 \text{ miles}}$. □

2. James attends this meeting at Kennedy Space Center. He writes top secret information on the JWST's blueprint on a small piece of paper, and puts it in his phone case. Katharine is a thief with an extensive criminal record, and decides to rob James in broad daylight for the top secret papers. This turns out to be quite the terrible decision, as James has 10 large bodyguards protecting him. Katharine is immediately arrested and sentenced to 5 life sentences in supermax prison. The supermax prison is the shape of a square with a diagonal of 100 yards. What is the area of the supermax prison?

Solution. The side length of the supermax prison is $100/\sqrt{2} = 50\sqrt{2}$ yards, so the area of the supermax prison is $(50\sqrt{2} \text{ yards})^2 = \boxed{5000 \text{ yards}^2}$. □

3. James wants to change the official name of the telescope from James Webb Space Telescope to "James Zhang Space Telescope". NASA has decided that in order to make this change, James must evaluate this expression:

$$(26 + 45(235 + 12 - 555/3)/9) \cdot 1 + 714/7 + (-113)^0.$$

What is the value of this expression?

Solution. This is equal to

$$26 + 5(235 + 12 - 185) + 102 + 1 = 129 + 5 \cdot 62 = 129 + 310 = \boxed{439}.$$

□

4. Unfortunately, James was unable to solve the problem, and his dreams were crushed. Yangyang decided to bake him a large moon pie in the shape of a cylinder. While making the pie, Yangyang got a little hungry and ate 1/9 of the pie. What percent of the pie is remaining after Yangyang eats his portion (rounded to the nearest hundredth)?

Solution. The percent of the pie remaining is

$$1 - \frac{1}{9} = \frac{8}{9} = \frac{800}{9}\% \approx \boxed{88.89\%}.$$

□

5. The NASA engineers are on a lunch break! It just happens to be Arib's birthday, and there are three boxes with a dozen donuts in each. The smell of the donuts immediately caught William's attention, and he broke Usain Bolt's record while running to the donuts! He took a whole box for himself and eats all the donuts he has. Nelson grabbed 3 donuts, but was only able to eat 1. Nelson then offered his remaining donuts to Bruce, who ate both of the donuts. Jiayi then took 1/3 of the remaining donuts. How many donuts remain?

Solution. Initially, there are $3 \cdot 12 = 36$ donuts in total. William and Nelson take 12 and 3 donuts, respectively, and all 15 donuts are eaten, leaving $36 - 15 = 21$ donuts. Jiayi then takes $21/3 = 7$ donuts, so $21 - 7 = \boxed{14 \text{ donuts}}$ remain. □

6. The JWST consists of 18 equal-sized hexagonal-shaped mirror segments. Wesley is closely analyzing these hexagons and finds that the side of one hexagonal mirror is 2 feet. What is the surface area of the mirror segments of the JWST? (Hint: The equation for the area of a hexagon is $\frac{3\sqrt{3}}{2}s^2$, where s is the side length).

Solution. The area of each of the mirror segments is $\frac{3\sqrt{3}}{2}(2 \text{ ft})^2 = 6\sqrt{3} \text{ ft}^2$, so the total area of all 18 mirror segments is $18 \cdot 6\sqrt{3} \text{ ft}^2 = \boxed{108\sqrt{3} \text{ ft}^2}$. □

7. While on lunch break, Nima and Caleb decide to have a one-versus-one basketball match right next to the telescope! What could go wrong?! The base of the basketball hoop is a rectangular prism with dimensions 12 inches × 3 feet × 40 inches. What is the volume of the base of the basketball hoop?

Solution. The volume of the base is

$$(1 \text{ ft})(3 \text{ ft})(10/3 \text{ ft}) = \boxed{10 \text{ ft}^3 = 17280 \text{ in}^3}.$$

□

8. Aaron is trying to find the specific dimensions of the entire basketball hoop (Ball is life). The pole of the basketball hoop is a cylinder and has a base radius of 2 inches and a height of 11 feet. The hoop is 1 foot below the top of the pole. What is the volume of the pole?

Solution. The radius of the pole is $2/12 = 1/6$ ft, so the volume of the pole is

$$(1/6 \text{ ft})^2(11 \text{ ft})\pi = \boxed{\frac{11\pi}{36} \text{ ft}^3}.$$

□

9. Nima's dream is to one day touch the rim of the basketball hoop. If Nima's arm reaches 76 inches above the ground and the rim is 120 inches high, how high does Nima need to jump to touch the rim?

Solution. Nima needs to jump 120 in − 76in = $\boxed{44 \text{ in}}$ to touch the rim. □

10. The one-versus-one between Nima and Caleb is at game point for Caleb. Caleb crosses Nima and makes him fall on one of the hexagonal panels. Unfortunately, Nima's force while falling was too great and all 18 panels on the JWST need to be repaired. If each panel costs $\$250,000$ to repair and insurance only pays for 40% of the total repair costs, how much does Nima have to pay to repair the 18 panels given that he pays the remaining costs?

Solution. The total cost of the 18 panels is $18 \cdot \$250,000 = \$4,500,000$. Since insurance pays 40%, Nima pays 60%, for an answer of $0.6 \cdot \$4,500,000 = \boxed{\$2,700,000}$. □

11. After inspecting the broken panels, they turned out to be faulty from the beginning! Nima is named a hero after revealing the black mold growing inside of one of the mirrors! The mold grew into a peculiar shape of 2 rectangles and a square all put together! The dimensions of the rectangles are 2 in × 5 in and 1 ft × 4 in, and the square has a side length of 3 in. What is the area of the black mold?

Solution. The total area of the black mold is

$$2 \cdot 5 + 12 \cdot 4 + 3^2 = 10 + 48 + 9 = \boxed{67 \text{ in}^2}.$$

□

12. The JWST is being launched on the Ariane 5 Rocket at the Guiana Space Centre in French Guiana. However, Hadriel spots a storm coming and calls off the launch for the time being. It is currently 10 : 42 AM and the storm will begin in 1 hour and 13 minutes. If the storm is predicted to stay above the launch site for 5 hours and 49 minutes, at what time will the storm pass so that the JWST can be launched?

Solution. The storm passes 6 hours and 62 minutes, or 7 hours and 2 minutes after the current time, which is just $\boxed{5 : 44 \text{ PM}}$. □

13. Once the storm passes, Linsey gives the final go-ahead for approval to launch the JWST. At the T-10 second mark on the countdown, the electricity suddenly goes out! Yejun is tasked with fixing the electricity and sprints to the electricity room. However, the room is password locked and requires a four-digit code with only non-negative, single-digit integers for each digit. There is a hint for the password on the door. It reads "The first digit is not a natural number, the second digit is the largest possible single-digit number, the third digit is the first odd prime number, the last digit is the only even prime number" What is the code?

Solution. For the first digit, note that the only nonnegative number that is not natural is 0. For the second digit, the largest single-digit integer is 9. The third and fourth digits are just 3 and 2, respectively, so the code is $\boxed{0932}$. □

14. After Yejun successfully opens the door, Rohan is standing in the room with pliers and a bunch of broken wires in his hands (He is the imposter)! Rohan then says "Solve this problem to fix the wires instantly: What is the largest prime factor of 2022?"

Solution. Note that 2022 factors as $2 \cdot 3 \cdot 337$, so the answer is $\boxed{337}$. □

15. The Electrical system is back up, and the JWST is now launched. Given that the JWST needs to travel 62 miles to reach the Kármán Line, how far does it need to travel in feet?

Solution. There are 5280 feet in a mile, so the answer is $62 \cdot 5280 = \boxed{327360 \text{ feet}}$. □

16. Yimo, Cyrus, and Lillian's dream is to someday become astrophysicists! Out of curiosity, they decide to take their spaceship, the "Ye Machine" to see the JWST for themselves. If they need to travel 1.5 million kilometers to reach the telescope, how far are they from the JWST in centimeters?

Solution. There are 100 centimeters in a meter and 1000 meters in a kilometer, so there are 10^5 centimeters in a kilometer. Note that 1.5 million equals $1.5 \cdot 10^6$, so the distance in centimeters is $1.5 \cdot 10^6 \cdot 10^5 = \boxed{1.5 \cdot 10^{11} \text{ cm}}$. □

17. Yimo, Cyrus, and Lillian are in a time crunch and they only have 200 hours to reach the JWST. Assuming they move in a straight line towards the JWST, what is the minimum average speed they have to travel at in order to arrive within the time limit? (Use the distance from the previous question)

Solution. If they use all 200 hours to reach the JWST, they will travel at the minimum average speed of

$$\frac{1.5 \cdot 10^6 \text{ km}}{200 \text{ hr}} = \frac{1.5 \cdot 10^6 \text{ km}}{2 \cdot 10^2 \text{ hr}} = 0.75 \cdot 10^4 \text{ km/hr} = \boxed{7500 \text{ km/hr}}.$$

□

18. Oh no! On the way to the telescope, Cyrus notices that the "Ye Machine" doesn't have enough fuel to reach the JWST. If the "Ye Machine" is at $(2, 4)$ and the gas station is at $(3, 7)$, how far is the group from the gas station?

Solution. The distance between these two points is

$$\sqrt{(2-3)^2 + (4-7)^3} = \sqrt{1^2 + 3^2} = \boxed{\sqrt{10}}.$$

□

19. Heewon can solve an $N \times N \times N$ Rubik's cube in $f(N)$ seconds where $f(N)$ is the Nth Fibonacci number given by $f(1) = 0$, $f(2) = 1$, and $f(N) = f(N-1) + f(N-2)$ for $N > 1$. How long, in seconds, will it take Heewon to solve a $10 \times 10 \times 10$ Rubik's cube?

Solution. We can compute the first ten terms of the sequence to be $0, 1, 1, 2, 3, 5, 8, 13, 21, 34$, so the answer is $\boxed{34 \text{ s}}$. □

20. The JWST has a 5-layer rectangular sunshield that measures 70 feet × 45 feet. If the gap between each layer is 10 inches, what is the volume of the space between the combined gaps of the 5 layers?

Solution. There are 4 gaps between the five layers, and each gap is $10/12 = 5/6$ ft, so the answer is

$$4(5/6 \text{ ft})(70 \text{ ft})(45 \text{ ft}) = \boxed{10500 \text{ ft}^3}.$$

□

21. Ryan is tasked with temperature control, and reported that the JWST is currently at 7 degrees Kelvin. What is the temperature that JWST is at in Celsius (The conversion from Kelvin to Celsius is $\text{K} = \text{C} + 273.15$ where K is the temperature in Kelvins and C is the temperature in Celsius).

Solution. The temperature in Celcius is just $7 - 273.15 = \boxed{-266.15^\circ\text{C}}$. □

22. Oh no! Nicholas and Farhana have discovered an asteroid heading straight towards the JWST! If they calculated that it will take $30 \times (45-3)/14 \times 7 + 3$ hours for the asteroid to hit the telescope, how long does the JWST have until being annihilated from existence?

Solution. The 7 and 14 cancel to give one half, so the expression equals

$$30 \cdot 42/2 + 3 = 30 \cdot 21 + 3 = \boxed{633 \text{ hours}}.$$

□

23. If Linda and Khawla are able to take 47 pictures each hour with the JWST, using the information from question 22, how many pictures can they take before the asteroid hits the telescope?

Solution. They can take a total of $47 \cdot 633 = \boxed{29751 \text{ pictures}}$. □

24. The pictures are then analyzed by Jay who adjusts the color wavelengths so that humans can see the pictures. JWST only detects infrared wavelengths around 1000 nanometers (nm). If the visible light wavelengths range from 380 nm to 740 nm. What is the range of the visible light wavelengths?

Solution. The range is just $740 \text{ nm} - 380 \text{ nm} = \boxed{360 \text{ nm}}$. □

25. While on a cruise ship, Cruz claims that he is able to spot the JWST even though it is 1.5 million kilometers away. He convinces Grace to look up in the sky for it, but she only has her eyes focused on a seagull flying. The seagull is flying 9 miles above the cruise ship in the opposite direction at a constant 21 mph, and the cruise ship is traveling in the opposite direction from the seagull at a constant 39 mph. After 40 minutes from the point where the seagull is directly above Grace, what is the distance between the seagull and the cruise?

Solution. The seagull is traveling at $21 + 39 = 60$ mph in the opposite direction relative to the cruise ship. Then after 40 minutes, the horizontal distance between the seagull and the ship is 40 miles, so the total distance is $\sqrt{9^2 + 40^2} = \boxed{41 \text{ miles}}$. □

26. Chanith and Shreeyan (Group 1) are in a race against Miaohan and Nonoko (Group 2) to count the number of stars in the sky that they can see. Chanith and Shreeyan collectively count 19874 stars while Miaohan and Nonoko each count 9947 stars. What is the absolute difference between the number of stars the two groups counted?

Solution. This is just

$$|19874 - 2 \cdot 9947| = |19874 - 19894| = |-20| = \boxed{20}.$$

□

27. Elijah says that the answer to the next question is not C. David says that the answer to the next question is A. Elijah and David are both lying.

Solution. Looking at the next question, the answer is $\boxed{\text{D}}$. □

28. The answer to the previous question is D.

Solution. Looking at the previous question, we see that Elijah is lying when he says the answer is not C, so the answer is $\boxed{\text{C}}$. □

29. Auska loves rowing! NASA has developed new advanced technology which allows anyone in space to row within a 2^7 mile radius around the JWST. Given the space is 2 dimensional, what is the area of space that someone can row in?

Solution. The area is just a circle with radius 2^7 miles, which is just $\boxed{2^{14} \cdot \pi \text{ miles}^2}$. □

30. If you are seeing this problem, James and Nima did not do a good job of writing this test. How many distinct letters does "James Webb Space Telescope" have?

Solution. It has distinct letters J, A, M, E, S, W, B, P, C, T, L, and O, for a total of $\boxed{12 \text{ letters}}$. □