Algebra Round 2 Problems and Solutions

Chiles Mini Mu

2022-2023

1. This is just $\lceil \frac{103}{3} \rceil = \boxed{35}$.

2. We can rearrange to get $(x-6)^2 = 0$, so $x = \boxed{6}$.

3. The angle between the arms of a clock when it is hour h and minute m is $|30h - 5.5m|$. This is equal to $360 - 33$ when $h = 12, m = 6$, which makes the smaller angle between the arms 33 degrees. This is clearly the earliest time after $11:55$ am when this angle is formed, so the answer is $\boxed{12:06 \text{ pm}}$.

4. This is just

$$3 \text{ bags} \cdot \frac{45 \text{ mins}}{10 \text{ bags}} = \boxed{13.5 \text{ bags}}.$$

5. Lilly packs bags at a rate $\frac{10}{45}$ bags per minute, while Aaron packs bags at a rate of $\frac{4}{22.5} = \frac{8}{45}$ bags per minute. Then their combined rate is $\frac{18}{45} = \frac{2}{5}$ bags per minute, so it takes them $3 \div \frac{2}{5} = \frac{15}{2} = \boxed{7.5 \text{ minutes}}$ to pack 3 bags.

6. We solve $455 = \frac{5}{9}(x-32)$ for x :

$$x - 32 = \frac{9}{5} \cdot 455 = 819 \implies x = 32 + 819 = \boxed{851}.$$

7. Note that $x = \sqrt{2-x}$, so squaring and rearranging gives $0 = x^2 + x - 2 = (x-1)(x+2)$. Since x must be positive, the answer is $x = \boxed{1}$.

8. One can solve this by setting $P(x) = x^2 + ax + b$, and solving the resulting system of equations. But this is more fun: Note that $x = 4, 7$ are the roots of $P(x) + x - 11$, and since P has leading coefficient 1, we have

$$P(x) = (x-4)(x-7) - x + 11 = x^2 - 12x + 39.$$

Then $P(23) = 23^2 - 12 \cdot 23 + 39 = \boxed{292}$.

9. We have

$$\frac{420\sqrt{3}}{\sqrt{6}} = \frac{420}{\sqrt{2}} = \boxed{210\sqrt{2}}.$$

10. The distance between $(11, 15)$ and the line $4x - 3y + 2$ is given by

$$\frac{|4 \cdot 11 - 3 \cdot 15 + 2|}{\sqrt{4^2 + 3^2}} = \boxed{\frac{1}{5}}.$$

11. Note that the relative speed of Bruce with respect to the ball is $17 - 12 = 5$ mph. Then the answer is just

$$50 \text{ ft} \cdot \frac{1\text{mile}}{5280 \text{ ft}} \cdot \frac{1 \text{ hr}}{5 \text{ miles}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} \approx 6.8 \text{ s},$$

so the answer is $\boxed{7 \text{ seconds}}$.

12. After 5 hours and 50 minutes, Bruce is at $5(20 - 6) = 70$ feet. After 7 hours, he goes to 90 feet and then slides down to 84 feet. The remaining 16 feet takes him another $16/20$ hours to get out of the hole. $7 + 16/20 = 7.8$, so the answer is $\boxed{7.8 \text{ hours}}$.

13. From Vieta, $a + b = 7, ab = 1$, so

$$a^2 + b^2 = (a + b)^2 - 2ab = 7^2 - 2 \cdot 1 = 49 - 2 = \boxed{47}.$$

14. This shortest distance is just the distance between $(3, 2)$ and the point $(-5, 4)$ reflected over the line $y = 0$, which is $(-5, -4)$. Then the distance is

$$\sqrt{(3 - (-5))^2 + (2 - (-4))^2} = \sqrt{8^2 + 6^2} = \boxed{10}.$$

15. This is just $\binom{5+3}{3} = \binom{8}{3} = \boxed{56}$.

16. We have

$$70 \text{ marz} \cdot \frac{4 \text{ moks}}{20 \text{ marz}} \cdot \frac{2 \text{ mas}}{7 \text{ moks}} \cdot \frac{2 \text{ mers}}{1 \text{ mas}} = \frac{70 \cdot 4}{5 \cdot 7} \text{ mers} = \boxed{8 \text{ mers}}.$$

17. The answer is just $\left(\frac{3}{2}\right)^5 - 1 = \frac{243}{32} - 1 = \boxed{\frac{211}{32}}$.

18. The answer is just

$$20 \text{ kg} \cdot 9 \div 4 = \boxed{45 \text{ kg}}.$$

19. He either selects at least two rocky planets or at least two gas planets, so from symmetry, the answer is just $\boxed{1/2}$.

20. We have that

$$16t^2 = 2 \cdot 5280 \implies t^2 = 660,$$

so

$$t = \sqrt{660} \text{ s} \approx \sqrt{676} \text{ s} = \boxed{24 \text{ s}},$$

as desired.

21. Let r, s denote the roots of the equation. From Vieta, $r + s = 22, rs = 23$, so

$$\begin{aligned} r^3 + s^3 &= (r + s)(r^2 + s^2 - rs) = (r + s)((r + s)^2 - 3rs) \\ &= 22(22^2 - 3 \cdot 23) = 22 \cdot 415 = \boxed{9130}, \end{aligned}$$

as desired.

22. The vertices of the bounded region are $(2, 0), (2, 10), (0, 16), (-2, 10), (-2, 0)$, forming a rectangle topped by a triangle. Then the area is

$$4 \cdot 10 + 4 \cdot 6/2 = 40 + 12 = \boxed{52},$$

as desired.

23. When only Nelson is working, it takes $168/6 = 28$ minutes to fill the spaceship. When both Arib and Nelson are working, they gather $2 \cdot 6 + 9 = 21$ rocks every two minutes, so it takes them $2 \cdot 168/21 = 16$ minutes to fill the spaceship. Then the answer is $28 - 16 = \boxed{12 \text{ mins}}$.

24. Note that $-x^2 + 17x = x(17 - x)$, so the first rock lands 17 feet away from the spaceship. Furthermore, $-x^2 + 17x + 390$ factors as $(30 - x)(17 + x)$, so the second rock lands 30 feet away from the spaceship. Then the answer is $30 - 17 = \boxed{13 \text{ ft}}$.

25. The first term of the arithmetic sequence is 12, and the common difference is $17 - 12 = 5$. Then the eighth term is $12 + 5 \cdot 7 = 47$, so the sum of the first eight terms is $8(12 + 47)/2 = \boxed{236 \text{ ft}}$.

26. This is just $11!/3! = \boxed{6652800}$.

27. Suppose we have prime $p = n^2 - 2$. Note that $p = 2$ works when $n = 2$, so suppose p is odd. Then n must be odd, so we need only check $3^2 - 2 = 7, 5^2 - 2 = 23, 7^2 - 2 = 47, 9^2 - 2 = 79$, all of which are prime. Then the answer is just

$$2 + 7 + 23 + 47 + 79 = \boxed{158}.$$

28. The total number of signatures he gets is

$$\begin{aligned}\sum_{n=0}^{100} n(100 - n) = 100 \sum_{n=0}^{100} - \sum_{n=0}^{100} n^2 &= (100)(100)(101)/2 - (100)(101)(201)/6 \\ &= (100)(101)(100/2 - 201/6) = (100)(101)(33/2) = 166,650.\end{aligned}$$

Therefore he does **not** get enough signatures, and was off by

$$200,000 - 166,650 = \boxed{33,350}$$

signatures.

29. On Earth, d is maximized when $t = 3/10$, which gives $d = 9/20$ m $= 45$ cm. On Mercury, d is maximized when $t = 3/4$, which gives $d = 9/8$ m $= 112.5$ cm. Then the answer is $112.5 - 45 = \boxed{67.5 \text{ cm}}$.

30. We have $2023 = 3747_8$, so the answer is $3 + 7 + 4 + 7 = \boxed{21}$.

31. It takes Hadriel $(1.8 \cdot 10^9)/10^6 = 1.8 \cdot 10^3 = 1800$ seconds to reach Uranus. It takes Rohan

$$\frac{1}{2}\left(\frac{1.8 \cdot 10^9}{5 \cdot 10^5} + \frac{1.8 \cdot 10^9}{1.5 \cdot 10^6}\right) = \frac{1}{2}(3600 + 1200) = 2400 \text{ s}.$$

Thus, **Hadriel** arrives first, by $2400 - 1800 = \boxed{600 \text{ s}}$.

32. Note that the quadratic $ax^2 + bx + c$ has discriminant $b^2 - 4ac$. Note that perfect squares are either equivalent to 0 or 1 modulo 4, so $n = b^2 - 4ac$ must also be equivalent to either 0 or 1 modulo 4. Then the answer is just $100/2 = \boxed{50}$.