制御系構成特論 レポート課題

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問題

厳密モデル $\tilde{P}(s)$，ノミナルモデル $P_n(s)$ が

$$\begin{aligned}\tilde{P}(s) &= \frac{2}{(s+1)(s-2)} \quad (1)\\ P_n(s) &= \frac{1}{s+1} \quad (2)\end{aligned}$$

で表されるとき，左既約分解的不確かさ $\Delta N_l(s)$，$\Delta D_l(s)$，加法的不確かさ $\Delta_a(s)$，乗法的不確かさ $\Delta_m(s)$ を求めよ．ただし，

$$\begin{aligned}N_l(s) &= \frac{n_1}{s+4} \quad (3)\\ D_l(s) &= \frac{d_1}{s+4} \quad (4)\\ \Delta N_l(s) &= \frac{n_2}{(s+2)(s+3)} \quad (5)\\ \Delta D_l(s) &= \frac{d_2}{(s+2)(s+3)} \quad (6)\end{aligned}$$

とする．

解答

$\Delta_a(s)$，$\tilde{P}(s)$ および $P_n(s)$ は以下の関係式を満たす．

$$\tilde{P}(s) = P_n(s) - \Delta_a(s) \quad (7)$$

これを式変形すると $\Delta_a(s)$

$$\begin{aligned}\Delta_a(s) &= \tilde{P}(s) - P_n(s)\\ &= \frac{2}{(s+1)(s-2)} - \frac{1}{s+1}\\ &= \frac{-s+4}{(s+1)(s-2)} \quad (8)\end{aligned}$$

を得る．また，$\Delta_m(s)$，$\tilde{P}(s)$ および $P_n(s)$ は以下の関係式を満たす．

$$\tilde{P}(s) = (1+\Delta_m(s))P_n(s) \quad (9)$$

これを式変形すると $\Delta_m(s)$

$$\begin{aligned}\Delta_m(s) &= \tilde{P}(s)P_n^{-1}(s)-1 &(10)\\ &= \frac{2(s+1)}{(s+1)(s-2)}-1 &(11)\\ &= \frac{-s+4}{s-2} &(12)\end{aligned}$$

を得る．

次に，$\Delta N_l(s)$，$\Delta D_l(s)$ を求める．$D_l(s), N_l(s)$ の分母は $(s+4)$ であるので，それぞれ

$$\begin{aligned}D_l(s) &= \frac{d_1}{s+4} &(13)\\ N_l(s) &= \frac{n_1}{s+4} &(14)\end{aligned}$$

とおく．すると

$$\begin{aligned}P_n(s) &= D_l^{-1}(s)N_l(s) &(15)\\ \frac{1}{s+1} &= \frac{s+4}{d}\cdot\frac{n}{s+4} &(16)\end{aligned}$$

より，$d_1 = s+1, n_1 = 1$ となり

$$\begin{aligned}D_l(s) &= \frac{s+1}{s+4} &(17)\\ N_l(s) &= \frac{1}{s+4} &(18)\end{aligned}$$

が求まる．また，$\Delta N_l(s), \Delta D_l(s)$ の分母は $(s+2)(s+3)$ であるのでそれぞれ

$$\begin{aligned}\Delta N_l &= \frac{n_2}{(s+2)(s+3)} &(19)\\ \Delta D_l &= \frac{d_2}{(s+2)(s+3)} &(20)\end{aligned}$$

とおく．ここで，$\Delta N_l(s), \Delta D_l(s)$ は

$$\begin{aligned}\Delta N_l(s) &= \tilde{N}_l(s) - N_l(s) &(21)\\ \Delta D_l(s) &= \tilde{D}_l(s) - D_l(s) &(22)\end{aligned}$$

で表される．(21) 式より，

$$\begin{aligned}\frac{n_2}{(s+2)(s+3)} &= \tilde{N}_l(s) - \frac{1}{s+4}\\ \tilde{N}_l(s) &= \frac{n_2(s+4)+(s+2)(s+3)}{(s+2)(s+3)(s+4)} &(23)\end{aligned}$$

を得る．同様に（22）式より，

$$
\begin{aligned}
\frac{d_2}{(s+2)(s+3)} &= \tilde{D}_l(s) - \frac{s+1}{s+4} \\
\tilde{D}_l(s) &= \frac{d_2(s+4)+(s+1)(s+2)(s+3)}{(s+2)(s+3)(s+4)} \qquad (24)
\end{aligned}
$$

を得る．$\tilde{P}(s), \tilde{D}_l(s), \tilde{N}_l(s)$ の関係式より，

$$
\begin{aligned}
\tilde{P}(s) &= \tilde{D}_l^{-1}(s)\tilde{N}_l(s) \qquad (25) \\
\frac{2}{(s+1)(s-2)} &= \frac{(s+2)(s+3)(s+4)}{d_2(s+4)+(s+1)(s+2)(s+3)} \cdot \frac{n_2(s+4)+(s+2)(s+3)}{(s+2)(s+3)(s+4)} \\
&= \frac{n_2(s+4)+(s+2)(s+3)}{d_2(s+4)+(s+1)(s+2)(s+3)} \qquad (26)
\end{aligned}
$$

となり，

$$
n_2(s+4) = 2-(s+2)(s+3) \qquad (27)
$$

$$
d_2(s+4) = (s+1)(s-2)-(s+1)(s+2)(s+3) \qquad (28)
$$

を得る．(21) 式より，$\Delta N_l(s)$ は

$$
\begin{aligned}
\Delta N_l(s) &= \tilde{N}_l(s) - N_l(s) \\
&= \frac{n_2(s+4)+(s+2)(s+3)}{(s+2)(s+3)(s+4)} - \frac{1}{s+4} \\
&= \frac{2-(s+2)(s+3)+(s+2)(s+3)}{(s+2)(s+3)(s+4)} - \frac{1}{s+4} \\
&= \frac{-(s+1)}{(s+2)(s+3)} \qquad (29)
\end{aligned}
$$

となり，また，(22) 式より $\Delta D_l(s)$ は，

$$
\begin{aligned}
\Delta D_l(s) &= \tilde{D}_l(s) - D_l(s) \\
&= \frac{d_2(s+4)+(s+1)(s+2)(s+3)}{(s+2)(s+3)(s+4)} - \frac{s+1}{s+4} \\
&= \frac{(s+1)(s-2)-(s+1)(s+2)(s+3)+(s+1)(s+2)(s+3)}{(s+2)(s+3)(s+4)} - \frac{s+1}{s+4} \\
&= \frac{(s+1)(s-2)-(s+1)(s+2)(s+3)}{(s+2)(s+3)(s+4)} \\
&= \frac{-s^3-5s^2-12s-8}{(s+2)(s+3)(s+4)} \qquad (30)
\end{aligned}
$$

となる．