制御系構成特論 レポート課題3

機械知能工学専攻　16344217　津上 祐典

課題1.

$$P(s) = \frac{1}{s-1} \tag{1}$$

のとき，また

$$N_r = N_l = \frac{1}{s+1} \tag{2}$$

$$D_r = D_l = \frac{s-1}{s+1} \tag{3}$$

であるとき，ベズー等式の一般解を求めよ．

解答

ベズーの等式は

$$X_r N_r + Y_r D_r = I_m \tag{4}$$

$$N_l X_l + D_l Y_l = I_p \tag{5}$$

で表される．N_r，N_l，D_r，D_l を代入すると，

$$X_r \cdot \frac{1}{s+1} + Y_r \cdot \frac{s-1}{s+1} = 1 \tag{6}$$

$$X_l \cdot \frac{1}{s+1} + Y_l \cdot \frac{s-1}{s+1} = 1 \tag{7}$$

となり，

$$X_r = X_l = 2 \tag{8}$$

$$Y_r = Y_l = 1 \tag{9}$$

を得る．これよりベズー等式を満足する解 $\bar{X}_r, \bar{Y}_r, \bar{X}_l, \bar{Y}_l$ を

$$\bar{X}_r = \bar{X}_l = 2 = \frac{2s+4}{s+2} \tag{10}$$

$$\bar{Y}_r = \bar{Y}_l = 1 = \frac{s+2}{s+2} \tag{11}$$

とおく．また，自由パラメータ Q, R を

$$Q = R = -\frac{s+1}{s+2} \tag{12}$$

とするとベズー等式の一般解は，それぞれ

$$\begin{aligned} X_r &= \bar{X}_r + QD_l \\ &= \frac{2s+4}{s+2} - \left(\frac{s+1}{s+2}\right)\frac{s-1}{s+1} \\ &= \frac{s+5}{s+2} \end{aligned} \tag{13}$$

$$\begin{aligned} Y_r &= \bar{Y}_r - QN_l \\ &= \frac{s+2}{s+2} + \left(\frac{s+1}{s+2}\right)\frac{1}{s+1} \\ &= \frac{s+3}{s+2} \end{aligned} \tag{14}$$

$$\begin{aligned} X_l &= \bar{X}_l + D_rR \\ &= \frac{2s+4}{s+2} + \frac{s-1}{s+1}\cdot\left(-\frac{s+1}{s+2}\right) \\ &= \frac{s+5}{s+2} \end{aligned} \tag{15}$$

$$\begin{aligned} Y_l &= \bar{Y}_l - N_rR \\ &= \frac{s+2}{s+2} + \frac{1}{s+1}\left(\frac{s+1}{s+2}\right) \\ &= \frac{s+3}{s+2} \end{aligned} \tag{16}$$

となる．

課題 2.

$$P(s) = \frac{1}{s-1} = [1, 1, 1, 0] \tag{17}$$

において，$H = F = 2$ としたときの 2 重既約分解表現を求めよ．

解答

$P(s)$ をドイルの記法で表現すると，

$$P(s) = \frac{1}{s-1} = \left[\begin{array}{c|c} 1 & 1 \\ \hline 1 & 0 \end{array}\right] = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{18}$$

となる．ここで

$$A_H = A - HC = 1 - 2 \cdot 1 = -1 \tag{19}$$

$$A_F = A - BF = 1 - 1 \cdot 2 = -1 \tag{20}$$

$$B_H = B - HD = 1 - 2 \cdot 0 = 1 \tag{21}$$

$$C_F = C - DF = 1 - 0 \cdot 2 = 1 \tag{22}$$

であるから二重既約分解表現すると

$$\left[\begin{array}{c|c} Y_r & X_r \\ \hline -N_l & D_l \end{array}\right] = \left[\begin{array}{c|cc} A_H & B_H & H \\ \hline F & I_m & 0 \\ -C & -D & I_p \end{array}\right] = \left[\begin{array}{c|cc} -1 & 1 & 2 \\ \hline 2 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right] \tag{23}$$

$$\left[\begin{array}{c|c} D_r & -X_l \\ \hline N_r & Y_l \end{array}\right] = \left[\begin{array}{c|cc} A_F & B & H \\ \hline -F & I_m & 0 \\ C_F & -D & I_p \end{array}\right] = \left[\begin{array}{c|cc} -1 & 1 & 2 \\ \hline -2 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right] \tag{24}$$

となる．

課題3.

$$P(s) = \frac{1}{s-1} = [1,1,1,0] = \left[\begin{array}{c|c} 1 & 1 \\ \hline 1 & 0 \end{array}\right] \tag{25}$$

において，正規化右・左既約分解表現を求めよ．

解答

はじめにリッカチ方程式を解くと

$$X(A - BR^{-1}D^{\mathrm{T}}C) + (A - BR^{-1}D^{\mathrm{T}}C)^{\mathrm{T}}X - XBR^{-1}B^{\mathrm{T}}X + C^{\mathrm{T}}\tilde{R}^{-1}C = 0 \tag{26}$$

$$X(1 - 1\cdot 1\cdot 0\cdot 1) + (1 - 1\cdot 1\cdot 0\cdot 1)X - X\cdot 1\cdot 1\cdot 1\cdot X + 1\cdot 1\cdot 1 = 0 \tag{27}$$

$$-X^2 + 2X + 1 = 0 \tag{28}$$

$X > 0$ より解は $X = 1 + \sqrt{2}$ となる．また，Y に関するリッカチ方程式は

$$(A - BD^{\mathrm{T}}\tilde{R}^{-1}C)Y + Y(A - BD^{\mathrm{T}}\tilde{R}^{-1}C)^{\mathrm{T}} - YC^{\mathrm{T}}\tilde{R}^{-1}CY + BR^{-1}B^{\mathrm{T}} = 0 \tag{29}$$

$$(1 - 1\cdot 0\cdot 1\cdot 1)Y + Y(1 - 1\cdot 0\cdot 1\cdot 1) - Y\cdot 1\cdot 1\cdot 1\cdot Y + 1\cdot 1\cdot 1 = 0 \tag{30}$$

$$-Y^2 + 2Y + 1 = 0 \tag{31}$$

となり，解は $Y > 0$ より，$Y = 1 + \sqrt{2}$ となる．得られた X, Y より，

$$F = R^{-1}(D^{\mathrm{T}}C + B^{\mathrm{T}}X) = 1(0\cdot 1 + 1\cdot X) = 1 + \sqrt{2} \tag{32}$$

$$H = (BD^{\mathrm{T}} + YC^{\mathrm{T}})\tilde{R}^{-1} = (1\cdot 0 + Y\cdot 1) = 1 + \sqrt{2} \tag{33}$$

$$A_F = A - BF = 1 - 1 - \sqrt{2} = -\sqrt{2} \tag{34}$$

$$A_H = A - HC = 1 - 1 - \sqrt{2} = -\sqrt{2} \tag{35}$$

$$C_F = C + DF = 1 \tag{36}$$

$$B_H = B + DH = 1 \tag{37}$$

が求まり正規化右，左規約分解表現はそれぞれ，

$$
\begin{bmatrix} D_r \\ N_r \end{bmatrix} = \left[\begin{array}{c|c} A_F & BR^{-1/2} \\ \hline -F & R^{-1/2} \\ C_F & DR^{-1/2} \end{array}\right] = \left[\begin{array}{c|c} -\sqrt{2} & 1 \\ \hline -1-\sqrt{2} & 1 \\ 1 & 0 \end{array}\right] \tag{38}
$$

$$
\begin{bmatrix} D_l \\ N_l \end{bmatrix} = \left[\begin{array}{c|cc} A_H & B_H & -H \\ \hline \tilde{R}^{-1/2}C & \tilde{R}^{-1/2}D & \tilde{R}^{-1/2} \end{array}\right] = \left[\begin{array}{c|cc} -\sqrt{2} & 1 & -1-\sqrt{2} \\ \hline 1 & 0 & 1 \end{array}\right] \tag{39}
$$

となる．