Name Answer Key ______________________________ **Math 226 Test 3 Review**

Show all of your work when solving the following problems.

Give a counterexample in each case in which the relation does not satisfy one of the properties: reflexive, symmetric, transitive and antisymmetric.

1) $R_1 = \{(0,0),(0,1),(0,3),(1,1),(1,0),(2,3),(3,3)\}$

R_1 is not reflexive. $(2,2) \notin R_1$

R_1 is not symmetric. $(0,3) \in R_1$ but $(3,0) \notin R_1$

R_1 is not transitive $(1,0) \in R_1 \wedge (0,3) \in R_1$ but $(1,3) \notin R_1$

R_1 is not antisymmetric $(1,0) \in R_1 \wedge (0,1) \in R_1$ but $0 \neq 1$

2) $R_2 = \{(0,0),(0,1),(1,1),(1,2),(2,2),(2,3)\}$

R_2 is not reflexive because $(3,3) \notin R_2$

R_2 is not symmetric because $(0,1) \in R_2$ but $(1,0) \notin R_2$

R_2 is not transitive because $(0,1) \in R_2 \wedge (1,2) \in R_2$ but $(0,2) \notin R_2$

R_2 is antisymmetric because it does not contain pairs of symmetric elements, for example $(0,1) \in R_2$ but $(1,0) \notin R_2$, $(1,2) \in R_2$ but $(2,1) \notin R_2$

3) $R_3 = \{(2,3),(3,2)\}$

R_3 is not reflexive $(2,2) \notin R_3$

R_3 is symmetric $(2,3) \in R_3 \wedge (3,2) \in R_3$

R_3 is not transitive $(2,3) \in R_3 \wedge (3,2) \in R_3$ but $(2,2) \notin R_3$

R_3 is not antisymmetric $(2,3) \in R_3 \wedge (3,2) \in R_3$ but $2 \neq 3$.

For problems 4-7, let A be a set with eight elements.

4) How many binary relations are there on A?

There are $8 \times 8 = 64$ possible pairs (a,b) with $a + b$ in A

There are 2^{64} possible relations on A (subsets of the set with 64 pairs).

5) How many binary relations on A are reflexive?

A relation is reflective if $(a,a) \in R \ \forall a \in A$.
There are 8 possible pairs of the form (a,a).

That means all the reflexive relations must contain all 8 pairs of the form (a,a). In addition, they may contain any possible pair of the form (a,b) s.t. $a \neq b$.
There are $8 \cdot 7 = 56$ possible such pairs, and so there are 2^{56} possible subsets that include all pairs of the form (a,a) and any any subset of pairs of the form (a,b) w $a \neq b$.

6) How many binary relations on A are symmetric?

A relation is symmetric if it contains (b,a) whenever it contains (a,b).

There are $8 \cdot 7$ possible pairs (a,b) with $a \neq b$

Smaller example to help understand

For example, on a set with 3 (not 8) elements, there are $3 \cdot 2$ possible pairs. Suppose $A = \{1,2,3\}$ The total number of pairs with $a \neq b$ are $\{(1,2), (1,3), \cancel{(2,1)}, (2,3), \cancel{(3,1)}, \cancel{(3,2)}\}$
There are 2^6 possible subsets of this set, however if a relation contains $(1,2)$, it must also contain $(2,1)$. So to count the number of unique subsets we can remove the 2nd pair. We see that we can form 2^3 subsets of the set $\{(1,2), (1,3), (2,3)\}$, or $2^{\frac{3 \cdot 2}{2}}$

Similarly, for a set A with 8 elements we can form $2^{\frac{8\cdot 7}{2}}$ distinct subsets.

Symmetric relations may also include reflexive elements such as $(1,1)$, $(2,2)$, etc.

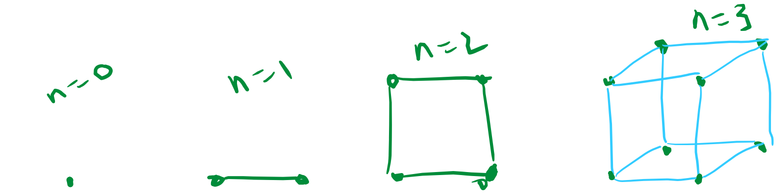
There are 8 possible pairs of elements (a,b) where $a=b$

So there are 2^8 possible subsets containing elements of the type (a,a).

A symmetric relation can contain any element of the type (a,a) AND any pair (a,b), $a \neq b$ as long as (b,a) is also present.

So in total there are $2^8 \cdot 2^{\frac{8\cdot 7}{2}} = 2^{8+\frac{8\cdot 7}{2}} = 2^{36}$ possible binary relations that are symmetric.

7) Q_n has $n \cdot 2^{n-1}$ edges and 2^n vertices.

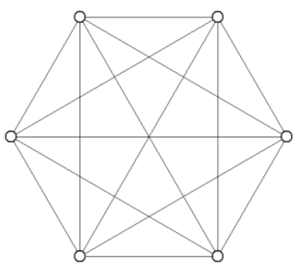


1 vertex $= 2^0$ | 2 vertices $= 2^1$ | 4 vertices $= 2^2$ | 8 vertices $= 2^3$

0 edges $= 0 \cdot 2^{-1}$ | 1 edge $= 1 \cdot 2^0$ | 4 edges $= 2 \cdot 2^1$ | 12 edges $= 3 \cdot 2^2$

8) Find a graph representation for the graph K_6.

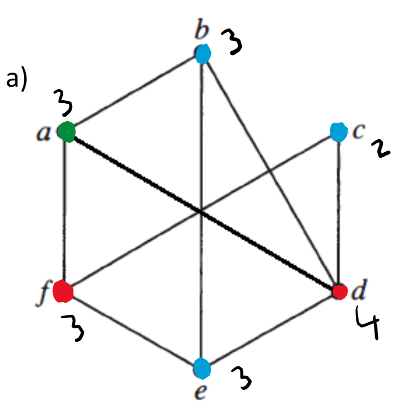
The ordered pair representation of K6 is (v,e) or (6, 5+4+3+2+1) or (6,15).

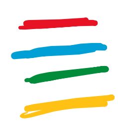


9) What is the chromatic number for each of the following graphs? That is, what is the fewest number of colors given to vertices so that no two adjacent vertices are the same color.

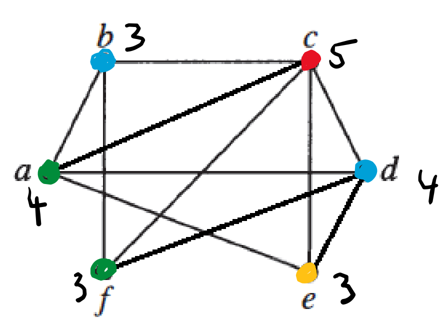
Color Priority List

a)





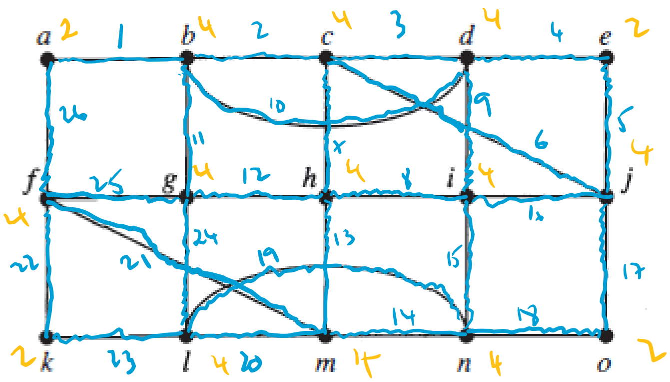
b)



The chromatic number of this graph is 3.

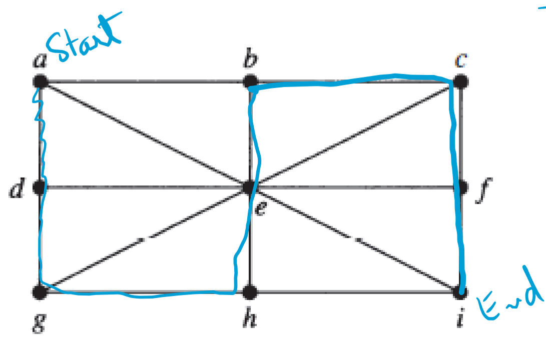
The chromatic number of this graph is 4.

10) Does the graph contain an Euler Path or Euler circuit? Explain how you know. If one exists, find it.



An Euler circuit exists because all vertices are even

11) Is there a Hamilton Circuit in the following graph (if so, show it)? If not, is there a Hamilton Path (if so, show it)?



There is no Hamilton circuit

A Hamilton Path exists.

12)Construct the **Dual graph** of the following map, then color it and the map with as few colors as possible.

Priority List of Colors