Michael Chillemi quiz 4

1. $f(t) = \frac{7}{9}t^6 - 2t^4 + 4t$

$6\left(\frac{7}{9}\right)$ $\frac{6}{1}\left(\frac{7}{9}\right)$

$\frac{42 \div 3}{9 \div 3} = \frac{14}{3}$

$\frac{d}{dt}\ \frac{7}{9}t^6 - 2t^4 + 4t$

$6\left(\frac{7}{9}\right)t^5 - (4)(2)t^3 + 4$

$F'(t) = \frac{14}{3}t^5 - 8t^3 + 4$

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2. $h(x) = (x-3)(4x+6)$

$(x-3)(4x+6)$

$4x^2 + 6x - 12x - 18$

$\frac{d}{dx}\ 4x^2 - 6x - 18$

$= (2)(4)x - 6$

$h'(x) = 8x - 6$

3. $y = 4e^x + \frac{8}{\sqrt[3]{x}}$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} 4e^x + \frac{d}{dx} \frac{8}{\sqrt[3]{x}}$

$\frac{1}{a} = a^{-1}$ exponent

$\frac{d}{dx} 4e^x = 4e^x$ ✓

$\frac{d}{dx} \frac{8}{\sqrt[3]{x}} = 8 \frac{d}{dx} \left(\frac{1}{\sqrt[3]{x}} \right)$

$= x^{-\frac{1}{3}}$

$= 8\left(\frac{-1}{3} x^{-\frac{1}{3}-1}\right)$

$= \frac{-8}{3x^{4/3}}$

$y' = 4e^x - \frac{8}{3x^{4/3}}$

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4 $y = \frac{8x^2 + 6x + 6}{\sqrt{x}}$

quotient rule

$\left(\frac{f}{g}\right)' \frac{f \cdot g - g \cdot f}{g^2}$

$\frac{d}{dx} \frac{(8x^2+6x+6)\sqrt{x} - \frac{d}{dx}(\sqrt{x})(8x^2+6x+6)}{(\sqrt{x})^2}$

$= 16 + 6$

$\frac{d}{dx} = \sqrt{x} = \frac{1}{2\sqrt{x}}$

$= \frac{16x+6(\sqrt{x}) - \left(\frac{1}{2\sqrt{x}}\right)(8x^2+6x+6)}{(\sqrt{x})^2}$

$y' = \frac{12x^2 + 3x - 3}{x\sqrt{x}}$

5. $y = x^4 + 6x^2 - x$ $(1, 6)$

$$\frac{d}{dx} x^4 + 6x^2 - x$$

$$\boxed{4x^3 + 12x - 1}$$

$f'(x) = 4x^3 + 12x - 1$

$f(1) = 4(1)^3 + 12(1) - 1$

$4 + 12 - 1$

$= 15$

m = slope

$y - y_1 = m(x - x_1)$

$(x_1, y_1) = (1, 6)$

$m = 15$

$y - 6 = 15(x - 1)$

$y - 6 = 15x - 15$

$+6 \quad +6$

$\boxed{y = 15x - 9}$

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6. $b(r) = \sqrt{r} + \sqrt[8]{r}$

$\frac{d}{dr} r^{\frac{1}{2}} + r^{\frac{1}{8}}$

$= \frac{1}{2} r^{\frac{1}{2}-1} + \frac{1}{8} r^{\frac{1}{8}-1}$

$= \frac{1}{2} r^{-\frac{1}{2}} + \frac{1}{8} r^{-\frac{7}{8}}$

$\boxed{b'(r) = \frac{1}{2r^{\frac{1}{2}}} + \frac{1}{8r^{7/8}}}$

$b''(r) = \frac{1}{2} r^{-\frac{1}{2}} + \frac{1}{8} r^{-\frac{7}{8}}$

$-\frac{1}{2}\left(\frac{1}{2}\right) r^{-\frac{1}{2}-1} + \left(-\frac{7}{8}\right)\left(\frac{1}{8}\right) r^{-\frac{7}{8}-1}$

b''

$b'' =$

$\boxed{b'' = -\frac{1}{4r^{3/2}} - \frac{7}{64 r^{\frac{15}{8}}}}$

7. $y = x^2 - 7x + 1$ $\quad x - 3y = 5$

$y = mx + b$

$x = a$

$x = 5 + 3y$

$3y = x - 5$

$y = \frac{1}{3}x - \frac{5}{3}$

$\frac{d}{dx} = 2x - 7$

$2a - 7$

$\frac{-1}{f'(a)} = -\frac{1}{2a-7}$

$-\frac{1}{2a-7} = \frac{1}{3}$

$2a - 7 = -3$

$+3 \quad +3$

$2a - 4 = 0$

$+4 \quad +4$

$\frac{2a}{2} = \frac{4}{2}$

$a = 2$

$(2, f(2))$

$(2, (2)^2 - 7(2) + 1)$

$(2, -9)$

$y - y_1 = m(x - x_1)$

$y - (-9) = \frac{1}{3}(x - 2)$

$y + 9 = \frac{x}{3} - \frac{2}{3} - 9$

-9

$y = \frac{x}{3} - \frac{29}{3}$

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6. $f(t) = \frac{3t}{7 + t^2}$

$= 3\frac{d}{dx}\left(\frac{t}{7 + t^2}\right)$

$= 3 \cdot \frac{d}{dt}(t)\frac{(7 + t^2) - \frac{d}{dt}(7 + t^2)t}{(7 + t^2)^2}$

$= 3 \cdot \frac{1 \cdot (7 + t^2) - 2tt}{(7 + t^2)^2}$

$= \frac{3(7 - t^2)}{(7 + t^2)^2}$

$= \frac{21 - 3t^2}{(7 + t^2)^2}$

9. $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 7y^3)$

$= \frac{d}{dy}\left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 7y^3) + \frac{d}{dy}(y + 7y^3)\left(\frac{1}{y^2} - \frac{3}{y^4}\right)$

$\frac{d}{dy}\left(\frac{1}{y^2} - \frac{3}{y^4}\right) = -\frac{2}{y^3} + \frac{12}{y^5}$

$\frac{d}{dy}(y + 7y^3) = 1 + 21y^2$

$= \left(-\frac{2}{y^3} + \frac{12}{y^5}\right)(y + 7y^3) + (1 + 21y^2)\left(\frac{1}{y^2} - \frac{3}{y^4}\right)$

$= \frac{20}{y^2} + \frac{9}{y^4} + 7$

10 $z = u^{3/2}(u + ce^u)$

$\frac{d}{du} u^{3/2}(u + ce^u)$

$\frac{3}{2} u^{\frac{3}{2} - \frac{2}{2}}(u + ce^u)$

$z' = \frac{3\sqrt{u}(ce^u + u)}{2} + u^{\frac{3}{2}}(ce^u + 1)$

$\frac{3}{2} u^{\frac{3}{2}} + ce^u$

$\frac{3}{2} u^2 + ce^u$

11. $y = 5xe^x \qquad (0,0)$

$\frac{d}{dx} 5xe^x$

$5x\frac{d}{dx}e^x + e^x\frac{d}{dx}5x$

$5x\frac{d}{dx}e^x + e^x(5)\frac{d}{dx}(x)$

$= 5xe^x + 5e^x$

$5e^x(x+1)$

$\frac{dy}{dx} = 5e^x(x+1)$

$\left.\frac{dy}{dx}\right|_{(0,0)} = 5e^0(0+1)$

$5(1)$

5

$y - y_1 = m(x - x_1)$

$y - 0 = 5(x-0)$

$y = 5x$

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 0 = \frac{-1}{5}(x-0)$

$y = -\frac{1}{5}x$

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10. $z = u^{3/2}(u + ce^u)$

$\frac{d}{du}\left(u^{3/2}(u+ce^u)\right)$

$= (ce^u + u)\left(\frac{d}{dy}(u^{3/2})\right) + u^{3/2}\left(\frac{d}{dy}(ce^u + u)\right)$

$= \frac{3}{2}\sqrt{u}(ce^u + u) + c\frac{d}{du}(e^u) + \frac{d}{du}(u)\,u^{3/2}$

$= \frac{3}{2}\sqrt{u}(ce^u + u) + u^{3/2}\left(\frac{d}{du}(u) + e^u c\right)$

$z' = \frac{3\sqrt{u}(ce^u + u)}{2} + u^{\left(\frac{3}{2}\right)}(ce^u + 1)$