

1. Assume p : a and b are two even positive integers

~~Assume~~ q : Sum of a and b is even number

If a & b are even, it does not

So even positive number means $a = b = 2n$ mean they =

So Sum of a and b is $(2n) + (2n) = (4n) = 2(2n)$

is even.

2. ~~Case 1~~

$$P(1) \quad 1^2 \geq 1 \quad n^2 \geq n$$

$$1 \geq 1$$

one is equal to one you use the exhaustive proof.

3. Suppose $m^2 = n^2$ $m^2 = n^2$ does not imply $m = n \neq 0$!

therefore $m - n \neq 0$ or $m + n \neq 0$

contradiction! $(m - n)(m + n) \neq 0$

So $m + n \neq 0$ and $m - n \neq 0$ $m^2 + mn - nm - n^2 \neq 0$

$m \neq -n$ and $m \neq n$ $m^2 - n^2 \neq 0$

if $m = n$ or $m = -n$ then $m^2 = n^2$

$$m^2 = n^2$$

thus $m^2 = n^2$ if and only if $m = n$ or $m = -n$

4. Assume n is an integer and n is odd by the definition there is an integer k such that

$$n = 2k + 1 \quad \checkmark$$

$$n^3 + 5 = (2k + 1)^3 + 5 = (8k^3 + 12k^2 + 6k + 1) + 5$$

$$= 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3) \quad \checkmark$$

thus we can find $l = 4k^3 + 6k^2 + 3k + 3$

such that $n^3 + 5 = 2l$

it means $n^3 + 5$ is even. \checkmark

5. $13 \bmod 3$

$$13 / 3 = 4.\overline{3}$$

$$4 \times 3 = 12$$

$$13 - 12 = 1$$

$$\boxed{13 \bmod 3 = 1} \quad \checkmark$$

$$6. -97 \bmod 11$$

$$(-1)$$

$$-97 = 11(-9) + 2$$

$$-97/11 = -8.81$$

$$\text{no. } \dots$$

$$11 \cdot -8 = -88$$

$$-97 + 88 = 9$$

$$-97 \bmod 11 = 9$$

$$\begin{array}{r} -9 \\ 11 \overline{) -97} \\ \underline{99} \\ 2 \end{array}$$

$$7. 21, 43, 55$$

pairwise relatively prime

$$21 = 3 \cdot 7$$

$$43 = 43 \cdot 1$$

$$55 = 5 \cdot 11$$

$$\gcd(21, 43) = 1$$

$$\gcd(21, 55) = 1$$

$$\gcd(43, 55) = 1$$



All are relatively prime because

\gcd is equal to 1.