1 Deviation

Calculate the mean with the following:

$$\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{1}$$

The variance of a sample of measurements y_n is the sum of the square of the differences between the measurements and their mean, divided by $n - 1$.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2 \tag{2}$$

The standard deviation of a sample of measurements is calculated as:

$$s = \sqrt{s^2} \tag{3}$$

2 Combinations

An ordered arrangement of r distinct objects is called a permutation. The number of ways of ordering n distinct objects taken r at a time will be designated as the following:

$$P_r^n = \frac{n!}{(n-r)!} \tag{4}$$

The number of ways of partitioning n distinct objects into k distinct groups:

$$P_n^n = N = \frac{n!}{n_1! n_2! ... n_k!} \tag{5}$$

The number of combinations of n objects taken r at a time is the number of subsets, each of size r, that can be formed from the n objects, denoted as:

$$C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!} \tag{6}$$

3 Conditional Probability

The conditional probability of an event A, given that an event B has occured, is equal to:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{7}$$

The probability of the intersection of two events A and B is:

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) && (8) \\ &= P(B)P(B|A) && (9) \end{aligned}$$

4 Independent Rules

A and B are independent if:

$$\begin{aligned} P(A|B) &= P(A) && (10) \\ P(B|A) &= P(B) && (11) \\ P(A \cap B) &= P(A)P(B) && (12) \end{aligned}$$

5 General Addition Rule

Two arbitrary events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{13}$$

If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B) \tag{14}$$

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3)$$
$$-P(A_2 \cup A_3) + P(A_1 \cap A_2 \cap A_3)$$

6 The Theorem of Total Probability

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + ... + P(A|B_n)P(B_n) \tag{15}$$

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i) \tag{16}$$

7 Bayes Theorem

For two events A and B in sample space S, with $P(A) > 0$ and $P(B) > 0$,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \tag{17}$$

If $0 < P(B) < 1$, we may write by the Theorem of Total Probability.

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')} \tag{18}$$

Reminder about Conditional Probability $P(A|B)$

$$P(A|B) = \frac{B(A \cap B)}{P(B)} \tag{19}$$

Chapter 3

The probabilities assigned to the individual points in the set Y in must carry all the probability followed, which is fillowed by the probability axioms as all the probability from S has been mapped into Y.

$$\sum_{y \;\epsilon\; Y} P(Y = y) = 1 \tag{20}$$

8 Probability Mass Function

The probability mass function (pmf) for Y, p(.), is the mathematical function that records how probability is distributed across points R. If $y \;\epsilon\; Y, p(y) > 0$, otherwise $p(y) = 0$

$$p(y) = P(Y = y) \tag{21}$$

9 Binomial Probability Distribution

A random variable Y is said to have a binomial distribution based on n trials with success probability p iff

$$p(y) = \binom{n}{y} p^y q^{n-y} \tag{22}$$

$$y = 0, 1, 2, ..., n \quad 0 \leq p \leq 1 \tag{23}$$

Let Y be a binomial random variable based on n trials and success probability p. Then:

$$\mu = EY = np \quad \sigma^2 = VY = npq \tag{24}$$

10 Geometric Probability Distribution

A random variable Y is said to have a geometric probability distribution iff

$$p(y) = q^{y-1}p \quad y = 1, 2, 3, ..., \quad 0 \leq p \leq 1 \tag{25}$$

If Y is a random variable with a geometric distribution:

$$\mu = E(Y) = \frac{1}{p} \quad \sigma^2 = V(Y) = \frac{1-p}{p^2} \tag{26}$$

11 The Poisson Probability Distribution

A random variable Y is said to have a Poisson probability distribution iff

$$p(y) = \frac{\lambda^y}{y!}e^{-\lambda} \quad y = 0, 1, 2, ..., \lambda > 0 \tag{27}$$

If Y is a random variable possessing a Poisson distribution with parameter λ, then:

$$\mu = E(Y) = \lambda \quad \sigma^2 = V(Y) = \lambda \tag{28}$$

12 Tchebysheff's Theorem

Let Y be a random variable with mean μ and finite variable σ^2. Then, for any constant $k > 0$,

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad or \quad P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2} \tag{29}$$

Chapter 4

13 Random Variable

Let Y denote any random variable. The distribution function of Y, denoted by F(y), is such that $F(y) = P(Y <= y)$ for $-\infty < y < \infty$
Properties of a Distribution Function: If $F(y)$ is a distribution function, then

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to -\infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y.

A random variable Y with distribution function $F(y)$ is said to be continuous if $F(y)$ is continuous, for $-\infty < y < \infty^2$

If the random variable Y has the density function $f(y)$ and $a < b$, then the probability that Y falls in the interval $[a, b]$ is

$$P(a <= Y <= b) = \int_a^b f(y)\ dy \tag{30}$$

Let $g(y)$ be a function of Y; then the expected value of $g(Y)$ is given by:

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y) f(y) dy \tag{31}$$

Let c be a constant and let $g(Y)$, $g_1(Y)$, $g_2(Y)$, ..., $g_k(Y)$ be functions of a continuous random variable Y. Then the following results hold:

1. $E(c) = c$
2. $E[cg(Y)] = cE[g(Y)]$
3. E$E[g_1(Y) + g_2(Y) + ... + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + ... + E[g_k(Y)]$