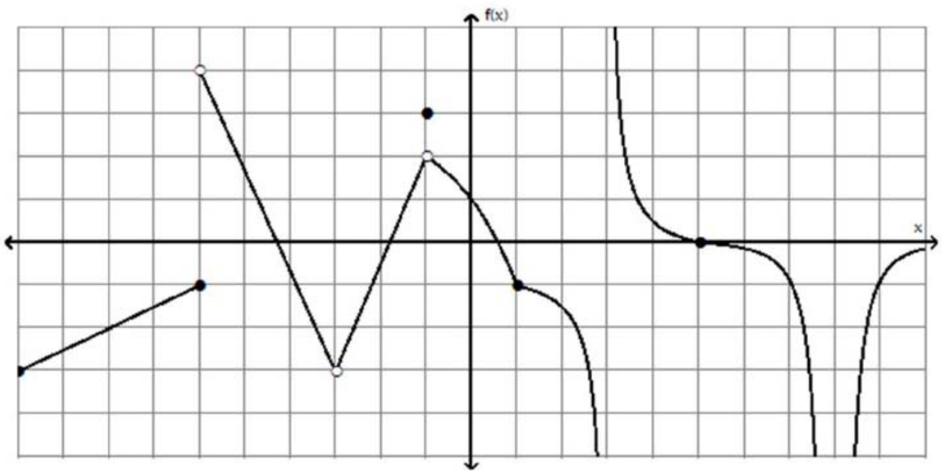
MATH 171
Test 1 In-Class Review Worksheet

Name: ______________________________ **Date:** ___________

Topics: Chapter 1, 2.1 – 2.7

Instructions: Please solve all the problems below. We will NOT be able to go over every problem in class. These review problems are additional practice problems. Use your notes, quizzes, homework problems and the Department's Test 1 review to study for Exam 1.

1. Use the given graph of f to state the value of each quantity, if it exists.



$a)\ \lim_{x\to -6^-} f(x) =$

$b)\ \lim_{x\to -6^+} f(x) =$

$c)\ \lim_{x\to -6} f(x) =$

$d)\ f(-6) =$

$e)\ f(-3) =$

$f)\ \lim_{x\to 3^+} f(x) =$

$g)\ \lim_{x\to 3^-} f(x) =$

$h)\ \lim_{x\to 3} f(x) =$

$i)\ \lim_{x\to 8^-} f(x) =$

$j)\ \lim_{x\to 8^+} f(x) =$

$k)\ \lim_{x\to \infty} f(x) =$

$l)\ \lim_{x\to 5} f(x) =$

2. Use the graph of f from question 1, to answer the following questions:

a) State each x – value at which $f(x)$ is discontinuous. State the type of discontinuity for each given x –value.

b) State each x – value at which f is **NOT** differentiable. Explain.

3. Given the following limits, calculate the limits below, if they exist. You **must show** how you arrived to your answer by using the limit laws.

$$\lim_{x\to 1} g(x) = -8 \qquad \lim_{x\to 1} h(x) = -4$$

$a)\ \lim_{x\to 1} \dfrac{2}{h(x)}$ $\qquad$ $b)\ \lim_{x\to 1} \sqrt[3]{g(x)}$

$c)\ \lim_{x\to 1}[5g(x) - 2h(x)]$ $\qquad$ $d)\ \lim_{x\to 1} g(x)h(x)$

4. Evaluate the given limits.

$a)\ \lim_{x\to 1} \sin\left(\dfrac{\pi x}{2}\right) =$ $\qquad$ $b)\ \lim_{x\to 4} \dfrac{\sqrt{x+1}}{x-4}$

$c)\ \lim_{h\to 4} \dfrac{\sqrt{h+5}-3}{h-4} =$ $\qquad$ $d)\ \lim_{x\to 0}\ e^x\cos(2x) =$

$e)\ \lim_{x\to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x} =$ $\qquad$ $f)\ \lim_{x\to 3} \dfrac{x^2 - x - 6}{x^2 - 5x + 6} =$

$g)\ \lim_{x\to\infty} \dfrac{6x^2 - 2x - 1}{2x^2 + 3x + 2}$ $\qquad$ $h)\ \lim_{x\to\infty} 9e^{-x} + 3$

$i)\ \lim_{x\to 2^+} \dfrac{x-3}{x-2}$ $\qquad$ $j)\ \lim_{x\to 0^-} 1 + \dfrac{1}{x}$

$k)\ \lim_{x\to 0} \dfrac{\sin(7x)}{x}$ $\qquad$ $l)\ \lim_{x\to 0} \dfrac{1-\cos(4x)}{x}$

$m)\ \lim_{x\to 0} \dfrac{x+2}{\cos(x)}$ $\qquad$ $n)\ \lim_{x\to \frac{\pi}{2}} \sin^2 x + \cos^2 x$

$o)\ Given\ h(x) = \dfrac{|x+8|}{2x+16}\ , find\ the\ following:$

$Make\ sure\ to\ define\ the\ function\ as\ a\ piecewise\ function.$

$i)\ \lim_{x\to -8^+} h(x)$ $\qquad$ $ii)\ \lim_{x\to -8^-} h(x)$ $\qquad$ $iii)\ \lim_{x\to -8} h(x)$ $\qquad$ $iv)\ \lim_{x\to 1} h(x)$ $\qquad$ $v)\ \lim_{x\to -11} h(x)$

$$p)\ \lim_{x\to 3} f(x), \quad f(x) = \begin{cases} \dfrac{x+2}{2} & x \le 3 \\ \dfrac{12-2x}{3} & x > 3 \end{cases}$$

5. Use the Squeeze Theorem to find $\lim_{x\to 0} f(x)$ given that $4 - x^2 \le f(x) \le 4 + x^2$.

6. Give the precise mathematical definition for continuity at a point *a* by providing ALL three conditions.

7. Determine if the function is continuous at the indicated value. If the function is not continuous then determine the type of discontinuity. Make sure to support your answer by using the three conditions of continuity.

$a)\ g(x) = 7x^2 - 3x^3 + 4x + 5 \qquad x = 1$

$b)\ h(x) = \dfrac{4x^2 + 3x - 1}{4x - 1} \qquad x = \dfrac{1}{4}$

$c)\ f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & x \neq 5 \\ 0 & x = 5 \end{cases} \qquad x = 5$

$d)\ f(x) = \begin{cases} \dfrac{e^x}{e^x + 1} & x \leq 0 \\ x^2 + 1 & x > 0 \end{cases} \qquad x = 0$

8. For what value of the constant *c* is the function f continuous on (-∞, ∞)?

$a)\ f(x) = \begin{cases} e^{2x+c} & x \geq 0 \\ x + 2 & x < 0 \end{cases}$

$b)\ f(x) = \begin{cases} \sin(x + c) & x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$

9. Determine where the given function is continuous. Make sure to write your answer using interval notation.

$a)\ f(x) = 4 - 3x^3 + 4x^5$

$b)\ g(x) = \dfrac{10}{-3x^2 - 5x + 2}$

$c)\ h(x) = \sqrt{5 - 9x}$

$d)\ y = \cos(x)$

10. Use the Intermediate Value Theorem to show that the function $f(x) = 4x^3 - 6x^2 + 3x - 2$ has a zero in the interval [1,2].

11. Find the equation of the tangent line to the curve at the given point.

$a)\ y = 3x^2 - x \ \ at\ (1,2)$

$b)\ y = \dfrac{8}{x + 4} \ \ at\ (0,2)$

12. Suppose that the height of a projectile is shot vertically upward from a height of 60 feet with an initial velocity of 64 ft per second and is given by $h(t) = -16t^2 + 64t + 80$.

a) Compute the height of the object for $t = 0$.

b) What is the **average velocity** of the projectile for each of the following time intervals?
i. [0,1] ii. [0,0.001]

c) What is the **instantaneous velocity** at $t = 0$?

13. Given a function $f(x)$, write the mathematical definition of its derivative.

14.
a) Use the **definition of the derivative** to find $f'(x)$ where $f(x) = \sqrt{1-9x}$.

b) Find an equation of the tangent line to the curve $f(x) = \sqrt{1-9x}$ at the point $(0,1)$.

15. Sketch the graph of one possible function $f(x)$ for which ALL of the following conditions are true. Write down what each condition tells you about the graph of $f(x)$.

$a)\ \lim_{x\to -4} f(x) = \infty$

$b)\ \lim_{x\to \pm\infty} f(x) = 0$

$c)\ f(0) = 2$

$d)\ \lim_{x\to 2^+} f(x) = -\infty$

$e)\ \lim_{x\to 2^-} f(x) = \infty$

16. Investigate $\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1}$ graphically, numerically and algebraically. Provide all the evidence necessary.

Note: Use converge for the graphical evidence.

Graphical Evidence:

Numerical Evidence:

Left Hand Limit:

x	*f(x)*

Right Hand Limit:

x	*f(x)*

Algebraic Evidence:

Final Conclusion:

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Answer Key

1. **a.** – 1

b. 4

c. DNE

d. – 1

e. undefined

f. ∞

g. $-\infty$

h. DNE

i. $-\infty$

j. $-\infty$

k. 0

l. 0

g. 3

h. 3

i. ∞

j. $-\infty$

k. 7

l. 0

m. 2

n. 1

o. **i.** ½

ii. – ½

iii. DNE since left-hand and right-hand limits are not equal

iv. ½

v. – ½

p. DNE since left-hand and right-hand limits are not equal

2. **a.** x = –6 (jump); x = –3, –1 (removable); x = 3, 8 (infinite)

b. x = –6, –3, –1, 3, 8 (discontinuous); x = 1 (cusp)

3. **a.** –1/2

b. –2

c. –32

d. 32

4. **a.** 1

b. DNE

c. 1/6

d. 1

e. –1/16

f. 5

5. Since $\lim\limits_{x\to 0}\left(4-x^2\right)=\lim\limits_{x\to 0}\left(4+x^2\right)=0,\ \lim\limits_{x\to 0} f(x)=0.$ (Since f is squeezed between the two functions for all values of x.)

6. A function f is continuous at $x = a$ if

i. $f(a)$ exists

ii. $\lim\limits_{x\to a} f(x)$ exists

iii. $\lim\limits_{x\to a} f(x)= f(a)$

7. **a.** Continuous at x = 1 since $\lim\limits_{x\to 1} g(x)=g(1)=13$

b. Not continuous at x = ¼ since $h\left(\frac{1}{4}\right)$ is not defined. Removable discontinuity since $\lim_{x\to 1/4} h(x) = \frac{5}{4}$.

c. Not continuous at x = 5 since $\lim_{x\to 5} f(x) = 10$ but $f(5) = 0$. Removable discontinuity.

d. Not continuous at x = 0 since $\lim_{x\to 0^-} f(x) = \frac{1}{2}$ but $\lim_{x\to 0^+} f(x) = 1$. (So $\lim_{x\to 0} f(x)$ does not exist.) Jump discontinuity. Note: We could say that f is continuous from the left at x = 0 since f(0) = ½ .

8. a. $c = \ln 2$. (Solve the equation $e^c = 2$ to make left-hand and right-hand limits equal at 0.)

b. $c = k\pi - 1$. (Solve the equation $\sin(1+c) = 0$ to make left-hand and right-hand limits equal at 1.)

9. a. $(-\infty, \infty)$

b. $(-\infty, -2) \cup (-2, 1/3) \cup (1/3, \infty)$

c. $(-\infty, 5/9]$

d. $(-\infty, \infty)$

10. Since f(1) = –1 and f(2) = 12, there exists a number $c \in (1,2)$ such that f(c) = 0. In other words, f has a zero in the interval (1, 2).

11. a. $y = 5(x-1) + 2$

b. $y = -\frac{1}{2}x + 2$

12. a. 60 feet

b. i. 48 feet/sec

ii. 63.984 feet/sec

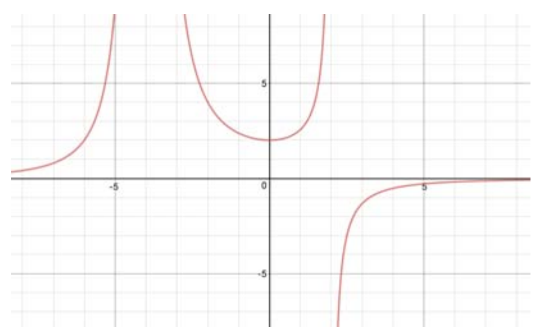
c. 64 feet/sec

13.
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(x) = \lim_{a\to x} \frac{f(a) - f(x)}{a - x}$$

14. a.
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{1-9(x+h)} - \sqrt{1-9x}}{h}$$
$$= \lim_{h\to 0} \frac{-9}{\sqrt{1-9(x+h)} + \sqrt{1-9x}}$$
(after multiplying by the conjugate and simplifying)
$$= \frac{-9}{2\sqrt{1-9x}}$$

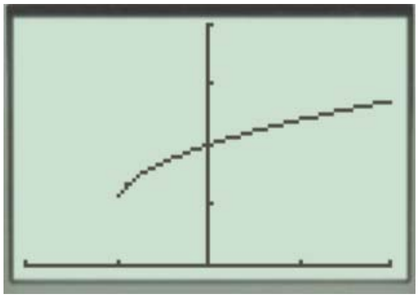
b. $y = -\frac{9}{2}x + 1$

15.



16.

Graphical Evidence



(Window [–2, 2, 1] X [0, 4, 1])

Numerical Evidence

Left-hand limit:

x	$f(x)$
–1	1
–0.1	1.9487
–0.01	1.995
–0.001	1.9995

Right-hand limit:

x	$f(x)$
1	2.4142
0.1	2.0488
0.01	2.005
0.001	2.0005

Algebraic Evidence:

$$\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}$$
$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+1}+1\right)}{x}$$
$$=\lim_{x\to 0}\sqrt{x+1}+1$$
$$=2$$

Final Conclusion:

All evidence suggests that

$$\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}=2.$$