Michael Chillemi midterm

1. $\neg(p \wedge q)$ and $p \leftrightarrow q$

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \wedge q$	$p \leftrightarrow q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	T

Not equivalent

b. $\neg((p \rightarrow q) \wedge \neg p)$ and $\neg q \wedge p$

$\neg((p \rightarrow q) \wedge \neg p) \equiv \neg q \wedge p$

$\neg(\neg p \vee \neg q) \equiv \neg q \wedge p$

$\neg q \wedge p \equiv \neg q \wedge p$

They are equivalent

3. $\neg((p \rightarrow q) \wedge r)$

$\neg((p \rightarrow q) \wedge r) \equiv (p \wedge \neg q) \vee \neg r)$

$\equiv (p \wedge \neg q \vee \neg r)$

b. $\neg[\exists x, \forall y \exists z, P(x,y) \wedge \neg P(x,z)$
$(\forall x, \exists y, \forall z, \neg P(x,y) \vee P(x,z)$

4. $P(x)$ = I will give you 5 dollars
$\neg P(x)$ = I will not give you 5 dollars

I will give you 5 dollars or I will not give you 5 dollars.

This is a true statement because the first statement can be true and the other false which makes it true. Or You do not get 5 dollars and the premise is false which also makes it true.

b "If it snows today, then I will Ski tomorrow"

Contra positive - "If I do not Ski tomorrow then it did not snow today".

c. "No student in this class passes Discrete mathematics"

Negation - ~~"No students in this class pass discrete mathematics~~

"there exists a student that passes discrete math".

5.

a) $\exists_x \; W(mark, x)$

Mark has visited some website

b $\exists_x, \forall_y, (x \neq pete) \wedge [W(pete, y) \rightarrow W(x, y)]$

'Someone that is not pete has visited the same website someone else has

c $\forall_x, \exists_y \quad W(\neg x, y)$

6. $N(x)$: x read their notes
$C(x)$: x is in this class
$T(x)$ x read their textbook

a) $\exists x \quad C(x) \wedge T(x) \wedge N(x)$

b) $\forall x \quad C(x) \wedge T(x) \wedge N(x)$

7. $\exists x, \quad (S(x) \wedge L(x)) \longrightarrow D(x)$

8 $\quad q(x, y) \qquad x + y = x - y$

a) $\exists x \; \forall y \quad q(x, y)$

$x = 0$ doesn't work

$x = 1$ doesn't work

False

b $\exists y \; \forall x \quad q(x, y)$

$x + y = x - y$

$y = 0 \qquad x = 1$

$1 + 0 = 1 - 0$

$1 = 1$

True

c $\forall x, \forall y \quad q(x, y)$

$x + y = x - y$

False

9. a) not valid

failing on negating the premise

b) valid

10. for real numbers x and y
if $x+y \geq 2$ then $x \geq 1$ or $y \geq 1$

proof by contradiction If $x < 1$ and $y < 1$ then $x+y < 2$

11. proof by contradiction
"If x is irrational then $x+1$ is irrational"

Assume x is irrational
$x+1$ is rational

12. if n^2+5 is even the n is odd

~~Direct proof~~

Assume $n = 2k+1$ $k \in \mathbb{Z}$
consider $n^2+5 = (2k+1)^2+5$
$= 4k^2+4k+2+5$
$(2k+1)(2k+1)$
$4k^2+2k+2k+2$
$4k^2+4k+2$

proof by contrapositive
if n is even then n^2+5 is odd

Assume $n = 2k$ $k \in \mathbb{Z}$
Consider
$n^2+5 = (2k)^2+5$
$= 4k+5$
$= 4k+4+1$
$2(2k+2)+1$
$r = 2k+2$
$r \in \mathbb{Z}$
$= 2r+1$
Hence n^2+5 is odd by def.

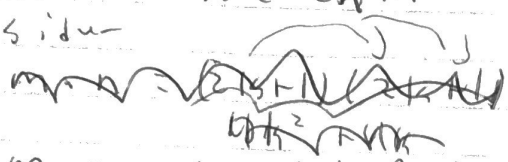
13. The product of 2 odd numbers is odd

Direct proof

assume $m = 2k+1 \quad k \in \mathbb{Z}$

$n = 2l+1 \quad l \in \mathbb{Z}$

consider



$m \cdot n = (2k+1)(2l+1)$

$= 4kl + 2k + 2l + 1$

$= 2(2kl + k + l) + 1$

$r = 2kl + k + l$

$r \in \mathbb{Z}$ $\quad = 2r + 1$

Hence the product of two odd numbers is odd by def.

14. for $x \quad 2x > x$

$\forall x \quad 2x > x$

assume $a \in \mathbb{Z}$

$b \in \mathbb{Z}$

consider $2x > x \qquad x = 2$

$2(2) > (2)$

$4 > 2$ ✓

$x = -2 \qquad 2(-2) > (-2)$

$-4 > -2$ ✗ false

Disprove

15. $A \cap B$

$A \cap B = \emptyset$

$|B \cup C| = 9$

$B \cup C = \{1, 2, 3, 4, 5, 7, 9, 11, 13\}$

$A \times D = \{(x, w), (x, x), (y, w), (y, x), (z, w), (z, x)\}$

$P(A) = \{\{\}, x, y, z, \{x, y\}, \{x, z\}$
$\{y, z\}, \{x, y, z\}$

$D - A = \{w\}$

16. $S - R = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$S \cap R = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

17 $S \subseteq T$

a) $S \cap T$ every element in set S is in set T

a $S \cap T = T$

b) $S - T$

HW A

20 $2 - 4(2 - 4x)$ $g(g(x))$

$2 - 8 + 16x$

$-6 + 16x$

$(f \circ g) - 2$

$(x^2 + 1 + 2 - 4(x))(-2)$

$(x^2 + 1 - 4(x) + 2)$

$-2x^2 + 8x - 2$

21 $h(x) = (2x+10)^2 + 5$ $f(g(x))$

$f(x) = x^2 + 5$

$g(x) = 2x + 10$

18 $\bigcup_{i=1}^{100} A_i \{1, \ldots, 100\}$: $= A_1$ $\bigcap_{i=1}^{100} A_i = \{100\} = A_{100}$