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Quiz 5

1. $y = \dfrac{7+\sin x}{7x+\cos x}$

quotient rule $\dfrac{f'g-g'f}{g^2}$

$$\frac{dy}{dx} = \frac{(7x+\cos x)\cdot\frac{d}{dx}(7+\sin x)-(7+\sin x)\cdot\frac{d}{dx}(7x+\cos x)}{(7x+\cos x)^2}$$

$$= \frac{(7x+\cos x)(\cos x)-(7+\sin x)(7-\sin x)}{(7x+\cos x)^2}$$

$(7+\sin x)(7-\sin x)$

$= 49-\sin^2 x$

$$y' = \frac{7x + \cos x + \cos^2 x - 49 + \sin^2 x}{(7x+\cos x)^2}$$

$$y' = \frac{7x\cos x - 48}{(7x+\cos x)^2}$$

2. $f(\theta) = \dfrac{\sec\theta}{5+\sec\theta}$ $\quad f'(\theta)\dfrac{d}{d\theta}\left(\dfrac{\sec\theta}{5+\sec\theta}\right)$

$$= \frac{(5+\sec\theta)\cdot\frac{d}{d\theta}\sec\theta - \sec\theta\cdot\frac{d}{d\theta}(5+\sec\theta)}{(5+\sec\theta)^2}$$

$$= \frac{(5+\sec\theta)(\sec\theta\cdot\tan\theta)-\sec\theta(\theta+\sec\theta\cdot\tan\theta)}{(5+\sec\theta)^2}$$

$$= \frac{(\sec\theta\cdot\tan\theta)(5+\sec\theta-\sec\theta)}{(5+\sec\theta)^2}$$

$$= \frac{5\sec\theta\cdot\tan\theta}{(5+\sec\theta)^2}$$

3 $f(x) = \frac{\tan x - 1}{\sec x}$ $f'(x) = \frac{\sec x(\sec^2 x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x}$

$= \frac{\sec x((\sec^2 x) - (\tan x - 1)\tan x)}{\sec^2 x}$

$= \frac{\sec^2 x - \tan^2 x + \tan}{\sec x}$

$= \frac{1 + \tan x}{\sec x}$ $= \frac{1}{\sec x} + \frac{\tan x}{\sec x}$

$= \cos x + \sin x$

b) $f(x) = \frac{\tan x - 1}{\sec} = \frac{\tan x}{\sec x} - \frac{1}{\sec x} = \sin x - \cos x$

$f(x) = \sin x - \cos x$ $f'(x) = \cos x - (-\sin x)$

they ~~m~~ $= \cos x + \sin x$

they are equal

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4. $f(t) = \sqrt[9]{1 + \tan t}$

$\frac{1}{9}((1 + \tan(t))^{\frac{1}{9} - 1}(0 + \sec^2 t) = \frac{1}{9}\sec^2 t(1 + \tan(t))^{-\frac{8}{9}}$

$= \frac{\sec^2(t)}{9(1 + \tan(t))^{\frac{8}{9}}}$

5. $y = (5x-2)^4 (9x^2-2)^{-3}$

$$\frac{d}{dx}\left[\frac{(5x-2)^4}{(9x^2-2)^3}\right]$$

$$= \frac{\frac{d}{dx}\left[(5x-2)^4\right] \cdot (9x^2-2)^3 - (5x-2)^4 \cdot \frac{d}{dx}\left[(9x^2-2)^3\right]}{((9x^2-2)^3)^2}$$

$$= \frac{4(5x-2)^3 \cdot \frac{d}{dx}[5x-2] \cdot (9x^2-2)^2 - (5x-2)^4 \cdot 3(9x^2-2)^2 \cdot \frac{d}{dx}[9x^2-2]}{(9x^2-2)6}$$

$$= \frac{4(5x-2)^3\left(5 \cdot \frac{d}{dx}[x] + \frac{d}{dx}[-2]\right)(9x^2-2)^3 - 5(x-2)^4 \cdot 3(9x^2-2)^2\left(9 \cdot \frac{d}{dx}[x^2] + \frac{d}{dx}[-2]\right)}{(9x^2-2)^6}$$

$$= \frac{4(5x-2)^3(5 \cdot 1+0)(9x^2-2)^3 - (5x-2)^4 \cdot 3(9x^2-2)^2(9 \cdot 2x+0)}{(9x^2-2)^6}$$

$$= \frac{20(5x-2)^3(5 \cdot 1+0)(9x^2-2)^3 - 54x(5x-2)^4(9x^2-2)^2}{(9x^2-2)^6}$$

$$= \frac{20(5x-2)^3}{(9x^2-2)^3} - \frac{54x(5x-2)^4}{(9x^2-2)^4}$$

$$y' = -\frac{2(5x-2)^3(45x^2-54x+20)}{(9x^2-2)^4}$$

6 $y = \left(\frac{x^2+4}{x^2-4}\right)^4$ $\frac{d}{dx}(y) = \frac{d}{dx}\left(\left(\frac{x^2+4}{x^2-4}\right)^4\right)$

$= -\frac{64x(x^2+4)^3}{(x^2-4)^5}$

$y' = -\frac{64x(x^2+4)^3}{(x^2-4)^5}$

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7. $y = e^{k \tan \sqrt{5x}}$

$= e^{k \tan(\sqrt{5}\sqrt{x})} \cdot \frac{d}{dx}\left[k \tan(\sqrt{5}(\sqrt{x}))\right]$

$= e^{k \tan(\sqrt{5}\sqrt{x})} \cdot k \frac{d}{dx}\left[\tan(\sqrt{5}\sqrt{x})\right]$

$= e^{k \tan(\sqrt{5}\sqrt{x})} \sec^2(\sqrt{5}\sqrt{x})\, k \cdot \frac{d}{dx}\left[\sqrt{5}\sqrt{x}\right)$

$= e^{k \tan(\sqrt{5}\sqrt{x})} \sec^2(\sqrt{5}\sqrt{x})\, k \cdot \frac{1}{2} x^{\frac{1}{2}-1}$

$y'(x) = \frac{\sqrt{5}\, k \sec^2(\sqrt{5}\sqrt{x})\, e^{k \tan(\sqrt{5}\sqrt{x})}}{2\sqrt{x}}$

8. $y = 10^{\sin(\pi x)}$

$\frac{d}{dx}(a^x) = a^x \log a$

$$\frac{d}{dx}\left[10^{\sin(\pi x)}\right]$$

$$= \ln(10) \cdot 10^{\sin(\pi x)} \cdot \frac{d}{dx}\left[\sin(\pi x)\right]$$

$$= \ln(10) \cdot 10^{\sin(\pi x)} \cos(\pi x) \cdot \frac{d}{dx}[\pi x]$$

$$= \ln(10) \cdot 10^{\sin(\pi x)} \cos(\pi x) \cdot \pi \cdot \frac{d}{dx}[x]$$

$$= \pi \ln(10) \cdot 10^{\sin(\pi x)} \cos(\pi x) \cdot 1$$

$$= \pi \ln(10) \cdot 10^{\sin(\pi x)} \cos(\pi x)$$

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10. $y = 9^{8^{x^2}}$ $\frac{d}{dx}[f(x)] = f'(x) =$

$= \ln(9) \cdot 9^{8^{x^2}} \cdot \frac{d}{dx}[8^{x^2}]$

$= \ln(9) \cdot 9^{8^{x^2}} \ln(8) \cdot 8^{x^2} \cdot \frac{d}{dx}[x^2]$

$= \ln(9) \cdot 9^{8^{x^2}} \ln(8) \cdot 8^{x^2} \cdot 2x$

$= 2\ln(8)\ln(9)x \cdot 8^{x^2} \cdot 9^{8^{x^2}}$

11. if $F(x) = f(g(x))$, where $f(-4) = 9$, $f'(-4) = 3$, $f'(-3) = 5$, $g(-3) = -4$ and $g'(-3) = 8$, find $F'(-3)$

$$f'(x) = f'(g(x)) \frac{d}{dx} g(x)$$

$$f'(x) = f'(g(x)) \cdot g'(x)$$

$$f'(-3) = f'(g(-3)) \cdot g'(-3)$$

$$f'(g(-3)) \cdot 8$$

$$f'(-4) \cdot 8$$

$$3 \cdot 8 = 24$$

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9. $\sqrt{5x + \sqrt{5x + \sqrt{5x}}}$

$$\frac{d}{du}(\sqrt{u}) = \frac{1}{2\sqrt{u}}$$

$$= \frac{d}{du}(\sqrt{u}) \frac{d}{dx}\left(5x + \sqrt{5x + \sqrt{5x}}\right)$$

$$= \frac{1}{2\sqrt{u}}\left(5 + \frac{10\sqrt{x} + \sqrt{5}}{4\sqrt{x}\sqrt{5x + \sqrt{5}\sqrt{x}}}\right)$$

$$\frac{d}{dx}\left(5x + \sqrt{5x + \sqrt{5x}}\right) = 5 + \frac{10\sqrt{x} + \sqrt{5}}{4\sqrt{x}\sqrt{5x + \sqrt{5}\sqrt{x}}}$$

$$= \frac{1}{2\sqrt{5x + \sqrt{5x + \sqrt{5x}}}}\left(5 + \frac{10\sqrt{x} + \sqrt{5}}{4\sqrt{x}\sqrt{5x + \sqrt{5}\sqrt{x}}}\right)$$

$$= \frac{20\sqrt{x}\sqrt{5x + \sqrt{5}\sqrt{x}} + 10\sqrt{x} + \sqrt{5}}{8\sqrt{x}\sqrt{5x + \sqrt{5}\sqrt{x}}\sqrt{5x + \sqrt{5x + \sqrt{5}\sqrt{x}}}}$$

12. $x = e^{\sqrt{t}}$ $f'(t) = \frac{e^{\sqrt{t}}}{2\sqrt{t}}$

$y = t - \ln(t^5)$

$t = 1$

$g'(t) = 1 - \left(\frac{1}{t^5}\right) 5t^4 = 1 - \frac{5}{t}$

$m = \frac{g'(t)}{f'(t)}$ $\frac{1 - \frac{5}{t}}{\frac{e^{\sqrt{t}}}{2\sqrt{t}}}$ $\frac{2t - 10}{\cancel{2}\sqrt{t}}$

$(x_0, y_0) =$

for $t = 1$ $m = \frac{-8}{e}$ $(e^{\sqrt{1}}, 1 - \ln 1^5) =$

$(e, 1)$

$y(x) = \frac{-8}{e} x + 9$

13.

horizontal

$\frac{dy}{dx} = \frac{dx}{dt} \cdot \frac{dt}{dx}$

$x = 2t^3 + 3t^2 - 36t$
$y = 2t^3 + 3t^2 + 1$ $\Big\}$ $\frac{dx}{dt} = 2t^3 + 3t^2 - 36t$

$= 6t^2 + 6t - 36$

$\frac{dy}{dt}\left(\frac{1}{dx/dt}\right)$

$\frac{dy}{dt} = 2t^3 + 3t^2 + 1$

$6t^2 + 6t$

$\frac{dy}{dt} = \frac{6t^2 + 6t}{6t^2 + 6t - 36}$ $\frac{dy}{dt} = 0$

$6t^2 + 6t =$ factor $6t \rightarrow$

for $t = 0$ $(x, y) =$

$6t(t+1) = 0$

Smallest $= t = -1$

$t(t+1) = 0$ $t = 0, t = -1$

$x = 2t^3 + 3t^2 - 36t$

$y = 2t^3 + 3t^2 + 1$

$-2 + 3 + 1 = 2$

$2(-1)^3 + 3(-1)^2 - 36(-1)$

$-2 + 3 + 36 = 0$

$= 37$

$(37, 2)$
Smallest

largest $t = 0$ we get $=$

$y = 2t^3 + 3t + 1$

$0 + 0 + 1 = 1$

$x = 2t^3 + 3t^2 - 36t$

$0 + 0 \cdot 0$

largest
$(0, 1)$

13. b) $\frac{dy}{dx} = \infty$ when parallel slope $= \infty$

$\frac{dy}{dx} = \infty \rightarrow \frac{6t^2}{6} + \frac{6t}{6} - \frac{36}{6} = 0$

$t^2 + t - 6 = 0 \rightarrow (t-2)(t+3) = 0$

$t = 2, -3$]

Smallest $= -3.$

$x = 2t^3 + 3t^2 - 36t$

$2(-3)^3 + 3(-3)^2 - 36(-3) =$

$-54 + 27 + 108 = 81$

$y = 2t^3 + 3t^2 + 1$

$2(-3)^3 + 3(-3)^2 + 1$

$-54 + 27 + 1 = -26$

Smallest $= (81, -26)$

$t = 2$

greatest

$x = 2t^3 + 3t^2 - 36t$

$2(2)^3 + 3(2) - 36(2)$

$y = 2t^3 + 3t^2 + 1$

$2(2) + 3(2)^2 + 1$

$16 + 12 + 1 = 29$

Largest $-44, 29$