Michael Chillemi quiz 1

1. $\left(\dfrac{7x^{5/2}y^{5}}{x^{4}y^{-3/2}}\right)^{-2}$

$= \dfrac{1}{\left(\dfrac{7x^{5/2}y^{5}}{x^{4}y^{-3/2}}\right)^{2}}$

$= \dfrac{1}{\dfrac{(7x^{5/2}y^{5})^{2}}{(x^{4}y^{-3/2})^{2}}}$

$= \dfrac{1}{\dfrac{(7)^{2}(x^{5/2})^{2}(y^{5})^{2}}{(x^{4})^{2}(y^{-3/2})^{2}}}$

$= \dfrac{1}{\dfrac{49x^{5}y^{10}}{x^{8}y^{-3}}}$

$= \dfrac{1}{\dfrac{\dfrac{49x^{5}y^{10}}{x^{8}}}{y^{1/3}}}$

$= \dfrac{1}{\dfrac{49x^{8}y^{10}y^{3}}{x^{8}}}$

$= \dfrac{1}{\dfrac{49y^{13}}{x^{3}}}$

$= \dfrac{1\cdot x^{3}}{49y^{13}}$

$= \dfrac{x^{3}}{49y^{13}}$

2. a) $f(x) = \dfrac{64 - e^{x^2}}{1 - e^{64-x^2}}$

Set denominator to zero

$1 - e^{64-x^2} = 0$

$e^{64-x^2} = 1$

$e^{64-x^2} = e^{0}$

$64 - x^2 = 0$

$64 = x^2$

$\sqrt{64} = \sqrt{x^2}$

$8 \pm = x$

$(-\infty, -8) \cup (-8, 8) \cup (8, \infty)$

b) $f(x) = \dfrac{8+x}{e^{\cos x}}$

$-8 \le \cos x \le 8$

$e^{\cos x} > 0$

All Real numbers

$(-\infty, \infty)$

3 $-\frac{7\pi}{12}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(-x) = -\sin(x)$

$\sin\left(-\frac{7\pi}{12}\right) = -\sin\left(\frac{7\pi}{12}\right) = -\sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= -\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$-\sin\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos(-x) = \cos(x)$

$\cos\left(-\frac{7\pi}{12}\right)$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ $\quad \cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

$\tan\left(-\frac{7\pi}{12}\right)$

$-\tan\left(\frac{7\pi}{12}\right) = -\tan\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$ $\quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$-\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \frac{\frac{\pi}{3} + \frac{\pi}{4}}{1 - \left(\frac{\pi}{3}\right)\left(\frac{\pi}{4}\right)} = \frac{\frac{\pi}{3} + \frac{\pi}{4}}{1 - \frac{\pi^2}{12}}$

7. $\lim_{x \to 3} f(x) = 1$ $\quad \lim_{x \to 3} [1 + 2(-5)]$

$\lim g(x) = -5$ $\quad 1 + -10$

$\lim_{x \to 3} h(x) = 0$ $\quad -9$

b) $\lim_{x \to 3} [-5^3]$

-125

c) $\lim_{x \to 3} \sqrt{1}$

1

D) $\lim_{x \to 3} \frac{3(1)}{-5}$

$\frac{3}{-5}$

e) $\lim_{x \to 3} \frac{g(x)}{h(x)}$

$\lim_{x \to 3} \frac{5}{0}$

Does not exist

f) $\lim_{x \to 3} \frac{g(x)\, h(x)}{f(x)}$

$\frac{(-5)(0)}{(1)}$

$\frac{0}{1} = 0$

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8 $\lim_{x \to 8} (2 + \sqrt[3]{x})(5 - 6x^2 + x^3)$

$(2 + \sqrt[3]{8})(5 - 6(8)^2 + 8^3)$

$(2 + 2)(5 - 6 \cdot 64 + 512)$

$(4) \cdot (133)$

532

9. $\lim_{x \to 1} \frac{x^2 - 4x + 3}{x - 1}$ $\frac{(x-3)(\cancel{x-1})}{\cancel{x-1}}$

$1 - 4 + 3$ $\frac{(x-3)}{1} = (x-3)$ $x - 3 = 0$ $x = 3$

$\frac{(1)^2 - 4(1) + 3}{(1) - 1} = \frac{0}{0}$ indeterminate form

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10 $\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$ $\frac{(1+0)^2 - 1}{0}$

$\frac{1^2 - 1}{0}$ $\frac{1-1}{0}$ $\frac{0}{0}$

$= \frac{(1+h)(1+h) - 1}{h}$

$\lim_{h \to 0} = \frac{(2+h)h}{h} = \lim_{h \to 0} (2+h)$

$= 2$

$= \frac{1 + h + h + h^2 - 1}{h}$

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11. $\lim_{x \to -5} \frac{x+5}{x^3 + 125}$ $\frac{-5 + 5}{-5 + 125}$ $\frac{0}{0}$

$a^3 + b^3 = (a+b) - ab + b^2)$

$\lim_{x \to -5} = \frac{x+5}{x^3 + 5^3}$ $\lim_{x \to -5} \frac{\cancel{x+5}}{(\cancel{x+5})(x^2 - 5x + 5^2)}$

$\lim_{x \to 5} = \frac{1}{(x^2 - 5x + 25)}$ $\lim_{x \to 5} \frac{1}{(-5)^2 - 5(-5) + 25}$

$\lim_{x \to 5} = \frac{1}{25 + 25 + 25}$ $= \frac{1}{75}$

12 $\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$ $\frac{\sqrt{9+0}-3}{0}$

$\frac{\sqrt{9}-3}{0}$ $\frac{3-3}{0}$ $\frac{0}{0}$

$\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}$

$\lim_{h \to 0} \frac{(\sqrt{9+h})^{\cancel{2}}-3^2}{h(\sqrt{9+h}+3)}$ $\lim_{h \to 0} \frac{9+h-9}{h(\sqrt{9+h}+3)}$

$\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$ $\lim_{h \to 0} \frac{1}{\sqrt{9+h}+3}$

$\lim_{h \to 0} \frac{1}{\sqrt{9+0}+3}$ $\frac{1}{3+3} = \frac{1}{6}$

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13 $\lim_{t \to 0} \left(\frac{7}{t} - \frac{7}{t^2+t}\right)$ $\left(\frac{7}{0} - \frac{7}{0^2+0}\right)$ $\frac{7}{0} - \frac{7}{0}$

$\frac{0}{0}$ $\lim_{t \to 0} \left(\frac{7}{t} - \frac{7}{t^2+t}\right) = \frac{7}{t} - \frac{7}{t(t+1)}$

$= \frac{7(t+1)}{t(t+1)} - \frac{7}{t(t+1)} = \frac{7(t+1)-7}{t(t+1)} = \frac{7t+\cancel{7}-\cancel{7}}{t(t+1)}$

$= \frac{7\cancel{t}}{\cancel{t}(t+1)}$ $\lim_{t \to 0} \frac{7}{t+1} = \frac{7}{0+1} = 7$

14. $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) = \frac{1}{0\sqrt{1+0}} - \frac{1}{0}$

$= \frac{0}{0}$ $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) = \frac{1}{t\sqrt{1+t}} - \frac{1 \cdot \sqrt{1+t}}{t\sqrt{1+t}}$

$= \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \cdot \frac{(1+\sqrt{1+t})}{(1+\sqrt{1+t})} = \frac{(1-1-t)}{t\sqrt{1+t}(1+\sqrt{1+t})}$

$= \frac{-\cancel{t}}{\cancel{t}\sqrt{1+t}(1+\sqrt{1+t})} = \frac{-1}{\sqrt{1+1}(1+\sqrt{1+t})}$

$\lim_{t \to 0} \frac{-1}{(1+\sqrt{1+0})(\sqrt{1+0})} = \frac{1}{(1+1)1} = -1/2$

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15 if $3x-3 \leq f(x) \leq x^2-3x+6$, for $x \geq 0$ find $\lim_{x \to 3} f(x)$

$\lim_{x \to 3} (3x-3) = (3(3)-3)$

$= (9-3)$

$= 6$

$\lim_{x \to 3} x^2-3x+6 = 3^2-3(3)+6$

$= \cancel{9} - \cancel{9} + 6$

$= 6$

16. $\lim_{x \to -7} \frac{4x + 28}{|x + 7|} = \frac{4(-7) + 28}{|-7 + 7|}$

$= \frac{-28 + 28}{14} \quad \frac{0}{14} \qquad |x + 7| = -(x + 7)$

$\lim_{x \to 7^-} \frac{4x + 28}{-(x+7)} \quad \frac{4(x \cancel{+7})}{-(x \cancel{+7})} = -4$

$\lim_{x \to 7^+} = \frac{4x + 28}{|x + 7|} \quad \frac{4(x \cancel{+7})}{(x \cancel{+7})} = 4$

Does Not exist

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$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 7 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 2 & \text{if } x > 2 \end{cases}$$

a) when $x \to 1^-$ $\quad x < 1 \quad = 1$

b) when $x \to 1^+$ $\quad 1 < x \leq 2 \quad = 2 - (1^2)$

$= 2 - 1$

$= 1$

c) when $g(1)$ $\quad x = 1 \quad = 7$

d) when $x \to 2^-$ $\quad 2 - (2^2) = 2 - 4$

$= -2$

e) when $x \to 2^+$ $\quad x - 2 \quad 2 - 2 = 0$

f) when $x \to 2$ it does not exist