1 1

1.0.1

$$A_{(ij)} = \frac{1}{2}(A_{ij} + A_{ji})$$

1.0.2

$$A_{[ij]} = \frac{1}{2}(A_{ij} - A_{ji})$$

: $A^T = A,\ A_{ij} = A_{ji}$: $A^T = -A,\ A_{ij} = -A_{ji}$
. . :

$$A_{[ij]} + A_{(ij)} = A_{ij}$$

1.0.3 1

$$[A_{ij}] = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 3 & 5 & 2 \end{pmatrix}$$

1.0.4 2

R^n. .
. : n^2 . : $\frac{n^2+n}{2}$. : $\frac{n^2-n}{2}$

1.0.5 3

$= 273 + 6 + 1 = 10$ 1

1.0.6 4

A_{ij} - . B^{ij} - . . $A_{ij}B^{ij} = 0$

$$i, j = 1, 2: \; A_{11}B^{11} + A_{12}B^{12} + A_{21}B^{21} + A_{22}B^{22} = 0$$

1.0.7 5

, $A_{ij}B^{[ij]}$ A.

$$A_{ij}B^{[ij]} = (A_{(ij)} + A_{[ij]})B^{[ij]} = \underbrace{A_{(ij)}B^{[ij]}}_{0} + A_{[ij]}B^{[ij]}$$

1. [!definition]

$$\delta^i_k = \begin{cases} 1, i = k \\ 0, i \neq k \end{cases}$$

2. [!definition] -

$$\begin{vmatrix} a^1 & b^1 & c^1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \epsilon_{ijk}a^i b^j c^k$$

1.0.8 6

$$n!$$

1.0.9 7

$$\epsilon_{ij}\epsilon^{ik} = a^k_j$$

1.0.10 8

2 2

2.1

2.2

2.2.1

2.3

[!definition] , .

$$(1,0): \ |a| = \sqrt{\langle a,a\rangle},\ \langle a,a\rangle = g_{ij}a^i a^j$$

2.3.1

3 3

[!important] $\iff$ $\iff$ $\iff$

3.1

J^{ij} - , .
- $E_i = \frac{1}{2}J^{ij}\omega_i\omega_j = \frac{1}{2}\omega J\omega^T$ - $L^i = J^{ij}\omega_j,\ L = J\omega^T$
ω - () , $|\omega| =$.

3.1.1

3.2

3.2.1

$$\vec{a}\otimes\vec{b}, \vec{b}\otimes\vec{a}, \vec{a}\wedge\vec{b},\ \vec{a} = (1,2,3),\ \vec{b} = (-1,1,-1)$$

3.2.2 $(2,0)$ R^n ?

- . $(n^2 > 2n, (n > 2))$. R^2 - .

3.2.3

3.2.4 , 3- .

4 4

4.1

[!definition]

4.1.1

+ 3dx$^2) \wedge (dx^1 - dx^2))$ R^2 R^3

1 $*((dx^1 - 3dx^2) \wedge (dx^1 - dx^2) \wedge (dx^1 - 4dx^3))$ R^3, R^4

$2 *((-2dx^1 + 2dx^2) \wedge (dx^1 \wedge dx^2 + 4dx^1 \wedge dx^3))$ R^3 R^4

3, 1 2- R^3 $\tilde{\omega}_a = a_i dx^i$, $\tilde{\omega}_b = b_{ij} dx^i \wedge dx^j$ a, b (2-).

4.2 ()

1. [!definition] $\tilde{d}$ - , , . $\tilde{\alpha}$ - p-, $\tilde{\beta}, \tilde{\gamma}$ - q-. 1. $\tilde{d}(\tilde{\beta} + \tilde{\gamma}) = \tilde{d}\tilde{\beta} + \tilde{d}\tilde{\gamma}$ - 2. $\tilde{d}(\tilde{\alpha} \wedge \tilde{\beta}) = d\tilde{\alpha} \wedge \tilde{\beta} + (-1)^p \tilde{\alpha} \wedge \tilde{d}\tilde{\beta}$ 3. $\tilde{d}\tilde{d}\tilde{\alpha} = 0$

   f - 0- $\tilde{d}f = df = \frac{\partial f}{\partial x^1} dx^1 + \frac{\partial f}{\partial x^2} dx^2$ - 1- $\tilde{d}^2 f = \frac{\partial^2 f}{\partial x^1 \partial x^1}(dx^1)^2 + 2\frac{\partial^2 f}{\partial x^1 \partial x^2} dx^1 dx^2 + \frac{\partial^2 f}{\partial x^2 \partial x^2}(dx^2)^2 = 0$ - 2-

$$\tilde{d}(f\tilde{d}\tilde{g}) = \tilde{d}f \wedge \tilde{d}\tilde{g}$$

4.2.1

5 5

,

[!definition] .

$\tilde{\alpha}$ - p- p- , u - p-

$y = y(x)$:

[!important]

$\tilde{\alpha}$ - p-, $\tilde{d}\tilde{\alpha}$ - $p+1$-, U - $p+1$- , ∂U - U, p-

5.0.1

R^1, 0- $f(x)$, $n = 1$, $p = 0$

5.0.2 3

, - R^4? R^4.

R^4:

5.0.3 4

,

, $\vec{a}$ $\tilde{\alpha} = *\vec{a}$.

[!definition]

5.0.4 5

:

5.1 /

6 6

[!definition] p- - $(p, 0)$.

6.0.1

$(1,0)$ - $(2,0)$ - $(3,0)$ -

[!definition] : $\langle \vec{x}, \vec{y} \rangle = g_{ij} x^i y^j =$ inv : $\langle \vec{x}, \vec{y} \rangle = g_{ij} g_{kj} x^{ik} y^{jl} =$ inv ()

[!definition]

$$|x| = \sqrt{\langle x, x\rangle} = V \text{ (-)}$$

$v = v_1 \wedge v_2 \wedge \cdots \wedge v_n$, $v = v_1 \wedge v_2 \wedge \cdots \wedge v_n$,

6.0.2 1

$u = u_1 \wedge u_2$.

6.0.3 2

$$u_1 = (1, 4, 3, 0)^T,\ u_2 = (1, 2, 0, 1)^T.$$

6.0.4 3

, $u_1 = (1, 2, 3, 0)^T,\ u_2 = (1, 1, -1, 0)^T,\ u_3 = (1, 1, 0, 2)^T.$

6.0.5 4

$$\alpha(t) = \left(t, \tfrac{t^2}{2} - t, \tfrac{t^3}{3}, \tfrac{t^4}{4}\right)^T \quad t = 0.$$

6.0.6 “”

$F_{\mu\nu}F^{\mu\nu}$ $\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\sigma\rho}$ E H.

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & H_3 & -H_2 \\ E_2 & -H_3 & 0 & H_1 \\ E_3 & H_2 & -H_1 & 0 \end{pmatrix}$$