1 1

1.0.1

$$A_{(ij)} = \frac{1}{2}(A_{ij} + A_{ji})$$

1.0.2

$$A_{[ij]} = \frac{1}{2}(A_{ij} - A_{ji})$$

: $A^T = A,\ A_{ij} = A_{ji}$: $A^T = -A,\ A_{ij} = -A_{ji}$

. . :

$$A_{[ij]} + A_{(ij)} = A_{ij}$$

1.0.3 1

$$[A_{ij}] = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 3 & 5 & 2 \end{pmatrix}$$

1.0.4 2

R^n. .

. : n^2 . : $\frac{n^2+n}{2}$. : $\frac{n^2-n}{2}$

1.0.5 3

1. $3^3 = 27$
2. $3 + 6 + 1 = 10$
3. 1

1.0.6 4

A_{ij} - . B^{ij} - . . $A_{ij}B^{ij}=0$

$$i,j=1,2:\ A_{11}B^{11}+A_{12}B^{12}+A_{21}B^{21}+A_{22}B^{22}=0$$

1.0.7 5

, $A_{ij}B^{[ij]}$ $A.$

$$A_{ij}B^{[ij]}=(A_{(ij)}+A_{[ij]})B^{[ij]}=\underbrace{A_{(ij)}B^{[ij]}}_{0}+A_{[ij]}B^{[ij]}$$

[!definition]

$$\delta^i_k=\begin{cases}1, i=k\\0, i\neq k\end{cases}$$

[!definition] -

$$\begin{vmatrix}a^1 & b^1 & c^1\\a^2 & b^2 & c^2\\a^3 & b^3 & c^3\end{vmatrix}=\epsilon_{ijk}a^ib^jc^k$$

1.0.8 6

$$n!$$

1.0.9 7

$$\epsilon_{ij}\epsilon^{ik}=a^k_j$$

1.0.10 8

2 2

2.1

2.2

2.2.1

2.3

[!definition] , .

$$(1,0):\ \ |a| = \sqrt{\langle a, a\rangle},\ \ \langle a, a\rangle = g_{ij}a^i a^j$$

2.3.1

3 3

[!important] $\iff$ $\iff$ $\iff$

3.1

J^{ij} - , .
- $E_i = \frac{1}{2} J^{ij}\omega_i\omega_j = \frac{1}{2}\omega J \omega^T$ - $L^i = J^{ij}\omega_j,\ L = J\omega^T$
ω - () , $|\omega| =$.

3.1.1

3.2

3.2.1

$$\vec{a}\otimes\vec{b}, \vec{b}\otimes\vec{a}, \vec{a}\wedge\vec{b},\ \vec{a} = (1,2,3),\ \vec{b} = (-1,1,-1)$$

3.2.2 $(2,0)$ R^n ?

- . $(n^2 > 2n, (n > 2))$. R^2 - .

3.2.3

3.2.4 , 3- .

4 4

4.1

[!definition]

4.1.1

1. $*((2dx^1 + 3dx^2) \wedge (dx^1 - dx^2))$ R^2 R^3

2. $*((dx^1 - 3dx^2) \wedge (dx^1 - dx^2) \wedge (dx^1 - 4dx^3))$ R^3, R^4

3. $*((-2dx^1+2dx^2)\wedge(dx^1\wedge dx^2+4dx^1\wedge dx^3))$ R^3 R^4

4. , 1 2- R^3 $\tilde{\omega}_a = a_i dx^i,\ \tilde{\omega}_b = b_{ij} dx^i \wedge dx^j$ a, b (2-).

4.2 ()

[!definition] $\tilde{d}$ - , , . $\tilde{\alpha}$ - p-, $\tilde{\beta},\tilde{\gamma}$ - q-. 1. $\tilde{d}(\tilde{\beta}+\tilde{\gamma}) = \tilde{d}\tilde{\beta}+\tilde{d}\tilde{\gamma}$ - 2. $\tilde{d}(\tilde{\alpha}\wedge\tilde{\beta}) = d\tilde{\alpha}\wedge\tilde{\beta}+(-1)^p\tilde{\alpha}\wedge\tilde{d}\tilde{\beta}$ 3. $\tilde{d}\tilde{d}\tilde{\alpha}=0$

f - 0- $\tilde{d}f = df = \frac{\partial f}{\partial x^1}dx^1+\frac{\partial f}{\partial x^2}dx^2$ - 1- $\tilde{d}^2 f = \frac{\partial^2 f}{\partial x^1 \partial x^1}(dx^1)^2 + 2\frac{\partial^2 f}{\partial x^1 \partial x^2}dx^1 dx^2 + \frac{\partial^2 f}{\partial x^2 \partial x^2}(dx^2)^2 = 0$ - 2-

$$\tilde{d}(f\tilde{d}\tilde{g}) = \tilde{d}f\wedge\tilde{d}\tilde{g}$$

4.2.1

5 5

,

[!definition] .

$\tilde{\alpha}$ - p- p- , u - p-

$y = y(x)$:

[!important]

$\tilde{\alpha}$ - p-, $\tilde{d}\tilde{\alpha}$ - $p + 1$-, U - $p + 1$- , ∂U - U, p-

5.0.1

R^1, 0- $f(x)$, $n = 1$, $p = 0$

5.0.2 3

, - R^4? R^4.

R^4:

5.0.3 4

,

, $\vec{a}$ $\tilde{\alpha} = *\vec{a}$.

[!definition]

5.0.4 5

:

5.1 /

1.
2. , ,
3. (4) , ,
4.

6 6

[!definition] p- - $(p, 0)$.

6.0.1

$(1,0)$ - $(2,0)$ - $(3,0)$ -

[!definition] : $\langle \vec{x}, \vec{y} \rangle = g_{ij} x^i y^j = \text{inv} : \langle \vec{x}, \vec{y} \rangle = g_{ij} g_{kj} x^{ik} y^{jl} =$ inv ()

[!definition]

$$|x| = \sqrt{\langle x, x \rangle} = V \text{ (-)}$$

$v = v_1 \wedge v_2 \wedge \cdots \wedge v_n$, $v = v_1 \wedge v_2 \wedge \cdots \wedge v_n$,

6.0.2 1

$u = u_1 \wedge u_2$.

6.0.3 2

$u_1 = (1, 4, 3, 0)^T$, $u_2 = (1, 2, 0, 1)^T$.

6.0.4 3

, $u_1 = (1, 2, 3, 0)^T$, $u_2 = (1, 1, -1, 0)^T$, $u_3 = (1, 1, 0, 2)^T$.

6.0.5 4

$\alpha(t) = \left(t, \frac{t^2}{2} - t, \frac{t^3}{3}, \frac{t^4}{4}\right)^T \quad t = 0.$

6.0.6 “”

$F_{\mu\nu}F^{\mu\nu}$ $\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\sigma\rho}$ E H.

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & H_3 & -H_2 \\ E_2 & -H_3 & 0 & H_1 \\ E_3 & H_2 & -H_1 & 0 \end{pmatrix}$$