Applied Category Theory

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Why?

Why should you care about studying the coming content and applying it to your field?

Physicists:

- The particles in the standard model *are* irreducible representations. So rep theory is crucial to you.
- Monoidal categories give a good framework for understanding QM.

Computer scientists:

- It gives a framework for the Curry-Howard correspondence (proofs are programs).

Me:

- Began as a study of "analogies" and turned into a study of nifty algebraic gadgets.
- It's written in a *wild* language, and learning languages is fun.

Plan

The **first goal** is to define monoidal categories with some context. The **second goal** is to describe a "skeletal" category defined by diagrammatics. To accomplish the first goal, we will study things including:

- Algebraic objects (groups, vector spaces, ...) and maps between them
- Subobjects, images, combining objects ($\times$, $\otimes$, $\oplus$, ...)
- Categories (Grp, Set, $PoSet$, $\mathbb{N}$, Vec, ...)

I'd like to have as little fat on this as necessary. That is, not get sidetracked studying, for instance, too much of the internal structure of these objects. I want to give many examples and try to build intuition. For the second goal we'll study things including:

- Representations and maps ($T(gv) = gT(v)$)
- Rep(D_3) in detail

Up to this point I have a strong vision of where we're going. After this we can go where the interest steers us.

This plan is incomplete and non-exhaustive.

Groups

This is the best onramp to categories I know of, so bear with me through some basics.

Definition 1. *A* **group** *is a triple*

$$(G, \mu, e)$$

where G is a set, $\mu : G \times G \to G$ is a function, and $e \in G$, such that

$$\forall a, b, c \in G, \quad \mu(\mu(a, b), c) = \mu(a, \mu(b, c)) \tag{Associativity}$$

$$\forall a \in G, \quad \mu(a, e) = \mu(e, a) = a \tag{Identity}$$

$$\forall a \in G, \exists b \in G, \quad \textit{such that} \quad \mu(a, b) = \mu(b, a) = e \tag{Inverse}$$