# EGH445 Modern Control
## Written Report Assignment

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***Abstract*— This technical report details the complete modeling, design & analysis of a complete output-feedback control system for the Translational Oscillator with Rotational Actuator system. This report entails the derivation of dynamics from Lagrange principles and linearisation through the 1st order Taylor series for this non-linear system. Both the non-linear and linearized models of the system were implemented on MATLAB/SIMULINK for simulation analysis. The design of a state-feedback controller through linear quadratic regulation and the design of an output-feedback controller through linear quadratic estimation was then computed following optimal control concepts. The combined system which formed a linear quadratic gaussian controller was then analyzed for its performance in terms of design stability, model uncertainty compensation, disturbance rejection, and noise attenuation. Finally, conclusions and recommendations were drawn from this report considering its whole design and derivation process. Overall, this report displays the conceptual and practical understanding of control system's engineering applied to the mentioned system for its stabilization within the designated constraints.**

### I. Introduction

The Translational Oscillator with Rotating Actuator System or 'TORA' for short is a system with a linear spring connecting a platform to a fixed frame. The platform has constrained vertical motion and moves horizontally as a result of the rotation of a proof mass through the actuation of a DC motor torque. This technical report documents the design and development of a complete output-feedback control system for the TORA model. It mainly entails the system modeling & linearisation, state-feedback controller design, observer design, output-feedback control system design, and its performance analysis along with MATLAB/SIMULINK simulations & results modeled in the continuous-time domain. The TORA system used for demonstration is depicted in fig.1,

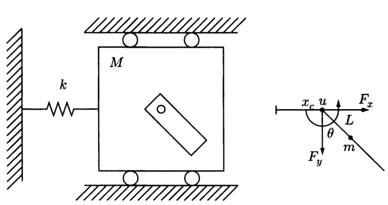


Fig. 1. Translational Oscillator with Rotating Actuator System

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### II. Modelling & Linearisation

As shown in fig.1, the TORA system consists of a platform of mass $M$ which can only move in the horizontal plane parallel to a linear spring with spring constant $k$ connected to a fixed frame of reference. A rotating proof mass $m$ is actuated on the platform by a DC motor. Located at a distance $l$ from its rotational axis, is its moment of inertia $J$ around its mass center. Actuated by the DC motor torque $\tau(t)$, the system moves in the horizontal direction parallel to the spring axis. The Lagrange equations of motion that describe the dynamics of the TORA system are given by,

$$(J + ml^2)\ddot{\theta} + (ml\cos\theta)\ddot{x}_c = \tau$$

$$(ml\cos\theta)\ddot{\theta} + (m + M)\ddot{x}_c = ml\dot{\theta}^2\sin\theta - kx_c$$

where $\theta$ is the angle, $\dot{\theta}$ is the angular velocity of the rotational actuator (measured counter-clockwise from positive $F_y$ axis), $x_c$ is the position, and $\dot{x}_c$ is the linear velocity of the translational oscillator (measured along positive $F_x$ axis) as shown in fig.1. Taking the state variables $x_1$, $x_2$ as angle, angular velocity of the rotating actuator and $x_3$, $x_4$ as position, linear velocity of the translational oscillator respectively, and $u$ as the input torque, we are able to model the TORA system in the state space form of $\dot{x} = f(x,u)$ and output $y = g(x,u)$ as given in (1),

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{(m+M)u - ml\cos x_1(mlx_2^2\sin x_1 - kx_3)}{(J+ml^2)(m+M) - m^2l^2\cos^2 x_1} \\ x_4 \\ \frac{-mlu\cos x_1 + (J+ml^2)(mlx_2^2\sin x_1 - kx_3)}{(J+ml^2)(m+M) - m^2l^2\cos^2 x_1}\end{bmatrix},\ y = \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{1}$$

The above state-space model is observed to be non-linear through the dependency of states in the dynamics matrix $f(x,u)$. Therefore this non-linear model of the TORA system has to be linearised about it's equilibrium points where the non-linear system behaves similar to a linear system with input (torque) equal to zero (i.e. $\tau = 0$). We denote the states and input at an equilibrium point by,

$$\bar{x} = \begin{bmatrix}\bar{x}_1 & \bar{x}_2 & \bar{x}_3 & \bar{x}_4\end{bmatrix}^T,\ \bar{u} = \bar{\tau}$$

At equilibrium, it is expected that the TORA system won't move unless acted by the DC motor torque. Therefore we can set the linear velocity and angular velocity to zero, and set any other terms that become zero, as a result, to get the state-space equations at the equilibrium point(s),

$$\begin{cases} \bar{x}_2 = 0 \\ \frac{ml\cos\bar{x}_1 k\bar{x}_3}{(J+ml^2)(m+M)-m^2l^2\cos^2\bar{x}_1} = 0 \\ \bar{x}_4 = 0 \\ \frac{-(J+ml^2)k\bar{x}_3}{(J+ml^2)(m+M)-m^2l^2\cos^2\bar{x}_1} = 0 \end{cases} \tag{2}$$

From the $2^{nd}$ & $4^{th}$ conditions in (2), we see that the equilibrium points depend only on state $\bar{x}_3$ (position of platform) being zero while there are no restrictions on $\bar{x}_1$, i.e. the platform can be in equilibrium at any angular displacement $x_1$. This means that this system has a continuum of infinite non-isolated equilibria. Therefore the equilibrium points can be given as,

$$\bar{x} = \begin{bmatrix}\bar{x}_1 & 0 & 0 & 0\end{bmatrix}^T \quad \forall\, \bar{x}_1 \in \mathbb{R}^{\circ}$$

Prior to performing linearisation, we must select at least two equilibrium points from the infinite space of $\bar{x}_1$ to design around. The $\cos\bar{x}_1$ terms in (2) suggest that the rotational actuator vibrations about $\bar{x}_1 = 0^{\circ}$ have most influence on the translational oscillator while at $\bar{x}_1 = 90^{\circ}$, they have the least influence [4]. Therefore building around these 2 instances, the equilibrium points were picked as,

$$\bar{x}_a = \begin{bmatrix}\bar{x}_{1a} & 0 & 0 & 0\end{bmatrix}^T \ ; \ \bar{x}_{1a} = 0^{\circ}$$

$$\bar{x}_b = \begin{bmatrix}\bar{x}_{1b} & 0 & 0 & 0\end{bmatrix}^T \ ; \ \bar{x}_{1b} = 90^{\circ}$$

We can now linearise the system about each of the equilibrium points $\bar{x}_a$ and $\bar{x}_b$. The linearised states, inputs and outputs of the system can be defined as,

$$\tilde{x} = x - \bar{x}$$

$$\tilde{u} = u - \bar{u} \ ; \ \bar{u} = 0 \therefore \tilde{u} = u$$

$$\tilde{y} = y - \bar{y}$$

Once performed, we are to obtain the linearised state-space model of the TORA system, $\dot{\tilde{x}} = A\tilde{x} + B\tilde{\tau}$ where the $A$ and $B$ matrices are defined as the Jacobian matrices shown below,

$$A = \left(\frac{\partial f}{\partial x}\right)^T\Bigg|_{x=\bar{x},\tau=\bar{\tau}} \ , \ B = \left(\frac{\partial f}{\partial \tau}\right)^T\Bigg|_{x=\bar{x},\tau=\bar{\tau}}$$

Using partial differentiation, the matrices were derived as,

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \alpha & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \beta & 0\end{bmatrix} \ , \ B = \begin{bmatrix}0\\ \gamma\\ 0\\ \eta\end{bmatrix}$$

where the constants,

$$\alpha = \frac{mlcos\bar{x}_1 k}{\epsilon} \ , \ \beta = \frac{-(J+ml^2)k}{\epsilon}$$

$$\gamma = \frac{m+M}{\epsilon} \ , \ \eta = \frac{-mlcos(\bar{x}_1)}{\epsilon}$$

$$\epsilon = (J+ml^2)(m+M) - m^2l^2\cos^2\bar{x}_1$$

The model parameters used in the formulation were,

| Model Parameter | Value |
|---|---|
| $m$ | 0.096 $[kg]$ |
| $M$ | 1.3608 $[kg]$ |
| $J$ | 0.0002175 $[kg \cdot m^2]$ |
| $l$ | 1.00 $[m]$ |
| $k$ | 186.30 $[units]$ |

The linearized LTI state-space matrices were obtained as such for equilibrium point 'a' and 'b' along with their linearized system dynamics and are given below,

$$EP \text{ "}a\text{"} \longrightarrow \dot{\tilde{x}}_a = A_a\tilde{x} + B_a\tilde{\tau}$$

$$A_a = \begin{bmatrix}0 & 1.00 & 0 & 0\\ 0 & 0 & 136.57 & 0\\ 0 & 0 & 0 & 1.00\\ 0 & 0 & -136.88 & 0\end{bmatrix} \ , \ B_a = \begin{bmatrix}0\\ 11.12\\ 0\\ -0.73\end{bmatrix}$$

$$EP \text{ "}b\text{"} \longrightarrow \dot{\tilde{x}}_b = A_b\tilde{x} + B_b\tilde{\tau}$$

$$A_b = \begin{bmatrix}0 & 1.00 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1.00\\ 0 & 0 & -127.88 & 0\end{bmatrix} \ , \ B_b = \begin{bmatrix}0\\ 10.39\\ 0\\ 0\end{bmatrix}$$

The 'C' and 'D' matrices remain the same measuring the states $x_2$ and $x_4$ as outputs at both equilibrium points with zero feed-through terms,

$$C = \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \ , \ D = \begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}$$

The stability of the linearised open-loop systems were analysed by calculating the eigenvalues of the system dynamics $\lambda_i(A_{EP})$,

$$\lambda(A_a) = \begin{bmatrix}0\\ 0\\ 0+11.70i\\ 0-11.70i\end{bmatrix} \ , \ \lambda(A_b) = \begin{bmatrix}0\\ 0\\ 0+11.31i\\ 0-11.31i\end{bmatrix}$$

According to Lyapunov's first ("indirect") method, both these linearised open-loop systems cannot be concluded as either stable or unstable i.e. stability cannot be guaranteed as of now as all eigenvalues lie on the imaginary axis of the complex plane.

## III. State-Feedback Control Design

For both the linearized model and the non-linear models of the TORA system, a state-feedback controller was to be designed to regulate the states towards the equilibrium points $\bar{x}_a$ & $\bar{x}_b$. Before that, the controllability matrix '$C^{AB}$' must be calculated and checked for it's rank. The system is completely controllable if and only if the controllability matrix is full rank (i.e. rank equal to number of states). The controllability matrix was defined in this instance for both equilibrium points as,

$$C^{AB} = \begin{bmatrix}B & AB & A^2B & A^3B\end{bmatrix}$$

However, upon the check on controllablility, it was found that system $\dot{\tilde{x}}_a$ was completely controllable while $\dot{\tilde{x}}_b$ was not completely controllable due to rank deficiency. The controllability matrix of equilibrium point "a" is shown below,

$$C_a^{AB} = \begin{bmatrix} 0 & 11.12 & 0 & -100.12 \\ 11.12 & 0 & -100.12 & 0 \\ 0 & -0.73 & 0 & 100.35 \\ -0.73 & 0 & 100.35 & 0 \end{bmatrix}$$

Therefore the state-feedback controller was implemented only for the system at EP "a". For the linearized model, the state-feedback controller was implemented in the form $u = -K\tilde{x}$ where $\tilde{x} = x - \bar{x}$ as derived earlier. For the non-linear model, the state-feedback controller was designed to be $u = -Kx$ such that the desired closed loop system response becomes,

$$\dot{x} = Ax + B(-Kx) = Ax - BKx = (A - BK)x$$

Both these state-feedback controllers perform zero set-point regulation driving all states of the system to zero. The controller gains 'K' was found by incorporating linear quadratic regulation (LQR) design which minimizes the cost function,

$$J(x_0, u) = \int_0^\infty x^T Q_{con} x + u^T R_{con} u \ dt$$

where '$Q_{con}$' is a symmetric positive semi-definite matrix while '$R_{con}$' is a positive definite matrix. The gain is found by solving for the algebraic Riccati equation (ARE) given below for 'S' which is a positive semi-definite matrix where $S = S^T >= 0$

$$A^T S + SA - (SB \cdot R_{con}^{-1} \cdot B^T S) + Q_{con} >= 0$$

where gain 'K' is given by the equation,

$$K = R_{con}^{-1} B^T S$$

The matrices '$Q_{con}$' and '$R_{con}$' was tuned to penalize the magnitude of the states and the magnitude of the control actions respectively. Overall through trial and error testing/tuning, the states were penalized more than the control actions which resulted in the states '$x$' to remain small. The choice of '$Q_{con}$' and '$R_{con}$' and the resulted controller gain values are given below,

$$Q_{con} = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.025 & 0 & 0 \\ 0 & 0 & 250.00 & 0 \\ 0 & 0 & 0 & 7.50 \end{bmatrix} , \ R_{con} = 0.01$$

$$K = \begin{bmatrix} 5.00 & 2.23 & -294.77 & -14.47 \end{bmatrix}$$

The closed-loop dynamics of the linearized system with the state feedback controller $\lambda_i(A - BK)$ were seen to be stable as all the eigenvalues had negative real components,

$$\lambda_1 = -3.25 \ , \ \lambda_{2,3} = -4.53 \pm 8.60i \ , \ \lambda_4 = -23.13$$

Therefore, the state-feedback controller was designed as,

$$u = -\begin{bmatrix} 5.00 & 2.23 & -294.77 & -14.47 \end{bmatrix} x$$

The non-linear and linearized models of the TORA system were modelled on MATLAB/SIMULINK initially along with the zero-state regulation controller with the desired gain values for 'K'. The initial conditions of the states were specified as,

$$x_0 = \begin{bmatrix} 20^\circ & 0 & 0.1 & 0 \end{bmatrix}^T$$

The results below were obtained once the simulation was run for 5 seconds and as seen, all states settled to zero in less than 1.5 seconds while the linearized model was accurate to follow the non-linear model about the equilibrium point in terms of all states as well as the input torque.

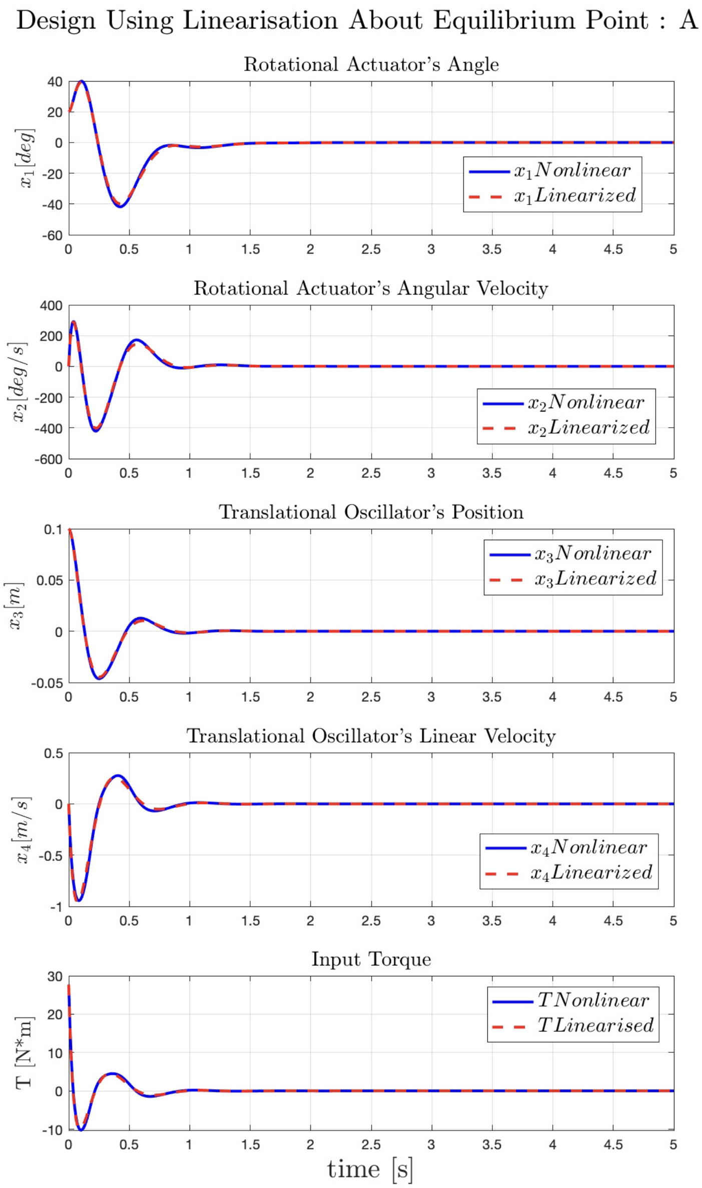


Fig. 2. TORA Linearized vs Nonlinear Models LQR Controlled Zero-State Regulation of States & Input Torque

## IV. Output-Feedback Control Design

When measuring from a system, at times we will not have access to the state information. Therefore knowing the input '$u$' and output '$y$' of the system we should be able to estimate the states '$x$'. For this purpose, an observer was required to be designed for both the non-linear model and the linearized model. Before the design process, the observability matrix was to be checked for its full rank,

$$O^{AC} = \begin{bmatrix} C & CA & CA^2 & CA^3 \end{bmatrix}^T$$

It was noticed that the system was not completely observable for the current output of states $x_2$ (Angular Velocity) and $x_4$ (Linear Velocity) due to rank deficiency. Therefore, the observer was designed for the remaining states $x_1$ (Angle) and $x_3$ (Position) as they were completely observable. This is evident from the principle that a positional vector can be used to track the respective velocity vector while the vice versa tracking doesn't occur due to lack of initial conditions. Given below is the observability matrix for the observable states,

$$O^{AC}_{x_1,x_3} = \begin{bmatrix} 1.00 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 \\ 0 & 1.00 & 0 & 0 \\ 0 & 0 & 0 & 1.00 \\ 0 & 0 & 136.57 & 0 \\ 0 & 0 & -136.88 & 0 \\ 0 & 0 & 0 & 136.57 \\ 0 & 0 & 0 & -136.88 \end{bmatrix}$$

For the output-feedback controller, the implementation was of the form of a full-order Luenberger observer with the closed-loop error dynamics as,

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \ ; \ y = Cx \tag{3}$$

$$\dot{e} = (A - LC)e \ ; \ e = x - \hat{x} \ (error)$$

and the state-feedback controller from the estimated states was given as,

$$u = -K\hat{x}$$

Both output-feedback controllers perform state estimation to produce a model of the regulated responses. The observer gains 'L' was found by incorporating linear quadratic estimation (LQE) design using a Kalman filter which minimizes the trace of the mean square value of the state estimation error covariance,

$$J(u, y) = trace(P(t)) \ ; \ P(t) = E[(x - \hat{x})(x - \hat{x})^T]$$

The kalman gain 'L' was found by solving the algebraic Riccati equation (ARE) given below for 'P' which is always symmetric matrix such that $P^T = P$,

$$AP + PA^T - (PC^T \cdot R_{obs}^{-1} \cdot CP) + Q_{obs} = 0$$

where Kalman gain 'L' is given by the equation,

$$L = PC^T R_{obs}^{-1}$$

The matrices '$Q_{obs}$' and '$R_{obs}$' here are the process and measurement noise covariance matrices respectively. They were tuned to penalize the noise in the states and the noise in the measurements accordingly. It was expected that the states had more noise than the measurements due to the unit constant disturbance given into the system states through the filter itself and therefore the process noise covariance matrix was weighted higher.

The choice of '$Q_{obs}$' and '$R_{obs}$' through testing/tuning and the resulted observer gain values are given below,

$$Q_{obs} = \begin{bmatrix} 500.00 & 0 & 0 & 0 \\ 0 & 500.00 & 0 & 0 \\ 0 & 0 & 200.00 & 0 \\ 0 & 0 & 0 & 800.00 \end{bmatrix}$$

$$R_{obs} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

$$L = \begin{bmatrix} 224.76 & 259.08 & 0.20 & -24.87 \\ 0.20 & 97.46 & 142.66 & 176.36 \end{bmatrix}^T$$

The closed loop dynamics of the system with the output feedback controller $\lambda_i(A - LC)$ was seen to be stable as all the eigenvalues had negative real componenets,

$$\lambda_5 = -223.61 \ , \ \lambda_6 = -140.43 \ , \ \lambda_7 = -1.13 \ , \ \lambda_8 = -2.26$$

Therefore, this observer gain 'L' was included in the output feedback controller mentioned in (3). The non-linear and linearized models of the TORA system which were previously modelled on MATLAB/SIMULINK were added with the designed observer with the new output matrix to reflect only the measured states '$x_1$' and '$x_3$' along with the new output initial condition matrix as given below,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \ , \ D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The results in. fig.3 were obtained once the simulation was run for 5 seconds and as seen all 4 states were measured well through the observer although the output-feedback controller was designed to measure 'angle' and 'position' only.

## V. Performance Analysis

The stability of the designed system was analyzed. At different initial conditions, the closed-loop system was measured to see if it was able to regulate towards zero-state. This was done with aim of finding the maximum possible boundary value of angle and position which was able to successfully regulate the TORA system towards equilibrium. It was found that the system was successfully able to regulate the response and stabilize from any given initial angle ($x_1$ between $0°$ and $\pm 180°$) with other states set to 0. The maximum time taken to regulate and reach a steady-state with any initial angle was slightly less than 2 seconds with any state of the system which proves the stable design of the state-feedback controller. fig.4 shows the instance when the TORA model was simulated for the worst-case of angle scenario.

As seen from fig.4, there are slight undershoots & overshoots initially but this is expected due to the rotational actuator performing a fully controlled swing from the upright position to the downwards position.

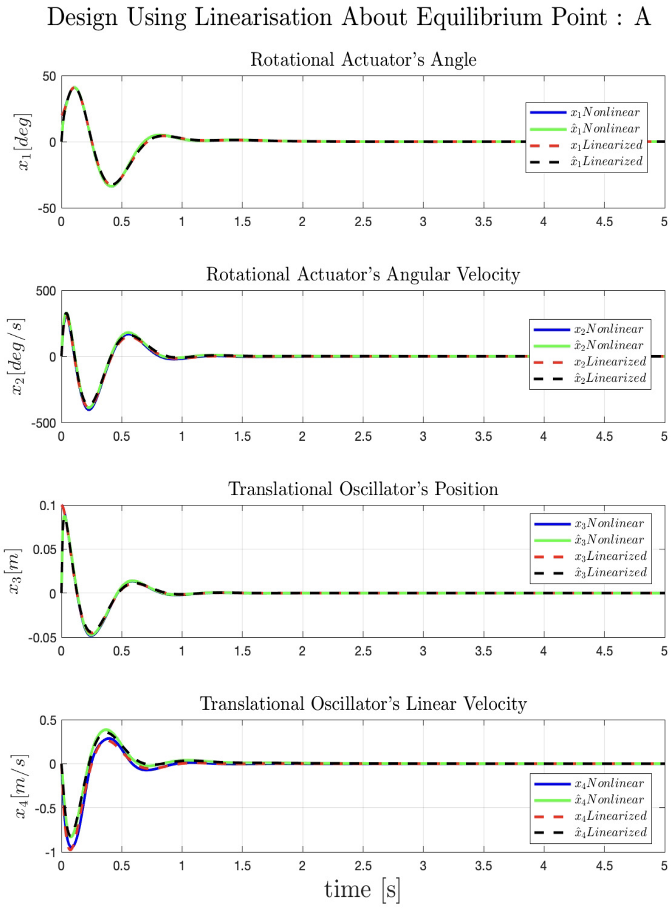


Fig. 3. TORA Linearized vs Nonlinear Models LQE Controlled Estimation of States

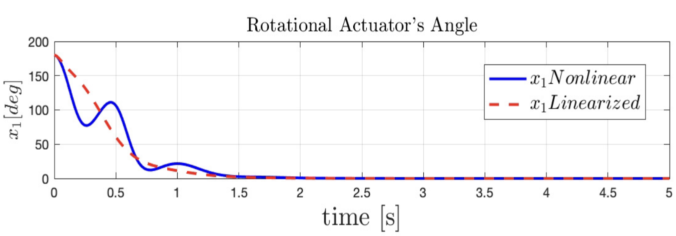


Fig. 4. TORA Linearized vs Nonlinear Models Regulation of Angle State from Initial Condition of $180°$

The state-feedback controller was able to successfully regulate the system states to zero with initial displacements until $0.3m$ while having other states set to 0 as initial conditions. The maximum time taken to regulate and reach zero-state was less than 3 seconds with any state of the system which again proves a fairly stable design of the state-feedback controller. The worst case of position scenario regulation is shown in fig.5. Again there were a few overshooting oscillations present in the response and this was expected as the initial position has moved further away from the system's linearized position at equilibrium point "a" hence caused such behavior as it was harder to regulate,

Next the choice of optimal control was compared to the desired closed-loop pole placement design approach. A state-

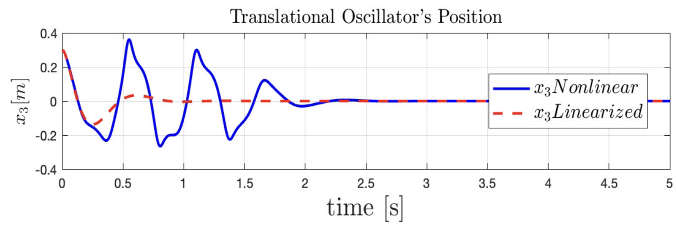


Fig. 5. TORA Linearized vs Nonlinear Models Regulation of Position State from Initial Condition of $0.3m$

feedback controller with closed-loop stable poles located at,

$$\lambda_{1,2} = -2.00 \pm 2.10i \ , \ \lambda_{3,4} = -20.00 \pm 20.97i$$

were selected with the desire of behaving like a $2^{nd}$ order under-damped system with 5% overshoot & 2s settling time. The resulting regulation response was compared against the previously designed controller which used linear quadratic regulation. The same initial conditions of $20°$ for angle and $0.1m$ for position were provided and the plot is seen in fig.6.

As seen from the plot, the system regulates towards zero-state, yet giving higher overshoot magnitudes. When comparing fig.2 response to fig.6, fig.2 (LQR controlled) is much more desirable as the TORA response is allowed to regulate smoothly with less settling time of 1.5 seconds and much lower overshoot magnitudes. The input torque '$\tau$' needed for actuation of the TORA system is approx. 4 times higher than the optimal control approach making the pole placement design more costly to implement. This can be realized from the resulting regulation controller gain values,

$$K = \begin{bmatrix} 4.96 & 2.60 & -1113.45 & -20.60 \end{bmatrix}$$

Therefore the optimal control choice for the regulation controller can be considered as a better performing approach for the TORA system.

Further, the choice of optimal estimation was also compared to the desired closed-loop pole placement design approach. A full-order Luenberger observer with closed-loop stable poles located at,

$$\lambda_{5,6} = -40.00 \pm 2.10i \ , \ \lambda_{7,8} = -200.00 \pm 20.97i$$

were selected to be 10 to 20 times further from the regulation controller poles so that the estimation error is compensated on the regulation controller's performance. The resulting estimation response was compared against the previously designed Kalman filter which used linear quadratic estimation. The resulting plot is seen in fig.7 while having the LQ regulation controller.

As seen from the plot, the system estimates the states again giving higher overshoots in the response. The previous response in fig.3 (LQE controlled) is much more preferable compared to the pole placement design approach. This is due to the higher error present in the estimation along with the observer gains complementing the error due to amplification,

$$L = \begin{bmatrix} 221.97 & 7612.17 & -43.27 & -9789.82 \\ -60.02 & -2375.59 & 258.03 & 11617.36 \end{bmatrix}^T$$

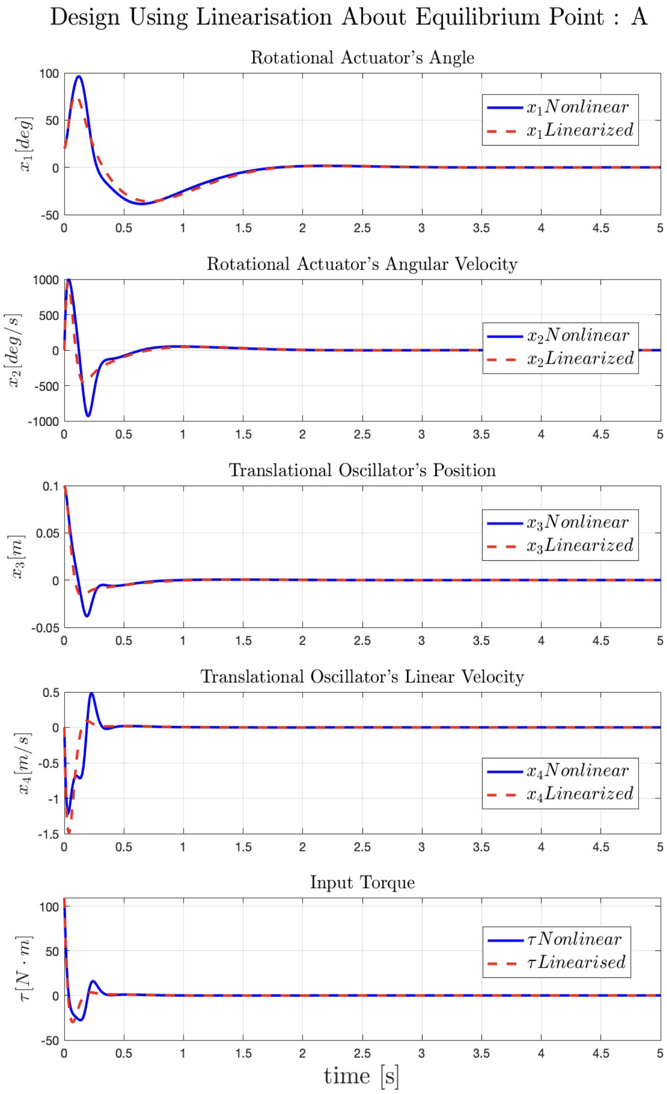


Fig. 6. TORA Linearized vs Nonlinear Models Pole Placement Regulation of States & Input Torque

Therefore the optimal estimation choice for the observer can be considered as a better performing approach for the TORA system in terms of cost-effectiveness & better performance.

To test and analyse the disturbance rejection and noise attenuation levels of the kalman filter observer, the system was augmented with constant process disturbances and measurement noise in the form of two matrices 'G' and 'H' respectively which carried the values,

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; H = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

The designed linear quadratic estimator was tested for its estimation capabilities and it was able to successfully estimate all of the states even while having disturbed processes and noisy sensors. Throughout the estimation, the process response was trusted more than the sensor response and this can be seen in the 10:1 relationship present between the

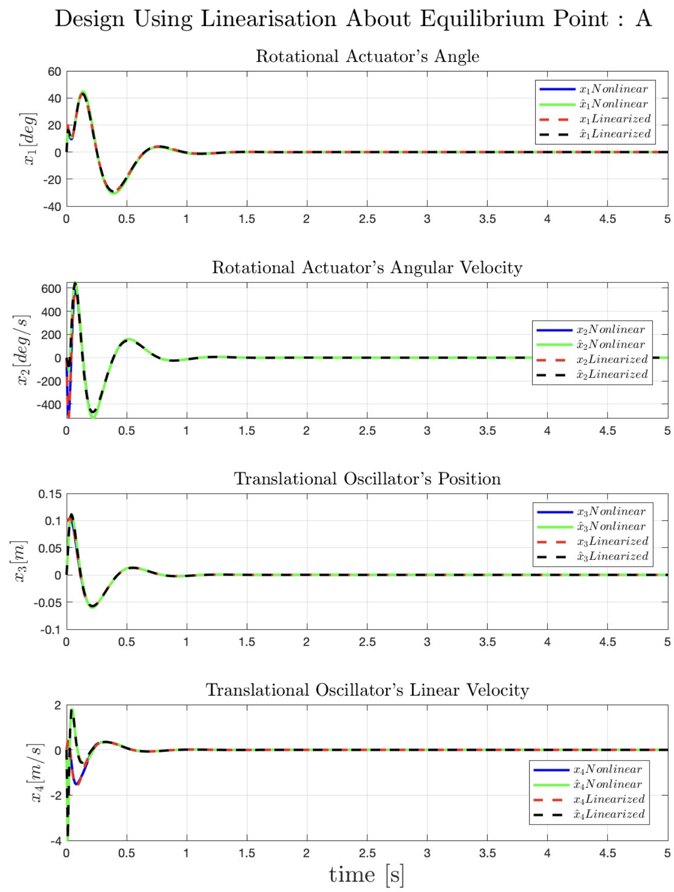


Fig. 7. TORA Linearized vs Nonlinear Models Pole Placement Estimation of States

'G' & 'H' matrices. Therefore it is evident that the Kalman filter observer was successfully able to reject disturbances and attenuate noise which can occur while the states of the system are being estimated from its input & output.

## VI. Conclusion & Recommendations

In conclusion, through linearization of non-linear dynamics, state-feedback controller design & observer design, a complete output-feedback LQG control system was designed for the TORA system. The optimal regulation & estimation was seen to perform better with the TORA model over the pole placement approaches with the trade-off being that system is slightly prone to model uncertainties. Further, it is safe to state that the designed output-feedback controller provides stable regulation & estimation capabilities yet being cost-effective with actuator realization. Moreover, on a practical setup of the TORA system, there would be dampening forces & other external disturbances acting on the system which can critically change the system dynamics. As a recommendation, the robustness issues of the current regulator can be hurdled through '$H\infty$' control design along with '$\mu$' synthesis as this design approach finds the most robust gain values to uncertainty while optimally controlling.

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## References

[1] B. Luders "16.30/31 Feedback Control Systems, Fall 2010 — Recitation # 10" MIT OpenCourseWare - Astronautics and Aeronautics Courses, November 2010.

[2] J. Doyle "Guaranteed Margins for LQG Regulators", IEEE Transactions on Automatic Control, vol. ac-23, no. 4, August 1978.

[3] K. Hassan "Nonlinear Control" Pearson Education Limited, 2015

[4] M. Tavakoli, H. Taghirad, and M. Abrishamchian "Identification and Robust H∞ Control of the Rotational/Translational Actuator System", International Journal of Control, Automation, and Systems, vol. 3, no. 3, pp. 387-396, September 2005.

## Appendix

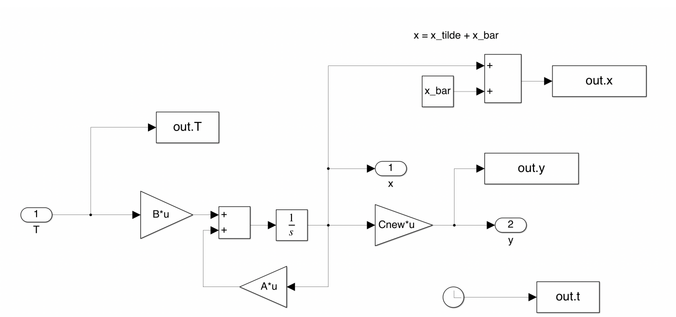


Fig. 8. TORA Linearised System Model SIMULINK

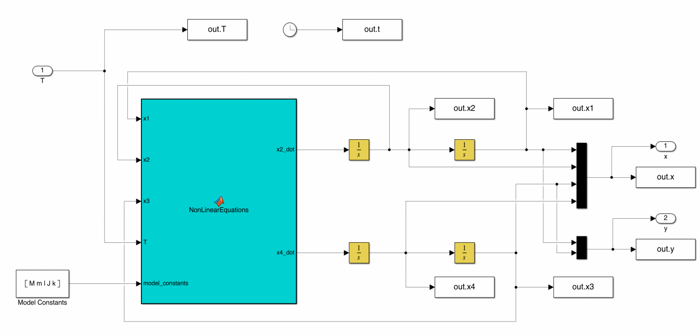


Fig. 9. TORA Nonlinear System Model on SIMULINK

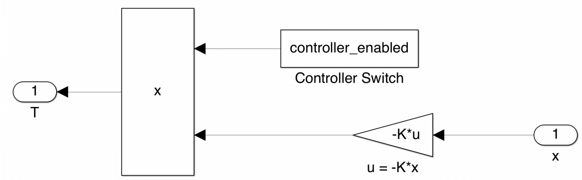


Fig. 10. TORA Linearized Model Regulator on SIMULINK

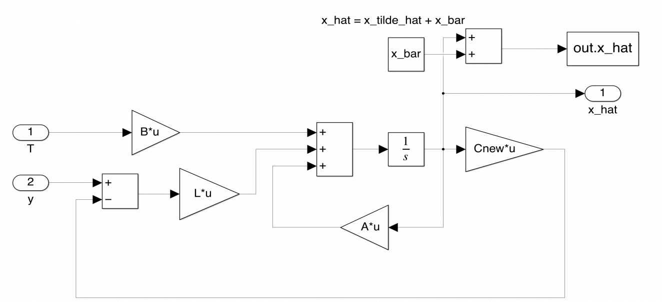


Fig. 11. TORA Linearized Model Observer on SIMULINK

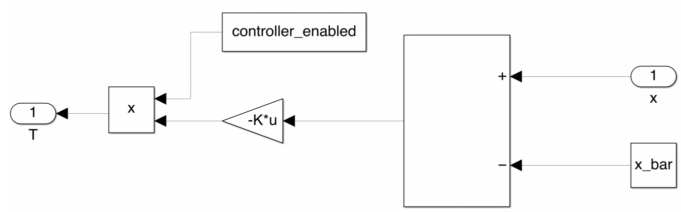


Fig. 12. TORA Nonlinear Model Regulator on SIMULINK

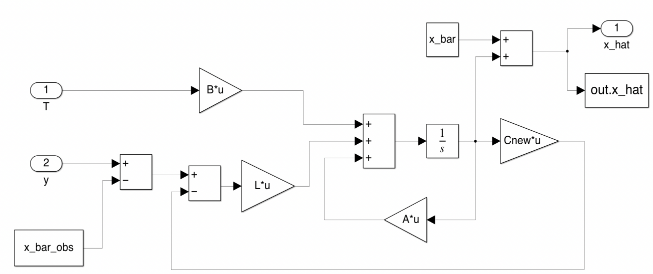


Fig. 13. TORA Nonlinear Model Observer on SIMULINK