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Gilbert Fritz

Matrikelnummer 0827276

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Betreuung: Univ.-Prof. Dipl.-Ing. Dr.techn. Günther Raidl

Mitwirkung: Univ.Ass. Dipl.-Ing. Dr.techn. Dr. Bin Hu

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Gilbert Fritz
Schlosshofer Straße 49/18

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Abstract

The Partition Coloring Problem (PCP) is a generalization of the the classical Vertex Coloring Problem (VCP), partitioning the set of nodes into clusters and seeking a coloring for the subgraph induced by selecting exactly one node from each cluster.

Diese Arbeit beschäftigt sich mit dem Partition Coloring Problem (PCP). Es handelt sich dabei um eine Generalisierung des Knotenfärbungsproblems und ist ein Optimierungsproblem der Komplexitätsklasse $\mathcal{NP}$.

Gegeben ist ein Graph, dessen Knotenmenge in disjunkte Partitionen unterteilt ist. Aus jeder Partition muss ein Knoten gewählt werden. Der durch die gewählten Knoten induzierte Subgraph soll unter der Bedingung eingefärbt werden, dass kein zueinander adjazentes Knotenpaar die gleiche Farbe annimmt. Ziel ist es, die Gesamtanzahl der verwendeten Farben - die sogenannte chromatische Zahl - zu minimieren.

Zur Lösung dieses Problems sollen mittels heuristischer Verfahren initiale Lösungen erstellt und diese mittels Tabusuche und wiederholter, partieller Neueinfärbung verbessert werden. Das Problem der Neueinfärbung wird mit unterschiedlichen Ansätzen gelöst.

Kurzfassung

Todo

Contents

CHAPTER 1

Introduction

1.1 Motivation

In order to obey the emerging demand for advanced broadband Internet applications such as video-conferences, high performance computing and others, extensive networks capacities have to be achieved. Links in optical networks operate much faster than their currently available electronic counterparts. Combined with the technique of Wavelength Division Multiplexing (WDM), which permits the simultaneous transmission of different channels along the same fiber [30], these so called Wavelength Routed Optical Networks (WRON's) are promising candidates for providing a flexible transport backbone network [22]. They bring out new problems in coordination of wavelengths usage [29]. One of them is the Routing and Wavelength Assignment problem (RWA), which consists in routing a set of light-paths and assigning a wavelength to each of them. The variant where all connection requirements are known beforehand and which aims to minimize the amount of used wavelength is called min-RWA and found to be $\mathcal{NP}$-hard [37].

Assigning wavelengths to one out of many paths for each connection requirement is equivalent to the $\mathcal{NP}$-hard **Partition Coloring Problem (PCP)** [25], also known as **Partition Graph Coloring Problem (PGCP)** which is subject of this thesis. Given a graph consisting of a clustered set of vertices and a set of edges, the aim is to select one vertex per cluster and for each vertex in the induced subgraph assign a color in the way that the overall number of colors – which in this context is said to be the chromatic number – is minimized. If each cluster holds only one vertex, the problem reduces to the standard Vertex Coloring Problem (VCP), which is used for a wide range of applications as scheduling, register allocation, pattern matching and others and has been studied extensively. In contrast, only a few papers have been published on PCP so far.

1.2 Guide to the Thesis

Definitions from graph theory and basic concepts which are required for the analysis of the Partition Coloring Problem are introduced in Chapter 2. Afterwards, Chapter 3 defines the PCP as well as the min-RWA formally and comments their computational complexity. Previous works and related research done so far is presented in Chapter 4. Chapter 5 provides details of the approach developed for the PCP and Chapter 6 presents its experimental results. Chapter 8 summarizes the knowledge achieved within this thesis, brings the considered approach into question and finally proposes a possible further work.

CHAPTER 2

Preliminaries

This chapter introduces theoretical fundamentals like definitions, terms and methods, that are necessary for analysing the PCP. The presented notations will be used consistently in this thesis.

2.1 Optimization Problems and Complexity

Since this thesis deals with an optimization problem and the analysis of a solution method, it needs to consider some complexity theory, which is an important field of computer science. Some definitions and explanations of optimization problems and complexity are given in this section. For a more detailed insight into these topics, the reader is referred to [15, 23, 33].
In general an optimization problem is the problem of finding the best solution among all feasible solutions. Depending on weather the variables are continuous or discrete, the optimization problem is said to be a continuous optimization problem or a combinatorial optimization problem (COP). Since the PCP belongs to the latter category, this thesis will not cover further explanations on continuous optimization problems. For information on that topic, the reader is referred to [21, 32]. Most of the following the definitions have been introduced in [6, 12].

Definition 1 (Combinatorial Optimization Problem) *A Combinatorial Optimization Problem $P = (S, f)$ can be defined by:*

- *A set of variables $X = \{x_1, x_2, \dots x_n\}$;*
- *variable domains $D_1, \dots, D_n$;*
- *constraints among the variables;*
- *an objective function f to be minimized*[1]*, where $f : D_1 \times D_2 \times \dots D_n \to \mathbb{R}^+$;*

[1]Maximizing an objective function f is the same as minimizing $-f$

The set of all feasible assignments is $S = \{s = \{(x_1, v_1), (x_2, v_2), \dots (x_n, v_n)\}\} \mid v_i \in D_i$, s *satisfies all the constraints* $\}$
For each COP P there exists a corresponding decision problem D, i.e. a problem whose output is either *YES* or *NO*. The complexity of D determines the complexity of P.

Definition 2 (Decision Problem) *The decision problem D for a Combinatorial Optimization Problem P asks if, for a given solution $s \in S$, there exists a solution $s' \in S$, such that $f(s')$ is better than $f(s)$: for a minimization problem this means $f(s') < f(s)$ and for a maximization problem $f(s) > f(s')$.*

An important issue that comes up when considering combinatorial optimization problems is the classification problems by their difficulty . To categorize problems into easy and difficult ones, the class of problems that are solvable in polynomial time by a deterministic touring machine and problems that are solvable in polynomial time by a nondeterministic touring machine are considered. This thesis prefers to describe the characteristics of $\mathcal{P}$ and $\mathcal{NP}$ at a more intuitive level, rather than formalizing the classes via Turing Machines. An examples for a problem in $\mathcal{P}$ is single source shortest path.

Definition 3 (Complexity class $\mathcal{P}$) *A problem is in $\mathcal{P}$ iff it can be solved by an algorithm in polynomial time.*

The complexity class $\mathcal{NP}$ is associated with hard problems. $\mathcal{NP}$ stands for "nondeterministic polynomial time", where "nondeterministic" is a way to express that solutions are guessed. The class $\mathcal{NP}$ is restricted to Decision Problems.

Definition 4 (Complexity class $\mathcal{NP}$) *A decision problem is in $\mathcal{NP}$ iff any given solution of the problem can be verified in polynomial time.*

The definition above states, that the solutions for problems in $\mathcal{NP}$ do not require to be calculated in polynomial time, but the solutions need to be verified in polynomial time. Therefore $\mathcal{P} \subseteq \mathcal{NP}$ holds (slightly abusing notation by restricting $\mathcal{P}$ to decision problems) [12].

Definition 5 ($\mathcal{NP}$-optimization Problem) *A COP is a $\mathcal{NP}$-optimization problem (NPOP) if the corresponding decision problem is in $\mathcal{NP}$.*

As an example the decision variant of the standard Vertex Coloring Problem (VCP) is considered, which asks weather a graph G is colorable within k colors or not. An algorithm has to verify for each vertex v colored with color c_v, if all its neighbors are colored with a color different to c_v and further count the number of distinct colors to check weather the number of colors is lower or equal to k. This can be done in time $\mathcal{O}(|V^2|)$, where V is the set of vertices.
The decision variant of VCP and many other decision problems are at least as difficult as any problem in $\mathcal{NP}$. These problems are said to be $\mathcal{NP}$-hard. Giving a polynomial-time reduction from an $\mathcal{NP}$-hard problem to a particular problem shows that this problem is $\mathcal{NP}$-hard, too. Such a reduction links the considered problem to the known $\mathcal{NP}$-hard problem in such a way that if and only if the considered problem can be solved in polynomial time also the $\mathcal{NP}$-hard problem to which it has been reduced can. [12] To gain a more detailed insight into that topic, the reader is referenced to [39].

Definition 6 ($\mathcal{NP}$-hard problems) *A problem is called $\mathcal{NP}$-hard iff it is at least as difficult as any problem in $\mathcal{NP}$, i.e., each problem in $\mathcal{NP}$ can be reduced to it.*

A lot of optimization problems are $\mathcal{NP}$-hard but not in $\mathcal{NP}$. For example, the optimization variant of the VCP, which searches for the minimum chromatic number is clearly at least as hard at its decision variant described above. Since the output is a number rather than a decision, it is not in $\mathcal{NP}$. $\mathcal{NP}$-hard problems that are also in $\mathcal{NP}$ are called $\mathcal{NP}$-complete. Many decision variants of $\mathcal{NP}$-hard problems like the one of VCP are $\mathcal{NP}$-complete. For the specific case of VCP, which is a generalization of PCP and therefore relevant for this thesis, its complexity is analysed in more detail in chapter 3.

Definition 7 ($\mathcal{NP}$-complete problems) *A problem is $\mathcal{NP}$-complete iff it is $\mathcal{NP}$-hard and in $\mathcal{NP}$.*

Informally, $\mathcal{NP}$-complete problems are the hardest problems in the class $\mathcal{NP}$. If there is an algorithm that solves any $\mathcal{NP}$-complete problem in polynomial time, then every problem in $\mathcal{NP}$ can be solved in polynomial time. So far no polynomial time deterministic algorithm has been found to solve one of them.

Theorem 1 *If any $\mathcal{NP}$-complete problem can be solved by a polynomial-time deterministic algorithm, then $\mathcal{P} = \mathcal{NP}$. If any problem in $\mathcal{NP}$ cannot be solved by a polynomial-time deterministic algorithm, then $\mathcal{NP}$-complete problems are not in $\mathcal{P}$.*

Most computer scientists assume that $\mathcal{P} \neq \mathcal{NP}$, although it has not been proven yet. The question $\mathcal{P} = \mathcal{NP}$ is one of the most prominent unresolved questions in the field of complexity theory, since a proof would imply a huge impact on any other discipline in discrete mathematics and computer science.

Nevertheless, for some $\mathcal{NP}$-complete problems of this class it is possible develop algorithms that have an average-case polynomial time complexity, despite having exponential time complexity in worst case. For other problems in this class, approximation algorithms can be found that return solutions in polynomial time with a guarantee of a specific solution quality. The development and analysis of approximation algorithms is an important field of research. The following definitions are taken from [40].

Definition 8 (Approximation algorithm) *An α-approximation algorithm for an optimization problem is a polynomial-time algorithm that for all instances of the problem produces a solution whose value is within a factor of α of the value of an optimal solution.*

The α is called approximation ratio or performance guarantee of the α-approximation algorithm. For minimization problems $\alpha > 1$ and for maximization problems $\alpha < 1$ holds. For example, a $1/2$-approximation algorithm for a maximization problem always returns a solution in polynomial time, that is at least half as good as the optimal solution. For some problems there even exist polynomial time algorithms, whose approximation ratio can be given as parameter. They have so called polynomial-time approximation schemes.

Definition 9 (Polynomial-time approximation scheme) *A polynomial-time approximation scheme (PTAS) is a family of algorithms $\{A_\epsilon\}$, where there is an algorithm for each $\epsilon > 0$, such that A_ϵ is a $(1 + \epsilon)$-approximation algorithm (for minimization problems) or a $(1 - \epsilon)$-approximation algorithm (for maximization problems).*

2.2 Graph Theory Definitions

Graphs are mathematical structures used to model relations and processes of interconnected objects. They are one of the most important objects for the study of discrete mathemathics and can be used to model a wide range of problems that appear in physics, biology, linguistics, information technology and other disciplines. Most of the following definitions are take from [17].

Definition 10 (Graph) *A graph is a tuple $G = (V, E)$, where V denotes the set of nodes and $E \subseteq V \times V$ denotes the set of edges. An edge from node i to j is denoted by $\{i, j\}$. We call a graph simple, if it does not contain multiple edges, i.e. more than one edge between the same nodes, or loops, i.e. edges $\{i, i\}$.*

Definition 11 (Directed graph) *A directed graph or digraph is a tuple $D = (V, A)$, where V denotes the set of nodes and $A \subseteq V \times V$ denotes the set of arcs or directed edges. An arc from node i to j is denoted by (i, j). We call a directed graph simple, if it does not contain multiple arcs, i.e. more than one arc between the same nodes, or loops, i.e. arcs (i, i).*

Unless declared explicitly, this thesis considers only simple, undirected graphs $G = (V, E)$.

Definition 12 (Subgraph) *Given a graph $G = (V, E)$, $G' = (V', E')$ is called a subgraph if $V' \subseteq V$, $E' \subseteq E$ and $E' \subseteq V' \times V'$. If $V' = V$ we call G' a spanning subgraph or factor.*

Definition 13 (Deletion of a node) *Given a graph $G = (V, E)$, $G - v = (V \backslash v, E \backslash \{e \mid v \in e\})$.*

Definition 14 (Adjacency and incidence) *Two nodes x and y are called adjacent, if they share an edge e, i.e. $\exists e = \{x, y\} \in E$. Two edges e and f are called adjacent, if they share a node x, i.e. $e \cap f = x$. A node v is called incident to an edge e, if $v \in e$.*

Definition 15 (Node degree) *The degree of a node v in an undirected graph G, denoted by $d(v)$, is the number of edges, that are incident to the node v, i.e. in E there exists an edge $\{v, x\}$. The number of outgoing arcs (v, x) from a node v in a directed graph D is called out-degree and is denoted by $d^+(v)$, the number of ingoing arcs (x, v) to a node v is called in-degree and is denoted by $d^-(v)$.*

Lemma 1 (Handshaking Lemma)

$$\sum_{v \in V} d(v) = 2 \cdot |E|$$

Proof. As every edge $\{i, j\}$ is incident to exactly two nodes, namely i and j, it is counted one time at $d(i)$ and one time at $d(j)$. So the sum over all node degrees is exactly two times the number of edges. □

Corollary 1 (Directed graph) *The number of nodes with odd node degree is even.*

Proof. This immediately follows from the handshaking lemma. □

Lemma 2

$$\sum_{v \in V} d^+(v) = \sum_{v \in V} d^-(v) = 2 \cdot |A|$$

Proof. As every arc (i, j) has exactly one "in-node" and one "out-node", it follows, that the sum of all out-degrees equals the sum of all in-degrees and hence the number of arcs. □

Definition 16 (Maximum and minimum node degree) $\triangle(G) = max\{d(v) \mid v \in V\}$ *denotes the maximum node degree in a graph.* $\delta(G) = min\{d(v) \mid v \in V\}$ *denotes the minimum node degree in a graph.*

Definition 17 (Neighborhood of a node) *The neighborhood of a node* $v \in V$ *is denoted by* $N(v) = \{x \mid \{v, x\} \in E\}$*. In the directed case the neighborhood consists of all nodes that are reachable from* v*, i.e.* $N(v) = \{x \mid (v, x) \in A\}$*.*

Definition 18 (Walk) *A sequence* $v_0, e_1, v_1, e_2, \ldots, e_n, v_n$ *with* $n \geq 0$ *is called a walk, if for all* v_i *with* $i \neq 0$ *exists an* $e_i = \{v_{i-1}, v_i\} \in E$*.*

Definition 19 (Directed walk) *A sequence* $v_0, a_1, v_1, a_2, \ldots, a_n, v_n$ *with* $n \geq 0$ *is called a directed walk, if for all* v_i *with* $i \neq 0$ *exists an* $a_i = (v_{i-1}, v_i) \in A$*.*

Definition 20 (Trail) *A sequence* $v_0, e_1, v_1, e_2, \ldots, e_n, v_n$ *with* $n \geq 0$ *is called a trail, if for all* v_i *with* $i \neq 0$ *exists an* $e_i = \{v_{i-1}, v_i\} \in E$ *and all* e_i *are distinct.*

Definition 21 (Directed Trail) *A sequence* $v_0, a_1, v_1, a_2, \ldots, a_n, v_n$ *with* $n \geq 0$ *is called a directed trail, if for all* v_i *with* $i \neq 0$ *exists an* $a_i = (v_{i-1}, v_i) \in A$ *and all* a_i *are distinct.*

Definition 22 (Path) *A sequence* $v_0, e_1, v_1, e_2, \ldots, e_n, v_n$ *with* $n \geq 0$ *is called a path, if for all* v_i *with* $i \neq 0$ *exists an* $e_i = \{v_{i-1}, v_i\} \in E$ *and all* v_i *are distinct.*

Definition 23 (Directed path) *A sequence* $v_0, a_1, v_1, a_2, \ldots, a_n, v_n$ *with* $n \geq 0$ *is called a directed path, if for all* v_i *with* $i \neq 0$ *exists an* $a_i = (v_{i-1}, v_i) \in A$ *and all* v_i *are distinct.*

Definition 24 (Length of a path) *Given a path* $P = v_0, e_1, v_1, e_2, \ldots, e_n, v_n = P(v_0, v_n)$*, the length is the number of edges and denoted by* $l(P) = n$*, analogously for the directed case.*

Definition 25 (Cycle) *A cycle is a path, where $v_0 = v_n$.*

Definition 26 (Acyclic graph) *A graph is called acyclic if it does not contain a cycle.*

Definition 27 (Bipartite Graph) *todo*

Theorem 2 *Let $W = W(v_0, v_n)$ be a walk, then there is a subsequence $P = P(v_0, v_n) \subseteq W(v_0, v_n)$ such that P is a path.*

Proof. We know that for a path holds $v_i \neq v_j, \forall i < j$. Suppose for W holds that $v_i = v_j$ for arbitrary $i < j$. Then $W' = v_0, e_1, v1, \ldots, v_i, e_{j+1}, v_{j+1}, \ldots, v_n$ is a walk with $i - j$ less edges and W' is a subsequence of W. Applying this until $\forall i, j : v_i \neq v_j$ yields a path from v_0 to v_n. This of course also holds for the directed case. □

Definition 28 (Network) *A network $N = (G, c)$ consists of a graph $G = (V, E)$ and a cost function $c : E(G) \longrightarrow \mathbb{R} \geq 0$, which assigns each edge e a nonnegative value c_e . Networks are also called weighted graphs.*

Definition 29 (Costs of a graph) *The cost c_G of a graph G is the sum of its edge costs, i.e. $c_G = \sum_{e \in E} c_e$.*

Definition 30 (k-coloring) *A k-coloring is a function $f : V \to \{1, \ldots, k\}$ such that for every edge $\{u, v\} \in E$ we have $f(u) \neq f(v)$.*

Definition 31 (Chromatic Number) *Let $G = (V, E)$ be a graph. We state that c is a (proper) k-coloring of G if all the vertices in V are colored using k colors such that no two adjacent vertices have the same color. The chromatic number is defined as the minimum k for which there exists a (proper) k-coloring of G.*

Definition 32 (Truth Assigment) *A truth assignment is a choice of $true$ or $false$ for each variable, i.e., a function $v : X \to \{true, false\}$.*

Definition 33 (Conjunctive Normal Form) *A conjunctive normal form (CNF) formula is a conjunction of clauses: $C_1 \wedge C_2 \wedge \ldots \wedge C_k$*

Definition 34 (Satisfying Assignment) *A truth assignment is a satisfying assignment if it makes every clause $true$.*

2.3 Metaheuristics

As defined before, a COP consists of finding an optimum among a set of feasible solutions. In almost every case this is of huge size compared to the size of the instance. Solving a COP exactly means finding the optimal solution out of that set. Since for $\mathcal{NP}$-complete problems no algorithm that performs in polynomial time could be found yet, scientists try to find algorithms that approximate optimal solutions.
In general there exist two classes of approximation methods: construction and improvement heuristics. As its name states, the former construct a solution from scratch by adding components until the solution is complete. Usually these algorithms perform fastest but often return a solution quality that is inferior to the ones returned by improvement heuristics. Used as initial solution for an improvement heuristic, the construction heuristic may return an infeasible solution. An improvement heuristic starts out from a feasible solution and iteratively tries to replace it by a better/feasible one that is derived from the current solution.
Improvement heuristics can be be divided into two groups, population based and local search based heuristics [6]. Examples of the former are Ant Colony Optimization (ACO), Evolutionary Computation (EC) including Genetic Algorithms (GA) and of the latter Iterated Local Search (ILS), Simulated Annealing (SA) and Tabu Search (TS). TS is described in more detail in section 9 and as this thesis does not use population based algorithms, it excludes further descriptions of that group. For a more detailed insight into the field of population based algorithms, the reader is referred to [16].
All these methods form a relatively new group of heuristics and are summed up by the term *metaheuristics*, which was first introduced in [18]. In 1996 Osman and Laporte provided a formal definition of metaheuristics [31]:

Definition 35 (Metaheuristic) *A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions.*

Any COP can be solved by any kind of metaheuristic. The famous no free lunch theorem [41] states, that over all possible problems, there is no heuristic that performs better than any other heuristic including random search. According to the theorem, if a strategy performs better in one subarea, it performs worse in another. By acquiring and including problem specific knowledge, it is possible to develop strategies for classes of problems that perform better than others [10]. In that context, Gebhard defined in [34]:

Definition 36 *Metaheuristic algorithms make no assumptions on the problem and (in theory) can be applied on any optimization problem. They define an abstract order of instructions which lead to improved or feasible solutions. In almost any case these instructions must be implemented using problem specific knowledge.*

Basic Local Search

Basic local search (LS) is an improvement heuristic that iteratively tries to replace the solution by a better one that is located in an appropriately defined neighborhood structure of the current solution [6]. Definitions 37,39,40 are taken from [28]. Algorithm 1 outlines the basic local search procedure.

Algorithm 1: Basic Local Search

Input: A COP $P = (S, f)$
Output: A feasible Solution s
$s \leftarrow$ GenerateInitialSolution(S);
$improved \leftarrow true$;
while $improved$ **do**
    $s' \leftarrow$ Improve($\mathcal{N}(s)$);
    if $f(s')$ *NOT better than* $f(s)$ **then**
        $improved \leftarrow false$;
    else
        $s \leftarrow s'$;
return s

Definition 37 (Neighborhood structure) *A neighborhood structure is a function $\mathcal{N} : S \rightarrow 2^S$ that assigns to every $s \in S$ a set of neighbors $\mathcal{N}(s) \subseteq S$. $\mathcal{N}(s)$ is called the neighborhood of s.*

Definition 38 (Move) *In the context of examining a search space by using a neighborhood structure and given an actual solution $s \in S$, a move is the acceptance of a solution $s' \in \mathcal{N}(s)$ as the new actual solution.*

Iteratively improving the solution by choosing the first neighbor $s_f \in \mathcal{N}(s) \mid f(s_f) < f(s)$ for a minimization problem is called *First Fit* (FF), choosing the best neighbor $s_b \in \mathcal{N}(s) \mid f(s_b) < f(x), \forall x \in \mathcal{N}(s)$ is called *Best Fit* (BF). In the basic version of LS, both variants stop if no better solution can be found, which is called a local minimum. As this thesis is about the PCP which is a minimization problem, optimum and minimum are used equivalently.

Definition 39 (Local minimum) *A locally minimal solution (or local minimum) with respect to a neighborhood structure $\mathcal{N}$ is a solution $\hat{s}$ such that $\forall s \in \mathcal{N}(\hat{s}) : f(\hat{s}) \leq f(s)$. We call $\hat{s}$ a strict locally minimal solution if $\forall s \in \mathcal{N}(\hat{s}) : f(\hat{s}) < f(s)$.*

Definition 40 (Global minimum) *A global minimum (optimum) of a minimizing combinatorial optimization problem is a solution, such that $f(\hat{s}) \leq f(s), \forall x \in X$. Therefore a global optimum is a local optimum for all neighborhood structures $\mathcal{N}$.*

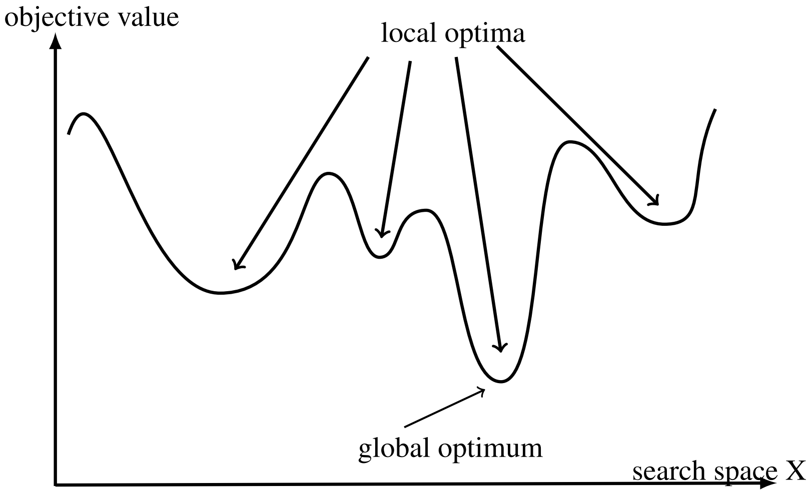

Figure 2.1: Local and global optima for a minimization problem

Stopping at a local minimum results in quite unsatisfactory solutions, therefore methods have been developed to escape from such a local minimum in order to find the global minimum. Figure 9 provides an idea of local and global minima to the reader. A simple technique is to start the LS from different initial solutions repeatedly, which is not very efficient, since the search information from preceding searches is not used. In order to avoid stopping at a local minima, metaheuristic algorithms use more complex search techniques and termination conditions including maximum number of iterations or CPU time.

Tabu Search

The simple Tabu Search (TS) outlined in listing 2 chooses the best fitting solution out of the neighborhood $\mathcal{N}(s)$. In contrast to LS, uphill moves are accepted. To prevent cycles, a short term memory – the so called tabu list – stores recently visited solution, which are excluded from the neighbourhood of the current solution. The tabu list is of a specified size and therefore the algorithm has to remove solutions from it, which is usually done in a FIFO order. Most implementations of TS – like the one implemented in this thesis – do not store whole solutions, but solution attributes like moves or differences between solutions. The size of the list is a crucial parameter in TS and has to be chosen wisely, since if it is chosen too short the algorithm may stuck in cycles of higher order more likely and if it is chosen too long, the search space is restricted too much. The algorithm terminates if a specified termination criterion is met.

Algorithm 2: Tabu Search

Input: A COP $P = (S, f)$
Output: A feasible Solution s

$s \leftarrow$ GenerateInitialSolution(S);
$TabuList \leftarrow \emptyset$;
while *NOT termination criterion* **do**
    $s' \leftarrow$ ChooseBestOf($\mathcal{N}(s) \setminus TabuList$);
    Update($TabuList$);
return s

2.4 Integer Linear Programming

Linear Programming (LP) or Mathematical Programming is an important field of operations research. Formulating a Linear Program is a mathematical way to define a COP and consists of defining the following inequations and equations: an objective function, a set of constraints on the variables x and a set of constraints on their domain, which are all inequations. All LPs can be written in the following form:

$$\begin{aligned} max. \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & x \in \mathbb{R}^n \end{aligned}$$

Restrictions on the domain of the variables dedicates the LP into one of the following categories. Be x the set of variables:

1. Linear Program (LP): $x \in \mathbb{R}^n$
2. Integer Linear Program (ILP): $x \in \mathbb{Z}^n$
3. Binary Integer Linear Program (BIP): $x \in \{0, 1\}^n$
4. Mixed Integer Linear Program (MIP): variables can be restricted to different domains

Optimizing the objective function with respect to the constraints means to solve the problem exactly which is in general $\mathcal{NP}$-hard. It has to be considered that classifying the problem into complexity classes only gives a worst case analysis of the running time any algorithm would require. State-of-the-art solvers like CPLEX [1], GUROBI [2], XPRESS [3] use the simplex or the interior point algorithm combined with solution pruning methods as Branch and Bound or Branch and Cut, what makes them acquire relatively good average running times and makes LP a widely used method in operations research today. For example, for the Traveling Salesman Problem the optimal tour through 15112 Cities was calculated in 2001 [3].

[1] http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/
[2] http://www.gurobi.com/
[3] www.solver.com/xpress-solver-engine

CHAPTER 3

Problem Definition

The Partition Coloring Problem (PCP) is a generalization of the Standard Vertex Coloring Problem (VCP) and has initially been considered by Li and Simha in [25]. The problem arose from considering the join problem of routing and wavelength assignment in Wavelength Division Multiplexing optical networks. It is therefore a subproblem of a variant of the Wavelength Routing and Assignment Problem (RWA), namely the min-RWA problem. This chapter aims to give formal definitions and examples of VCP, PCP and min-RWA and reasons about their computational complexities.

3.1 Standard Vertex Coloring Problem

As this thesis refers to VCP in various contexts and for the sake of completeness, this section provides a formal definition, although the problem has already been explained as an example in chapter 2.
Given a non-directed graph $G = (V, E)$, the VCP consists in assigning a color to each node in V, such that no adjacent nodes have the same color. The aim is to minimize the chromatic number, i.e. the total number of colors used. Figure 3.1 shows a simple graph colored with three colors.

3.2 Partition Coloring Problem

As many Network Design Problems (NDPs), the VCP can be generalized by partitioning the vertex set V into clusters $V_k, k \in K$, and expressing feasibility constraints in terms of the clusters instead of individual nodes [13]. One resulting Generalized Network Design Problem (GNDP) is the PCP[1]. A formal definition of PCP follows:

[1]Due to Fereman's definition the PCP is an "Exactly" GNDP, since it requires the solution to select exactly one vertex per each cluster.

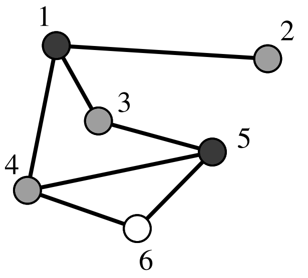

Figure 3.1: A graph with 6 nodes, colored optimally with 3 colors

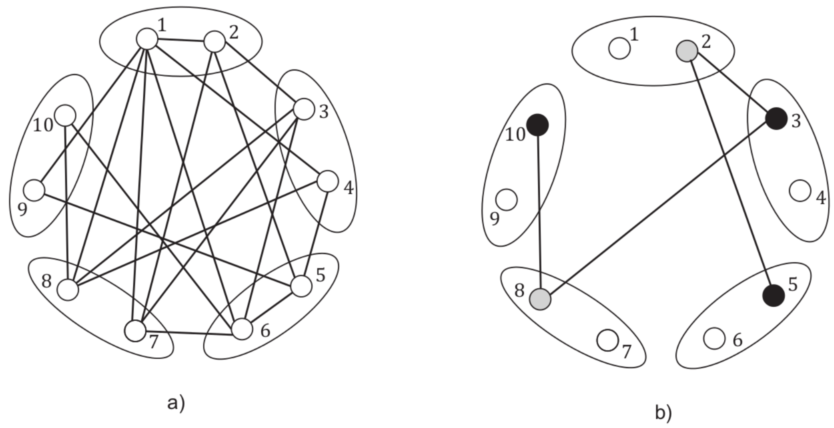

Figure 3.2: a) Shows a problem instance and b) a solution with two colors.

Let $G = (V, E)$ be a non-directed graph and V partitioned into q mutually exclusive, nonempty subsets $V_1, V_2, \ldots, V_q$, where $V_i \cap V_j = \emptyset, \forall i, j = 1, \ldots, q,\ i \neq j$. We refer to $V_1, V_2, \ldots, V_q$ as the components of the partition. The PCP consists in finding a subset $V' \subset V$ such that $|V' \cap V_i| = 1, \forall i = 1, \ldots, q$ (i.e., V' contains one node of each component V_i), and the chromatic number of the graph G' induced by V' is minimum.

Figure 3.2 shows an example of an instance with 5 clusters, each holding 2 nodes and its solution with a chromatic number of 2.

3.3 Wavelength Routing and Assignment Problem

Wavelength Division Multiplexing is a technique that allows a single optical link to transfer multiple data streams simultaneously by using distinct wavelengths for each data stream. Data

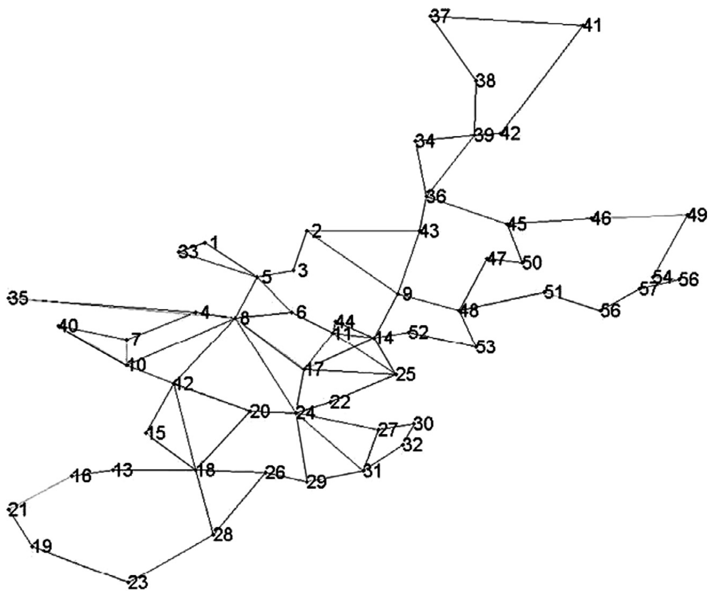

Figure 3.3: Instance of an optical network with 57 vertices and 85 edges. Extracted from the European optical transport network [5].

is transferred along a route of linked physical network routing devices (nodes). An all optical connection between two nodes is called lightpath. Assuming the so called wavelength continuity constraint [27] means to assume that the same wavelength has to be kept over all physical links along the route (i.e. it can not be converted by any node), so the lightpath has to be set up with one wavelength from the source to the destination node. It follows that any two paths, having at least one link in common, have to use different wavelengths, in order to enable the common link(s) to transfer data simultaneously. As an example of how such a network looks like, figure 3.3 shows an extract of the European optical transport network.

In general, the Wavelength Routing and Assignment Problem (RWA) consists of an undirected network Graph $N = (V, E)$, where nodes represent network routing devices and edges represent full duplex (optical) links, i.e. links supporting data transmission in both directions. Further, a set of source-destination pairs (or connection requests) $C = \{(s_1, d_2), \ldots, (s_k, d_k) \mid s_i, d_i \in V\}$ and a set of wavelengths $\Lambda = \{\alpha_1, \ldots, \alpha_m\}$ is given. If all connection requests are known in advance, the RWA is said to be static, otherwise dynamic [29]. The static RWA can further be distinguished by characteristics of its objective. In the context of this thesis, only the min-RWA problem is relevant, which is a static version of RWA aiming to select exactly one path and one wavelength for each pair $(s, d) \in C$, in the way that the number of wavelength $|\Lambda|$ is minimized under the continuity constraint and its consequences, i.e. if any two paths have at least one edge in common, distinct wavelengths have to be assigned to them. The problem can be decomposed into two subproblems:

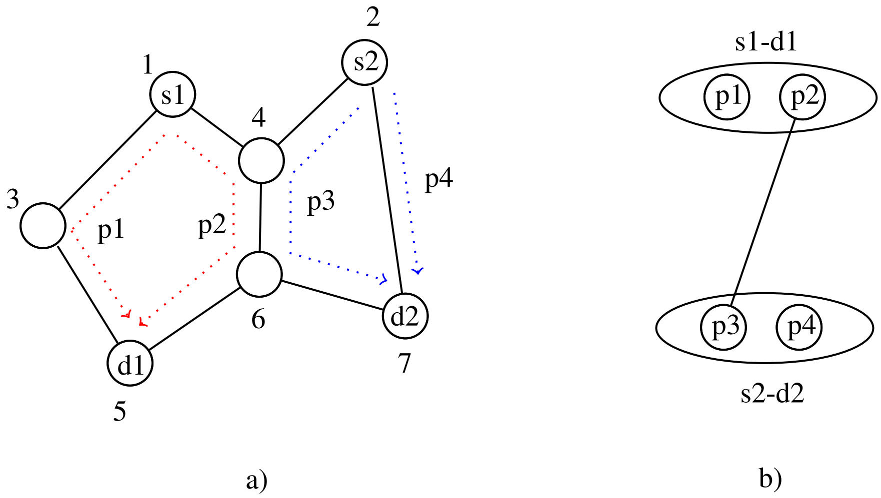

Figure 3.4: a) A graph with two source-destination pairs and two paths each. Since paths p2 and p3 share edge {4,6}, they are not allowed to use the same wavelength and therefore the corresponding nodes are adjacent in b) the resulting PCP.

1. routing: finding a set of paths $P_{s,d}$ for each source-destination pair $(s, d) \in C$
2. wavelength assignment: selecting exactly one path in $P_{s,d}$ and one wavelength α_i for each pair $(s, d) \in C$, in the way that the number of wavelength is minimized under the continuity constraint.

Considering the demands of real world instances, it is clear that the computed paths should be relatively short. The first subproblem can be solved in polynomial time by any single source shortest path algorithm like Dijkstra's or the B*-algorithm. If $|P_{s,d}| = 1, \forall(s, d) \in C$, i.e. there is exactly one path considered for each source-destination pair, the second subproblem can be transformed into VCP, in any other case $\exists(s, d) \in C : |P_{s,d}| > 1$ into PCP. The transformation consists in considering each source-destination pair (s, d) as a cluster and and each path in $P_{s,d}$ as a node. In the case that two paths share at least one edge, the corresponding nodes are adjacent. Selecting a node-color pair out of a cluster in PCP is equivalent to selecting a path for a source-destination pair and assign a wavelength to it. Figure 3.4 demonstrates the transformation by example.

3.4 Problem Complexity

The decision variant of VCP asks for whether a graph can be colored within k colors or not. For $k = 2$ the answer can be computed in linear time by checking if the graph is bipartite. For $k \geq 3$, a certificate for a decision can only be given by a valid coloring. In chapter 2 it has been stated that the decision variant of VCP belongs to the class $\mathcal{NP}$. It has been proven in the early 1970s by Cook and Levin, that any problem in $\mathcal{NP}$ can be reduced to SAT. In complexity theory, SAT is one the most prominent NPOs in $\mathcal{NP}$ and it is widely assumed that there does not exist an algorithm that solves it exactly in polynomial time. This assumption is closely linked to the question $\mathcal{P} \neq \mathcal{NP}$.

Definition 41 (SAT) *Given a set of clauses $C_1, \ldots, C_k$ in CNF over a set of variables $X = \{x_1, \ldots, x_n\}$, the SAT problem asks if there exist a satisfying assignment.*

Theorem 3 (Cook Levin) *Any problem in $\mathcal{NP}$ can be reduced in polynomial time by a deterministic Touring machine to SAT.*

Generating a k-coloring of a graph $G = (V, E)$ can be reduced to SAT as follows: For each possible node-color assignment, introduce a boolean variable $x_{vc}, v \in V, c \in \{1, \ldots, k\}$. Considering the following formulas:

$$\bigvee_{1 \leq c \leq k} p_{vc} \qquad (v \in V) \tag{3.1}$$

$$\neg(p_{vc} \wedge p_{vd}) \qquad (v \in V, 1 \leq i < j \leq k), \tag{3.2}$$

$$\neg(p_{vc} \wedge p_{wc}) \qquad (\{v, w\} \in E, 1 \leq i \leq k). \tag{3.3}$$

Finding an assignment satisfying these formulas is a one-to-one correspondence to finding a k-coloring of graph G.

When PCP is decomposed into two phases – the node selection phase and the coloring phase – it becomes clear that the coloring phase is equivalent to VCP, therefore $VCP \leq_P PCP$ holds. Li and Simha show in [25] another way to reduce PCP to VCP.

Theorem 4 *PCP is $\mathcal{NP}$-hard.*

Similarly, min-RWA can be decomposed into two phases, where the assignment phase is equivalent to PCP, as shown in section 3.3. A proof of $\mathcal{NP}$-hardness has been provided by Erlebach and Jansen in [37].

Theorem 5 *min-RWA is $\mathcal{NP}$-hard.*

CHAPTER 4

Previous Works

This chapter provides an overview on previous works dealing with the Partition Coloring Problem. While the VCP has been studied extensively, only a few papers have been published on the PCP. So far, two exact approaches and three heuristics have been presented.

4.1 Exact Approaches

In 2010, Frota and Ribeiro presented a Branch-and-Cut algorithm for the PCP in [14], which is based on a generalization of the 0-1 formulation for the VCP proposed in [26, 43], called formulation of representatives. The branching strategy to decompose the problem into two subproblems is based on Mehrotra and Trick's branching rule [1], that branches on two non-adjacent vertices. Improvements of linear relaxation bounds have been achieved by generalizing the original family of valid inequalites [26, 43], based on *External cuts* and *Internal cuts*. For their experiments they used an AMD-Atlon machine with a 1.8 GHz clock and one Gbyte of RAM memory. Within 2 hours, each instance with up to 80 nodes and density of 0.5 could be solved to optimality. For instances with 90 nodes, only the ones with density ≥ 0.5 could be solved to optimality in the same time. It is remarkable, that the algorithm performs worst on instances with a density between 0.3 and 0.5.
One year later, Hoshino et al. published a paper describing a Branch-and-Price algorithm, that performs "far superior to the branch-and-cut algorithm in all instance classes tested" [20]. It uses a new formulation that combines the main ingredients of the formulation of representatives used in [14] and the classical independent set formulation presented in [1]. Campelo et al. previously proposed a combination of these formulations for the VCP in [8]. For solving the pricing problem, which is equivalent to the classical maximum weighted independent set problem (MWIS), two different algorithms are used depending on the density of the graph. On a Pentium Core2 Quad 2.83 GHz with 8 Gb of RAM, an instance with 706 vertices and 101.600 edges could have been solved, but the authors did not provide a hint on the time required.

4.2 Heuristical Approaches

Li and Simha introduced the PCP in [25] as a subproblem of the min-RWA problem, proofed that it is as hard as VCP (which is $\mathcal{NP}$-complete [**?**] and proposed adaptions of the coloring heuristics

In an earlier publication Li et al. [22] proved that the selective graph coloring problem is as hard as standard vertex coloring. They also proposed extensions of well-known vertex coloring heuristics to the partition coloring problem and ap- plied these heuristics to some instances of the routing and wavelength assignment problem. This paper also cites a lot of papers that deal with theoretical aspects of the routing and wavelength assignment problem.

2. Noronha, Ribiero (2006): Tabusearch

Noronha et al. [34] propose a heuristic for solving the Partition Graph Color- ing Problem based on tabu search.

3. Hu, Raidl, Pop (2013): Memetic algorithm

We proposed a memetic algorithm (MA) for the partition graph coloring problem that uses two distinct solution representations. For maintaining a diverse popula- tion and to keep the computational effort for genetic operators low, we use a full solution representation for crossover and mutation. In contrast, we use a more compact and incomplete solution representation during local search. Both repre- sentations work well in combination in the MA. During local search, we observed that minimizing the number of colors results in many solutions of equal quality. Therefore, we use a second evaluation criterion based on the number of conflicts when using one color less. Computational experiments on common benchmark instances sets show that although the MA is not always able to find the optimal solutions, it produces solid results with very low run-times and therefore has excellent scalability when it comes to large instances. For future work, we want to consider a further incomplete solution represen- tation which is based on characterizing the colors of the clusters. The challenge will be to develop efficient algorithms for choosing the nodes in the clusters that are compatible with the color assignments. We also want to consider further eval- uation criteria besides color and conflicts so that more fine-tuned measurements depending on specific situations are possible.

CHAPTER 5

Problem Solving Approach

In this chapter, the algorithms and models of the hybrid approach for the PCP will be described and analyzed in detail. First in section 5.1, the main procedure is explained. Section 5.2 then analyses the two different construction heuristics used in this work, namely *OneStepCD* and *DANGER*. The improvement phase is split into two parts: algorithms, that assign a new, but not mandatorily feasible coloring to a chosen set of nodes, here consisting of one random-, one heuristic- and two exact approaches, are applied in the first phase. In the following, this thesis will refer to these algorithms to as the "recoloring" algorithms 5.3. As for the second phase, section 5.4 presents the tabu search, which tries to find a coloring that makes the conflict-prone solution feasibly created by one of the recoloring algorithms. Additionally, two variants, one of them consisting in a modified ILP formulation and the other one in adding lately recolored areas to the tabu list, are considered in subsections of 5.3 and 5.4.

5.1 Main Procedure

The idea of the approach in general is to start from a feasible solution, pick a color and eliminate it. For each color c, the set of nodes colored with c is exchanged with other nodes in the respecting clusters as well as recolored without considering color c. Using this strategy, it might not be possible to find a feasible solution. Therefore, infeasible solutions are accepted, i.e. solutions including at least one conflict, that is a pair of adjacent nodes $\{i, j\} \in E : i, j \in V$ colored with the same color. One of the two nodes is chosen and in the following referred to as the "conflicting node"[1], its encasing cluster to as the "conflicting cluster". Further, a cluster is said to be of color c if and only if it contains a selected node colored with color c. The conflicting clusters form the starting points for the tabu search, that tries to find an alternative color for each of them. This process eventually produces further conflicts, which then again have to be eliminated. If no feasible coloring can be found within a specified number of iterations, the next color $c + 1$ is considered. If all conflicts can be eliminated, the algorithm has successfully decreased the

[1] How the node is chosen is shown in detail in 5.3

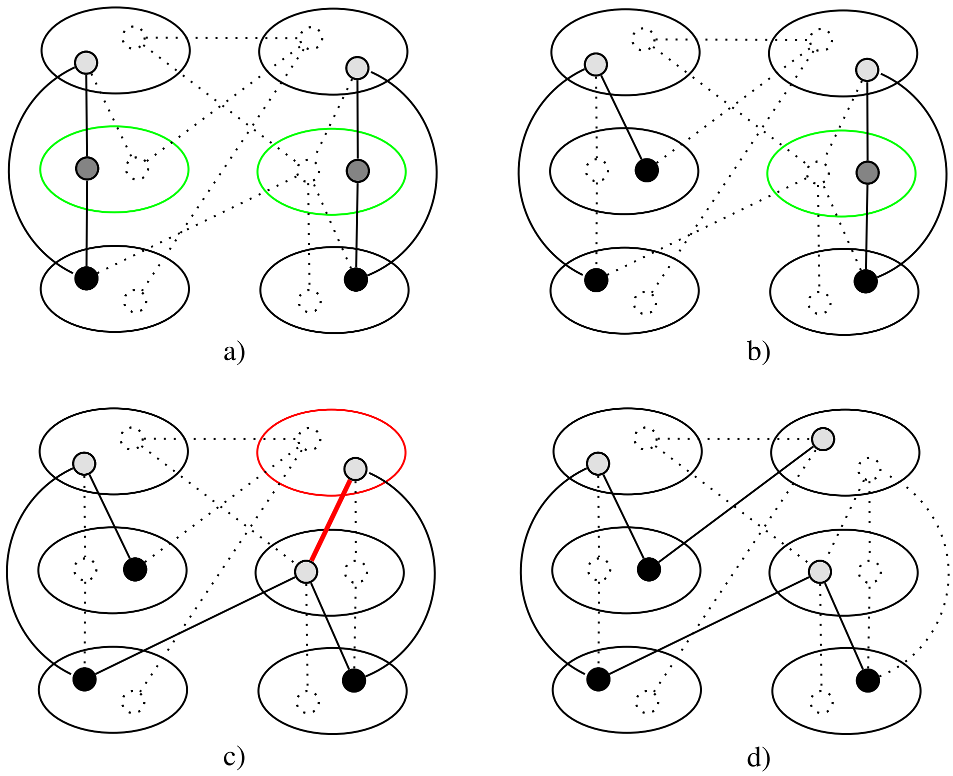

Figure 5.1: a) a feasible solution with 3 colors; b,c) recoloring phase: dark grey is intended to be eliminated. An infeasible solution with one conflict results; d) elimintation of the conflict by tabu search

chromatic number and repeats the whole process.
This approach differs from the strategy presented in [30] mainly by the effort that is investigated in the recoloring phase. There, Noronha et.al. reassign colors in a random way and therefore produce a random number of conflicts. The main innovation presented in this work is the minimization of the number of conflicts produced by an advanced recoloring algorithm, with the intention to increase the chance of eliminating these conflicts by tabu search.
An example of a possible sequence of steps performed by the algorithm is given in Figure 5.1. There, a feasible solution is shown in figure 5.1a). The color dark grey is chosen to be eliminated. In the following two steps, for each darkgrey cluster, another node is chosen and colored with any color other than dark grey. After recoloring, the resulting graph shown in 5.1c) is infeasible. Then, starting from the conflicting cluster outlined in red, the tabu search looks up the node-color pair inside that cluster, that causes the fewest conflicts. In 5.1d), a node-color pair is found, which does not produce any new conflicts. If this solution would have produced conflicts again, the local search would go on searching until a feasible solution was found or the maximum amount of iteration was reached. Listing 3 provides an overview of the algorithm that has been implemented. The algorithm takes an instance P of PCP, an algorithm $INITIAL$ comput-

ing an initial solution, as well as a recoloring algorithm $RECOLOR$ as input. As described in 3, an instance of PCP consists of an uncolored graph $G = (V, E)$, where V is divided into k clusters. Parameter $INITIAL$ can be any algorithm that creates a feasible solution for PCP. Two of them have been taken into account in this work and are described in section 5.3, others are proposed e.g. in [25].
In line 1, the initial solution is calculated and assigned to s. The chromatic number of s is assigned to $cmax$ in line 2. Line 3 initializes an empty set X. In line 5 a copy s_c of s is created where all nodes in colored with c are recolored by the algorithm $RECOLOR$, excluding color c. The tuple consisting of a tentative solution s_c and the set of conflicting nodes R_c are added to X in line 6. In line 7 the elements are sorted, such that the tuples with the fewest conflicting nodes are first. The loop in line 9 creates a tentative solution s'_c for each solution $s_c : c \in \{1, \ldots, k\}$, applies the tabu search and breaks in case of a feasible assignment could be found. If so, the chromatic number $cmax$ is reduced and the whole process is repeated with s'_c as new solution. If the tabu search could not find a feasible solution for all the solutions in X, the algorithm returns the latest feasible solution s.

Algorithm 3: PCP Hybrid

Input: A problem instance $\mathcal{P}$, an algorithm $INITIAL$ and an algorithm $RECOLOR$
Output: A feasible Solution s

1 $s \leftarrow INITIAL(\mathcal{P})$;
2 $cmax \leftarrow$ the chromatic number of s;
3 List $X \leftarrow \emptyset$;
4 **for** $c = 1, \ldots, cmax$ **do**
5   Let $\langle s_c, R_c \rangle$ be the solution and its conflicts created by applying $RECOLOR(s, c)$;
6     $X \leftarrow X \cup \langle s_c, R_c \rangle$
7 Sort elements in X ascendingly by $|R_i|$;
8 $reduction \leftarrow$ false;
9 **for** $\langle s_c, R_c \rangle \in X$ **do**
10     $s'_c \leftarrow TabuSearch(s_c, R_c)$;
11   **if** s'_c *is free of conflicts* **then**
12         $reduction \leftarrow$ true;
13   break;
14 **if** $reduction$ **then**
15     $s \leftarrow s'_c$;
16     $cmax = cmax - 1$;
17   **goto** line 3;
18 **return** s;

5.2 Constructional Heuristics

Construction heuristics create a solution from scratch. This section presents the two construction heuristics implemented by the author.

OneStepCD

OneStepCD is the best performing out of six constructional greedy algorithms for PCP presented by Li and Simha in [25]. The algorithms have originally been constructed for VCP by [7] and have been adapted to PCP. Listing 4 shows the algorithm.

The main criterion considered for selecting and coloring the next node is the so called *color degree* or *saturation degree*, which is defined as the amount of different colors of nodes adjacent to the considered node. The idea behind *OneStepCD* is first for each (unselected) cluster to select the node with the lowest color degree. Out of the resulting set, select the node with the highest color degree and color it with the lowest possible color. This procedure is repeated until all clusters are colored, i.e. hold one selected and colored node.

Intuitively, this approach leads to good results, since for *selecting* a node it would not be efficient to choose a node out of the cluster with a high color degree, while for *coloring* with each iteration it gets more and more dangerous to not color the node with highest degree.

Algorithm 4: OneStepCD

Input: A problem instance P
Output: A feasible Coloring s

Remove from G all edges $(i, j) \in E : i, j \in V_k$ for some $k = 1, \ldots, q$;
Set $s \leftarrow \emptyset$;
while $|s| < q$ **do**
    Set $X \leftarrow \emptyset$;
    for $k = 1, \ldots, q : V_k \cap s = \emptyset$ **do**
        $X \leftarrow X \cup argmin\{CD(i) : i \in V_k\}$;
    $x \leftarrow argmax\{CD(i) : i \in X\}$;
    $s \leftarrow s \cup \{x\}$;
    Assign the minimum possible color to x;
return s;

DANGER

The *DANGER* heuristic is a method to color a graph introduced by Parker et.al. in [19]. It has originaly also been constructed for VCP and therefore does not concern about the peculiarities of PCP. In this work, inquiries have been made to adapt the *DANGER* heuristic to PCP by slotting an algorithm in ahead, which selects one node per cluster, in order to color the resulting subgraph as usual.

Listing 5 shows the preprocessing algorithm. In line 3, the node v is chosen which has the minimum weighted sum of the already selected adjacent nodes $s(v)$ and the unselected but still selectable adjacent nodes $u(v)$. The weights c_s and c_u are constants. In line 4, the selected node is added to the set of selected nodes V' and in line 5, all the nodes contained in the cluster $p(v)$ enveloping v are removed from V.

Algorithm 5: Greedy Nodeselection

Input: A problem instance P
Output: A subproblem P' with $G' \subseteq G$
1 $V' \leftarrow \emptyset$;
2 **while** $|V| > 0$ **do**
3 $\quad v \leftarrow v \in V : min\{c_s s(v) + c_u u(v)\}$;
4 $\quad V' \leftarrow V' \cup v$;
5 $\quad V \setminus V_{p(v)}$;

Once one node per cluster is selected, the remaining problem is equivalent to VCP and *DANGER* can be applied in its original version. In contrast to *OneStepCD*, *DANGER* requires the allowed set of colors as parameter and is successful if and only if the graph can be colored within the given amount of colors. This makes the algorithm less flexible and requires it to run several times in order to explore the lowest size of the set. *DANGER* is based on two formulas *Node Danger* and *Color Danger*. The former decides at each iteration which node shall be colored next. For every node, the algorithm evaluates how dangerous it is, *not* to color node i in this iteration. The node i is chosen, that maximizes the following term:

$$NodeDanger(i) = F(different\_colored(i)) + k_u \cdot uncolored(i) + k_a \frac{share(i)}{avail(i)}$$

Where k_u, k_a are nonnegative constants, $different\_colored(i)$ denotes the color degree of node i, $uncolored(i)$ is the number of yet uncolored nodes adjacent to i, $avail(i)$ is the number of colors available and $share(i)$ denotes the number of available colors that is also available to all uncolored neighbors of i. If F is the identity function and $k_u = 1$ and $k_a = 0$, the danger of node i represents the scarcity of colors that may potentially be assigned to its neighbors. As it turned out that the "real" danger does not depend that much on the number of uncolored neighbors [19], other values are used for the constants and F is given as the following, monotonic increasing function:

$$F(y) = \frac{C}{(max\_color - y)^k}$$

The boundary on the domain of usable colors is denoted as max_color and given as paramter to *DANGER*. As the value of

$different\_colored$ grows, its influence is emphasized compared to the other parameters. The proposed values for the constants are $C = 1.0, k = 1.0, k_u = 1, k_a = 0.33$.

Once a node to color is chosen, the color danger is calculated by a quite similar concept. A color is dangerous to the node i if it is:

- attractive to an uncolored node with a large value of $different\_colored$,
- attractive to an uncolored node with many uncolored neighbors,
- infrequently used (a color that is used extensively is prefered) [19]

Let $diff\_neighbors(c)$ be a function returning over all uncolored nodes having c available, the maximum number of neighbors colored with c. Let the node achieving this maximum be denoted by i_c, then $uncolored(i_c)$ returns its amount of uncolored neighbors. A function $num(c)$ denotes the number of nodes colored with color c. Further, let k_1, k_2, k_3, k_4 be nonnegative constants with proposed values of $1.0, 1.0, 0.5, 0.025$, respectively. Then the danger of color c is:

$$ColorDanger(i) = \frac{k_1}{(max\_color - diff\_neighbours(c))^{k_2}} + k_3 \cdot uncolored(n_c) - k_4 \cdot num(c)$$

The concept of danger has not been intended to act as construction heuristic only. Instead, it has been used to be part of an improving heuristic, using tabu branch and bound and backtracking to reassign colors, as well as methods to prune branches from the assignment tree [2].

5.3 Recoloring

As explained above, the process of finding a new coloring for a set of clusters, omitting the actual color is of high relevance for this work. The recoloring algorithms intend to minimize the conflicts that arise from the new coloring, in order to increase the chance for the local search to eliminate them. An adaption of the already presented construction heuristic *OneStepCD* and two ILP models are shown in the following. A method assigning random colors has been implemented, too.

OneStepCD Adaption

The main idea behind the algorithm is to use the same strategy as the construction heuristic but restrict the colors to the domain $\{1, \ldots, cmax\} \setminus c$, where c is the color that is intended to be eliminated. If no color in the domain can establish a feasible solution, the color that produces the minimum amount of conflicts is chosen.
Listing 6 shows the algorithm in detail. A feasible solution s and the color c that has to be eliminated is given as input. Lines 1-3 create a duplicate s' of s and "uncolor" all clusters of color c. Then, for all uncolored clusters, the same procedure as for the standard *OneStepCD* is applied, with the only difference that if no feasible color can be found for a node, the color that produces the fewest conflicts is chosen. The potentially infeasible solution s' is returned.

[2]While the exploration phase of this thesis, the author has considered to adapt a similar strategy to PCP.

Algorithm 6: OneStepCD Recoloring

Input: A feasible solution s, a color c

Output: A possibly infeasible solution s' not using c

Create a duplicate s' of s;
Let $V'_{col(c)}$ be the set of nodes of all clusters colored with c in s';
Uncolor all nodes in $V'_{col(c)}$;
while *uncolored clusters exist in S'* **do**
    Set $X \leftarrow \emptyset$;
    for $v \in V'_{col(c)}$ **do**
        $X \leftarrow X \cup argmin\{CD(i) : i \in V_{p(v)}\}$;
    $z \leftarrow argmax\{CD(x) : x \in X\}$;
    $cmin \leftarrow$ the minimum colour that can be assigned to z without producing a conflict;
    if $cmin > cmax$ **then**
        $cmin \leftarrow argmin\{conflicts(z, i) : i \in \{1, \ldots, cmax\} \setminus c\}$.
    color z with $cmin$;
return s';

ILP1: minimizing conflicts

The following ILP model solves the problem of recoloring exactly in terms of minimizing the number of produced conflicts. Let $Q = Q_1, \ldots, Q_q$ be the set of uncolored clusters and let $C = \{1, \ldots, cmax\}$ be the set of allowed colors. The 3-dimensional array of binary variables X denote for each cluster $p \in Q$, if the node-color pair $(v \in Q_p, c \in C)$ is selected. Let M be a 3-dimensional array of constants, storing for each cluster $p \in Q$ the number conflicts that would occur by selecting the pair $(v \in Q_p, c \in C)$. E denotes the set of edges.

$$\begin{aligned}
\underset{X}{\text{minimize}} \quad & \sum_{p \in Q} \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} \cdot M_{pvc} && && (1)\\
\text{subject to} \quad & \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} = 1, && \forall p \in Q && (2)\\
& X_{pvc} + X_{quc} \leq 1, && \forall ((p,v),(q,u)) \in E, \forall c \in C && (3)\\
& X_{pvc} \in \{0,1\}, && \forall p \in Q, \forall v \in Q_p, \forall c \in C && (4)
\end{aligned}$$

In (1), the objective function is declared as the sum of the conflicts of all selected node-color pairs. The set of constraints in (2) demand, that for each cluster there has to be one node-color pair selected. Further, inequations (3) prevent conflicts between clusters in Q.
An advantage of this model is that the matrix M does not have to be created each time the ILP is initialized. By updating the information each time a node is colored, it can in fact be made constantly available.

ILP2: minimizing conflicting nodes

After recoloring, the tabu search starts eliminating the set of "conflicting clusters", i.e. the clusters involved in conflicts, that have *not* been recolored by the latest recoloring process[3], i.e. is not in Q. Let this set be denoted by R. Testing the effect of minimizing $|R|$ as a part of the whole algorithm is an important concerns of this work. In this context, figure 13 shows a case where *ILP1* would not decide optimally in terms of minimizing the number of conflicting clusters. The circles represent clusters inside of set Q and R, respectively. The connections between the clusters denote conflicts. Assuming that there exists a coloring that produces 2 conflicts and $|R| = 2$ 13a and a coloring that produces 3 conflicts, but $|R| = 1$ 13b, *ILP1* would decide on the first. Therefore a second ILP model has been designed, which minimizes the number of conflicting clusters $|R|$. Let $color[(p, v)]$ be a function returning the color of the

[3] In case of recoloring algorithms that allow conflicts between recolored clusters, additionally one cluster per conflict is added to R.

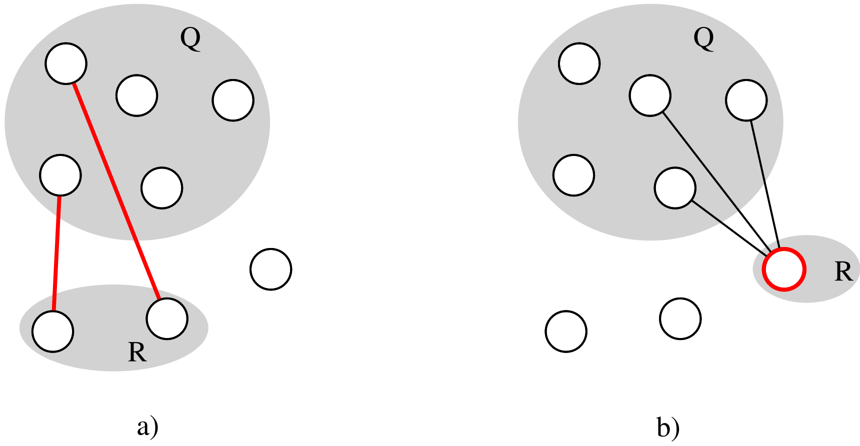

Figure 5.2: Circles denote clusters and a connection between them denote a conflict. *ILP1* would choose a solution with 2 conflicts (a) over a solution with 3 conflicts, which produces fewer conflicting clusters (b).

node v in partition p.

$$\begin{aligned}
\underset{Z}{\text{minimize}} \quad & \sum_{p \in V \setminus Q} \sum_{v \in V_p} \sum_{c \in C} Z_{pvc} && && (1)\\
\text{subject to} \quad & Z_{pvc} \geq X_{quc}, && \forall((p,v),(q,u)) \in E : (p,v) \notin Q, (q,u) \in Q, c = \mathit{color}[(p,v)] && (2)\\
& \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} = 1, && \forall p \in Q && (3)\\
& X_{pvc} + X_{quc} \leq 1, && \forall((p,v),(q,u)) \in E, \forall c \in C && (4)\\
& X_{pvc} \in \{0,1\}, && \forall p \in Q, \forall v \in Q_p, \forall c \in C && (5)
\end{aligned}$$

The array of binary variables Z holds one variable for each cluster connected to any cluster in Q. The objective function (1) minimizes the sum of all variables Z. The variable Z_{pvc} is set to one, if and only if the variable X_{quc} is set to one and the node $u \in Q$ has a connection to node $v \in V \setminus Q$ (2). Constraints (3) and (4) are the same as in *ILP1*, forcing one selected node-color pair to be selected per cluster and preventing conflicts inside Q.

ILP variants

Furthermore, variants of both ILP models have been considered, that omit the constraints ensuring the absence of conflicts between clusters inside Q. The intention here is to reduce the pruning of the solution space in order to allow a larger set of possibly good solutions. These variants will in the following be denoted as *ILP1** and *ILP2**, respectively.

5.4 Tabu Search

As stated above, the purpose of the tabu search is to eliminate the set of "conflicting clusters" R. Given the infeasible solution s, its neighborhood $\mathcal{N}(s)$ consists of the set of all solutions established by exchanging the selected node and/or color of the conflicting clusters. The node-color pair that produces the fewest conflicts is chosen, the resulting solution is accepted and the set R is updated. Therefore it is said to be a best first (BF) strategy. The chosen move is put on the tabu list for a random number of iterations in a specified interval. An aspiration criterion has been used, which means in this context that if the tabued move is allowed if it would lead to zero conflicts. The search terminates if either a feasible solution could have been established or a specified limit of iteration has been reached.

Algorithm 7: TabuSearch

Input: An infeasible solution s, the set of conflicting clusters R, an interval $interval$
Output: A Solution $\bar{s}$

1 Set $cmax \leftarrow$ the chromatic number of s;
2 Set $iter \leftarrow 0$;
3 Set $\bar{s} \leftarrow s$;
4 **while** $|R| > 0$ *and* $iter < maxiter$ **do**
5   Set $minConflicts \leftarrow \infty$;
6   **for** $p \in R$ **do**
7   **for** $v \in p$ *and* ***for*** $c = 1, \ldots, cmax$ **do**
8   Obtain a tentative solution s' by selecting and coloring node v with color c in $\bar{s}$;
9   **if** $conflicts(s') = 0$ **then**
10                 $\bar{s} \leftarrow s', \bar{v} \leftarrow v, \bar{c} \leftarrow c, \bar{p} \leftarrow p$;
11   break both loops;
12   **else if** *the pair* (v, c) *is not in the tabu list* **then**
13   **if** $conflicts(s') < minConflicts$ **then**
14                     $minConflicts \leftarrow conflicts(s')$;
15                     $\bar{s} \leftarrow s', \bar{v} \leftarrow v, \bar{c} \leftarrow c, \bar{p} \leftarrow p$;
16     $tabutenure$ = random number in $interval$;
17   insert pair $(\bar{v}, \bar{c})$ in the tabu list for $tabutenure$ iterations;
18     $R \leftarrow R \setminus \bar{p}$;
19   Let $R_{\bar{v}}$ be the set of clusters conflicting with $(\bar{v}, \bar{c})$;
20     $R \leftarrow R \cup R_{\bar{v}}$;
21 **return** $\bar{s}$;

Listing 7 outlines the procedure. The loops starting in line 6 and 7 are determining the move producting the minimum amount of conflicts. The condition in line 9 implements the aspiration criterion. The chosen node-color pair is added to the tabulist for $tabutenure$ iterations in line

17. The cluster p enclosing the chosen node is removed from the set of conflicting nodes R in line 18 and all clusters $R_{\bar{v}}$ conflicting with p are added to it in line 19 and 20. If the algorithm terminates because of too much iterations, the algorithm returns an infeasible solution $\bar{s}$.

Variant: protected recolored area

One implemented variant protects the set of clusters Q that has been recolored recently from being changed for a defined number of iterations, intending to gain benefits from its optimal coloring in the future. That is realized by putting all node-color pairs of all clusters in Q on the tabu list before the tabu search starts.

CHAPTER 6

Computational Results

This chapter provides information about the implementation, testing environment, instances used for evaluation and the computational results. Different methods which have been presented in chapter 5 and various parameters are compared to each other and to results of previous works [14, 25, 30].

6.1 Implementation Details and Testing Environment

The program has been implemented in Java and compiled with the JDK compiler version 1.7.025. For reasons of runtime comparability it has been designed to execute on a single thread, although the recoloring for each set of clusters of same color makes the program highly suitable to be processed in a parallel way. For the implementation of abstract data structures no other libraries than the ones provides by the JDK have been used. For solving the ILPs described in **??**, ILOG CPLEX version 12.5 has been used, which is by now one of the fastest CP solvers available [4]. It is written in C++, provides facades to Java, Python, .NET, Matlab, Excel and supports comfortable usage of integer variables and a wide set of constraints and solving strategies.
All tests have been performed on a Pentium i5 DualCore, 2.5 GHz, 8GB RAM, with Linux Mint 14 and OpenJDK Runtime Environment (IcedTea 2.3.9) installed.

6.2 Instances

Instances of different size, nodes per cluster ratio and density have been evaluated. The instances have been generated randomly by the authors of [14] and evaluated in [14], [35] and [11]. For reasons of better comparability to previous works, instances have been pooled to sets of same size and density, respectively. All of them contain 2 nodes per cluster.
Furthermore four larger instances with density of 0.5 and sizes of 500, 1000, 1500 and 2000 nodes provided by the authors of [30] have been evaluated and compared. In all of the instances,

the nodeset is divided into 500 clusters, holding 1,2,3 and 4 nodes, respectively. Note that the instance with 1 node per cluster is identical to VCP.

6.3 Results

In the following section preliminarily and final results as well as comparison to results of previous works are presented. There have been preceding tests performed to select the most competitive ranges of parameters for the test.

OneStepCD vs DANGER

To find out what method is more efficient to generate an initial solution, OneStepCD or DANGER combined with the prefixed node selection algorithm presented in chapter 5, tests have been performed on all instances. As mentioned above, one major disadvantage of DANGER is that a domain of colors has to be given as parameter and the algorithm then tries to color the graph only using colors in that domain. So in order to find out the lower bound of the domain, several runs have to be performed, which multiplies the runtime. Table **??**

Conflicting nodes

As an intermediate result the numbers of conflicting nodes per each recoloring produced by the different recoloring algorithms have been recorded and compared to each other. In tables 6.1 and 6.2 the results for sets of different size respectively density are presented. Each set contains five instances. Table 6.3 presents the results for the four larger instances. Each of the following tables uses the same notation, where $cnodes/recoloring$ denotes the average amount of conflicting nodes per recoloring. Each column presents results for one of the four main recoloring methods discussed in section 5.3 namely $Random$, $OneStepCD$, the ILP model minimizing the amount of conflicts $ILP1$ and the one minimizing the amount of conflicting nodes $ILP2$. Since for these experiments a constant size for the tabu list has been used, $HYBRID\text{-}PCP$ is deterministic except the case when random recoloring is used. Therefore for random recoloring the average of ten runs per instance and recoloring has been calculated.

It can be seen that a large number of nodes and as well as a low density lead to a high amount of conflicts per recoloring. The differences between the results for $Random$ and $ILP2$ grow to a factor of over 7 for the larger instances.

Table 6.1: Sets of different size containing five instances each.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	$\overline{cnodes/recoloring}$	$cnodes/recoloring$	$cnodes/recoloring$	$cnodes/recoloring$
20	0.5	3.69	2.25	1.60	**1.36**
40	0.5	7.33	3.85	3.21	**2.29**
60	0.5	10.21	4.99	4.21	**2.83**
70	0.5	11.30	5.84	4.56	**3.27**
80	0.5	12.69	6.04	4.97	**3.41**
90	0.5	12.32	5.93	4.64	**3.38**
100	0.5	14.91	7.16	5.23	**3.92**
120	0.5	15.53	6.44	5.07	**3.38**

Table 6.2: Sets of different density containing five instances each.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	$\overline{cnodes/recoloring}$	$cnodes/recoloring$	$cnodes/recoloring$	$cnodes/recoloring$
90	0.1	15.71	9.50	6.61	**5.65**
90	0.2	16.70	7.99	6.36	**4.87**
90	0.3	15.94	7.60	5.48	**4.03**
90	0.4	14.73	6.16	4.75	**3.41**
90	0.5	13.51	5.93	4.94	**3.43**
90	0.6	11.78	5.20	4.39	**2.84**
90	0.7	9.60	4.61	3.90	**2.44**
90	0.8	7.70	3.66	3.04	**2.05**
90	0.9	5.56	2.69	2.34	**1.74**

Table 6.3: Evaluation of the four larger instances. $ILP2$ produces over 7 times less conflicting nodes than $RANDOM$.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	$\overline{cnodes/recoloring}$	$cnodes/recoloring$	$cnodes/recoloring$	$cnodes/recoloring$
500	0.5	35.13	7.89	7.88	**5.02**
1000	0.5	39.87	9.15	7.74	**5.15**
1500	0.5	44.67	11.52	8.12	**6.02**
2000	0.5	46.81	12.29	4.75	**6.42**

Parameter Tests

After performing tests for the recoloring algorithms, tests on the whole $HYBRID\text{-}PCP$ algorithm have been performed. Here, different parameters have been compared by terms of the final result: the chromatic number. For each set of instances experiments with different recoloring algorithms, various ranges of tabu list lengths as well as various boundaries for the maximum number of iterations have been performed.

Tables 6.4 to 6.20 show the results of the instances provided in [14]. In tables 6.21 to 6.24 results of the large instances are shown. Again, the four main recoloring strategies are denoted by $Random, OneStepCD$, $ILP1$ and $ILP2$. The size of the tabu list for each iteration is a random number between the lower and upper bound given as $TabuTenure$, where C' is number of colors allowed for the actual solution. Because of that indeterminism 5 runs per each configuration have been performed. The maximum number of iterations used as stopping criterion is set as $q \cdot (C') \cdot ItMax$, where q is the amount of clusters. In tables 6.21 to 6.24 the values of the parameters $TabuTenure$ and $ItMax$ have been chosen similar to the ones used in [30].

When comparing the different recoloring algorithms in terms of the chromatic numbers resulting from the whole $HYBRID\text{-}PCP$ process, it can be seen that these results neither exhibit an improvement similar to the preliminary ones, nor any significant improvement at all. The differences of runtimes between heuristical and exact recoloring methods become visible especially on larger instances. For most instances except the four large ones a $TabuTenure$ of $U[1.0C', 4.0C']$ and $U[0.0C', 5.0C']$ has shown to lead to best results. For the larger instances, a $TabuTenure$ of $U[0.0C', 0.5C']$ fits best, which approves the results in [30]. Moreover it can be observed over all instances that $ItMax > 20$ does not lead to significant improvements.

Table 6.4: Results for a set of 5 instances of size 90 and density 0.1

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**3.0**	0.000	0.047	**3.0**	0.000	0.001	**3.0**	0.000	0.040	**3.0**	0.000	0.224
	$U[0.5C', 1.0C']$	**3.0**	0.000	0.016	**3.0**	0.000	0.001	**3.0**	0.000	0.029	**3.0**	0.000	0.211
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.010	**3.0**	0.000	0.001	**3.0**	0.000	0.033	**3.0**	0.000	0.197
	$U[0.0C', 5.0C']$	**3.0**	0.000	0.006	**3.0**	0.000	0.001	**3.0**	0.000	0.028	**3.0**	0.000	0.204
	$U[5.0C', 10.0C']$	**3.0**	0.000	0.005	**3.0**	0.000	0.001	**3.0**	0.000	0.023	**3.0**	0.000	0.224
	$U[10.0C', 20.0C']$	**3.0**	0.000	0.004	**3.0**	0.000	0.001	**3.0**	0.000	0.025	**3.0**	0.000	0.188
10	$U[0.0C', 0.5C']$	**2.9**	0.050	0.010	3.0	0.000	0.004	3.0	0.000	0.029	3.0	0.000	0.204
	$U[0.5C', 1.0C']$	**2.9**	0.050	0.007	**2.9**	0.050	0.005	3.0	0.000	0.030	**2.9**	0.050	0.186
	$U[1.0C', 4.0C']$	**2.9**	0.050	0.005	**2.9**	0.050	0.005	3.0	0.000	0.030	3.0	0.000	0.198
	$U[0.0C', 5.0C']$	3.0	0.000	0.005	2.9	0.050	0.005	3.0	0.000	0.029	**2.8**	0.000	0.183
	$U[5.0C', 10.0C']$	3.0	0.000	0.006	**2.9**	0.050	0.005	3.0	0.000	0.030	**2.9**	0.050	0.198
	$U[10.0C', 20.0C']$	3.0	0.000	0.005	**2.9**	0.050	0.005	3.0	0.000	0.027	3.0	0.000	0.209
20	$U[0.0C', 0.5C']$	3.0	0.000	0.008	**2.9**	0.050	0.009	**2.9**	0.050	0.034	3.0	0.000	0.213
	$U[0.5C', 1.0C']$	3.0	0.000	0.008	**2.9**	0.050	0.008	3.0	0.000	0.031	3.0	0.000	0.194
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.008	**3.0**	0.000	0.009	**3.0**	0.000	0.030	**3.0**	0.000	0.240
	$U[0.0C', 5.0C']$	3.0	0.000	0.008	3.0	0.000	0.008	**2.9**	0.050	0.030	3.0	0.000	0.215
	$U[5.0C', 10.0C']$	3.0	0.000	0.008	**2.8**	0.000	0.009	3.0	0.000	0.032	2.9	0.050	0.214
	$U[10.0C', 20.0C']$	**2.9**	0.050	0.009	3.0	0.000	0.008	3.0	0.000	0.031	3.0	0.000	0.202
50	$U[0.0C', 0.5C']$	**3.0**	0.000	0.017	**3.0**	0.000	0.018	**3.0**	0.000	0.040	**3.0**	0.000	0.200
	$U[0.5C', 1.0C']$	3.0	0.000	0.021	3.0	0.000	0.018	**2.9**	0.050	0.042	3.0	0.000	0.213
	$U[1.0C', 4.0C']$	3.0	0.000	0.018	3.0	0.000	0.018	3.0	0.000	0.038	**2.9**	0.050	0.225
	$U[0.0C', 5.0C']$	**2.9**	0.050	0.019	3.0	0.000	0.018	**2.9**	0.050	0.041	3.0	0.000	0.235
	$U[5.0C', 10.0C']$	3.0	0.000	0.017	3.0	0.000	0.017	**2.9**	0.050	0.040	3.0	0.000	0.201
	$U[10.0C', 20.0C']$	**2.8**	0.000	0.021	2.9	0.050	0.020	2.9	0.050	0.041	2.9	0.050	0.223

Table 6.5: Results for a set of 5 instances of size 90 and density 0.2

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	4.0	0.000	0.007	3.9	0.050	0.004	**3.8**	0.000	0.070	4.0	0.000	0.377
	$U[0.5C', 1.0C']$	4.0	0.000	0.004	3.9	0.050	0.004	**3.6**	0.800	0.062	4.0	0.000	0.354
	$U[1.0C', 4.0C']$	4.0	0.000	0.003	**3.9**	0.050	0.004	**3.9**	0.050	0.064	4.0	0.000	0.357
	$U[0.0C', 5.0C']$	**3.9**	0.050	0.003	4.0	0.000	0.003	4.0	0.000	0.064	**3.9**	0.050	0.373
	$U[5.0C', 10.0C']$	4.0	0.000	0.003	**3.9**	0.050	0.004	4.0	0.000	0.067	4.0	0.000	0.354
	$U[10.0C', 20.0C']$	**3.9**	0.050	0.003	4.0	0.000	0.003	**3.9**	0.050	0.064	4.0	0.000	0.349
10	$U[0.0C', 0.5C']$	3.9	0.050	0.015	**3.8**	0.000	0.016	3.9	0.050	0.075	3.9	0.050	0.364
	$U[0.5C', 1.0C']$	3.9	0.050	0.014	3.9	0.050	0.015	**3.8**	0.000	0.079	4.0	0.000	0.373
	$U[1.0C', 4.0C']$	3.9	0.050	0.014	3.9	0.050	0.014	**3.8**	0.000	0.078	**3.8**	0.000	0.445
	$U[0.0C', 5.0C']$	3.9	0.050	0.015	**3.8**	0.000	0.015	**3.8**	0.000	0.080	**3.8**	0.000	0.439
	$U[5.0C', 10.0C']$	3.9	0.050	0.015	**3.8**	0.000	0.015	**3.8**	0.000	0.080	**3.8**	0.000	0.446
	$U[10.0C', 20.0C']$	**3.8**	0.000	0.017	3.9	0.050	0.016	**3.8**	0.000	0.079	**3.8**	0.000	0.455
20	$U[0.0C', 0.5C']$	4.0	0.000	0.025	**3.8**	0.000	0.028	3.9	0.050	0.091	**3.8**	0.000	0.436
	$U[0.5C', 1.0C']$	**3.8**	0.000	0.028	**3.8**	0.000	0.027	3.9	0.050	0.088	3.9	0.050	0.412
	$U[1.0C', 4.0C']$	3.9	0.050	0.028	3.9	0.050	0.027	**3.8**	0.000	0.091	**3.8**	0.000	0.417
	$U[0.0C', 5.0C']$	**3.8**	0.000	0.029	3.9	0.050	0.027	**3.8**	0.000	0.087	3.9	0.050	0.438
	$U[5.0C', 10.0C']$	**3.8**	0.000	0.028	**3.8**	0.000	0.028	**3.8**	0.000	0.090	**3.8**	0.000	0.460
	$U[10.0C', 20.0C']$	**3.8**	0.000	0.028	**3.8**	0.000	0.028	**3.8**	0.000	0.091	**3.8**	0.000	0.477
50	$U[0.0C', 0.5C']$	**3.8**	0.000	0.062	4.0	0.000	0.058	**3.8**	0.000	0.130	3.9	0.050	0.438
	$U[0.5C', 1.0C']$	**3.8**	0.000	0.066	3.9	0.050	0.059	3.9	0.050	0.117	3.9	0.050	0.459
	$U[1.0C', 4.0C']$	**3.8**	0.000	0.064	3.9	0.050	0.059	3.9	0.050	0.123	3.9	0.050	0.439
	$U[0.0C', 5.0C']$	3.9	0.050	0.063	**3.8**	0.000	0.068	**3.8**	0.000	0.123	**3.8**	0.000	0.459
	$U[5.0C', 10.0C']$	**3.8**	0.000	0.066	**3.8**	0.000	0.065	**3.8**	0.000	0.132	**3.8**	0.000	0.431
	$U[10.0C', 20.0C']$	**3.8**	0.000	0.071	**3.8**	0.000	0.067	**3.8**	0.000	0.128	**3.8**	0.000	0.445

Table 6.6: Results for a set of 5 instances of size 90 and density 0.3

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**5.0**	0.000	0.007	**5.0**	0.000	0.005	**5.0**	0.000	0.105	**5.0**	0.000	0.424
	$U[0.5C', 1.0C']$	**5.0**	0.000	0.005	**5.0**	0.000	0.006	**5.0**	0.000	0.098	**5.0**	0.000	0.430
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.005	**5.0**	0.000	0.005	**5.0**	0.000	0.099	**5.0**	0.000	0.426
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.006	**5.0**	0.000	0.007	**5.0**	0.000	0.095	**5.0**	0.000	0.417
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.005	**5.0**	0.000	0.005	**5.0**	0.000	0.097	**5.0**	0.000	0.430
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.005	**5.0**	0.000	0.005	**5.0**	0.000	0.091	**5.0**	0.000	0.373
10	$U[0.0C', 0.5C']$	**5.0**	0.000	0.028	**5.0**	0.000	0.029	**5.0**	0.000	0.115	**5.0**	0.000	0.436
	$U[0.5C', 1.0C']$	**5.0**	0.000	0.029	**5.0**	0.000	0.030	**5.0**	0.000	0.119	**5.0**	0.000	0.465
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.029	**5.0**	0.000	0.029	**5.0**	0.000	0.115	**5.0**	0.000	0.436
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.029	**5.0**	0.000	0.029	**5.0**	0.000	0.115	**5.0**	0.000	0.449
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.029	**5.0**	0.000	0.029	**5.0**	0.000	0.126	**5.0**	0.000	0.400
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.030	**5.0**	0.000	0.030	**5.0**	0.000	0.118	**5.0**	0.000	0.378
20	$U[0.0C', 0.5C']$	**5.0**	0.000	0.052	**5.0**	0.000	0.055	**5.0**	0.000	0.138	**5.0**	0.000	0.436
	$U[0.5C', 1.0C']$	**5.0**	0.000	0.052	**5.0**	0.000	0.053	**5.0**	0.000	0.141	**5.0**	0.000	0.431
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.055	**5.0**	0.000	0.055	**5.0**	0.000	0.137	**5.0**	0.000	0.439
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.055	**5.0**	0.000	0.055	**5.0**	0.000	0.149	**5.0**	0.000	0.435
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.055	**5.0**	0.000	0.055	**5.0**	0.000	0.143	**5.0**	0.000	0.515
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.056	**5.0**	0.000	0.059	**5.0**	0.000	0.150	**5.0**	0.000	0.447
50	$U[0.0C', 0.5C']$	**5.0**	0.000	0.127	**5.0**	0.000	0.126	**5.0**	0.000	0.205	**5.0**	0.000	0.513
	$U[0.5C', 1.0C']$	**5.0**	0.000	0.125	**5.0**	0.000	0.128	**5.0**	0.000	0.212	**5.0**	0.000	0.525
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.128	**5.0**	0.000	0.129	**5.0**	0.000	0.217	**5.0**	0.000	0.526
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.128	**5.0**	0.000	0.133	**5.0**	0.000	0.215	**5.0**	0.000	0.481
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.136	**5.0**	0.000	0.131	**5.0**	0.000	0.223	**5.0**	0.000	0.538
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.139	**5.0**	0.000	0.139	**5.0**	0.000	0.223	**5.0**	0.000	0.532

Table 6.7: Results for a set of 5 instances of size 90 and density 0.4

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**6.0**	0.000	0.009	**6.0**	0.000	0.008	**6.0**	0.000	0.169	**6.0**	0.000	0.474
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.010	**6.0**	0.000	0.009	**6.0**	0.000	0.167	**6.0**	0.000	0.492
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.009	**6.0**	0.000	0.008	**6.0**	0.000	0.168	**6.0**	0.000	0.484
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.008	**6.0**	0.000	0.008	**6.0**	0.000	0.164	**6.0**	0.000	0.467
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.009	**6.0**	0.000	0.009	6.1	0.050	0.147	**6.0**	0.000	0.503
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.009	**6.0**	0.000	0.009	**6.0**	0.000	0.159	**6.0**	0.000	0.474
10	$U[0.0C', 0.5C']$	**6.0**	0.000	0.054	**6.0**	0.000	0.054	**6.0**	0.000	0.200	**6.0**	0.000	0.536
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.052	**6.0**	0.000	0.050	**6.0**	0.000	0.196	**6.0**	0.000	0.512
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.052	**6.0**	0.000	0.052	**6.0**	0.000	0.204	**6.0**	0.000	0.534
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.053	**6.0**	0.000	0.052	**6.0**	0.000	0.196	**6.0**	0.000	0.523
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.053	**6.0**	0.000	0.055	**6.0**	0.000	0.185	**6.0**	0.000	0.526
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.055	**6.0**	0.000	0.054	**6.0**	0.000	0.193	**6.0**	0.000	0.519
20	$U[0.0C', 0.5C']$	**6.0**	0.000	0.099	**6.0**	0.000	0.098	**6.0**	0.000	0.247	**6.0**	0.000	0.546
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.097	**6.0**	0.000	0.095	**6.0**	0.000	0.243	**6.0**	0.000	0.562
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.100	**6.0**	0.000	0.102	**6.0**	0.000	0.243	**6.0**	0.000	0.578
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.100	**6.0**	0.000	0.101	**6.0**	0.000	0.247	**6.0**	0.000	0.567
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.101	**6.0**	0.000	0.101	**6.0**	0.000	0.250	**6.0**	0.000	0.578
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.103	**6.0**	0.000	0.103	**6.0**	0.000	0.252	**6.0**	0.000	0.605
50	$U[0.0C', 0.5C']$	**6.0**	0.000	0.233	**6.0**	0.000	0.233	**6.0**	0.000	0.396	**6.0**	0.000	0.722
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.238	**6.0**	0.000	0.230	**6.0**	0.000	0.384	**6.0**	0.000	0.706
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.240	**6.0**	0.000	0.237	**6.0**	0.000	0.384	**6.0**	0.000	0.714
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.244	**6.0**	0.000	0.243	**6.0**	0.000	0.384	**6.0**	0.000	0.725
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.247	**6.0**	0.000	0.245	**6.0**	0.000	0.393	**6.0**	0.000	0.689
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.256	**6.0**	0.000	0.254	**6.0**	0.000	0.395	**6.0**	0.000	0.721

Table 6.8: Results for a set of 5 instances of size 90 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	7.2	0.000	0.015	7.2	0.000	0.014	**7.1**	0.050	0.216	7.2	0.000	0.617
	$U[0.5C', 1.0C']$	**7.2**	0.000	0.017	**7.2**	0.100	0.014	**7.2**	0.000	0.204	7.4	0.000	0.560
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.014	7.1	0.050	0.015	7.2	0.100	0.199	7.1	0.050	0.632
	$U[0.0C', 5.0C']$	**7.1**	0.050	0.014	7.2	0.000	0.014	**7.1**	0.050	0.211	**7.1**	0.050	0.606
	$U[5.0C', 10.0C']$	7.3	0.050	0.014	**7.0**	0.000	0.016	7.2	0.100	0.194	7.2	0.000	0.583
	$U[10.0C', 20.0C']$	**7.2**	0.100	0.015	7.5	0.050	0.014	7.4	0.100	0.193	7.4	0.100	0.567
10	$U[0.0C', 0.5C']$	**7.0**	0.000	0.098	**7.0**	0.000	0.095	7.1	0.050	0.285	7.1	0.050	0.700
	$U[0.5C', 1.0C']$	**7.0**	0.000	0.108	7.2	0.000	0.086	**7.0**	0.000	0.278	7.1	0.050	0.709
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.089	**7.0**	0.000	0.090	**7.0**	0.000	0.288	**7.0**	0.000	0.719
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.088	**7.0**	0.000	0.089	**7.0**	0.000	0.300	**7.0**	0.000	0.716
	$U[5.0C', 10.0C']$	**7.0**	0.000	0.096	**7.0**	0.000	0.094	**7.0**	0.000	0.299	**7.0**	0.000	0.719
	$U[10.0C', 20.0C']$	7.1	0.050	0.097	7.1	0.050	0.097	**7.0**	0.000	0.313	**7.0**	0.000	0.709
20	$U[0.0C', 0.5C']$	7.1	0.050	0.191	7.1	0.050	0.185	**7.0**	0.000	0.375	7.2	0.000	0.755
	$U[0.5C', 1.0C']$	7.1	0.050	0.193	**7.0**	0.000	0.178	7.1	0.050	0.363	7.1	0.050	0.789
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.172	**7.0**	0.000	0.173	**7.0**	0.000	0.367	**7.0**	0.000	0.763
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.172	**7.0**	0.000	0.174	**7.0**	0.000	0.369	**7.0**	0.000	0.787
	$U[5.0C', 10.0C']$	**7.0**	0.000	0.178	**7.0**	0.000	0.182	**7.0**	0.000	0.379	**7.0**	0.000	0.796
	$U[10.0C', 20.0C']$	**7.0**	0.000	0.205	7.1	0.050	0.185	7.1	0.050	0.386	7.1	0.050	0.792
50	$U[0.0C', 0.5C']$	**7.0**	0.000	0.433	7.1	0.050	0.409	7.1	0.050	0.617	**7.0**	0.000	1.071
	$U[0.5C', 1.0C']$	7.2	0.000	0.436	**7.0**	0.000	0.459	7.2	0.000	0.577	**7.0**	0.000	1.073
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.412	**7.0**	0.000	0.418	**7.0**	0.000	0.597	**7.0**	0.000	1.015
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.408	**7.0**	0.000	0.419	**7.0**	0.000	0.600	**7.0**	0.000	1.018
	$U[5.0C', 10.0C']$	**7.0**	0.000	0.426	**7.0**	0.000	0.428	**7.0**	0.000	0.618	**7.0**	0.000	1.023
	$U[10.0C', 20.0C']$	**7.0**	0.000	0.478	**7.0**	0.000	0.442	**7.0**	0.000	0.681	**7.0**	0.000	1.039

Table 6.9: Results for a set of 5 instances of size 90 and density 0.6

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**8.8**	0.100	0.022	9.0	0.000	0.021	**8.8**	0.100	0.264	**8.8**	0.100	0.729
	$U[0.5C', 1.0C']$	**8.7**	0.050	0.021	**8.7**	0.050	0.021	8.8	0.100	0.253	**8.7**	0.050	0.739
	$U[1.0C', 4.0C']$	8.7	0.050	0.020	8.6	0.100	0.022	8.7	0.050	0.268	**8.5**	0.150	0.774
	$U[0.0C', 5.0C']$	**8.6**	0.200	0.022	8.7	0.050	0.020	8.7	0.150	0.272	8.9	0.050	0.697
	$U[5.0C', 10.0C']$	8.8	0.100	0.021	8.8	0.100	0.021	**8.7**	0.050	0.256	8.9	0.050	0.708
	$U[10.0C', 20.0C']$	**8.9**	0.050	0.022	9.0	0.000	0.020	**8.9**	0.050	0.245	9.0	0.000	0.679
10	$U[0.0C', 0.5C']$	8.6	0.100	0.160	8.8	0.100	0.146	8.6	0.100	0.392	**8.5**	0.050	0.943
	$U[0.5C', 1.0C']$	8.7	0.050	0.144	**8.4**	0.100	0.157	8.5	0.150	0.422	8.6	0.100	0.891
	$U[1.0C', 4.0C']$	**8.2**	0.000	0.162	8.3	0.050	0.169	**8.2**	0.000	0.429	8.3	0.050	0.972
	$U[0.0C', 5.0C']$	**8.2**	0.000	0.171	**8.2**	0.000	0.148	**8.2**	0.000	0.445	8.3	0.050	0.955
	$U[5.0C', 10.0C']$	**8.4**	0.100	0.187	**8.4**	0.100	0.169	8.5	0.050	0.422	8.6	0.100	0.916
	$U[10.0C', 20.0C']$	8.8	0.000	0.158	8.8	0.000	0.159	8.7	0.050	0.400	**8.5**	0.050	0.928
20	$U[0.0C', 0.5C']$	8.8	0.100	0.297	8.8	0.100	0.287	**8.6**	0.100	0.552	**8.6**	0.000	1.053
	$U[0.5C', 1.0C']$	8.6	0.000	0.302	**8.5**	0.050	0.297	**8.5**	0.050	0.556	8.8	0.100	0.954
	$U[1.0C', 4.0C']$	**8.2**	0.000	0.306	**8.2**	0.000	0.307	**8.2**	0.000	0.567	**8.2**	0.000	1.137
	$U[0.0C', 5.0C']$	**8.2**	0.000	0.305	8.3	0.050	0.300	**8.2**	0.000	0.559	**8.2**	0.000	1.115
	$U[5.0C', 10.0C']$	8.3	0.050	0.325	8.3	0.050	0.326	**8.2**	0.000	0.600	8.4	0.100	1.110
	$U[10.0C', 20.0C']$	8.6	0.000	0.313	8.5	0.050	0.324	**8.4**	0.000	0.588	8.7	0.050	1.046
50	$U[0.0C', 0.5C']$	8.8	0.000	0.728	8.8	0.100	0.682	**8.7**	0.050	0.980	8.8	0.100	1.377
	$U[0.5C', 1.0C']$	**8.6**	0.100	0.768	**8.6**	0.100	0.684	8.7	0.050	0.892	**8.6**	0.100	1.436
	$U[1.0C', 4.0C']$	**8.2**	0.000	0.690	**8.2**	0.000	0.710	**8.2**	0.000	0.951	**8.2**	0.000	1.507
	$U[0.0C', 5.0C']$	**8.2**	0.000	0.716	**8.2**	0.000	0.757	**8.2**	0.000	0.984	**8.2**	0.000	1.526
	$U[5.0C', 10.0C']$	8.3	0.050	0.745	**8.2**	0.000	0.833	**8.2**	0.000	1.056	**8.2**	0.000	1.630
	$U[10.0C', 20.0C']$	8.4	0.100	0.860	8.4	0.100	0.797	**8.3**	0.050	1.102	8.5	0.050	1.560

Table 6.10: Results for a set of 5 instances of size 90 and density 0.7

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	10.6	0.100	0.037	**10.2**	0.100	0.034	10.5	0.050	0.301	10.4	0.200	0.865
	$U[0.5C', 1.0C']$	**10.2**	0.100	0.037	10.5	0.050	0.032	10.4	0.100	0.310	**10.2**	0.000	0.906
	$U[1.0C', 4.0C']$	**10.1**	0.050	0.038	**10.1**	0.050	0.034	10.3	0.050	0.340	**10.1**	0.050	0.927
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.037	10.1	0.050	0.031	10.4	0.100	0.317	10.1	0.050	0.904
	$U[5.0C', 10.0C']$	10.4	0.100	0.034	10.5	0.150	0.034	**10.2**	0.100	0.328	**10.2**	0.100	0.921
	$U[10.0C', 20.0C']$	10.6	0.100	0.035	**10.5**	0.050	0.034	10.8	0.100	0.287	**10.5**	0.050	0.809
10	$U[0.0C', 0.5C']$	**10.4**	0.100	0.270	**10.4**	0.200	0.265	10.8	0.100	0.462	10.5	0.150	1.070
	$U[0.5C', 1.0C']$	**10.0**	0.000	0.309	**10.0**	0.000	0.264	10.2	0.000	0.556	10.2	0.100	1.113
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.233	**10.0**	0.000	0.231	**10.0**	0.000	0.523	**10.0**	0.000	1.120
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.229	**10.0**	0.000	0.230	**10.0**	0.000	0.536	**10.0**	0.000	1.129
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.248	**10.0**	0.000	0.241	**10.0**	0.000	0.528	**10.0**	0.000	1.139
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.311	**10.0**	0.000	0.300	**10.0**	0.000	0.588	**10.0**	0.000	1.230
20	$U[0.0C', 0.5C']$	10.5	0.050	0.502	**10.4**	0.100	0.527	10.5	0.250	0.821	10.6	0.100	1.237
	$U[0.5C', 1.0C']$	**10.0**	0.000	0.462	10.1	0.050	0.517	10.2	0.100	0.805	**10.0**	0.000	1.464
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.445	**10.0**	0.000	0.446	**10.0**	0.000	0.726	**10.0**	0.000	1.336
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.446	**10.0**	0.000	0.443	**10.0**	0.000	0.741	**10.0**	0.000	1.338
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.471	**10.0**	0.000	0.467	**10.0**	0.000	0.782	**10.0**	0.000	1.357
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.493	**10.0**	0.000	0.532	**10.0**	0.000	0.824	**10.0**	0.000	1.432
50	$U[0.0C', 0.5C']$	**10.4**	0.100	1.183	10.6	0.100	1.099	10.5	0.050	1.501	10.5	0.150	2.083
	$U[0.5C', 1.0C']$	10.1	0.050	1.215	10.1	0.050	1.338	10.2	0.000	1.554	**10.0**	0.000	2.200
	$U[1.0C', 4.0C']$	**10.0**	0.000	1.077	**10.0**	0.000	1.098	**10.0**	0.000	1.377	**10.0**	0.000	1.973
	$U[0.0C', 5.0C']$	**10.0**	0.000	1.078	**10.0**	0.000	1.086	**10.0**	0.000	1.403	**10.0**	0.000	1.990
	$U[5.0C', 10.0C']$	**10.0**	0.000	1.135	**10.0**	0.000	1.135	**10.0**	0.000	1.428	**10.0**	0.000	2.022
	$U[10.0C', 20.0C']$	**10.0**	0.000	1.197	**10.0**	0.000	1.193	**10.0**	0.000	1.471	**10.0**	0.000	2.066

Table 6.11: Results for a set of 5 instances of size 90 and density 0.8

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**12.6**	0.100	0.058	12.8	0.000	0.056	13.0	0.000	0.464	13.1	0.150	1.040
	$U[0.5C', 1.0C']$	12.7	0.050	0.046	**12.6**	0.000	0.052	12.9	0.050	0.442	12.8	0.000	1.106
	$U[1.0C', 4.0C']$	12.6	0.100	0.049	**12.5**	0.050	0.048	12.7	0.050	0.467	12.7	0.050	1.109
	$U[0.0C', 5.0C']$	**12.5**	0.150	0.051	12.8	0.000	0.046	12.7	0.050	0.449	12.6	0.100	1.144
	$U[5.0C', 10.0C']$	12.7	0.050	0.051	**12.5**	0.050	0.058	12.6	0.100	0.457	12.8	0.000	1.100
	$U[10.0C', 20.0C']$	12.8	0.000	0.050	12.8	0.000	0.051	12.8	0.000	0.429	**12.7**	0.050	1.119
10	$U[0.0C', 0.5C']$	**12.7**	0.150	0.491	12.9	0.050	0.443	13.1	0.050	0.752	12.9	0.250	1.450
	$U[0.5C', 1.0C']$	12.6	0.100	0.410	**12.4**	0.100	0.440	12.5	0.050	0.819	12.6	0.100	1.490
	$U[1.0C', 4.0C']$	12.2	0.000	0.408	**12.0**	0.000	0.426	**12.0**	0.000	0.893	12.3	0.050	1.543
	$U[0.0C', 5.0C']$	12.1	0.050	0.390	12.1	0.050	0.429	**12.0**	0.000	0.903	12.1	0.050	1.600
	$U[5.0C', 10.0C']$	**12.2**	0.000	0.451	12.3	0.050	0.452	12.3	0.050	0.862	12.3	0.050	1.549
	$U[10.0C', 20.0C']$	12.5	0.050	0.437	**12.4**	0.100	0.459	12.5	0.150	0.852	12.5	0.050	1.546
20	$U[0.0C', 0.5C']$	12.9	0.050	0.750	12.9	0.050	0.907	**12.8**	0.100	1.173	12.9	0.050	1.885
	$U[0.5C', 1.0C']$	**12.4**	0.100	0.817	12.5	0.050	0.769	12.5	0.150	1.242	12.5	0.050	1.935
	$U[1.0C', 4.0C']$	**12.0**	0.000	0.770	**12.0**	0.000	0.918	**12.0**	0.000	1.271	**12.0**	0.000	2.063
	$U[0.0C', 5.0C']$	**12.0**	0.000	0.826	12.1	0.050	0.838	**12.0**	0.000	1.211	12.1	0.050	1.979
	$U[5.0C', 10.0C']$	**12.1**	0.050	0.828	12.2	0.100	0.820	**12.1**	0.050	1.344	12.2	0.000	2.034
	$U[10.0C', 20.0C']$	12.4	0.100	0.931	12.5	0.050	0.795	12.4	0.200	1.318	**12.3**	0.050	2.066
50	$U[0.0C', 0.5C']$	13.0	0.100	2.108	13.0	0.100	2.076	**12.7**	0.050	2.560	13.0	0.100	2.841
	$U[0.5C', 1.0C']$	**12.5**	0.050	1.994	12.6	0.100	2.112	**12.5**	0.050	2.412	12.6	0.100	2.883
	$U[1.0C', 4.0C']$	**12.0**	0.000	1.869	**12.0**	0.000	1.872	**12.0**	0.000	2.291	**12.0**	0.000	3.119
	$U[0.0C', 5.0C']$	**12.0**	0.000	1.833	**12.0**	0.000	1.858	**12.0**	0.000	2.284	**12.0**	0.000	3.065
	$U[5.0C', 10.0C']$	12.1	0.050	2.030	12.1	0.050	2.243	12.1	0.050	2.536	**12.0**	0.000	3.217
	$U[10.0C', 20.0C']$	12.2	0.100	2.106	12.3	0.050	2.166	**12.0**	0.000	3.000	12.1	0.050	3.505

Table 6.12: Results for a set of 5 instances of size 90 and density 0.9

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	17.1	0.050	0.089	16.9	0.150	0.091	**16.8**	0.200	0.493	16.9	0.150	1.026
	$U[0.5C', 1.0C']$	16.4	0.100	0.084	**16.2**	0.100	0.076	16.4	0.000	0.505	**16.2**	0.000	1.179
	$U[1.0C', 4.0C']$	16.1	0.050	0.081	**16.0**	0.100	0.078	16.1	0.050	0.523	16.3	0.050	1.141
	$U[0.0C', 5.0C']$	16.3	0.050	0.077	16.1	0.050	0.078	16.3	0.150	0.493	**16.0**	0.100	1.218
	$U[5.0C', 10.0C']$	**16.1**	0.050	0.083	16.4	0.000	0.080	16.3	0.050	0.489	16.2	0.000	1.153
	$U[10.0C', 20.0C']$	**16.1**	0.050	0.085	16.2	0.100	0.085	**16.1**	0.050	0.536	16.3	0.050	1.179
10	$U[0.0C', 0.5C']$	**16.6**	0.100	0.895	17.0	0.100	0.766	16.7	0.150	1.072	17.2	0.100	1.597
	$U[0.5C', 1.0C']$	**16.1**	0.050	0.656	16.3	0.350	0.648	16.2	0.000	1.119	16.2	0.100	1.723
	$U[1.0C', 4.0C']$	15.9	0.050	0.682	**15.8**	0.000	0.696	**15.8**	0.000	1.122	**15.8**	0.000	1.848
	$U[0.0C', 5.0C']$	**15.8**	0.000	0.679	**15.8**	0.000	0.652	**15.8**	0.000	1.083	**15.8**	0.000	1.867
	$U[5.0C', 10.0C']$	15.9	0.050	0.699	**15.8**	0.000	0.740	**15.8**	0.000	1.197	**15.8**	0.000	1.925
	$U[10.0C', 20.0C']$	16.0	0.100	0.735	16.0	0.000	0.737	16.1	0.050	1.158	**15.9**	0.050	1.852
20	$U[0.0C', 0.5C']$	16.6	0.200	1.627	16.9	0.150	1.467	**16.4**	0.000	1.939	17.0	0.100	2.123
	$U[0.5C', 1.0C']$	16.4	0.100	1.283	**16.2**	0.100	1.412	16.3	0.050	1.643	**16.2**	0.100	2.317
	$U[1.0C', 4.0C']$	**15.8**	0.000	1.308	15.9	0.050	1.235	**15.8**	0.000	1.742	**15.8**	0.000	2.422
	$U[0.0C', 5.0C']$	**15.8**	0.000	1.302	15.9	0.050	1.378	**15.8**	0.000	1.715	**15.8**	0.000	2.406
	$U[5.0C', 10.0C']$	15.9	0.050	1.372	**15.8**	0.000	1.448	**15.8**	0.000	1.815	**15.8**	0.000	2.455
	$U[10.0C', 20.0C']$	16.0	0.000	1.413	**15.8**	0.000	1.478	**15.8**	0.000	1.880	15.9	0.050	2.513
50	$U[0.0C', 0.5C']$	17.0	0.100	3.405	**16.9**	0.050	3.484	**16.9**	0.150	3.922	17.1	0.150	4.539
	$U[0.5C', 1.0C']$	16.2	0.100	3.441	16.4	0.100	3.133	**16.1**	0.050	3.523	**16.1**	0.050	4.228
	$U[1.0C', 4.0C']$	**15.8**	0.000	3.019	**15.8**	0.000	3.062	**15.8**	0.000	3.485	**15.8**	0.000	4.214
	$U[0.0C', 5.0C']$	**15.8**	0.000	3.173	**15.8**	0.000	3.071	**15.8**	0.000	3.459	**15.8**	0.000	4.229
	$U[5.0C', 10.0C']$	**15.8**	0.000	3.263	**15.8**	0.000	3.300	**15.8**	0.000	3.770	**15.8**	0.000	4.329
	$U[10.0C', 20.0C']$	**15.8**	0.000	3.451	15.9	0.050	3.436	15.9	0.050	3.800	**15.8**	0.000	4.562

Table 6.13: Results for a set of 5 instances of size 20 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.023	**3.0**	0.000	0.034
	$U[0.5C', 1.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.021	**3.0**	0.000	0.031
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.021	**3.0**	0.000	0.032
	$U[0.0C', 5.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.018	**3.0**	0.000	0.036
	$U[5.0C', 10.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.029	**3.0**	0.000	0.035
	$U[10.0C', 20.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.021	**3.0**	0.000	0.036
10	$U[0.0C', 0.5C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.026	**3.0**	0.000	0.031
	$U[0.5C', 1.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.025	**3.0**	0.000	0.038
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.023	**3.0**	0.000	0.037
	$U[0.0C', 5.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.027	**3.0**	0.000	0.035
	$U[5.0C', 10.0C']$	**3.0**	0.000	0.001	**3.0**	0.000	0.001	**3.0**	0.000	0.026	**3.0**	0.000	0.034
	$U[10.0C', 20.0C']$	**3.0**	0.000	0.002	**3.0**	0.000	0.001	**3.0**	0.000	0.025	**3.0**	0.000	0.033
20	$U[0.0C', 0.5C']$	**3.0**	0.000	0.009	**3.0**	0.000	0.001	**3.0**	0.000	0.029	**3.0**	0.000	0.036
	$U[0.5C', 1.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.001	**3.0**	0.000	0.023	**3.0**	0.000	0.037
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.001	**3.0**	0.000	0.025	**3.0**	0.000	0.036
	$U[0.0C', 5.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.001	**3.0**	0.000	0.024	**3.0**	0.000	0.037
	$U[5.0C', 10.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.001	**3.0**	0.000	0.022	**3.0**	0.000	0.034
	$U[10.0C', 20.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.002	**3.0**	0.000	0.021	**3.0**	0.000	0.035
50	$U[0.0C', 0.5C']$	**3.0**	0.000	0.005	**3.0**	0.000	0.003	**3.0**	0.000	0.030	**3.0**	0.000	0.038
	$U[0.5C', 1.0C']$	**3.0**	0.000	0.004	**3.0**	0.000	0.003	**3.0**	0.000	0.025	**3.0**	0.000	0.034
	$U[1.0C', 4.0C']$	**3.0**	0.000	0.003	**3.0**	0.000	0.003	**3.0**	0.000	0.023	**3.0**	0.000	0.035
	$U[0.0C', 5.0C']$	**3.0**	0.000	0.004	**3.0**	0.000	0.003	**3.0**	0.000	0.022	**3.0**	0.000	0.036
	$U[5.0C', 10.0C']$	**3.0**	0.000	0.004	**3.0**	0.000	0.003	**3.0**	0.000	0.030	**3.0**	0.000	0.036
	$U[10.0C', 20.0C']$	**3.0**	0.000	0.004	**3.0**	0.000	0.004	**3.0**	0.000	0.024	**3.0**	0.000	0.036

Table 6.14: Results for a set of 5 instances of size 40 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**4.0**	0.000	0.002	**4.0**	0.000	0.001	**4.0**	0.000	0.073	4.1	0.050	0.119
	$U[0.5C', 1.0C']$	**4.0**	0.000	0.002	**4.0**	0.000	0.001	**4.0**	0.000	0.075	**4.0**	0.000	0.128
	$U[1.0C', 4.0C']$	**4.0**	0.000	0.001	**4.0**	0.000	0.001	**4.0**	0.000	0.076	**4.0**	0.000	0.127
	$U[0.0C', 5.0C']$	**4.0**	0.000	0.001	**4.0**	0.000	0.001	**4.0**	0.000	0.069	**4.0**	0.000	0.128
	$U[5.0C', 10.0C']$	**4.0**	0.000	0.001	**4.0**	0.000	0.001	**4.0**	0.000	0.069	**4.0**	0.000	0.127
	$U[10.0C', 20.0C']$	**4.0**	0.000	0.001	**4.0**	0.000	0.001	**4.0**	0.000	0.070	**4.0**	0.000	0.136
10	$U[0.0C', 0.5C']$	**4.0**	0.000	0.007	**4.0**	0.000	0.006	**4.0**	0.000	0.078	4.1	0.050	0.130
	$U[0.5C', 1.0C']$	**4.0**	0.000	0.006	**4.0**	0.000	0.007	**4.0**	0.000	0.076	**4.0**	0.000	0.136
	$U[1.0C', 4.0C']$	**4.0**	0.000	0.006	**4.0**	0.000	0.006	**4.0**	0.000	0.078	**4.0**	0.000	0.137
	$U[0.0C', 5.0C']$	**4.0**	0.000	0.006	**4.0**	0.000	0.006	**4.0**	0.000	0.077	**4.0**	0.000	0.139
	$U[5.0C', 10.0C']$	**4.0**	0.000	0.006	**4.0**	0.000	0.006	**4.0**	0.000	0.082	**4.0**	0.000	0.131
	$U[10.0C', 20.0C']$	**4.0**	0.000	0.007	**4.0**	0.000	0.007	**4.0**	0.000	0.076	**4.0**	0.000	0.133
20	$U[0.0C', 0.5C']$	**4.0**	0.000	0.011	**4.0**	0.000	0.012	**4.0**	0.000	0.078	**4.0**	0.000	0.139
	$U[0.5C', 1.0C']$	**4.0**	0.000	0.012	**4.0**	0.000	0.012	**4.0**	0.000	0.089	**4.0**	0.000	0.144
	$U[1.0C', 4.0C']$	**4.0**	0.000	0.012	**4.0**	0.000	0.013	**4.0**	0.000	0.082	**4.0**	0.000	0.138
	$U[0.0C', 5.0C']$	**4.0**	0.000	0.011	**4.0**	0.000	0.011	**4.0**	0.000	0.079	**4.0**	0.000	0.137
	$U[5.0C', 10.0C']$	**4.0**	0.000	0.012	**4.0**	0.000	0.012	**4.0**	0.000	0.081	**4.0**	0.000	0.137
	$U[10.0C', 20.0C']$	**4.0**	0.000	0.013	**4.0**	0.000	0.012	**4.0**	0.000	0.081	**4.0**	0.000	0.144
50	$U[0.0C', 0.5C']$	**4.0**	0.000	0.029	**4.0**	0.000	0.027	**4.0**	0.000	0.091	4.1	0.050	0.148
	$U[0.5C', 1.0C']$	**4.0**	0.000	0.028	**4.0**	0.000	0.027	**4.0**	0.000	0.098	**4.0**	0.000	0.154
	$U[1.0C', 4.0C']$	**4.0**	0.000	0.026	**4.0**	0.000	0.026	**4.0**	0.000	0.088	**4.0**	0.000	0.156
	$U[0.0C', 5.0C']$	**4.0**	0.000	0.026	**4.0**	0.000	0.026	**4.0**	0.000	0.100	**4.0**	0.000	0.156
	$U[5.0C', 10.0C']$	**4.0**	0.000	0.029	**4.0**	0.000	0.028	**4.0**	0.000	0.093	**4.0**	0.000	0.154
	$U[10.0C', 20.0C']$	**4.0**	0.000	0.030	**4.0**	0.000	0.030	**4.0**	0.000	0.101	**4.0**	0.000	0.153

Table 6.15: Results for a set of 5 instances of size 60 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	5.6	0.100	0.005	5.3	0.050	0.004	**5.2**	0.100	0.128	5.5	0.050	0.261
	$U[0.5C', 1.0C']$	5.6	0.100	0.006	**5.4**	0.100	0.004	**5.4**	0.000	0.117	**5.4**	0.100	0.266
	$U[1.0C', 4.0C']$	5.7	0.050	0.004	5.3	0.150	0.004	**5.2**	0.100	0.122	5.4	0.100	0.265
	$U[0.0C', 5.0C']$	5.3	0.150	0.004	5.3	0.050	0.004	**5.2**	0.100	0.125	5.6	0.100	0.248
	$U[5.0C', 10.0C']$	5.7	0.050	0.004	**5.3**	0.150	0.004	5.4	0.100	0.114	**5.3**	0.050	0.277
	$U[10.0C', 20.0C']$	5.9	0.050	0.004	5.8	0.100	0.004	**5.4**	0.100	0.110	5.8	0.000	0.220
10	$U[0.0C', 0.5C']$	5.4	0.100	0.024	**5.1**	0.050	0.026	5.3	0.050	0.139	5.2	0.100	0.298
	$U[0.5C', 1.0C']$	**5.1**	0.050	0.026	5.2	0.100	0.024	**5.1**	0.050	0.142	**5.1**	0.050	0.323
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.024	**5.0**	0.000	0.024	**5.0**	0.000	0.148	**5.0**	0.000	0.306
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.023	**5.0**	0.000	0.023	**5.0**	0.000	0.144	**5.0**	0.000	0.323
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.023	**5.0**	0.000	0.024	**5.0**	0.000	0.151	**5.0**	0.000	0.323
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.028	5.2	0.100	0.027	**5.0**	0.000	0.157	5.2	0.100	0.309
20	$U[0.0C', 0.5C']$	**5.2**	0.000	0.045	5.3	0.050	0.047	**5.2**	0.000	0.157	5.3	0.050	0.319
	$U[0.5C', 1.0C']$	**5.1**	0.050	0.052	5.3	0.150	0.042	5.2	0.100	0.156	5.3	0.050	0.324
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.043	**5.0**	0.000	0.042	**5.0**	0.000	0.164	**5.0**	0.000	0.341
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.044	**5.0**	0.000	0.043	**5.0**	0.000	0.167	**5.0**	0.000	0.339
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.045	**5.0**	0.000	0.043	**5.0**	0.000	0.171	**5.0**	0.000	0.338
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.046	**5.0**	0.000	0.051	**5.0**	0.000	0.168	**5.0**	0.000	0.358
50	$U[0.0C', 0.5C']$	**5.0**	0.000	0.117	**5.0**	0.000	0.118	5.2	0.100	0.214	5.1	0.050	0.392
	$U[0.5C', 1.0C']$	5.2	0.100	0.115	**5.1**	0.050	0.118	**5.1**	0.050	0.228	5.3	0.050	0.374
	$U[1.0C', 4.0C']$	**5.0**	0.000	0.096	**5.0**	0.000	0.097	**5.0**	0.000	0.218	**5.0**	0.000	0.401
	$U[0.0C', 5.0C']$	**5.0**	0.000	0.099	**5.0**	0.000	0.101	**5.0**	0.000	0.216	**5.0**	0.000	0.394
	$U[5.0C', 10.0C']$	**5.0**	0.000	0.103	**5.0**	0.000	0.103	**5.0**	0.000	0.230	**5.0**	0.000	0.409
	$U[10.0C', 20.0C']$	**5.0**	0.000	0.119	**5.0**	0.000	0.114	**5.0**	0.000	0.229	**5.0**	0.000	0.403

Table 6.16: Results for a set of 5 instances of size 70 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**6.0**	0.000	0.006	**6.0**	0.000	0.006	**6.0**	0.000	0.132	**6.0**	0.000	0.362
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.006	**6.0**	0.000	0.006	6.1	0.050	0.127	6.1	0.050	0.338
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.006	**6.0**	0.000	0.006	**6.0**	0.000	0.134	6.1	0.050	0.347
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.007	6.1	0.050	0.006	**6.0**	0.000	0.130	**6.0**	0.000	0.348
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.006	**6.0**	0.000	0.006	**6.0**	0.000	0.132	6.1	0.050	0.332
	$U[10.0C', 20.0C']$	6.2	0.000	0.006	**6.0**	0.000	0.006	6.1	0.050	0.120	**6.0**	0.000	0.356
10	$U[0.0C', 0.5C']$	**6.0**	0.000	0.032	**6.0**	0.000	0.032	**6.0**	0.000	0.160	**6.0**	0.000	0.372
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.033	**6.0**	0.000	0.032	**6.0**	0.000	0.154	**6.0**	0.000	0.379
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.032	**6.0**	0.000	0.033	**6.0**	0.000	0.150	**6.0**	0.000	0.379
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.032	**6.0**	0.000	0.031	**6.0**	0.000	0.155	**6.0**	0.000	0.381
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.033	**6.0**	0.000	0.032	**6.0**	0.000	0.155	**6.0**	0.000	0.378
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.034	**6.0**	0.000	0.034	**6.0**	0.000	0.153	**6.0**	0.000	0.370
20	$U[0.0C', 0.5C']$	**6.0**	0.000	0.061	**6.0**	0.000	0.061	**6.0**	0.000	0.179	**6.0**	0.000	0.401
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.059	**6.0**	0.000	0.058	**6.0**	0.000	0.179	**6.0**	0.000	0.400
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.062	**6.0**	0.000	0.060	**6.0**	0.000	0.185	**6.0**	0.000	0.417
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.060	**6.0**	0.000	0.060	**6.0**	0.000	0.187	**6.0**	0.000	0.417
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.063	**6.0**	0.000	0.062	**6.0**	0.000	0.191	**6.0**	0.000	0.396
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.065	**6.0**	0.000	0.065	**6.0**	0.000	0.196	**6.0**	0.000	0.412
50	$U[0.0C', 0.5C']$	**6.0**	0.000	0.144	**6.0**	0.000	0.141	**6.0**	0.000	0.261	**6.0**	0.000	0.463
	$U[0.5C', 1.0C']$	**6.0**	0.000	0.139	**6.0**	0.000	0.136	**6.0**	0.000	0.267	**6.0**	0.000	0.493
	$U[1.0C', 4.0C']$	**6.0**	0.000	0.142	**6.0**	0.000	0.141	**6.0**	0.000	0.271	**6.0**	0.000	0.487
	$U[0.0C', 5.0C']$	**6.0**	0.000	0.147	**6.0**	0.000	0.144	**6.0**	0.000	0.273	**6.0**	0.000	0.476
	$U[5.0C', 10.0C']$	**6.0**	0.000	0.153	**6.0**	0.000	0.152	**6.0**	0.000	0.281	**6.0**	0.000	0.495
	$U[10.0C', 20.0C']$	**6.0**	0.000	0.157	**6.0**	0.000	0.156	**6.0**	0.000	0.282	**6.0**	0.000	0.492

Table 6.17: Results for a set of 5 instances of size 80 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**6.7**	0.050	0.009	**6.7**	0.050	0.009	7.0	0.000	0.148	6.8	0.100	0.423
	$U[0.5C', 1.0C']$	7.0	0.000	0.008	6.9	0.050	0.009	7.0	0.000	0.145	**6.8**	0.000	0.442
	$U[1.0C', 4.0C']$	**6.8**	0.000	0.008	6.9	0.050	0.008	7.0	0.000	0.147	**6.8**	0.100	0.436
	$U[0.0C', 5.0C']$	**6.8**	0.000	0.010	7.0	0.000	0.009	7.0	0.000	0.143	6.9	0.050	0.414
	$U[5.0C', 10.0C']$	6.9	0.050	0.008	**6.7**	0.050	0.009	6.9	0.050	0.157	6.8	0.000	0.443
	$U[10.0C', 20.0C']$	7.0	0.000	0.008	7.0	0.000	0.009	**6.9**	0.050	0.147	7.0	0.000	0.410
10	$U[0.0C', 0.5C']$	6.7	0.050	0.053	6.8	0.100	0.051	**6.5**	0.050	0.217	6.6	0.100	0.515
	$U[0.5C', 1.0C']$	**6.5**	0.050	0.061	6.8	0.100	0.050	6.8	0.100	0.200	6.6	0.000	0.515
	$U[1.0C', 4.0C']$	6.5	0.050	0.053	**6.4**	0.000	0.059	**6.4**	0.100	0.217	**6.4**	0.000	0.536
	$U[0.0C', 5.0C']$	6.5	0.050	0.053	6.4	0.000	0.058	6.5	0.050	0.202	**6.3**	0.050	0.550
	$U[5.0C', 10.0C']$	**6.4**	0.000	0.063	6.6	0.100	0.055	**6.4**	0.100	0.218	**6.4**	0.000	0.536
	$U[10.0C', 20.0C']$	6.8	0.100	0.061	**6.6**	0.000	0.061	7.0	0.000	0.187	6.7	0.150	0.500
20	$U[0.0C', 0.5C']$	6.8	0.100	0.090	6.7	0.050	0.096	**6.6**	0.100	0.255	6.7	0.150	0.564
	$U[0.5C', 1.0C']$	**6.7**	0.050	0.100	**6.7**	0.150	0.096	6.9	0.050	0.230	7.0	0.000	0.498
	$U[1.0C', 4.0C']$	**6.2**	0.100	0.109	6.6	0.000	0.105	6.3	0.050	0.271	6.4	0.000	0.597
	$U[0.0C', 5.0C']$	**6.1**	0.050	0.116	6.3	0.050	0.111	6.3	0.050	0.264	6.3	0.050	0.620
	$U[5.0C', 10.0C']$	6.5	0.050	0.109	6.4	0.000	0.113	6.4	0.000	0.272	**6.3**	0.050	0.634
	$U[10.0C', 20.0C']$	6.8	0.100	0.114	**6.7**	0.050	0.130	6.8	0.000	0.249	**6.7**	0.050	0.580
50	$U[0.0C', 0.5C']$	6.7	0.150	0.240	6.9	0.050	0.219	**6.6**	0.000	0.404	**6.6**	0.000	0.677
	$U[0.5C', 1.0C']$	**6.6**	0.000	0.260	**6.6**	0.100	0.251	6.7	0.150	0.389	**6.6**	0.000	0.711
	$U[1.0C', 4.0C']$	6.3	0.050	0.255	**6.1**	0.050	0.284	**6.1**	0.050	0.448	6.2	0.000	0.768
	$U[0.0C', 5.0C']$	6.2	0.000	0.263	6.2	0.000	0.270	**6.1**	0.050	0.475	6.2	0.000	0.750
	$U[5.0C', 10.0C']$	6.3	0.050	0.270	6.3	0.050	0.279	6.4	0.000	0.414	**6.2**	0.000	0.795
	$U[10.0C', 20.0C']$	6.4	0.000	0.303	**6.3**	0.150	0.329	6.6	0.100	0.426	**6.3**	0.050	0.844

Table 6.18: Results for a set of 5 instances of size 90 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	8.2	0.027	0.021	8.2	0.038	0.023	**8.2**	0.055	0.225	8.3	0.044	0.613
	$U[0.5C', 1.0C']$	8.1	0.016	0.020	8.1	0.033	0.021	8.1	0.050	0.226	**8.1**	0.050	0.666
	$U[1.0C', 4.0C']$	**8.0**	0.038	0.021	8.1	0.016	0.021	8.1	0.027	0.227	8.0	0.016	0.667
	$U[0.0C', 5.0C']$	8.0	0.038	0.022	8.1	0.005	0.021	8.1	0.027	0.229	**8.0**	0.033	0.672
	$U[5.0C', 10.0C']$	**8.1**	0.033	0.021	8.1	0.038	0.022	8.1	0.016	0.222	8.1	0.022	0.641
	$U[10.0C', 20.0C']$	8.2	0.016	0.021	8.2	0.016	0.021	8.2	0.016	0.226	**8.1**	0.027	0.628
10	$U[0.0C', 0.5C']$	**8.1**	0.038	0.165	8.2	0.055	0.150	8.2	0.033	0.347	8.2	0.027	0.776
	$U[0.5C', 1.0C']$	**7.9**	0.022	0.160	8.0	0.038	0.155	8.0	0.050	0.359	8.0	0.033	0.787
	$U[1.0C', 4.0C']$	7.8	0.011	0.155	7.8	0.016	0.159	**7.8**	0.011	0.389	7.9	0.016	0.819
	$U[0.0C', 5.0C']$	7.9	0.016	0.158	7.9	0.011	0.156	**7.8**	0.000	0.380	**7.8**	0.011	0.835
	$U[5.0C', 10.0C']$	7.9	0.022	0.159	7.9	0.022	0.165	7.9	0.022	0.376	**7.9**	0.016	0.832
	$U[10.0C', 20.0C']$	8.0	0.033	0.170	8.0	0.038	0.169	**7.9**	0.027	0.390	8.0	0.027	0.814
20	$U[0.0C', 0.5C']$	**8.1**	0.050	0.332	8.2	0.044	0.319	8.2	0.077	0.525	8.2	0.038	0.892
	$U[0.5C', 1.0C']$	8.0	0.055	0.309	8.1	0.038	0.292	**8.0**	0.027	0.505	8.0	0.033	0.930
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.298	7.8	0.005	0.315	7.8	0.011	0.518	7.8	0.005	0.968
	$U[0.0C', 5.0C']$	7.8	0.005	0.293	7.8	0.005	0.305	**7.8**	0.011	0.517	**7.8**	0.000	0.982
	$U[5.0C', 10.0C']$	**7.8**	0.005	0.320	**7.8**	0.005	0.332	7.8	0.000	0.530	7.9	0.016	0.994
	$U[10.0C', 20.0C']$	7.9	0.011	0.322	**7.9**	0.027	0.348	8.0	0.022	0.528	7.9	0.005	0.987
50	$U[0.0C', 0.5C']$	**8.1**	0.044	0.834	8.2	0.050	0.767	8.2	0.072	0.932	8.2	0.050	1.324
	$U[0.5C', 1.0C']$	8.0	0.044	0.723	8.0	0.027	0.715	**7.9**	0.016	0.913	8.0	0.044	1.321
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.710	**7.8**	0.000	0.710	**7.8**	0.000	0.915	7.8	0.000	1.366
	$U[0.0C', 5.0C']$	**7.8**	0.011	0.716	**7.8**	0.011	0.721	**7.8**	0.000	0.929	7.8	0.005	1.378
	$U[5.0C', 10.0C']$	7.8	0.000	0.764	**7.8**	0.005	0.784	7.8	0.011	0.963	7.8	0.011	1.432
	$U[10.0C', 20.0C']$	7.9	0.022	0.814	7.9	0.011	0.844	7.9	0.016	1.028	**7.8**	0.011	1.481

Table 6.19: Results for a set of 5 instances of size 100 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**7.6**	0.100	0.018	7.8	0.000	0.017	**7.6**	0.100	0.240	7.8	0.100	0.735
	$U[0.5C', 1.0C']$	7.8	0.000	0.017	7.8	0.000	0.016	7.7	0.050	0.234	**7.6**	0.100	0.801
	$U[1.0C', 4.0C']$	**7.7**	0.050	0.017	7.8	0.000	0.016	7.8	0.000	0.226	**7.7**	0.050	0.767
	$U[0.0C', 5.0C']$	7.8	0.000	0.016	7.8	0.000	0.016	7.8	0.000	0.218	**7.7**	0.050	0.774
	$U[5.0C', 10.0C']$	7.9	0.050	0.016	7.8	0.000	0.017	**7.7**	0.050	0.223	7.8	0.000	0.716
	$U[10.0C', 20.0C']$	8.0	0.000	0.016	8.0	0.000	0.016	**7.9**	0.050	0.219	8.0	0.000	0.711
10	$U[0.0C', 0.5C']$	7.8	0.100	0.104	**7.6**	0.100	0.111	**7.6**	0.100	0.325	7.7	0.050	0.834
	$U[0.5C', 1.0C']$	**7.6**	0.000	0.102	**7.6**	0.100	0.103	7.7	0.050	0.321	7.7	0.050	0.859
	$U[1.0C', 4.0C']$	7.2	0.000	0.120	**7.1**	0.050	0.122	**7.1**	0.050	0.375	7.2	0.100	1.019
	$U[0.0C', 5.0C']$	7.1	0.050	0.124	**7.0**	0.000	0.118	7.1	0.050	0.373	7.1	0.050	1.022
	$U[5.0C', 10.0C']$	7.4	0.100	0.138	**7.3**	0.050	0.136	7.4	0.200	0.357	7.4	0.200	0.929
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.110	**7.8**	0.000	0.114	**7.8**	0.000	0.321	**7.8**	0.000	0.840
20	$U[0.0C', 0.5C']$	7.6	0.100	0.210	7.8	0.000	0.196	**7.3**	0.050	0.466	7.6	0.100	0.963
	$U[0.5C', 1.0C']$	7.5	0.050	0.229	7.6	0.100	0.201	7.5	0.150	0.430	**7.3**	0.150	1.045
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.233	**7.0**	0.000	0.232	**7.0**	0.000	0.483	**7.0**	0.000	1.139
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.232	**7.0**	0.000	0.227	7.2	0.100	0.460	**7.0**	0.000	1.110
	$U[5.0C', 10.0C']$	7.3	0.050	0.259	**7.1**	0.050	0.261	7.3	0.050	0.471	7.3	0.050	1.061
	$U[10.0C', 20.0C']$	7.8	0.000	0.212	7.8	0.000	0.216	**7.5**	0.150	0.471	**7.5**	0.150	1.003
50	$U[0.0C', 0.5C']$	7.8	0.100	0.478	7.7	0.050	0.502	**7.5**	0.150	0.729	7.6	0.100	1.272
	$U[0.5C', 1.0C']$	**7.2**	0.100	0.658	7.4	0.100	0.582	7.6	0.100	0.695	7.6	0.100	1.239
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.544	**7.0**	0.000	0.523	**7.0**	0.000	0.764	**7.0**	0.000	1.427
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.521	**7.0**	0.000	0.538	**7.0**	0.000	0.758	**7.0**	0.000	1.422
	$U[5.0C', 10.0C']$	**7.0**	0.000	0.622	7.1	0.050	0.582	**7.0**	0.000	0.827	7.1	0.050	1.455
	$U[10.0C', 20.0C']$	7.7	0.050	0.539	**7.5**	0.050	0.579	7.7	0.050	0.735	7.6	0.100	1.303

Table 6.20: Results for a set of 5 instances of size 120 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**8.7**	0.050	0.032	9.0	0.000	0.029	8.9	0.050	0.349	8.9	0.050	1.201
	$U[0.5C', 1.0C']$	**8.8**	0.100	0.030	9.0	0.000	0.028	8.9	0.050	0.337	8.9	0.050	1.213
	$U[1.0C', 4.0C']$	**8.7**	0.050	0.032	8.9	0.050	0.029	9.0	0.000	0.341	8.9	0.050	1.173
	$U[0.0C', 5.0C']$	9.0	0.000	0.028	**8.7**	0.050	0.031	8.8	0.100	0.348	8.8	0.000	1.211
	$U[5.0C', 10.0C']$	9.0	0.000	0.029	**8.9**	0.050	0.029	9.0	0.000	0.337	9.0	0.000	1.139
	$U[10.0C', 20.0C']$	**9.0**	0.000	0.031	**9.0**	0.000	0.030	**9.0**	0.000	0.349	**9.0**	0.000	1.144
10	$U[0.0C', 0.5C']$	8.9	0.050	0.202	**8.6**	0.000	0.242	8.8	0.000	0.513	8.8	0.100	1.412
	$U[0.5C', 1.0C']$	**8.7**	0.050	0.237	8.9	0.050	0.192	**8.7**	0.050	0.532	**8.7**	0.050	1.390
	$U[1.0C', 4.0C']$	**8.5**	0.050	0.216	8.6	0.000	0.205	8.6	0.000	0.528	8.6	0.000	1.473
	$U[0.0C', 5.0C']$	8.6	0.000	0.211	8.6	0.000	0.208	8.6	0.000	0.525	**8.5**	0.050	1.498
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.218	**8.6**	0.000	0.213	**8.6**	0.000	0.554	8.7	0.050	1.433
	$U[10.0C', 20.0C']$	8.9	0.050	0.221	**8.8**	0.100	0.231	8.9	0.050	0.535	**8.8**	0.000	1.388
20	$U[0.0C', 0.5C']$	8.7	0.050	0.436	8.8	0.100	0.388	8.8	0.000	0.718	**8.6**	0.000	1.677
	$U[0.5C', 1.0C']$	8.8	0.100	0.376	8.7	0.050	0.413	**8.6**	0.000	0.760	8.9	0.050	1.494
	$U[1.0C', 4.0C']$	8.6	0.000	0.408	8.6	0.000	0.390	**8.5**	0.050	0.744	**8.5**	0.050	1.694
	$U[0.0C', 5.0C']$	8.6	0.000	0.411	8.6	0.000	0.394	**8.5**	0.050	0.740	**8.5**	0.050	1.679
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.438	**8.6**	0.000	0.434	**8.6**	0.000	0.750	**8.6**	0.000	1.667
	$U[10.0C', 20.0C']$	8.9	0.050	0.421	8.8	0.000	0.434	**8.6**	0.000	0.787	8.8	0.000	1.666
50	$U[0.0C', 0.5C']$	8.8	0.100	0.944	**8.6**	0.000	0.977	**8.6**	0.000	1.342	8.7	0.050	2.152
	$U[0.5C', 1.0C']$	8.7	0.050	1.009	8.7	0.050	0.947	8.7	0.050	1.374	**8.6**	0.000	2.193
	$U[1.0C', 4.0C']$	8.6	0.000	0.949	8.6	0.000	0.943	**8.5**	0.050	1.312	8.6	0.000	2.212
	$U[0.0C', 5.0C']$	**8.4**	0.100	1.113	8.6	0.000	0.965	8.5	0.050	1.354	8.5	0.050	2.249
	$U[5.0C', 10.0C']$	8.6	0.000	0.998	**8.5**	0.050	1.047	8.6	0.000	1.300	8.6	0.000	2.232
	$U[10.0C', 20.0C']$	**8.6**	0.000	1.060	**8.6**	0.000	1.117	8.7	0.050	1.363	**8.6**	0.000	2.318

Table 6.21: Results for the instance *dsjc500.5-1*, 500 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$
1	$U[0.25C', 0.75C']$	53.00	0.000	32.675	52.40	0.240	45.955	**52.00**	0.000	41.061	52.80	0.160	131.369
	$U[0.0C', 1.0C']$	53.00	0.000	31.822	53.00	0.000	31.307	**52.60**	0.240	38.698	53.00	0.000	124.405
	$U[0.0C', 0.5C']$	**52.00**	0.000	49.998	52.60	0.240	40.508	**52.00**	0.000	47.886	52.80	0.160	133.227
	$U[0.5C', 1.0C']$	52.60	0.240	34.443	52.40	0.240	34.702	53.00	0.000	34.162	**52.00**	0.000	151.490
	$U[0.25C', 1.0C']$	53.00	0.000	33.513	**52.40**	0.240	34.969	53.00	0.000	35.207	53.00	0.000	126.708
	$U[0.0C', 0.75C']$	**52.40**	0.240	37.164	**52.40**	0.240	42.677	**52.40**	0.240	43.436	**52.40**	0.20	137.571
5	$U[0.25C', 0.75C']$	52.00	0.000	175.018	**51.60**	0.240	222.313	52.00	0.000	167.028	52.00	0.000	263.785
	$U[0.0C', 1.0C']$	**51.40**	0.240	192.997	52.00	0.000	166.878	52.00	0.000	168.842	52.00	0.000	270.638
	$U[0.0C', 0.5C']$	**51.40**	0.240	188.598	**51.40**	0.240	194.900	**51.40**	0.240	234.086	52.00	0.000	267.352
	$U[0.5C', 1.0C']$	52.00	0.000	167.531	**51.40**	0.240	197.548	**51.40**	0.250	191.290	**51.40**	0.250	301.400
	$U[0.25C', 1.0C']$	**51.40**	0.240	227.016	**51.40**	0.240	190.509	**51.40**	0.240	226.595	52.00	0.000	273.516
	$U[0.0C', 0.75C']$	52.00	0.000	166.399	**51.00**	0.000	243.717	51.80	0.160	188.605	51.80	0.160	323.258
10	$U[0.25C', 0.75C']$	**51.00**	0.000	460.830	**51.00**	0.000	499.770	51.80	0.160	385.627	**51.00**	0.000	580.569
	$U[0.0C', 1.0C']$	**51.00**	0.000	445.430	**51.00**	0.000	441.279	**51.00**	0.000	556.089	**51.00**	0.000	1920.230
	$U[0.0C', 0.5C']$	**51.00**	0.000	438.898	**51.00**	0.000	506.921	**51.00**	0.000	491.451	**51.00**	0.000	574.642
	$U[0.5C', 1.0C']$	**51.00**	0.000	434.163	51.40	0.240	368.134	**51.00**	0.000	467.440	**51.00**	0.000	617.425
	$U[0.25C', 1.0C']$	51.80	0.160	365.680	**51.00**	0.000	424.486	51.60	0.240	365.924	51.40	0.240	441.895
	$U[0.0C', 0.75C']$	**51.00**	0.000	425.001	**51.00**	0.000	526.258	51.40	0.240	353.324	51.40	0.240	479.716

Table 6.22: Results for the instance *dsjc500.5-2*, 1000 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$
1	$U[0.25C', 0.75C']$	47.60	0.240	72.492	**47.00**	0.000	69.580	**47.00**	0.000	121.744	48.00	0.000	702.030
	$U[0.0C', 1.0C']$	**47.00**	0.000	77.980	**47.00**	0.000	78.475	**47.00**	0.000	126.041	48.00	0.000	722.381
	$U[0.0C', 0.5C']$	**47.00**	0.000	77.742	47.60	0.240	71.457	**47.00**	0.000	118.374	48.00	0.000	647.080
	$U[0.5C', 0.1C']$	47.20	0.160	80.400	**47.00**	0.000	72.520	47.40	0.240	102.338	**47.00**	0.000	761.492
	$U[0.25C', 1.0C']$	47.20	0.160	66.099	**47.00**	0.000	85.592	**47.00**	0.000	118.374	47.40	0.240	752.992
	$U[0.0C', 0.75C']$	47.60	0.240	74.900	47.20	0.160	65.508	**47.00**	0.000	118.374	47.60	0.240	669.113
5	$U[0.25C', 0.75C']$	**47.00**	0.000	329.858	**47.00**	0.000	322.690	**47.00**	0.000	341.401	**47.00**	0.000	911.078
	$U[0.0C', 1.0C']$	**47.00**	0.000	314.639	**47.00**	0.000	312.814	**47.00**	0.000	329.006	**47.00**	0.000	909.702
	$U[0.0C', 0.5C']$	**47.00**	0.000	336.046	**47.00**	0.000	368.138	**47.00**	0.000	363.048	**47.00**	0.000	893.171
	$U[0.5C', 0.1C']$	47.00	0.000	329.366	**46.60**	0.240	355.754	47.00	0.000	369.581	47.00	0.000	939.086
	$U[0.25C', 1.0C']$	47.00	0.000	320.355	**46.60**	0.240	504.976	47.00	0.000	312.525	47.00	0.000	888.474
	$U[0.0C', 0.75C']$	**47.00**	0.000	319.387	**47.00**	0.000	321.539	**47.00**	0.000	381.417	**47.00**	0.000	899.100
10	$U[0.25C', 0.75C']$	**47.00**	0.000	650.358	**47.00**	0.000	600.810	**47.00**	0.000	782.341	**47.00**	0.000	1340.031
	$U[0.0C', 1.0C']$	**46.40**	0.240	657.860	**46.40**	0.240	771.054	47.00	0.000	842.114	47.00	0.000	1112.001
	$U[0.0C', 0.5C']$	47.00	0.000	651.221	**46.60**	0.240	712.167	47.00	0.000	666.702	47.00	0.000	1260.525
	$U[0.5C', 0.1C']$	47.00	0.000	610.881	46.40	0.240	655.999	**46.00**	0.000	1090.255	47.00	0.000	1332.405
	$U[0.25C', 1.0C']$	**46.40**	0.240	811.390	47.00	0.000	618.188	47.00	0.000	699.847	47.00	0.000	1201.992
	$U[0.0C', 0.75C']$	47.00	0.000	630.789	**46.60**	0.240	700.052	47.00	0.000	947.012	47.00	0.000	1417.935

Table 6.23: Results for the instance *dsjc500.5-3*, 1500 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.25C', 0.75C']$	**45.00**	0.000	115.018	**45.00**	0.000	115.547	**45.00**	0.000	205.556	**45.00**	0.000	3110.445
	$U[0.0C', 1.0C']$	**45.00**	0.000	124.909	**45.00**	0.000	119.600	**45.00**	0.000	245.091	**45.00**	0.000	3101.009
	$U[0.0C', 0.5C']$	**45.00**	0.000	117.858	**45.00**	0.000	130.135	**45.00**	0.000	199.366	**45.00**	0.000	2960.582
	$U[0.5C', 1.0C']$	45.00	0.000	115.410	45.00	0.000	113.664	45.00	0.00	210.217	**44.60**	0.240	3227.712
	$U[0.25C', 1.0C']$	**45.00**	0.000	113.382	**45.00**	0.000	119.870	**45.00**	0.000	182.176	**45.00**	0.000	3298.511
	$U[0.0C', 0.75C']$	**45.00**	0.000	113.593	**45.00**	0.000	114.424	45.40	0.240	224.569	**45.00**	0.000	3205.006
5	$U[0.25C', 0.75C']$	**44.00**	0.000	775.920	44.20	0.160	644.067	**44.00**	0.000	712.253	**44.00**	0.000	3801.001
	$U[0.0C', 1.0C']$	**44.00**	0.000	692.584	44.40	0.240	590.315	**44.00**	0.000	736.866	**44.00**	0.000	3989.886
	$U[0.0C', 0.5C']$	**44.00**	0.000	887.549	44.60	0.240	569.560	**44.00**	0.000	770.456	**44.00**	0.000	4252.012
	$U[0.5C', 1.0C']$	44.40	0.240	535.744	**44.00**	0.000	981.468	44.80	0.160	631.256	44.60	0.240	3525.024
	$U[0.25C', 1.0C']$	44.80	0.160	623.681	44.60	0.240	547.132	**44.00**	0.000	705.102	**44.00**	0.000	3612.583
	$U[0.0C', 0.75C']$	44.60	0.240	716.675	44.40	0.240	564.862	**44.00**	0.000	681.207	**44.00**	0.000	4328.666
10	$U[0.25C', 0.75C']$	**44.00**	0.000	1711.281	**44.00**	0.000	1284.477	**44.00**	0.000	991.903	**44.00**	0.000	4385.033
	$U[0.0C', 1.0C']$	**44.00**	0.000	1609.692	44.20	0.160	1398.949	**44.00**	0.000	1013.891	**44.00**	0.000	4441.113
	$U[0.0C', 0.5C']$	**44.00**	0.000	1504.814	**44.00**	0.000	1624.962	**44.00**	0.000	1271.812	**44.00**	0.000	4641.228
	$U[0.5C', 1.0C']$	44.60	0.240	1156.730	**44.00**	0.000	1739.053	44.40	0.240	1163.476	**44.00**	0.000	4935.751
	$U[0.25C', 1.0C']$	**44.00**	0.000	1282.196	**44.00**	0.000	1772.908	44.60	0.240	1351.233	**44.00**	0.000	5011.565
	$U[0.0C', 0.75C']$	**44.00**	0.000	1928.361	**44.00**	0.000	1792.404	44.40	0.240	1291.088	**44.00**	0.000	4505.852

Table 6.24: Results for the instance *dsjc500.5-4*, 2000 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.25C', 0.75C']$	43.40	0.240	200.361	**43.00**	0.000	178.387	**43.00**	0.000	311.715	43.20	0.160	7530.236
	$U[0.0C', 1.0C']$	43.20	0.160	185.824	**43.00**	0.000	179.494	**43.00**	0.000	341.112	43.80	0.160	7012.431
	$U[0.0C', 0.5C']$	44.00	0.000	143.937	43.60	0.240	186.869	**43.00**	0.000	333.319	43.80	0.160	7171.777
	$U[0.5C', 1.0C']$	**43.00**	0.000	214.545	43.60	0.240	142.063	**43.00**	0.000	338.309	43.60	0.240	7439.572
	$U[0.25C', 1.0C']$	43.60	0.240	145.004	**43.00**	0.000	251.143	**43.00**	0.000	302.122	44.00	0.000	7621.502
	$U[0.0C', 0.75C']$	43.60	0.240	218.761	43.20	0.160	187.378	**43.00**	0.000	335.834	43.40	0.240	7478.121
5	$U[0.25C', 0.75C']$	**43.00**	0.000	780.378	**43.00**	0.000	963.153	**43.00**	0.000	978.203	**43.00**	0.000	8956.555
	$U[0.0C', 1.0C']$	**42.40**	0.240	849.693	42.80	0.160	771.423	43.00	0.000	1122.429	42.80	0.160	8701.683
	$U[0.0C', 0.5C']$	**43.00**	0.000	808.077	**43.00**	0.000	848.376	**43.00**	0.000	1006.318	**43.00**	0.000	8622.681
	$U[0.5C', 1.0C']$	**43.00**	0.000	781.823	**43.00**	0.000	823.835	**43.00**	0.000	958.897	**43.00**	0.000	9115.008
	$U[0.25C', 1.0C']$	**43.00**	0.000	776.229	**43.00**	0.000	806.833	**43.00**	0.000	989.006	**43.00**	0.000	9012.904
	$U[0.0C', 0.75C']$	**43.00**	0.000	823.599	**43.00**	0.000	809.126	**43.00**	0.000	1007.065	**43.00**	0.000	8848.775
10	$U[0.25C', 0.75C']$	**43.00**	0.000	1619.182	**43.00**	0.000	1711.629	**43.00**	0.000	1674.249	**43.00**	0.000	9246.012
	$U[0.0C', 1.0C']$	43.00	0.000	1570.523	43.00	0.000	1628.829	**42.80**	0.160	1569.004	43.00	0.000	9006.112
	$U[0.0C', 0.5C']$	43.00	0.000	1532.691	**42.40**	0.240	2145.856	43.00	0.000	1762.306	43.00	0.000	9176.079
	$U[0.5C', 1.0C']$	**42.80**	0.180	1554.850	43.00	0.000	1558.656	**42.80**	0.160	1710.961	**43.00**	0.000	9047.702
	$U[0.25C', 1.0C']$	43.00	0.000	1567.553	**42.60**	0.240	1799.667	43.00	0.000	1821.938	43.00	0.000	9312.528
	$U[0.0C', 0.75C']$	**43.00**	0.000	1529.261	**43.00**	0.000	1554.409	**43.00**	0.000	1701.039	**43.00**	0.000	9199.499

Variants

As discussed in section 5.3, variants for both ILPs have been created by removing the inequation that restricts conflicts inside the recolored set of clusters. In tables 6.25 to 6.30 the standard ILPs marked as $ILP1$ and $ILP2$ are compared to their variants $ILP1*$ and $ILP2*$ by evaluating three instances of different size as well as three instances of different density. It can be seen that removing the aforementioned constraint does not increase the solution quality.

Furthermore experiments have been performed, placing the recently recolored set of clusters on the tabu list for a specified number of iterations (see 5.3). In tables 6.31 to 6.36 sets diversing in size and density have been evaluated fixing $ItMax = 5$. The parameter $TTRecolored$ sets the number of iterations as $Tabusize = TTRecolored \cdot C'$ for the set of node-color pairs of the recolored set of clusters to remain on the tabu list. Again, no significant impacts on the final results can be observed by applying this kind of variation.

Table 6.25: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.7 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$
1	$U[0.0C', 0.5C']$	**10.3**	0.150	0.312	10.6	0.100	0.135	10.5	0.150	0.823	10.4	0.100	0.602
	$U[0.5C', 1.0C']$	10.4	0.000	0.302	10.4	0.100	0.131	**10.2**	0.100	0.875	10.3	0.050	0.617
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.326	10.1	0.050	0.151	10.2	0.100	0.868	10.2	0.000	0.628
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.337	10.1	0.050	0.138	10.2	0.100	0.917	10.3	0.050	0.632
	$U[5.0C', 10.0C']$	10.3	0.150	0.308	**10.2**	0.000	0.142	10.4	0.100	0.820	10.3	0.050	0.610
	$U[10.0C', 20.0C']$	**10.5**	0.150	0.295	10.9	0.050	0.112	10.6	0.100	0.793	10.7	0.050	0.545
10	$U[0.0C', 0.5C']$	**10.2**	0.100	0.542	10.7	0.050	0.295	10.7	0.150	0.952	10.3	0.050	0.799
	$U[0.5C', 1.0C']$	10.2	0.100	0.508	**10.1**	0.050	0.356	10.2	0.100	1.096	10.2	0.000	0.800
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.493	**10.0**	0.000	0.314	**10.0**	0.000	1.098	**10.0**	0.000	0.827
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.493	**10.0**	0.000	0.319	**10.0**	0.000	1.081	**10.0**	0.000	0.823
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.500	**10.0**	0.000	0.322	**10.0**	0.000	1.109	**10.0**	0.000	0.828
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.531	10.1	0.050	0.362	**10.0**	0.000	1.114	**10.0**	0.000	0.871
20	$U[0.0C', 0.5C']$	10.4	0.100	0.738	10.7	0.150	0.491	**10.3**	0.150	1.327	10.4	0.200	0.991
	$U[0.5C', 1.0C']$	**10.1**	0.050	0.745	**10.1**	0.050	0.587	**10.1**	0.050	1.345	**10.1**	0.050	1.023
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.673	**10.0**	0.000	0.499	**10.0**	0.000	1.281	**10.0**	0.000	0.993
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.682	**10.0**	0.000	0.494	**10.0**	0.000	1.290	**10.0**	0.000	1.001
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.690	**10.0**	0.000	0.517	**10.0**	0.000	1.289	**10.0**	0.000	1.013
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.721	**10.0**	0.000	0.548	**10.0**	0.000	1.320	**10.0**	0.000	1.048
50	$U[0.0C', 0.5C']$	10.3	0.150	1.386	10.6	0.100	1.105	10.7	0.150	1.694	**10.2**	0.000	1.709
	$U[0.5C', 1.0C']$	10.3	0.050	1.346	**10.0**	0.000	1.199	**10.0**	0.000	1.948	10.2	0.100	1.526
	$U[1.0C', 4.0C']$	**10.0**	0.000	1.207	**10.0**	0.000	1.060	**10.0**	0.000	1.778	**10.0**	0.000	1.523
	$U[0.0C', 5.0C']$	**10.0**	0.000	1.211	**10.0**	0.000	1.058	**10.0**	0.000	1.812	**10.0**	0.000	1.533
	$U[5.0C', 10.0C']$	**10.0**	0.000	1.231	**10.0**	0.000	1.093	**10.0**	0.000	1.837	**10.0**	0.000	1.553
	$U[10.0C', 20.0C']$	**10.0**	0.000	1.295	**10.0**	0.000	1.176	**10.0**	0.000	1.910	**10.0**	0.000	1.677

Table 6.26: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.8 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	13.1	0.150	0.394	**12.8**	0.000	0.215	12.9	0.050	1.045	**12.8**	0.000	0.851
	$U[0.5C', 1.0C']$	**12.7**	0.050	0.439	12.8	0.100	0.204	**12.7**	0.050	1.088	**12.7**	0.050	0.848
	$U[1.0C', 4.0C']$	12.7	0.050	0.426	12.8	0.000	0.213	**12.6**	0.000	1.081	12.7	0.050	0.870
	$U[0.0C', 5.0C']$	12.7	0.050	0.446	12.7	0.050	0.221	12.8	0.000	1.048	**12.5**	0.050	0.882
	$U[5.0C', 10.0C']$	**12.7**	0.050	0.426	12.8	0.000	0.203	**12.7**	0.050	1.094	12.8	0.000	0.832
	$U[10.0C', 20.0C']$	**12.8**	0.000	0.426	**12.8**	0.000	0.195	**12.8**	0.100	1.069	**12.8**	0.000	0.850
10	$U[0.0C', 0.5C']$	12.8	0.000	0.720	**12.7**	0.050	0.546	12.9	0.050	1.294	13.0	0.000	1.049
	$U[0.5C', 1.0C']$	12.7	0.050	0.691	12.5	0.150	0.556	12.5	0.050	1.417	**12.4**	0.000	1.208
	$U[1.0C', 4.0C']$	12.2	0.100	0.746	12.1	0.050	0.543	12.2	0.100	1.508	**12.0**	0.000	1.321
	$U[0.0C', 5.0C']$	12.2	0.100	0.765	**12.1**	0.050	0.521	12.2	0.100	1.522	12.2	0.000	1.218
	$U[5.0C', 10.0C']$	12.4	0.000	0.745	**12.1**	0.050	0.580	12.3	0.050	1.525	12.2	0.000	1.284
	$U[10.0C', 20.0C']$	**12.4**	0.000	0.779	**12.4**	0.100	0.529	12.5	0.050	1.439	12.5	0.150	1.208
20	$U[0.0C', 0.5C']$	13.0	0.000	0.930	13.1	0.150	0.787	12.8	0.200	1.671	**12.7**	0.050	1.435
	$U[0.5C', 1.0C']$	**12.5**	0.050	1.068	12.7	0.050	0.779	12.8	0.000	1.571	12.6	0.000	1.398
	$U[1.0C', 4.0C']$	12.1	0.050	1.101	**12.0**	0.000	0.949	12.2	0.000	1.778	**12.0**	0.000	1.620
	$U[0.0C', 5.0C']$	**12.0**	0.000	1.178	**12.0**	0.000	0.848	12.1	0.050	1.876	**12.0**	0.000	1.634
	$U[5.0C', 10.0C']$	12.2	0.000	1.088	12.2	0.100	0.858	12.2	0.000	1.813	**12.1**	0.050	1.657
	$U[10.0C', 20.0C']$	**12.2**	0.100	1.170	12.5	0.050	0.899	12.6	0.100	1.771	**12.2**	0.000	1.692
50	$U[0.0C', 0.5C']$	12.9	0.050	1.930	13.1	0.050	1.870	13.0	0.100	2.468	**12.8**	0.100	2.313
	$U[0.5C', 1.0C']$	12.8	0.000	1.716	**12.6**	0.100	1.776	12.7	0.050	2.487	12.7	0.050	2.294
	$U[1.0C', 4.0C']$	**12.0**	0.000	2.094	**12.0**	0.000	1.753	**12.0**	0.000	2.765	**12.0**	0.000	2.463
	$U[0.0C', 5.0C']$	**12.0**	0.000	2.054	**12.0**	0.000	1.824	**12.0**	0.000	2.791	**12.0**	0.000	2.687
	$U[5.0C', 10.0C']$	**12.0**	0.000	2.201	12.1	0.050	1.844	12.1	0.050	2.837	**12.0**	0.000	2.696
	$U[10.0C', 20.0C']$	12.3	0.150	2.140	**12.2**	0.100	2.080	12.3	0.050	2.962	**12.2**	0.100	2.676

Table 6.27: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.9 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**16.7**	0.250	0.450	17.0	0.100	0.235	16.9	0.250	1.001	16.8	0.300	0.877
	$U[0.5C', 1.0C']$	16.4	0.100	0.477	16.3	0.050	0.279	**16.2**	0.000	1.139	16.6	0.100	0.909
	$U[1.0C', 4.0C']$	**16.1**	0.050	0.513	16.3	0.050	0.266	16.2	0.000	1.125	16.3	0.050	0.996
	$U[0.0C', 5.0C']$	**16.0**	0.000	0.534	16.1	0.050	0.300	16.2	0.000	1.112	16.2	0.000	0.990
	$U[5.0C', 10.0C']$	**16.1**	0.050	0.510	16.3	0.050	0.252	16.2	0.000	1.096	16.2	0.000	0.998
	$U[10.0C', 20.0C']$	16.4	0.000	0.473	**16.1**	0.050	0.294	16.2	0.000	1.101	16.2	0.000	0.994
10	$U[0.0C', 0.5C']$	**16.6**	0.300	0.994	16.8	0.300	0.830	16.8	0.300	1.561	16.7	0.050	1.484
	$U[0.5C', 1.0C']$	**16.1**	0.050	0.938	16.2	0.300	0.790	16.3	0.050	1.584	16.2	0.000	1.474
	$U[1.0C', 4.0C']$	**15.8**	0.000	1.019	**15.8**	0.000	0.758	**15.8**	0.000	1.731	**15.8**	0.000	1.571
	$U[0.0C', 5.0C']$	**15.8**	0.000	1.005	15.9	0.050	0.763	**15.8**	0.000	1.694	16.0	0.100	1.517
	$U[5.0C', 10.0C']$	**15.8**	0.000	1.103	15.9	0.050	0.809	**15.8**	0.000	1.754	**15.8**	0.000	1.603
	$U[10.0C', 20.0C']$	16.1	0.050	1.003	**15.8**	0.000	0.949	16.0	0.000	1.701	16.0	0.000	1.563
20	$U[0.0C', 0.5C']$	17.0	0.200	1.491	17.1	0.250	1.384	17.1	0.150	2.114	**16.7**	0.050	1.980
	$U[0.5C', 1.0C']$	16.2	0.000	1.430	16.2	0.100	1.361	**16.0**	0.000	2.203	16.3	0.150	1.877
	$U[1.0C', 4.0C']$	**15.8**	0.000	1.499	**15.8**	0.000	1.289	**15.8**	0.000	2.181	**15.8**	0.000	2.039
	$U[0.0C', 5.0C']$	**15.8**	0.000	1.532	**15.8**	0.000	1.328	**15.8**	0.000	2.177	**15.8**	0.000	2.127
	$U[5.0C', 10.0C']$	15.9	0.050	1.546	**15.8**	0.000	1.370	**15.8**	0.000	2.227	**15.8**	0.000	2.178
	$U[10.0C', 20.0C']$	16.0	0.000	1.585	16.0	0.000	1.337	**15.8**	0.000	2.273	16.0	0.000	2.105
50	$U[0.0C', 0.5C']$	**16.7**	0.350	3.210	16.8	0.100	3.289	17.2	0.200	3.570	17.2	0.000	3.566
	$U[0.5C', 1.0C']$	16.2	0.000	2.842	16.1	0.050	2.740	16.3	0.250	3.523	**16.0**	0.000	3.637
	$U[1.0C', 4.0C']$	**15.8**	0.000	2.937	**15.8**	0.000	2.775	**15.8**	0.000	3.617	**15.8**	0.000	3.526
	$U[0.0C', 5.0C']$	**15.8**	0.000	2.948	**15.8**	0.000	2.766	**15.8**	0.000	3.688	**15.8**	0.000	3.526
	$U[5.0C', 10.0C']$	**15.8**	0.000	3.241	**15.8**	0.000	2.838	**15.8**	0.000	3.762	**15.8**	0.000	3.697
	$U[10.0C', 20.0C']$	**15.8**	0.000	3.457	**15.8**	0.000	3.065	**15.8**	0.000	3.903	**15.8**	0.000	3.831

Table 6.28: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.5 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	8.2	0.055	0.216	8.2	0.050	0.100	8.3	0.044	0.604	**8.1**	0.050	0.459
	$U[0.5C', 1.0C']$	8.1	0.011	0.226	8.1	0.038	0.099	**8.0**	0.033	0.662	8.1	0.033	0.464
	$U[1.0C', 4.0C']$	8.0	0.027	0.223	8.1	0.016	0.102	8.1	0.033	0.652	**8.0**	0.022	0.473
	$U[0.0C', 5.0C']$	8.1	0.027	0.225	8.0	0.022	0.101	8.0	0.022	0.662	**8.0**	0.033	0.480
	$U[5.0C', 10.0C']$	8.1	0.033	0.221	8.1	0.050	0.102	8.1	0.027	0.646	**8.1**	0.033	0.467
	$U[10.0C', 20.0C']$	8.2	0.033	0.219	**8.1**	0.038	0.100	8.2	0.038	0.633	8.2	0.033	0.453
10	$U[0.0C', 0.5C']$	8.2	0.033	0.354	8.2	0.061	0.238	**8.1**	0.072	0.780	8.1	0.061	0.592
	$U[0.5C', 1.0C']$	8.0	0.022	0.350	8.0	0.050	0.241	8.0	0.033	0.786	**8.0**	0.033	0.608
	$U[1.0C', 4.0C']$	7.8	0.011	0.376	**7.8**	0.000	0.250	7.9	0.016	0.835	7.9	0.016	0.632
	$U[0.0C', 5.0C']$	7.9	0.022	0.378	**7.9**	0.005	0.240	**7.9**	0.016	0.820	7.9	0.011	0.631
	$U[5.0C', 10.0C']$	7.9	0.016	0.378	7.9	0.005	0.252	**7.9**	0.022	0.830	7.9	0.011	0.632
	$U[10.0C', 20.0C']$	8.0	0.027	0.376	8.0	0.016	0.255	**7.9**	0.027	0.825	8.0	0.033	0.627
20	$U[0.0C', 0.5C']$	**8.1**	0.050	0.508	8.2	0.050	0.412	8.2	0.066	0.933	8.2	0.027	0.711
	$U[0.5C', 1.0C']$	8.0	0.022	0.506	8.0	0.038	0.370	**8.0**	0.038	0.936	8.0	0.038	0.734
	$U[1.0C', 4.0C']$	7.8	0.005	0.520	7.8	0.000	0.392	7.8	0.000	0.971	**7.8**	0.005	0.782
	$U[0.0C', 5.0C']$	7.8	0.005	0.517	**7.8**	0.000	0.395	**7.8**	0.000	0.973	7.8	0.011	0.786
	$U[5.0C', 10.0C']$	**7.9**	0.005	0.533	7.9	0.011	0.408	7.9	0.011	0.976	7.9	0.000	0.778
	$U[10.0C', 20.0C']$	**7.9**	0.016	0.534	7.9	0.033	0.430	**7.9**	0.016	0.985	7.9	0.016	0.778
50	$U[0.0C', 0.5C']$	**8.1**	0.038	0.973	8.2	0.094	0.825	8.2	0.033	1.393	8.1	0.072	1.208
	$U[0.5C', 1.0C']$	8.0	0.033	0.890	8.0	0.022	0.802	8.0	0.027	1.351	**8.0**	0.027	1.181
	$U[1.0C', 4.0C']$	**7.8**	0.011	0.922	7.8	0.005	0.798	7.8	0.005	1.384	7.8	0.005	1.185
	$U[0.0C', 5.0C']$	**7.8**	0.005	0.919	7.8	0.000	0.805	7.8	0.000	1.386	7.8	0.005	1.181
	$U[5.0C', 10.0C']$	**7.8**	0.005	0.958	7.8	0.000	0.848	7.8	0.000	1.440	**7.8**	0.005	1.242
	$U[10.0C', 20.0C']$	7.9	0.016	1.022	**7.8**	0.011	0.903	7.9	0.005	1.482	7.9	0.016	1.291

Table 6.29: ILP variants compared on a set of 5 instances with 100 nodes and a density of 0.5 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	**7.8**	0.000	0.221	8.0	0.000	0.085	**7.8**	0.000	0.756	**7.8**	0.000	0.527
	$U[0.5C', 1.0C']$	**7.7**	0.050	0.216	7.8	0.100	0.105	7.8	0.000	0.744	7.8	0.000	0.531
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.232	7.9	0.050	0.088	**7.8**	0.000	0.744	**7.8**	0.000	0.529
	$U[0.0C', 5.0C']$	7.8	0.100	0.223	7.8	0.000	0.087	7.8	0.100	0.735	**7.7**	0.050	0.577
	$U[5.0C', 10.0C']$	**7.7**	0.050	0.237	**7.7**	0.050	0.096	7.9	0.050	0.699	7.9	0.050	0.527
	$U[10.0C', 20.0C']$	8.0	0.000	0.216	7.9	0.050	0.087	**7.8**	0.100	0.733	7.9	0.050	0.490
10	$U[0.0C', 0.5C']$	**7.6**	0.100	0.321	**7.6**	0.100	0.194	**7.6**	0.100	0.877	7.8	0.000	0.615
	$U[0.5C', 1.0C']$	7.6	0.000	0.324	**7.5**	0.150	0.181	7.7	0.150	0.858	**7.5**	0.150	0.700
	$U[1.0C', 4.0C']$	7.1	0.050	0.359	**7.0**	0.000	0.214	**7.0**	0.000	1.034	7.1	0.050	0.789
	$U[0.0C', 5.0C']$	7.2	0.100	0.376	**7.0**	0.000	0.206	**7.0**	0.000	1.050	7.2	0.100	0.747
	$U[5.0C', 10.0C']$	7.5	0.050	0.349	7.6	0.100	0.195	**7.4**	0.200	0.940	**7.4**	0.000	0.700
	$U[10.0C', 20.0C']$	7.8	0.000	0.320	7.8	0.000	0.181	7.8	0.000	0.831	**7.7**	0.050	0.631
20	$U[0.0C', 0.5C']$	**7.6**	0.200	0.443	**7.6**	0.100	0.270	7.7	0.050	0.952	**7.6**	0.100	0.780
	$U[0.5C', 1.0C']$	7.6	0.100	0.414	**7.5**	0.150	0.299	7.8	0.000	0.923	7.7	0.050	0.727
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.483	**7.0**	0.000	0.331	7.1	0.050	1.128	**7.0**	0.000	0.861
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.492	**7.0**	0.000	0.342	**7.0**	0.000	1.138	**7.0**	0.000	0.885
	$U[5.0C', 10.0C']$	7.5	0.150	0.438	**7.1**	0.050	0.326	7.3	0.050	1.071	**7.1**	0.050	0.870
	$U[10.0C', 20.0C']$	**7.7**	0.050	0.429	**7.7**	0.050	0.311	**7.7**	0.050	0.949	**7.7**	0.050	0.760
50	$U[0.0C', 0.5C']$	7.6	0.000	0.716	**7.5**	0.150	0.572	**7.5**	0.250	1.300	7.6	0.100	1.027
	$U[0.5C', 1.0C']$	**7.4**	0.100	0.758	7.5	0.050	0.642	7.6	0.100	1.269	7.6	0.100	1.043
	$U[1.0C', 4.0C']$	**7.0**	0.000	0.803	**7.0**	0.000	0.627	**7.0**	0.000	1.402	**7.0**	0.000	1.206
	$U[0.0C', 5.0C']$	**7.0**	0.000	0.774	**7.0**	0.000	0.672	**7.0**	0.000	1.418	**7.0**	0.000	1.188
	$U[5.0C', 10.0C']$	7.1	0.050	0.829	7.1	0.050	0.723	**7.0**	0.000	1.500	7.1	0.050	1.246
	$U[10.0C', 20.0C']$	**7.3**	0.150	0.816	**7.3**	0.150	0.756	7.5	0.050	1.399	7.5	0.050	1.169

Table 6.30: ILP variants compared on a set of 5 instances with 120 nodes and a density of 0.5 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$	$\overline{obj}$	sd	$\overline{time}(s)$
1	$U[0.0C', 0.5C']$	8.8	0.100	0.355	8.9	0.050	0.126	8.9	0.050	1.222	**8.7**	0.050	0.938
	$U[0.5C', 1.0C']$	**8.8**	0.000	0.360	**8.8**	0.000	0.150	8.9	0.050	1.192	**8.8**	0.100	0.923
	$U[1.0C', 4.0C']$	**8.7**	0.050	0.366	8.9	0.050	0.132	8.9	0.050	1.149	8.9	0.050	0.875
	$U[0.0C', 5.0C']$	**8.8**	0.000	0.356	8.9	0.050	0.126	9.0	0.000	1.183	**8.8**	0.100	0.934
	$U[5.0C', 10.0C']$	9.0	0.000	0.335	8.9	0.050	0.133	**8.6**	0.000	1.330	8.9	0.050	0.892
	$U[10.0C', 20.0C']$	**9.0**	0.000	0.326	**9.0**	0.000	0.121	**9.0**	0.000	1.172	**9.0**	0.000	0.831
10	$U[0.0C', 0.5C']$	**8.7**	0.050	0.511	**8.7**	0.050	0.317	8.8	0.100	1.396	8.8	0.000	1.065
	$U[0.5C', 1.0C']$	**8.6**	0.000	0.565	8.7	0.050	0.326	**8.6**	0.000	1.497	8.7	0.050	1.140
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.541	**8.6**	0.000	0.326	**8.6**	0.000	1.447	**8.6**	0.000	1.129
	$U[0.0C', 5.0C']$	8.6	0.000	0.541	8.6	0.000	0.319	**8.5**	0.050	1.486	8.6	0.000	1.136
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.566	**8.6**	0.000	0.341	**8.6**	0.000	1.451	8.8	0.100	1.089
	$U[10.0C', 20.0C']$	8.9	0.050	0.538	**8.8**	0.000	0.355	8.9	0.050	1.361	8.9	0.050	1.111
20	$U[0.0C', 0.5C']$	8.7	0.050	0.716	9.0	0.000	0.468	**8.6**	0.000	1.696	**8.6**	0.000	1.340
	$U[0.5C', 1.0C']$	8.6	0.000	0.726	8.6	0.000	0.576	**8.5**	0.050	1.670	8.6	0.000	1.325
	$U[1.0C', 4.0C']$	8.6	0.000	0.714	**8.5**	0.050	0.532	**8.5**	0.050	1.697	8.6	0.000	1.367
	$U[0.0C', 5.0C']$	8.6	0.000	0.728	8.6	0.000	0.513	**8.5**	0.050	1.699	8.6	0.000	1.356
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.726	**8.6**	0.000	0.543	**8.6**	0.000	1.659	**8.6**	0.000	1.395
	$U[10.0C', 20.0C']$	**8.6**	0.000	0.787	8.9	0.050	0.533	8.8	0.000	1.675	8.9	0.050	1.262
50	$U[0.0C', 0.5C']$	9.0	0.000	1.156	**8.7**	0.050	1.065	**8.7**	0.050	2.217	8.8	0.100	1.829
	$U[0.5C', 1.0C']$	8.8	0.000	1.213	8.7	0.050	1.081	8.8	0.000	2.165	**8.6**	0.000	1.958
	$U[1.0C', 4.0C']$	**8.3**	0.050	1.405	8.5	0.050	1.111	8.5	0.050	2.289	8.6	0.000	1.872
	$U[0.0C', 5.0C']$	8.6	0.000	1.282	**8.5**	0.050	1.154	8.6	0.000	2.169	8.6	0.000	1.928
	$U[5.0C', 10.0C']$	**8.6**	0.000	1.322	**8.6**	0.000	1.109	**8.6**	0.000	2.190	**8.6**	0.000	1.952
	$U[10.0C', 20.0C']$	**8.6**	0.000	1.480	8.7	0.050	1.179	**8.6**	0.000	2.350	8.7	0.050	1.947

Table 6.31: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.7 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	10.3	0.050	0.222	**10.4**	0.000	0.479	10.3	0.150	1.081
	$U[0.5C', 1.0C']$	10.1	0.050	0.141	10.1	0.050	0.432	**10.0**	0.000	1.102
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.113	**10.0**	0.000	0.410	**10.0**	0.000	1.042
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.114	**10.0**	0.000	0.414	**10.0**	0.000	1.056
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.122	**10.0**	0.000	0.435	**10.0**	0.000	1.072
	$U[10.0C', 20.0C']$	10.2	0.100	0.142	10.1	0.050	0.425	10.2	0.100	1.020
0.3	$U[0.0C', 0.5C']$	10.4	0.000	0.126	10.7	0.150	0.359	10.5	0.150	0.940
	$U[0.5C', 1.0C']$	**10.0**	0.000	0.122	10.2	0.000	0.410	10.2	0.100	1.032
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.110	**10.0**	0.000	0.434	**10.0**	0.000	1.032
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.121	**10.0**	0.000	0.411	**10.0**	0.000	1.046
	$U[5.0C', 10.0C']$	10.1	0.050	0.124	**10.0**	0.000	0.440	**10.0**	0.000	1.053
	$U[10.0C', 20.0C']$	10.2	0.100	0.133	**10.0**	0.000	0.446	**10.0**	0.000	1.070
0.5	$U[0.0C', 0.5C']$	10.6	0.100	0.112	**10.4**	0.200	0.383	10.2	0.100	1.024
	$U[0.5C', 1.0C']$	10.2	0.000	0.119	**10.0**	0.000	0.419	10.3	0.050	0.983
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.114	**10.0**	0.000	0.404	**10.0**	0.000	1.020
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.131	**10.0**	0.000	0.414	**10.0**	0.000	1.057
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.118	**10.0**	0.000	0.430	**10.0**	0.000	1.058
	$U[10.0C', 20.0C']$	10.2	0.100	0.134	10.1	0.050	0.416	10.2	0.000	1.009
1.0	$U[0.0C', 0.5C']$	10.4	0.000	0.117	10.7	0.050	0.348	10.5	0.150	0.972
	$U[0.5C', 1.0C']$	10.2	0.000	0.114	10.1	0.050	0.418	**10.0**	0.000	1.081
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.113	**10.0**	0.000	0.413	**10.0**	0.000	1.050
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.112	**10.0**	0.000	0.400	**10.0**	0.000	1.058
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.122	**10.0**	0.000	0.415	**10.0**	0.000	1.077
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.161	10.1	0.050	0.421	10.1	0.050	1.117
2.0	$U[0.0C', 0.5C']$	10.6	0.000	0.117	10.6	0.100	0.367	**10.1**	0.050	1.088
	$U[0.5C', 1.0C']$	10.1	0.050	0.132	10.1	0.050	0.407	10.3	0.050	0.997
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.109	**10.0**	0.000	0.414	**10.0**	0.000	1.061
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.116	**10.0**	0.000	0.415	**10.0**	0.000	1.132
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.123	**10.0**	0.000	0.425	**10.0**	0.000	1.095
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.140	10.2	0.100	0.394	10.2	0.100	1.013
5.0	$U[0.0C', 0.5C']$	10.5	0.050	0.120	**10.4**	0.100	0.397	10.5	0.150	0.934
	$U[0.5C', 1.0C']$	**10.0**	0.000	0.112	10.1	0.050	0.414	10.2	0.100	1.031
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.110	**10.0**	0.000	0.411	**10.0**	0.000	1.043
	$U[0.0C', 5.0C']$	10.1	0.050	0.111	**10.0**	0.000	0.421	**10.0**	0.000	1.040
	$U[5.0C', 10.0C']$	**10.0**	0.000	0.127	**10.0**	0.000	0.420	**10.0**	0.000	1.044
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.137	10.1	0.050	0.431	**10.0**	0.000	1.088
10.0	$U[0.0C', 0.5C']$	**10.2**	0.000	0.146	10.5	0.150	0.395	10.6	0.100	0.905
	$U[0.5C', 1.0C']$	10.1	0.050	0.128	10.1	0.050	0.439	10.2	0.100	1.027
	$U[1.0C', 4.0C']$	**10.0**	0.000	0.114	**10.0**	0.000	0.416	**10.0**	0.000	1.060
	$U[0.0C', 5.0C']$	**10.0**	0.000	0.111	**10.0**	0.000	0.415	**10.0**	0.000	1.045
	$U[5.0C', 10.0C']$	10.1	0.050	0.118	**10.0**	0.000	0.426	**10.0**	0.000	1.057
	$U[10.0C', 20.0C']$	**10.0**	0.000	0.145	10.2	0.100	0.414	10.1	0.050	1.053

Table 6.32: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.8 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	12.8	0.000	0.220	12.9	0.150	0.576	12.9	0.050	1.212
	$U[0.5C', 1.0C']$	**12.3**	0.050	0.207	**12.4**	0.100	0.646	12.6	0.000	1.328
	$U[1.0C', 4.0C']$	12.3	0.050	0.195	**12.2**	0.100	0.623	12.4	0.000	1.362
	$U[0.0C', 5.0C']$	**12.1**	0.050	0.224	12.3	0.050	0.616	12.3	0.050	1.386
	$U[5.0C', 10.0C']$	12.5	0.050	0.199	12.5	0.050	0.601	12.4	0.100	1.413
	$U[10.0C', 20.0C']$	12.8	0.000	0.182	12.6	0.100	0.610	12.7	0.050	1.284
0.3	$U[0.0C', 0.5C']$	12.8	0.000	0.201	**12.8**	0.000	0.563	13.0	0.100	1.201
	$U[0.5C', 1.0C']$	12.7	0.050	0.184	12.6	0.000	0.588	12.6	0.100	1.319
	$U[1.0C', 4.0C']$	12.3	0.050	0.192	12.4	0.000	0.597	12.2	0.000	1.413
	$U[0.0C', 5.0C']$	12.2	0.100	0.195	**12.2**	0.100	0.641	**12.2**	0.000	1.415
	$U[5.0C', 10.0C']$	**12.3**	0.050	0.201	12.6	0.100	0.601	**12.2**	0.000	1.417
	$U[10.0C', 20.0C']$	**12.6**	0.100	0.194	**12.4**	0.100	0.651	**12.5**	0.050	1.332
0.5	$U[0.0C', 0.5C']$	13.0	0.100	0.189	13.0	0.100	0.559	12.9	0.050	1.225
	$U[0.5C', 1.0C']$	12.5	0.150	0.200	12.5	0.050	0.609	**12.5**	0.050	1.329
	$U[1.0C', 4.0C']$	12.2	0.000	0.195	**12.2**	0.000	0.639	12.3	0.050	1.437
	$U[0.0C', 5.0C']$	12.3	0.050	0.202	**12.2**	0.100	0.632	12.3	0.050	1.396
	$U[5.0C', 10.0C']$	12.6	0.100	0.195	12.4	0.100	0.649	12.4	0.100	1.372
	$U[10.0C', 20.0C']$	12.8	0.000	0.181	12.7	0.050	0.602	12.7	0.050	1.273
1.0	$U[0.0C', 0.5C']$	13.0	0.100	0.187	12.9	0.050	0.557	12.8	0.000	1.247
	$U[0.5C', 1.0C']$	12.8	0.000	0.164	12.6	0.000	0.604	**12.5**	0.050	1.375
	$U[1.0C', 4.0C']$	12.3	0.050	0.185	**12.2**	0.100	0.628	12.4	0.000	1.368
	$U[0.0C', 5.0C']$	12.2	0.000	0.196	**12.2**	0.000	0.646	12.4	0.000	1.353
	$U[5.0C', 10.0C']$	12.5	0.050	0.210	12.4	0.100	0.628	12.6	0.100	1.299
	$U[10.0C', 20.0C']$	12.7	0.050	0.186	12.7	0.050	0.594	12.6	0.100	1.307
2.0	$U[0.0C', 0.5C']$	**12.7**	0.150	0.194	**12.8**	0.000	0.585	13.0	0.300	1.220
	$U[0.5C', 1.0C']$	12.8	0.000	0.159	12.5	0.050	0.602	12.6	0.000	1.306
	$U[1.0C', 4.0C']$	12.3	0.050	0.179	**12.2**	0.100	0.662	12.4	0.000	1.364
	$U[0.0C', 5.0C']$	**12.1**	0.050	0.198	**12.2**	0.100	0.645	12.4	0.000	1.375
	$U[5.0C', 10.0C']$	12.4	0.100	0.200	12.3	0.150	0.654	12.5	0.050	1.343
	$U[10.0C', 20.0C']$	**12.6**	0.100	0.187	12.6	0.100	0.597	12.7	0.050	1.267
5.0	$U[0.0C', 0.5C']$	13.0	0.000	0.171	**12.8**	0.100	0.582	**12.7**	0.050	1.278
	$U[0.5C', 1.0C']$	12.4	0.000	0.187	12.8	0.000	0.564	12.6	0.100	1.288
	$U[1.0C', 4.0C']$	**12.1**	0.050	0.210	12.4	0.100	0.617	12.4	0.000	1.351
	$U[0.0C', 5.0C']$	12.3	0.050	0.196	12.3	0.050	0.637	12.4	0.000	1.359
	$U[5.0C', 10.0C']$	12.4	0.000	0.194	**12.2**	0.000	0.665	**12.2**	0.000	1.446
	$U[10.0C', 20.0C']$	12.7	0.050	0.195	12.7	0.050	0.614	12.7	0.050	1.262
10.0	$U[0.0C', 0.5C']$	12.8	0.100	0.189	13.1	0.050	0.522	12.9	0.050	1.231
	$U[0.5C', 1.0C']$	12.4	0.100	0.188	12.6	0.100	0.567	12.7	0.050	1.243
	$U[1.0C', 4.0C']$	**12.1**	0.050	0.195	**12.2**	0.000	0.610	**12.1**	0.050	1.445
	$U[0.0C', 5.0C']$	12.3	0.050	0.204	12.3	0.050	0.574	12.3	0.050	1.403
	$U[5.0C', 10.0C']$	12.4	0.100	0.196	12.5	0.050	0.591	12.3	0.050	1.410
	$U[10.0C', 20.0C']$	12.7	0.050	0.182	12.7	0.050	0.575	12.6	0.100	1.355

Table 6.33: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.9 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	16.9	0.250	0.371	16.7	0.050	0.721	17.2	0.300	1.234
	$U[0.5C', 1.0C']$	16.3	0.050	0.309	**16.1**	0.050	0.731	16.3	0.050	1.365
	$U[1.0C', 4.0C']$	16.0	0.000	0.297	**15.8**	0.000	0.774	15.9	0.050	1.518
	$U[0.0C', 5.0C']$	16.0	0.100	0.294	15.9	0.050	0.737	16.0	0.100	1.482
	$U[5.0C', 10.0C']$	16.0	0.000	0.306	16.1	0.050	0.743	**15.9**	0.050	1.515
	$U[10.0C', 20.0C']$	**16.0**	0.100	0.336	16.0	0.000	0.781	16.1	0.050	1.424
0.3	$U[0.0C', 0.5C']$	**16.6**	0.100	0.365	16.8	0.100	0.700	17.2	0.100	1.226
	$U[0.5C', 1.0C']$	16.5	0.050	0.296	16.3	0.050	0.683	16.2	0.000	1.387
	$U[1.0C', 4.0C']$	**15.8**	0.000	0.299	15.9	0.050	0.773	15.9	0.050	1.515
	$U[0.0C', 5.0C']$	**15.8**	0.000	0.337	15.9	0.050	0.757	16.0	0.100	1.463
	$U[5.0C', 10.0C']$	16.0	0.100	0.337	15.9	0.050	0.814	16.0	0.000	1.449
	$U[10.0C', 20.0C']$	16.1	0.050	0.330	16.1	0.050	0.836	**16.0**	0.100	1.477
0.5	$U[0.0C', 0.5C']$	16.9	0.250	0.345	**16.5**	0.150	0.782	17.3	0.150	1.166
	$U[0.5C', 1.0C']$	16.2	0.100	0.310	**16.1**	0.050	0.721	16.2	0.100	1.402
	$U[1.0C', 4.0C']$	16.1	0.050	0.313	15.9	0.050	0.774	**15.8**	0.000	1.532
	$U[0.0C', 5.0C']$	16.1	0.150	0.300	**15.8**	0.000	0.788	**15.8**	0.000	1.492
	$U[5.0C', 10.0C']$	16.0	0.000	0.311	16.1	0.050	0.733	**15.9**	0.050	1.493
	$U[10.0C', 20.0C']$	16.2	0.000	0.332	16.1	0.050	0.784	16.1	0.050	1.450
1.0	$U[0.0C', 0.5C']$	16.9	0.150	0.319	16.7	0.250	0.778	17.2	0.100	1.190
	$U[0.5C', 1.0C']$	16.3	0.050	0.293	**16.1**	0.050	0.679	**16.0**	0.100	1.482
	$U[1.0C', 4.0C']$	15.9	0.050	0.339	16.0	0.100	0.756	16.0	0.000	1.422
	$U[0.0C', 5.0C']$	16.0	0.100	0.302	15.9	0.050	0.809	**15.8**	0.000	1.533
	$U[5.0C', 10.0C']$	**15.9**	0.050	0.336	15.9	0.050	0.776	**15.9**	0.050	1.481
	$U[10.0C', 20.0C']$	**16.0**	0.000	0.344	**15.9**	0.050	0.808	16.1	0.050	1.470
2.0	$U[0.0C', 0.5C']$	16.9	0.150	0.327	17.0	0.100	0.659	**16.7**	0.050	1.357
	$U[0.5C', 1.0C']$	16.3	0.050	0.310	16.3	0.050	0.696	16.4	0.100	1.359
	$U[1.0C', 4.0C']$	15.9	0.050	0.319	16.0	0.100	0.756	15.9	0.050	1.486
	$U[0.0C', 5.0C']$	15.9	0.050	0.323	15.9	0.050	0.772	15.9	0.050	1.485
	$U[5.0C', 10.0C']$	16.0	0.000	0.303	**15.8**	0.000	0.840	16.0	0.000	1.443
	$U[10.0C', 20.0C']$	**16.0**	0.000	0.329	16.1	0.050	0.752	**16.0**	0.100	1.519
5.0	$U[0.0C', 0.5C']$	16.9	0.150	0.299	16.6	0.100	0.725	17.2	0.100	1.157
	$U[0.5C', 1.0C']$	**16.1**	0.050	0.295	16.2	0.000	0.676	16.3	0.050	1.311
	$U[1.0C', 4.0C']$	15.9	0.050	0.286	**15.8**	0.000	0.790	16.0	0.100	1.402
	$U[0.0C', 5.0C']$	**15.8**	0.000	0.292	15.9	0.050	0.774	16.0	0.000	1.408
	$U[5.0C', 10.0C']$	16.0	0.100	0.333	15.9	0.050	0.797	16.0	0.000	1.424
	$U[10.0C', 20.0C']$	16.1	0.050	0.311	16.0	0.000	0.775	**16.0**	0.000	1.444
10.0	$U[0.0C', 0.5C']$	16.8	0.100	0.314	**16.5**	0.250	0.790	17.1	0.150	1.161
	$U[0.5C', 1.0C']$	16.2	0.000	0.289	16.3	0.050	0.719	16.5	0.050	1.269
	$U[1.0C', 4.0C']$	16.0	0.100	0.279	15.9	0.050	0.779	**15.8**	0.000	1.469
	$U[0.0C', 5.0C']$	**15.8**	0.000	0.301	15.9	0.050	0.803	**15.8**	0.000	1.523
	$U[5.0C', 10.0C']$	**15.9**	0.050	0.310	15.9	0.050	0.792	**15.9**	0.050	1.473
	$U[10.0C', 20.0C']$	16.1	0.050	0.319	**15.9**	0.050	0.820	**16.0**	0.100	1.450

Table 6.34: ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	**8.2**	0.027	0.095	8.3	0.055	0.247	**8.2**	0.027	0.678
	$U[0.5C', 1.0C']$	8.2	0.055	0.080	8.1	0.083	0.269	**8.1**	0.027	0.717
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.098	**7.8**	0.000	0.317	7.9	0.027	0.785
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	**7.8**	0.000	0.318	8.0	0.055	0.750
	$U[5.0C', 10.0C']$	8.0	0.000	0.092	8.0	0.055	0.300	7.9	0.027	0.771
	$U[10.0C', 20.0C']$	8.2	0.000	0.082	8.1	0.027	0.276	**8.1**	0.000	0.743
0.3	$U[0.0C', 0.5C']$	**8.2**	0.055	0.083	**8.2**	0.027	0.259	**8.2**	0.083	0.677
	$U[0.5C', 1.0C']$	8.2	0.083	0.078	8.1	0.000	0.274	8.2	0.000	0.679
	$U[1.0C', 4.0C']$	8.0	0.055	0.082	7.9	0.027	0.289	**7.8**	0.000	0.802
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	7.9	0.027	0.292	8.0	0.000	0.749
	$U[5.0C', 10.0C']$	8.0	0.027	0.090	8.0	0.027	0.286	8.0	0.083	0.762
	$U[10.0C', 20.0C']$	8.2	0.055	0.083	8.2	0.027	0.261	8.2	0.055	0.682
0.5	$U[0.0C', 0.5C']$	**8.2**	0.027	0.080	**8.2**	0.083	0.273	8.3	0.027	0.667
	$U[0.5C', 1.0C']$	**8.1**	0.055	0.083	**8.0**	0.027	0.280	**8.1**	0.027	0.700
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.081	**7.8**	0.000	0.312	7.9	0.027	0.771
	$U[0.0C', 5.0C']$	7.9	0.027	0.093	7.9	0.027	0.293	**7.9**	0.027	0.772
	$U[5.0C', 10.0C']$	8.0	0.055	0.094	8.0	0.055	0.289	8.1	0.055	0.717
	$U[10.0C', 20.0C']$	8.2	0.027	0.084	**8.0**	0.027	0.299	8.2	0.055	0.687
1.0	$U[0.0C', 0.5C']$	**8.2**	0.027	0.081	8.3	0.000	0.261	**8.2**	0.027	0.675
	$U[0.5C', 1.0C']$	8.2	0.000	0.076	8.2	0.055	0.269	**8.1**	0.000	0.727
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.093	7.9	0.027	0.297	8.0	0.027	0.731
	$U[0.0C', 5.0C']$	7.9	0.027	0.090	7.9	0.027	0.300	**7.9**	0.027	0.786
	$U[5.0C', 10.0C']$	**7.8**	0.000	0.097	**7.9**	0.027	0.310	8.0	0.000	0.738
	$U[10.0C', 20.0C']$	**8.1**	0.027	0.088	8.1	0.055	0.288	**8.1**	0.027	0.718
2.0	$U[0.0C', 0.5C']$	**8.2**	0.027	0.079	8.3	0.055	0.251	8.3	0.055	0.666
	$U[0.5C', 1.0C']$	**8.1**	0.027	0.078	**8.0**	0.027	0.272	8.2	0.027	0.669
	$U[1.0C', 4.0C']$	7.9	0.027	0.084	7.9	0.027	0.298	7.9	0.027	0.776
	$U[0.0C', 5.0C']$	**7.8**	0.000	0.089	7.9	0.027	0.294	**7.9**	0.027	0.767
	$U[5.0C', 10.0C']$	7.9	0.027	0.092	**7.9**	0.027	0.310	8.1	0.055	0.735
	$U[10.0C', 20.0C']$	**8.1**	0.027	0.097	**8.0**	0.000	0.317	**8.1**	0.000	0.740
5.0	$U[0.0C', 0.5C']$	**8.2**	0.055	0.083	8.3	0.055	0.247	**8.2**	0.027	0.685
	$U[0.5C', 1.0C']$	**8.1**	0.027	0.075	8.1	0.083	0.270	8.2	0.055	0.695
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.093	**7.8**	0.000	0.305	8.1	0.055	0.724
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	**7.8**	0.000	0.318	8.0	0.055	0.760
	$U[5.0C', 10.0C']$	8.0	0.055	0.093	8.1	0.055	0.289	**7.8**	0.000	0.785
	$U[10.0C', 20.0C']$	8.2	0.055	0.094	8.1	0.000	0.293	8.2	0.055	0.683
10.0	$U[0.0C', 0.5C']$	**8.2**	0.083	0.080	**8.2**	0.055	0.271	**8.2**	0.083	0.700
	$U[0.5C', 1.0C']$	**8.1**	0.027	0.079	8.1	0.055	0.268	**8.1**	0.055	0.726
	$U[1.0C', 4.0C']$	**7.8**	0.000	0.093	8.0	0.083	0.285	8.0	0.055	0.767
	$U[0.0C', 5.0C']$	7.9	0.027	0.104	**7.8**	0.000	0.302	8.0	0.055	0.770
	$U[5.0C', 10.0C']$	8.0	0.027	0.086	8.0	0.055	0.293	7.9	0.027	0.758
	$U[10.0C', 20.0C']$	8.2	0.055	0.083	8.1	0.027	0.282	**8.1**	0.027	0.723

Table 6.35: ILP variants compared on a set of 5 instances with 100 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	7.6	0.100	0.059	**7.5**	0.050	0.291	7.9	0.050	0.837
	$U[0.5C', 1.0C']$	7.6	0.100	0.062	7.7	0.050	0.273	7.7	0.150	0.880
	$U[1.0C', 4.0C']$	**7.1**	0.050	0.073	**7.1**	0.050	0.326	**7.2**	0.000	0.958
	$U[0.0C', 5.0C']$	**7.1**	0.050	0.070	**7.2**	0.100	0.310	7.3	0.150	0.955
	$U[5.0C', 10.0C']$	7.7	0.050	0.061	7.7	0.050	0.283	7.6	0.000	0.883
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.059	7.8	0.000	0.263	7.8	0.000	0.840
0.3	$U[0.0C', 0.5C']$	7.8	0.000	0.054	7.8	0.100	0.268	7.5	0.150	0.905
	$U[0.5C', 1.0C']$	**7.4**	0.100	0.065	7.6	0.100	0.289	7.8	0.000	0.836
	$U[1.0C', 4.0C']$	7.2	0.100	0.080	7.2	0.100	0.307	7.3	0.050	1.002
	$U[0.0C', 5.0C']$	7.4	0.100	0.064	7.4	0.100	0.298	7.4	0.100	0.945
	$U[5.0C', 10.0C']$	7.8	0.000	0.061	7.7	0.050	0.275	7.6	0.000	0.882
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.059	7.7	0.050	0.279	7.8	0.000	0.811
0.5	$U[0.0C', 0.5C']$	7.8	0.000	0.056	**7.5**	0.050	0.301	7.5	0.150	0.900
	$U[0.5C', 1.0C']$	7.6	0.100	0.058	**7.5**	0.150	0.291	7.6	0.100	0.854
	$U[1.0C', 4.0C']$	7.4	0.100	0.071	7.5	0.050	0.293	**7.2**	0.000	0.993
	$U[0.0C', 5.0C']$	7.2	0.100	0.067	**7.2**	0.000	0.317	7.2	0.100	0.975
	$U[5.0C', 10.0C']$	7.6	0.100	0.062	7.6	0.100	0.289	7.6	0.000	0.901
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.062	7.8	0.000	0.280	7.8	0.000	0.881
1.0	$U[0.0C', 0.5C']$	7.5	0.150	0.063	**7.5**	0.050	0.300	7.6	0.100	0.891
	$U[0.5C', 1.0C']$	7.6	0.100	0.060	**7.5**	0.150	0.290	7.6	0.100	0.880
	$U[1.0C', 4.0C']$	7.3	0.050	0.072	7.3	0.050	0.311	**7.2**	0.100	0.974
	$U[0.0C', 5.0C']$	7.3	0.050	0.069	7.3	0.050	0.309	7.3	0.050	0.952
	$U[5.0C', 10.0C']$	7.6	0.000	0.067	7.6	0.100	0.282	7.8	0.000	0.813
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.062	7.8	0.000	0.276	7.8	0.100	0.822
2.0	$U[0.0C', 0.5C']$	**7.4**	0.200	0.063	7.8	0.000	0.262	**7.3**	0.050	0.964
	$U[0.5C', 1.0C']$	7.6	0.100	0.061	**7.5**	0.150	0.295	7.7	0.050	0.900
	$U[1.0C', 4.0C']$	7.3	0.050	0.069	7.2	0.000	0.331	7.3	0.050	0.931
	$U[0.0C', 5.0C']$	**7.1**	0.050	0.073	7.4	0.100	0.295	7.1	0.050	1.017
	$U[5.0C', 10.0C']$	**7.5**	0.150	0.068	**7.3**	0.150	0.309	7.7	0.050	0.858
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.062	7.8	0.000	0.259	**7.7**	0.050	0.857
5.0	$U[0.0C', 0.5C']$	7.5	0.050	0.066	7.6	0.100	0.280	7.8	0.000	0.844
	$U[0.5C', 1.0C']$	7.6	0.100	0.060	**7.5**	0.150	0.287	**7.5**	0.150	0.931
	$U[1.0C', 4.0C']$	7.2	0.100	0.068	7.4	0.100	0.293	**7.2**	0.000	0.946
	$U[0.0C', 5.0C']$	7.6	0.000	0.062	7.4	0.100	0.302	7.2	0.100	0.959
	$U[5.0C', 10.0C']$	**7.5**	0.050	0.070	7.8	0.000	0.262	7.6	0.100	0.875
	$U[10.0C', 20.0C']$	**7.8**	0.000	0.063	7.8	0.000	0.275	**7.7**	0.050	0.863
10.0	$U[0.0C', 0.5C']$	7.7	0.050	0.056	7.7	0.050	0.280	7.8	0.000	0.838
	$U[0.5C', 1.0C']$	7.7	0.050	0.055	7.6	0.100	0.286	7.6	0.100	0.888
	$U[1.0C', 4.0C']$	7.5	0.050	0.063	7.3	0.150	0.306	7.3	0.050	0.963
	$U[0.0C', 5.0C']$	7.2	0.100	0.070	7.3	0.050	0.321	**7.0**	0.000	1.008
	$U[5.0C', 10.0C']$	7.8	0.000	0.059	7.7	0.050	0.277	**7.5**	0.050	0.899
	$U[10.0C', 20.0C']$	7.9	0.050	0.061	**7.6**	0.100	0.288	7.9	0.050	0.822

Table 6.36: ILP variants compared on a set of 5 instances with 120 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$	$\overline{obj}$	sd	$\overline{time}$
0.0	$U[0.0C', 0.5C']$	8.8	0.000	0.119	**8.6**	0.000	0.462	8.9	0.050	1.328
	$U[0.5C', 1.0C']$	8.7	0.050	0.119	8.8	0.100	0.435	8.7	0.050	1.392
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.115	**8.6**	0.000	0.453	**8.6**	0.000	1.438
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.112	**8.6**	0.000	0.450	**8.6**	0.000	1.387
	$U[5.0C', 10.0C']$	8.8	0.000	0.116	8.7	0.050	0.448	8.7	0.050	1.400
	$U[10.0C', 20.0C']$	**8.8**	0.000	0.131	9.0	0.000	0.410	9.0	0.000	1.306
0.3	$U[0.0C', 0.5C']$	8.7	0.050	0.113	8.8	0.100	0.432	**8.6**	0.000	1.440
	$U[0.5C', 1.0C']$	8.7	0.050	0.119	8.7	0.050	0.446	8.7	0.050	1.395
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.114	**8.6**	0.000	0.442	**8.6**	0.000	1.436
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.119	**8.6**	0.000	0.446	**8.6**	0.000	1.434
	$U[5.0C', 10.0C']$	8.7	0.050	0.127	8.7	0.050	0.452	**8.6**	0.000	1.438
	$U[10.0C', 20.0C']$	9.0	0.000	0.112	**8.9**	0.050	0.428	9.0	0.000	1.338
0.5	$U[0.0C', 0.5C']$	9.0	0.000	0.105	8.7	0.050	0.445	8.7	0.050	1.371
	$U[0.5C', 1.0C']$	8.7	0.050	0.117	**8.6**	0.000	0.442	8.7	0.050	1.407
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.118	**8.6**	0.000	0.440	**8.6**	0.000	1.472
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.117	**8.6**	0.000	0.441	**8.6**	0.000	1.470
	$U[5.0C', 10.0C']$	8.7	0.050	0.125	8.7	0.050	0.442	8.7	0.050	1.421
	$U[10.0C', 20.0C']$	9.0	0.000	0.115	9.0	0.000	0.413	**8.8**	0.000	1.365
1.0	$U[0.0C', 0.5C']$	8.8	0.000	0.113	**8.6**	0.000	0.466	8.8	0.000	1.356
	$U[0.5C', 1.0C']$	**8.5**	0.050	0.136	**8.6**	0.000	0.453	8.8	0.100	1.353
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.116	**8.6**	0.000	0.441	**8.6**	0.000	1.449
	$U[0.0C', 5.0C']$	8.7	0.050	0.114	**8.6**	0.000	0.455	**8.6**	0.000	1.465
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.126	8.8	0.100	0.436	**8.6**	0.000	1.417
	$U[10.0C', 20.0C']$	9.0	0.000	0.115	9.0	0.000	0.419	8.9	0.050	1.358
2.0	$U[0.0C', 0.5C']$	**8.6**	0.000	0.127	8.8	0.100	0.430	8.9	0.050	1.318
	$U[0.5C', 1.0C']$	8.6	0.000	0.120	8.9	0.050	0.408	**8.5**	0.050	1.443
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.117	**8.6**	0.000	0.451	**8.6**	0.000	1.425
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.118	**8.6**	0.000	0.445	**8.6**	0.000	1.455
	$U[5.0C', 10.0C']$	8.7	0.050	0.121	8.8	0.100	0.428	8.8	0.000	1.349
	$U[10.0C', 20.0C']$	9.0	0.000	0.116	9.0	0.000	0.412	8.9	0.050	1.339
5.0	$U[0.0C', 0.5C']$	8.7	0.050	0.117	8.8	0.100	0.437	8.7	0.050	1.421
	$U[0.5C', 1.0C']$	8.7	0.050	0.116	8.7	0.050	0.442	8.6	0.000	1.410
	$U[1.0C', 4.0C']$	**8.6**	0.100	0.119	**8.6**	0.000	0.455	**8.6**	0.000	1.444
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.120	**8.6**	0.000	0.452	**8.6**	0.000	1.429
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.125	**8.6**	0.000	0.464	8.8	0.100	1.394
	$U[10.0C', 20.0C']$	9.0	0.000	0.113	9.0	0.000	0.420	8.9	0.050	1.332
10.0	$U[0.0C', 0.5C']$	8.7	0.050	0.115	8.8	0.100	0.423	8.8	0.000	1.343
	$U[0.5C', 1.0C']$	8.7	0.050	0.117	8.7	0.050	0.429	8.6	0.000	1.410
	$U[1.0C', 4.0C']$	**8.6**	0.000	0.115	**8.6**	0.000	0.452	**8.6**	0.000	1.426
	$U[0.0C', 5.0C']$	**8.6**	0.000	0.115	**8.6**	0.000	0.455	**8.6**	0.000	1.430
	$U[5.0C', 10.0C']$	**8.6**	0.000	0.132	8.8	0.000	0.434	**8.6**	0.000	1.439
	$U[10.0C', 20.0C']$	9.0	0.000	0.114	9.0	0.000	0.419	9.0	0.000	1.284

Comparison to previous works

Since $RANDOM$ performs best in terms of runtime and is not significantly inferior to other methods in terms of quality, its results are compared to those of [14] and [35] in tables 6.37 and 6.38. $MA2$ denotes one of the memetic algorithm variants by [35], described in chapter 4. It can be seen that the strategy presented in this theses performs better in both runtime and solution quality. This leads to the assumption, that for solving the PCP local search based metaheuristics are superior to population based approaches.

In 6.39 results are compared to results presented in [30]. The table shows, that for instances of smaller size and for a smaller local search iteration limit the strategy of this theses is superior. The largest instance with 2000 nodes is solved better by the strategy of Noronha et.al.

Table 6.37: Instances of different node size evaluated with $RANDOM$ recoloring and compared to the results presented in [35].

Instance set		B & C		Random (10 runs/inst)			MA2		
nodes	density	LB	UB	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$
20	0.5	3	3	**3.00**	0.00	0.01	**3.00**	0.00	0.14
40	0.5	4	4	**4.00**	0.00	0.02	**4.00**	0.00	0.60
60	0.5	5	5	**5.00**	0.00	0.06	5.63	0.49	2.00
70	0.5	6	6	**6.00**	0.00	0.08	6.06	0.24	3.33
80	0.5	6	6	**6.27**	0.13	0.15	6.94	0.29	4.90
90	0.5	6	7	7.88	0.17	0.36	**7.55**	0.50	7.49
100	0.5	6	7	**7.12**	0.01	0.32	7.93	0.30	11.04
120	0.5	7	8	**8.64**	0.19	0.52	9.22	0.43	21.05

Table 6.38: Instances with 90 nodes and of different density evaluated with $RANDOM$ recoloring and compared to the results presented in [35].

Instance set		B & C		Random (10 runs/inst)			MA2		
nodes	density	LB	UB	$\overline{obj}$	sd	$\overline{time(s)}$	$\overline{obj}$	sd	$\overline{time(s)}$
90	0.1	2	3	**3.00**	0.00	0.02	3.09	0.29	1.37
90	0.2	3	4	**3.80**	0.15	0.03	4.41	0.49	3.24
90	0.3	4	5	**5.00**	0.00	0.06	5.52	0.56	4.90
90	0.4	5	6	**6.00**	0.00	0.11	6.79	0.83	6.54
90	0.5	6	7	**7.00**	0.00	0.18	7.55	0.50	7.49
90	0.6	8	8	**8.28**	0.15	0.31	10.50	0.87	11.95
90	0.7	10	10	**10.00**	0.00	0.45	12.39	1.12	14.83
90	0.8	12	12	**12.05**	0.14	0.80	15.18	0.80	20.98
90	0.9	16	16	**15.80**	0.15	1.23	17.27	0.98	45.75

Table 6.39: The four large instances evaluated with $RANDOM$ recoloring and compared to the results presented in [30].

Parameters		DSJC500.5-1		DSJC500.5-2		DSJC500.5-3		DSJC500.5-4	
ItMax	TabuTenure	Random	Noronha	Random	Noronha	Random	Noronha	Random	Noronha
1	$U[0.25C', 0.75C']$	**53.0**	53.5	**47.6**	47.8	45.0	**44.8**	**43.4**	43.5
	$U[0.0C', 1.0C']$	**53.0**	53.7	**47.0**	47.5	**45.0**	45.4	**43.2**	43.6
	$U[0.0C', 0.5C']$	**52.0**	53.1	**47.0**	47.3	45.0	**44.6**	44.0	**42.8**
	$U[0.5C', 1.0C']$	**52.6**	54.2	**47.2**	48.1	**45.0**	45.8	**43.0**	43.9
	$U[0.25C', 1.0C']$	**53.0**	53.8	**47.2**	47.9	**45.0**	45.5	**43.6**	**43.6**
	$U[0.0C', 0.75C']$	**52.4**	53.3	**47.6**	47.5	45.0	**44.8**	43.6	**43.0**
5	$U[0.25C', 0.75C']$	**52.0**	52.7	47.0	**46.8**	**44.0**	44.4	43.0	**42.8**
	$U[0.0C', 1.0C']$	**51.4**	52.9	47.0	**46.8**	**44.0**	44.7	**42.4**	**42.7**
	$U[0.0C', 0.5C']$	**51.4**	52.2	47.0	**46.1**	44.0	**43.7**	43.0	**42.0**
	$U[0.5C', 1.0C']$	**52.0**	53.3	**47.0**	47.7	**44.4**	44.9	**43.0**	**43.0**
	$U[0.25C', 1.0C']$	**51.4**	53.0	**47.0**	47.3	44.8	**44.7**	43.0	**42.9**
	$U[0.0C', 0.75C']$	**52.0**	52.5	47.0	**46.6**	44.6	**44.0**	43.0	**42.4**
10	$U[0.25C', 0.75C']$	**51.0**	52.5	47.0	**46.7**	**44.0**	**44.0**	43.0	**42.4**
	$U[0.0C', 1.0C']$	**51.0**	52.3	**46.4**	46.7	**44.0**	44.2	43.0	**42.7**
	$U[0.0C', 0.5C']$	**51.0**	51.3	47.0	**45.9**	44.0	**43.3**	43.0	**42.0**
	$U[0.5C', 1.0C']$	**51.0**	53.0	**47.0**	47.3	**44.6**	44.8	**42.8**	43.0
	$U[0.25C', 1.0C']$	**51.8**	**52.8**	**46.4**	46.9	**44.0**	44.2	43.0	**42.8**
	$U[0.0C', 0.75C']$	**51.0**	52.2	47.0	**46.2**	44.0	**43.9**	43.0	**42.2**

CHAPTER 7

Critical Reflection and Outlook

7.1 Critical Reflection

Selecting and optimizing the coloring of a subset of clusters regardless of their location in the graph does not tackle the problem in an efficient way. The selection does not take into account any features of the graph like regional density, although dense subgraphs involve the most danger of increasing the chromatic number by being colored with a suboptimal coloring. Considering graph features being crucial for a good selection of clusters, the selection presented in this thesis is done in a random way and therefore an optimal, partial recoloring can not be integrated in the solution by the tabu search more probably than any random coloring.

7.2 Future Works

Future works could consider a more suggestive selection of the clusters to be recolored. Rather than selecting all clusters of the same color, the set could be chosen by criteria of regional density. Putting effort in optimizing the coloring of these regions – e.g. by the use of exact methods – could lead to results of higher quality.

On finding dense subgraphs

Finding dense subgraphs is a intensively studied problem in graph theory and became more relevant in recent years because of its application to social network graphs. As long as there are no boundaries set on the size of the densest subgraph, it can be found in polynomial time, despite the fact that there are exponentially many subgraphs to consider [24, 44]. Additionaly, Charikar [9] showed a 2 approximation to the densest subgraph problem in linear time using a very simple greedy algorithm which was previously studied by Asahiro et. al. [42]). The densest k-subgraph problem (*DkS*), which finds the densest subgraph of size k is shown to be $\mathcal{NP}$-hard [38, 44]. For the densest at-most-k-subgraph problem (*DamkS*), which searches for the densest subgraph of maximum size k (and therefore is a relaxation of *DkS*), Andersen et.al. [2] showed that if there exists a α approximation for *DamkS*, then there exists a $\mathcal{O}(\alpha^2)$ approximisation for *DkS*, indicating that this problem is quite hard as well. Khuller and Saha showed that approximating *DamkS* is as hard as *DkS* within a constant factor [36], specifically an α approximation for *DamkS*, implies a 4α approximation for *DkS*. A number of polynomial time greedy heuristics for *DkS* are proposed in Asahiro et.al. [42].

Algorithm proposal

Algorithm 8 proposes a procedure for the discussed approach. The graph G, a recoloring algorithm $RECOLOR$ like these presented in 5.3 and two integers used to parameterize the search for dense subgraphs are taken as input. In line 1 an initial solution is calculated and its chromatic number is assigned to $cmax$ in line 2. In line 3 an algorithm is called that returns up to $maxSubgraphs$ subgraphs with a maximum size of $denseMaxSize$. Line 4 to line 6 recolor all found subgraphs by applying $RECOLOR$ and all remaining nodes colored with colors $cmax$ randomly, all with $cmax - 1$ colors. In line 8 the tabusearch tries to eliminate all resulting conflicts and puts the recolored regions on the tabulist for a number of iterations as presented in **??**. Line 9 to 11 accept the new solution in case of feasablity and starts searching for dense regions again.

Algorithm 8: PCP HYBRID DENSERECOLORING

Input: An uncolored Graph $G = (V, E)$, a recoloring-algorithm $RECOLOR$, two integers $maxSubgraphs$ and $denseMaxSize$

Output: A feasible Solution s

1 Set $S \leftarrow ONESTEPCD(G)$;
2 Set $cmax \leftarrow$ the chromatic number of s;
3 Set $D \leftarrow FINDDENSESUBGRAPHS(s, maxSubgraphs, denseMaxSize)$;
4 Let s' be the solution after recoloring all subgraphs in D with $RECOLOR$ and $cmax - 1$ colors;
5 Let R be the set of all remaining nodes in V colored with $cmax$;
6 Let s' be the solution after recoloring R randomly with $cmax - 1$ colors;
7 Let C' be the set of nodes involved into color conflicts in s';
8 $s' \leftarrow TABUSEARCH(s', D \cup R, C')$;
9 **if** S' *is free of conflicts* **then**
10     $s \leftarrow s'$;
11   **goto** line 2;
12 **return** s;

CHAPTER 8

Summary

The PCP is a quite recently proposed COP which generalizes the classical VCP by considering the possiblity to select subsets of nodes. While for the VCP much research has been done, only a few papers about the PCP has been published so far. In this work a strategy is presented that creates an initial solution by a heuristical algorithm and improves the solution quality by recoloring sets of nodes of same color before eliminating the resulting conflicts by applying a tabu search. It has been tried to enhance the algorithm presented in [30] by substituting the process of random recoloring by more sophisticated algorithms in order to minimize the number of resulting conflicts. Therefor a variation of the $ONESTEPCD$ algorithm [25] and two ILPs were used. A local search algorithm then tries to eliminate all these conflicting nodes to create a feasible solution. Furthermore experiments with variations of the ILPs and a mechanism that puts the most recently recolored subgraph on the tabulist for an amount of iteration in order to protect the coloring of that subgraph from being overwritten have been done.

The results have shown that more sophisticated recoloring algorithms can reduce the number of conflicts dramatically. For the instances used, a random recoloring produces an amount of conflicting nodes up to 7.5 times higher than an optimized recoloring does. The fact that this gap is not reflected significantly in the final results leads to the conclusion that for the presented strategy the tabu search is much more relevant than the recoloring process. Finally an alternative strategy, that is suspected by the author to be more suitable for sophisticated recoloring methods has been proposed.

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