

# A Hybrid Algorithm for the Partition Coloring Problem

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# A Hybrid Algorithm for the Partition Coloring Problem

MASTER'S THESIS

submitted in partial fulfillment of the requirements for the degree of

**Diplom-Ingenieurin**

in

**Computational Intelligence**

by

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# Abstract

Todo



# Kurzfassung

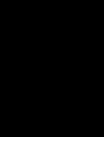
Todo



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# Introduction

## 1.1 Motivation

In order to obey the emerging demand for advanced broadband Internet applications such as video-conferences, high performance computing and others, extensive networks capacities have to be achieved. Links in optical networks operate much faster than their currently available electronic counterparts. Combined with the technique of Wavelength Division Multiplexing (WDM), which permits the simultaneous transmission of different channels along the same fiber [10], these so called Wavelength Routed Optical Networks (WRON's) are promising candidates for providing a flexible transport backbone network [7]. They bring out new problems in coordination of wavelengths usage [3]. One of them is the Routing and Wavelength Assignment Problem (RWA), which consists in routing a set of light-paths and assigning a wavelength to each of them. The variant where all connection requirements are known beforehand and which aims to minimize the amount of used wavelength is called min-RWA and found to be  $\mathcal{NP}$ -hard [13].

Assigning wavelengths to one out of many paths for each connection requirement is equivalent to the  $\mathcal{NP}$ -hard **Partition Coloring Problem (PCP)** [9] which is subject of this thesis. Given a graph consisting of a clustered set of vertices and a set of edges, the aim is to select one vertex per cluster and for each vertex in the resulting set assign a color in the way that the overall number of colors – which in this context is said to be the chromatic number – is minimized. If each cluster holds only one vertex, the problem reduces to the Standard Vertex Coloring Problem (VCP), which has been studied extensively and is used for a wide range of applications as scheduling, register allocation, pattern matching and others. In contrast, only a few papers have been published on PCP so far.

The aim of this thesis is to explore a solution method that has not been subject of publications on PCP yet, in order to achieve better approximation methods for the PCP.

## **1.2 Guide to the Thesis**

Necessary theoretical fundamentals like definitions, terms and methods are introduced in Chapter 2. Afterwards, Chapter 3 defines the PCP as well as the min-RWA formally and comments their computational complexity. Previous works and related research done so far is presented in Chapter 4. Chapter 5 provides details of the approach developed for the PCP and Chapter 6 presents its experimental results. Chapter 8 summarizes the knowledge achieved within this thesis, brings the considered approach into question and finally proposes a possible further work.



# CHAPTER 2

## Preliminaries

### 2.1 Graph Theory Definitions

### 2.2 Metaheuristics



## Problem Definition

### 3.1 Partition Coloring Problem

Let  $G = (V, E)$  be a non-directed graph and  $V$  partitioned into  $q$  subsets  $V_1, V_2, \dots, V_q$ , where  $V_i \cap V_j = \emptyset, \forall i, j = 1, \dots, q, i \neq j$ . We refer to  $V_1, V_2, \dots, V_q$  as the components of the partition. The PCP consists in finding a subset  $V' \subset V$  such that  $|V' \cap V_i| = 1, \forall i = 1, \dots, q$  (i.e.,  $V'$  contains one node of each component  $V_i$ ), and the chromatic number of the graph induced in  $G$  by  $V'$  is minimum.

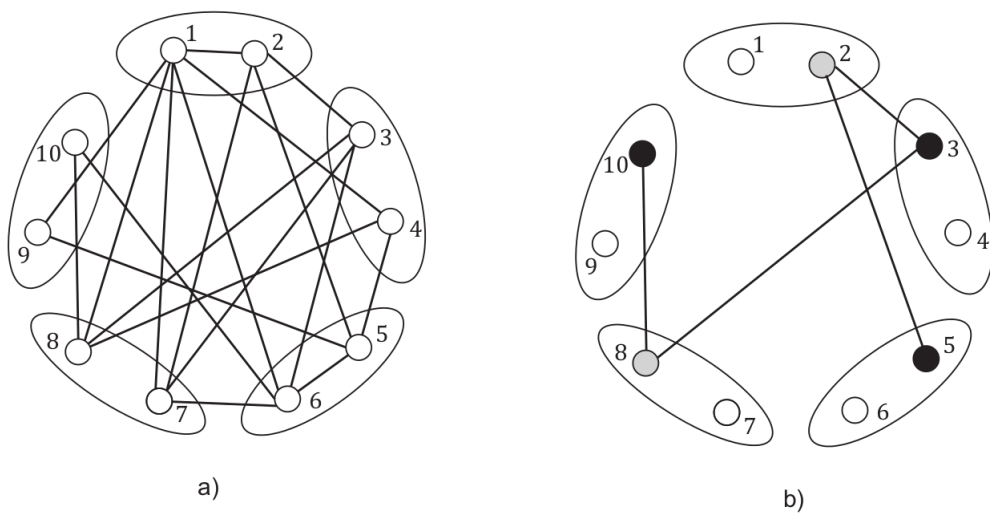
Figure 3.1 shows an example of an instance with 10 nodes and a density of about 0.2, where density is defined as the probability for each pair of nodes being connected by an edge.

### 3.2 Wavelength Routing and Assignment Problem

The PCP has initially been considered by Li and Simha in [9] and arises from considering the join problem of routing and wavelength assignment in WDM (Wavelength Division Multiplexing) optical networks.

### 3.3 Complexity

This problem is clearly a generalization of the graph colouring problem. Li and Simha [9] have shown that the decision version of PCP is NP-complete.



**Figure 3.1:** (a) Shows a problem instance and (b) its solution.

# CHAPTER 4

## Previous Works

### 4.1 Exact Approaches

### 4.2 Heuristical Approaches



# Problem Solving Approach

---

**Algorithm 1:** PCP Hybrid

---

**Input:** An uncolored Graph  $G = (V, E)$ , a recoloring-algorithm *RECOLOR*  
**Output:** A feasible Solution  $S$

- 1 Set  $S \leftarrow OneStepCD(G)$ ;
- 2 Set  $cmax \leftarrow$  the chromatic number of  $S$ ;
- 3 Set  $X \leftarrow \emptyset$ ;
- 4 **for**  $c = 1, \dots, cmax$  **do**
- 5     Let  $V_c$  be the set of nodes coloured by the colour  $c$ ;
- 6     Uncolor all nodes in  $V_c$ ;
- 7      $S_c \leftarrow RECOLOR(V_c, cmax - 1)$ ;
- 8     Let  $C$  be the set of all nodes involved in color conflicts of  $S_c$ ;
- 9      $X \leftarrow X \cup (S_c, V_c, C_c)$
- 10 Sort  $X$  ascendingly by  $|C_c|$ ;
- 11 Set  $reduction \leftarrow$  false;
- 12 **for**  $(S_c, V_c, C_c) \in X$  **do**
- 13      $S'_c \leftarrow TabuSearch(S_c, V_c, C_c)$ ;
- 14     **if**  $S'_c$  is free of conflicts **then**
- 15          $reduction \leftarrow$  true;
- 16         **break**;
- 17 **if**  $reduction$  **then**
- 18      $S \leftarrow S_c$ ;
- 19      $cmax = cmax - 1$ ;
- 20     **goto** line 3;
- 21 **return**  $S$ ;

---

## 5.1 Constructional Heuristics

---

**Algorithm 2:** OneStepCD

---

**Input:** An uncolored Graph  $G = (V, E)$

**Output:** A feasible Coloring  $V'$

```
1 Remove from G all edges  $(i, j) \in E : i, j \in V_k$  for some  $k = 1, \dots, q$ ;
2 Set  $V' \leftarrow \emptyset$ ;
3 while  $|V'| < q$  do
4   Set  $X \leftarrow \emptyset$ ;
5   for  $k = 1, \dots, q : V_k \cap V' = \emptyset$  do
6     Set  $X \leftarrow X \cup \operatorname{argmin}\{CD(i) : i \in V_k\}$ ;
7   Set  $x \leftarrow \operatorname{argmax}\{CD(i) : i \in X\}$ ;
8   Set  $V' \leftarrow V' \cup \{x\}$ ;
9   Assign the minimum possible colour to x;
10  Remove from G all nodes in  $V_{c(x)} \setminus \{x\}$ ;
11 return  $V'$ ;
```

---

## 5.2 Recoloring

**Random**

**OneStepCD**



---

**Algorithm 3:** OneStepCD Recoloring

---

**Input:** A partial Solution  $P$ , a number of maximum colours  $cmax$

**Output:** A feasible Solution  $S$

```
1 Let  $U$  be the set of uncolored nodes in  $P$ ;  
2 Set  $S \leftarrow \emptyset$ ;  
3 while  $|U| > 0$  do  
4   Set  $X \leftarrow \emptyset$ ;  
5   for  $u \in U$  do  
6      $X \leftarrow X \cup \operatorname{argmin}\{CD(i) : i \in V_{c(u)}\}$ ;  
7   Set  $x \leftarrow \operatorname{argmax}\{CD(i) : i \in X\}$ ;  
8   Set  $cmin \leftarrow$  the minimum possible colour that can be assigned to  $x$ ;  
9   if  $cmin \geq cmax$  then  
10     $cmin \leftarrow$  the color that produces the fewest conflicts.  
11  Assign  $cmin$  to  $x$ ;  
12   $S \leftarrow S \cup \{x\}$ ;  
13   $U \leftarrow U \setminus V_{c(x)}$ ;  
14 return  $V'$ ;
```

---

### ILP minimizing conflicts

Let  $Q = Q_1, \dots, Q_q$  be the set of Clusters. Every cluster  $Q_p$  consists of a set of nodes. Let  $C = \{1, \dots, cmax\}$  be the set of allowed colors. Let  $M$  be a 3-dimensional array of constants, storing for every cluster  $p \in Q$ , the number conflicts that would occur by selecting the pair  $(v \in Q_p, c \in C)$ .  $E$  denotes the set of edges and  $P[v]$  the cluster of node  $v$ .

$$\underset{X}{\text{minimize}} \quad \sum_{p \in Q} \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} * M_{pvc} \quad (1)$$

$$[h] \text{ subject to } \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} = 1, \quad \forall p \in Q \quad (2)$$

$$X_{pvc} + X_{quc} \leq 1, \quad \forall ((p, v), (q, u)) \in E, \forall c \in C \quad (3)$$

$$X_{pvc} \in \{0, 1\}, \quad \forall p \in Q, \forall v \in Q_p, \forall c \in C \quad (4)$$

### ILP minimizing conflicting nodes

Let  $U$  be the set of uncolored nodes in uncolored clusters and  $color[(p, v)]$  the color of the node  $v$  in partition  $p$ .

$$\underset{Z}{\text{minimize}} \quad \sum_{p \in Q} \sum_{v \in Q_p} \sum_{c \in C} Z_{pvc} \quad (1)$$

$$\text{subject to } Z_{pvc} \geq X_{quc}, \quad \forall ((p, v), (q, u)) \in E : (p, v) \notin U, (q, u) \in U, c = color[(p, v)] \quad (2)$$

$$[h] \quad \sum_{v \in Q_p} \sum_{c \in C} X_{pvc} = 1, \quad \forall p \in Q \quad (3)$$

$$X_{pvc} + X_{quc} \leq 1, \quad \forall ((p, v), (q, u)) \in E, \forall c \in C \quad (4)$$

$$X_{pvc} \in \{0, 1\}, \quad \forall p \in Q, \forall v \in Q_p, \forall c \in C \quad (5)$$

## 5.3 Tabu Search

## 5.4 Variants

---

**Algorithm 4:** TabuSearch

---

**Input:** An infeasible solution  $S$ , the set of previously recolored nodes  $R$ , the set of conflicting nodes  $C$

**Output:** A Solution  $\bar{S}$

```
1 Set  $C \leftarrow C \setminus R$ ;  
2 Set  $cmax \leftarrow$  the chromatic number of  $S$ ;  
3 Set  $iter \leftarrow 0$ ;  
4 Set  $minConflicts \leftarrow \infty$ ;  
5 Set  $\bar{S} \leftarrow S$ ;  
6 while  $|C| > 0$  and  $iter < maxiter$  do  
7   for  $V_{c(u)} : u \in C$  do  
8     for  $v \in V_{c(u)}$  and for  $c = 1, \dots, cmax$  do  
9       Obtain a tentative solution  $S'$  by selecting and coloring node  $v$  with color  $c$  in  
        $\bar{S}$ ;  
10      if  $conflicts(S') = 0$  then  
11         $\bar{S} \leftarrow S', \bar{v} \leftarrow v, \bar{c} \leftarrow c$ ;  
12        goto line 16;  
13      else if the pair  $v, c$  is not in the tabu list then  
14        if  $conflicts(S') < minConflicts$  then  
15           $minConflicts \leftarrow conflicts(S')$ ;  
16           $\bar{S} \leftarrow S', \bar{v} \leftarrow v, \bar{c} \leftarrow c$ ;  
17      insert pair  $\bar{v}, \bar{c}$  in the tabu list for TabuTenure iterations;  
18       $C \leftarrow C \setminus u$ ;  
19      Let  $C_{\bar{v}}$  be the set of nodes conflicting with  $\bar{v}$ ;  
20       $C \leftarrow C \cup C_{\bar{v}}$ ;  
21 return  $\bar{S}$ ;
```

---



# Computational Results

This chapter provides information about the implementation, testing environment, instances used for evaluation and the computational results for different methods and parameters presented in chapter 5 are compared to each other and to results of [6, 9, 10].

## 6.1 Implementation Details and Testing Environment

The program has been implemented in Java and compiled with the JDK compiler version 1.7.0\_25. For reasons of runtime comparability it has been designed to execute on a single thread, although the recoloring for each set of clusters of same color makes the program highly suitable to be processed in a parallel way. For the implementation of abstract data structures no other libraries than the ones provided by the JDK have been used. For solving the ILPs described in 5.4, ILOG CPLEX version 12.5 has been used, which is by now one of the fastest CP solvers available [2]. It is written in C++, provides facades to Java, Python, .NET, Matlab, Excel and supports comfortable usage of integer variables and a wide set of constraints and solving strategies. All tests have been performed on a Pentium i5 DualCore, 2.5 GHz, 8GB RAM, with Linux Mint 14 and OpenJDK Runtime Environment (IcedTea 2.3.9) installed.

## 6.2 Instances

Instances of different size, nodes per cluster ratio and density have been evaluated. Generated randomly and used by the authors of [6], the instances have also been evaluated in [11] and [5]. For reasons of better comparability to previous works, instances have been pooled to sets of same size respectively density. Furthermore four larger instances with constant density of 0.5 and different sizes of 500, 1000, 1500 and 2000 nodes provided by the authors of [10] have been evaluated and compared.

## 6.3 Results

In the following section preliminarily and final results as well as comparison to results of previous works are presented. There have been preceding tests to select the most competitive ranges of parameters used in the tables.

### Conflicting Nodes

As an intermediate result the numbers of conflicting nodes per each recoloring produced by the different recoloring algorithms have been recorded and compared to each other. Since for these experiments a constant length has been used for the tabu list, *HYBRID-PCP* is deterministic except the case when random recoloring is used. Therefore for random recoloring the average of ten runs per instance and recoloring has been calculated.

In tables 6.1 and 6.2 the results for sets of different size respectively density are presented. Each set contains five instances. Table 6.3 presents the results for the four larger instances. It can be seen that a large number of nodes and as well as a low density lead to a high amount of conflicts per recoloring. The difference between the results for *RANDOM* and *ILP2* grows to a factor of over 7.5 on the larger instances.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>
20	0.5	3.69	2.25	1.60	<b>1.36</b>
40	0.5	7.33	3.85	3.21	<b>2.29</b>
60	0.5	10.21	4.99	4.21	<b>2.83</b>
70	0.5	11.30	5.84	4.56	<b>3.27</b>
80	0.5	12.69	6.04	4.97	<b>3.41</b>
90	0.5	12.32	5.93	4.64	<b>3.38</b>
100	0.5	14.91	7.16	5.23	<b>3.92</b>
120	0.5	15.53	6.44	5.07	<b>3.38</b>

**Table 6.1:** Sets of different size containing five instances each. *cnodes/recoloring* denotes the average amount of conflicting nodes per recoloring.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>
90	0.1	15.71	9.50	6.61	<b>5.65</b>
90	0.2	16.70	7.99	6.36	<b>4.87</b>
90	0.3	15.94	7.60	5.48	<b>4.03</b>
90	0.4	14.73	6.16	4.75	<b>3.41</b>
90	0.5	13.51	5.93	4.94	<b>3.43</b>
90	0.6	11.78	5.20	4.39	<b>2.84</b>
90	0.7	9.60	4.61	3.90	<b>2.44</b>
90	0.8	7.70	3.66	3.04	<b>2.05</b>
90	0.9	5.56	2.69	2.34	<b>1.74</b>

**Table 6.2:** Sets of different density containing five instances each.

Instance set		Random (10 runs/inst)	OneStepCD	ILP1	ILP2
nodes	density	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>	<i>cnodes/recoloring</i>
500	0.5	35.13	7.89	7.88	<b>5.02</b>
1000	0.5	39.87	9.15	7.74	<b>5.15</b>
1500	0.5	44.67	11.52	8.12	<b>6.02</b>
2000	0.5	46.81	12.29	4.75	<b>6.42</b>

**Table 6.3:** Evaluation of the four larger instances. *ILP2* produces about 7.5 times less conflicting nodes than *RANDOM*.

## Final Results

For each set of instances experiments with various ranges of tabu list lengths as well as various boundaries for the maximum number of iterations have been performed. The size of the tabu list for each insertion is a random number between the lower and upper bound given as *TabuTenure*, where  $C'$  is the tentative number of colors. Because of that indeterminism 10 runs per instance have been performed. The maximum number of iterations used as stopping criterion is set as  $maxIter = q * (C') * F_{end}$ , where  $q$  is the amount of clusters. Tables 6.4 to 6.20 show the results of the instances provided in [6]. In tables 6.21 to 6.24 results of the large instances are shown, where the values of the parameters *TabuTenure* and  $F_{end}$  have been chosen similar to the ones used in [10]. The following tables are labeled like the names of each instance, indicating its size and density, e.g. *pcpn90p1* indicates an instance of 90 nodes and density 0.1.

The final results do not exhibit an improvement similar to the preliminary results or any significant improvement at all. Especially on larger instances the dramatic differences between the runtimes of the exact and non-exact methods becomes visible. For most instances except the four large ones a *TabuTenure* of  $U[1.0C', 4.0C']$  and  $U[0.0C', 5.0C']$  has shown to lead to best results. For the larger instances, a *TabuTenure* of  $U[0.0C', 0.5C']$  fits best, which approves the results in [10].

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.047	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.040	<b>3.0</b>	0.000	0.224
	$U[0.5C', 1.0C']$	<b>3.0</b>	0.000	0.016	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.029	<b>3.0</b>	0.000	0.211
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.010	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.033	<b>3.0</b>	0.000	0.197
	$U[0.0C', 5.0C']$	<b>3.0</b>	0.000	0.006	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.028	<b>3.0</b>	0.000	0.204
	$U[5.0C', 10.0C']$	<b>3.0</b>	0.000	0.005	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.023	<b>3.0</b>	0.000	0.224
	$U[10.0C', 20.0C']$	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.025	<b>3.0</b>	0.000	0.188
10	$U[0.0C', 0.5C']$	<b>2.9</b>	0.050	0.010	3.0	0.000	0.004	3.0	0.000	0.029	3.0	0.000	0.204
	$U[0.5C', 1.0C']$	<b>2.9</b>	0.050	0.007	<b>2.9</b>	0.050	0.005	3.0	0.000	0.030	<b>2.9</b>	0.050	0.186
	$U[1.0C', 4.0C']$	<b>2.9</b>	0.050	0.005	<b>2.9</b>	0.050	0.005	3.0	0.000	0.030	3.0	0.000	0.198
	$U[0.0C', 5.0C']$	3.0	0.000	0.005	2.9	0.050	0.005	3.0	0.000	0.029	<b>2.8</b>	0.000	0.183
	$U[5.0C', 10.0C']$	3.0	0.000	0.006	<b>2.9</b>	0.050	0.005	3.0	0.000	0.030	<b>2.9</b>	0.050	0.198
	$U[10.0C', 20.0C']$	3.0	0.000	0.005	<b>2.9</b>	0.050	0.005	3.0	0.000	0.027	3.0	0.000	0.209
20	$U[0.0C', 0.5C']$	3.0	0.000	0.008	<b>2.9</b>	0.050	0.009	<b>2.9</b>	0.050	0.034	3.0	0.000	0.213
	$U[0.5C', 1.0C']$	3.0	0.000	0.008	<b>2.9</b>	0.050	0.008	3.0	0.000	0.031	3.0	0.000	0.194
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.008	<b>3.0</b>	0.000	0.009	<b>3.0</b>	0.000	0.030	<b>3.0</b>	0.000	0.240
	$U[0.0C', 5.0C']$	3.0	0.000	0.008	3.0	0.000	0.008	<b>2.9</b>	0.050	0.030	3.0	0.000	0.215
	$U[5.0C', 10.0C']$	3.0	0.000	0.008	<b>2.8</b>	0.000	0.009	3.0	0.000	0.032	2.9	0.050	0.214
	$U[10.0C', 20.0C']$	<b>2.9</b>	0.050	0.009	3.0	0.000	0.008	3.0	0.000	0.031	3.0	0.000	0.202
50	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.017	<b>3.0</b>	0.000	0.018	<b>3.0</b>	0.000	0.040	<b>3.0</b>	0.000	0.200
	$U[0.5C', 1.0C']$	3.0	0.000	0.021	3.0	0.000	0.018	<b>2.9</b>	0.050	0.042	3.0	0.000	0.213
	$U[1.0C', 4.0C']$	3.0	0.000	0.018	3.0	0.000	0.018	3.0	0.000	0.038	<b>2.9</b>	0.050	0.225
	$U[0.0C', 5.0C']$	<b>2.9</b>	0.050	0.019	3.0	0.000	0.018	<b>2.9</b>	0.050	0.041	3.0	0.000	0.235
	$U[5.0C', 10.0C']$	3.0	0.000	0.017	3.0	0.000	0.017	<b>2.9</b>	0.050	0.040	3.0	0.000	0.201
	$U[10.0C', 20.0C']$	<b>2.8</b>	0.000	0.021	2.9	0.050	0.020	2.9	0.050	0.041	2.9	0.050	0.223

**Table 6.4:** Results for a set of 5 instances of size 90 and density 0.1



Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	4.0	0.000	0.007	3.9	0.050	0.004	<b>3.8</b>	0.000	0.070	4.0	0.000	0.377
	$U[0.5C', 1.0C']$	4.0	0.000	0.004	3.9	0.050	0.004	<b>3.6</b>	0.800	0.062	4.0	0.000	0.354
	$U[1.0C', 4.0C']$	4.0	0.000	0.003	<b>3.9</b>	0.050	0.004	<b>3.9</b>	0.050	0.064	4.0	0.000	0.357
	$U[0.0C', 5.0C']$	<b>3.9</b>	0.050	0.003	4.0	0.000	0.003	4.0	0.000	0.064	<b>3.9</b>	0.050	0.373
	$U[5.0C', 10.0C']$	4.0	0.000	0.003	<b>3.9</b>	0.050	0.004	4.0	0.000	0.067	4.0	0.000	0.354
	$U[10.0C', 20.0C']$	<b>3.9</b>	0.050	0.003	4.0	0.000	0.003	<b>3.9</b>	0.050	0.064	4.0	0.000	0.349
10	$U[0.0C', 0.5C']$	3.9	0.050	0.015	<b>3.8</b>	0.000	0.016	3.9	0.050	0.075	3.9	0.050	0.364
	$U[0.5C', 1.0C']$	3.9	0.050	0.014	3.9	0.050	0.015	<b>3.8</b>	0.000	0.079	4.0	0.000	0.373
	$U[1.0C', 4.0C']$	3.9	0.050	0.014	3.9	0.050	0.014	<b>3.8</b>	0.000	0.078	<b>3.8</b>	0.000	0.445
	$U[0.0C', 5.0C']$	3.9	0.050	0.015	<b>3.8</b>	0.000	0.015	<b>3.8</b>	0.000	0.080	<b>3.8</b>	0.000	0.439
	$U[5.0C', 10.0C']$	3.9	0.050	0.015	<b>3.8</b>	0.000	0.015	<b>3.8</b>	0.000	0.080	<b>3.8</b>	0.000	0.446
	$U[10.0C', 20.0C']$	<b>3.8</b>	0.000	0.017	3.9	0.050	0.016	<b>3.8</b>	0.000	0.079	<b>3.8</b>	0.000	0.455
20	$U[0.0C', 0.5C']$	4.0	0.000	0.025	<b>3.8</b>	0.000	0.028	3.9	0.050	0.091	<b>3.8</b>	0.000	0.436
	$U[0.5C', 1.0C']$	<b>3.8</b>	0.000	0.028	<b>3.8</b>	0.000	0.027	3.9	0.050	0.088	3.9	0.050	0.412
	$U[1.0C', 4.0C']$	3.9	0.050	0.028	3.9	0.050	0.027	<b>3.8</b>	0.000	0.091	<b>3.8</b>	0.000	0.417
	$U[0.0C', 5.0C']$	<b>3.8</b>	0.000	0.029	3.9	0.050	0.027	<b>3.8</b>	0.000	0.087	3.9	0.050	0.438
	$U[5.0C', 10.0C']$	<b>3.8</b>	0.000	0.028	<b>3.8</b>	0.000	0.028	<b>3.8</b>	0.000	0.090	<b>3.8</b>	0.000	0.460
	$U[10.0C', 20.0C']$	<b>3.8</b>	0.000	0.028	<b>3.8</b>	0.000	0.028	<b>3.8</b>	0.000	0.091	<b>3.8</b>	0.000	0.477
50	$U[0.0C', 0.5C']$	<b>3.8</b>	0.000	0.062	4.0	0.000	0.058	<b>3.8</b>	0.000	0.130	3.9	0.050	0.438
	$U[0.5C', 1.0C']$	<b>3.8</b>	0.000	0.066	3.9	0.050	0.059	3.9	0.050	0.117	3.9	0.050	0.459
	$U[1.0C', 4.0C']$	<b>3.8</b>	0.000	0.064	3.9	0.050	0.059	3.9	0.050	0.123	3.9	0.050	0.439
	$U[0.0C', 5.0C']$	3.9	0.050	0.063	<b>3.8</b>	0.000	0.068	<b>3.8</b>	0.000	0.123	<b>3.8</b>	0.000	0.459
	$U[5.0C', 10.0C']$	<b>3.8</b>	0.000	0.066	<b>3.8</b>	0.000	0.065	<b>3.8</b>	0.000	0.132	<b>3.8</b>	0.000	0.431
	$U[10.0C', 20.0C']$	<b>3.8</b>	0.000	0.071	<b>3.8</b>	0.000	0.067	<b>3.8</b>	0.000	0.128	<b>3.8</b>	0.000	0.445

**Table 6.5:** Results for a set of 5 instances of size 90 and density 0.2

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>5.0</b>	0.000	0.007	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.105	<b>5.0</b>	0.000	0.424
	$U[0.5C', 1.0C']$	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.006	<b>5.0</b>	0.000	0.098	<b>5.0</b>	0.000	0.430
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.099	<b>5.0</b>	0.000	0.426
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.006	<b>5.0</b>	0.000	0.007	<b>5.0</b>	0.000	0.095	<b>5.0</b>	0.000	0.417
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.097	<b>5.0</b>	0.000	0.430
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.005	<b>5.0</b>	0.000	0.091	<b>5.0</b>	0.000	0.373
10	$U[0.0C', 0.5C']$	<b>5.0</b>	0.000	0.028	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.115	<b>5.0</b>	0.000	0.436
	$U[0.5C', 1.0C']$	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.030	<b>5.0</b>	0.000	0.119	<b>5.0</b>	0.000	0.465
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.115	<b>5.0</b>	0.000	0.436
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.115	<b>5.0</b>	0.000	0.449
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.029	<b>5.0</b>	0.000	0.126	<b>5.0</b>	0.000	0.400
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.030	<b>5.0</b>	0.000	0.030	<b>5.0</b>	0.000	0.118	<b>5.0</b>	0.000	0.378
20	$U[0.0C', 0.5C']$	<b>5.0</b>	0.000	0.052	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.138	<b>5.0</b>	0.000	0.436
	$U[0.5C', 1.0C']$	<b>5.0</b>	0.000	0.052	<b>5.0</b>	0.000	0.053	<b>5.0</b>	0.000	0.141	<b>5.0</b>	0.000	0.431
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.137	<b>5.0</b>	0.000	0.439
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.149	<b>5.0</b>	0.000	0.435
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.055	<b>5.0</b>	0.000	0.143	<b>5.0</b>	0.000	0.515
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.056	<b>5.0</b>	0.000	0.059	<b>5.0</b>	0.000	0.150	<b>5.0</b>	0.000	0.447
50	$U[0.0C', 0.5C']$	<b>5.0</b>	0.000	0.127	<b>5.0</b>	0.000	0.126	<b>5.0</b>	0.000	0.205	<b>5.0</b>	0.000	0.513
	$U[0.5C', 1.0C']$	<b>5.0</b>	0.000	0.125	<b>5.0</b>	0.000	0.128	<b>5.0</b>	0.000	0.212	<b>5.0</b>	0.000	0.525
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.128	<b>5.0</b>	0.000	0.129	<b>5.0</b>	0.000	0.217	<b>5.0</b>	0.000	0.526
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.128	<b>5.0</b>	0.000	0.133	<b>5.0</b>	0.000	0.215	<b>5.0</b>	0.000	0.481
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.136	<b>5.0</b>	0.000	0.131	<b>5.0</b>	0.000	0.223	<b>5.0</b>	0.000	0.538
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.139	<b>5.0</b>	0.000	0.139	<b>5.0</b>	0.000	0.223	<b>5.0</b>	0.000	0.532

**Table 6.6:** Results for a set of 5 instances of size 90 and density 0.3

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.008	<b>6.0</b>	0.000	0.169	<b>6.0</b>	0.000	0.474
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.010	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.167	<b>6.0</b>	0.000	0.492
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.008	<b>6.0</b>	0.000	0.168	<b>6.0</b>	0.000	0.484
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.008	<b>6.0</b>	0.000	0.008	<b>6.0</b>	0.000	0.164	<b>6.0</b>	0.000	0.467
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.009	6.1	0.050	0.147	<b>6.0</b>	0.000	0.503
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.009	<b>6.0</b>	0.000	0.159	<b>6.0</b>	0.000	0.474
10	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.054	<b>6.0</b>	0.000	0.054	<b>6.0</b>	0.000	0.200	<b>6.0</b>	0.000	0.536
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.052	<b>6.0</b>	0.000	0.050	<b>6.0</b>	0.000	0.196	<b>6.0</b>	0.000	0.512
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.052	<b>6.0</b>	0.000	0.052	<b>6.0</b>	0.000	0.204	<b>6.0</b>	0.000	0.534
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.053	<b>6.0</b>	0.000	0.052	<b>6.0</b>	0.000	0.196	<b>6.0</b>	0.000	0.523
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.053	<b>6.0</b>	0.000	0.055	<b>6.0</b>	0.000	0.185	<b>6.0</b>	0.000	0.526
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.055	<b>6.0</b>	0.000	0.054	<b>6.0</b>	0.000	0.193	<b>6.0</b>	0.000	0.519
20	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.099	<b>6.0</b>	0.000	0.098	<b>6.0</b>	0.000	0.247	<b>6.0</b>	0.000	0.546
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.097	<b>6.0</b>	0.000	0.095	<b>6.0</b>	0.000	0.243	<b>6.0</b>	0.000	0.562
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.100	<b>6.0</b>	0.000	0.102	<b>6.0</b>	0.000	0.243	<b>6.0</b>	0.000	0.578
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.100	<b>6.0</b>	0.000	0.101	<b>6.0</b>	0.000	0.247	<b>6.0</b>	0.000	0.567
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.101	<b>6.0</b>	0.000	0.101	<b>6.0</b>	0.000	0.250	<b>6.0</b>	0.000	0.578
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.103	<b>6.0</b>	0.000	0.103	<b>6.0</b>	0.000	0.252	<b>6.0</b>	0.000	0.605
50	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.233	<b>6.0</b>	0.000	0.233	<b>6.0</b>	0.000	0.396	<b>6.0</b>	0.000	0.722
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.238	<b>6.0</b>	0.000	0.230	<b>6.0</b>	0.000	0.384	<b>6.0</b>	0.000	0.706
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.240	<b>6.0</b>	0.000	0.237	<b>6.0</b>	0.000	0.384	<b>6.0</b>	0.000	0.714
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.244	<b>6.0</b>	0.000	0.243	<b>6.0</b>	0.000	0.384	<b>6.0</b>	0.000	0.725
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.247	<b>6.0</b>	0.000	0.245	<b>6.0</b>	0.000	0.393	<b>6.0</b>	0.000	0.689
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.256	<b>6.0</b>	0.000	0.254	<b>6.0</b>	0.000	0.395	<b>6.0</b>	0.000	0.721

**Table 6.7:** Results for a set of 5 instances of size 90 and density 0.4

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	7.2	0.000	0.015	7.2	0.000	0.014	<b>7.1</b>	0.050	0.216	7.2	0.000	0.617
	$U[0.5C', 1.0C']$	<b>7.2</b>	0.000	0.017	<b>7.2</b>	0.100	0.014	<b>7.2</b>	0.000	0.204	7.4	0.000	0.560
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.014	7.1	0.050	0.015	7.2	0.100	0.199	7.1	0.050	0.632
	$U[0.0C', 5.0C']$	<b>7.1</b>	0.050	0.014	7.2	0.000	0.014	<b>7.1</b>	0.050	0.211	<b>7.1</b>	0.050	0.606
	$U[5.0C', 10.0C']$	7.3	0.050	0.014	<b>7.0</b>	0.000	0.016	7.2	0.100	0.194	7.2	0.000	0.583
	$U[10.0C', 20.0C']$	<b>7.2</b>	0.100	0.015	7.5	0.050	0.014	7.4	0.100	0.193	7.4	0.100	0.567
10	$U[0.0C', 0.5C']$	<b>7.0</b>	0.000	0.098	<b>7.0</b>	0.000	0.095	7.1	0.050	0.285	7.1	0.050	0.700
	$U[0.5C', 1.0C']$	<b>7.0</b>	0.000	0.108	7.2	0.000	0.086	<b>7.0</b>	0.000	0.278	7.1	0.050	0.709
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.089	<b>7.0</b>	0.000	0.090	<b>7.0</b>	0.000	0.288	<b>7.0</b>	0.000	0.719
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.088	<b>7.0</b>	0.000	0.089	<b>7.0</b>	0.000	0.300	<b>7.0</b>	0.000	0.716
	$U[5.0C', 10.0C']$	<b>7.0</b>	0.000	0.096	<b>7.0</b>	0.000	0.094	<b>7.0</b>	0.000	0.299	<b>7.0</b>	0.000	0.719
	$U[10.0C', 20.0C']$	7.1	0.050	0.097	7.1	0.050	0.097	<b>7.0</b>	0.000	0.313	<b>7.0</b>	0.000	0.709
20	$U[0.0C', 0.5C']$	7.1	0.050	0.191	7.1	0.050	0.185	<b>7.0</b>	0.000	0.375	7.2	0.000	0.755
	$U[0.5C', 1.0C']$	7.1	0.050	0.193	<b>7.0</b>	0.000	0.178	7.1	0.050	0.363	7.1	0.050	0.789
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.172	<b>7.0</b>	0.000	0.173	<b>7.0</b>	0.000	0.367	<b>7.0</b>	0.000	0.763
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.172	<b>7.0</b>	0.000	0.174	<b>7.0</b>	0.000	0.369	<b>7.0</b>	0.000	0.787
	$U[5.0C', 10.0C']$	<b>7.0</b>	0.000	0.178	<b>7.0</b>	0.000	0.182	<b>7.0</b>	0.000	0.379	<b>7.0</b>	0.000	0.796
	$U[10.0C', 20.0C']$	<b>7.0</b>	0.000	0.205	7.1	0.050	0.185	7.1	0.050	0.386	7.1	0.050	0.792
50	$U[0.0C', 0.5C']$	<b>7.0</b>	0.000	0.433	7.1	0.050	0.409	7.1	0.050	0.617	<b>7.0</b>	0.000	1.071
	$U[0.5C', 1.0C']$	7.2	0.000	0.436	<b>7.0</b>	0.000	0.459	7.2	0.000	0.577	<b>7.0</b>	0.000	1.073
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.412	<b>7.0</b>	0.000	0.418	<b>7.0</b>	0.000	0.597	<b>7.0</b>	0.000	1.015
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.408	<b>7.0</b>	0.000	0.419	<b>7.0</b>	0.000	0.600	<b>7.0</b>	0.000	1.018
	$U[5.0C', 10.0C']$	<b>7.0</b>	0.000	0.426	<b>7.0</b>	0.000	0.428	<b>7.0</b>	0.000	0.618	<b>7.0</b>	0.000	1.023
	$U[10.0C', 20.0C']$	<b>7.0</b>	0.000	0.478	<b>7.0</b>	0.000	0.442	<b>7.0</b>	0.000	0.681	<b>7.0</b>	0.000	1.039

**Table 6.8:** Results for a set of 5 instances of size 90 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>8.8</b>	0.100	0.022	9.0	0.000	0.021	<b>8.8</b>	0.100	0.264	<b>8.8</b>	0.100	0.729
	$U[0.5C', 1.0C']$	<b>8.7</b>	0.050	0.021	<b>8.7</b>	0.050	0.021	8.8	0.100	0.253	<b>8.7</b>	0.050	0.739
	$U[1.0C', 4.0C']$	8.7	0.050	0.020	8.6	0.100	0.022	8.7	0.050	0.268	<b>8.5</b>	0.150	0.774
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.200	0.022	8.7	0.050	0.020	8.7	0.150	0.272	8.9	0.050	0.697
	$U[5.0C', 10.0C']$	8.8	0.100	0.021	8.8	0.100	0.021	<b>8.7</b>	0.050	0.256	8.9	0.050	0.708
	$U[10.0C', 20.0C']$	<b>8.9</b>	0.050	0.022	9.0	0.000	0.020	<b>8.9</b>	0.050	0.245	9.0	0.000	0.679
10	$U[0.0C', 0.5C']$	8.6	0.100	0.160	8.8	0.100	0.146	8.6	0.100	0.392	<b>8.5</b>	0.050	0.943
	$U[0.5C', 1.0C']$	8.7	0.050	0.144	<b>8.4</b>	0.100	0.157	8.5	0.150	0.422	8.6	0.100	0.891
	$U[1.0C', 4.0C']$	<b>8.2</b>	0.000	0.162	8.3	0.050	0.169	<b>8.2</b>	0.000	0.429	8.3	0.050	0.972
	$U[0.0C', 5.0C']$	<b>8.2</b>	0.000	0.171	<b>8.2</b>	0.000	0.148	<b>8.2</b>	0.000	0.445	8.3	0.050	0.955
	$U[5.0C', 10.0C']$	<b>8.4</b>	0.100	0.187	<b>8.4</b>	0.100	0.169	8.5	0.050	0.422	8.6	0.100	0.916
	$U[10.0C', 20.0C']$	8.8	0.000	0.158	8.8	0.000	0.159	8.7	0.050	0.400	<b>8.5</b>	0.050	0.928
20	$U[0.0C', 0.5C']$	8.8	0.100	0.297	8.8	0.100	0.287	<b>8.6</b>	0.100	0.552	<b>8.6</b>	0.000	1.053
	$U[0.5C', 1.0C']$	8.6	0.000	0.302	<b>8.5</b>	0.050	0.297	<b>8.5</b>	0.050	0.556	8.8	0.100	0.954
	$U[1.0C', 4.0C']$	<b>8.2</b>	0.000	0.306	<b>8.2</b>	0.000	0.307	<b>8.2</b>	0.000	0.567	<b>8.2</b>	0.000	1.137
	$U[0.0C', 5.0C']$	<b>8.2</b>	0.000	0.305	8.3	0.050	0.300	<b>8.2</b>	0.000	0.559	<b>8.2</b>	0.000	1.115
	$U[5.0C', 10.0C']$	8.3	0.050	0.325	8.3	0.050	0.326	<b>8.2</b>	0.000	0.600	8.4	0.100	1.110
	$U[10.0C', 20.0C']$	8.6	0.000	0.313	8.5	0.050	0.324	<b>8.4</b>	0.000	0.588	8.7	0.050	1.046
50	$U[0.0C', 0.5C']$	8.8	0.000	0.728	8.8	0.100	0.682	<b>8.7</b>	0.050	0.980	8.8	0.100	1.377
	$U[0.5C', 1.0C']$	<b>8.6</b>	0.100	0.768	<b>8.6</b>	0.100	0.684	8.7	0.050	0.892	<b>8.6</b>	0.100	1.436
	$U[1.0C', 4.0C']$	<b>8.2</b>	0.000	0.690	<b>8.2</b>	0.000	0.710	<b>8.2</b>	0.000	0.951	<b>8.2</b>	0.000	1.507
	$U[0.0C', 5.0C']$	<b>8.2</b>	0.000	0.716	<b>8.2</b>	0.000	0.757	<b>8.2</b>	0.000	0.984	<b>8.2</b>	0.000	1.526
	$U[5.0C', 10.0C']$	8.3	0.050	0.745	<b>8.2</b>	0.000	0.833	<b>8.2</b>	0.000	1.056	<b>8.2</b>	0.000	1.630
	$U[10.0C', 20.0C']$	8.4	0.100	0.860	8.4	0.100	0.797	<b>8.3</b>	0.050	1.102	8.5	0.050	1.560

**Table 6.9:** Results for a set of 5 instances of size 90 and density 0.6

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	10.6	0.100	0.037	<b>10.2</b>	0.100	0.034	10.5	0.050	0.301	10.4	0.200	0.865
	$U[0.5C', 1.0C']$	<b>10.2</b>	0.100	0.037	10.5	0.050	0.032	10.4	0.100	0.310	<b>10.2</b>	0.000	0.906
	$U[1.0C', 4.0C']$	<b>10.1</b>	0.050	0.038	<b>10.1</b>	0.050	0.034	10.3	0.050	0.340	<b>10.1</b>	0.050	0.927
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.037	10.1	0.050	0.031	10.4	0.100	0.317	10.1	0.050	0.904
	$U[5.0C', 10.0C']$	10.4	0.100	0.034	10.5	0.150	0.034	<b>10.2</b>	0.100	0.328	<b>10.2</b>	0.100	0.921
	$U[10.0C', 20.0C']$	10.6	0.100	0.035	<b>10.5</b>	0.050	0.034	10.8	0.100	0.287	<b>10.5</b>	0.050	0.809
10	$U[0.0C', 0.5C']$	<b>10.4</b>	0.100	0.270	<b>10.4</b>	0.200	0.265	10.8	0.100	0.462	10.5	0.150	1.070
	$U[0.5C', 1.0C']$	<b>10.0</b>	0.000	0.309	<b>10.0</b>	0.000	0.264	10.2	0.000	0.556	10.2	0.100	1.113
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.233	<b>10.0</b>	0.000	0.231	<b>10.0</b>	0.000	0.523	<b>10.0</b>	0.000	1.120
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.229	<b>10.0</b>	0.000	0.230	<b>10.0</b>	0.000	0.536	<b>10.0</b>	0.000	1.129
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.248	<b>10.0</b>	0.000	0.241	<b>10.0</b>	0.000	0.528	<b>10.0</b>	0.000	1.139
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.311	<b>10.0</b>	0.000	0.300	<b>10.0</b>	0.000	0.588	<b>10.0</b>	0.000	1.230
20	$U[0.0C', 0.5C']$	10.5	0.050	0.502	<b>10.4</b>	0.100	0.527	10.5	0.250	0.821	10.6	0.100	1.237
	$U[0.5C', 1.0C']$	<b>10.0</b>	0.000	0.462	10.1	0.050	0.517	10.2	0.100	0.805	<b>10.0</b>	0.000	1.464
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.445	<b>10.0</b>	0.000	0.446	<b>10.0</b>	0.000	0.726	<b>10.0</b>	0.000	1.336
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.446	<b>10.0</b>	0.000	0.443	<b>10.0</b>	0.000	0.741	<b>10.0</b>	0.000	1.338
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.471	<b>10.0</b>	0.000	0.467	<b>10.0</b>	0.000	0.782	<b>10.0</b>	0.000	1.357
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.493	<b>10.0</b>	0.000	0.532	<b>10.0</b>	0.000	0.824	<b>10.0</b>	0.000	1.432
50	$U[0.0C', 0.5C']$	<b>10.4</b>	0.100	1.183	10.6	0.100	1.099	10.5	0.050	1.501	10.5	0.150	2.083
	$U[0.5C', 1.0C']$	10.1	0.050	1.215	10.1	0.050	1.338	10.2	0.000	1.554	<b>10.0</b>	0.000	2.200
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	1.077	<b>10.0</b>	0.000	1.098	<b>10.0</b>	0.000	1.377	<b>10.0</b>	0.000	1.973
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	1.078	<b>10.0</b>	0.000	1.086	<b>10.0</b>	0.000	1.403	<b>10.0</b>	0.000	1.990
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	1.135	<b>10.0</b>	0.000	1.135	<b>10.0</b>	0.000	1.428	<b>10.0</b>	0.000	2.022
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	1.197	<b>10.0</b>	0.000	1.193	<b>10.0</b>	0.000	1.471	<b>10.0</b>	0.000	2.066

**Table 6.10:** Results for a set of 5 instances of size 90 and density 0.7

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>12.6</b>	0.100	0.058	12.8	0.000	0.056	13.0	0.000	0.464	13.1	0.150	1.040
	$U[0.5C', 1.0C']$	12.7	0.050	0.046	<b>12.6</b>	0.000	0.052	12.9	0.050	0.442	12.8	0.000	1.106
	$U[1.0C', 4.0C']$	12.6	0.100	0.049	<b>12.5</b>	0.050	0.048	12.7	0.050	0.467	12.7	0.050	1.109
	$U[0.0C', 5.0C']$	<b>12.5</b>	0.150	0.051	12.8	0.000	0.046	12.7	0.050	0.449	12.6	0.100	1.144
	$U[5.0C', 10.0C']$	12.7	0.050	0.051	<b>12.5</b>	0.050	0.058	12.6	0.100	0.457	12.8	0.000	1.100
	$U[10.0C', 20.0C']$	12.8	0.000	0.050	12.8	0.000	0.051	12.8	0.000	0.429	<b>12.7</b>	0.050	1.119
10	$U[0.0C', 0.5C']$	<b>12.7</b>	0.150	0.491	12.9	0.050	0.443	13.1	0.050	0.752	12.9	0.250	1.450
	$U[0.5C', 1.0C']$	12.6	0.100	0.410	<b>12.4</b>	0.100	0.440	12.5	0.050	0.819	12.6	0.100	1.490
	$U[1.0C', 4.0C']$	12.2	0.000	0.408	<b>12.0</b>	0.000	0.426	<b>12.0</b>	0.000	0.893	12.3	0.050	1.543
	$U[0.0C', 5.0C']$	12.1	0.050	0.390	12.1	0.050	0.429	<b>12.0</b>	0.000	0.903	12.1	0.050	1.600
	$U[5.0C', 10.0C']$	<b>12.2</b>	0.000	0.451	12.3	0.050	0.452	12.3	0.050	0.862	12.3	0.050	1.549
	$U[10.0C', 20.0C']$	12.5	0.050	0.437	<b>12.4</b>	0.100	0.459	12.5	0.150	0.852	12.5	0.050	1.546
20	$U[0.0C', 0.5C']$	12.9	0.050	0.750	12.9	0.050	0.907	<b>12.8</b>	0.100	1.173	12.9	0.050	1.885
	$U[0.5C', 1.0C']$	<b>12.4</b>	0.100	0.817	12.5	0.050	0.769	12.5	0.150	1.242	12.5	0.050	1.935
	$U[1.0C', 4.0C']$	<b>12.0</b>	0.000	0.770	<b>12.0</b>	0.000	0.918	<b>12.0</b>	0.000	1.271	<b>12.0</b>	0.000	2.063
	$U[0.0C', 5.0C']$	<b>12.0</b>	0.000	0.826	12.1	0.050	0.838	<b>12.0</b>	0.000	1.211	12.1	0.050	1.979
	$U[5.0C', 10.0C']$	<b>12.1</b>	0.050	0.828	12.2	0.100	0.820	<b>12.1</b>	0.050	1.344	12.2	0.000	2.034
	$U[10.0C', 20.0C']$	12.4	0.100	0.931	12.5	0.050	0.795	12.4	0.200	1.318	<b>12.3</b>	0.050	2.066
50	$U[0.0C', 0.5C']$	13.0	0.100	2.108	13.0	0.100	2.076	<b>12.7</b>	0.050	2.560	13.0	0.100	2.841
	$U[0.5C', 1.0C']$	<b>12.5</b>	0.050	1.994	12.6	0.100	2.112	<b>12.5</b>	0.050	2.412	12.6	0.100	2.883
	$U[1.0C', 4.0C']$	<b>12.0</b>	0.000	1.869	<b>12.0</b>	0.000	1.872	<b>12.0</b>	0.000	2.291	<b>12.0</b>	0.000	3.119
	$U[0.0C', 5.0C']$	<b>12.0</b>	0.000	1.833	<b>12.0</b>	0.000	1.858	<b>12.0</b>	0.000	2.284	<b>12.0</b>	0.000	3.065
	$U[5.0C', 10.0C']$	12.1	0.050	2.030	12.1	0.050	2.243	12.1	0.050	2.536	<b>12.0</b>	0.000	3.217
	$U[10.0C', 20.0C']$	12.2	0.100	2.106	12.3	0.050	2.166	<b>12.0</b>	0.000	3.000	12.1	0.050	3.505

**Table 6.11:** Results for a set of 5 instances of size 90 and density 0.8

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	17.1	0.050	0.089	16.9	0.150	0.091	<b>16.8</b>	0.200	0.493	16.9	0.150	1.026
	$U[0.5C', 1.0C']$	16.4	0.100	0.084	<b>16.2</b>	0.100	0.076	16.4	0.000	0.505	<b>16.2</b>	0.000	1.179
	$U[1.0C', 4.0C']$	16.1	0.050	0.081	<b>16.0</b>	0.100	0.078	16.1	0.050	0.523	16.3	0.050	1.141
	$U[0.0C', 5.0C']$	16.3	0.050	0.077	16.1	0.050	0.078	16.3	0.150	0.493	<b>16.0</b>	0.100	1.218
	$U[5.0C', 10.0C']$	<b>16.1</b>	0.050	0.083	16.4	0.000	0.080	16.3	0.050	0.489	16.2	0.000	1.153
	$U[10.0C', 20.0C']$	<b>16.1</b>	0.050	0.085	16.2	0.100	0.085	<b>16.1</b>	0.050	0.536	16.3	0.050	1.179
10	$U[0.0C', 0.5C']$	<b>16.6</b>	0.100	0.895	17.0	0.100	0.766	16.7	0.150	1.072	17.2	0.100	1.597
	$U[0.5C', 1.0C']$	<b>16.1</b>	0.050	0.656	16.3	0.350	0.648	16.2	0.000	1.119	16.2	0.100	1.723
	$U[1.0C', 4.0C']$	15.9	0.050	0.682	<b>15.8</b>	0.000	0.696	<b>15.8</b>	0.000	1.122	<b>15.8</b>	0.000	1.848
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	0.679	<b>15.8</b>	0.000	0.652	<b>15.8</b>	0.000	1.083	<b>15.8</b>	0.000	1.867
	$U[5.0C', 10.0C']$	15.9	0.050	0.699	<b>15.8</b>	0.000	0.740	<b>15.8</b>	0.000	1.197	<b>15.8</b>	0.000	1.925
	$U[10.0C', 20.0C']$	16.0	0.100	0.735	16.0	0.000	0.737	16.1	0.050	1.158	<b>15.9</b>	0.050	1.852
20	$U[0.0C', 0.5C']$	16.6	0.200	1.627	16.9	0.150	1.467	<b>16.4</b>	0.000	1.939	17.0	0.100	2.123
	$U[0.5C', 1.0C']$	16.4	0.100	1.283	<b>16.2</b>	0.100	1.412	16.3	0.050	1.643	<b>16.2</b>	0.100	2.317
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	1.308	15.9	0.050	1.235	<b>15.8</b>	0.000	1.742	<b>15.8</b>	0.000	2.422
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	1.302	15.9	0.050	1.378	<b>15.8</b>	0.000	1.715	<b>15.8</b>	0.000	2.406
	$U[5.0C', 10.0C']$	15.9	0.050	1.372	<b>15.8</b>	0.000	1.448	<b>15.8</b>	0.000	1.815	<b>15.8</b>	0.000	2.455
	$U[10.0C', 20.0C']$	16.0	0.000	1.413	<b>15.8</b>	0.000	1.478	<b>15.8</b>	0.000	1.880	15.9	0.050	2.513
50	$U[0.0C', 0.5C']$	17.0	0.100	3.405	<b>16.9</b>	0.050	3.484	<b>16.9</b>	0.150	3.922	17.1	0.150	4.539
	$U[0.5C', 1.0C']$	16.2	0.100	3.441	16.4	0.100	3.133	<b>16.1</b>	0.050	3.523	<b>16.1</b>	0.050	4.228
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	3.019	<b>15.8</b>	0.000	3.062	<b>15.8</b>	0.000	3.485	<b>15.8</b>	0.000	4.214
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	3.173	<b>15.8</b>	0.000	3.071	<b>15.8</b>	0.000	3.459	<b>15.8</b>	0.000	4.229
	$U[5.0C', 10.0C']$	<b>15.8</b>	0.000	3.263	<b>15.8</b>	0.000	3.300	<b>15.8</b>	0.000	3.770	<b>15.8</b>	0.000	4.329
	$U[10.0C', 20.0C']$	<b>15.8</b>	0.000	3.451	15.9	0.050	3.436	15.9	0.050	3.800	<b>15.8</b>	0.000	4.562

**Table 6.12:** Results for a set of 5 instances of size 90 and density 0.9

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.023	<b>3.0</b>	0.000	0.034
	$U[0.5C', 1.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.021	<b>3.0</b>	0.000	0.031
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.021	<b>3.0</b>	0.000	0.032
	$U[0.0C', 5.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.018	<b>3.0</b>	0.000	0.036
	$U[5.0C', 10.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.029	<b>3.0</b>	0.000	0.035
	$U[10.0C', 20.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.021	<b>3.0</b>	0.000	0.036
10	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.026	<b>3.0</b>	0.000	0.031
	$U[0.5C', 1.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.025	<b>3.0</b>	0.000	0.038
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.023	<b>3.0</b>	0.000	0.037
	$U[0.0C', 5.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.027	<b>3.0</b>	0.000	0.035
	$U[5.0C', 10.0C']$	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.026	<b>3.0</b>	0.000	0.034
	$U[10.0C', 20.0C']$	<b>3.0</b>	0.000	0.002	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.025	<b>3.0</b>	0.000	0.033
20	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.009	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.029	<b>3.0</b>	0.000	0.036
	$U[0.5C', 1.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.023	<b>3.0</b>	0.000	0.037
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.025	<b>3.0</b>	0.000	0.036
	$U[0.0C', 5.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.024	<b>3.0</b>	0.000	0.037
	$U[5.0C', 10.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.001	<b>3.0</b>	0.000	0.022	<b>3.0</b>	0.000	0.034
	$U[10.0C', 20.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.002	<b>3.0</b>	0.000	0.021	<b>3.0</b>	0.000	0.035
50	$U[0.0C', 0.5C']$	<b>3.0</b>	0.000	0.005	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.030	<b>3.0</b>	0.000	0.038
	$U[0.5C', 1.0C']$	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.025	<b>3.0</b>	0.000	0.034
	$U[1.0C', 4.0C']$	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.023	<b>3.0</b>	0.000	0.035
	$U[0.0C', 5.0C']$	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.022	<b>3.0</b>	0.000	0.036
	$U[5.0C', 10.0C']$	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.003	<b>3.0</b>	0.000	0.030	<b>3.0</b>	0.000	0.036
	$U[10.0C', 20.0C']$	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.004	<b>3.0</b>	0.000	0.024	<b>3.0</b>	0.000	0.036

**Table 6.13:** Results for a set of 5 instances of size 20 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>4.0</b>	0.000	0.002	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.073	<b>4.1</b>	0.050	0.119
	$U[0.5C', 1.0C']$	<b>4.0</b>	0.000	0.002	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.075	<b>4.0</b>	0.000	0.128
	$U[1.0C', 4.0C']$	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.076	<b>4.0</b>	0.000	0.127
	$U[0.0C', 5.0C']$	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.069	<b>4.0</b>	0.000	0.128
	$U[5.0C', 10.0C']$	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.069	<b>4.0</b>	0.000	0.127
	$U[10.0C', 20.0C']$	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.001	<b>4.0</b>	0.000	0.070	<b>4.0</b>	0.000	0.136
10	$U[0.0C', 0.5C']$	<b>4.0</b>	0.000	0.007	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.078	<b>4.1</b>	0.050	0.130
	$U[0.5C', 1.0C']$	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.007	<b>4.0</b>	0.000	0.076	<b>4.0</b>	0.000	0.136
	$U[1.0C', 4.0C']$	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.078	<b>4.0</b>	0.000	0.137
	$U[0.0C', 5.0C']$	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.077	<b>4.0</b>	0.000	0.139
	$U[5.0C', 10.0C']$	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.006	<b>4.0</b>	0.000	0.082	<b>4.0</b>	0.000	0.131
	$U[10.0C', 20.0C']$	<b>4.0</b>	0.000	0.007	<b>4.0</b>	0.000	0.007	<b>4.0</b>	0.000	0.076	<b>4.0</b>	0.000	0.133
20	$U[0.0C', 0.5C']$	<b>4.0</b>	0.000	0.011	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.078	<b>4.0</b>	0.000	0.139
	$U[0.5C', 1.0C']$	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.089	<b>4.0</b>	0.000	0.144
	$U[1.0C', 4.0C']$	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.013	<b>4.0</b>	0.000	0.082	<b>4.0</b>	0.000	0.138
	$U[0.0C', 5.0C']$	<b>4.0</b>	0.000	0.011	<b>4.0</b>	0.000	0.011	<b>4.0</b>	0.000	0.079	<b>4.0</b>	0.000	0.137
	$U[5.0C', 10.0C']$	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.081	<b>4.0</b>	0.000	0.137
	$U[10.0C', 20.0C']$	<b>4.0</b>	0.000	0.013	<b>4.0</b>	0.000	0.012	<b>4.0</b>	0.000	0.081	<b>4.0</b>	0.000	0.144
50	$U[0.0C', 0.5C']$	<b>4.0</b>	0.000	0.029	<b>4.0</b>	0.000	0.027	<b>4.0</b>	0.000	0.091	<b>4.1</b>	0.050	0.148
	$U[0.5C', 1.0C']$	<b>4.0</b>	0.000	0.028	<b>4.0</b>	0.000	0.027	<b>4.0</b>	0.000	0.098	<b>4.0</b>	0.000	0.154
	$U[1.0C', 4.0C']$	<b>4.0</b>	0.000	0.026	<b>4.0</b>	0.000	0.026	<b>4.0</b>	0.000	0.088	<b>4.0</b>	0.000	0.156
	$U[0.0C', 5.0C']$	<b>4.0</b>	0.000	0.026	<b>4.0</b>	0.000	0.026	<b>4.0</b>	0.000	0.100	<b>4.0</b>	0.000	0.156
	$U[5.0C', 10.0C']$	<b>4.0</b>	0.000	0.029	<b>4.0</b>	0.000	0.028	<b>4.0</b>	0.000	0.093	<b>4.0</b>	0.000	0.154
	$U[10.0C', 20.0C']$	<b>4.0</b>	0.000	0.030	<b>4.0</b>	0.000	0.030	<b>4.0</b>	0.000	0.101	<b>4.0</b>	0.000	0.153

**Table 6.14:** Results for a set of 5 instances of size 40 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	5.6	0.100	0.005	5.3	0.050	0.004	<b>5.2</b>	0.100	0.128	5.5	0.050	0.261
	$U[0.5C', 1.0C']$	5.6	0.100	0.006	<b>5.4</b>	0.100	0.004	<b>5.4</b>	0.000	0.117	<b>5.4</b>	0.100	0.266
	$U[1.0C', 4.0C']$	5.7	0.050	0.004	5.3	0.150	0.004	<b>5.2</b>	0.100	0.122	5.4	0.100	0.265
	$U[0.0C', 5.0C']$	5.3	0.150	0.004	5.3	0.050	0.004	<b>5.2</b>	0.100	0.125	5.6	0.100	0.248
	$U[5.0C', 10.0C']$	5.7	0.050	0.004	<b>5.3</b>	0.150	0.004	5.4	0.100	0.114	<b>5.3</b>	0.050	0.277
	$U[10.0C', 20.0C']$	5.9	0.050	0.004	5.8	0.100	0.004	<b>5.4</b>	0.100	0.110	5.8	0.000	0.220
10	$U[0.0C', 0.5C']$	5.4	0.100	0.024	<b>5.1</b>	0.050	0.026	5.3	0.050	0.139	5.2	0.100	0.298
	$U[0.5C', 1.0C']$	<b>5.1</b>	0.050	0.026	5.2	0.100	0.024	<b>5.1</b>	0.050	0.142	<b>5.1</b>	0.050	0.323
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.024	<b>5.0</b>	0.000	0.024	<b>5.0</b>	0.000	0.148	<b>5.0</b>	0.000	0.306
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.023	<b>5.0</b>	0.000	0.023	<b>5.0</b>	0.000	0.144	<b>5.0</b>	0.000	0.323
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.023	<b>5.0</b>	0.000	0.024	<b>5.0</b>	0.000	0.151	<b>5.0</b>	0.000	0.323
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.028	5.2	0.100	0.027	<b>5.0</b>	0.000	0.157	5.2	0.100	0.309
20	$U[0.0C', 0.5C']$	<b>5.2</b>	0.000	0.045	5.3	0.050	0.047	<b>5.2</b>	0.000	0.157	5.3	0.050	0.319
	$U[0.5C', 1.0C']$	<b>5.1</b>	0.050	0.052	5.3	0.150	0.042	5.2	0.100	0.156	5.3	0.050	0.324
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.043	<b>5.0</b>	0.000	0.042	<b>5.0</b>	0.000	0.164	<b>5.0</b>	0.000	0.341
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.044	<b>5.0</b>	0.000	0.043	<b>5.0</b>	0.000	0.167	<b>5.0</b>	0.000	0.339
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.045	<b>5.0</b>	0.000	0.043	<b>5.0</b>	0.000	0.171	<b>5.0</b>	0.000	0.338
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.046	<b>5.0</b>	0.000	0.051	<b>5.0</b>	0.000	0.168	<b>5.0</b>	0.000	0.358
50	$U[0.0C', 0.5C']$	<b>5.0</b>	0.000	0.117	<b>5.0</b>	0.000	0.118	5.2	0.100	0.214	5.1	0.050	0.392
	$U[0.5C', 1.0C']$	5.2	0.100	0.115	<b>5.1</b>	0.050	0.118	<b>5.1</b>	0.050	0.228	5.3	0.050	0.374
	$U[1.0C', 4.0C']$	<b>5.0</b>	0.000	0.096	<b>5.0</b>	0.000	0.097	<b>5.0</b>	0.000	0.218	<b>5.0</b>	0.000	0.401
	$U[0.0C', 5.0C']$	<b>5.0</b>	0.000	0.099	<b>5.0</b>	0.000	0.101	<b>5.0</b>	0.000	0.216	<b>5.0</b>	0.000	0.394
	$U[5.0C', 10.0C']$	<b>5.0</b>	0.000	0.103	<b>5.0</b>	0.000	0.103	<b>5.0</b>	0.000	0.230	<b>5.0</b>	0.000	0.409
	$U[10.0C', 20.0C']$	<b>5.0</b>	0.000	0.119	<b>5.0</b>	0.000	0.114	<b>5.0</b>	0.000	0.229	<b>5.0</b>	0.000	0.403

**Table 6.15:** Results for a set of 5 instances of size 60 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.132	<b>6.0</b>	0.000	0.362
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.006	6.1	0.050	0.127	6.1	0.050	0.338
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.134	6.1	0.050	0.347
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.007	6.1	0.050	0.006	<b>6.0</b>	0.000	0.130	<b>6.0</b>	0.000	0.348
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.006	<b>6.0</b>	0.000	0.132	6.1	0.050	0.332
	$U[10.0C', 20.0C']$	6.2	0.000	0.006	<b>6.0</b>	0.000	0.006	6.1	0.050	0.120	<b>6.0</b>	0.000	0.356
10	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.160	<b>6.0</b>	0.000	0.372
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.033	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.154	<b>6.0</b>	0.000	0.379
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.033	<b>6.0</b>	0.000	0.150	<b>6.0</b>	0.000	0.379
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.031	<b>6.0</b>	0.000	0.155	<b>6.0</b>	0.000	0.381
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.033	<b>6.0</b>	0.000	0.032	<b>6.0</b>	0.000	0.155	<b>6.0</b>	0.000	0.378
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.034	<b>6.0</b>	0.000	0.034	<b>6.0</b>	0.000	0.153	<b>6.0</b>	0.000	0.370
20	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.061	<b>6.0</b>	0.000	0.061	<b>6.0</b>	0.000	0.179	<b>6.0</b>	0.000	0.401
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.059	<b>6.0</b>	0.000	0.058	<b>6.0</b>	0.000	0.179	<b>6.0</b>	0.000	0.400
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.062	<b>6.0</b>	0.000	0.060	<b>6.0</b>	0.000	0.185	<b>6.0</b>	0.000	0.417
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.060	<b>6.0</b>	0.000	0.060	<b>6.0</b>	0.000	0.187	<b>6.0</b>	0.000	0.417
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.063	<b>6.0</b>	0.000	0.062	<b>6.0</b>	0.000	0.191	<b>6.0</b>	0.000	0.396
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.065	<b>6.0</b>	0.000	0.065	<b>6.0</b>	0.000	0.196	<b>6.0</b>	0.000	0.412
50	$U[0.0C', 0.5C']$	<b>6.0</b>	0.000	0.144	<b>6.0</b>	0.000	0.141	<b>6.0</b>	0.000	0.261	<b>6.0</b>	0.000	0.463
	$U[0.5C', 1.0C']$	<b>6.0</b>	0.000	0.139	<b>6.0</b>	0.000	0.136	<b>6.0</b>	0.000	0.267	<b>6.0</b>	0.000	0.493
	$U[1.0C', 4.0C']$	<b>6.0</b>	0.000	0.142	<b>6.0</b>	0.000	0.141	<b>6.0</b>	0.000	0.271	<b>6.0</b>	0.000	0.487
	$U[0.0C', 5.0C']$	<b>6.0</b>	0.000	0.147	<b>6.0</b>	0.000	0.144	<b>6.0</b>	0.000	0.273	<b>6.0</b>	0.000	0.476
	$U[5.0C', 10.0C']$	<b>6.0</b>	0.000	0.153	<b>6.0</b>	0.000	0.152	<b>6.0</b>	0.000	0.281	<b>6.0</b>	0.000	0.495
	$U[10.0C', 20.0C']$	<b>6.0</b>	0.000	0.157	<b>6.0</b>	0.000	0.156	<b>6.0</b>	0.000	0.282	<b>6.0</b>	0.000	0.492

**Table 6.16:** Results for a set of 5 instances of size 70 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>6.7</b>	0.050	0.009	<b>6.7</b>	0.050	0.009	7.0	0.000	0.148	6.8	0.100	0.423
	$U[0.5C', 1.0C']$	7.0	0.000	0.008	6.9	0.050	0.009	7.0	0.000	0.145	<b>6.8</b>	0.000	0.442
	$U[1.0C', 4.0C']$	<b>6.8</b>	0.000	0.008	6.9	0.050	0.008	7.0	0.000	0.147	<b>6.8</b>	0.100	0.436
	$U[0.0C', 5.0C']$	<b>6.8</b>	0.000	0.010	7.0	0.000	0.009	7.0	0.000	0.143	6.9	0.050	0.414
	$U[5.0C', 10.0C']$	6.9	0.050	0.008	<b>6.7</b>	0.050	0.009	6.9	0.050	0.157	6.8	0.000	0.443
	$U[10.0C', 20.0C']$	7.0	0.000	0.008	7.0	0.000	0.009	<b>6.9</b>	0.050	0.147	7.0	0.000	0.410
10	$U[0.0C', 0.5C']$	6.7	0.050	0.053	6.8	0.100	0.051	<b>6.5</b>	0.050	0.217	6.6	0.100	0.515
	$U[0.5C', 1.0C']$	<b>6.5</b>	0.050	0.061	6.8	0.100	0.050	6.8	0.100	0.200	6.6	0.000	0.515
	$U[1.0C', 4.0C']$	6.5	0.050	0.053	<b>6.4</b>	0.000	0.059	<b>6.4</b>	0.100	0.217	<b>6.4</b>	0.000	0.536
	$U[0.0C', 5.0C']$	6.5	0.050	0.053	6.4	0.000	0.058	6.5	0.050	0.202	<b>6.3</b>	0.050	0.550
	$U[5.0C', 10.0C']$	<b>6.4</b>	0.000	0.063	6.6	0.100	0.055	<b>6.4</b>	0.100	0.218	<b>6.4</b>	0.000	0.536
	$U[10.0C', 20.0C']$	6.8	0.100	0.061	<b>6.6</b>	0.000	0.061	7.0	0.000	0.187	6.7	0.150	0.500
20	$U[0.0C', 0.5C']$	6.8	0.100	0.090	6.7	0.050	0.096	<b>6.6</b>	0.100	0.255	6.7	0.150	0.564
	$U[0.5C', 1.0C']$	<b>6.7</b>	0.050	0.100	<b>6.7</b>	0.150	0.096	6.9	0.050	0.230	7.0	0.000	0.498
	$U[1.0C', 4.0C']$	<b>6.2</b>	0.100	0.109	6.6	0.000	0.105	6.3	0.050	0.271	6.4	0.000	0.597
	$U[0.0C', 5.0C']$	<b>6.1</b>	0.050	0.116	6.3	0.050	0.111	6.3	0.050	0.264	6.3	0.050	0.620
	$U[5.0C', 10.0C']$	6.5	0.050	0.109	6.4	0.000	0.113	6.4	0.000	0.272	<b>6.3</b>	0.050	0.634
	$U[10.0C', 20.0C']$	6.8	0.100	0.114	<b>6.7</b>	0.050	0.130	6.8	0.000	0.249	<b>6.7</b>	0.050	0.580
50	$U[0.0C', 0.5C']$	6.7	0.150	0.240	6.9	0.050	0.219	<b>6.6</b>	0.000	0.404	<b>6.6</b>	0.000	0.677
	$U[0.5C', 1.0C']$	<b>6.6</b>	0.000	0.260	<b>6.6</b>	0.100	0.251	6.7	0.150	0.389	<b>6.6</b>	0.000	0.711
	$U[1.0C', 4.0C']$	6.3	0.050	0.255	<b>6.1</b>	0.050	0.284	<b>6.1</b>	0.050	0.448	6.2	0.000	0.768
	$U[0.0C', 5.0C']$	6.2	0.000	0.263	6.2	0.000	0.270	<b>6.1</b>	0.050	0.475	6.2	0.000	0.750
	$U[5.0C', 10.0C']$	6.3	0.050	0.270	6.3	0.050	0.279	6.4	0.000	0.414	<b>6.2</b>	0.000	0.795
	$U[10.0C', 20.0C']$	6.4	0.000	0.303	<b>6.3</b>	0.150	0.329	6.6	0.100	0.426	<b>6.3</b>	0.050	0.844

**Table 6.17:** Results for a set of 5 instances of size 80 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	8.2	0.027	0.021	8.2	0.038	0.023	<b>8.2</b>	0.055	0.225	8.3	0.044	0.613
	$U[0.5C', 1.0C']$	8.1	0.016	0.020	8.1	0.033	0.021	8.1	0.050	0.226	<b>8.1</b>	0.050	0.666
	$U[1.0C', 4.0C']$	<b>8.0</b>	0.038	0.021	8.1	0.016	0.021	8.1	0.027	0.227	8.0	0.016	0.667
	$U[0.0C', 5.0C']$	8.0	0.038	0.022	8.1	0.005	0.021	8.1	0.027	0.229	<b>8.0</b>	0.033	0.672
	$U[5.0C', 10.0C']$	<b>8.1</b>	0.033	0.021	8.1	0.038	0.022	8.1	0.016	0.222	8.1	0.022	0.641
	$U[10.0C', 20.0C']$	8.2	0.016	0.021	8.2	0.016	0.021	8.2	0.016	0.226	<b>8.1</b>	0.027	0.628
10	$U[0.0C', 0.5C']$	<b>8.1</b>	0.038	0.165	8.2	0.055	0.150	8.2	0.033	0.347	8.2	0.027	0.776
	$U[0.5C', 1.0C']$	<b>7.9</b>	0.022	0.160	8.0	0.038	0.155	8.0	0.050	0.359	8.0	0.033	0.787
	$U[1.0C', 4.0C']$	7.8	0.011	0.155	7.8	0.016	0.159	<b>7.8</b>	0.011	0.389	7.9	0.016	0.819
	$U[0.0C', 5.0C']$	7.9	0.016	0.158	7.9	0.011	0.156	<b>7.8</b>	0.000	0.380	<b>7.8</b>	0.011	0.835
	$U[5.0C', 10.0C']$	7.9	0.022	0.159	7.9	0.022	0.165	7.9	0.022	0.376	<b>7.9</b>	0.016	0.832
	$U[10.0C', 20.0C']$	8.0	0.033	0.170	8.0	0.038	0.169	<b>7.9</b>	0.027	0.390	8.0	0.027	0.814
20	$U[0.0C', 0.5C']$	<b>8.1</b>	0.050	0.332	8.2	0.044	0.319	8.2	0.077	0.525	8.2	0.038	0.892
	$U[0.5C', 1.0C']$	8.0	0.055	0.309	8.1	0.038	0.292	<b>8.0</b>	0.027	0.505	8.0	0.033	0.930
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.298	7.8	0.005	0.315	7.8	0.011	0.518	7.8	0.005	0.968
	$U[0.0C', 5.0C']$	7.8	0.005	0.293	7.8	0.005	0.305	<b>7.8</b>	0.011	0.517	<b>7.8</b>	0.000	0.982
	$U[5.0C', 10.0C']$	<b>7.8</b>	0.005	0.320	<b>7.8</b>	0.005	0.332	7.8	0.000	0.530	7.9	0.016	0.994
	$U[10.0C', 20.0C']$	7.9	0.011	0.322	<b>7.9</b>	0.027	0.348	8.0	0.022	0.528	7.9	0.005	0.987
50	$U[0.0C', 0.5C']$	<b>8.1</b>	0.044	0.834	8.2	0.050	0.767	8.2	0.072	0.932	8.2	0.050	1.324
	$U[0.5C', 1.0C']$	8.0	0.044	0.723	8.0	0.027	0.715	<b>7.9</b>	0.016	0.913	8.0	0.044	1.321
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.710	<b>7.8</b>	0.000	0.710	<b>7.8</b>	0.000	0.915	7.8	0.000	1.366
	$U[0.0C', 5.0C']$	<b>7.8</b>	0.011	0.716	<b>7.8</b>	0.011	0.721	<b>7.8</b>	0.000	0.929	7.8	0.005	1.378
	$U[5.0C', 10.0C']$	7.8	0.000	0.764	<b>7.8</b>	0.005	0.784	7.8	0.011	0.963	7.8	0.011	1.432
	$U[10.0C', 20.0C']$	7.9	0.022	0.814	7.9	0.011	0.844	7.9	0.016	1.028	<b>7.8</b>	0.011	1.481

**Table 6.18:** Results for a set of 5 instances of size 90 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>7.6</b>	0.100	0.018	7.8	0.000	0.017	<b>7.6</b>	0.100	0.240	7.8	0.100	0.735
	$U[0.5C', 1.0C']$	7.8	0.000	0.017	7.8	0.000	0.016	7.7	0.050	0.234	<b>7.6</b>	0.100	0.801
	$U[1.0C', 4.0C']$	<b>7.7</b>	0.050	0.017	7.8	0.000	0.016	7.8	0.000	0.226	<b>7.7</b>	0.050	0.767
	$U[0.0C', 5.0C']$	7.8	0.000	0.016	7.8	0.000	0.016	7.8	0.000	0.218	<b>7.7</b>	0.050	0.774
	$U[5.0C', 10.0C']$	7.9	0.050	0.016	7.8	0.000	0.017	<b>7.7</b>	0.050	0.223	7.8	0.000	0.716
	$U[10.0C', 20.0C']$	8.0	0.000	0.016	8.0	0.000	0.016	<b>7.9</b>	0.050	0.219	8.0	0.000	0.711
10	$U[0.0C', 0.5C']$	7.8	0.100	0.104	<b>7.6</b>	0.100	0.111	<b>7.6</b>	0.100	0.325	7.7	0.050	0.834
	$U[0.5C', 1.0C']$	<b>7.6</b>	0.000	0.102	<b>7.6</b>	0.100	0.103	7.7	0.050	0.321	7.7	0.050	0.859
	$U[1.0C', 4.0C']$	7.2	0.000	0.120	<b>7.1</b>	0.050	0.122	<b>7.1</b>	0.050	0.375	7.2	0.100	1.019
	$U[0.0C', 5.0C']$	7.1	0.050	0.124	<b>7.0</b>	0.000	0.118	7.1	0.050	0.373	7.1	0.050	1.022
	$U[5.0C', 10.0C']$	7.4	0.100	0.138	<b>7.3</b>	0.050	0.136	7.4	0.200	0.357	7.4	0.200	0.929
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.110	<b>7.8</b>	0.000	0.114	<b>7.8</b>	0.000	0.321	<b>7.8</b>	0.000	0.840
20	$U[0.0C', 0.5C']$	7.6	0.100	0.210	7.8	0.000	0.196	<b>7.3</b>	0.050	0.466	7.6	0.100	0.963
	$U[0.5C', 1.0C']$	7.5	0.050	0.229	7.6	0.100	0.201	7.5	0.150	0.430	<b>7.3</b>	0.150	1.045
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.233	<b>7.0</b>	0.000	0.232	<b>7.0</b>	0.000	0.483	<b>7.0</b>	0.000	1.139
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.232	<b>7.0</b>	0.000	0.227	7.2	0.100	0.460	<b>7.0</b>	0.000	1.110
	$U[5.0C', 10.0C']$	7.3	0.050	0.259	<b>7.1</b>	0.050	0.261	7.3	0.050	0.471	7.3	0.050	1.061
	$U[10.0C', 20.0C']$	7.8	0.000	0.212	7.8	0.000	0.216	<b>7.5</b>	0.150	0.471	<b>7.5</b>	0.150	1.003
50	$U[0.0C', 0.5C']$	7.8	0.100	0.478	7.7	0.050	0.502	<b>7.5</b>	0.150	0.729	7.6	0.100	1.272
	$U[0.5C', 1.0C']$	<b>7.2</b>	0.100	0.658	7.4	0.100	0.582	7.6	0.100	0.695	7.6	0.100	1.239
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.544	<b>7.0</b>	0.000	0.523	<b>7.0</b>	0.000	0.764	<b>7.0</b>	0.000	1.427
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.521	<b>7.0</b>	0.000	0.538	<b>7.0</b>	0.000	0.758	<b>7.0</b>	0.000	1.422
	$U[5.0C', 10.0C']$	<b>7.0</b>	0.000	0.622	7.1	0.050	0.582	<b>7.0</b>	0.000	0.827	7.1	0.050	1.455
	$U[10.0C', 20.0C']$	7.7	0.050	0.539	<b>7.5</b>	0.050	0.579	7.7	0.050	0.735	7.6	0.100	1.303

**Table 6.19:** Results for a set of 5 instances of size 100 and density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>8.7</b>	0.050	0.032	9.0	0.000	0.029	8.9	0.050	0.349	8.9	0.050	1.201
	$U[0.5C', 1.0C']$	<b>8.8</b>	0.100	0.030	9.0	0.000	0.028	8.9	0.050	0.337	8.9	0.050	1.213
	$U[1.0C', 4.0C']$	<b>8.7</b>	0.050	0.032	8.9	0.050	0.029	9.0	0.000	0.341	8.9	0.050	1.173
	$U[0.0C', 5.0C']$	9.0	0.000	0.028	<b>8.7</b>	0.050	0.031	8.8	0.100	0.348	8.8	0.000	1.211
	$U[5.0C', 10.0C']$	9.0	0.000	0.029	<b>8.9</b>	0.050	0.029	9.0	0.000	0.337	9.0	0.000	1.139
	$U[10.0C', 20.0C']$	<b>9.0</b>	0.000	0.031	<b>9.0</b>	0.000	0.030	<b>9.0</b>	0.000	0.349	<b>9.0</b>	0.000	1.144
10	$U[0.0C', 0.5C']$	8.9	0.050	0.202	<b>8.6</b>	0.000	0.242	8.8	0.000	0.513	8.8	0.100	1.412
	$U[0.5C', 1.0C']$	<b>8.7</b>	0.050	0.237	8.9	0.050	0.192	<b>8.7</b>	0.050	0.532	<b>8.7</b>	0.050	1.390
	$U[1.0C', 4.0C']$	<b>8.5</b>	0.050	0.216	8.6	0.000	0.205	8.6	0.000	0.528	8.6	0.000	1.473
	$U[0.0C', 5.0C']$	8.6	0.000	0.211	8.6	0.000	0.208	8.6	0.000	0.525	<b>8.5</b>	0.050	1.498
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.218	<b>8.6</b>	0.000	0.213	<b>8.6</b>	0.000	0.554	8.7	0.050	1.433
	$U[10.0C', 20.0C']$	8.9	0.050	0.221	<b>8.8</b>	0.100	0.231	8.9	0.050	0.535	<b>8.8</b>	0.000	1.388
20	$U[0.0C', 0.5C']$	8.7	0.050	0.436	8.8	0.100	0.388	8.8	0.000	0.718	<b>8.6</b>	0.000	1.677
	$U[0.5C', 1.0C']$	8.8	0.100	0.376	8.7	0.050	0.413	<b>8.6</b>	0.000	0.760	8.9	0.050	1.494
	$U[1.0C', 4.0C']$	8.6	0.000	0.408	8.6	0.000	0.390	<b>8.5</b>	0.050	0.744	<b>8.5</b>	0.050	1.694
	$U[0.0C', 5.0C']$	8.6	0.000	0.411	8.6	0.000	0.394	<b>8.5</b>	0.050	0.740	<b>8.5</b>	0.050	1.679
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.438	<b>8.6</b>	0.000	0.434	<b>8.6</b>	0.000	0.750	<b>8.6</b>	0.000	1.667
	$U[10.0C', 20.0C']$	8.9	0.050	0.421	8.8	0.000	0.434	<b>8.6</b>	0.000	0.787	8.8	0.000	1.666
50	$U[0.0C', 0.5C']$	8.8	0.100	0.944	<b>8.6</b>	0.000	0.977	<b>8.6</b>	0.000	1.342	8.7	0.050	2.152
	$U[0.5C', 1.0C']$	8.7	0.050	1.009	8.7	0.050	0.947	8.7	0.050	1.374	<b>8.6</b>	0.000	2.193
	$U[1.0C', 4.0C']$	8.6	0.000	0.949	8.6	0.000	0.943	<b>8.5</b>	0.050	1.312	8.6	0.000	2.212
	$U[0.0C', 5.0C']$	<b>8.4</b>	0.100	1.113	8.6	0.000	0.965	8.5	0.050	1.354	8.5	0.050	2.249
	$U[5.0C', 10.0C']$	8.6	0.000	0.998	<b>8.5</b>	0.050	1.047	8.6	0.000	1.300	8.6	0.000	2.232
	$U[10.0C', 20.0C']$	<b>8.6</b>	0.000	1.060	<b>8.6</b>	0.000	1.117	8.7	0.050	1.363	<b>8.6</b>	0.000	2.318

**Table 6.20:** Results for a set of 5 instances of size 120 and density 0.5



Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.25C', 0.75C']$	53.00	0.000	32.675	52.40	0.240	45.955	<b>52.00</b>	0.000	41.061	52.80	0.160	131.369
	$U[0.0C', 1.0C']$	53.00	0.000	31.822	53.00	0.000	31.307	<b>52.60</b>	0.240	38.698	53.00	0.000	124.405
	$U[0.0C', 0.5C']$	<b>52.00</b>	0.000	49.998	52.60	0.240	40.508	<b>52.00</b>	0.000	47.886	52.80	0.160	133.227
	$U[0.5C', 1.0C']$	52.60	0.240	34.443	52.40	0.240	34.702	53.00	0.000	34.162	<b>52.00</b>	0.000	151.490
	$U[0.25C', 1.0C']$	53.00	0.000	33.513	<b>52.40</b>	0.240	34.969	53.00	0.000	35.207	53.00	0.000	126.708
	$U[0.0C', 0.75C']$	<b>52.40</b>	0.240	37.164	<b>52.40</b>	0.240	42.677	<b>52.40</b>	0.240	43.436	<b>52.40</b>	0.20	137.571
5	$U[0.25C', 0.75C']$	52.00	0.000	175.018	<b>51.60</b>	0.240	222.313	52.00	0.000	167.028	52.00	0.000	263.785
	$U[0.0C', 1.0C']$	<b>51.40</b>	0.240	192.997	52.00	0.000	166.878	52.00	0.000	168.842	52.00	0.000	270.638
	$U[0.0C', 0.5C']$	<b>51.40</b>	0.240	188.598	<b>51.40</b>	0.240	194.900	<b>51.40</b>	0.240	234.086	52.00	0.000	267.352
	$U[0.5C', 1.0C']$	52.00	0.000	167.531	<b>51.40</b>	0.240	197.548	<b>51.40</b>	0.250	191.290	<b>51.40</b>	0.250	301.400
	$U[0.25C', 1.0C']$	<b>51.40</b>	0.240	227.016	<b>51.40</b>	0.240	190.509	<b>51.40</b>	0.240	226.595	52.00	0.000	273.516
	$U[0.0C', 0.75C']$	52.00	0.000	166.399	<b>51.00</b>	0.000	243.717	51.80	0.160	188.605	51.80	0.160	323.258
10	$U[0.25C', 0.75C']$	<b>51.00</b>	0.000	460.830	<b>51.00</b>	0.000	499.770	51.80	0.160	385.627	<b>51.00</b>	0.000	580.569
	$U[0.0C', 1.0C']$	<b>51.00</b>	0.000	445.430	<b>51.00</b>	0.000	441.279	<b>51.00</b>	0.000	556.089	<b>51.00</b>	0.000	1920.230
	$U[0.0C', 0.5C']$	<b>51.00</b>	0.000	438.898	<b>51.00</b>	0.000	506.921	<b>51.00</b>	0.000	491.451	<b>51.00</b>	0.000	574.642
	$U[0.5C', 1.0C']$	<b>51.00</b>	0.000	434.163	51.40	0.240	368.134	<b>51.00</b>	0.000	467.440	<b>51.00</b>	0.000	617.425
	$U[0.25C', 1.0C']$	51.80	0.160	365.680	<b>51.00</b>	0.000	424.486	51.60	0.240	365.924	51.40	0.240	441.895
	$U[0.0C', 0.75C']$	<b>51.00</b>	0.000	425.001	<b>51.00</b>	0.000	526.258	51.40	0.240	353.324	51.40	0.240	479.716

**Table 6.21:** Results for the instance *dsjc500.5-1*, 500 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.25C', 0.75C']$	47.60	0.240	72.492	<b>47.00</b>	0.000	69.580	<b>47.00</b>	0.000	121.744	48.00	0.000	702.030
	$U[0.0C', 1.0C']$	<b>47.00</b>	0.000	77.980	<b>47.00</b>	0.000	78.475	<b>47.00</b>	0.000	126.041	48.00	0.000	722.381
	$U[0.0C', 0.5C']$	<b>47.00</b>	0.000	77.742	47.60	0.240	71.457	<b>47.00</b>	0.000	118.374	48.00	0.000	647.080
	$U[0.5C', 0.1C']$	47.20	0.160	80.400	<b>47.00</b>	0.000	72.520	47.40	0.240	102.338	<b>47.00</b>	0.000	761.492
	$U[0.25C', 1.0C']$	47.20	0.160	66.099	<b>47.00</b>	0.000	85.592	<b>47.00</b>	0.000	118.374	47.40	0.240	752.992
	$U[0.0C', 0.75C']$	47.60	0.240	74.900	47.20	0.160	65.508	<b>47.00</b>	0.000	118.374	47.60	0.240	669.113
5	$U[0.25C', 0.75C']$	<b>47.00</b>	0.000	329.858	<b>47.00</b>	0.000	322.690	<b>47.00</b>	0.000	341.401	<b>47.00</b>	0.000	911.078
	$U[0.0C', 1.0C']$	<b>47.00</b>	0.000	314.639	<b>47.00</b>	0.000	312.814	<b>47.00</b>	0.000	329.006	<b>47.00</b>	0.000	909.702
	$U[0.0C', 0.5C']$	<b>47.00</b>	0.000	336.046	<b>47.00</b>	0.000	368.138	<b>47.00</b>	0.000	363.048	<b>47.00</b>	0.000	893.171
	$U[0.5C', 0.1C']$	47.00	0.000	329.366	<b>46.60</b>	0.240	355.754	47.00	0.000	369.581	47.00	0.000	939.086
	$U[0.25C', 1.0C']$	47.00	0.000	320.355	<b>46.60</b>	0.240	504.976	47.00	0.000	312.525	47.00	0.000	888.474
	$U[0.0C', 0.75C']$	<b>47.00</b>	0.000	319.387	<b>47.00</b>	0.000	321.539	<b>47.00</b>	0.000	381.417	<b>47.00</b>	0.000	899.100
10	$U[0.25C', 0.75C']$	<b>47.00</b>	0.000	650.358	<b>47.00</b>	0.000	600.810	<b>47.00</b>	0.000	782.341	<b>47.00</b>	0.000	1340.031
	$U[0.0C', 1.0C']$	<b>46.40</b>	0.240	657.860	<b>46.40</b>	0.240	771.054	47.00	0.000	842.114	47.00	0.000	1112.001
	$U[0.0C', 0.5C']$	47.00	0.000	651.221	<b>46.60</b>	0.240	712.167	47.00	0.000	666.702	47.00	0.000	1260.525
	$U[0.5C', 0.1C']$	47.00	0.000	610.881	46.40	0.240	655.999	<b>46.00</b>	0.000	1090.255	47.00	0.000	1332.405
	$U[0.25C', 1.0C']$	<b>46.40</b>	0.240	811.390	47.00	0.000	618.188	47.00	0.000	699.847	47.00	0.000	1201.992
	$U[0.0C', 0.75C']$	47.00	0.000	630.789	<b>46.60</b>	0.240	700.052	47.00	0.000	947.012	47.00	0.000	1417.935

**Table 6.22:** Results for the instance *dsjc500.5-2*, 1000 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.25C', 0.75C']$	<b>45.00</b>	0.000	115.018	<b>45.00</b>	0.000	115.547	<b>45.00</b>	0.000	205.556	<b>45.00</b>	0.000	3110.445
	$U[0.0C', 1.0C']$	<b>45.00</b>	0.000	124.909	<b>45.00</b>	0.000	119.600	<b>45.00</b>	0.000	245.091	<b>45.00</b>	0.000	3101.009
	$U[0.0C', 0.5C']$	<b>45.00</b>	0.000	117.858	<b>45.00</b>	0.000	130.135	<b>45.00</b>	0.000	199.366	<b>45.00</b>	0.000	2960.582
	$U[0.5C', 1.0C']$	45.00	0.000	115.410	45.00	0.000	113.664	45.00	0.00	210.217	<b>44.60</b>	0.240	3227.712
	$U[0.25C', 1.0C']$	<b>45.00</b>	0.000	113.382	<b>45.00</b>	0.000	119.870	<b>45.00</b>	0.000	182.176	<b>45.00</b>	0.000	3298.511
	$U[0.0C', 0.75C']$	<b>45.00</b>	0.000	113.593	<b>45.00</b>	0.000	114.424	45.40	0.240	224.569	<b>45.00</b>	0.000	3205.006
5	$U[0.25C', 0.75C']$	<b>44.00</b>	0.000	775.920	44.20	0.160	644.067	<b>44.00</b>	0.000	712.253	<b>44.00</b>	0.000	3801.001
	$U[0.0C', 1.0C']$	<b>44.00</b>	0.000	692.584	44.40	0.240	590.315	<b>44.00</b>	0.000	736.866	<b>44.00</b>	0.000	3989.886
	$U[0.0C', 0.5C']$	<b>44.00</b>	0.000	887.549	44.60	0.240	569.560	<b>44.00</b>	0.000	770.456	<b>44.00</b>	0.000	4252.012
	$U[0.5C', 1.0C']$	44.40	0.240	535.744	<b>44.00</b>	0.000	981.468	44.80	0.160	631.256	44.60	0.240	3525.024
	$U[0.25C', 1.0C']$	44.80	0.160	623.681	44.60	0.240	547.132	<b>44.00</b>	0.000	705.102	<b>44.00</b>	0.000	3612.583
	$U[0.0C', 0.75C']$	44.60	0.240	716.675	44.40	0.240	564.862	<b>44.00</b>	0.000	681.207	<b>44.00</b>	0.000	4328.666
10	$U[0.25C', 0.75C']$	<b>44.00</b>	0.000	1711.281	<b>44.00</b>	0.000	1284.477	<b>44.00</b>	0.000	991.903	<b>44.00</b>	0.000	4385.033
	$U[0.0C', 1.0C']$	<b>44.00</b>	0.000	1609.692	44.20	0.160	1398.949	<b>44.00</b>	0.000	1013.891	<b>44.00</b>	0.000	4441.113
	$U[0.0C', 0.5C']$	<b>44.00</b>	0.000	1504.814	<b>44.00</b>	0.000	1624.962	<b>44.00</b>	0.000	1271.812	<b>44.00</b>	0.000	4641.228
	$U[0.5C', 1.0C']$	44.60	0.240	1156.730	<b>44.00</b>	0.000	1739.053	44.40	0.240	1163.476	<b>44.00</b>	0.000	4935.751
	$U[0.25C', 1.0C']$	<b>44.00</b>	0.000	1282.196	<b>44.00</b>	0.000	1772.908	44.60	0.240	1351.233	<b>44.00</b>	0.000	5011.565
	$U[0.0C', 0.75C']$	<b>44.00</b>	0.000	1928.361	<b>44.00</b>	0.000	1792.404	44.40	0.240	1291.088	<b>44.00</b>	0.000	4505.852

**Table 6.23:** Results for the instance *dsjc500.5-3*, 1500 nodes, density 0.5

Parameters		Random			OneStepCD			ILP1			ILP2		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.25C', 0.75C']$	43.40	0.240	200.361	<b>43.00</b>	0.000	178.387	<b>43.00</b>	0.000	311.715	43.20	0.160	7530.236
	$U[0.0C', 1.0C']$	43.20	0.160	185.824	<b>43.00</b>	0.000	179.494	<b>43.00</b>	0.000	341.112	43.80	0.160	7012.431
	$U[0.0C', 0.5C']$	44.00	0.000	143.937	43.60	0.240	186.869	<b>43.00</b>	0.000	333.319	43.80	0.160	7171.777
	$U[0.5C', 1.0C']$	<b>43.00</b>	0.000	214.545	43.60	0.240	142.063	<b>43.00</b>	0.000	338.309	43.60	0.240	7439.572
	$U[0.25C', 1.0C']$	43.60	0.240	145.004	<b>43.00</b>	0.000	251.143	<b>43.00</b>	0.000	302.122	44.00	0.000	7621.502
	$U[0.0C', 0.75C']$	43.60	0.240	218.761	43.20	0.160	187.378	<b>43.00</b>	0.000	335.834	43.40	0.240	7478.121
5	$U[0.25C', 0.75C']$	<b>43.00</b>	0.000	780.378	<b>43.00</b>	0.000	963.153	<b>43.00</b>	0.000	978.203	<b>43.00</b>	0.000	8956.555
	$U[0.0C', 1.0C']$	<b>42.40</b>	0.240	849.693	42.80	0.160	771.423	43.00	0.000	1122.429	42.80	0.160	8701.683
	$U[0.0C', 0.5C']$	<b>43.00</b>	0.000	808.077	<b>43.00</b>	0.000	848.376	<b>43.00</b>	0.000	1006.318	<b>43.00</b>	0.000	8622.681
	$U[0.5C', 1.0C']$	<b>43.00</b>	0.000	781.823	<b>43.00</b>	0.000	823.835	<b>43.00</b>	0.000	958.897	<b>43.00</b>	0.000	9115.008
	$U[0.25C', 1.0C']$	<b>43.00</b>	0.000	776.229	<b>43.00</b>	0.000	806.833	<b>43.00</b>	0.000	989.006	<b>43.00</b>	0.000	9012.904
	$U[0.0C', 0.75C']$	<b>43.00</b>	0.000	823.599	<b>43.00</b>	0.000	809.126	<b>43.00</b>	0.000	1007.065	<b>43.00</b>	0.000	8848.775
10	$U[0.25C', 0.75C']$	<b>43.00</b>	0.000	1619.182	<b>43.00</b>	0.000	1711.629	<b>43.00</b>	0.000	1674.249	<b>43.00</b>	0.000	9246.012
	$U[0.0C', 1.0C']$	43.00	0.000	1570.523	43.00	0.000	1628.829	<b>42.80</b>	0.160	1569.004	43.00	0.000	9006.112
	$U[0.0C', 0.5C']$	43.00	0.000	1532.691	<b>42.40</b>	0.240	2145.856	43.00	0.000	1762.306	43.00	0.000	9176.079
	$U[0.5C', 1.0C']$	<b>42.80</b>	0.180	1554.850	43.00	0.000	1558.656	<b>42.80</b>	0.160	1710.961	<b>43.00</b>	0.000	9047.702
	$U[0.25C', 1.0C']$	43.00	0.000	1567.553	<b>42.60</b>	0.240	1799.667	43.00	0.000	1821.938	43.00	0.000	9312.528
	$U[0.0C', 0.75C']$	<b>43.00</b>	0.000	1529.261	<b>43.00</b>	0.000	1554.409	<b>43.00</b>	0.000	1701.039	<b>43.00</b>	0.000	9199.499

**Table 6.24:** Results for the instance *dsjc500.5-4*, 2000 nodes, density 0.5

## Variants

As discussed in section 5.4, variants for both ILPs have been created by removing the inequation that restricts conflicts inside the recolored set of clusters. In tables 6.25 to 6.30 the standard ILPs marked as *ILP1* and *ILP2* are compared to their variants *ILP1\** and *ILP2\** by evaluating three instances of different size as well as three instances of different density. It can be seen that removing the aforementioned constraint does not increase the solution quality.

Furthermore experiments have been performed, placing the recently recolored set of clusters on the tabu list. In tables 6.31 to 6.36 sets diversing in size and density have been evaluated using  $F_{max} = 5$ . The parameter *TTRecolored* sets the number of iterations as  $Tabu_{size} = TTRecolored * C'$  for all the node-color pairs of the recolored set of clusters to remain on the tabu list.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>10.3</b>	0.150	0.312	10.6	0.100	0.135	10.5	0.150	0.823	10.4	0.100	0.602
	$U[0.5C', 1.0C']$	10.4	0.000	0.302	10.4	0.100	0.131	<b>10.2</b>	0.100	0.875	10.3	0.050	0.617
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.326	10.1	0.050	0.151	10.2	0.100	0.868	10.2	0.000	0.628
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.337	10.1	0.050	0.138	10.2	0.100	0.917	10.3	0.050	0.632
	$U[5.0C', 10.0C']$	10.3	0.150	0.308	<b>10.2</b>	0.000	0.142	10.4	0.100	0.820	10.3	0.050	0.610
	$U[10.0C', 20.0C']$	<b>10.5</b>	0.150	0.295	10.9	0.050	0.112	10.6	0.100	0.793	10.7	0.050	0.545
10	$U[0.0C', 0.5C']$	<b>10.2</b>	0.100	0.542	10.7	0.050	0.295	10.7	0.150	0.952	10.3	0.050	0.799
	$U[0.5C', 1.0C']$	10.2	0.100	0.508	<b>10.1</b>	0.050	0.356	10.2	0.100	1.096	10.2	0.000	0.800
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.493	<b>10.0</b>	0.000	0.314	<b>10.0</b>	0.000	1.098	<b>10.0</b>	0.000	0.827
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.493	<b>10.0</b>	0.000	0.319	<b>10.0</b>	0.000	1.081	<b>10.0</b>	0.000	0.823
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.500	<b>10.0</b>	0.000	0.322	<b>10.0</b>	0.000	1.109	<b>10.0</b>	0.000	0.828
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.531	10.1	0.050	0.362	<b>10.0</b>	0.000	1.114	<b>10.0</b>	0.000	0.871
20	$U[0.0C', 0.5C']$	10.4	0.100	0.738	10.7	0.150	0.491	<b>10.3</b>	0.150	1.327	10.4	0.200	0.991
	$U[0.5C', 1.0C']$	<b>10.1</b>	0.050	0.745	<b>10.1</b>	0.050	0.587	<b>10.1</b>	0.050	1.345	<b>10.1</b>	0.050	1.023
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.673	<b>10.0</b>	0.000	0.499	<b>10.0</b>	0.000	1.281	<b>10.0</b>	0.000	0.993
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.682	<b>10.0</b>	0.000	0.494	<b>10.0</b>	0.000	1.290	<b>10.0</b>	0.000	1.001
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.690	<b>10.0</b>	0.000	0.517	<b>10.0</b>	0.000	1.289	<b>10.0</b>	0.000	1.013
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.721	<b>10.0</b>	0.000	0.548	<b>10.0</b>	0.000	1.320	<b>10.0</b>	0.000	1.048
50	$U[0.0C', 0.5C']$	10.3	0.150	1.386	10.6	0.100	1.105	10.7	0.150	1.694	<b>10.2</b>	0.000	1.709
	$U[0.5C', 1.0C']$	10.3	0.050	1.346	<b>10.0</b>	0.000	1.199	<b>10.0</b>	0.000	1.948	10.2	0.100	1.526
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	1.207	<b>10.0</b>	0.000	1.060	<b>10.0</b>	0.000	1.778	<b>10.0</b>	0.000	1.523
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	1.211	<b>10.0</b>	0.000	1.058	<b>10.0</b>	0.000	1.812	<b>10.0</b>	0.000	1.533
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	1.231	<b>10.0</b>	0.000	1.093	<b>10.0</b>	0.000	1.837	<b>10.0</b>	0.000	1.553
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	1.295	<b>10.0</b>	0.000	1.176	<b>10.0</b>	0.000	1.910	<b>10.0</b>	0.000	1.677

**Table 6.25:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.7 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	13.1	0.150	0.394	<b>12.8</b>	0.000	0.215	12.9	0.050	1.045	<b>12.8</b>	0.000	0.851
	$U[0.5C', 1.0C']$	<b>12.7</b>	0.050	0.439	12.8	0.100	0.204	<b>12.7</b>	0.050	1.088	<b>12.7</b>	0.050	0.848
	$U[1.0C', 4.0C']$	12.7	0.050	0.426	12.8	0.000	0.213	<b>12.6</b>	0.000	1.081	12.7	0.050	0.870
	$U[0.0C', 5.0C']$	12.7	0.050	0.446	12.7	0.050	0.221	12.8	0.000	1.048	<b>12.5</b>	0.050	0.882
	$U[5.0C', 10.0C']$	<b>12.7</b>	0.050	0.426	12.8	0.000	0.203	<b>12.7</b>	0.050	1.094	12.8	0.000	0.832
	$U[10.0C', 20.0C']$	<b>12.8</b>	0.000	0.426	<b>12.8</b>	0.000	0.195	<b>12.8</b>	0.100	1.069	<b>12.8</b>	0.000	0.850
10	$U[0.0C', 0.5C']$	12.8	0.000	0.720	<b>12.7</b>	0.050	0.546	12.9	0.050	1.294	13.0	0.000	1.049
	$U[0.5C', 1.0C']$	12.7	0.050	0.691	12.5	0.150	0.556	12.5	0.050	1.417	<b>12.4</b>	0.000	1.208
	$U[1.0C', 4.0C']$	12.2	0.100	0.746	12.1	0.050	0.543	12.2	0.100	1.508	<b>12.0</b>	0.000	1.321
	$U[0.0C', 5.0C']$	12.2	0.100	0.765	<b>12.1</b>	0.050	0.521	12.2	0.100	1.522	<b>12.2</b>	0.000	1.218
	$U[5.0C', 10.0C']$	12.4	0.000	0.745	<b>12.1</b>	0.050	0.580	12.3	0.050	1.525	12.2	0.000	1.284
	$U[10.0C', 20.0C']$	<b>12.4</b>	0.000	0.779	<b>12.4</b>	0.100	0.529	12.5	0.050	1.439	12.5	0.150	1.208
20	$U[0.0C', 0.5C']$	13.0	0.000	0.930	13.1	0.150	0.787	12.8	0.200	1.671	<b>12.7</b>	0.050	1.435
	$U[0.5C', 1.0C']$	<b>12.5</b>	0.050	1.068	12.7	0.050	0.779	12.8	0.000	1.571	12.6	0.000	1.398
	$U[1.0C', 4.0C']$	12.1	0.050	1.101	<b>12.0</b>	0.000	0.949	12.2	0.000	1.778	<b>12.0</b>	0.000	1.620
	$U[0.0C', 5.0C']$	<b>12.0</b>	0.000	1.178	<b>12.0</b>	0.000	0.848	12.1	0.050	1.876	<b>12.0</b>	0.000	1.634
	$U[5.0C', 10.0C']$	12.2	0.000	1.088	12.2	0.100	0.858	12.2	0.000	1.813	<b>12.1</b>	0.050	1.657
	$U[10.0C', 20.0C']$	<b>12.2</b>	0.100	1.170	12.5	0.050	0.899	12.6	0.100	1.771	<b>12.2</b>	0.000	1.692
50	$U[0.0C', 0.5C']$	12.9	0.050	1.930	13.1	0.050	1.870	13.0	0.100	2.468	<b>12.8</b>	0.100	2.313
	$U[0.5C', 1.0C']$	12.8	0.000	1.716	<b>12.6</b>	0.100	1.776	12.7	0.050	2.487	12.7	0.050	2.294
	$U[1.0C', 4.0C']$	<b>12.0</b>	0.000	2.094	<b>12.0</b>	0.000	1.753	<b>12.0</b>	0.000	2.765	<b>12.0</b>	0.000	2.463
	$U[0.0C', 5.0C']$	<b>12.0</b>	0.000	2.054	<b>12.0</b>	0.000	1.824	<b>12.0</b>	0.000	2.791	<b>12.0</b>	0.000	2.687
	$U[5.0C', 10.0C']$	<b>12.0</b>	0.000	2.201	12.1	0.050	1.844	12.1	0.050	2.837	<b>12.0</b>	0.000	2.696
	$U[10.0C', 20.0C']$	12.3	0.150	2.140	<b>12.2</b>	0.100	2.080	12.3	0.050	2.962	<b>12.2</b>	0.100	2.676

**Table 6.26:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.8 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>16.7</b>	0.250	0.450	17.0	0.100	0.235	16.9	0.250	1.001	16.8	0.300	0.877
	$U[0.5C', 1.0C']$	16.4	0.100	0.477	16.3	0.050	0.279	<b>16.2</b>	0.000	1.139	16.6	0.100	0.909
	$U[1.0C', 4.0C']$	<b>16.1</b>	0.050	0.513	16.3	0.050	0.266	16.2	0.000	1.125	16.3	0.050	0.996
	$U[0.0C', 5.0C']$	<b>16.0</b>	0.000	0.534	16.1	0.050	0.300	16.2	0.000	1.112	16.2	0.000	0.990
	$U[5.0C', 10.0C']$	<b>16.1</b>	0.050	0.510	16.3	0.050	0.252	16.2	0.000	1.096	16.2	0.000	0.998
	$U[10.0C', 20.0C']$	16.4	0.000	0.473	<b>16.1</b>	0.050	0.294	16.2	0.000	1.101	16.2	0.000	0.994
10	$U[0.0C', 0.5C']$	<b>16.6</b>	0.300	0.994	16.8	0.300	0.830	16.8	0.300	1.561	16.7	0.050	1.484
	$U[0.5C', 1.0C']$	<b>16.1</b>	0.050	0.938	16.2	0.300	0.790	16.3	0.050	1.584	16.2	0.000	1.474
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	1.019	<b>15.8</b>	0.000	0.758	<b>15.8</b>	0.000	1.731	<b>15.8</b>	0.000	1.571
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	1.005	15.9	0.050	0.763	<b>15.8</b>	0.000	1.694	16.0	0.100	1.517
	$U[5.0C', 10.0C']$	<b>15.8</b>	0.000	1.103	15.9	0.050	0.809	<b>15.8</b>	0.000	1.754	<b>15.8</b>	0.000	1.603
	$U[10.0C', 20.0C']$	16.1	0.050	1.003	<b>15.8</b>	0.000	0.949	16.0	0.000	1.701	16.0	0.000	1.563
20	$U[0.0C', 0.5C']$	17.0	0.200	1.491	17.1	0.250	1.384	17.1	0.150	2.114	<b>16.7</b>	0.050	1.980
	$U[0.5C', 1.0C']$	16.2	0.000	1.430	16.2	0.100	1.361	<b>16.0</b>	0.000	2.203	16.3	0.150	1.877
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	1.499	<b>15.8</b>	0.000	1.289	<b>15.8</b>	0.000	2.181	<b>15.8</b>	0.000	2.039
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	1.532	<b>15.8</b>	0.000	1.328	<b>15.8</b>	0.000	2.177	<b>15.8</b>	0.000	2.127
	$U[5.0C', 10.0C']$	15.9	0.050	1.546	<b>15.8</b>	0.000	1.370	<b>15.8</b>	0.000	2.227	<b>15.8</b>	0.000	2.178
	$U[10.0C', 20.0C']$	16.0	0.000	1.585	16.0	0.000	1.337	<b>15.8</b>	0.000	2.273	16.0	0.000	2.105
50	$U[0.0C', 0.5C']$	<b>16.7</b>	0.350	3.210	16.8	0.100	3.289	17.2	0.200	3.570	17.2	0.000	3.566
	$U[0.5C', 1.0C']$	16.2	0.000	2.842	16.1	0.050	2.740	16.3	0.250	3.523	<b>16.0</b>	0.000	3.637
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	2.937	<b>15.8</b>	0.000	2.775	<b>15.8</b>	0.000	3.617	<b>15.8</b>	0.000	3.526
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	2.948	<b>15.8</b>	0.000	2.766	<b>15.8</b>	0.000	3.688	<b>15.8</b>	0.000	3.526
	$U[5.0C', 10.0C']$	<b>15.8</b>	0.000	3.241	<b>15.8</b>	0.000	2.838	<b>15.8</b>	0.000	3.762	<b>15.8</b>	0.000	3.697
	$U[10.0C', 20.0C']$	<b>15.8</b>	0.000	3.457	<b>15.8</b>	0.000	3.065	<b>15.8</b>	0.000	3.903	<b>15.8</b>	0.000	3.831

**Table 6.27:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.9 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	8.2	0.055	0.216	8.2	0.050	0.100	8.3	0.044	0.604	<b>8.1</b>	0.050	0.459
	$U[0.5C', 1.0C']$	8.1	0.011	0.226	8.1	0.038	0.099	<b>8.0</b>	0.033	0.662	8.1	0.033	0.464
	$U[1.0C', 4.0C']$	8.0	0.027	0.223	8.1	0.016	0.102	8.1	0.033	0.652	<b>8.0</b>	0.022	0.473
	$U[0.0C', 5.0C']$	8.1	0.027	0.225	8.0	0.022	0.101	8.0	0.022	0.662	<b>8.0</b>	0.033	0.480
	$U[5.0C', 10.0C']$	8.1	0.033	0.221	8.1	0.050	0.102	8.1	0.027	0.646	<b>8.1</b>	0.033	0.467
	$U[10.0C', 20.0C']$	8.2	0.033	0.219	<b>8.1</b>	0.038	0.100	8.2	0.038	0.633	8.2	0.033	0.453
10	$U[0.0C', 0.5C']$	8.2	0.033	0.354	8.2	0.061	0.238	<b>8.1</b>	0.072	0.780	8.1	0.061	0.592
	$U[0.5C', 1.0C']$	8.0	0.022	0.350	8.0	0.050	0.241	8.0	0.033	0.786	<b>8.0</b>	0.033	0.608
	$U[1.0C', 4.0C']$	7.8	0.011	0.376	<b>7.8</b>	0.000	0.250	7.9	0.016	0.835	7.9	0.016	0.632
	$U[0.0C', 5.0C']$	7.9	0.022	0.378	<b>7.9</b>	0.005	0.240	<b>7.9</b>	0.016	0.820	7.9	0.011	0.631
	$U[5.0C', 10.0C']$	7.9	0.016	0.378	7.9	0.005	0.252	<b>7.9</b>	0.022	0.830	7.9	0.011	0.632
	$U[10.0C', 20.0C']$	8.0	0.027	0.376	8.0	0.016	0.255	<b>7.9</b>	0.027	0.825	8.0	0.033	0.627
20	$U[0.0C', 0.5C']$	<b>8.1</b>	0.050	0.508	8.2	0.050	0.412	8.2	0.066	0.933	8.2	0.027	0.711
	$U[0.5C', 1.0C']$	8.0	0.022	0.506	8.0	0.038	0.370	<b>8.0</b>	0.038	0.936	8.0	0.038	0.734
	$U[1.0C', 4.0C']$	7.8	0.005	0.520	7.8	0.000	0.392	7.8	0.000	0.971	<b>7.8</b>	0.005	0.782
	$U[0.0C', 5.0C']$	7.8	0.005	0.517	<b>7.8</b>	0.000	0.395	<b>7.8</b>	0.000	0.973	7.8	0.011	0.786
	$U[5.0C', 10.0C']$	<b>7.9</b>	0.005	0.533	7.9	0.011	0.408	7.9	0.011	0.976	7.9	0.000	0.778
	$U[10.0C', 20.0C']$	<b>7.9</b>	0.016	0.534	7.9	0.033	0.430	<b>7.9</b>	0.016	0.985	7.9	0.016	0.778
50	$U[0.0C', 0.5C']$	<b>8.1</b>	0.038	0.973	8.2	0.094	0.825	8.2	0.033	1.393	8.1	0.072	1.208
	$U[0.5C', 1.0C']$	8.0	0.033	0.890	8.0	0.022	0.802	8.0	0.027	1.351	<b>8.0</b>	0.027	1.181
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.011	0.922	7.8	0.005	0.798	7.8	0.005	1.384	7.8	0.005	1.185
	$U[0.0C', 5.0C']$	<b>7.8</b>	0.005	0.919	7.8	0.000	0.805	7.8	0.000	1.386	7.8	0.005	1.181
	$U[5.0C', 10.0C']$	<b>7.8</b>	0.005	0.958	7.8	0.000	0.848	7.8	0.000	1.440	<b>7.8</b>	0.005	1.242
	$U[10.0C', 20.0C']$	7.9	0.016	1.022	<b>7.8</b>	0.011	0.903	7.9	0.005	1.482	7.9	0.016	1.291

**Table 6.28:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.5 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)	obj	sd	time(s)
1	$U[0.0C', 0.5C']$	<b>7.8</b>	0.000	0.221	8.0	0.000	0.085	<b>7.8</b>	0.000	0.756	<b>7.8</b>	0.000	0.527
	$U[0.5C', 1.0C']$	<b>7.7</b>	0.050	0.216	7.8	0.100	0.105	7.8	0.000	0.744	7.8	0.000	0.531
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.232	7.9	0.050	0.088	<b>7.8</b>	0.000	0.744	<b>7.8</b>	0.000	0.529
	$U[0.0C', 5.0C']$	7.8	0.100	0.223	7.8	0.000	0.087	7.8	0.100	0.735	<b>7.7</b>	0.050	0.577
	$U[5.0C', 10.0C']$	<b>7.7</b>	0.050	0.237	<b>7.7</b>	0.050	0.096	7.9	0.050	0.699	7.9	0.050	0.527
	$U[10.0C', 20.0C']$	8.0	0.000	0.216	7.9	0.050	0.087	<b>7.8</b>	0.100	0.733	7.9	0.050	0.490
10	$U[0.0C', 0.5C']$	<b>7.6</b>	0.100	0.321	<b>7.6</b>	0.100	0.194	<b>7.6</b>	0.100	0.877	7.8	0.000	0.615
	$U[0.5C', 1.0C']$	7.6	0.000	0.324	<b>7.5</b>	0.150	0.181	7.7	0.150	0.858	<b>7.5</b>	0.150	0.700
	$U[1.0C', 4.0C']$	7.1	0.050	0.359	<b>7.0</b>	0.000	0.214	<b>7.0</b>	0.000	1.034	7.1	0.050	0.789
	$U[0.0C', 5.0C']$	7.2	0.100	0.376	<b>7.0</b>	0.000	0.206	<b>7.0</b>	0.000	1.050	7.2	0.100	0.747
	$U[5.0C', 10.0C']$	7.5	0.050	0.349	7.6	0.100	0.195	<b>7.4</b>	0.200	0.940	<b>7.4</b>	0.000	0.700
	$U[10.0C', 20.0C']$	7.8	0.000	0.320	7.8	0.000	0.181	7.8	0.000	0.831	<b>7.7</b>	0.050	0.631
20	$U[0.0C', 0.5C']$	<b>7.6</b>	0.200	0.443	<b>7.6</b>	0.100	0.270	7.7	0.050	0.952	<b>7.6</b>	0.100	0.780
	$U[0.5C', 1.0C']$	7.6	0.100	0.414	<b>7.5</b>	0.150	0.299	7.8	0.000	0.923	7.7	0.050	0.727
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.483	<b>7.0</b>	0.000	0.331	7.1	0.050	1.128	<b>7.0</b>	0.000	0.861
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.492	<b>7.0</b>	0.000	0.342	<b>7.0</b>	0.000	1.138	<b>7.0</b>	0.000	0.885
	$U[5.0C', 10.0C']$	7.5	0.150	0.438	<b>7.1</b>	0.050	0.326	7.3	0.050	1.071	<b>7.1</b>	0.050	0.870
	$U[10.0C', 20.0C']$	<b>7.7</b>	0.050	0.429	<b>7.7</b>	0.050	0.311	<b>7.7</b>	0.050	0.949	<b>7.7</b>	0.050	0.760
50	$U[0.0C', 0.5C']$	7.6	0.000	0.716	<b>7.5</b>	0.150	0.572	<b>7.5</b>	0.250	1.300	7.6	0.100	1.027
	$U[0.5C', 1.0C']$	<b>7.4</b>	0.100	0.758	7.5	0.050	0.642	7.6	0.100	1.269	7.6	0.100	1.043
	$U[1.0C', 4.0C']$	<b>7.0</b>	0.000	0.803	<b>7.0</b>	0.000	0.627	<b>7.0</b>	0.000	1.402	<b>7.0</b>	0.000	1.206
	$U[0.0C', 5.0C']$	<b>7.0</b>	0.000	0.774	<b>7.0</b>	0.000	0.672	<b>7.0</b>	0.000	1.418	<b>7.0</b>	0.000	1.188
	$U[5.0C', 10.0C']$	7.1	0.050	0.829	7.1	0.050	0.723	<b>7.0</b>	0.000	1.500	7.1	0.050	1.246
	$U[10.0C', 20.0C']$	<b>7.3</b>	0.150	0.816	<b>7.3</b>	0.150	0.756	7.5	0.050	1.399	7.5	0.050	1.169

**Table 6.29:** ILP variants compared on a set of 5 instances with 100 nodes and a density of 0.5 each.

Parameters		ILP1			ILP1*			ILP2			ILP2*		
ItMax	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i> (s)	<i>obj</i>	<i>sd</i>	<i>time</i> (s)	<i>obj</i>	<i>sd</i>	<i>time</i> (s)	<i>obj</i>	<i>sd</i>	<i>time</i> (s)
1	$U[0.0C', 0.5C']$	<b>8.8</b>	0.100	0.355	8.9	0.050	0.126	8.9	0.050	1.222	<b>8.7</b>	0.050	0.938
	$U[0.5C', 1.0C']$	<b>8.8</b>	0.000	0.360	<b>8.8</b>	0.000	0.150	8.9	0.050	1.192	<b>8.8</b>	0.100	0.923
	$U[1.0C', 4.0C']$	<b>8.7</b>	0.050	0.366	8.9	0.050	0.132	8.9	0.050	1.149	8.9	0.050	0.875
	$U[0.0C', 5.0C']$	<b>8.8</b>	0.000	0.356	8.9	0.050	0.126	9.0	0.000	1.183	<b>8.8</b>	0.100	0.934
	$U[5.0C', 10.0C']$	9.0	0.000	0.335	8.9	0.050	0.133	<b>8.6</b>	0.000	1.330	8.9	0.050	0.892
	$U[10.0C', 20.0C']$	<b>9.0</b>	0.000	0.326	<b>9.0</b>	0.000	0.121	<b>9.0</b>	0.000	1.172	<b>9.0</b>	0.000	0.831
10	$U[0.0C', 0.5C']$	<b>8.7</b>	0.050	0.511	<b>8.7</b>	0.050	0.317	8.8	0.100	1.396	8.8	0.000	1.065
	$U[0.5C', 1.0C']$	<b>8.6</b>	0.000	0.565	8.7	0.050	0.326	<b>8.6</b>	0.000	1.497	8.7	0.050	1.140
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.541	<b>8.6</b>	0.000	0.326	<b>8.6</b>	0.000	1.447	<b>8.6</b>	0.000	1.129
	$U[0.0C', 5.0C']$	8.6	0.000	0.541	8.6	0.000	0.319	<b>8.5</b>	0.050	1.486	8.6	0.000	1.136
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.566	<b>8.6</b>	0.000	0.341	<b>8.6</b>	0.000	1.451	8.8	0.100	1.089
	$U[10.0C', 20.0C']$	8.9	0.050	0.538	<b>8.8</b>	0.000	0.355	8.9	0.050	1.361	8.9	0.050	1.111
20	$U[0.0C', 0.5C']$	8.7	0.050	0.716	9.0	0.000	0.468	<b>8.6</b>	0.000	1.696	<b>8.6</b>	0.000	1.340
	$U[0.5C', 1.0C']$	8.6	0.000	0.726	8.6	0.000	0.576	<b>8.5</b>	0.050	1.670	8.6	0.000	1.325
	$U[1.0C', 4.0C']$	8.6	0.000	0.714	<b>8.5</b>	0.050	0.532	<b>8.5</b>	0.050	1.697	8.6	0.000	1.367
	$U[0.0C', 5.0C']$	8.6	0.000	0.728	8.6	0.000	0.513	<b>8.5</b>	0.050	1.699	8.6	0.000	1.356
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.726	<b>8.6</b>	0.000	0.543	<b>8.6</b>	0.000	1.659	<b>8.6</b>	0.000	1.395
	$U[10.0C', 20.0C']$	<b>8.6</b>	0.000	0.787	8.9	0.050	0.533	8.8	0.000	1.675	8.9	0.050	1.262
50	$U[0.0C', 0.5C']$	9.0	0.000	1.156	<b>8.7</b>	0.050	1.065	<b>8.7</b>	0.050	2.217	8.8	0.100	1.829
	$U[0.5C', 1.0C']$	8.8	0.000	1.213	8.7	0.050	1.081	8.8	0.000	2.165	<b>8.6</b>	0.000	1.958
	$U[1.0C', 4.0C']$	<b>8.3</b>	0.050	1.405	8.5	0.050	1.111	8.5	0.050	2.289	8.6	0.000	1.872
	$U[0.0C', 5.0C']$	8.6	0.000	1.282	<b>8.5</b>	0.050	1.154	8.6	0.000	2.169	8.6	0.000	1.928
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	1.322	<b>8.6</b>	0.000	1.109	<b>8.6</b>	0.000	2.190	<b>8.6</b>	0.000	1.952
	$U[10.0C', 20.0C']$	<b>8.6</b>	0.000	1.480	8.7	0.050	1.179	<b>8.6</b>	0.000	2.350	8.7	0.050	1.947

**Table 6.30:** ILP variants compared on a set of 5 instances with 120 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	10.3	0.050	0.222	<b>10.4</b>	0.000	0.479	10.3	0.150	1.081
	$U[0.5C', 1.0C']$	10.1	0.050	0.141	10.1	0.050	0.432	<b>10.0</b>	0.000	1.102
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.113	<b>10.0</b>	0.000	0.410	<b>10.0</b>	0.000	1.042
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.114	<b>10.0</b>	0.000	0.414	<b>10.0</b>	0.000	1.056
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.122	<b>10.0</b>	0.000	0.435	<b>10.0</b>	0.000	1.072
	$U[10.0C', 20.0C']$	10.2	0.100	0.142	10.1	0.050	0.425	10.2	0.100	1.020
0.3	$U[0.0C', 0.5C']$	10.4	0.000	0.126	10.7	0.150	0.359	10.5	0.150	0.940
	$U[0.5C', 1.0C']$	<b>10.0</b>	0.000	0.122	10.2	0.000	0.410	10.2	0.100	1.032
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.110	<b>10.0</b>	0.000	0.434	<b>10.0</b>	0.000	1.032
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.121	<b>10.0</b>	0.000	0.411	<b>10.0</b>	0.000	1.046
	$U[5.0C', 10.0C']$	10.1	0.050	0.124	<b>10.0</b>	0.000	0.440	<b>10.0</b>	0.000	1.053
	$U[10.0C', 20.0C']$	10.2	0.100	0.133	<b>10.0</b>	0.000	0.446	<b>10.0</b>	0.000	1.070
0.5	$U[0.0C', 0.5C']$	10.6	0.100	0.112	<b>10.4</b>	0.200	0.383	10.2	0.100	1.024
	$U[0.5C', 1.0C']$	10.2	0.000	0.119	<b>10.0</b>	0.000	0.419	10.3	0.050	0.983
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.114	<b>10.0</b>	0.000	0.404	<b>10.0</b>	0.000	1.020
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.131	<b>10.0</b>	0.000	0.414	<b>10.0</b>	0.000	1.057
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.118	<b>10.0</b>	0.000	0.430	<b>10.0</b>	0.000	1.058
	$U[10.0C', 20.0C']$	10.2	0.100	0.134	10.1	0.050	0.416	10.2	0.000	1.009
1.0	$U[0.0C', 0.5C']$	10.4	0.000	0.117	10.7	0.050	0.348	10.5	0.150	0.972
	$U[0.5C', 1.0C']$	10.2	0.000	0.114	10.1	0.050	0.418	<b>10.0</b>	0.000	1.081
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.113	<b>10.0</b>	0.000	0.413	<b>10.0</b>	0.000	1.050
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.112	<b>10.0</b>	0.000	0.400	<b>10.0</b>	0.000	1.058
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.122	<b>10.0</b>	0.000	0.415	<b>10.0</b>	0.000	1.077
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.161	10.1	0.050	0.421	10.1	0.050	1.117
2.0	$U[0.0C', 0.5C']$	10.6	0.000	0.117	10.6	0.100	0.367	<b>10.1</b>	0.050	1.088
	$U[0.5C', 1.0C']$	10.1	0.050	0.132	10.1	0.050	0.407	10.3	0.050	0.997
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.109	<b>10.0</b>	0.000	0.414	<b>10.0</b>	0.000	1.061
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.116	<b>10.0</b>	0.000	0.415	<b>10.0</b>	0.000	1.132
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.123	<b>10.0</b>	0.000	0.425	<b>10.0</b>	0.000	1.095
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.140	10.2	0.100	0.394	10.2	0.100	1.013
5.0	$U[0.0C', 0.5C']$	10.5	0.050	0.120	<b>10.4</b>	0.100	0.397	10.5	0.150	0.934
	$U[0.5C', 1.0C']$	<b>10.0</b>	0.000	0.112	10.1	0.050	0.414	10.2	0.100	1.031
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.110	<b>10.0</b>	0.000	0.411	<b>10.0</b>	0.000	1.043
	$U[0.0C', 5.0C']$	10.1	0.050	0.111	<b>10.0</b>	0.000	0.421	<b>10.0</b>	0.000	1.040
	$U[5.0C', 10.0C']$	<b>10.0</b>	0.000	0.127	<b>10.0</b>	0.000	0.420	<b>10.0</b>	0.000	1.044
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.137	10.1	0.050	0.431	<b>10.0</b>	0.000	1.088
10.0	$U[0.0C', 0.5C']$	<b>10.2</b>	0.000	0.146	10.5	0.150	0.395	10.6	0.100	0.905
	$U[0.5C', 1.0C']$	10.1	0.050	0.128	10.1	0.050	0.439	10.2	0.100	1.027
	$U[1.0C', 4.0C']$	<b>10.0</b>	0.000	0.114	<b>10.0</b>	0.000	0.416	<b>10.0</b>	0.000	1.060
	$U[0.0C', 5.0C']$	<b>10.0</b>	0.000	0.111	<b>10.0</b>	0.000	0.415	<b>10.0</b>	0.000	1.045
	$U[5.0C', 10.0C']$	10.1	0.050	0.118	<b>10.0</b>	0.000	0.426	<b>10.0</b>	0.000	1.057
	$U[10.0C', 20.0C']$	<b>10.0</b>	0.000	0.145	10.2	0.100	0.414	10.1	0.050	1.053

**Table 6.31:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.7 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	12.8	0.000	0.220	12.9	0.150	0.576	12.9	0.050	1.212
	$U[0.5C', 1.0C']$	<b>12.3</b>	0.050	0.207	<b>12.4</b>	0.100	0.646	12.6	0.000	1.328
	$U[1.0C', 4.0C']$	12.3	0.050	0.195	<b>12.2</b>	0.100	0.623	12.4	0.000	1.362
	$U[0.0C', 5.0C']$	<b>12.1</b>	0.050	0.224	12.3	0.050	0.616	12.3	0.050	1.386
	$U[5.0C', 10.0C']$	12.5	0.050	0.199	12.5	0.050	0.601	12.4	0.100	1.413
	$U[10.0C', 20.0C']$	12.8	0.000	0.182	12.6	0.100	0.610	12.7	0.050	1.284
0.3	$U[0.0C', 0.5C']$	12.8	0.000	0.201	<b>12.8</b>	0.000	0.563	13.0	0.100	1.201
	$U[0.5C', 1.0C']$	12.7	0.050	0.184	12.6	0.000	0.588	12.6	0.100	1.319
	$U[1.0C', 4.0C']$	12.3	0.050	0.192	12.4	0.000	0.597	12.2	0.000	1.413
	$U[0.0C', 5.0C']$	12.2	0.100	0.195	<b>12.2</b>	0.100	0.641	<b>12.2</b>	0.000	1.415
	$U[5.0C', 10.0C']$	<b>12.3</b>	0.050	0.201	12.6	0.100	0.601	<b>12.2</b>	0.000	1.417
	$U[10.0C', 20.0C']$	<b>12.6</b>	0.100	0.194	<b>12.4</b>	0.100	0.651	<b>12.5</b>	0.050	1.332
0.5	$U[0.0C', 0.5C']$	13.0	0.100	0.189	13.0	0.100	0.559	12.9	0.050	1.225
	$U[0.5C', 1.0C']$	12.5	0.150	0.200	12.5	0.050	0.609	<b>12.5</b>	0.050	1.329
	$U[1.0C', 4.0C']$	12.2	0.000	0.195	<b>12.2</b>	0.000	0.639	12.3	0.050	1.437
	$U[0.0C', 5.0C']$	12.3	0.050	0.202	<b>12.2</b>	0.100	0.632	12.3	0.050	1.396
	$U[5.0C', 10.0C']$	12.6	0.100	0.195	12.4	0.100	0.649	12.4	0.100	1.372
	$U[10.0C', 20.0C']$	12.8	0.000	0.181	12.7	0.050	0.602	12.7	0.050	1.273
1.0	$U[0.0C', 0.5C']$	13.0	0.100	0.187	12.9	0.050	0.557	12.8	0.000	1.247
	$U[0.5C', 1.0C']$	12.8	0.000	0.164	12.6	0.000	0.604	<b>12.5</b>	0.050	1.375
	$U[1.0C', 4.0C']$	12.3	0.050	0.185	<b>12.2</b>	0.100	0.628	12.4	0.000	1.368
	$U[0.0C', 5.0C']$	12.2	0.000	0.196	<b>12.2</b>	0.000	0.646	12.4	0.000	1.353
	$U[5.0C', 10.0C']$	12.5	0.050	0.210	12.4	0.100	0.628	12.6	0.100	1.299
	$U[10.0C', 20.0C']$	12.7	0.050	0.186	12.7	0.050	0.594	12.6	0.100	1.307
2.0	$U[0.0C', 0.5C']$	<b>12.7</b>	0.150	0.194	<b>12.8</b>	0.000	0.585	13.0	0.300	1.220
	$U[0.5C', 1.0C']$	12.8	0.000	0.159	12.5	0.050	0.602	12.6	0.000	1.306
	$U[1.0C', 4.0C']$	12.3	0.050	0.179	<b>12.2</b>	0.100	0.662	12.4	0.000	1.364
	$U[0.0C', 5.0C']$	<b>12.1</b>	0.050	0.198	<b>12.2</b>	0.100	0.645	12.4	0.000	1.375
	$U[5.0C', 10.0C']$	12.4	0.100	0.200	12.3	0.150	0.654	12.5	0.050	1.343
	$U[10.0C', 20.0C']$	<b>12.6</b>	0.100	0.187	12.6	0.100	0.597	12.7	0.050	1.267
5.0	$U[0.0C', 0.5C']$	13.0	0.000	0.171	<b>12.8</b>	0.100	0.582	<b>12.7</b>	0.050	1.278
	$U[0.5C', 1.0C']$	12.4	0.000	0.187	12.8	0.000	0.564	12.6	0.100	1.288
	$U[1.0C', 4.0C']$	<b>12.1</b>	0.050	0.210	12.4	0.100	0.617	12.4	0.000	1.351
	$U[0.0C', 5.0C']$	12.3	0.050	0.196	12.3	0.050	0.637	12.4	0.000	1.359
	$U[5.0C', 10.0C']$	12.4	0.000	0.194	<b>12.2</b>	0.000	0.665	<b>12.2</b>	0.000	1.446
	$U[10.0C', 20.0C']$	12.7	0.050	0.195	12.7	0.050	0.614	12.7	0.050	1.262
10.0	$U[0.0C', 0.5C']$	12.8	0.100	0.189	13.1	0.050	0.522	12.9	0.050	1.231
	$U[0.5C', 1.0C']$	12.4	0.100	0.188	12.6	0.100	0.567	12.7	0.050	1.243
	$U[1.0C', 4.0C']$	<b>12.1</b>	0.050	0.195	<b>12.2</b>	0.000	0.610	<b>12.1</b>	0.050	1.445
	$U[0.0C', 5.0C']$	12.3	0.050	0.204	12.3	0.050	0.574	12.3	0.050	1.403
	$U[5.0C', 10.0C']$	12.4	0.100	0.196	12.5	0.050	0.591	12.3	0.050	1.410
	$U[10.0C', 20.0C']$	12.7	0.050	0.182	12.7	0.050	0.575	12.6	0.100	1.355

**Table 6.32:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.8 each.



Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	16.9	0.250	0.371	16.7	0.050	0.721	17.2	0.300	1.234
	$U[0.5C', 1.0C']$	16.3	0.050	0.309	<b>16.1</b>	0.050	0.731	16.3	0.050	1.365
	$U[1.0C', 4.0C']$	16.0	0.000	0.297	<b>15.8</b>	0.000	0.774	15.9	0.050	1.518
	$U[0.0C', 5.0C']$	16.0	0.100	0.294	15.9	0.050	0.737	16.0	0.100	1.482
	$U[5.0C', 10.0C']$	16.0	0.000	0.306	16.1	0.050	0.743	<b>15.9</b>	0.050	1.515
	$U[10.0C', 20.0C']$	<b>16.0</b>	0.100	0.336	16.0	0.000	0.781	16.1	0.050	1.424
0.3	$U[0.0C', 0.5C']$	<b>16.6</b>	0.100	0.365	16.8	0.100	0.700	17.2	0.100	1.226
	$U[0.5C', 1.0C']$	16.5	0.050	0.296	16.3	0.050	0.683	16.2	0.000	1.387
	$U[1.0C', 4.0C']$	<b>15.8</b>	0.000	0.299	15.9	0.050	0.773	15.9	0.050	1.515
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	0.337	15.9	0.050	0.757	16.0	0.100	1.463
	$U[5.0C', 10.0C']$	16.0	0.100	0.337	15.9	0.050	0.814	16.0	0.000	1.449
	$U[10.0C', 20.0C']$	16.1	0.050	0.330	16.1	0.050	0.836	<b>16.0</b>	0.100	1.477
0.5	$U[0.0C', 0.5C']$	16.9	0.250	0.345	<b>16.5</b>	0.150	0.782	17.3	0.150	1.166
	$U[0.5C', 1.0C']$	16.2	0.100	0.310	<b>16.1</b>	0.050	0.721	16.2	0.100	1.402
	$U[1.0C', 4.0C']$	16.1	0.050	0.313	15.9	0.050	0.774	<b>15.8</b>	0.000	1.532
	$U[0.0C', 5.0C']$	16.1	0.150	0.300	<b>15.8</b>	0.000	0.788	<b>15.8</b>	0.000	1.492
	$U[5.0C', 10.0C']$	16.0	0.000	0.311	16.1	0.050	0.733	<b>15.9</b>	0.050	1.493
	$U[10.0C', 20.0C']$	16.2	0.000	0.332	16.1	0.050	0.784	16.1	0.050	1.450
1.0	$U[0.0C', 0.5C']$	16.9	0.150	0.319	16.7	0.250	0.778	17.2	0.100	1.190
	$U[0.5C', 1.0C']$	16.3	0.050	0.293	<b>16.1</b>	0.050	0.679	<b>16.0</b>	0.100	1.482
	$U[1.0C', 4.0C']$	15.9	0.050	0.339	16.0	0.100	0.756	16.0	0.000	1.422
	$U[0.0C', 5.0C']$	16.0	0.100	0.302	15.9	0.050	0.809	<b>15.8</b>	0.000	1.533
	$U[5.0C', 10.0C']$	<b>15.9</b>	0.050	0.336	15.9	0.050	0.776	<b>15.9</b>	0.050	1.481
	$U[10.0C', 20.0C']$	<b>16.0</b>	0.000	0.344	<b>15.9</b>	0.050	0.808	16.1	0.050	1.470
2.0	$U[0.0C', 0.5C']$	16.9	0.150	0.327	17.0	0.100	0.659	<b>16.7</b>	0.050	1.357
	$U[0.5C', 1.0C']$	16.3	0.050	0.310	16.3	0.050	0.696	16.4	0.100	1.359
	$U[1.0C', 4.0C']$	15.9	0.050	0.319	16.0	0.100	0.756	15.9	0.050	1.486
	$U[0.0C', 5.0C']$	15.9	0.050	0.323	15.9	0.050	0.772	15.9	0.050	1.485
	$U[5.0C', 10.0C']$	16.0	0.000	0.303	<b>15.8</b>	0.000	0.840	16.0	0.000	1.443
	$U[10.0C', 20.0C']$	<b>16.0</b>	0.000	0.329	16.1	0.050	0.752	<b>16.0</b>	0.100	1.519
5.0	$U[0.0C', 0.5C']$	16.9	0.150	0.299	16.6	0.100	0.725	17.2	0.100	1.157
	$U[0.5C', 1.0C']$	<b>16.1</b>	0.050	0.295	16.2	0.000	0.676	16.3	0.050	1.311
	$U[1.0C', 4.0C']$	15.9	0.050	0.286	<b>15.8</b>	0.000	0.790	16.0	0.100	1.402
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	0.292	15.9	0.050	0.774	16.0	0.000	1.408
	$U[5.0C', 10.0C']$	16.0	0.100	0.333	15.9	0.050	0.797	16.0	0.000	1.424
	$U[10.0C', 20.0C']$	16.1	0.050	0.311	16.0	0.000	0.775	<b>16.0</b>	0.000	1.444
10.0	$U[0.0C', 0.5C']$	16.8	0.100	0.314	<b>16.5</b>	0.250	0.790	17.1	0.150	1.161
	$U[0.5C', 1.0C']$	16.2	0.000	0.289	16.3	0.050	0.719	16.5	0.050	1.269
	$U[1.0C', 4.0C']$	16.0	0.100	0.279	15.9	0.050	0.779	<b>15.8</b>	0.000	1.469
	$U[0.0C', 5.0C']$	<b>15.8</b>	0.000	0.301	15.9	0.050	0.803	<b>15.8</b>	0.000	1.523
	$U[5.0C', 10.0C']$	<b>15.9</b>	0.050	0.310	15.9	0.050	0.792	<b>15.9</b>	0.050	1.473
	$U[10.0C', 20.0C']$	16.1	0.050	0.319	<b>15.9</b>	0.050	0.820	<b>16.0</b>	0.100	1.450

**Table 6.33:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.9 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	<b>8.2</b>	0.027	0.095	8.3	0.055	0.247	<b>8.2</b>	0.027	0.678
	$U[0.5C', 1.0C']$	8.2	0.055	0.080	8.1	0.083	0.269	<b>8.1</b>	0.027	0.717
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.098	<b>7.8</b>	0.000	0.317	7.9	0.027	0.785
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	<b>7.8</b>	0.000	0.318	8.0	0.055	0.750
	$U[5.0C', 10.0C']$	8.0	0.000	0.092	8.0	0.055	0.300	7.9	0.027	0.771
	$U[10.0C', 20.0C']$	8.2	0.000	0.082	8.1	0.027	0.276	<b>8.1</b>	0.000	0.743
0.3	$U[0.0C', 0.5C']$	<b>8.2</b>	0.055	0.083	<b>8.2</b>	0.027	0.259	<b>8.2</b>	0.083	0.677
	$U[0.5C', 1.0C']$	8.2	0.083	0.078	8.1	0.000	0.274	8.2	0.000	0.679
	$U[1.0C', 4.0C']$	8.0	0.055	0.082	7.9	0.027	0.289	<b>7.8</b>	0.000	0.802
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	7.9	0.027	0.292	8.0	0.000	0.749
	$U[5.0C', 10.0C']$	8.0	0.027	0.090	8.0	0.027	0.286	8.0	0.083	0.762
	$U[10.0C', 20.0C']$	8.2	0.055	0.083	8.2	0.027	0.261	8.2	0.055	0.682
0.5	$U[0.0C', 0.5C']$	<b>8.2</b>	0.027	0.080	<b>8.2</b>	0.083	0.273	8.3	0.027	0.667
	$U[0.5C', 1.0C']$	<b>8.1</b>	0.055	0.083	<b>8.0</b>	0.027	0.280	<b>8.1</b>	0.027	0.700
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.081	<b>7.8</b>	0.000	0.312	7.9	0.027	0.771
	$U[0.0C', 5.0C']$	7.9	0.027	0.093	7.9	0.027	0.293	<b>7.9</b>	0.027	0.772
	$U[5.0C', 10.0C']$	8.0	0.055	0.094	8.0	0.055	0.289	8.1	0.055	0.717
	$U[10.0C', 20.0C']$	8.2	0.027	0.084	<b>8.0</b>	0.027	0.299	8.2	0.055	0.687
1.0	$U[0.0C', 0.5C']$	<b>8.2</b>	0.027	0.081	8.3	0.000	0.261	<b>8.2</b>	0.027	0.675
	$U[0.5C', 1.0C']$	8.2	0.000	0.076	8.2	0.055	0.269	<b>8.1</b>	0.000	0.727
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.093	7.9	0.027	0.297	8.0	0.027	0.731
	$U[0.0C', 5.0C']$	7.9	0.027	0.090	7.9	0.027	0.300	<b>7.9</b>	0.027	0.786
	$U[5.0C', 10.0C']$	<b>7.8</b>	0.000	0.097	<b>7.9</b>	0.027	0.310	8.0	0.000	0.738
	$U[10.0C', 20.0C']$	<b>8.1</b>	0.027	0.088	8.1	0.055	0.288	<b>8.1</b>	0.027	0.718
2.0	$U[0.0C', 0.5C']$	<b>8.2</b>	0.027	0.079	8.3	0.055	0.251	8.3	0.055	0.666
	$U[0.5C', 1.0C']$	<b>8.1</b>	0.027	0.078	<b>8.0</b>	0.027	0.272	8.2	0.027	0.669
	$U[1.0C', 4.0C']$	7.9	0.027	0.084	7.9	0.027	0.298	7.9	0.027	0.776
	$U[0.0C', 5.0C']$	<b>7.8</b>	0.000	0.089	7.9	0.027	0.294	<b>7.9</b>	0.027	0.767
	$U[5.0C', 10.0C']$	7.9	0.027	0.092	<b>7.9</b>	0.027	0.310	8.1	0.055	0.735
	$U[10.0C', 20.0C']$	<b>8.1</b>	0.027	0.097	<b>8.0</b>	0.000	0.317	<b>8.1</b>	0.000	0.740
5.0	$U[0.0C', 0.5C']$	<b>8.2</b>	0.055	0.083	8.3	0.055	0.247	<b>8.2</b>	0.027	0.685
	$U[0.5C', 1.0C']$	<b>8.1</b>	0.027	0.075	8.1	0.083	0.270	8.2	0.055	0.695
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.093	<b>7.8</b>	0.000	0.305	8.1	0.055	0.724
	$U[0.0C', 5.0C']$	7.9	0.027	0.091	<b>7.8</b>	0.000	0.318	8.0	0.055	0.760
	$U[5.0C', 10.0C']$	8.0	0.055	0.093	8.1	0.055	0.289	<b>7.8</b>	0.000	0.785
	$U[10.0C', 20.0C']$	8.2	0.055	0.094	8.1	0.000	0.293	8.2	0.055	0.683
10.0	$U[0.0C', 0.5C']$	<b>8.2</b>	0.083	0.080	<b>8.2</b>	0.055	0.271	<b>8.2</b>	0.083	0.700
	$U[0.5C', 1.0C']$	<b>8.1</b>	0.027	0.079	8.1	0.055	0.268	<b>8.1</b>	0.055	0.726
	$U[1.0C', 4.0C']$	<b>7.8</b>	0.000	0.093	8.0	0.083	0.285	8.0	0.055	0.767
	$U[0.0C', 5.0C']$	7.9	0.027	0.104	<b>7.8</b>	0.000	0.302	8.0	0.055	0.770
	$U[5.0C', 10.0C']$	8.0	0.027	0.086	8.0	0.055	0.293	7.9	0.027	0.758
	$U[10.0C', 20.0C']$	8.2	0.055	0.083	8.1	0.027	0.282	<b>8.1</b>	0.027	0.723

**Table 6.34:** ILP variants compared on a set of 5 instances with 90 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	7.6	0.100	0.059	<b>7.5</b>	0.050	0.291	7.9	0.050	0.837
	$U[0.5C', 1.0C']$	7.6	0.100	0.062	7.7	0.050	0.273	7.7	0.150	0.880
	$U[1.0C', 4.0C']$	<b>7.1</b>	0.050	0.073	<b>7.1</b>	0.050	0.326	<b>7.2</b>	0.000	0.958
	$U[0.0C', 5.0C']$	<b>7.1</b>	0.050	0.070	<b>7.2</b>	0.100	0.310	7.3	0.150	0.955
	$U[5.0C', 10.0C']$	7.7	0.050	0.061	7.7	0.050	0.283	7.6	0.000	0.883
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.059	7.8	0.000	0.263	7.8	0.000	0.840
0.3	$U[0.0C', 0.5C']$	7.8	0.000	0.054	7.8	0.100	0.268	7.5	0.150	0.905
	$U[0.5C', 1.0C']$	<b>7.4</b>	0.100	0.065	7.6	0.100	0.289	7.8	0.000	0.836
	$U[1.0C', 4.0C']$	7.2	0.100	0.080	7.2	0.100	0.307	7.3	0.050	1.002
	$U[0.0C', 5.0C']$	7.4	0.100	0.064	7.4	0.100	0.298	7.4	0.100	0.945
	$U[5.0C', 10.0C']$	7.8	0.000	0.061	7.7	0.050	0.275	7.6	0.000	0.882
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.059	7.7	0.050	0.279	7.8	0.000	0.811
0.5	$U[0.0C', 0.5C']$	7.8	0.000	0.056	<b>7.5</b>	0.050	0.301	7.5	0.150	0.900
	$U[0.5C', 1.0C']$	7.6	0.100	0.058	<b>7.5</b>	0.150	0.291	7.6	0.100	0.854
	$U[1.0C', 4.0C']$	7.4	0.100	0.071	7.5	0.050	0.293	<b>7.2</b>	0.000	0.993
	$U[0.0C', 5.0C']$	7.2	0.100	0.067	<b>7.2</b>	0.000	0.317	7.2	0.100	0.975
	$U[5.0C', 10.0C']$	7.6	0.100	0.062	7.6	0.100	0.289	7.6	0.000	0.901
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.062	7.8	0.000	0.280	7.8	0.000	0.881
1.0	$U[0.0C', 0.5C']$	7.5	0.150	0.063	<b>7.5</b>	0.050	0.300	7.6	0.100	0.891
	$U[0.5C', 1.0C']$	7.6	0.100	0.060	<b>7.5</b>	0.150	0.290	7.6	0.100	0.880
	$U[1.0C', 4.0C']$	7.3	0.050	0.072	7.3	0.050	0.311	<b>7.2</b>	0.100	0.974
	$U[0.0C', 5.0C']$	7.3	0.050	0.069	7.3	0.050	0.309	7.3	0.050	0.952
	$U[5.0C', 10.0C']$	7.6	0.000	0.067	7.6	0.100	0.282	7.8	0.000	0.813
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.062	7.8	0.000	0.276	7.8	0.100	0.822
2.0	$U[0.0C', 0.5C']$	<b>7.4</b>	0.200	0.063	7.8	0.000	0.262	<b>7.3</b>	0.050	0.964
	$U[0.5C', 1.0C']$	7.6	0.100	0.061	<b>7.5</b>	0.150	0.295	7.7	0.050	0.900
	$U[1.0C', 4.0C']$	7.3	0.050	0.069	7.2	0.000	0.331	7.3	0.050	0.931
	$U[0.0C', 5.0C']$	<b>7.1</b>	0.050	0.073	7.4	0.100	0.295	7.1	0.050	1.017
	$U[5.0C', 10.0C']$	<b>7.5</b>	0.150	0.068	<b>7.3</b>	0.150	0.309	7.7	0.050	0.858
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.062	7.8	0.000	0.259	<b>7.7</b>	0.050	0.857
5.0	$U[0.0C', 0.5C']$	7.5	0.050	0.066	7.6	0.100	0.280	7.8	0.000	0.844
	$U[0.5C', 1.0C']$	7.6	0.100	0.060	<b>7.5</b>	0.150	0.287	<b>7.5</b>	0.150	0.931
	$U[1.0C', 4.0C']$	7.2	0.100	0.068	7.4	0.100	0.293	<b>7.2</b>	0.000	0.946
	$U[0.0C', 5.0C']$	7.6	0.000	0.062	7.4	0.100	0.302	7.2	0.100	0.959
	$U[5.0C', 10.0C']$	<b>7.5</b>	0.050	0.070	7.8	0.000	0.262	7.6	0.100	0.875
	$U[10.0C', 20.0C']$	<b>7.8</b>	0.000	0.063	7.8	0.000	0.275	<b>7.7</b>	0.050	0.863
10.0	$U[0.0C', 0.5C']$	7.7	0.050	0.056	7.7	0.050	0.280	7.8	0.000	0.838
	$U[0.5C', 1.0C']$	7.7	0.050	0.055	7.6	0.100	0.286	7.6	0.100	0.888
	$U[1.0C', 4.0C']$	7.5	0.050	0.063	7.3	0.150	0.306	7.3	0.050	0.963
	$U[0.0C', 5.0C']$	7.2	0.100	0.070	7.3	0.050	0.321	<b>7.0</b>	0.000	1.008
	$U[5.0C', 10.0C']$	7.8	0.000	0.059	7.7	0.050	0.277	<b>7.5</b>	0.050	0.899
	$U[10.0C', 20.0C']$	7.9	0.050	0.061	<b>7.6</b>	0.100	0.288	7.9	0.050	0.822

**Table 6.35:** ILP variants compared on a set of 5 instances with 100 nodes and a density of 0.5 each.

Parameters		OneStepCD			ILP1			ILP2		
RecoloredTT	TabuTenure	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>	<i>obj</i>	<i>sd</i>	<i>time</i>
0.0	$U[0.0C', 0.5C']$	8.8	0.000	0.119	<b>8.6</b>	0.000	0.462	8.9	0.050	1.328
	$U[0.5C', 1.0C']$	8.7	0.050	0.119	8.8	0.100	0.435	8.7	0.050	1.392
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.115	<b>8.6</b>	0.000	0.453	<b>8.6</b>	0.000	1.438
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.112	<b>8.6</b>	0.000	0.450	<b>8.6</b>	0.000	1.387
	$U[5.0C', 10.0C']$	8.8	0.000	0.116	8.7	0.050	0.448	8.7	0.050	1.400
	$U[10.0C', 20.0C']$	<b>8.8</b>	0.000	0.131	9.0	0.000	0.410	9.0	0.000	1.306
0.3	$U[0.0C', 0.5C']$	8.7	0.050	0.113	8.8	0.100	0.432	<b>8.6</b>	0.000	1.440
	$U[0.5C', 1.0C']$	8.7	0.050	0.119	8.7	0.050	0.446	8.7	0.050	1.395
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.114	<b>8.6</b>	0.000	0.442	<b>8.6</b>	0.000	1.436
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.119	<b>8.6</b>	0.000	0.446	<b>8.6</b>	0.000	1.434
	$U[5.0C', 10.0C']$	8.7	0.050	0.127	8.7	0.050	0.452	<b>8.6</b>	0.000	1.438
	$U[10.0C', 20.0C']$	9.0	0.000	0.112	<b>8.9</b>	0.050	0.428	9.0	0.000	1.338
0.5	$U[0.0C', 0.5C']$	9.0	0.000	0.105	8.7	0.050	0.445	8.7	0.050	1.371
	$U[0.5C', 1.0C']$	8.7	0.050	0.117	<b>8.6</b>	0.000	0.442	8.7	0.050	1.407
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.118	<b>8.6</b>	0.000	0.440	<b>8.6</b>	0.000	1.472
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.117	<b>8.6</b>	0.000	0.441	<b>8.6</b>	0.000	1.470
	$U[5.0C', 10.0C']$	8.7	0.050	0.125	8.7	0.050	0.442	8.7	0.050	1.421
	$U[10.0C', 20.0C']$	9.0	0.000	0.115	9.0	0.000	0.413	<b>8.8</b>	0.000	1.365
1.0	$U[0.0C', 0.5C']$	8.8	0.000	0.113	<b>8.6</b>	0.000	0.466	8.8	0.000	1.356
	$U[0.5C', 1.0C']$	<b>8.5</b>	0.050	0.136	<b>8.6</b>	0.000	0.453	8.8	0.100	1.353
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.116	<b>8.6</b>	0.000	0.441	<b>8.6</b>	0.000	1.449
	$U[0.0C', 5.0C']$	8.7	0.050	0.114	<b>8.6</b>	0.000	0.455	<b>8.6</b>	0.000	1.465
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.126	8.8	0.100	0.436	<b>8.6</b>	0.000	1.417
	$U[10.0C', 20.0C']$	9.0	0.000	0.115	9.0	0.000	0.419	8.9	0.050	1.358
2.0	$U[0.0C', 0.5C']$	<b>8.6</b>	0.000	0.127	8.8	0.100	0.430	8.9	0.050	1.318
	$U[0.5C', 1.0C']$	8.6	0.000	0.120	8.9	0.050	0.408	<b>8.5</b>	0.050	1.443
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.117	<b>8.6</b>	0.000	0.451	<b>8.6</b>	0.000	1.425
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.118	<b>8.6</b>	0.000	0.445	<b>8.6</b>	0.000	1.455
	$U[5.0C', 10.0C']$	8.7	0.050	0.121	8.8	0.100	0.428	8.8	0.000	1.349
	$U[10.0C', 20.0C']$	9.0	0.000	0.116	9.0	0.000	0.412	8.9	0.050	1.339
5.0	$U[0.0C', 0.5C']$	8.7	0.050	0.117	8.8	0.100	0.437	8.7	0.050	1.421
	$U[0.5C', 1.0C']$	8.7	0.050	0.116	8.7	0.050	0.442	8.6	0.000	1.410
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.100	0.119	<b>8.6</b>	0.000	0.455	<b>8.6</b>	0.000	1.444
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.120	<b>8.6</b>	0.000	0.452	<b>8.6</b>	0.000	1.429
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.125	<b>8.6</b>	0.000	0.464	8.8	0.100	1.394
	$U[10.0C', 20.0C']$	9.0	0.000	0.113	9.0	0.000	0.420	8.9	0.050	1.332
10.0	$U[0.0C', 0.5C']$	8.7	0.050	0.115	8.8	0.100	0.423	8.8	0.000	1.343
	$U[0.5C', 1.0C']$	8.7	0.050	0.117	8.7	0.050	0.429	8.6	0.000	1.410
	$U[1.0C', 4.0C']$	<b>8.6</b>	0.000	0.115	<b>8.6</b>	0.000	0.452	<b>8.6</b>	0.000	1.426
	$U[0.0C', 5.0C']$	<b>8.6</b>	0.000	0.115	<b>8.6</b>	0.000	0.455	<b>8.6</b>	0.000	1.430
	$U[5.0C', 10.0C']$	<b>8.6</b>	0.000	0.132	8.8	0.000	0.434	<b>8.6</b>	0.000	1.439
	$U[10.0C', 20.0C']$	9.0	0.000	0.114	9.0	0.000	0.419	9.0	0.000	1.284

**Table 6.36:** ILP variants compared on a set of 5 instances with 120 nodes and a density of 0.5 each.

### Comparison to previous works

Since *RANDOM* performs best in terms of runtime and is not significantly inferior to other methods in terms of quality, its results are compared to those of [6] and [11] in tables 6.37 and 6.38. In 6.39 results are compared to [10].

Instance set		B & C		Random (10 runs/inst)			MA2		
nodes	density	LB	UB	$\overline{obj}$	$sd$	$time(s)$	$\overline{obj}$	$sd$	$time(s)$
20	0.5	3	3	<b>3.00</b>	0.00	0.01	<b>3.00</b>	0.00	0.14
40	0.5	4	4	<b>4.00</b>	0.00	0.02	<b>4.00</b>	0.00	0.60
60	0.5	5	5	<b>5.00</b>	0.00	0.06	5.63	0.49	2.00
70	0.5	6	6	<b>6.00</b>	0.00	0.08	6.06	0.24	3.33
80	0.5	6	6	<b>6.27</b>	0.13	0.15	6.94	0.29	4.90
90	0.5	6	7	7.88	0.17	0.36	<b>7.55</b>	0.50	7.49
100	0.5	6	7	<b>7.12</b>	0.01	0.32	7.93	0.30	11.04
120	0.5	7	8	<b>8.64</b>	0.19	0.52	9.22	0.43	21.05

**Table 6.37:** TODO

Instance set		B & C		Random (10 runs/inst)			MA2		
nodes	density	LB	UB	$\overline{obj}$	$sd$	$time(s)$	$\overline{obj}$	$sd$	$time(s)$
90	0.1	2	3	<b>3.00</b>	0.00	0.02	3.09	0.29	1.37
90	0.2	3	4	<b>3.80</b>	0.15	0.03	4.41	0.49	3.24
90	0.3	4	5	<b>5.00</b>	0.00	0.06	5.52	0.56	4.90
90	0.4	5	6	<b>6.00</b>	0.00	0.11	6.79	0.83	6.54
90	0.5	6	7	<b>7.00</b>	0.00	0.18	7.55	0.50	7.49
90	0.6	8	8	<b>8.28</b>	0.15	0.31	10.50	0.87	11.95
90	0.7	10	10	<b>10.00</b>	0.00	0.45	12.39	1.12	14.83
90	0.8	12	12	<b>12.05</b>	0.14	0.80	15.18	0.80	20.98
90	0.9	16	16	<b>15.80</b>	0.15	1.23	17.27	0.98	45.75

**Table 6.38:** pcpn120

Parameters		DSJC500.5-1		DSJC500.5-2		DSJC500.5-3		DSJC500.5-4	
ItMax	TabuTenure	Random	Noronha	Random	Noronha	Random	Noronha	Random	Noronha
1	$U[0.25C', 0.75C']$	<b>53.0</b>	53.5	<b>47.6</b>	47.8	45.0	<b>44.8</b>	<b>43.4</b>	43.5
	$U[0.0C', 1.0C']$	<b>53.0</b>	53.7	<b>47.0</b>	47.5	<b>45.0</b>	45.4	<b>43.2</b>	43.6
	$U[0.0C', 0.5C']$	<b>52.0</b>	53.1	<b>47.0</b>	47.3	45.0	<b>44.6</b>	44.0	<b>42.8</b>
	$U[0.5C', 1.0C']$	<b>52.6</b>	54.2	<b>47.2</b>	48.1	<b>45.0</b>	45.8	<b>43.0</b>	43.9
	$U[0.25C', 1.0C']$	<b>53.0</b>	53.8	<b>47.2</b>	47.9	<b>45.0</b>	45.5	<b>43.6</b>	<b>43.6</b>
	$U[0.0C', 0.75C']$	<b>52.4</b>	53.3	<b>47.6</b>	47.5	45.0	<b>44.8</b>	43.6	<b>43.0</b>
5	$U[0.25C', 0.75C']$	<b>52.0</b>	52.7	47.0	<b>46.8</b>	<b>44.0</b>	44.4	43.0	<b>42.8</b>
	$U[0.0C', 1.0C']$	<b>51.4</b>	52.9	47.0	<b>46.8</b>	<b>44.0</b>	44.7	<b>42.4</b>	<b>42.7</b>
	$U[0.0C', 0.5C']$	<b>51.4</b>	52.2	47.0	<b>46.1</b>	44.0	<b>43.7</b>	43.0	<b>42.0</b>
	$U[0.5C', 1.0C']$	<b>52.0</b>	53.3	<b>47.0</b>	47.7	<b>44.4</b>	44.9	<b>43.0</b>	<b>43.0</b>
	$U[0.25C', 1.0C']$	<b>51.4</b>	53.0	<b>47.0</b>	47.3	44.8	<b>44.7</b>	43.0	<b>42.9</b>
	$U[0.0C', 0.75C']$	<b>52.0</b>	52.5	47.0	<b>46.6</b>	44.6	<b>44.0</b>	43.0	<b>42.4</b>
10	$U[0.25C', 0.75C']$	<b>51.0</b>	52.5	47.0	<b>46.7</b>	<b>44.0</b>	<b>44.0</b>	43.0	<b>42.4</b>
	$U[0.0C', 1.0C']$	<b>51.0</b>	52.3	<b>46.4</b>	46.7	<b>44.0</b>	44.2	43.0	<b>42.7</b>
	$U[0.0C', 0.5C']$	<b>51.0</b>	51.3	47.0	<b>45.9</b>	44.0	<b>43.3</b>	43.0	<b>42.0</b>
	$U[0.5C', 1.0C']$	<b>51.0</b>	53.0	<b>47.0</b>	47.3	<b>44.6</b>	44.8	<b>42.8</b>	43.0
	$U[0.25C', 1.0C']$	<b>51.8</b>	<b>52.8</b>	<b>46.4</b>	46.9	<b>44.0</b>	44.2	43.0	<b>42.8</b>
	$U[0.0C', 0.75C']$	<b>51.0</b>	52.2	47.0	<b>46.2</b>	44.0	<b>43.9</b>	43.0	<b>42.2</b>

**Table 6.39:** in1

# Critical Reflection and Outlook

## 7.1 Critical Reflection

Selecting and optimizing the coloring of a subset of clusters regardless of their location in the graph does not tackle the problem in an efficient way. The selection does not take into account any features of the graph like regional density, although dense subgraphs involve the most danger of increasing the chromatic number by being colored with a suboptimal coloring. Considering graph features being crucial for a good selection of clusters, the selection presented in this thesis is done in a random way and therefore an optimal, partial recoloring can not be integrated in the solution by the tabu search more probably than any random coloring.

## 7.2 Future Works

Future works could consider a more suggestive selection of the clusters to be recolored. Rather than selecting all clusters of the same color, the set could be chosen by criteria of regional density. Putting effort in optimizing the coloring of these regions – e.g. by the use of exact methods – could lead to results of higher quality.

### On finding dense subgraphs

Finding dense subgraphs is a intensively studied problem in graph theory and became more relevant in recent years because of its application to social network graphs. As long as there are no boundaries set on the size of the densest subgraph, it can be found in polynomial time, despite the fact that there are exponentially many subgraphs to consider [8, 16]. Additionally, Charikar [4] showed a 2 approximation to the densest subgraph problem in linear time using a very simple greedy algorithm which was previously studied by Asahiro et. al. [15]). The densest  $k$ -subgraph problem ( $DkS$ ), which finds the densest subgraph of size  $k$  is shown to be  $\mathcal{NP}$ -hard [14, 16]. For the densest at-most- $k$ -subgraph problem ( $DamkS$ ), which searches for the densest subgraph of maximum size  $k$  (and therefore is a relaxation of  $DkS$ ), Andersen

et.al. [1] showed that if there exists a  $\alpha$  approximation for *DamkS*, then there exists a  $\mathcal{O}(\alpha^2)$  approximation for *DkS*, indicating that this problem is quite hard as well. Khuller and Saha showed that approximating *DamkS* is as hard as *DkS* within a constant factor [12], specifically an  $\alpha$  approximation for *DamkS*, implies a  $4\alpha$  approximation for *DkS*. A number of polynomial time greedy heuristics for *DkS* are proposed in Asahiro et.al. [15].

### Algorithm proposal

Algorithm 5 proposes a procedure for the discussed approach. The graph  $G$ , a recoloring algorithm *RECOLOR* like these presented in 5.2 and two integers used to parameterize the search for dense subgraphs are taken as input. In line 1 an initial solution is calculated and its chromatic number is assigned to  $cmax$  in line 2. In line 3 an algorithm is called that returns up to  $maxSubgraphs$  subgraphs with a maximum size of  $denseMaxSize$ . Line 4 to line 6 recolor all found subgraphs by applying *RECOLOR* and all remaining nodes colored with colors  $cmax$  randomly, all with  $cmax - 1$  colors. In line 8 the tabusearch tries to eliminate all resulting conflicts and puts the recolored regions on the tabulist for a number of iterations as presented in [TODO]. Line 9 to 11 accept the new solution in case of feasibility and starts searching for dense regions again.

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#### Algorithm 5: PCP HYBRID DENSERECOLORING

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**Input:** An uncolored Graph  $G = (V, E)$ , a recoloring-algorithm *RECOLOR*, two integers  $maxSubgraphs$  and  $denseMaxSize$

**Output:** A feasible Solution  $S$

- 1 Set  $S \leftarrow ONESTEPCD(G)$ ;
  - 2 Set  $cmax \leftarrow$  the chromatic number of  $S$ ;
  - 3 Set  $D \leftarrow FINDDENSESUBGRAPHS(S, maxSubgraphs, denseMaxSize)$ ;
  - 4 Let  $S'$  be the solution after recoloring all subgraphs in  $D$  with *RECOLOR* and  $cmax - 1$  colors;
  - 5 Let  $R$  be the set of all remaining nodes in  $V$  colored with  $cmax$ ;
  - 6 Let  $S'$  be the solution after recoloring  $R$  randomly with  $cmax - 1$  colors;
  - 7 Let  $C'$  be the set of nodes involved into color conflicts in  $S'$ ;
  - 8  $S' \leftarrow TABUSEARCH(S', D \cup R, C')$ ;
  - 9 **if**  $S'$  is free of conflicts **then**
  - 10      $S \leftarrow S'$ ;
  - 11     **goto** line 2;
  - 12 **return**  $S$ ;
-



## Summary

The PCP is a quite recently proposed COP which generalizes the classical VCP by considering the possibility to select subsets of nodes. While for the VCP much research has been done, only a few papers about the PCP has been published so far. In this work a strategy is presented that creates an initial solution by a heuristical algorithm and improves the solution quality by recoloring sets of nodes of same color before eliminating the resulting conflicts by applying a tabu search. It has been tried to enhance the algorithm presented in [10] by substituting the process of random recoloring by more sophisticated algorithms in order to minimize the number of resulting conflicts. Therefore variation of the *ONESTEP*CD algorithm [9] and two ILPs were used. A local search algorithm then tries to eliminate all these conflicting nodes to create a feasible solution. Furthermore experiments with variations of the ILPs and a mechanism that puts the most recently recolored subgraph on the tabulist for an amount of iteration in order to protect the coloring of that subgraph from being overwritten have been done.

The results have shown that more sophisticated recoloring algorithms can reduce the number of conflicts dramatically. For the instances used, a random recoloring produces an amount of conflicting nodes up to 7.5 times higher than an optimized recoloring does. The fact that this gap is not reflected significantly in the final results leads to the conclusion that for the presented strategy the tabu search is much more relevant than the recoloring process. Finally an alternative strategy, that is suspected by the author to be more suitable for sophisticated recoloring methods has been proposed.



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