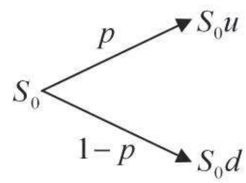
Question 2:
Binomial Model

So now we will be covering the binomial model in derivative pricing.

Let's say we have a stock whose price is currently S0. After 1 unit of time the price can go up by a factor of 'u' or go down by a factor of 'd' = 1/u. Assume you have access to a bank account that lets you borrow and lend at a rate r (denote (1+r) by R i.e. value of 1$ a year later with interest rate r)– this means you can borrow 1$ from the bank and you will have to pay R$ after 1 unit of time and if you have 1$ you can keep it in the bank and get R$ after 1 unit of time. In general, after n units of time you will have to give/get from the bank $R^n$$. You can buy the stock and sell in the future and also short sell the stock which is sell now and buy later

Now, a call option is a derivative product where the buyer of the option has the right but not the obligation to buy the stock at the predetermined price called the strike price 'K' after N units of time. What this means is the person will get max (S_N - K, 0) at time N, where S_N is the price of the stock at t=N.

<u>The objective is to determine the price of this option.</u>
Consider the following example, where N=1:
Where, $S_1 = u * S_0$; $S_2 = d * S_0$ and $d = \frac{1}{u}$



1. To calculate p, write S_1 and S_2 in terms of S_0 as discounted expected value and derive p in terms of R, u and d.
2. Find the general solution at t=k in terms of S_{k-1}. Calculate the price of the European Call option at each step as discounted expected value of two extending nodes
   i.e. $[\, C_{upperNode} * p + C_{lowerNode} * (1-p)] * \frac{1}{R}$
3. In regards to the discounting, as $100 today is equivalent to $100*($R^n$) after n years, similarly with the interest rate of 6%, R= 1.06, we have the present day value as $100 / ($R^n$) for $100 after n year.

Now consider S_0 =100, u=1.25 and R=1.1. Let Strike Price K = 100.
Therefore at t=1, Payoff at S_1 = 25 and Payoff at S_2 = 0.
Let's say you buy x units of stock and put y units in the bank at t=0 and this portfolio at t=1 has the same payoff as the option.

S_1 *x + R*y = 25 => 125x + 1.1y = 25
S_2 *x + R*y = 0 => 80x + 1.1y = 0

Solving these we get x = 5/9 = 0.56 and y= -4000/99 = -40.404.
At t=0, the price of the portfolio x*S_0 +y = 15.15

Now if you price the option more than this value you can sell the option and buy the above portfolio for 15.28 and since at t=1 make free money and buy the option and sell the portfolio if the option price is less than the above price.
Write a code that can find a general solution for N periods and hence determine the price of the EUROPEAN option.
Hint: Find the general solution at t=k in terms of S_{k-1} and extend it.

Input Specification:

input1: an integer S_0 , Initial price of the option
input2: a double u, the factor by which the option moves in an up move
input3: a double R = 1 + the interest rate at which you can borrow or lend from the bank.
input4: a double K, the Strike price of the option
input5: an integer N, the number of periods

Output Specification:
Return the price of the option rounded to 2 decimal places.