Unit-1 :- Mathematical logic

Preposition :-

A declarative sentence which is given in a context said to either true (or) False but not both

ex :-

→ The $\triangle^{le}$ has '3' sides (P)

→ rectangle consists of '2' sides (P)

→ 'x' is boy and a girl (N.P)

Note :-

ex. 3 is not a preposition

Negation :- (~)

A preposition obtained by inserting the word 'not' at a appropriate place is called negation

It is denoted by '~' (or) '¬'

ex :-

→ 'x' is a boy

⇒ 'x' is not a boy

logical connection :-

i) conjunction :- ('∧' → and)

Two preposition P & Q are connected by the word 'and' then the compound preposition is called conjunction.

denoted by ('∧') 'and'

Truth table

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction :- ('v')

Two preposition P & Q are connected by the word 'OR' then the compound preposition is called disjunction

denoted by ('v')('or')

Truth table:-

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F

Conditional :- ('→')

Two preposition P & Q are connected by the word 'THEN' then the compound preposition is called conditional statement

denoted by ('→') ('THEN')

Truth table:-

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional :- ('⟷')

Two preposition P & Q are connected by the word 'IF & only IF' then the compound preposition is called Biconditional statement

denoted by ('⟷') ('IF & only IF')

Truth table:-

P	Q	P ⟷ Q
T	T	T
T	F	F
F	T	F
F	F	T

ex:-

Q) If P: x is a boy

Q: y is a girl

Then P∧Q : x is a boy 'and' y is a girl

P v Q : x is a boy 'or' y is a girl

P → Q : x is a boy 'then' y is a girl

P ⟷ Q : x is a boy "if and only if" y is a girl

~P : x is 'not' a boy

1) $P \to Q \Leftrightarrow \sim P \vee Q$ ($\Leftrightarrow$ equal)

P	Q	~P	P → Q	~P v Q	P → Q ⟺ ~P v Q
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

2) $P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

P	Q	P ⟷ Q	P → Q	Q → P	(P → Q) ∧ (Q → P)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\therefore P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

iii) $(P \to Q) \to R$

P	Q	R	$P \to Q$	$(P \to Q) \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

Q) $\sim(P \to Q) \to (\sim Q)$

P	Q	$P \to Q$	$\sim(P \to Q)$	$\sim Q$	$\sim(P \to Q) \to (\sim Q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Tautology

Q) $(P \wedge (P \to Q)) \to Q$

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautology :-

A compound preposition which is true for all possible truth values is called Tautology denoted by 'T' (or) 'T_0'

contradiction :-

A compound preposition which is false for all possible Truth values is called contradiction denoted by 'F' (or) 'F_0'

ex:-

P	$\sim P$	$P \vee (\sim P)$	$P \wedge (\sim P)$
T	F	T	F
F	T	T	F

logical equivalence :-

two compound preposition are said to be logical equivalent if the truth values ~~ar.~~ of this two compound statements are same denoted ('$\Leftrightarrow$')

Q) $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$

P	Q	R	$P \wedge Q$	$Q \to R$	$P \to (Q \to R)$	$(P \wedge Q \to R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	F	T	T
F	F	F	F	T	T	T

$\therefore$ $P \to (Q \to R)$ and $(P \wedge Q) \to R$ are logical equivalent

Q) $P \to (Q \to P) \Leftrightarrow [\sim P \to (P \to Q)]$

P	Q	~P	P→Q	Q→P	P→(Q→P)	[~P→(P→Q)]
T	T	F	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

$\therefore P \to (Q \to P) \Leftrightarrow [\sim P \to (P \to Q)]$ are logical equivalent

$\therefore$ They are Tautology

Q) $\sim(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \sim(P \wedge Q)$

P	Q	P↔Q	~(P↔Q)	(P∨Q)	(P∧Q)	~(P∧Q)	∨∧
T	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	F	F	F	T	F

$\therefore \sim(P \leftrightarrow Q)$ and $(P \vee Q) \wedge \sim(P \wedge Q)$ are logical equivalent

$\sim(P \leftrightarrow Q) = \sim[(P \to Q) \wedge (Q \to P)]$

Represertment process:-

rules of logic:-

i) Law of Double negation :- $\sim(\sim P) \Leftrightarrow P$

ii) Idempotent law :- $P \wedge P \Leftrightarrow P$ &
$(P \vee P) \Leftrightarrow P$

iii) Inverse law :- $P \vee \sim P \Leftrightarrow T_0$ & $P \wedge \sim P \Leftrightarrow F_0$

iv) Absortion law:-
$P \vee (P \wedge Q) \Leftrightarrow P$ & $P \wedge (P \vee Q) \Leftrightarrow P$

v) Assocative law:-
$P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$ &
$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

vi) Distributive law:-
$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
&
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

** vii) Demorgan's law:-
$\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$
&
$\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

viii) Identity law:-
$P \vee F_0 \Leftrightarrow P$ & $P \wedge T_0 \Leftrightarrow P$

ix) Domianant Law:-
$P \vee T_0 \Leftrightarrow T_0$
$P \wedge F_0 \Leftrightarrow F_0$

process:-

i) Ellimate conditional & biconditionsal from the given preposition i.e
$P \to Q \Leftrightarrow \sim P \vee Q$

ii) apply laws of logic in any order to obtai obtained the required preposition

Q) using replacement process show that
$P \to Q \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

P	Q	R	$P \to Q$	$R \to Q$	$\wedge$	$P \vee Q$	$(P \vee Q) \to Q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

$\therefore (P \to Q) \wedge (R \to Q)$ and $(P \vee Q) \to Q$ are ~~equ~~ logical ~~equale~~ equivalent

(Logic. Law)

$(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

$\Leftrightarrow (\sim P \wedge \sim R) \vee Q$ [DL]

$\sim(P \vee R) \vee Q$ [DML]

$(P \vee R) \to Q$

Q) $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$

$P \to (Q \to R) \Leftrightarrow \sim P \vee (Q \to R)$

$\Leftrightarrow \sim P \vee (\sim Q \vee R)$

$(\sim P \vee \sim Q) \vee R$ [A.L]

$\sim(P \wedge Q) \vee R$ [DML]

$(P \wedge Q) \to R$

Q) $[\sim P \wedge (\sim Q \wedge R)] \vee [(Q \wedge R) \vee (P \wedge R)] \Leftrightarrow R$

$\Leftrightarrow [(\sim P \wedge \sim Q) \wedge R] \vee [(Q \vee P) \wedge R]$ [AL & DL]

$\Leftrightarrow [\sim(P \vee Q) \wedge R] \vee [(P \vee Q) \wedge R]$ [DML]

$\Leftrightarrow [\sim(P \vee Q) \vee (P \vee Q)] \wedge R$ [DL]

$\Leftrightarrow T_0 \wedge R$ [IL]

$\Leftrightarrow R$ [IL]

Q) $\sim(P \leftrightarrow Q) \Leftrightarrow (P \wedge \sim Q) \vee (Q \wedge \sim P)$ ~

$[(R \to Q) \wedge (Q$

$\sim[(P \to Q) \wedge (Q \to P)]$

$\sim[(\sim P \vee Q) \wedge (\sim Q \vee P)]$

$\Rightarrow (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Q) $P \to (Q \to P) \Leftrightarrow \sim P \to (P \to Q)$

LHS	RHS
$P \to (\sim Q \vee P)$	$\sim P \to (\sim P \vee Q)$
$\sim P \vee (\sim Q \vee P)$	$\sim(\sim P) \vee (\sim P \vee Q)$
$\sim P \vee (P \vee \sim Q)$	$P \vee (\sim P \vee Q)$
$(\sim P \vee P) \vee \sim Q$	$(P \vee \sim P) \vee Q$
$T_0 \vee \sim Q$	$T_0 \vee Q$
$= T_0$	T_0

$\therefore$ LHS = RHS

$\Rightarrow$ Tautological implication :- ('$\Rightarrow$')

A statement formula 'A' is said to be tautologically implied to other statement 'B' If and if $A \Rightarrow B$ is Tautology

It can be represented as '$A \Rightarrow B$' ('A' implie 'B')

1Q) Show that $\sim Q \wedge (P \to Q) \Rightarrow \sim P$ is tautologically implication using Truth table

P	Q	$\sim P$	$\sim Q$	$\sim Q (P \to Q)$	$\sim Q \wedge (P \to Q)$	$\to$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

$\therefore$ Hence proved

20) $(P \to Q) \Rightarrow (\sim Q \to (\sim P))$

P	Q	~P	~Q	P→Q	~Q→~P	→
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T ✓	T ✓	T ✓

⇒ Normal Form's:-

→ while constructing and comparing truth tables we can find whether two statements formula A & B are equivalent (or) not

→ It is difficult to construct T.T's has we increase atomic statement

→ A better method is to transfer the statement formulas A & B to some standard form A' & B' such that a simple comparision of A' & B' shows whether A ⇒ B

→ Let $A(P_1, P_2 \ldots P_n)$ be a statement formula where $P_1 P_2 \ldots P_n$ are atomic statements 'if' 'A' has truth value 'T' for ~~atleast~~ atleast one combination truth value assign to 'A' is said to be ~~sta~~ satisfiable

*→ It will be convent to use the word product in the place of conjunction (∧) and sum in the place of disjunction (∨)

→ The product of statement varaibles & their negation is called elementy product

→ The elementy products are

$P \wedge Q, \ \sim P \wedge Q, \ P \wedge \sim Q, \ \sim P \wedge \sim Q$

→ The sum of statement variables & their negation is called elementy sum

$P \vee Q, \ \sim P \vee Q, \ P \vee \sim Q, \ \sim P \vee \sim Q$

D.N.F (Disjunctive normal form):-

The formula which is equivalent to given formula & which consists of "sum of elementy products" is called DNF of given formula

extended distributive law:-

ex:- $(P \vee Q) \wedge (R \vee S)$

$[(P \vee Q) \wedge R] \vee [(P \vee Q) \wedge S]$

$[P \wedge R] \vee [Q \wedge R] \vee [P \wedge S] \vee [Q \wedge S]$

(or)

$[P \wedge R] \vee [Q \wedge R] \vee [P \wedge S] \vee Q$

Q) find DNF of $(\sim P \to R) \wedge (P \leftrightarrow Q)$ use "T.T"

Sol)

P	Q	R	~P	P↔Q	~P→R	∧
T	T	T	F	T	T	T ✓
T	T	F	F	T	T	T ✓
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T ✓
F	F	F	T	T	F	F

ii) DNF:-

only 'True' values

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

Q) obtained DNF of $P \to [(P \to Q) \wedge \sim(\sim Q \vee \sim P)]$

Sol) $P \to [(\sim P \vee Q) \wedge (Q \wedge P)]$

$\sim P \vee [(\sim P \vee Q) \wedge (Q \wedge P)]$

$\sim P \vee [(\sim P \vee Q) \wedge Q] \wedge [(\sim P \vee Q) \wedge P]$

$\sim P \vee [\sim P \vee (Q \wedge Q) \wedge (\sim P \vee P) \wedge Q]$

$\sim P \vee (\sim P \vee Q) \wedge (T_0 \wedge Q)$

$\sim P \vee (\sim P \vee Q) \wedge Q$

$(\sim P \vee \sim P) \vee (Q \wedge Q)$

$\sim (P \wedge P) \vee (Q \wedge Q)$

Q) $P \wedge (P \to Q)$ $\Rightarrow P \wedge \sim P \vee Q$

$P \wedge \sim P \vee Q$ $\Rightarrow (\sim P \wedge P) \vee (P \wedge Q)$

$\sim(P \vee Q) \wedge$ [illegible] $= (P \wedge \sim P) \vee Q$

$F_0 \vee Q$

Q) $\sim [P \to (Q \wedge R)]$

$\sim [\sim P \vee (Q \wedge R)]$

$P \wedge \sim (Q \wedge R)$

$P \wedge (\sim Q \vee \sim R)$

$(P \wedge \sim Q) \vee (P \wedge \sim R)$

$\Rightarrow$ CNF (conjuctive normal form)

The formula which is equivalent to given statement formula which consists of "products of elementry sum" is called CNF of given statement

Q) CNF of $(\sim P \to R) \wedge (P \leftrightarrow Q)$

P	Q	R	$\sim P$	$\sim P \to R$	$P \leftrightarrow Q$	$\wedge$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F ✓
T	F	F	F	T	F	F ✓
F	T	T	T	T	F	F ✓
F	T	F	T	F	F	F ✓
F	F	T	T	T	T	T
F	F	F	T	F	T	F ✓

i) CNF :- (only 'False values')

$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee \tilde{Q} \vee \tilde{R}) \wedge$

$(P \vee \sim Q \vee R) \wedge (P \vee Q \vee R)$

$\therefore$ True $\to \sim$

$\therefore$ False $\to$ same

$\Rightarrow$ There are '5' F's i.e

(T, F, F), (T, F, F), (F, T, T)

(F, T, F), (F, F, F)

$\to$ Take '$\sim$' for True and as it is for false

Q) Obtained CNF for $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

$[\sim(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to \sim(P \vee Q)]$

$[\sim(\sim P \vee Q) \vee (P \wedge Q)] \wedge [\sim(P \wedge Q) \vee \sim(P \vee Q)]$

$[(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q)]$

$[(P \vee Q) \wedge (P \vee Q)] \wedge \{\sim P \vee \sim Q\} \vee \sim P) \wedge (\sim P \vee \sim Q \vee \sim Q)$

$[(P \vee Q) \wedge (P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

$(P \vee Q) \wedge (\sim P \vee \sim Q)$

3) $P \wedge (P \to Q)$

$P \wedge (\sim P \vee Q)$

$(P \vee P) \wedge (\sim P \vee Q)$

4) $[Q \vee (P \wedge R)] \wedge \sim[(P \vee R) \wedge Q]$

$[(P \vee Q) \wedge (R \vee Q)] \wedge \sim[(P \wedge Q) \vee (R \wedge Q)]$

$[(P \vee Q) \wedge (R \vee Q)] \wedge [(\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)]$

$(P \vee Q) \wedge (R \vee Q) \wedge (\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)$

Min terms :-

minimum consists of conjunction of statement varaibles or their negation but only one appearing only one's

ex:-

1) min terms of P, Q, are

$(P \wedge Q)$, $(\sim P \wedge Q)$, $(P \wedge \sim Q)$, $(\sim P \wedge \sim Q)$

But not $(P \wedge P)$, $(P \wedge \sim P)$, $(Q \wedge \sim Q)$ etc

2) min-terms for P, Q, R are

$(P \wedge Q \wedge R)$, $(\sim P \wedge Q \wedge R)$, $(P \wedge \sim Q \wedge R)$, $(P \wedge Q \wedge \sim R)$, $(\sim P \wedge \sim Q \wedge R)$, $(P \wedge \sim Q \wedge \sim R)$ $(\sim P \wedge Q \wedge \sim R)$, $(\sim P \wedge \sim Q \wedge \sim R)$

Max -terms :-

max- terms consists of disjunction of statement variable or their negation but only one appearing only one's.

ex:-

1) $(P \vee Q)$, $(\sim P \vee Q)$, $(P \vee \sim Q)$, $(\sim P \vee \sim Q)$

But not $(P \vee P)$, $(P \vee \sim P)$, $(Q \vee \sim Q)$ etc

2) $(P \vee Q \vee R)$, $(\sim P \vee Q \vee R)$, $(P \vee \sim Q \vee R)$, $(P \vee Q \vee \sim R)$, $(\sim P \vee \sim Q \vee R)$, $(P \vee \sim Q \vee \sim R)$ $(\sim P \vee Q \vee \sim R)$, $(\sim P \vee \sim Q \vee \sim R)$

But not $(P \vee \sim P \vee R)$ etc.

principle disjunctive normal form :- (PDNF)

The formula which is equivalent to given statement formula consisting of disjunction of min -terms is called "PDNF" (or) sum of product canonorical form.

Q) find PDNF for

$\overset{A}{(P \wedge Q)} \vee \overset{B}{(\sim P \wedge R)} \vee \overset{C}{(Q \wedge R)}$

P	Q	R	$P \wedge Q$	$\sim P$	$\sim P \wedge R$	$(Q \wedge R)$	$A \vee B$	$A \vee B \vee C$
T	T	T	T	F	F	T	T	T ✓
T	T	F	T	F	F	F	T	T ✓
T	F	T	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F
F	T	T	F	T	T	T	T	T ✓
F	T	F	F	T	F	F	F	F
F	F	T	F	T	T	F	T	T ✓
F	F	F	F	T	F	F	F	F
			✓		✓	✓		

DNF

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$

Q) $(P \to Q) \land \sim(\sim Q \lor \sim P)$

P	Q	$\sim P$	$\sim Q$	$P \to Q$	$(\sim Q \lor \sim P)$	$\sim(\ )$	$\land$
T	T	F	F	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	F
F	F	T	T	T	T	F	F

Q) $P \to [(P \to Q) \land \sim(\sim Q \lor \sim P)]$ PDNF

P	Q	$P \to Q$	$\sim Q$	$\land \sim P$	$(\sim Q \lor \sim P)$	$\sim$	$P \to (\ \land$
T	T	T	F	F	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	F
F	F	T	T	T	T	F	F

$P \to (\ \land\ )$
T ✓
F
T ✓
T ✓

PDNF

$(P \land Q) \lor (\sim P \land Q) \lor (\sim P \land \sim Q)$

Q) $\sim P \lor Q$

P	Q	$\sim P$	$\sim P \lor Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

$(P \land Q) \lor (\sim P \land Q) \lor (\sim P \land \sim Q)$

Q) $P \to [(P \to Q) \land \sim(\sim Q \lor \sim P)]$

PDNF

$\because P \land T_0 \to$

$P \to [(\sim P \lor Q) \land (Q \land P)]$

$\sim P \lor [(\sim P \lor Q) \land (Q \land P)]$

$\sim P \lor [(\sim P \land Q \land P) \lor (Q \land Q \land P)]$

$\sim P \lor [(F_0 \land Q) \lor (Q \land P)]$

$\sim P \lor [F_0 \lor (Q \land P)]$

$\sim P \lor [P \land Q] \to$ DNF

$(\sim P \land T_0) \lor (P \land Q)$

$\sim P \land (Q \lor \sim Q) \lor (P \land Q)$

$(\sim P \land Q) \lor (\sim P \land \sim Q) \lor (P \land Q) \to$ PDNF

Q) $(P \leftrightarrow Q)$

$[(P \to Q) \land (Q \to P)]$

$[(\sim P \lor Q) \land (\sim Q \lor P)] \to$ PCNF

$\cancel{[\sim P \land (\sim Q \lor P) \lor (Q \land (\sim Q \lor P))]}$

$[(\sim P \land \sim Q) \lor (\sim P \land P) \lor (Q \land \sim Q) \lor (Q \land P)]$

$[(\sim P \land \sim Q) \lor (F_0 \lor F_0) \lor (Q \land P)]$

$[(\sim P \land \sim Q) \lor (P \land Q)]$

PCNF(principle conjunctive normal form):-

An equalent formula consisting of conjuction of max terms only is known as PCNF (or) product of sums concial form

1Q) find PCNF of

$(\sim P \rightarrow R) \wedge (Q \leftrightarrow P)$ using T.T

P	Q	R	$\sim P$	$\sim P \rightarrow R$	$Q \leftrightarrow P$	$\wedge$
T	T	T	F	T	T	T
T	T	F	F	T	F	T
T	F	T	F	T	F	F ✓
T	F	F	F	T	F	F ✓
F	T	T	T	T	F	F ✓
F	T	F	T	F	F	F ✓
F	F	T	T	T	T	T
F	F	F	T	F	T	F ✓

CNF:-

$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge$

$(P \vee \sim Q \vee R) \wedge (P \vee Q \vee R)$

2Q) $\sim(P \vee Q)$ i) T.T

P	Q	$P \vee Q$	$\sim$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

CNF:-

$(\sim P \vee \sim Q) \wedge (\sim P \vee Q) \wedge (P \vee \sim Q)$

3Q) find PCNF for $\sim(P \vee Q)$ with out using T.T

$\Rightarrow \sim(P \vee Q)$

$\Rightarrow \sim P \wedge \sim Q$

$\Rightarrow (\sim P \vee F_0) \wedge (\sim Q \vee F_0)$

$\Rightarrow [\sim P \vee (Q \wedge \sim Q)] \wedge [(\sim Q \vee (P \wedge \sim P)]$

$\Rightarrow (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (\sim Q \vee P) \wedge (\sim Q \wedge \sim P)$

$\Rightarrow (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (\sim Q \vee P)$

PCNF :-
$\because P \vee F_0 \Rightarrow P \vee (Q \wedge \sim Q)$

conversion from PDNF to PCNF

If PDNF of a given formula is known then PDNF of ~~this know~~ $\sim A$ will contain Disjunction of remaining min terms which don't appear in PDNF "find $\sim$(PDNF of $(\sim A)$)" which is "PCNF of A"

PDNF of $A \Rightarrow (\sim P \wedge \sim Q)$

" " $(\sim A) \Rightarrow (P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

$\sim($" " $(\sim A)) \Rightarrow \sim[(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)]$

PCNF of $A \Rightarrow (\sim P \vee \sim Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee Q)$

PCNF of $(\sim A) \Rightarrow (P \vee Q)$

$\sim($" " $(\sim A)) \Rightarrow \sim(P \vee Q)$

$\Rightarrow \sim P \wedge \sim Q$

convertion from PCNF to PDNF:-

If PCNF of a given formula A is known then PDNF of $\sim A$ will contain conjuntion of remaing max terms which don't appear in PCNF

Q) find PCNF from PDNF

$A \Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$

PDNF of $(\sim A) \Rightarrow (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

$\sim$(PDNF of $(\sim A)) \Rightarrow \sim[(P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

PCNF of $A \Rightarrow (\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee Q \vee R)$

Inference :-

The main aim of logic is to provide rules of inference to infer a conclusion from certain premises, the theory associated with rules of inference is known as inference theory

→ If a conclusion is derivated from a set of premises by using the accepted rules of reasoning then such a process of derivation is called a deduction (or) a formal proff and the argument is called a valid argument (or) conclusion is called valid conclusion.

→ They are two methods to decide whether the conclusion is valid or not

i) Truth table method

ii) without using truth table method

Truth table method:-

1Q) Determine wether the conclusion 'c' follows logically from the premises H_1 & H_2

i) $H_1: P \to Q$, $H_2: P$; $C: Q$

ii) $H_1: P \to Q$, $H_2: \sim P$; $C: Q$

iii) $H_1: P \to Q$, $H_2: \sim(P \wedge Q)$; $C: \sim P$

iv) $H_1: \sim P$, $H_2: P \leftrightarrow Q$; $C: \sim(P \wedge Q)$

v) $H_1: P \to Q$; $H_2: Q$; $C: P$

sol)

P	Q	$P \to Q$	$\sim P$	$\sim(P \wedge Q)$	$P \leftrightarrow Q$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	T	F
F	F	T	T	T	T

i) we observe that 1st row is only row in which both the premises have value 'T' the conclusion also has the value 'T' in that row

Hence it is valid

ii) we observe that 3rd & 4th rows in which both the premises have the value 'T', But the conclusion has the value 'F' in 4th row

Hence it is invalid

iii) we observe that 3rd & 4th rows in which both the premises have the value 'T', the conclusion also have the value 'T' in those rows

Hence it is valid

iv) we observe that 4th row is only row in which the both the premises has the value 'T' and conclusion also have the premises value 'T' in that row

Hence it is valid

v) we observe that 1st & 3rd rows is only row in which both the premises have the value 'T', But conclusion have the premises value 'F' in 3rd row.

Hence it is invalid

10) show that the conclusion C follows from the premises in the following cases

i) $H_1: P \to Q$ $C: P \to (P \wedge Q)$

ii) $H_1: \sim P$, $H_2: P \vee Q$ $C: Q$

iii) $H_1: \sim P \vee Q$, $H_2: \sim(Q \wedge \sim R)$, $H_3: \sim R$; $C: \sim P$

P	Q	~P	P → Q	R→(P∧Q)	P→(P∧Q)	(P∨Q)
T	F.	F	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	F	F	T
F	F	T	T	F	F	F

F

F i) valid → 3 rows

F ii) valid → 1 rows

F

P	Q	R	~P	~R	~P∨Q	(Q∧~R)	~C
T	T	T	F	F	T	F	T
T	T	F	F	T	T	F	F
T	F	T	F	F	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	T	T	F
F	T	F	T	T	F	F	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	F	T

iii) invalid → 1 row

method - 2 :-

Rules of inference :-

✓ If the formula is $P \to Q$

it converse is $Q \to P$ ✓

it's inverse is $\sim P \to \sim Q$ ✓

✓ it's contra positive is $\sim Q \to \sim P$

Note :-

→ given formula is equivalent to contra positive

→ converse is equivalent to inverse

*

$I_1: P \wedge Q \Rightarrow P$
$I_2: P \wedge Q \Rightarrow Q$ } (simplification) *

$I_3: P \Rightarrow P \vee Q$
$I_4: Q \Rightarrow P \vee Q$ } (additions)

$I_5: \sim P \Rightarrow P \to Q$

$I_6: Q \Rightarrow P \to Q$

$I_7: \sim(P \to Q) \Rightarrow P$

$I_8: \sim(P \to Q) \Rightarrow \sim Q$

I_9: $P, Q \Rightarrow P \wedge Q$

rule CP:- if we can derive 'S' from 'R' & a set of primsis then we can derive $R \to S$ from the set of primis alone

* I_{10}: $\sim P, P \vee Q \Rightarrow Q$ (Disjunctive syllogism)

* I_{11}: $P, P \to Q \Rightarrow Q$ (modus ponens)

* I_{12}: $\sim Q, P \to Q \Rightarrow \sim P$ (modus Tollens)

* I_{13}: $P \to Q, Q \to R \Rightarrow P \to R$ (Hypothetical syllogism)

* I_{14}: $P \vee Q, P \to R, Q \to R \Rightarrow R$ (dilemma)

1Q) Demostrate that 'R' is valid inference from the premsis $P \to Q, Q \to R$ & P

{1} $P \to Q$, Rule P

{2} P , Rule P

(1,2) {3} Q , I_{11} Rule T

{4} $Q \to R$, Rule P

(3,4) {5} R , I_{11} Rule T

(or)

{1} $P \to Q$

{2} $Q \to R$

(1,2) {3} $P \to R$, I_{13}, Rule T

{4} P . rules P

(3,4) {5} R, I_{11} rules T

2Q) $\sim P \to \sim(P \wedge \sim Q)$, $\sim Q \vee R$, $\sim R$

{1} $\sim(P \wedge \sim Q)$; Rule P

{2} $\sim P \vee Q$; Rule T (Demo , I_n)

{3} $P \to Q$; Rule T

{4} $\sim Q \vee R$; Rule P

{5} $Q \to R$; Rule T

(3,5) {6} $P \to R$; Rule T (I_{13})

{7} $\sim R$; Rule P

(6,7) {8} $\sim P$; Rule T (I_{12})

Hence Proved

3Q) Show that $R \vee S$ follows logically from the premsis $C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$, $(A \wedge \sim B) \to (R \vee S)$

$R \vee S \to C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$, $(A \wedge \sim B) \to (R \vee S)$

{1} $(C \vee D) \to \sim H$, Rule P

{2} $\sim H \to (A \wedge \sim B)$, Rule P

(1,2) {3} $(C \vee D) \to (A \wedge \sim B)$; Rule T, I_{13}

{4} $(A \wedge \sim B) \to (R \vee S)$; Rule P

(3,4) {5} $(C \vee D) \to (R \vee S)$; Rule T, I_{13}

{6} $C \vee D$; Rule P

(5,6) {7} $R \vee S$; Rule T, I_{11}

Hence proved

4Q) Show that $R \to S$:- can be derived from the premis

$P \to (Q \to S)$, $\sim R \vee P$, Q

{1} $\sim R \vee P$; Rule P

{2} R ; Rule P

(1,2) {3} P ; Rule T I_{10}

{4} $P \to (Q \to S)$; Rule P

(3,4) {5} $Q \to S$; Rule T, I_{11}

{6} Q ; Rule P

(5,6) {7} S ; Rule T I_{11}

(2,7) {8} $R \to S$; Rule C.P

5Q) Show that $P \to S$ can be derived from the premsis $\sim P \vee Q$, $\sim Q \vee R$, $R \to S$

{1} $\sim P \vee Q$; Rule P

{2} P ; Rule P

(1,2) {3} Q ; Rule T, I_{10}

{4} $\sim Q \vee R$; Rule P

(3,4) {5} R ; Rule T, I_{10}

{6} $R \to S$; Rule P

(5,6) {7} S ; Rule T, I_{11}

(2,7) {8} $P \to S$; Rule C.P

Hence proved

Consistency of premsis

A set of premsis $H_1, H_2 .. H_m$ is said to consistent if they conjunction has Truth value 'T' for some assignement of the truthvalues to the atomic variables.

Inconsistency of premsis:-

A set of premsis $H_1, H_2 .. H_m$ is said to inconsistent if they atleast one of the formula $H_1, H_2 ... H_m$ is false for every assignment of the truth values to the atomic variables.

Indirect method of proof:-

The notation of inconsistency is used in a procedure called proof of contradiction (or) indirect method of proof

Q) Show that the premsis are inconsistent $P \to Q$, $P \to R$, $Q \to \sim R$, P

{1} $P \to Q$; Rule P

{2} P ; Rule P

(1,2) {3} Q ; Rule T, I_{11}

{4} $Q \to \sim R$; Rule P

(3,4) {5} $\sim R$; Rule T, I_{11}

{6} $P \to R$; Rule P

(5,6) {7} $\sim P$; Rule T, I_{12}

(2,7) {8} $P \wedge \sim P$; Rule T

Q) Proof by Indirect method

$\sim Q, P \to Q, P \vee R \Rightarrow R$

{1} $P \vee R$; Rule P

{2} $\sim R$; Rule P

(1,2) {3} P ; Rule T, I_{10}

{4} $P \to Q$; Rule P

(3,4) {5} Q ; Rule T, I_{11}

{6} $\sim Q$; Rule P

(5, 6) {7} $Q \wedge \sim Q$; Rule T

3Q) $P \to Q, Q \to R, P \vee R \Rightarrow R$

It is a contradiction

∴ the conclusion follows

3Q) $P \to Q$, $Q \to R$, $P \vee R \Rightarrow R$

{1} $P \vee R$; Rule P

{2} $\sim R$; Rule P

(1, 2) {3} P ; Rule T, I_{10}

{4} $P \to Q$; Rule P

(3,4) {5} Q ; Rule T, I_{11}

{6} $Q \to R$; Rule P

{7} R ; Rule T, I_{11}

(2, 7) {8} $R \wedge \sim R$; Rule T

It is a contradiction

∴ the conclusion follows

(or)

{1} $P \to Q$

{2} $Q \to R$

{3} $P \to R$

{4} $\sim R$

{5} P

{6} $P \vee R$

{7} R

{8} $R \wedge \sim R$

The predicate logic :-

The logic based upon analysis of the predicates in any state is called predicate logic

eg ① The statement "'x' is student" can be denoted as $S(x)$

② "x is taller than y" can be denoted as $T(x, y)$

eg: '1' is 1 place predicate because it relates to only one object 'x'

eg: '2' is 2 place predicate since it deals belong's to 2 object (x, y)

Quantifiers :- Quantifiers :-

contain statements involves words that indicates quantity such as all some, none, one then this statements are called quantifiers

→ The two main quantifiers are $\exists x$ (some) & $\forall x$ (all)

→ The "quantifiers all" is called universal quantifier ($\forall x$) '→'

→ The quantifier some is called existential quantifier ($\exists x$) "∧"

① Something is good

$G(x)$: x is good

$\exists x, G(x)$

② everything is good

$G(x)$: x is good

$\forall x, G(x)$

③ Nothing is good

$\forall x, \sim G(x)$

$\forall$:- $\rightarrow$
$\exists$:- $\wedge$

④ Something is not good

$\exists(x), \sim G(x)$

⑤ All monkey has tail

$M(x)$:- x is monkey

$T(x)$: x has Tail

$\forall x, M(x) \rightarrow T(x)$

⑥ No monkey has tail

$\forall x, M(x) \rightarrow \sim T(x)$

⑦ Some monkey have tails

$\exists x; M(x) \wedge T(x)$

⑧ Some monkey have no Tails

$\exists x; M(x) \wedge \sim T(x)$

Rules of Generligation & specfication

i) universe specfication :- (US)

$\forall x, P(x) \Rightarrow P(T)$ for all t

ii) universe Generligation :- (UG)

$P(T)$ for all t $\Rightarrow \forall x, P(x)$

iii) existential specification :- (ES)

$\exists x, P(x) \rightarrow P(t)$ for some t

iv) existential Generalisation :- (EG)

$P(t)$ for some $t \rightarrow \exists x, P(x)$

1Q) verify the value the following argument

i) Tigers are dangeous animals

ii) There are some tigers

$\therefore$ there are some dangeous animals.

Let $T(x)$: x is a tiger

$D(x)$: x is dangers

(1) $\forall x, T(x) \rightarrow D(x)$

(2) $\exists x, T(x)$

conclusion can be converted to

$\exists x, D(x)$

{1} $\forall x, T(x) \rightarrow D(x)$, Rule P

(1) {2} $T(a) \rightarrow D(a)$; US

{3} $\exists x, T(x)$, Rule P

(3) {4} $T(a)$; Rule ES

(2, 4) {5} $D(a)$ Rule T, $\tilde{s}_{11}$

{6} $\exists x, D(x)$; EG

2Q) verify the validate of the following arguments

i) all integer are rational no.

ii) Some integer are power's of '2'

$\therefore$ some rational no. are power's of '2'

Let $I(x)$: x is integer

$R(x)$: x is rational no.

$P(x)$: x is power of 2

$$\frac{\begin{array}{l}\forall x\ I(x) \rightarrow R(x) \\ \exists x\ I(x) \wedge P(x)\end{array}}{\exists x\ R(x) \wedge P(x)}$$

{1} $\exists x, I(x) \wedge P(x)$; Rule P

{2} $I(a) \wedge P(a)$; ES

(2){3} $I(a)$, Rule T

(2) {4} $P(a)$, Rule T

{5} $\forall x, I(x) \rightarrow R(x)$; Rule P

{6} $I(a) \rightarrow R(a)$; US

(3,6) {7} $R(a)$; Rule T, I_{11}

(4,7) {8} $R(a) \wedge P(a)$; Rule T

{9} $\exists R(x) \wedge P(x)$; EG

hence proved

3Q) i) All man are ~~mortals~~ motal

ii) Socrates is a man

iii) $\therefore$ socrates is ~~mortals~~ motals

$M(x)$: x is a man

$O(x)$: x is mortal

S : Socrates

1) $\forall x [M(x) \rightarrow O(x)]$

2) $M(S)$

$O(S)$

{1} $\forall x [M(x) \rightarrow O(x)]$, Rule P

{2} $M(S) \rightarrow O(S)$, US

{3} $M(S)$, Rule P

{4} $O(S)$, Rule T, I_{11}

4Q) verify the validate of the following premsis

i) $\forall x [P(x) \rightarrow Q(x)]$

ii) $\forall x [R(x) \rightarrow \sim(Q(x))]$

iii) $\forall x [R(x)]$

$\forall x [\sim P(x)]$

{1} $\forall x [R(x) \rightarrow \sim Q(x)]$ Rule P

{2} $R(a) \rightarrow \sim Q(a)$ Rule US

{3} $\forall x [R(x)]$ Rule P

(1,3) {4} $\forall x [\sim Q(x)]$, Rule T, I_{11}

{5} $\forall x [P(x) \rightarrow Q(x)]$ Rule P

{6} $\forall x [\sim Q(x) \rightarrow \sim P(x)]$ contra positive

(4,6) {7} $\forall x [\sim P(x)]$ Rule T, I_{11}

Set theory

Set:-

A set is a collection of object or elements which share common property

-> A set is said to be well defind if it is possible to determine by means of ~~sub abstracts~~ certain rules

-> A set is denoted by capital letters [A, B, C] and elements are denoted by lower case letters

ex:-

-> set of rivers in a country
$A = \{x \mid x \text{ is a river in country}\}$

-> a set of even numbers.
i.e $A = \{0, 2, 4, 6, 8, 10\}$
$A = \{x \mid x \text{ is an even no.}\}$

-> $B = \{1, 2, 3, 4, 5, 6\}$
$= \{x \mid x \text{ is a rational no.}\}$

Subset:-

Let A & B be two sets then A is said to be a subset of B if every element of A is element of B,
'A' is said to be proper subset of B if 'A' is subset of B & there is atleast one element B which is not in 'A'

-> A is subset to B is denoted by $A \subseteq B$

-> If it is a proper subset it is denoted by $A \subset B$

ex:-

$A = \{1, 2, 3\}$

$B = \{0, 1, 2, 3, 4\}$

$A \subset B$

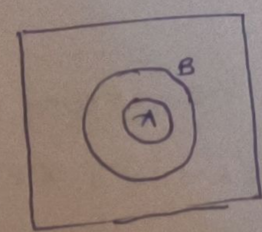
Venn diagrams:-

A simple & instructive way of representing a set is with the help of diagram known as venn diagrams

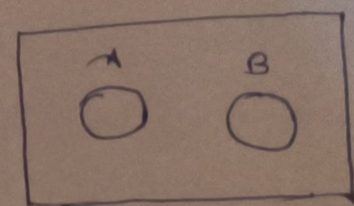
-> here a rectangle represents universal set any other set 'A' is represented by the interial of simple closed curve inside the rectangle usually a circle

ex:-

① A is subset to B is represented as in venn diagrams as $(A \subset B)$



② A is not subset to B $(A \not\subset B)$



properties:-

1) $A \subseteq A$

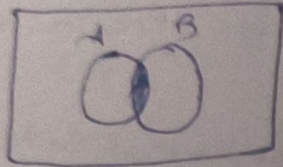
2) $A \subset B$ & $B \subset C$ then $A \subset C$

3) $A \subset B$ & $A \not\subset C$ then $B \not\subset C$

4) Intersection

The intersection of any two sets 'A' & 'B' return as $A \cap B$ is the set consists of all the elements which belongs to A & B both

$A \cap B = \{x / x \in A \ \& \ x \in B\}$

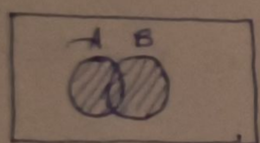


properties

1) $A \cap B = B \cap A$
2) $A \cap A = A$
3) $A \cap \emptyset = \emptyset$
4) $A \cap (B \cap C) = (A \cap B) \cap C$ (associative law)

union

for any two set 'A & B' the union of 'A, B' is denoted by $A \cup B = \{x / x \in A \text{ (or) } x \in B\}$

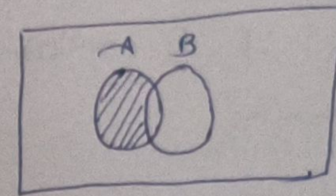


properties:-

1) $A \cup A = A$
2) $A \cup \emptyset = A$
3) $A \cup B = B \cup A$
4) $A \cup (B \cup C) = (A \cup B) \cup C$

Relative complement:-

Let 'A, B' be any two sets the relative complement of B in A is denoted by 'A-B' is the set containing all the elements of A which are not elements of B

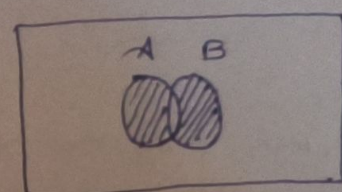


$A - B = \{x | x \in A \ \& \ x \in B\}$

Symmetric difference:-

Let 'A, B' be any two sets then the symmetric difference of A&B is denoted by $A \Delta B$ which consists of the elements of A & or B but not in both

$A \Delta B = (A - B) \cup (B - A)$
$(A \cup B) - (A \cap B)$



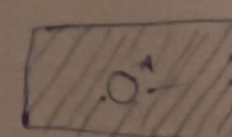
properties:-

1) $A \Delta B = B \Delta A$
2) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$
3) $A \Delta \emptyset = A$ (identity)
4) $A \Delta A = \emptyset$ (Inverse)
5) $(A \Delta B) = (A \cap B') \cup (B \cap A')$

complement:- (A' (or) A^c (or) $\bar{A}$)

suppose A is a set then complement of A is denoted by A' or A^c or $\bar{A}$ which consists of the all the element of universal set U which or not in 'A'

$A^c = \{x / x \in U \ \& \ x \notin A\}$



finite set:-

A set is said to be finite if it containing finite no. of different elements

ex:-

$A = \{1, 2, 3\}$

$B = \{a, b, c, d\}$

Infinite set:-

A set is said to be infinite set if it contains infinite distinct elements

ex:- $N = \{1, 2, 3, \ldots \infty\}$

Single-ton set:-

A set which contains only one element is called single ton set

$A = \{0\}$

$B = \{x / 2 < x < 4, x \in N\}$

Basic laws of set:-

1) Idempotent law:-

$A \cap A = A$

$A \cup A = A$

2) Associative law:-

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cup C) = (A \cup B) \cup C$

3) commutative law:-

$A \cup B = B \cup A$

$A \cap B = B \cap A$

4) Distributive law:-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

5) De morgan's law:-

$(A \cap B)' = A' \cup B'$

$(A \cup B)' = A' \cap B'$

6) Absorption law:-

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

7) complementary law:-

$\phi' = U$ (universal set)

$U' = \phi$

$(A')' = A$

Cartesian product:-

Let A & B be two sets then the cartesian product of A & B is denoted by $A \times B$, and it is defined that $A \times B = \{(a,b) / a \in A \;\&\; b \in B\}$

eg:-

$A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ $\qquad A \times B \neq B \times A$

$A \times B = \{(1,a), (1,b), (1,c), (1,d),$
$(2,a), (2,b)\ (2,c), (2,d),$
$(3,a), (3,b), (3,c), (3,d)\}$

Note:-

1) $A \times B \neq B \times A$

2) $A \times (B \times C) = (A \times B) \times C$

3) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

4) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Relations:-

A Relation 'R' from a set 'A' to set 'B' is subset of cartesian product of A&B

$\Rightarrow R = \{(x,y) / x \in A, y \in B \ \& \ x+y=10\}$

$A = \{1,2,3,4\}$, $B = \{5,6,7,8\}$

$R = \{(2,8), (3,7), (4,6)\}$

$\Rightarrow R = \{(x,y) / x^2+y^2<1\}$

Domain & range:-

Domain of R $= \{x \mid x \in A \ \& \ (x,y) \in R\}$ ($\because$ R is relation)

Range of R $= \{y \mid y \in B \ \& \ (x,y) \in R\}$

ex:-

if $A = \{2,3,4\}$ & $B = \{3,4,5,6,7\}$ & R is the relation $\because (a,b) \in R$ a divides b

$\because R = \{(2,4), (2,6), (3,3), (3,6), (4,4)\}$

Domain $= \{2,3,4\}$

Range $= \{3,4,6\}$

$\rightarrow$ ~~Domain of R=~~

Properties of relation:- (R S T)

A Relation 'R' on a set 'A' is said to be

i) ~~refelective~~ reflexive if xRx (or) $(x,x) \in R$ $\forall x \in A$

ii) irreflexive

$x \not R x$ (or) $(x,x) \notin R$ $\forall x \in R$

iii) Symmetric

if xRy ~~$\forall y$~~ $\Rightarrow yRx$ (or)

$(x,y) \in R \Rightarrow (y,x) \in R$ $\forall x,y \in A$

iv) anti symmetric

if $xRy \Rightarrow y \not R x$ (or)

$(x,y) \in R \Rightarrow (y,x) \notin R$ $\forall x,y \in A$

v) a-symmetric

neither symmetric nor anti-symmetric

vi) Transtive

$xRy \ \& \ yRz \Rightarrow xRz$ (or)

$(x,y) \in R \ \& \ (y,z) \in R$ Then $(x,z) \in R$ $\forall x,y,z \in R$

Q) If $A = \{1,2,3,4\}$

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (2,4), (3,3), (3,4), (4,4)\}$

sol) R_1 is reflexive, a-symmetric,

it is not transitive

Q) $R_2 = \{(2,2), (3,3) (4,4) (1,3), (3,1), (3,2)\}$

R_2 is a-symmetric

Power set:-

Let 'S' be a set then power set of 'S' is denoted by $P(S)$ and is defined by set of all possible subsets of 'S'

ex:-

$S = \{1,2,3\}$

$P(S) = \{(\phi), (1), (2), (3), (1,2), (1,3), (2,3), (1,2,3)\}$

→ The no. of subsets of 's' with 'n' elements is "2^n"

⇒ $2^n = 2^3 = 8$

operations on Relations:-

Let A and B be two sets then R is a relation from A to B which is a subset of $A \times B$

The operations on sets i.e union, intersection and complement can be applied to relations also

Let R & S be relations from A to B

i) $R \cap S = \{(a,b) \in A \times B \mid (a,b) \in R \ \& \ (a,b) \in S\}$ is called intersection of R & S

ii) $R \cup S = \{(a,b) \in A \times B / (a,b) \in R$ or $(a,b) \in S\}$ is called union of R and S

iii) $R - S = \{(a,b) \in A \times B \mid (a,b) \in R$ and $(a,b) \notin S\}$ is difference of R and S

iv) $R^c = \{(a,b) \in A \times B / (a,b) \notin R\}$ is called complement of R

⇒ Here $A \times B$ is a universal set

Q) Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ R and S be relations from A to B where

$R = \{(1,1), (2,2), (3,3)\}$ and

$S = \{(1,1), (1,2), (1,3), (1,4)\}$

find i) $R \cup S$ ii) $R \cap S$ iii) $R - S$ iv) R^c

i) $R \cup S = \{(1,1), (2,2), (3,3), (1,2), (1,3), (1,4)\}$

ii) $R \cap S = \{(1,1)\}$

iii) $R - S = \{(2,2), (3,3)\}$

iv) $R^c = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4)\}$

$S^c = \{(2,1), (2,3), (2,4), (3,1), (3,2), (3,4)\}$

Equivalence Relation:-

A relation R on a set x is called as equivalence relation if it is reflexive, symmetric & transitive

ex:-

1) Equality of numbers on a set of real numbers

$R = \{(x,y) / x = y$ for $x, y \in R\}$

2) Similarity of triangles on set of triangles

Q) Let $x = \{1, 2, \ldots 7\}$ and $R = \{(x,y) / x-y$ is divisible by $3\}$ in x

show that R is an equivalence relation.

i) for any $a \in x$, $a-a=0$ is divisible by 3

⇒ a,a ⇒ R is reflexive

ii) for any $a, b \in x$, $a-b$ is divisible by 3

$b-a$ is also

(or)

$(a-b)\%3 = 0$ also $(b-a)\%3 = 0$

if aRb then bRa ⇒ R is symmetric

iii) for any $a, b, c \in x$

$a-b$ is divisible by 3

$b-c$ is divisible by 3

then $a-c$

(or)

$a-b = 3m$

$b-a = 3n$

then $a-c = a-b+b-c = 3m+3n = 3(m+n)$

⇒ $a-c$ is divisible by 3 ⇒ aRc

∴ It is transitive

* Partition set:-

→ Let A be any non-empty set & $A_1, A_2 \cdots A_n$ are subsets of A

A set denoted by π is called partition of set A if

i) $A_i \cap A_j = \phi$ for $i \neq j$

ii) $\bigcup_{i=1}^{n} A_i = A$

→ If only (ii) satisifies we call it as cover of A

Ex:- Let $A = \{a, b, c, d, e, f, g, h, i, j\}$

$A_1 = \{a, b, c, d, e\}$ $\quad A_4 = \{a, b, c, d\}$

$A_2 = \{f, g, h\}$ $\quad A_5 = \{c\}$

$A_3 = \{i, j\}$

find partition of A

$\pi_1 = \{A_1, A_2, A_3\}$ is a partition of A

Since $A_1 \cap A_2 = \phi$, $A_2 \cap A_3 = \phi$, $A_1 \cap A_3 = \phi$ and

$A_1 \cup A_2 \cup A_3 = A$

$\pi_2 = \{A_3, A_4, A_2\}$ is not partition of A

$A_3 \cap A_4 = \phi$, $A_4 \cap A_2 = \phi$, $A_3 \cap A_2 = \phi$,

$A_2 \cup A_3 \cup A_4 \neq A$

Q) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A_1 = \{1, 2, 3, 4\}$; $A_2 = \{5, 6, 7\}$; $A_3 = \{4, 5, 7, 9\}$

$A_4 = \{4, 8, 10\}$; $A_5 = \{8, 9, 10\}$; $A_6 = \{1, 2, 3, 6, 8, 10\}$

which of the following are partition of A?

a) $\{A_1, A_2, A_3\}$ $\quad$ b) $\{A_1, A_3, A_5\}$

✓ c) $\{A_3, A_6\}$ $\quad$ d) $\{A_2, A_3, A_4\}$

$A_3 \cap A_6 = \phi$ and

$A_3 \cup A_6 = A$

* cover of a set:-

→ Let S be the given set and $A_1, A_2 \ldots A_n$ be subset of S then

$A = \{A_1, A_2, \ldots A_n\}$ is called cover of S if

$\bigcup_{i=1}^{n} A_i = S$

→ In above example, $\{A_1, A_2, A_3, A_6\}$ is a covers

Representation of relations :-

-> These are 2 alternating methods for representing relations

-> one method uses zero-one matrixe & the other method uses digraph

Relation matrix:-

If $A = \{a_1, a_2, \dots a_n\}$, $B = \{b_1, b_2, \dots b_m\}$ are finite sets containing n & m elements respectively and R is a relation from A to B, then relation R can be ~~relat~~ represented by an $n \times m$ matrix called relation matrix denoted by M_R

$M_R = [m_{ij}]$ where $m_{ij} = 1$ if $(a_i, b_j) \in R$

$m_{ij} = 0$ if $(a_i, b_j) \notin R$

-> Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3, b_4\}$ and

$R = \{(a_1, b_1), (a_1, b_4), (a_2, b_2), (a_2, b_3), (a_3, b_1), (a_3, b_3)\}$

Relation matrix, $M_R =$
$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{matrix} b_1 & b_2 & b_3 & b_4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Ex. R is a relation from A to B, $A = [a_1, a_2, a_3]$

$B = [b_1, b_2, b_3, b_4, b_5]$

$M_R =$
$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

find R

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}$

-> A relation matrix reflects some of the properties of the relation

i) R is reflexive if all the elements on the main diagonal of M_R are equal to 1

ii) R is symmetric if and only if $m_{ij} = m_{ji}$ i.e $M_R = M_R^T$ for all integers of i and j

iii) R is anti-symmetric if $m_{ij} = 1$ with $i \neq j$ then $m_{ji} = 0$

Ex:- If $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is, R reflexive, symmetric or anti-symmetric

Sol:-

All principal diag elements are 1

=> reflexive

$M_R = M_R^T$ => symmetric

not anti-symmetric => $m_{12} = 1$ and $m_{21} = 1$

* Digraph:-

-> A relation can be represented pictorially by drawing its digraph as follows

-> A small circle is drawn for each element of A and mark with the corresponding elements

-> These circles are called vertices

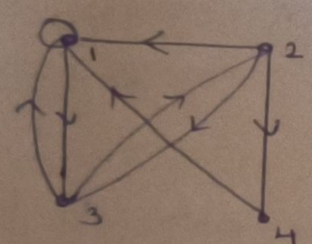
→ An arrow is drawn for the vertex a_i to a vertex a_j if $a_i R a_j$. This is called an edge (directed edge)

→ If (a,a) is an ordered pair in the relation which is represented by directed edge from a to a such an edge is called loop

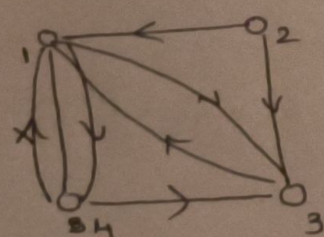
Ex:- If $R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$

on the set $A = \{1,2,3,4\}$

represent R with diagraph



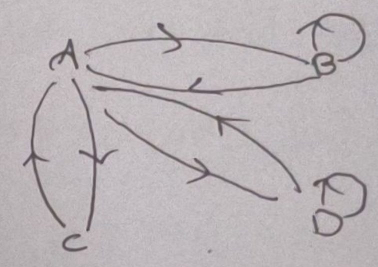
Ex:-



find R

$R = \{(1,4), (1,3), (2,2), (2,1), (2,3), (3,1), (4,1), (4,3)\}$

Q) Determine where the relation formed by the digraph is reflexive, symmetric, anti-symmetric & transitive



→ R is not reflexive since there is no loop for A and C

→ R is symmetric since for every vertix which is joined with another vertex also has an edge in opposite direction

→ R is not anti-symmetric since we have edge in opposite direction for distinct vertices

→ R is not transitive since there is an edge from C to A & A to B but no edge

* Inverse relation :-

→ If R is a relation, then $\tilde{R}$ is called inverse relation if $\tilde{R} - \{(a,b) / \text{if } (b,a) \in R\}$

→ The graph of $\tilde{R}$ is obtained from R by simply reversing arrows for each orderd pair of R

→ Relation matrix of $\tilde{R}$ is denoted by $M_{\tilde{R}} = (M_R)^T$

Boolean product :-

Boolean product of A & B denoted by $A \odot B$ =

$\{(a_i, c_j) \,/\, (a_i, b_k) \in A$ and $(b_k, c_j) \in B\}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \\ (0 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \\ (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \end{bmatrix}$$

$$\begin{bmatrix} 1 \vee 0 & 1 \vee 0 & 0 \vee 0 \\ 0 \vee 0 & 0 \vee 1 & 0 \vee 1 \\ 1 \vee 0 & 1 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Ex:-

~~Join & meet of A & B is denoted by $A \wedge B$~~

→ Join & meet of A & B is denoted by $A \wedge B$ is the zero - one matrix with $(i, j)^{th}$ entry as $a_{ij} \wedge b_{ij}$

Ex:- If R and S are relations defined on set A & B are represented by relation matrix

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \;\&\; M_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

find meet ($\wedge$) & join ($\vee$) of R & S

$$M_{R \cup S} = M_R \vee M_S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 1 & 0 \vee 0 & 0 \vee 1 \\ 1 \vee 0 & 0 \vee 1 & 0 \vee 1 \\ 0 \vee 1 & 1 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_R \wedge M_S \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Composition of Relation:-

If R, S is a relation from A to B, B to C composition of relation R & S is denoted by $S \circ R$ & is defined as $M_{S \circ R} = M_R \odot M_S$ where M_R is relation matrix of relation R & M_S is relation matrix of relation S

Q) find matrix representation of the relation $S \circ R$ when the matrix representing R & S are given by

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

matrix of $S \circ R \Rightarrow M_{S \circ R} = M_R \odot M_S$

$$= \begin{bmatrix} (1 \wedge 0) \vee (0 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) \\ (1 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 0) & (1 \wedge 0) \vee (1 \wedge 1) (0 \wedge 1) \\ (0 \wedge 1) \vee (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 0) (0 \wedge 0) & (0 \wedge 0) \vee (0 \wedge 1) (0 \wedge 1) \end{bmatrix}$$

$$\begin{bmatrix} 0\vee0\vee1 & 1\vee0\vee0 & 1\vee0\vee1 \\ 0\vee0\vee0 & 1\vee0\vee0 & 0\vee1\vee0 \\ 0\vee0\vee0 & 0\vee0\vee0 & 0\vee0\vee0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ 1) $(\tilde{\tilde{R}}) = R$

2) $R = S \iff \tilde{R} = \tilde{S}$

3) $R \le S \iff \tilde{R} \le \tilde{S}$

4) $R \tilde{\cup} S \iff \tilde{R} \cup \tilde{S}$

5) $R \tilde{\cap} S \iff \tilde{R} \cap \tilde{S}$

$\Rightarrow$ $M_{\widetilde{SoR}} = [M_{SoR}]^T$

Q) find M_{SoR}, $M_{\bar{R}}$, $M_{\bar{S}}$, $M_{\overline{SoR}}$ & ST ~~MoR~~ $M_{\bar{S}o\bar{R}} = M_{\overline{SoR}}$

where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

$M_{\bar{R}} = (M_R)^T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$M_{\bar{S}} = (M_S)^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$M_{SoR} = M_R \odot M_S$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_{\overline{SoR}} = (M_{SoR})^T$