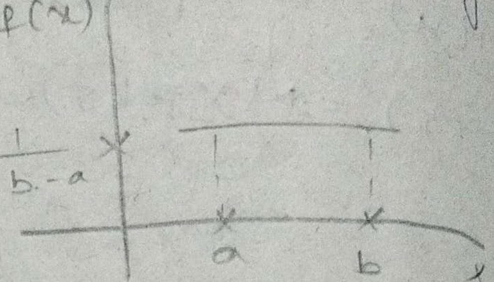
19/09/24 UNIT-3

Uniform Distribution: Let X be CRV of UD. The Prob. density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$



Mean of Uniform Distribution:

$$\text{Mean} - \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$= \int_{-\infty}^{a} x \cdot 0\,dx + \int_{a}^{b} x \cdot \frac{1}{b-a}\,dx + \int_{b}^{\infty} x \cdot 0\,dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right]_a^b = \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right]$$

$$= \frac{1}{\cancel{b-a}}\left(\frac{\cancel{(b-a)}(b+a)}{2}\right) = \frac{a+b}{2}$$

$$\text{mean} - \boxed{\mu = \frac{a+b}{2}}$$

Variance of Uniform Distribution:

$$\text{variance} - \sigma^2 = E(x^2) - [E(x)]^2 \quad \text{①}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$$

$$E(x^2) = \int_{-\infty}^{a} x^2 \cdot 0\, dx + \int_{a}^{b} x^2 \cdot \frac{1}{b-a}\, dx + \int_{b}^{\infty} x^2 \cdot 0\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right)$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right]$$ — sub in (1)

$$\sigma^2 = \frac{b^3-a^3}{3(b-a)} - \left(\frac{a+b}{2}\right)^2 = \frac{b^3-a^3}{3(b-a)} - \frac{(a+b)^2}{4}$$

$$\sigma^2 = \frac{4(b^3-a^3) - 3(b-a)(a^2+b^2+2ab)}{12(b-a)}$$

$$= \frac{4b^3-4a^3-3(a^2b+b^3+2ab^2-a^3-ab^2-2a^2b)}{12(b-a)}$$

$$= \frac{b^3-a^3-3(ab^2-a^2b)}{12(b-a)}$$

$$= \frac{b^3-a^3-3ab^2+3a^2b}{12(b-a)} = \frac{-(a-b)^3}{12(b-a)}$$

$$= \frac{(a-b)^{\cancel{3}}}{12\cancel{(a-b)}} = \frac{(a-b)^2}{12}$$

variance - $\boxed{\sigma^2 = \frac{(a-b)^2}{12}}$

Standard Deviation of UD:

$$\sigma = +\sqrt{var} = +\sqrt{\frac{(a-b)^2}{12}} = \frac{a-b}{2\sqrt{3}}$$

Normal Distribution:

Let 'x' be CRV, x is said to follow normal distribution if the PDF (prob. Density function) is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad \begin{array}{l} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{array}$$

x is called normal variate.

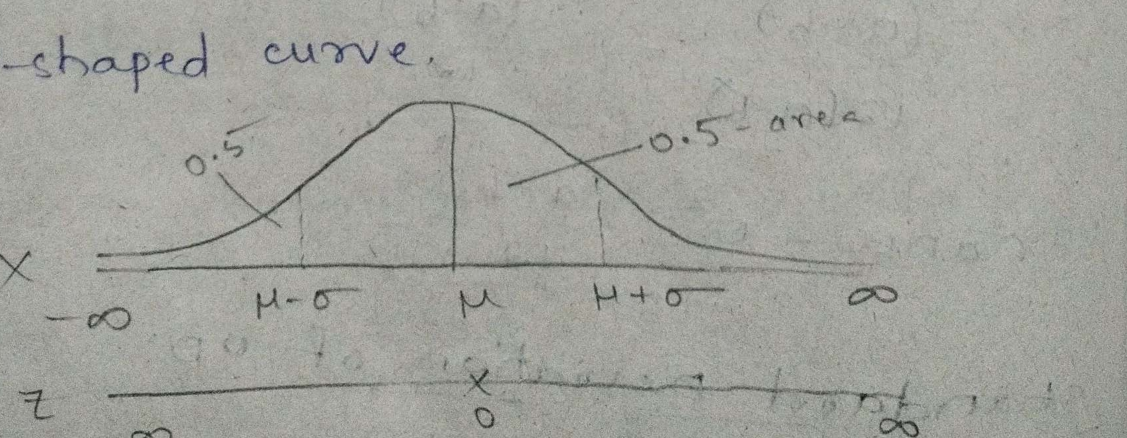
$z = \frac{x-\mu}{\sigma}$ _ standard normal variate

with mean = 1 and std. deviat^n $(\sigma) = 0$

then $f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < \infty$

Chief characteristics of Normal Distribution $f(x)$

1) The graph of the (normal ~~curve~~) in XY plane is called normal curve (or) bell-shaped curve.



2) The total area under normal curve is one. i.e., $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

3) The curve is symmetric about mean.

4) The points where the curve drops are called points of inflexion and are located at $\mu \pm \sigma$.

5) The area b/w $\mu-\sigma$ and $\mu+\sigma$ is 66.27%.

6) The area b/w $\mu-2\sigma$ and $\mu+2\sigma$ is 95.43%.

7) The area b/w $\mu-3\sigma$ and $\mu+3\sigma$ is 99.73%.

8) The curve has its max. value at $x=\mu$.

9) In Normal Distribution,

mean = median = mode.

10) $$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

NOTE: 1) $$\int_0^{\infty} e^{-x^2/2}\,dx = \sqrt{\frac{\pi}{2}}$$ (gamma function)

2) $$\int_{-\infty}^{\infty} z \cdot e^{-z^2/2}\,dz = 0$$ (it is an odd funct^n)

3) $$\int_{-\infty}^{\infty} z^2 \cdot e^{-z^2/2}\,dz = 2\int_0^{\infty} z^2 e^{-z^2/2}\,dz$$ (even funct^n)

Mean of Normal distribution:

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad \left(\text{let } z = \frac{x-\mu}{\sigma}, \; x = \mu + \sigma z, \; dx = \sigma\,dz\right)$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) e^{\frac{-z^2}{2}} (\cancel{\sigma}\,dz)$$

$$= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z \cdot e^{-z^2/2} dz$$

$$= \frac{\mu}{\sqrt{2\pi}} \times 2 \int_{0}^{\infty} e^{-z^2/2} dz + 0 \Rightarrow \frac{1}{\sqrt{2\pi}} \times \mu \times 2 \times \sqrt{\frac{\pi}{2}}$$

$$\boxed{\text{Mean} = \mu}$$

Variance of Normal Distribution:

$$\text{Variance} - \sigma^2 = E(x^2) - [E(x)]^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$\left(E(x^2) - \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx\right)$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $z = \frac{x-\mu}{\sigma} \Rightarrow x-\mu = \sigma z$

$dx = \sigma\, dz$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^2 e^{-\frac{z^2}{2}} \cancel{\sigma}\, dz = \frac{1}{\sqrt{2\pi}} \sigma^2 \int_{-\infty}^{\infty} z^2 e^{-z^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \sigma^2 \times 2 \int_0^{\infty} z^2 \cdot e^{-z^2/2} dz$$

put $\frac{z^2}{2} = t \Rightarrow z\,dz = dt$

$dz = \frac{dt}{z} = \frac{dt}{\sqrt{2t}}$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} 2t\, e^{-t} \frac{1}{\sqrt{2t}} dt$$

$$= \frac{4\sigma^2}{2\sqrt{\pi}} \int_0^{\infty} t^{1-\frac{1}{2}} \cdot e^{-t} dt = \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-t} \cdot t^{1/2} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-t} \cdot t^{3/2-1} dt$$

$\left(\because \Gamma n = \int_0^{\infty} e^{-x} x^{n-1} dx\right)$

$\left(\because \Gamma(n+1) = n\Gamma n\right)$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}\right) = \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\left(\frac{1}{2}+1\right)$$

$\left(\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right)$

$$= \frac{\cancel{2}\sigma^2}{\sqrt{\pi}} \frac{1}{\cancel{2}} \Gamma\left(\frac{1}{2}\right) = \frac{\sigma^2}{\sqrt{\pi}} \times \sqrt{\pi} \Rightarrow \boxed{\text{variance} = \sigma^2}$$

2019

Mode of Normal Distribution:

It is the value of x for which $f(x)$ is maximum. i.e., $f'(x)=0$

$f''(x)<0$

In N.D, $f(x)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ ——— ①

Diff. eq ① wrt 'x'

$$f'(x)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)} \times \frac{-1}{\cancel{2}}\times\cancel{2}\left(\frac{x-\mu}{\sigma}\right)\times\frac{1}{\sigma}$$ ——— ②

for $x=\mu$, $f'(\mu)=0$

Diff. eq ② wrt 'x'

$$f''(x)=\frac{-1}{\sigma^2\sqrt{2\pi}}\left[\frac{1}{\sigma}\times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}+\left(\frac{x-\mu}{\sigma}\right)e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\times\frac{1}{2}\times 2\left(\frac{x-\mu}{\sigma}\right)\right]$$

for $x=\mu$, $f''(\mu)<0$

$\therefore$ μ is mode of ND.

NOTE: $\int_a^b f(x)\,dx = 0$ then $a=b$

Median of ND:

Let 'M' be median of ND, then

$$\int_{-\infty}^{M} f(x)\,dx=\frac{1}{2}$$

$$\int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{M} f(x)\,dx = \frac{1}{2} \quad \text{--- (1)}$$

$$\int_{-\infty}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad \text{- put } \frac{x-\mu}{\sigma} = z$$

$x = \sigma z + \mu$

$dx = \sigma\,dz$

$x = \mu,\ z = 0$

$x = -\infty,\ z = -\infty$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2}\, \cancel{\sigma}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2}\, dz$$ (since above is an even funct^n limits are changed)

$$= \frac{1}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}} = \frac{1}{2}$$

Sub. in (1)

$$= \frac{1}{2} + \int_{\mu}^{M} f(x)\,dx = \frac{1}{2}$$

$$= \int_{\mu}^{M} f(x)\,dx = 0 \Rightarrow \int_{\mu}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 0$$

$$\therefore \boxed{M = \mu}$$

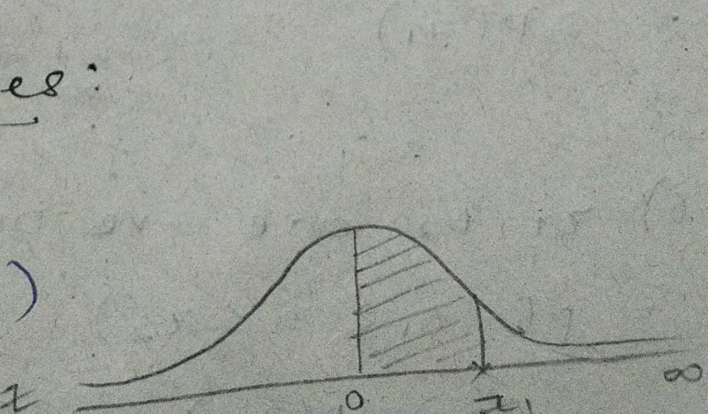
*-10M.

1) S.T in ND mean = mode = median.

<u>finding probabilities</u>:

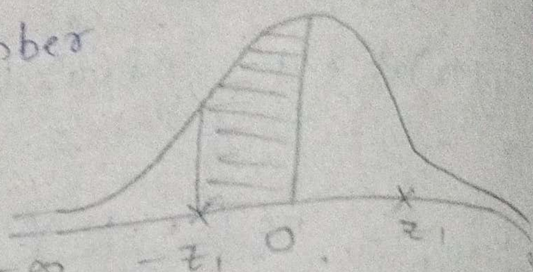
1) z_1 - the number

$p(z_1) = p(0 < z < z_1)$

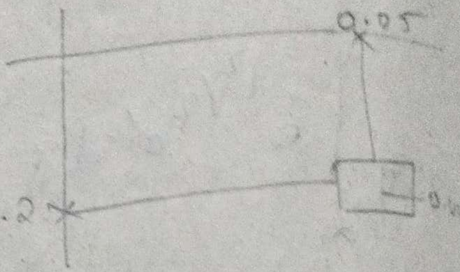


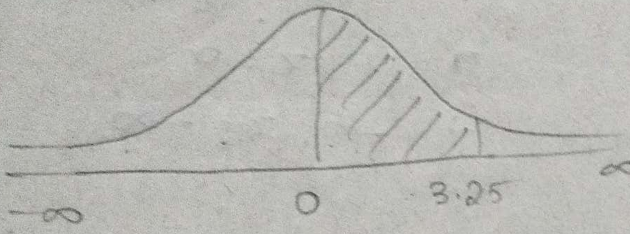
2) z_1 is negative number

$P(-z_1) = P(z_1)$



ex: $P(3.25) = 0.4994$

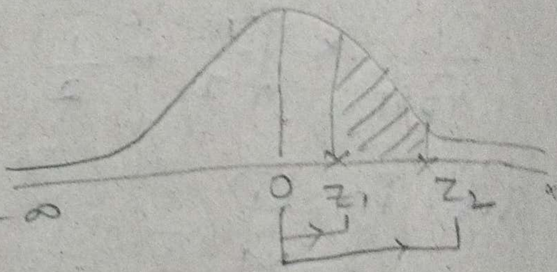




3) z_1, z_2 are +ve numbers

$P(z_1 < z < z_2)$

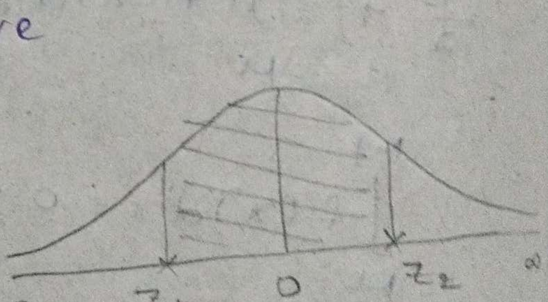
$= P(z_2) - P(z_1)$



4) z_1 is -ve, z_2 is +ve

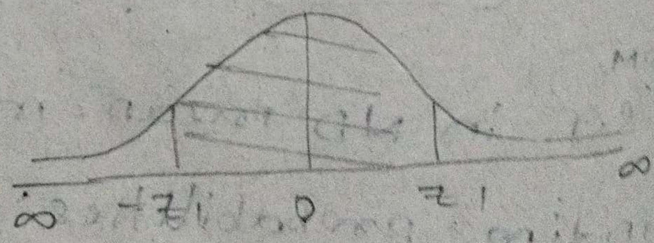
$P(-z_1 < z < z_2)$

$= P(z_1) + P(z_2)$



5) $P(-z_1 < z < z_1)$

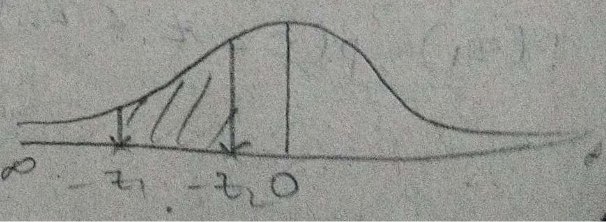
$= 2P(z_1)$



6) z_1, z_2 are -ve numbers

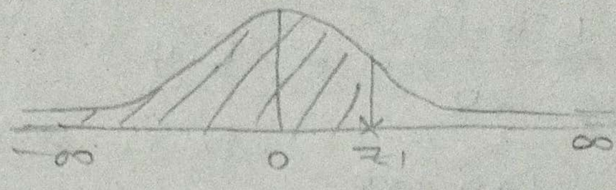
$= P(-z_1 < z < z_2)$

$= P(z_1) - P(z_2)$



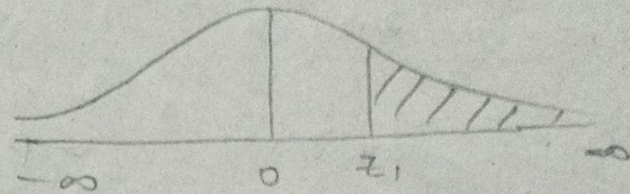
7) $P(z < z_1)$

$= 0.5 + P(z_1)$



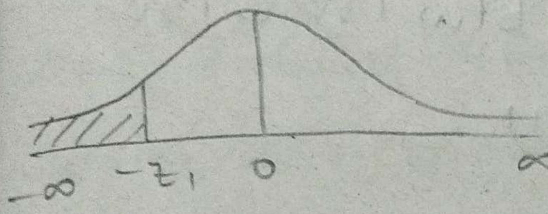
8) $P(z > z_1)$

$= 0.5 - P(z_1)$



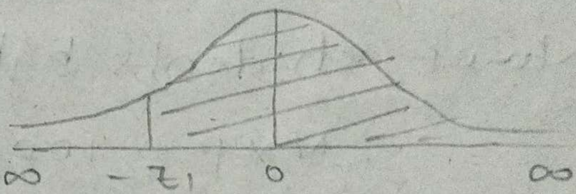
9) $P(z < -z_1)$

$= 0.5 - P(z_1)$



10) $P(z > -z_1)$

$= 0.5 + P(z_1)$



23/09

1) Given that the mean height of students in a class is 158 cm with std. deviation 20 cm. find how many students height lies b/w 150 & 170 cm if there are 100 students in the class.

No. of students $N = 100$

Mean - $\mu = 158$ cm

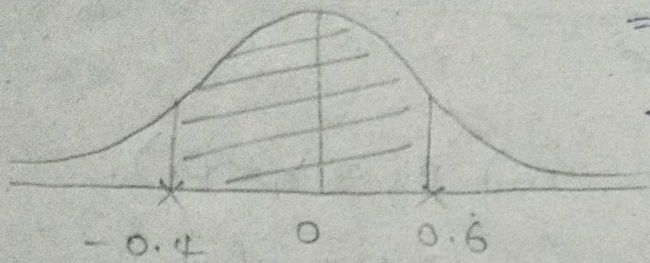
Std. dev - $\sigma = 20$ cm

$P(150 \le x \le 170)$

We know that, $z = \frac{x - \mu}{\sigma}$

$$z_1 = \frac{150-158}{20} = -0.4$$

$$z_2 = \frac{170-158}{20} = 0.6$$



$= P(-0.4 \le z \le 0.6)$

$= P(0.4) + P(0.6)$

$= 0.1554 + 0.2257$

$= 0.3811$

No. of students hgt lies b/w 150 & 170

$= N \times P = 100 \times 0.3811$

$\cong 38.$

2) A Manufacturer knows from the experience that the resistance of resistors he produces is normal with mean 100Ω and std. deviation 2Ω. what % of resistors will have resistance b/w 98Ω & 102Ω

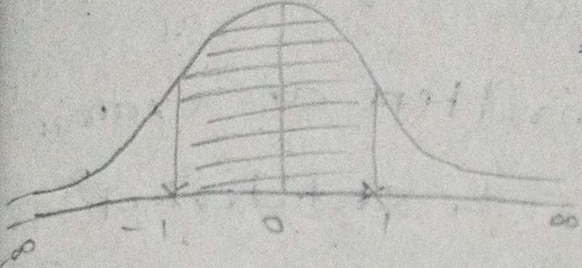
Mean $\mu = 100\Omega$

std. deviation $\sigma = 2\Omega$

$P(98 \le x \le 102)$

WKT, $z = \frac{x-\mu}{\sigma}$

$$z_1 = \frac{98-100}{2} = -1 \; ; \; z_2 = \frac{102-100}{2} = 1.$$



$= P(-1 \le z \le 1)$

$= 2P(1)$

$= 2 \times 0.3413$

$= 0.6826$

% of resistors having resistance b/w

98Ω & 102Ω $= 0.6826 \times 100$

$= 68\%$

3) In a test of 200 electrical bulbs it was found that the life of a particular make was normally distributed with avg 2040 hours and sd = 40 hrs. Estimate the no. of ~~bulb~~ bulbs likely to burn for more than 2140 hrs.

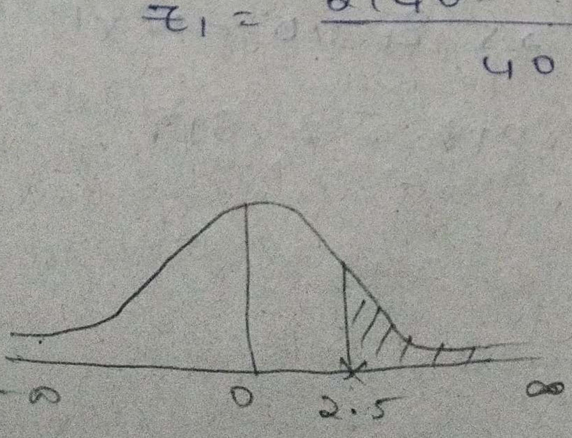
Mean $\mu = 2040$ hrs

Std. dev^n $= \sigma = 40$ hrs

$P(x \ge 2140)$

WKT, $z = \dfrac{x - \mu}{\sigma}$

$z_1 = \dfrac{2140 - 2040}{40} = 2.5$



$= P(z \ge 2.5)$

$= 0.5 - P(2.5)$

$= 0.5 - 0.4938$

$= 0.0062$

No. of bulbs $= 0.0062 \times 200 = 1.2 \simeq 1$

7) 1000 students have written an exam. The mean of test is 35 & std. deviation 5. Assuming the distribution to be normal find how many students

i) Lie b/w 25 & 40

ii) more than 40

iii) below 20

iv) more than 50

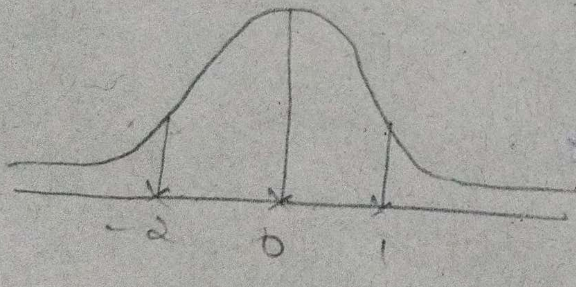
No. of students - $N = 1000$

mean - $\mu = 35$

std. deviation - $\sigma = 5$

i) $P(25 \le x \le 40)$

$$z_1 = \frac{25-35}{5} = -2 \; ; \; z_2 = \frac{40-35}{5} = 1$$



$= P(-2) + P(1)$

$= 0.4772 + 0.3413$

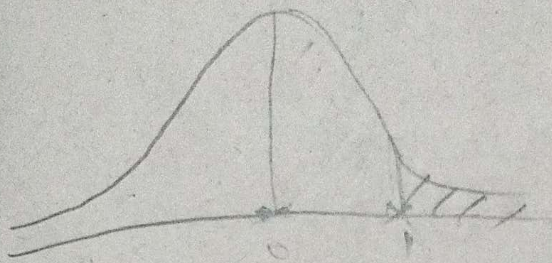
$= 0.8185$

No. of students lie b/w 25 & 40 $= N \times P$

$= 1000 \times 0.8185 = 818.5 \simeq 819$.

ii) $P(x \ge 40)$

$z = \frac{40-35}{5} = \frac{5}{5} = 1.$



$= P(z \geq 1) = 0.5 - P(1)$

$= -0.3413 + 0.5$

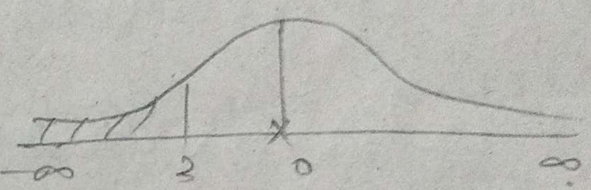
$= 0.1587$

No. of students more than 40 = $N \times P$

$= 1000 \times 0.1587 = 159.$

iii) $P(X \leq 20)$

$X = \frac{20-35}{5} = -3$



$P(X \leq -3) = 0.5 - P(3) = 0.001$

$N \times P = 1.$

iv) $P(X \geq 50)$

$z = \frac{50-35}{5} = 3$

$P(X \geq 3) = 0.5 - 0.4987$

$= 0.001$

$N \times P = 1.$

26/09

Q1) In a sample of 1000 cases the mean of certain test is 14 and sd is 2.5. How many students score

i) b/w 12 & 15

ii) above 18

iii) below 18

$N = 1000$

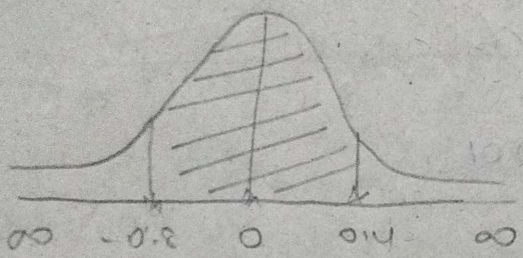
$\mu = 14$

$S.d - \sigma = 2.5$

1) $P(12 \le x \le 15)$

we've $z = \dfrac{x - \mu}{\sigma}$

$z_1 = \dfrac{12 - 14}{2.5} = -0.8$; $z_2 = \dfrac{15 - 14}{2.5} = 0.4$



$= P(-0.8 \le z \le 0.4)$

$= p(0.8) + p(0.4)$

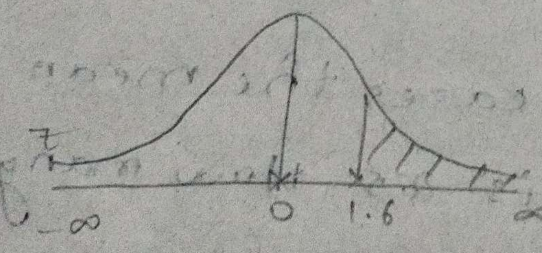
$= 0.1554 + 0.2881$

$= 0.4435$

No. of cases b/w 12 & 15 $= N \times P \simeq 443$

2) $P(x \ge 18)$

$z_1 = \dfrac{18 - 14}{2.5} = 1.6$



$= P(z \ge 1.6)$

$= 0.5 - P(1.6)$

$= 0.5 - 0.4452$

$= 0.0548$

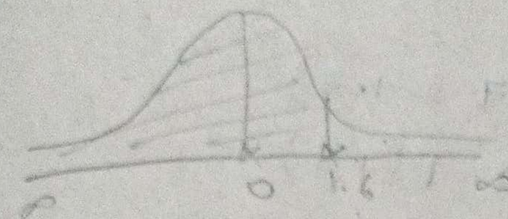
No. of case b/w above 18 is $N \times P$

$= 0.0548 \times 1000$

$= 54.8 \simeq 55$

3) $P(x \leq 18)$

$$z_1 = \frac{18-14}{2.5} = 1.6$$



$= P(z \leq 1.6)$

$= 0.5 + P(1.6)$

$= 0.5 + 0.4452$

$= 0.9452$

No. of cases below 18 is $N \times P$

$= 0.9452 \times 1000$

$= 945$

6) (1000 students have) If the masses of 300 students have mean 68 kg and sd 3 kg. How many students have masses

i) greater than 72 kg

ii) less than or equal to 64 kg

iii) b/w 65 and 71 kg

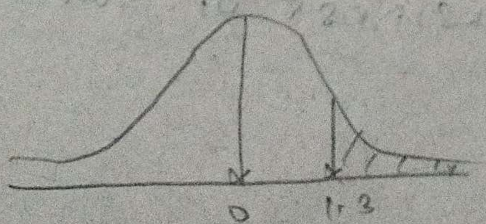
$N = 300$

$\mu = 68$

$sd = 3$

i) $P(x \geq 72)$

$$z_1 = \frac{x-\mu}{\sigma} = \frac{72-68}{3} = 1.3$$



$= P(z \geq 1.3)$

$= 0.5 - P(1.3)$

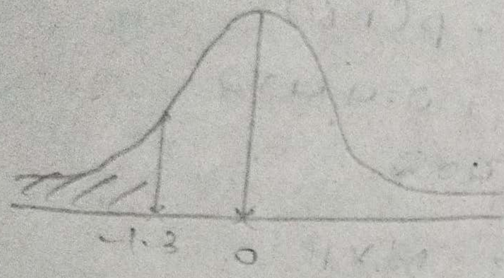
$= 0.5 - 0.4032$

$= 0.0968$

$N \times P = 29.04 \simeq 29.$

ii) $P(X \le 64)$

$z_1 = \frac{64-68}{3} = -1.3$



$= P(z \le -1.3)$

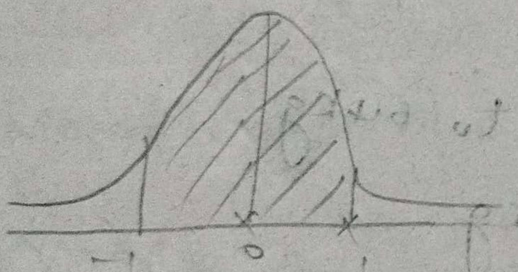
$= 0.5 - P(1.3)$

$= 0.0968$

no. of students with mass less than or equal to 64 kg is 29.

iii) $P(65 \le X \le 71)$

$z_1 = \frac{65-68}{3} = -1$ $\quad z_2 = \frac{71-68}{3} = 1$



$= P(-1 \le z \le 1)$

$= 2P(1)$

$= 0.6826$

no. of students b/w 65 & 71 is 204.78 ≈ 205

7) A sales tax officer reported on avg 500 business that he has to deal in the year is Rs. 36000 with s.d 10000. find no. of business of sales of

i) more than 40000 Rs

ii) % of business of sales in a range blw 30000 and 40000.

$N = 500$

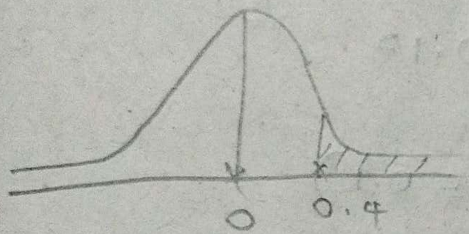
$\mu = ₹36000$

$s.d - \sigma = Rs. 10,000$

we've $z = \frac{x - \mu}{\sigma}$

i) $P(X \geq 40000)$

$z = \frac{40000 - 36000}{10000} = 0.4$



$= P(z \geq 0.4)$

$= 0.5 - P(0.4)$

$= 0.5 - 0.1554$

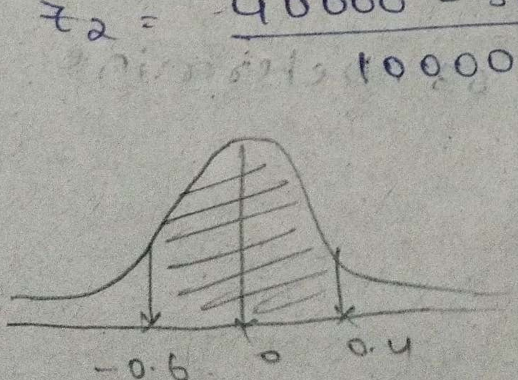
$= 0.3446$

no. of business of sales more than 40000 is $\simeq 172.3 = 172$.

ii) $P(30000 < x < 40000)$

$z_1 = \frac{30000 - 36000}{10000} = -0.6$

$z_2 = \frac{40000 - 36000}{10000} = 0.4$



$= P(-0.6 \leq z \leq 0.4)$

$= P(0.6) + P(0.4)$

$= 0.2257 + 0.1554$

$= 0.3811$

% of business of sales $= 0.3811 \times 100$

$= 38\%$.

8) The mean hgt of students in a coll. is 155 cm with sd = 15. What is the prob. that mean hgt of 36 students is less than 157 cm.

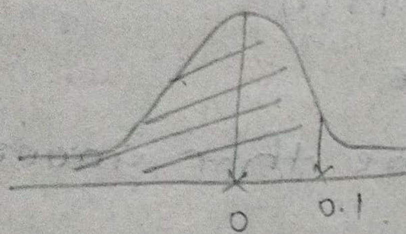
$N = 36$

$\mu = 155$

$sd = 15$

$P(x \le 157)$

$$z = \frac{x-\mu}{\sigma} = \frac{157-155}{15} = 0.13$$



$= P(z \le 0.1)$

$= 0.5 + p(0.13)$

$= 0.5 + \cancel{0.0398}\ 0.0517$

$= \cancel{0.5398}\ 0.5517$

Mean hgt of 36 students less than 157 is $0.5517 \times 36 = 19.86$

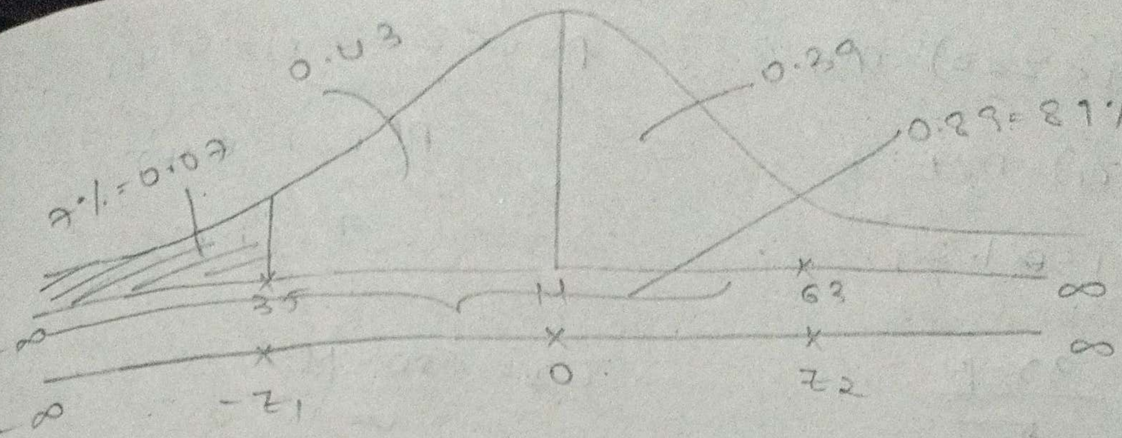
$\simeq 20.$

9) In ND, 7% of ~~terms~~ items are under 35 and 89% are under 63. Determine mean and variance

$P(0 < z < z_2) = 0.39$

$P(z_2) = 0.39$

$z_2 = 1.23$



$$P(-z_1 < z < 0) = 0.43$$

$$P(z_1) = 0.43$$

$$z_1 = -1.48$$

In ND, $z = \dfrac{x-\mu}{\sigma}$ $\qquad \dfrac{(1)}{(2)} \Rightarrow \dfrac{-1.48}{1.23} = \dfrac{35-\mu}{63-\mu}$

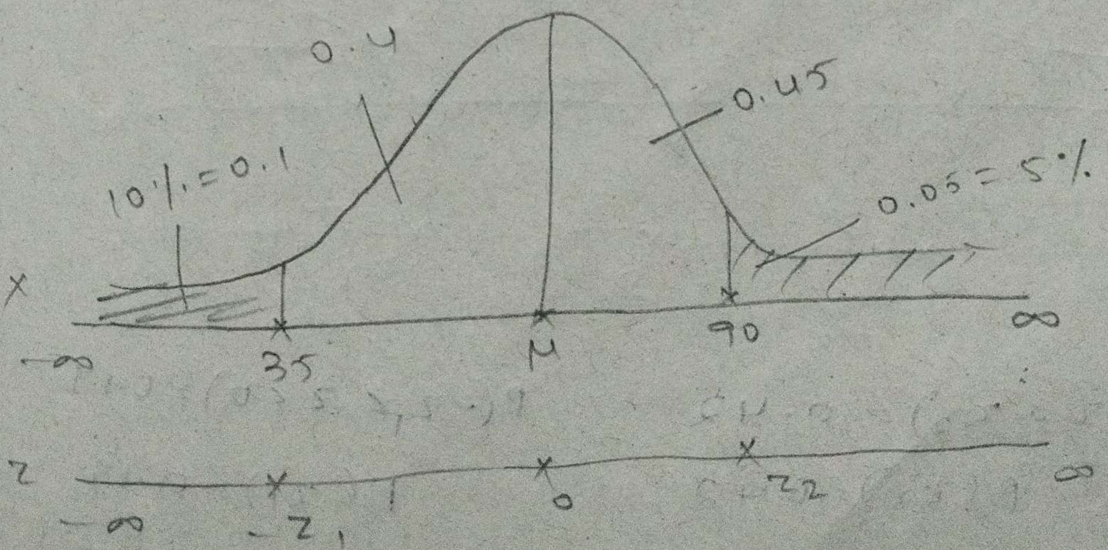
$$-1.48 = \frac{35-\mu}{\sigma} \quad \text{— (1)} \qquad\qquad -1.203 = \frac{35-\mu}{63-\mu}$$

$$+1.23 = \frac{63-\mu}{\sigma} \quad \text{— (2)}$$

$\cancel{\sigma = 112}$, $\mu = 50$, $\sigma = 10.33$

27/09

10) suppose 10% of ND is below 35 & 5% above 90. What are the values of μ & σ



$P(-z_1 < z < 0) = 0.4$

$P(z_1) = 0.4$

$z_1 = 1.29$

$-z_1 = \frac{35-\mu}{\sigma}$

$-1.29\sigma = 35-\mu$

$1.65\sigma = 90-\mu$

$\sigma = \frac{55}{2.94} = 18.707$

$\boxed{\sigma = 18.707}$

$P(0 < z < z_2) = 0.45$

$P(z_2) = 0.45$

$z_2 = 1.65$

$z_2 = \frac{90-\mu}{\sigma}$

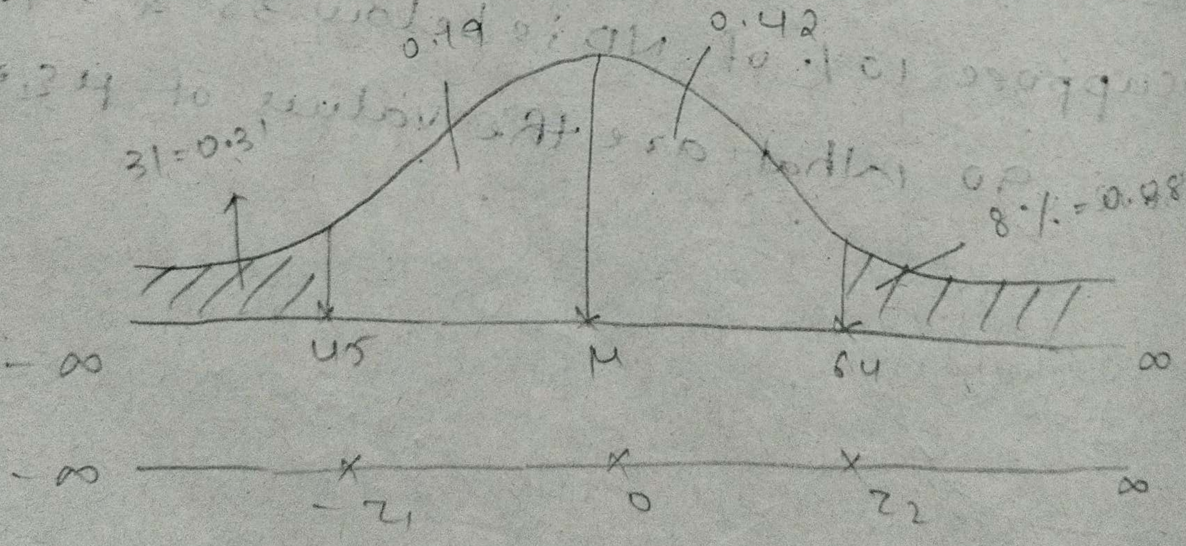
$1.65\sigma = 90-\mu$

$\mu = 35 + 1.29(18.707)$

$\mu = 35 + 24.136$

$\boxed{\mu = 59.136}$

11) In a ND, 31% of items are under 45 & 8% over 64. Find μ and variance of distribution.



$P(0 < z < z_2) = 0.42$

$P(z_2) = 0.42$

$z_2 = 1.41$

$P(-z_1 < z < 0) = 0.19$

$P(z_1) = 0.19$

$z_1 = 0.5$

$-z_1 = \frac{45-\mu}{\sigma}$, $z_2 = \frac{64-\mu}{\sigma}$

$-0.50\sigma = 45 - \cancel{\mu}$

$1.41\sigma = 64 - \cancel{\mu}$

$-1.91\sigma = -19$

$\sigma = 9.94$

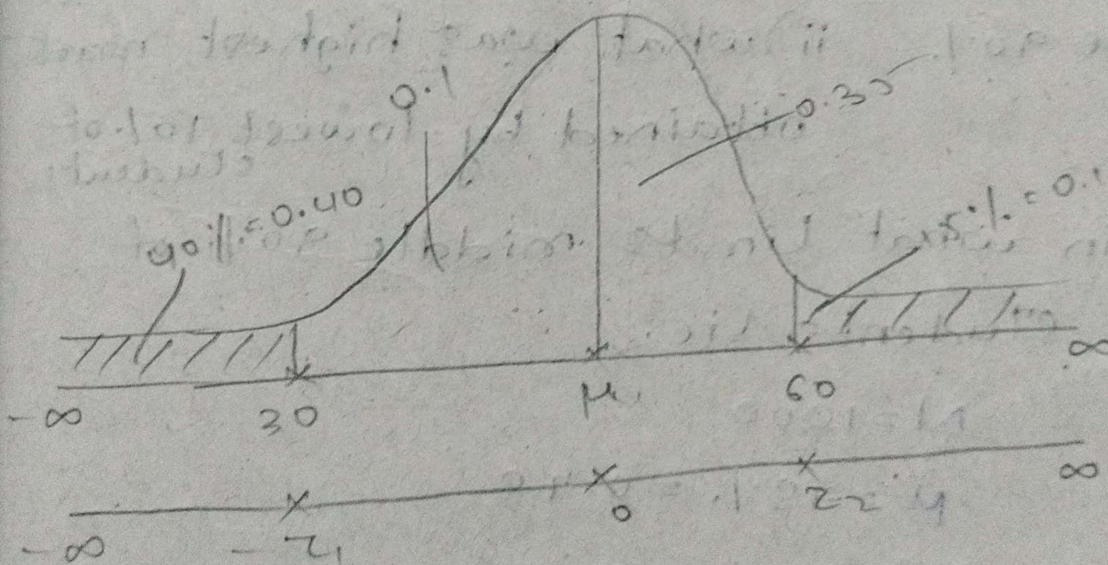
variance = 98.95

$1.41(9.94) = 64 - \mu$

$\mu = 64 - 14.0154$

$\boxed{\mu = 49.98}$

12) The marks obtained in statistics in a certain examination found to be normally distributed if 15% of students $\geq$ 60 marks, 40% < 30 marks. find μ, σ.



$P(-z_1 < z < 0) = 0.1$

$P(z_1) = 0.1$

$z_1 = 0.26$

$-z_1 = \frac{30-\mu}{\sigma}$

$P(0 < z < z_2) = 0.35$

$P(z_2) = 0.35$

$z_2 = 1.04$

$z_2 = \frac{60-\mu}{\sigma}$

$$-0.26\sigma = 30 - \mu$$

$$1.04\sigma = 60 - \mu$$

$$-1.30\sigma = -30$$

$$\sigma = \frac{30}{1.30}$$

$$\boxed{\sigma = 23.07}$$

$$\mu = 30 + 0.26(23.07)$$

$$\boxed{\mu = 35.99}$$

13) The marks obtained by 1000 students is normally distributed with mean 78% and sd 11%. Determine how many stud got marks

i) above 90%.

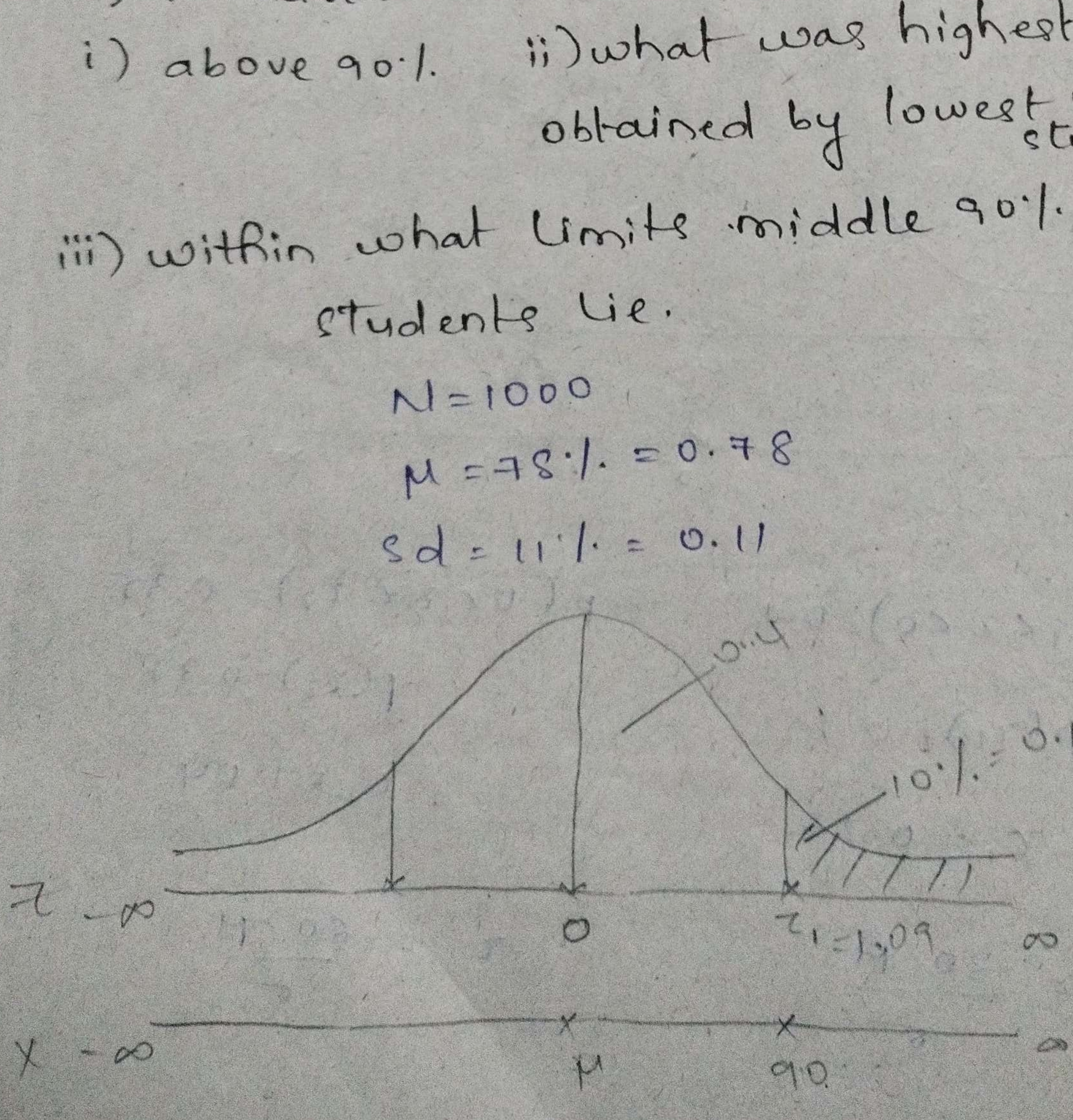
ii) what was highest mark obtained by lowest 10% of student

iii) within what limits middle 90% of students lie.

$N = 1000$

$\mu = 78\% = 0.78$

$sd = 11\% = 0.11$



i) $P(X > 90\%) = P(X > 0.9)$

$z_1 = \dfrac{90-78}{11} \Rightarrow z_1 = 1.09$

$P(z > 1.09) = 0.5 - P(1.09)$

$= 0.5 - 0.3621$

$= 0.1379$

No. of students scored more than 90%

$= N \times P = 1000 \times 0.1379$

$\simeq 138.$

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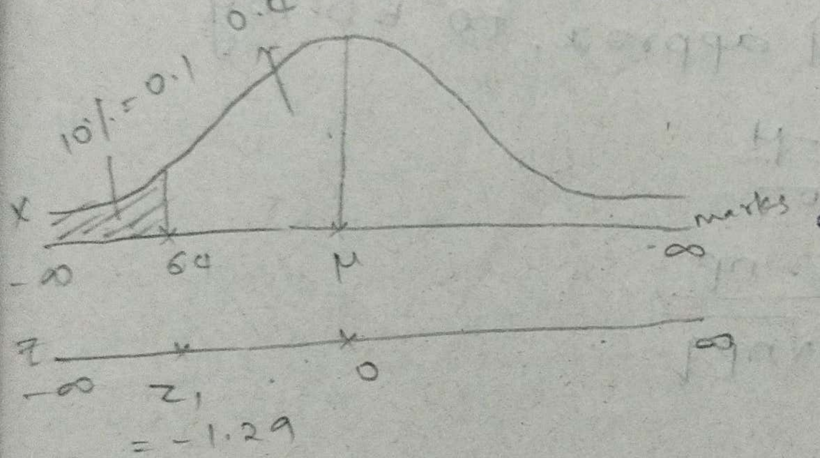
ii) highest mark by lowest 10%



$P(z_1) = 0.4$

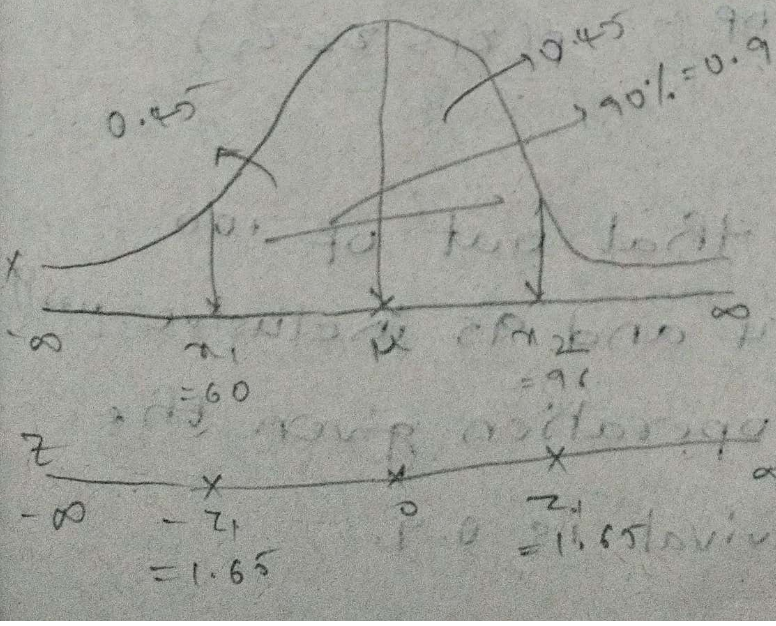
$z_1 = 1.29$

In ND,

$z = \dfrac{x-\mu}{\sigma}$

$-1.29 = \dfrac{x-78}{11} \Rightarrow x = 63.81 \Rightarrow \boxed{x \simeq 64}$

iii) b/w what limits 90% lie



$P(-z_1 < z < z_1) = 0.9$

$2P(z_1) = 0.9$

$P(z_1) = 0.45$

$z_1 = 1.65$

$$-z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_1 = \frac{x_2 - \mu}{\sigma}$$

binomial d normal

$$-1.65 = \frac{x_1 - 78}{11} \qquad 1.65 = \frac{x_2 - 78}{11}$$

$$\boxed{x_1 = 60} \qquad \boxed{x_2 = 96}$$

90% of students lie b/w 60 to 96 marks

Normal approximation to Binomial dist

In B.D, mean $\mu = np$

std. dev $\sigma = \sqrt{npq}$

we use normal approx. to BD by

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{x - np}{\sqrt{npq}}$$

$P(x_1 \le X \le x_2)$

$$z_1 = \frac{(x_1 - 0.5) - np}{\sqrt{npq}}$$

$$z_2 = \frac{(x_1 + 0.5) - np}{\sqrt{npq}} \qquad P(z_1 \le z \le z_2)$$

1) Find the prob. that out of 100 patients b/w 84 and 95 inclusive will survive heart operation given the chance of survival is 0.9.

mean $\mu = np = 90$

std. dev $\sigma = \sqrt{npq} = 3$

we use normal approx. to BD by

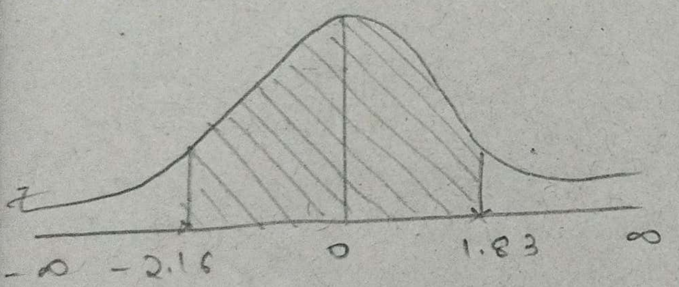
$$z = \frac{x - np}{\sqrt{npq}} = \frac{x-90}{3}$$

$p(84 \le X \le 95)$

$$z_1 = \frac{(84-0.5) - np}{3} = \frac{(84-0.5)-90}{3} = -2.16$$

$$z_2 = \frac{(95+0.5)-90}{3} = 1.83$$

$p(-2.16 \le z \le 1.83)$



$= p(2.16) + p(1.83)$

$= 0.4846 + 0.4664$

$= 0.951$

2) Find the probability by given work a student on correctly answer 25 to 30 MCQ's consisting of 80 questions. Assuming that each question has 4 options and only 1 is correct. and student has no knowledge of subject

$n = 80, \; p = 0.25$

$q = 1 - 0.25 = 0.75$

In B.D, mean $\mu = np = 20$

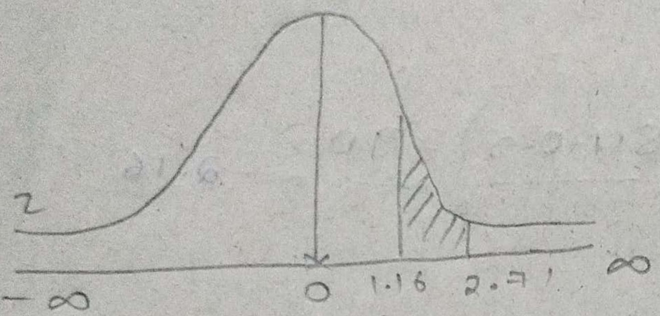
$\sigma = \sqrt{npq} = 3.87$

$x_1 = 25$, $x_2 = 30$

(25-0.5) (30+0.5)

$z_1 = \dfrac{25-20}{3.87} = \cancel{1.29} = 1.16$

$z_2 = \dfrac{30-20}{3.87} = \cancel{2.58} = 2.71$



$p(1.16 \le z \le 2.71)$

$= p(1.16) + p(2.71)$

$= -0.3770 + 0.4966$

$= 0.1196$.