8/8/24 **Unit-1 Probability & Random Variables**

Random Experiment :-

An experiment which consists of uncertain (or) unbaised is called a random experiment

Ex: Tossing a coin, Throwing a Dice.

Probability:-

Abbility to happen (or) chance of happening is called probability.

Ex: In tossing a coin the probability of getting head is 1/2 or 0.5.

Equally likely elements:

Events are said to be equally likely elements when there is no reason to expect any one of them rather than the other.

Ex: In throwing a die, all the events 1, 2, 3, 4, 5, 6 have equal chance of happening.

Exhaustive Events:

All possible events in any trail are known as exhaustive events.

Ex: i) In tosing a coin, there are 2 exhaustive events; head & tail.

ii) In throwing a die, there are 6 exhaustive events 1, 2, 3, 4, 5, 6.

mutually exclusive events:-

Events are said to be mutually exclusive if the happening of any one of them excluse the happening any one of the other i.e., no 2 (or) more events can happen in single trail.

sample space:

set of all possible events is called a sample space.

→ It is denoted by 'S'.

Ex: In throwing a die $S = \{1, 2, 3, 4, 5, 6\}$

sample events:

Each element in a sample space which cannot be further split is called a sample event.

classical defination of probability:-

Let S be the sample space and 'E' be the event then the probability of E is given by $P(E) = \frac{n(E)}{n(S)} = \frac{\text{fav. no. of cases}}{\text{Total no. of cases}}$

1) In throwing a die, what is Probability of getting an even number.

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

$n(E) = 3$ $\quad$ $n(S) = 6$

$P(E) = \frac{n(E)}{n(S)} = 3/6 = 1/2$

2) What is the probability of getting a diamond card from well suffeled cards.

$n(S) = 52$

Let 'E' be the event of getting diamond card

$n(E) = 13$

$P(E) = 13/52 = 1/4$

3) what is the probability of getting leap year to have 52 monday, 53 sundays.

366 days → 52 weeks, 2 days

$S = \{(S, M), (M, T), (T, W), (W, Th), (Th, F), (F, Sa), (Sa, Sun)\}$

$n(S) = 7$

Let E be event of getting 52 mon, 53 sun

$E = \{(Sat, Sun)\}$ $\quad$ $n(E) = 1$

$P(E) = 1/7$

* 52 mondays, 52 sundays

$E = \{(T, W), (W, Th), (Th, F), (F, Sat)\}$

$n(E) = 4$

$P(E) = 4/7$

9/8/24 & → × or → + At Least ≥ At most ≤

4) In a class there are 10 boys and 5 girls a committee of 4 students is to be selected from the class. Find the Probability for the committee to contain atleast 3 girls.

$n(S) = {}^{15}C_4 = 1365$

E → 3 girls and 1 boy or 4 girls and 0 boy

$n(E) = {}^5C_3 \times {}^{10}C_1 + {}^5C_4 \times {}^{10}C_0$

$= 10 \times 10 + 5$

$= 105$

$P(E) = \frac{105}{1365} = \frac{1}{13}$

5) Two unbaised dice are thrown. find the Probability that

i) Both the dice shows the same number

ii) The first dice shows 6

iii) The total of the numbers on the dice is 8.

iv) The total of the numbers on the dice is greatmer than 8.

v) The total of the numbers on the dice is 13.

vi) The total of the numbers on the dice between 2 to 12 (both inclusive)

$n(S) = 6^2 = 36$

i) Let E_1 be the event that shows the same number on both the dice

$E_1 = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$n(E_1) = 6$

$P(E_1) = \frac{6}{36} = \frac{1}{6}$

ii) Let E_2 be the event that shows 6 on the first dice

$E_2 = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(E_2) = 6$

$P(E_2) = \frac{6}{36} = \frac{1}{6}$

iii) $E_3 = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$n(E_3) = 5$

$P(E_3) = \frac{5}{36}$

iv) $E_4 = \{(3,6),(4,5),(5,4),(6,3),(4,6),(5,5),(6,4),$
$(5,6),(6,5),(6,6)\}$

$n(E_4) = 10$

$P(E_4) = \frac{10}{36} = \frac{5}{18}$

v) This is an impossible case
so the Probability is 0.

vi) This is an sure case
so the probability is 1.

6) A fair coin is tossed four times find the sample space corresponding to this random experiment also find the Probabilities that

i) more heads than tails occur

ii) Tails occur on the even number tosses.

$n(S) = 2^4 = 16$

$S = \{(H,H,H,H), (T,H,H,H), (H,T,H,H), (H,H,T,H),$
$(H,H,H,T), (H,H,T,T), (H,T,H,T), (H,T,T,H),$
$(T,H,H,T), (T,H,T,H), (T,T,H,H), (H,T,T,T),$
$(T,H,T,T), (T,T,H,T), (T,T,T,H), (T,T,T,T)\}$

i) Let E_1 be the event that occurs more heads than tails

$E_1 = \{(H,H,H,H), (T,H,H,H), (H,T,H,H), (H,H,T,H),$
$(H,H,H,T)\}$

$n(E_1) = 5$

$P(E_1) = \frac{5}{16}$

ii) Let E_2 be the event that occurs that Tails occur on the even number tosses.

$E_2 = \{(H,T,H,T), (H,T,T,T), (T,T,H,T), (T,T,T,T)\}$

$n(E_2) = 4$

$P(E_2) = \frac{4}{16} = \frac{1}{4}$

7) From 6 positive and 8 negative numbers 4 numbers are choosen at random and multiplied. what is the probability that

i) The product is +ve

ii) The product is -ve

$$n(S) = 1001$$

i) Let E_1 be the event that the Product is +ve.

$E_1 \rightarrow$ 2 positive and 2 negative or 4 positive or 4 negative

$$n(E_1) = {}^6C_2 \times {}^8C_2 + {}^6C_4 + {}^8C_4$$

$$= 505$$

$$P(E_1) = \frac{505}{1001}$$

ii) Let E_2 be the event that the product is -ve.

$E_2 \rightarrow$ 1 positive and 3 negative or 3 positive and 1 negative

$$n(E_2) = {}^6C_1 \times {}^8C_3 + {}^6C_3 \times {}^8C_1$$

$$= 496$$

$$P(E_2) = \frac{496}{1001}$$

8) A box contains tags numbered - $1, 2, \ldots\ldots n$. 2 tags are choosen at random. Find the probability that the number's on the tags are consecutive numbers.

$$n(S) = {}^{n}C_2$$

Let E be the event that 2 ~~no~~ tags are consecutive numbers.

$$E \to \{(1,2),(3,4)\ldots\ldots((n-1),n)\}$$

$$n(E) = {}^{n-1}C_1 = n-1$$

$$P(E) = \frac{n-1}{n!/(n-2)!\,2!}$$

$$= \frac{(n-1)(n-2)!\,2!}{n!}$$

$$= \frac{(n-1)!\,2}{n(n-1)!} = \frac{2}{n}$$

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9) A lot consists of 10 Good articles, 4 with minor defect and 2 with major defect. 2 articles are choosen from the lot at random. find the probability that

i) both are good ii) both have major defects

iii) atleast one is good

iv) atmost one is good

v) ~~a~~ exactly one is good

vi) neither has major defects

vii) niether is good.

$n(S) = {}^{16}C_2 = 120$

i) $n(E_1) = {}^{10}C_2 = 45$

$P(E_1) = \frac{45}{120} = \frac{3}{2}$

ii) $n(E_2) = {}^{2}C_2 = 1$

$P(E_2) = \frac{1}{120}$

iii) $E_3 \rightarrow$ 1 good and 1 Bad or 2 good and 0 bad

$n(E_3) = {}^{10}C_1 \times {}^{6}C_1 + {}^{10}C_2 \times {}^{6}C_0 = 105$

$P(E_3) = \frac{105}{120} = \frac{7}{8}$

iv) $E_4 \rightarrow$ 0 good ~~or~~ and 2 good ~~and~~ or 1 good ~~or~~ and 1 bad

$n(E_4) = {}^{10}C_0 \times {}^{6}C_2 + {}^{10}C_1 \times {}^{6}C_1 = 75$

$P(E_4) = \frac{75}{120} = \frac{5}{8}$

v) $E_5 \rightarrow$ 1 good and 1 bad

$n(E_5) = {}^{10}C_1 \times {}^{6}C_1 = 60$

$P(E_5) = \frac{60}{120} = \frac{1}{2}$

vi) $E_6 \rightarrow$ neglecting major

$n(E_6) = {}^{14}C_2 = 91$

$P(E_6) = \frac{91}{120}$

vii) $E_7 \rightarrow$ neglecting good

$n(E_7) = {}^{6}C_2 = 15$

$P(E_7) = \frac{15}{120} = \frac{1}{8}$

Properties of Probabilities:-

Let S be the sample space and E be the event then

1) $0 \leq P(E) \leq 1$

2) $P(S) = 1$, $P(\phi) = 0$

3) Let E_1 and E_2 be the mutally excusive events then probability of $P(E_1 \cup E_2)$ -

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Conditional Probability:-

The conditional probability of an event B assuming that event A has happened is denoted by $P(B/A)$. This is read as B given A and is defined by

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad [\text{Provided } P(A) \neq 0]$$

Similarly $P(A/B) = \frac{P(A \cap B)}{P(B)} \quad [P(B) \neq 0]$

Note: 1) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B) / n(S)}{n(A) / n(S)} = \frac{n(A \cap B)}{n(A)}$

2) Rewritting the defination of Conditional probability we get $P(A \cap B) = P(B/A) \cdot P(A)$; This Product therom for two events.

3) Product theorem can be extended to three events A, B, C as

$P(A \cap B \cap C) = P(C|A \cap B) \cdot P(B|A) \cdot P(A)$

Ex: When a fair dice is tossed find the probability of getting 1 given that an odd number has been obtained.

$A = \{1, 3, 5\}$, $B = \{1\}$

$A \cap B = \{1\}$

$n(A) = 3$; $n(A \cap B) = 1$

$$P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{1}{3}$$

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Theorem of Total probability:-

Let $B_1, B_2 \ldots B_n$ be a set of exhaustive and mutually exclusive events and another event 'A' associated with each B_i then the Probability of A is given by

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A|B_i)$$

Baye's theorem:-

Let $B_1, B_2 \ldots B_n$ be a set of exhaustive and mutually exclusive event and another event 'A' associated with each B_i, then the probability of any B_i given that 'A' has happend is given by

$$P(B_i|A) = \frac{P(B_i) \cdot P(A|B_i)}{\sum P(B_i) \cdot P(A|B_i)}$$

1 Q) companies B_1, B_2, B_3 produce 30%, 45%, 25% of the cars respectively. It is known that 2%, 3% and 2% of the cars produced from B_1, B_2, B_3 are defective

i) what is probability that a car purchased is Defective.

ii) If a car purchased is found to be defective what is the probability that this car is produced by company B_3.

Sol:- $P(B_1) = 30\% = \frac{3}{10}$

$P(B_2) = 45\% = \frac{45}{100} = \frac{9}{20}$

$P(B_3) = 25\% = \frac{25}{100} = \frac{1}{4}$

$P(D|B_3) = P(D|B_1) = 2\% = \frac{2}{100} = \frac{1}{50}$

$P(D|B_2) = \frac{3}{100}$

i) $P(D) = \sum P(B_i) \cdot P(D|B_i)$

$= \frac{3}{10} \times \frac{1}{50} + \frac{9}{20} \times \frac{3}{100} + \frac{1}{4} \times \frac{1}{50}$

$= \frac{3}{500} + \frac{27}{2000} + \frac{1}{200}$

$= \frac{12+27+10}{2000} = \frac{49}{2000} = 0.0245$

ii) $P(B_3|D) = \frac{P(B_3) \cdot P(D|B_3)}{P(D)} = \frac{\frac{1}{4} \times \frac{1}{50}}{\frac{49}{2000}} = \frac{1}{200} \times \frac{2000}{49}$

$= \frac{10}{49}$

Q2) Box-I contains (1W, 2R, 3G) balls

Box-II - 2W, 3R, 1G

Box III - 3W, 1R, 2G

i) 2 balls are drawn from a box choosen at random, they are found to be 1 white and 1 red.

ii) Determine the probability that the balls come from box-II ?

Sol :- $P(B_2/A) \neq P(B_1) = P(B_2) = P(B_3) = 1/3$

$$P(A|B_1) = \frac{1C_1 \times 2C_1}{6C_2} = \frac{1\times 2}{15} = \frac{2}{15}$$

$$P(A|B_2) = \frac{2C_1 \times 3C_1}{6C_2} = \frac{2\times 3}{15} = \frac{2}{5}$$

$$P(A|B_3) = \frac{3C_1 \times 1C_1}{6C_2} = \frac{3}{15} = \frac{1}{5}$$

$$P(B_2|A) = \frac{P(B_2)\cdot P(A|B_2)}{\Sigma P(B_i)\cdot P(A|B_i)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{5}}{\frac{1}{3}\times\frac{2}{15}+\frac{3}{5}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{5}}$$

$$= \frac{2/5}{\frac{2}{15}+\frac{6}{15}+\frac{3}{15}}$$

$$= \frac{2/5}{11/15}$$

$$= \frac{2}{5}\times\frac{15}{11} = \frac{6}{11}$$

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3) Suppose three companies x, y, z Produce T.V's. X produce twice as many as y while y and z produce the same number it is known that 2% of x, 2% of y and 4% of z are defective. all the T.V's produced and put into shop and one T.V is choosen at random.

i) what is the probability that the TV is defective.

ii) suppose the TV is defective what is the Probability that this TV is produced by company X.

$$P(x) = 2P(y) \quad , \quad P(y) = P(z)$$

$$P(x) + P(y) + P(z) = 1$$

$$2P(y) + P(y) + P(y) = 1$$

$$4P(y) = 1 \Rightarrow P(y) = \frac{1}{4}$$

$$P(x) = 2P(y) = 2\frac{1}{4} = \frac{1}{2} \quad , \quad P(z) = P(y) = \frac{1}{4}$$

$$P(D|x) = 2\% = \frac{1}{50} \quad P(D|y) = 2\% = \frac{1}{50}$$

$$P(D|z) = 4\% = \frac{1}{25}$$

i) $P(D) = P(x) \cdot P(D|x) + P(y)\, P(D|y) + P(z)\, P(D|z)$

$$= \frac{1}{2} \cdot \frac{1}{50} + \frac{1}{4} \cdot \frac{1}{50} + \frac{1}{4} \cdot \frac{1}{25} = \frac{1}{40}$$

ii) $$P(x|D) = \frac{P(x) \cdot P(D|x)}{P(D)} = \frac{1/2 \cdot 1/50}{1/40} = \frac{2}{5}$$

4) A bussinessman goes to hotels x, y, z 20, 50, 30 of the times respectively. it is known that 5%, 4%, 8% of the rooms in x, y, z hotels have faulty plumbing. what is the probability that bussiness man rooms having faulty plumbing is assinged to hotel z.

$$P(x) = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$P(y) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(z) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(F|x) = 5\% = \frac{5}{100} = \frac{1}{20}$$

$$P(F|y) = 4\% = \frac{4}{100} = \frac{1}{25}$$

$$P(F|z) = 8\% = \frac{8}{100} = \frac{2}{25}$$

$$P(F) = P(x) \cdot P(F|x) + P(y) \cdot P(F|y) + P(z) \cdot P(F|z)$$

$$= \frac{1}{5} \cdot \frac{1}{20} + \frac{1}{2} \cdot \frac{1}{25} + \frac{3}{10} \cdot \frac{2}{25}$$

$$= \frac{27}{500}$$

$$P(z|F) = \frac{P(z) \cdot P(F|z)}{P(F)}$$

$$= \frac{3/10 \cdot 2/25}{27/500}$$

$$= 4/9$$

5) The chances that the politician, Bussiness man, Accomedation will be appointed as vice chancellor of university are 0.5, 0.3, 0.2 respectively Probability that research is promoted by these persons if they are appointed as VC are 0.3, 0.7, 0.8

i) Probability that research is promoted

ii) if research is promoted what is probability that VC is an Accodemetion.

$$P(B_1) = 0.5, \; P(B_2) = 0.3, \; P(B_3) = 0.2$$

$$P(R|B_1) = 0.3 \quad P(R|B_2) = 0.7 \quad P(R|B_3) = 0.8$$

$$P(R) = P(B_1) \cdot P(R|B_1) + P(B_2) P(R|B_2) + P(B_3) P(R|B_3)$$

$$= (0.5)(0.3) + (0.3)(0.7) + (0.2)(0.8)$$

$$= 13/25$$

$$P(B_3|R) = \frac{P(B_3) \cdot P(R|B_3)}{P(R)}$$

$$= \frac{0.2 \times 0.8}{13/25}$$

$$= \frac{4}{13}$$

6) In a certain college 40% of men and 10% of women all are taller than 1.8m (6ft) further more in the college 60% of students are women if a student is selected at random and is taller than 1.8m then what is the probability that the student is women.

$P(M) = 40\% = \frac{2}{5}$

$P(W) = 60\% = \frac{3}{5}$

$P(T|\cancel{W}M) = 40\% = \frac{2}{5}$, $P(T|W) = 10\% = \frac{1}{10}$

$P(T) = P(M) \cdot P(T|M) + P(W) \cdot P(T|W)$

$= \frac{2}{5} \cdot \frac{2}{5} + \frac{3}{5} \cdot \frac{1}{10}$

$= 11/50$

$P(W|T) = \frac{P(W) \cdot P(T|W)}{P(T)}$

$= \frac{\frac{3}{5} \cdot \frac{1}{10}}{11/50}$

$= 3/11$

$= 0.\overline{27}$

Random Variables

Random variable:-

A Random variable is a variable which takes the values from the outcome of a random experiment.

Ex :- 1) In tossing two coins let us define a random variable x which counts the number of heads.

$S = \{(H,H), (T,H), (H,T), (T,T)\}$

$X = \{0, 1, 2\}$

2) In throwing two dice let us define a R.V y which is the sum of the numbers ~~of~~ on the faces of the dice.

$S = \{(1,1), (1,2) \ldots (1,6) \ldots (6,6)\}$

$Y = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

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Types of Random Variables :-

There are two types of random Variables

1) Discreate Random Variables

2) continuous Random variables

→ 1) Discreate Random variables :-

A random variable X which can take only a finite number of discreate values in an interval of domain is called a discreate random variable.

Ex:

In throwing 2 dice define a random variable x which is the max. no on the faces of the dice.

$x = \{1, 2, 3, 4, 5, 6\}$

→ 2) continous random variable:-

A random variable x which can take values continously. i.e all possible values in a given interval is called continous random variable.

Ex:

The weight or height of a randomly selected students.

Probability Distribution:- (Discrete only)

Suppose x is a R.V. The Probability Distribution of x is a set of all possible values of x along with there probabilities.

Ex:

1) In tossing 2 coins x is no of heads

$S = \{(H,H), (H,T), (T,H), (T,T)\}$ $x = \{0, 1, 2\}$

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

2) sum of two numbers on the dice

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Probability Distribution function:-

for a Discreate R.V X The real valued function $P(X=x) = P(x)$ is called Probability distribution function of X.

Ex:

In the above first example

$P(X=1) = P(1) = 1/2$

Properties:-

1) $0 \le P(x) \le 1$

2) $\Sigma P(x) = 1$

Cummilative Distribution function:-

Suppose X is a discreate R.V then the Cummilative Distribution function is defined by $F(x) = P(X \le x) = \sum_{x_i \le x} P(x_i)$

Ex:

In the above second example

$F(4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36} = \frac{1}{6}$

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Measures of Discreate random Variables:-

mean or Average or Expectation

If x is Discreate random variables the Expectation of x is given by

$$\mu = E(x) = \sum_{i=1}^{n} x_i p_i$$

Properties of mean:-

If X, Y are random variables (Discreate) and k is any constant then

1) $E(X+k) = E(X) + k$

2) $E(kX) = kE(X)$

3) $E(X \pm Y) = E(X) \pm E(Y)$

4) $E(kX+Y) = kE(X) + E(Y)$

5) If X, Y are independent then

$E(XY) = E(X) \cdot E(Y)$ (Different sample spaces Independent)

~~Proper~~ Varience:-

Varience of a discreate random variable is given by

$\sigma^2 = E(X-\mu)^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i$ (actual formula)

$= E(x^2) - \mu^2$ (For problem this is used)

$\Rightarrow E(x^2) = \sum_{i=1}^{n} x_i^2 p_i$

Properties of Varience:-

Suppose X, Y are Discreate R.V and 'a' is any constant then

1) $Var(ax) = a^2 Var(X)$

2) $Var(X \pm Y) = Var(X) \pm Var(Y)$

3) $Var(a) = 0$

4) $Var(X+a) = Var(X)$

Standard Deviation:-

Standard Deviation is the positive square root of varience.

i.e $\sigma = \sqrt{E(X-\mu)^2} = \sqrt{\sum_{i=1}^{n} (x_i-\mu)^2 p_i}$

1) Let X denoteds the no. of Heads in a single toss of four coins. determine the probability distribution and

i) $P(X<2)$

ii) $P(1<X<3)$

$n(S) = 2^4 = 16$ $\quad X = \{0,1,2,3,4\}$

X	0	1	2	3	4
P(X)	1/16	4/16	6/16	4/16	1/16

i) $P(X<2) = P(0)+P(1) = \frac{1}{16}+\frac{4}{16} = \frac{5}{16}$

ii) $P(1<X<3) = P(2)+P(3) = \frac{6}{16}+\frac{4}{16} = \frac{10}{16} = \frac{5}{8}$

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2) From a lot of 10 items consisting of 3 detectives, a sample of 4 items is drawn at random. Let the random variable X denotes the number of defective items in the sample find the probability distribution of X.

$n(S) = {}^{10}C_4$ $\quad X = \{0,1,2,3\}$

X	0	1	2	3
P(X)	1/6	1/2	3/10	1/30

0 Defective and 4 good

$$P(0) = \frac{^3C_0 \times {^7C_4}}{^{10}C_4} = \frac{1}{6}$$

1 Defective and 3 good

$$P(1) = \frac{^3C_1 \times {^7C_3}}{^{10}C_4} = \frac{1}{2}$$

2 Defective and 2 good

$$P(2) = \frac{^3C_2 \times {^7C_2}}{^{10}C_4} = \frac{3}{10}$$

3 defective and 1 good

$$P(3) = \frac{^3C_3 \times {^7C_1}}{^{10}C_4} = \frac{1}{30}$$

3) A random variable X has the following Probability Distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Determine i) K, ii) $P(X<6)$, $P(X\geq 6)$, $P(0<X<5)$
iii) mean (μ) iv) Varience (σ^2)

i) $K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$ $[\Sigma P_i = 1]$

$$10K^2+9K-1=0$$

$$K = 1/10, -1$$

$\therefore K = 1/10$ since $0<P<1$

ii) $P(X<6) = 1-P(X\geq 6) = 1-[P(6)+P(7)]$

$$= 1-[2K^2+7K^2+K]$$

$$\Rightarrow 1-9K^2+K = 1-9\left(\frac{1}{100}\right)+\frac{1}{10}$$

$$= \frac{81}{100}$$

$P(x \geq 6) = P(6) + P(7) = 2k^2 + 7k^2 + k = 9k^2 + k$

$= \frac{9}{100} + \frac{1}{10} = \frac{19}{100}$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= k + 2k + 2k + 3k = 8k = \frac{8}{10} = \frac{4}{5}$

iii) $\mu = E(x) = \sum x_i p_i$

$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 30k + 66k = \frac{30}{10} + \frac{66}{10} = \frac{366}{100}$

$= 3.66$

iv) $\sigma^2 = E(x^2) - \mu^2 = \sum x_i^2 p_i - \mu^2$

$= 0 + 1^2 \times k + 2^2 \times 2k + 3^2 \times 2k + 4^2 \times 3k + 5^2 \times k^2$
$+ 6^2 \times 2k^2 + 7^2(7k^2 + k) - (3.66)^2$

$= 0 + k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2$
$+ 49k - 13.3956$

$= 124k + 440k^2 - 13.3956$

$= \frac{124}{10} + \frac{440}{100} - 13.3956$

$= \frac{124}{10} + \frac{44}{10} - 13.3956$

$= 3.4044$

4) A R.V X has the following probability Distribution.

X	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

Determine: i) K ii) $P(X<4)$, $P(X\geq 5)$, $P(3<X\leq 6)$
iii) μ, iv) σ^2 v) what will be the min value of k so that $P(X\leq 2)>0.3$

i) $\Sigma Pi = 1$

$K+3K+5K+7K+9K+11K+13K = 1$

$49K = 1 \Rightarrow K = 1/49$

ii) $P(X<4) = 1-P(X\geq 4) = 1-[P(4)+P(5)+P(6)]$

$= 1-[9K+11K+13K] = 1-33K$

$= 1-\frac{33}{49} = \frac{16}{49}$

$P(X\geq 5) = P(5)+P(6) = 11K+13K = 24K = \frac{24}{49}$

$P(3<X\leq 6) = P(4)+P(5)+P(6) = 33K = \frac{33}{49}$

iii) $\mu = E(X) = \Sigma x_i p_i = 0+3K+10K+21K+36K+55K + 78K$

$= 203K = \frac{203}{49} = \frac{29}{7} = 4.142$

iv) $\sigma^2 = E(x^2)-\mu^2 = \Sigma x_i^2 p_i - \mu^2$

$= 0+3K+20K+63K+144K+275K+468K-(4.142)^2$

$= 973K - 17.156 = 973/49 - 17.156 = 2.701$

v) $P(X\leq 2)>0.3$

$P(0)+P(1)+P(2)>0.3$

$K+3K+5K>0.3$

$9K>0.3$

$K>\frac{0.3}{9} \Rightarrow K>\frac{1}{30}$

minimum value can be $1/30$

24/08/24 (continous Random variables)

Probability Density function:-

Consider a small interval $\left(x-\frac{dx}{2}, x+\frac{dx}{2}\right)$ of length 'dx', Round the point x.

Let $f(x)$ be any continious function of 'x' ~~such~~ that, $f(x)dx$ represents Probability that the variable 'x' falls in this interval, i.e, $f(x)dx = P\left(x-\frac{dx}{2} \leq x \leq x+\frac{dx}{2}\right)$

$f(x) \rightarrow$ Density function

Properties:-

1) $f(x) \geq 0 \ \forall x \in \mathbb{R}$

2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Cummilative Density function:-

Cummilative Density function of a continuous R.V is given by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$$

Properties:-

1) $0 \leq F(x) \leq 1 \ \forall x \in \mathbb{R}$

2) $F(-\infty) = 0$

3) $F(\infty) = 1$

4) $F'(x) = f(x)$

Measures of continous Random Variable

1) mean or Average or Expectation

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

2) median

median divides the entire Distribution into two equal parts. Let M be the median then

$$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = \frac{1}{2}$$

3) mode

mode is the value of x at which $f(x)$ is maximum. $f(x)$ is maximum when $f'(x) = 0$ and $f''(x) < 0$.

4) Varience :-

$$Var(x) = \sigma^2 = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx \quad \text{(actual formula)}$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 \quad \text{(formula used in Problems)}$$

5) standar deviation :-

$$S.D = \sigma = \sqrt{E(x-\mu)^2}$$

1) If a R.V has a probability density func^n

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

find i) $P(1 < X < 3)$

ii) $P(X > 0.5)$

i) $P(1 < x < 3) = \int_{1}^{3} f(x)\,dx$

$$= \int_{1}^{3} 2e^{-2x}\,dx = 2\left(\frac{e^{-2x}}{-2}\right)_1^3 = -\left[e^{-2x}\right]_1^3$$

$$= -\left[e^{-6} - e^{-2}\right] = e^{-2} - e^{-6}$$

ii) $P(X>0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{\infty} 2e^{-2x}\,dx$

$$= 2\left(\frac{e^{-2x}}{-2}\right)_{0.5}^{\infty} = -\left[e^{-\infty} - e^{-2(0.5)}\right]$$

$$= -(0 - e^{-1}) = e^{-1}$$

2) If the probability density function of a R.V is given by $f(x) = \begin{cases} K(1-x^2), & 0<x<1 \\ 0, & \text{Otherwise} \end{cases}$

Find the value of K and $P(0.1<X<0.2)$, $P(X>0.5)$

WKT $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 f(x)\,dx = 1$

$$\Rightarrow \int_0^1 K(1-x^2)\,dx = 1$$

$$\Rightarrow K\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$\Rightarrow K\left\{\left[1-\frac{1}{3}\right] - [0]\right\} = 1$$

$$\Rightarrow \frac{2}{3}K = 1 \Rightarrow K = \frac{3}{2}$$

$$P(0.1<X<0.2) = \int_{0.1}^{0.2} f(x)\,dx = \int_{0.1}^{0.2} \frac{3}{2}(1-x^2)\,dx$$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2}$$

$$= \frac{3}{2}\left\{\left[0.2 - \frac{(0.2)^3}{3}\right] - \left[0.1 - \frac{0.1^3}{3}\right]\right\}$$

$$= 0.1465$$

$$P(x>0.5) = \int_{0.5}^{1} f(x)\,dx = \int_{0.5}^{1} \frac{3}{2}(1-x^2)\,dx$$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.5}^{1}$$

$$= \frac{3}{2}\left\{\left[1-\frac{1}{3}\right] - \left[0.5 - \frac{(0.5)^3}{3}\right]\right\}$$

$$= 0.3125$$

29/08/24

3) If the probability density function of X is given by $f(x) = \begin{cases} \frac{1}{2}\sin x, & \text{for } 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$. find mean, median, mode and $P(0 < x < \pi/2)$

i) mean $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{\pi} x f(x)\,dx$

$$= \int_0^{\pi} \frac{1}{2}\sin x\,dx = \frac{1}{2}\int_0^{\pi} x \sin x\,dx$$

$\left[\because \int x f(x)\,dx = \left[x\int f(x)\,dx - \iint f(x)\,dx\,dx\right]\right]$

$\left[\because \int x^2 f(x)\,dx = \left[x^2\int f(x)\,dx - 2x\iint f(x)\,dx\,dx + 2\iiint f(x)\,dx\,dx\,dx\right]\right]$

$$= \frac{1}{2}\left[x(-\cos x) - (-\sin x)\right]_0^{\pi}$$

$$= \frac{1}{2}\left\{[-\pi\cos\pi + \sin\pi] - [0]\right\} \quad [\because \sin n\pi = 0] \quad [\cos n\pi = (-1)^n]$$

$$= \frac{1}{2}\left[(-\pi)(-1) + 0\right] = \frac{\pi}{2}$$

ii) median

Let M be the median.

$$\int_0^M f(x)\,dx = \frac{1}{2} \Rightarrow \int_0^M \frac{1}{2}\sin x\,dx = \frac{1}{2}$$

$$\Rightarrow (-\cos x)_0^M = 1$$

$$\Rightarrow -[\cos M - \cos 0] = 1$$

$$\Rightarrow -[\cos M - 1] = 1$$

$$\Rightarrow 1 - \cos M = 1$$

$$\Rightarrow \cos M = 0$$

$$\Rightarrow M = \frac{\pi}{2}$$

iii) Mode

$f'(x) = \frac{1}{2}\cos x \qquad f''(x) = -\frac{1}{2}\sin x$

$f'(x) = 0 \Rightarrow \frac{1}{2}\cos x = 0 \Rightarrow x = \frac{\pi}{2}$

$f''\left(\frac{\pi}{2}\right) = -\frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2} < 0$

since it is less than 0 . it is the maximum value.

mode $= \pi/2$

iv) $P(0 < x < \pi/2) = \int_0^{\pi/2} \frac{1}{2}\sin x \, dx$

$= \frac{1}{2}\int_0^{\pi/2} \sin x \, dx$

$= \frac{1}{2}(-\cos x)_0^{\pi/2}$

$= \frac{1}{2}[-\cos \pi/2 - (-\cos 0)]$

$= \frac{1}{2}[-\cos\pi/2 + \cos 0]$

$= \frac{1}{2}[0+1]$

$= \frac{1}{2}[1] = \frac{1}{2}$

$\therefore P(0 < x < \pi/2) = \frac{1}{2}$

4) A continous random variable has Probability density function

$$f(x) = \begin{cases} kxe^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

determine

i) k ii) mean iii) varience

i) $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_0^{\infty} kxe^{-\lambda x} = 1$

$$\Rightarrow \left[k \cdot x\left(\frac{e^{-\lambda x}}{-\lambda}\right) - \left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right)\right]_0^{\infty} = 1$$

$$\Rightarrow k\left[[0-0] - \left[0 - \frac{1}{\lambda^2}\right]\right] = 1$$

$$\Rightarrow k\frac{1}{\lambda^2} = 1 \Rightarrow \boxed{k = \lambda^2}$$

ii) $\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x(\lambda^2 xe^{-\lambda x})dx = \lambda^2 \int_0^{\infty} x^2 e^{-\lambda x}dx$

$$= \lambda^2 \left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2x\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 2\left(\frac{e^{-\lambda x}}{-\lambda^3}\right)\right]_0^{\infty}$$

$$= \lambda^2\left\{[0] - \left[0 - 0 - \frac{2}{\lambda^3}\right]\right\} = \lambda^2 \cdot \frac{2}{\lambda^3} = \frac{2}{\lambda}$$

iii) $\sigma^2 = E(x^2) - \mu^2 = \int_0^{\infty} x^2(\lambda^2 xe^{-\lambda x})dx - \mu^2$

$$= \lambda^2 \int_0^{\infty} x^3 e^{-\lambda x}dx - \mu^2$$

$$= \lambda^2\left[x^3\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 3x^2\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 6x\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) - 6\left(\frac{e^{-\lambda x}}{\lambda^4}\right)\right]_0^{\infty} - \mu^2$$

$$= \lambda^2\left[[0] - \left[0 - 0 + 0 - \frac{6}{\lambda^4}\right]\right] - \mu^2$$

$$= \lambda^2\left[\frac{6}{\lambda^4}\right] - \mu^2$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}$$

30/8/24

5) If X is a continous R.V and $Y = aX + b$ where Y is another continous R.V and a, b are constants, then prove that

i) $E(Y) = aE(X) + b$ ii) $Var(Y) = a^2 Var(X)$

i) $E(Y) = E(aX+b) = \int_{-\infty}^{\infty} (ax+b) f(x)dx$

$= \int_{-\infty}^{\infty} ax f(x)dx + \int_{-\infty}^{\infty} b f(x)dx$

$= a\int_{-\infty}^{\infty} x f(x)dx + b\int_{-\infty}^{\infty} f(x)dx$

$= aE(X) + b(1)$ $\left[\because \int_{-\infty}^{\infty} f(x)dx = 1\right]$

$= aE(X) + b$

ii) $Y = aX + b$ - ①

$E(Y) = aE(X) + b$ - ②

$\left[\because Var(Y) = E(Y-\mu)^2 = E[Y-E(Y)]^2\right]$

① - ② $\Rightarrow Y - E(Y) = (aX+b) - (aE(X)+b)$

$\Rightarrow Y - E(Y) = a(X - E(X))$

Squaring on both sides

$\Rightarrow [Y-E(Y)]^2 = a^2[X-E(X)]^2$

Taking Expectation on both sides

$\Rightarrow E[Y-E(Y)]^2 = a^2 E[X-E(X)]^2$

$\Rightarrow Var(Y) = a^2 Var(X)$

Important topics for this unit
- Definations
- Bayes theorem
- Discreate
- Continous