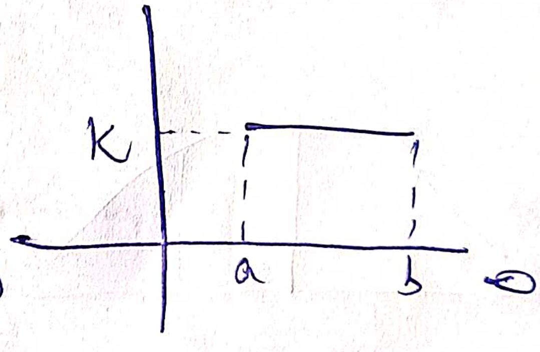
Uniform distribution:-

Uniform distribution is a continuous distribution. A random variable X is said to be uniformly distributed over the interval $-\infty < a < b < \infty$ If its probability density function is constant over the entire range. As the distribution is as rectangle it is also called as rectangular distribution

K = constant.

It is also called as rectangular distribution.



Note: To find k

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_a^b f(x)\,dx = 1$$

$$\int_a^b k\,dx = k(x)_a^b = 1$$

$$k(b-a) = 1$$

$$\boxed{k = 1/(b-a)}$$

→ Probability distribution function ⇒ $f(x) = \begin{cases} \dfrac{1}{b-a}, & a<x<b \\ 0, & \text{otherwise} \end{cases}$

Measures of uniform distribution:

1) Mean, μ:

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_a^b x k\,dx \Rightarrow k\left(\frac{x^2}{2}\right)_a^b = \frac{1}{b-a}\left(\frac{b^2-a^2}{2}\right)$$

$$= \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}} \Rightarrow \boxed{\frac{b+a}{2} = \mu}$$

2) Variance:

$$\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 = \int_a^b x^2 k\,dx - \mu^2$$

$$= k\left.\frac{x^3}{3}\right|_a^b + \mu^2 \Rightarrow \frac{b^3-a^3}{3(b-a)} - \frac{(a+b)^2}{4}$$

$$= \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} - \frac{(a+b)^2}{4}$$

$$= \frac{4(b^2+ab+a^2) - 3(a+b)^2}{12}$$

$$= \frac{4b^2+4ab+4a^2-3a^2-3b^2-6ab}{12}$$

$$= \frac{a^2-2ab+b^2}{12} = \boxed{\frac{(b-a)^2}{12} = \sigma^2}$$

To find the probability between a_1 to a_2:-

$$P(a_1 < x < a_2) = \int_{a_1}^{a_2} f(x)\,dx = \int_{a_1}^{a_2} \frac{1}{b-a}\,dx = \frac{1}{b-a}(x)_{a_1}^{a_2}$$

$$= \frac{a_2 - a_1}{b-a}.$$

1) A bus is uniformly late between 2 to 10 mins, how long can you expect wait. What is SD (σ).

If it's greater than 7 min late, you'll be late for work what is the probability of you being late.

Sol $a = 2,\ b = 10$

$\mu = ?$ (Expectation) $\quad f(x) = \frac{1}{b-a} = \frac{1}{10-2} = \frac{1}{8} = 0.125$

$\mu = \frac{a+b}{2} = \frac{2+10}{2} = 6$

$S.D = \sqrt{\sigma^2} \quad \sigma^2 = \frac{(b-a)^2}{12} = \frac{(10-2)^2}{12} = \frac{\cancel{64}^{16}}{\cancel{12}_{3}} = 5.33$

$= 2.309$

$P(x > 7) = \int_{7}^{10} f(x)\,dx = \int_{7}^{10} 0.125\,dx = 0.125 \times 3$
$= 0.375$

2) The amount of time a person must wait for a train to arrive in a certain town is uniformly distributed b/w 0 to 40. i) probability density funct?

ii) Draw a graph of $f(x)$

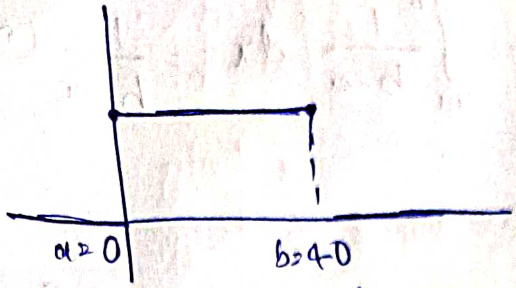
iii) What is the probability that a person must wait less than 8 mins

iv) What is the probability that a person must waite more than 30 mins.

v) find $P(10 < x < 20)$, $P(x > 45)$ vi) μ, σ^2

$a = 0,\ b = 40$ i) $f(x) = \frac{1}{b-a} = \frac{1}{40} = 0.025$

ii)



iii) $P(X<8) = \int_0^8 f(x)\,dx = \int_0^8 0.025\,dx = 0.025(8) = 0.2$

iv) $P(X>30) = \int_{30}^{40} f(x)\,dx = \int_{30}^{40} 0.025\,dx = 0.025 \times 10 = 0.25$

v) $P(10<X<26) = \int_{10}^{26} 0.025\,dx = 0.025 \times 16 = 0.4$

$P(X>45) = 0$

vi) $\mu = \frac{a+b}{2} = \frac{40}{2} = 20$ $\sigma^2 = \frac{(b-a)^2}{12} = \frac{(40)^2}{12} = \frac{40 \times 40}{12} = 133.3$

Standard Deviation $\Rightarrow \sigma = 11.54$

3) The amount of time that it takes a student to complete a chemistry test is uniformly distributed btw 20 & 45 min

i) probability density function ii) graph of $f(x)$

iii) what is the prob that a student will take >36

iv) " " " " " " " btw 26 & 35

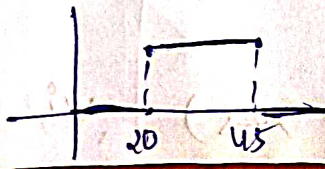
mins to complete the test

v) Determine μ, σ^2 vi) probability that student will take more than 50 min

Sol $a = 20$ $b = 45$

i) $f(x) = \frac{1}{45-20} = \frac{1}{25} = 0.04$

ii)



iii) $P(X > 36) = \int_{36}^{45} f(x)\,dx = \dfrac{45-36}{45-20} = 0.04 \times 9 = \underline{0.36}$

iv) $P(26 < X < 35) = \int_{26}^{35} f(x)\,dx = 9 \times 0.04 = \underline{0.36}$

v) $\mu = \dfrac{a+b}{2} = \dfrac{20+45}{2} = \dfrac{65}{2} = 32.5$

$\sigma^2 = \dfrac{(b-a)^2}{12} = \dfrac{(45-20)^2}{12} = \dfrac{25^2}{12} = \underline{52.08}$

vi) $P(X > 50) = \underline{0}$