25/11/24

# UNIT - 5 STOCHASTIC PROCESS

**Stochastic process:**

A stochastic process is a set of random variables $\{x_t\}$ depending on some real parameter "$t$".

→ The set is called "State space."

→ The elements in State space are called "States."

Ex: 1) In throwing a die, let $X_n$ be the max. no. of the dice in n throws. (i.e) if the die shows the no 2, 4, 1, 3, 6, 5 - - -

Then $X_1 = 2$, $X_2 = 4$, $X_3 = 4$, $X_4 = 4$, $X_5 = 6$.

Here state space is $\{1, 2, 3, 4, 5, 6\}$.

2) In tossing a coin, let $X_n$ be the no. of heads in 'n' tosses. (i.e) if tosses show T, H, H, T, T, - - - -

then $X_1 = 0$, $X_2 = 1$, $X_2 = 2$, $X_3 = 2$ - - - - - -

**MARKOV Process:-**

A stochastic process is said to be Markov's process given the value of $X_t$, the value of $X_v$ for $v > t$ does not depend on the value of $X_u$ for $u < t$.

(i.e) A random process in which the future value depends only on the present value but not on the past values.

**MARKOV chain:-**

A sequence of states $x_1, x_2$ - - - - is a Markov chain if $X_n$ is a markov process

**NOTE:-**

$$P\{X_n = x_n / X_{n-1} = x_{n-1}, X_{n-2} = X_{n-2} \text{ - - - - - } X_0 = x_0\}$$

$$= P\{X_n = x_n \mid X_{n-1} = x_{n-1}\}.$$

## Transition Probability Matrix :- (TPM)

→ The transition probability matrix is the arrangement of transition probabilities, $P_{ij}$ in rows & columns. Here $P_{ij}$ represents the probability that the next value is $j$ given that the present value is $i$.

. It is denoted by "P".

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \quad (1)$$

### Properties :-

1) tpm can be a square matrix [only]
2) The matrix values can be from 0 to 1.
3) Sum of each row is equal to 1.

Q) which of the following are Stochastic matrix?

1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ ×

Not square matrix

2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

Stochastic matrix

3) $\begin{bmatrix} 0 & 1 \\ 1/3 & 1/4 \end{bmatrix}$ ×

Sum ≠ 1

4) $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ ✓

Stochastic matrix

5) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ ×

-ve values

6) $\begin{bmatrix} 0 & 2 \\ 1/4 & 1/2 \end{bmatrix}$ ×

value > 1 & sum ≠ 1

7) $\begin{bmatrix} 1/3 & 2/3 & 4/3 \\ 1/2 & 1 & 1/2 \end{bmatrix}$ ×

8) $\begin{bmatrix} 15/16 & 1/16 \\ 2/3 & 1/3 \end{bmatrix}$ ✓

Stochastic matrix

## Regular Stochastic matrix:-

A stochastic matrix $P$ is said to be regular if all the entries of $P^m$ are +ve (for some +ve int. m)

NOTE :-

A stochastic matrix $P$ is not regular if
(1) 1 occurs in the principal main diagonal.

Which of the following matrices are regular?

1) $A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$ Not regular as 1 occurs in principle main diagonal.

2) $B = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$ $B^2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

$B^2 \Rightarrow \begin{bmatrix} 1/4+1/4+0 & 1/4+1/4 & 0 \\ 1/4+1/4 & 1/4+1/4 & 0 \\ 1/8+1/8+1/8 & 1/8+1/8+1/8 & 1/4 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 7/16 & 7/16 & 1/8 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 3/8 & 3/8 & 1/4 \end{bmatrix}$

No. of zeroes $\rightarrow$ same $\rightarrow$ do once again

$\rightarrow$ decrease $\rightarrow$ continue.

$\rightarrow$ Increase $\rightarrow$ Stop.

3) $C = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ $C^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$C^3 = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ $C^4 = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$

$$C^5 = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/8 & 1/2 & 3/8 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$ Regular with order = 5.

$m = 5$

$(\text{Stochastic})^n = \text{Stochastic}$

4) $$A = \begin{bmatrix} 0.75 & 0.25 & 0 \\ 0 & 0.5 & 0.5 \\ 0.6 & 0.4 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0.5625 & 0.3125 & 0.125 \\ 0.3 & 0.45 & 0.25 \\ 0.45 & 0.35 & 0.2 \end{bmatrix}$$ Regular with $m = 2$.

Q) Find the values of $x, y, z$ if $\begin{bmatrix} 0 & x & 1/3 \\ 0 & 0 & y \\ 1/3 & 1/4 & z \end{bmatrix}$ is Stochastic

$x + 1/3 = 1$

$x = 2/3$ $\quad y = 1$ $\quad 1/3 + 1/4 + z = 1$

$z = 1 - \frac{7}{12} = \frac{5}{12}$

Q) Find the values of $x, y, z$ if $\begin{bmatrix} 0 & 0.2 & x \\ x & 0.1 & y \\ 0.1 & 0.2 & z \end{bmatrix}$

$x + 0.2 = 1$ $\quad x + 0.1 + y = 1$

$x = 0.8$ $\quad y = 0.1$

$0.1 + 0.2 + z = 1$

$z = 0.7$

Q) An Alumini office of a clg finds on review that 80% of its students who contribute to annual fund 1 yr will also contribute next yr. And 30% of those who don't contribute 1 yr will contribute next yr. [TPM]

Let 0 be the state that he will contribute and 1 be the state that he will not contribute

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}_{2\times 2} \end{array}$$

27/11/24

Q) Suppose that the probability of a dry day (state 0) follows a rain day (state 1) is 1/3 and the prob. of a rain day follows a ~~rain~~ dry day is 1/2 then what is the probability that march 3rd is dry day & march 5th is dry day. Given march 1st. is dry day.

$$P = \begin{array}{c c} & \begin{array}{cc} \text{present} & \text{next} \\ 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix} \end{array}$$

$$P^2 = \begin{bmatrix} 5/12 & 7/12 \\ 7/18 & 11/18 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} \frac{173}{432} & \frac{259}{432} \\ \frac{259}{648} & \frac{389}{648} \end{bmatrix}$$

1 2 3 4 5

0   0   0

2

4 step Markov

$$\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{1}{4}+\frac{1}{6} & \frac{1}{4}+\frac{1}{3} \\ \frac{1}{6}+\frac{2}{9} & \frac{1}{6}+\frac{4}{9} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{6+4}{24} & \frac{7}{12} \\ \frac{3+4}{18} & \frac{3+8}{18} \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{12} & \frac{7}{12} \\ \frac{7}{18} & \frac{11}{18} \end{bmatrix}$$

So, the probability of March 3rd is dry day is $\underline{5/12}$ and the probability of March 5th is dry day is $\underline{173/432}$

IMP

Q) A raining process is considered as 2 state Markov chain. If it rains it considers to be state "0" and it doesnot rain the state is 1. tpm is given by

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

Find the prob that it will rain for 3 days from today assuming that it is raining today.

$P^2, P^3$

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \qquad P^2 = \begin{bmatrix} 11/25 & 14/25 \\ 7/25 & 18/25 \end{bmatrix} \qquad P^3 = \begin{bmatrix} 47/125 & 78/125 \\ 39/125 & 86/125 \end{bmatrix}$$

$$P^2 = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix} \end{array} \qquad P^3 = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.376 & 0.624 \\ 0.312 & 0.688 \end{bmatrix} \end{array}$$

$\Rightarrow$ $0.6 \times 0.44 \times 0.376 = 0.099264$.

continuous rain for 3 days [so multiply 3 prob's].

Q) consider the Markov chain tpm $P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ 0 & 3/4 & 1/4 & 0 \\ 1 & 1/4 & 1/2 & 1/4 \\ 2 & 0 & 3/4 & 1/4 \end{array}$

Find $P^2_{01}$ & $P(X_2 = 1, X_0 = 0)$

$P^2 = \begin{array}{c|ccc} & 0 & 1 & 2 \\ 0 & 5/8 & 5/16 & 1/16 \\ 1 & 5/16 & 1/2 & 3/16 \\ 2 & 3/16 & 9/16 & 1/4 \end{array}$ $\quad P^2_{01} = 5/16$

$\boxed{P(X_2 = 1, X_0 = 0) \Rightarrow P(X_2 = 1 / X_0 = 0) \cdot P(X_0 = 0)}$

$P(X_0 = i) = 1/3$ for, $(i = 0, 1, 2)$.

$P(X_2 = 1, X_0 = 0) = 5/16 \cdot 1/3 = 5/48$

5/12/24

Q) The tpm of Markov chain having 3 states 1, 2, 3. is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$. The initial probabilities $p^{(0)} = (0.7, 0.2, 0.1)$

i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

$P^2 = \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$

$\boxed{P(X_2 = 3) = \sum_{i=1}^{3} P(X_2 = 3 / X_0 = i) \times P(X_0 = i)}$

$P(X_2 = 3) = \sum_{i=1}^{3} P(X_2 = 3 | X_0 = i) \cdot P(X_0 = i)$

$= P(X_2 = 3 | X_0 = 1) P(X_0 = 1) + P(X_2 = 3 | X_0 = 2) P(X_0 = 2) +$
$P(X_2 = 3 | X_0 = 3) P(X_0 = 3)$

$= (0.26)(0.7) + (0.34)(0.2) + (0.29)(0.1) = \underline{0.279}$

ii) $P(X_1=3, X_0=2) = P(X_1=3/X_0=2)\cdot P(X_0=2)$

$= (0.2)(0.2)$

$= 0.04$

$P(X_2=3, X_1=3, X_0=2) = P(X_2=3/X_1=3, X_0=2)\, P(X_1=3, X_0=2)$

$= (0.3)(0.04)$

$= 12\times10^{-3}$

$= 0.012$

$P(X_3=2, X_2=3, X_1=3, X_0=2) = P(X_3=2/X_2=3, X_1=3, X_0=2)\; P(X_2=3, X_1=3, X_0=2)$

$= (0.4)(0.012)$

$= 48\times10^{-4}$

$= 0.0048$

12

Q) A fair die is tossed repeatedly if $X_n$ denotes max. number occurring in the first n tosses. Find tpm. Also find $P^2$, $P(X_2=6)$

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 2/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 3/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 4/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 6/6 \end{bmatrix} \end{array}$$

(1,1) (1,2)
(2,1) (2,2)
⋮
(6,1) (6,2)

$$P^2 = \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

$$P^2 = \frac{1}{36}\begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 4 & 5 & 7 & 9 & 11 \\ 0 & 0 & 9 & 7 & 9 & 11 \\ 0 & 0 & 0 & 16 & 9 & 11 \\ 0 & 0 & 0 & 0 & 25 & 11 \\ 0 & 0 & 0 & 0 & 0 & 36 \end{bmatrix}$$

$$P(X_2 = 6) = \sum_{i=1}^{6} P(X_2 = 6 \mid X_0 = i) \cdot P(X_0 = i)$$

$$\Rightarrow P(X_2 = 6 \mid X_0 = 1) P(X_0 = 1) + P(X_2 = 6 \mid X_0 = 2) P(X_0 = 2)$$
$$+ P(X_2 = 6 \mid X_0 = 3) P(X_0 = 3) + P(X_2 = 6 \mid X_0 = 4) P(X_0 = 4)$$
$$+ P(X_2 = 6 \mid X_0 = 4) P(X_0 = 4) + P(X_2 = 6 \mid X_0 = 5) P(X_0 = 5)$$

$$\Rightarrow \left(\frac{11}{36}\right)\left(\frac{1}{6}\right) + \left(\frac{11}{36}\right)\left(\frac{1}{6}\right) \times 4 + \frac{36}{36}$$

$$\Rightarrow \frac{11}{36}\left[\frac{1}{6} \times 5\right] + \frac{36}{36}$$

$$3\overline{6} \times 6 = 216$$

$$\Rightarrow \left[\frac{11}{36} \times \frac{5}{6}\right] + \frac{36}{36} = \frac{55}{216} + \frac{36}{} = \frac{91}{216}$$

IMP

3Q) An urn initially contains 5 Black balls & 5 White balls. The following exp is repeated indefinitely. A ball is drawn from the urn if the ball is white it is put back into urn otherwise it is left out. Let $X_n$ be the no. of black balls remaining in the urn after n draws.

i) Is $X_n$ a markov process
ii) Find tpm
iii) Find transition probabilities.
iv) Find 2 step tpm
v) What happens to $X_n$ as n approaches ∞ (Zero) use your ans to guess limit of $P^n$ as $n \to \infty$.

States : 0, 1, 2, 3, 4, 5

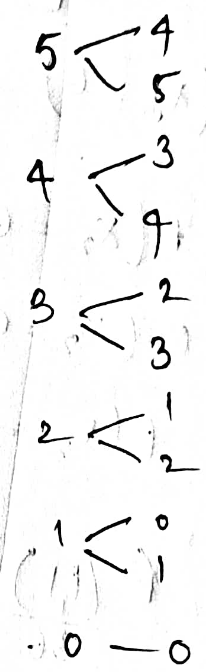
6x6 matrix

i) It is markov chain / process

The number $X_n$ of black balls independent of its past values. $X_{n+1}$ depends only on $X_n$ but not on remaining past values.

ii)

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 5/6 & 0 & 0 & 0 & 0 \\ 0 & 2/7 & 5/7 & 0 & 0 & 0 \\ 0 & 0 & 3/8 & 5/8 & 0 & 0 \\ 0 & 0 & 0 & 4/9 & 5/9 & 0 \\ 0 & 0 & 0 & 0 & \underset{(1/2)}{5/10} & \underset{(1/2)}{5/10} \end{bmatrix} \end{array}$$




iii) $P(X_n=0 / X_{n-1}=0) = 1$

$P(X_n=0 | X_{n-1}=1) = 1/6$ , $P(X_n=1 | X_{n-1}=1) = 5/6$.

$P(X_n=1 / X_{n-1}=2) = 2/7$, $P(X_n=2 | X_{n-1}=2) = 5/7$.

$P(X_n=2 | X_{n-1}=3) = 3/8$ , $P(X_n=3 | X_{n-1}=3) = 5/8$

$P(X_n=3 | X_{n-1}=4) = 4/9$ , $P(X_n=4 | X_{n-1}=4) = 5/9$.

$P(X_n=4 / X_{n-1}=5) = 5/10 = 1/2$ , $P(X_n=5 / X_{n-1}=5) = 5/10 = 1/2$

Remaining all are zero's.

iv) $P^2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 5/6 & 0 & 0 & 0 & 0 \\ 0 & 2/7 & 5/7 & 0 & 0 & 0 \\ 0 & 0 & 3/8 & 5/8 & 0 & 0 \\ 0 & 0 & 0 & 4/9 & 5/9 & 0 \\ 0 & 0 & 0 & 0 & \underset{(1/2)}{5/10} & \underset{(1/2)}{5/10} \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 5/6 & 0 & 0 & 0 & 0 \\ 0 & 2/7 & 5/7 & 0 & 0 & 0 \\ 0 & 0 & 3/8 & 5/8 & 0 & 0 \\ 0 & 0 & 0 & 4/9 & 5/9 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix} \end{array}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{11}{36} & \frac{25}{36} & 0 & 0 & 0 & 0 \\ \frac{2}{42} & \frac{65}{147} & \frac{25}{49} & 0 & 0 & 0 \\ 0 & \frac{6}{56} & \frac{225}{448} & \frac{25}{64} & 0 & 0 \\ 0 & 0 & \frac{12}{72} & \frac{85}{162} & \frac{25}{81} & 0 \\ 0 & 0 & 0 & \frac{20}{90} & \frac{19}{36} & 1/4 \end{bmatrix}$$

$\frac{20}{72} + \frac{20}{81}$

v) As $n \to \infty$ $X_n = 0$.

As $n \to \infty$

$$P^n = \begin{bmatrix} 1 & 0 & - & - & - & 0 \\ 1 & 0 & - & - & - & 0 \\ 1 & 0 & - & - & - & 0 \\ 1 & 0 & - & - & - & 0 \\ 1 & 0 & - & - & - & 0 \\ 1 & 0 & - & - & - & 0 \end{bmatrix}$$

VIMP

Q) A Gambler has Rs. 2. He bets one rupee at a time and wins one rupee with prob. 1/2. He stops playing if he losses 2 ruppe or win 4 rupees.

i) find tpm of the related Markov chain.

ii) What is the prob that he has lost his money at the end of 5 plays

iii) What is the prob that the game lasts more than 7 plays

i) Let $X_n$ be the amount exists with the gambler after $n$ plays

States are $0, 1, 2, 3, 4, 5, 6 \longrightarrow 7$ states.

tpm = P

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$1 < \begin{smallmatrix} 0 \\ 2 \end{smallmatrix}$

$2 < \begin{smallmatrix} 1 \\ 3 \end{smallmatrix}$

$3 < \begin{smallmatrix} 2 \\ 4 \end{smallmatrix}$

$4 < \begin{smallmatrix} 3 \\ 5 \end{smallmatrix}$

$5 < \begin{smallmatrix} 4 \\ 6 \end{smallmatrix}$

ii) $P(X_5) = 0$

$$p^{(0)} = (\overset{0}{0} \ 0 \ 1 \ 0 \ 0 \ 0 \ \overset{6}{0})$$

$$p^{(1)} = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0)\, P = p^{(0)} \cdot P = (1 \times 7)(7 \times 7) \rightarrow$$

$$= [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [0 \ 1/2 \ 0 \ 1/2 \ 0 \ 0 \ 0]$$

$p^2 = p^{(1)} \cdot p = [0 \ \ 1/2 \ \ 0 \ \ 1/2 \ \ 0 \ 0 \ 0] \, p$

$= \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \end{bmatrix}$

$p^3 = p^{(2)} \cdot p = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{8} & 0 & \frac{1}{8} & 0 \end{bmatrix}$

$p^4 = p^{(3)} \cdot p = \begin{bmatrix} \frac{3}{8} & 0 & \frac{5}{16} & 0 & \frac{4}{16} & 0 & \frac{1}{16} \end{bmatrix}$

$p^5 = p^{(4)} \cdot p = \begin{bmatrix} \frac{3}{8} & \frac{5}{32} & 0 & \frac{9}{32} & 0 & \frac{4}{32} & \frac{1}{16} \end{bmatrix}$

$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

$\frac{1}{8} + \frac{3}{16}$
$\frac{2+3}{16}$

$p^5$

The prob. that he lost his money after 5 plays $= \frac{3}{8}$.

iii) $p^6 = (p^{(5)}) \cdot p = \begin{bmatrix} \frac{29}{64} & 0 & \frac{\cancel{14}\,7}{\cancel{64}\,32} & 0 & \frac{13}{64} & 0 & \frac{1}{8} \end{bmatrix}$

$p^7 = p^{(6)} \cdot p = \begin{array}{c} \begin{bmatrix} \frac{29}{64} & \frac{\cancel{14}\,7}{\cancel{128}\,64} & 0 & \frac{27}{128} & 0 & \frac{13}{128} & \frac{1}{8} \end{bmatrix} \\ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \end{array}$

$= \frac{14}{128} + 0 + \frac{27}{128} + 0 + \frac{13}{128}$

$= \frac{\cancel{54}\,27}{\cancel{128}\,64} = \frac{27}{64}$

$\frac{3}{8} + \frac{5}{64}$
$= \frac{24+5}{64}$

$\frac{5}{64} + \frac{9}{64}$
$\frac{\cancel{14}}{\cancel{64}\,16}$

$\frac{14}{128} + \frac{13}{128}$
$27$
$\frac{27}{54}$

The prob. that the game lasts more than 7 plays $= \frac{27}{64}$.