Chi Ma 5241

cm3700 HW2

Problem 1

$\hat{\beta} \Rightarrow$ training data

$\hat{\beta}_{test} \Rightarrow$ test data

Prove: when $N \geq M > p$, $E[R_{tr}(\hat{\beta})] \leq E[R_{te}(\hat{\beta})]$

When $N = M \Rightarrow$ (population is same)

$$E[R_{tr}(\hat{\beta})] = E[R_{test}(\hat{\beta}_{test})] = \frac{N-p-1}{N}\sigma^2$$

Since $[R_{test}(\hat{\beta}_{test})] \leq E[R_{test}(\hat{\beta})]$

we have $\Rightarrow E[R_{tr}(\hat{\beta})] \leq E[R_{test}(\hat{\beta})]$

If $N > M$

$$E[R_{tr}(\hat{\beta})] = E\left[\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{\beta}^T x_i)^2\right] = \frac{1}{N}\sum_{i=1}^{N} E(y_i - \hat{\beta}^T x_i)^2$$

$$= E(y_i - \hat{\beta}^T x_i)^2 = \frac{1}{M}\sum_{i=1}^{M}(y_i - \hat{\beta}^T x_i)^2 \leq \frac{1}{M}\sum_{i=1}^{M} E(y_i - \hat{\beta}_{test}^T x_i)^2$$

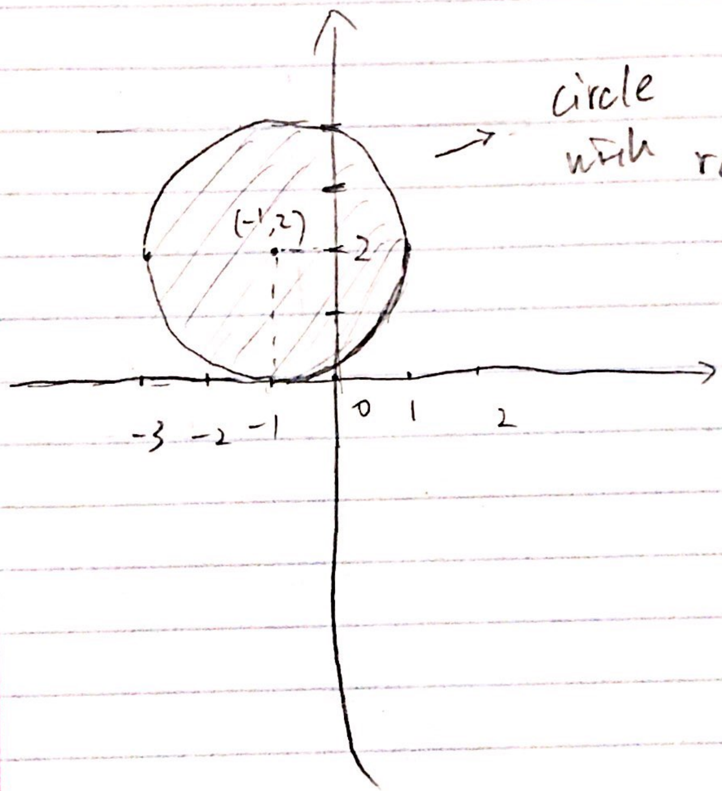
$$= \frac{1}{M}\sum_{i=1}^{M} E(\tilde{y}_i - \tilde{\beta}^T x_i)^2 \leq \frac{1}{M}\sum_{i=1}^{M} E(\tilde{y}_i - \hat{\beta}^T \tilde{x}_i)^2$$

$$= E\left(\frac{1}{M}\sum_{i=1}^{M}(\tilde{y} - \hat{\beta}^T \tilde{x}_i)^2\right) = E[R_{test}(\hat{\beta})]$$

Therefore $E[R_{tr}(\hat{\beta})] \leq E[R_{te}(\hat{\beta})]$

Problem 2

① $(1+X_1)^2+(2-X_2)^2=4$



circle center at (-1,2) with radius of 2

② Any points outside the circle is $(1+X_1)^2+(2-X_2)^2>4$ the points for which $(1+X_1)^2+(2-X_2)^2 \le 4$ are inside or on the circle (Shadow area)

③ (0,0) Blue
(-1,1) Red
(2,2) Blue
(3,8) Blue

④ $(1+X_1)^2+(2-X_2)^2=4$ => boundry decision

$1+2X_1+X_1^2+4-4X_2+X_2^2=4$

$X_1^2+2X_1-4X_2+X_2^2+1=0$

Therefore, the boundry decision is linear in terms of X_1^2, X_1, X_2^2, X_2.