2025Final

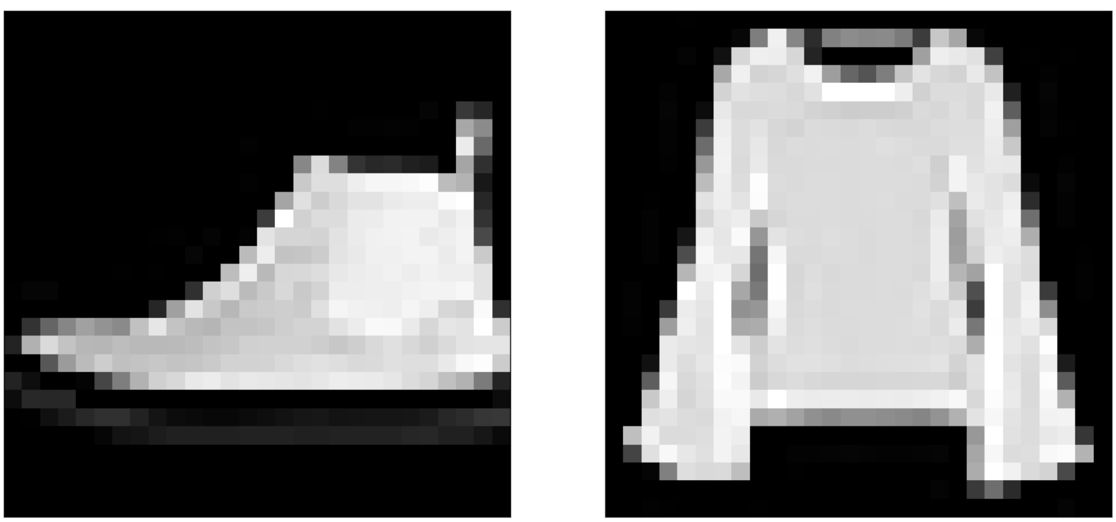
May 11, 2025

[16]:
```
import numpy as np
np.set_printoptions(precision=4)  # Print few decimal places
np.set_printoptions(suppress=True)  # Suppress scientific notation
import cvxpy as cp
import pandas as pd
from numpy.linalg import cholesky as llt
import matplotlib.pyplot as plt

X_train = pd.read_csv('/home/jovyan/work/Desktop/ORF307/X_train.csv').values
y_train = pd.read_csv('/home/jovyan/work/Desktop/ORF307/y_train.csv').values.
 ↪ravel()
X_test = pd.read_csv('/home/jovyan/work/Desktop/ORF307/X_test.csv').values
y_test = pd.read_csv('/home/jovyan/work/Desktop/ORF307/y_test.csv').values.
 ↪ravel()

n_train, m = X_train.shape
n_test, m = X_test.shape

print("-" * 50)
print("Fashion MNIST dataset")
print("-" * 50)
print(f"Number of features: {m}  ({int(np.sqrt(m))} x {int(np.sqrt(m))}␣
 ↪pixels)")
print(f"Training set:")
print(f"  • Samples: {n_train}")
print(f"  • Value range: [{X_train.min():.2f}, {X_train.max():.2f}]")
print(f"Test set:")
print(f"  • Samples: {n_test}")
print(f"  • Value range: [{X_train.min():.2f}, {X_train.max():.2f}]")

print(X_train.shape)
print(y_train)
print(y_train.shape)
```

```
--------------------------------------------------
Fashion MNIST dataset
--------------------------------------------------
Number of features: 784  (28 x 28 pixels)
```

```
Training set:
  • Samples: 5000
  • Value range: [0.00, 255.00]
Test set:
  • Samples: 1000
  • Value range: [0.00, 255.00]
(5000, 784)
[ 1. -1.  1. …  1. -1.  1.]
(5000,)
```

[17]:
```python
def visualize_images(*images):
    n_images = len(images)
    fig, axes = plt.subplots(1, n_images, figsize=(14,6))
    if n_images == 1:
        axes = [axes]
    for i, (ax, img) in enumerate(zip(axes, images)):
        img_reshaped = img.reshape(28, 28)
        im = ax.imshow(img_reshaped, cmap='gray')
        ax.set_xticks([])
        ax.set_yticks([])
    plt.tight_layout()

example_boot = X_train[3]
example_shirt = X_train[0]
visualize_images(example_boot, example_shirt)
```



[18]:
```python
#Part 1 - b

def error(X, y, a, b):
    y_pred = np.sign(X @ a + b)
```

```
    return np.mean(y_pred != y)

a = cp.Variable(784)
b = cp.Variable(1)

lamda = np.array([0.0001,0.0005,0.001,0.005,0.01,0.05])
asol = [] # where to store solutions of a for different values of lamda
bsol = [] # where to store solutions of b for different values of lamda

for val in lamda:
        objective = cp.Minimize(cp.sum(cp.maximum(0, (1 - cp.multiply(y_train,␣
 ↪(X_train @ a + b)))))/5000
                                + val * cp.norm(a,1))
        problem = cp.Problem(objective)
        problem.solve(solver = cp.CLARABEL)
        asol.append(a.value)
        bsol.append(b.value)
        print("test error for lamda(val):", error(X_test, y_test, a.value, b.
 ↪value))

#since the lowest test error was with lamda = .05, we will use that a and b␣
 ↪solution for other parts of this final
```

```
test error for lamda(val): 0.192
test error for lamda(val): 0.197
test error for lamda(val): 0.197
test error for lamda(val): 0.189
test error for lamda(val): 0.18
test error for lamda(val): 0.172
```

[3]:

```
#Using linear programming
a2 = cp.Variable(784)
b2 = cp.Variable(1)
s = cp.Variable(5000)
```

```
z = cp.Variable(784)

lamda = np.array([0.0001,0.0005,0.001,0.005,0.01,0.05])
a2sol = []
b2sol = []

for val in lamda:
        objective = cp.Minimize((1/5000) *cp.sum(s) + val * cp.sum(z))
        constraints = [cp.multiply((-1 * y_train), (X_train @ a2 + b2)) <= s -
 ↪1]
        constraints += [(-1 * s) <= 0]
        constraints += [ a2 <= z]
        constraints += [(-1 * a2) <= z]
        problem = cp.Problem(objective, constraints)
        problem.solve(solver = cp.CLARABEL)
        a2sol.append(a2.value)
        b2sol.append(b2.value)
        #print(error(X_test, y_test, a2.value, b2.value))
```

[13]:
```
#5 - checking dual -primal constraint relationship
print(z.value)
print(a2sol[5])
print(s.value)
```

```
[0.     0.     0.     0.     0.     0.0021 0.0074 0.0007 0.     0.0021
 0.0008 0.0021 0.003  0.003  0.0014 0.     0.0002 0.0031 0.0045 0.0018
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.0006 0.0009 0.0017 0.0042 0.0009 0.0023
 0.0002 0.0019 0.0001 0.0006 0.0014 0.0028 0.0002 0.0005 0.     0.0011
 0.0017 0.001  0.0016 0.     0.     0.     0.     0.     0.     0.
 0.0039 0.0026 0.0004 0.0026 0.     0.001  0.0004 0.0037 0.     0.0023
 0.0004 0.     0.     0.0006 0.     0.0011 0.0005 0.0002 0.     0.0025
 0.     0.     0.     0.     0.     0.     0.001  0.     0.0041 0.001
 0.001  0.0005 0.0054 0.     0.0001 0.0002 0.0009 0.0009 0.0004 0.0005
 0.0022 0.0009 0.0019 0.0014 0.0016 0.0017 0.0039 0.     0.0025 0.0003
 0.     0.     0.     0.     0.     0.     0.     0.     0.0003 0.0008
 0.0028 0.0002 0.0016 0.0009 0.0002 0.0019 0.0009 0.0003 0.0001 0.0007
 0.0004 0.001  0.0019 0.0013 0.0021 0.0015 0.0022 0.     0.     0.
 0.     0.     0.0024 0.0016 0.     0.     0.0013 0.0006 0.0017 0.
 0.     0.0002 0.0013 0.0022 0.0025 0.001  0.0002 0.0018 0.     0.0008
 0.0016 0.0022 0.     0.0004 0.     0.     0.002  0.     0.     0.
 0.     0.0019 0.0029 0.     0.0027 0.0026 0.0019 0.0009 0.0006 0.0002
 0.0009 0.0011 0.0001 0.0022 0.0038 0.0024 0.0008 0.0009 0.001  0.0016
 0.     0.     0.0005 0.     0.0028 0.     0.     0.0004 0.0009 0.
 0.0011 0.0013 0.     0.0021 0.     0.     0.001  0.0024 0.     0.
 0.0018 0.     0.     0.0004 0.     0.0004 0.0037 0.     0.0001 0.0006
 0.0008 0.0003 0.     0.     0.     0.     0.0009 0.     0.     0.0029
```

0.0006	0.0008	0.0009	0.	0.	0.	0.	0.0001	0.	0.
0.0007	0.0001	0.0008	0.	0.0028	0.0004	0.	0.	0.	0.
0.0012	0.	0.	0.0039	0.	0.0012	0.	0.0011	0.0012	0.0005
0.0025	0.0023	0.0003	0.	0.0024	0.	0.	0.0006	0.001	0.
0.0001	0.0018	0.002	0.	0.	0.0007	0.0012	0.	0.0011	0.
0.	0.	0.0025	0.0022	0.	0.0015	0.0007	0.0001	0.	0.0008
0.	0.0006	0.	0.	0.0004	0.0017	0.	0.	0.	0.0002
0.0032	0.0007	0.0009	0.	0.0009	0.	0.	0.	0.	0.
0.0004	0.0005	0.0009	0.0002	0.0008	0.0021	0.0006	0.	0.0003	0.0009
0.0042	0.0015	0.	0.0005	0.0006	0.	0.0018	0.0005	0.0007	0.0014
0.0007	0.	0.0027	0.0019	0.0035	0.0021	0.	0.0011	0.0004	0.003
0.	0.0015	0.0023	0.0005	0.0001	0.0026	0.0008	0.0008	0.0018	0.
0.	0.	0.	0.0004	0.0011	0.	0.0001	0.0014	0.0005	0.0008
0.	0.	0.0002	0.	0.	0.	0.	0.	0.0003	0.0003
0.	0.	0.0004	0.	0.0028	0.0038	0.0007	0.0023	0.	0.0013
0.0011	0.	0.0011	0.0021	0.0007	0.0002	0.	0.0013	0.0006	0.
0.	0.	0.	0.0004	0.0007	0.	0.	0.	0.	0.
0.0005	0.0013	0.0004	0.0022	0.0025	0.0009	0.0006	0.0003	0.	0.0012
0.	0.0013	0.	0.	0.0003	0.0002	0.0002	0.0008	0.0003	0.
0.0072	0.0024	0.0001	0.0008	0.0012	0.	0.0001	0.001	0.	0.0001
0.0001	0.0008	0.001	0.	0.	0.0001	0.001	0.0034	0.	0.0026
0.	0.0001	0.0003	0.0008	0.	0.	0.0002	0.	0.011	0.
0.	0.0007	0.	0.0006	0.0001	0.	0.0012	0.	0.	0.0008
0.0007	0.	0.0007	0.0001	0.	0.	0.	0.0005	0.0015	0.0006
0.0009	0.	0.	0.0005	0.	0.	0.0001	0.0009	0.	0.
0.	0.0006	0.	0.	0.0021	0.	0.0007	0.	0.001	0.0008
0.	0.0012	0.	0.0021	0.0012	0.0002	0.	0.	0.	0.0013
0.	0.	0.0013	0.0053	0.	0.	0.0003	0.	0.	0.0008
0.002	0.0029	0.0016	0.	0.0024	0.0011	0.	0.	0.0015	0.0014
0.002	0.	0.0005	0.0006	0.0005	0.0016	0.0013	0.0007	0.0031	0.0017
0.	0.	0.0037	0.	0.	0.0007	0.	0.	0.0032	0.0046
0.0007	0.0032	0.0014	0.0013	0.0003	0.0004	0.003	0.001	0.0001	0.0003
0.	0.	0.0016	0.	0.0023	0.0001	0.	0.0015	0.0005	0.
0.0018	0.	0.0012	0.0007	0.004	0.	0.	0.0027	0.0018	0.
0.	0.	0.0003	0.	0.0006	0.	0.001	0.	0.	0.0013
0.0007	0.	0.0032	0.	0.	0.0007	0.	0.	0.	0.0013
0.	0.	0.0013	0.0002	0.	0.0019	0.	0.0022	0.	0.0025
0.0022	0.	0.0001	0.0021	0.	0.0002	0.	0.0019	0.0014	0.0004
0.0021	0.0005	0.0013	0.	0.	0.	0.	0.001	0.001	0.
0.0001	0.0008	0.	0.0039	0.	0.0002	0.001	0.	0.	0.
0.0021	0.0006	0.0003	0.0003	0.0009	0.0019	0.0004	0.0017	0.	0.
0.0005	0.0008	0.	0.0047	0.	0.	0.0007	0.	0.	0.
0.001	0.	0.0034	0.0016	0.0022	0.	0.	0.	0.0005	0.
0.0027	0.003	0.0003	0.	0.0018	0.	0.0005	0.0003	0.002	0.
0.	0.	0.	0.	0.	0.0036	0.	0.0004	0.	0.0011
0.0004	0.	0.005	0.0006	0.	0.0017	0.	0.0013	0.0022	0.
0.0006	0.0003	0.0005	0.0007	0.	0.0003	0.0021	0.	0.	0.
0.0018	0.	0.	0.0007	0.0025	0.0008	0.001	0.001	0.0004	0.0011

```
 0.0002 0.0026 0.0024 0.0012 0.     0.     0.     0.0019 0.0006 0.0011
 0.0012 0.0014 0.0016 0.0015 0.     0.0042 0.0017 0.     0.     0.
 0.     0.0037 0.     0.0036 0.0015 0.     0.0013 0.0006 0.     0.0005
 0.     0.     0.0024 0.0012 0.     0.0002 0.0014 0.0009 0.     0.0005
 0.0011 0.0017 0.0014 0.0017 0.0006 0.0018 0.     0.     0.003  0.0025
 0.0001 0.0014 0.0009 0.     0.     0.0043 0.0012 0.0002 0.0001 0.
 0.0032 0.     0.0027 0.     0.0013 0.0009 0.0014 0.0034 0.0002 0.0004
 0.     0.     0.     0.    ]
[-0.      0.      0.      0.      0.      0.0021  0.0074 -0.0007 -0.
  0.0021  0.0008 -0.0021  0.003  -0.003  -0.0014  0.     -0.0002 -0.0031
  0.0045 -0.0018  0.      0.      0.      0.      0.      0.      0.
  0.      0.     -0.      0.      0.      0.      0.      0.0006  0.0009
  0.0017 -0.0042 -0.0009 -0.0023 -0.0002 -0.0019  0.0001  0.0006 -0.0014
 -0.0028  0.0002 -0.0005  0.      0.0011  0.0017  0.001   0.0016 -0.
 -0.     -0.     -0.     -0.      0.      0.      0.0039  0.0026  0.0004
  0.0026 -0.      0.001  -0.0004  0.0037 -0.      0.0023 -0.0004 -0.
 -0.     -0.0006 -0.      0.0011 -0.0005 -0.0002 -0.      0.0025  0.
 -0.     -0.     -0.     -0.     -0.      0.001  -0.     -0.0041 -0.001
  0.001  -0.0005  0.0054 -0.      0.0001  0.0002  0.0009  0.0009  0.0004
 -0.0005 -0.0022  0.0009 -0.0019  0.0014  0.0016  0.0017 -0.0039 -0.
 -0.0025 -0.0003 -0.     -0.     -0.      0.      0.     -0.      0.
  0.     -0.0003  0.0008 -0.0028 -0.0002 -0.0016 -0.0009  0.0002  0.0019
 -0.0009  0.0003  0.0001  0.0007 -0.0004  0.001  -0.0019  0.0013 -0.0021
  0.0015 -0.0022  0.     -0.     -0.     -0.     -0.      0.0024  0.0016
 -0.      0.      0.0013 -0.0006  0.0017  0.     -0.     -0.0002 -0.0013
 -0.0022  0.0025 -0.001   0.0002 -0.0018  0.     -0.0008  0.0016  0.0022
 -0.      0.0004  0.     -0.     -0.002  -0.      0.      0.      0.
  0.0019 -0.0029  0.      0.0027 -0.0026  0.0019 -0.0009  0.0006 -0.0002
  0.0009  0.0011  0.0001 -0.0022 -0.0038  0.0024  0.0008  0.0009  0.001
 -0.0016 -0.     -0.     -0.0005  0.     -0.0028  0.      0.     -0.0004
  0.0009 -0.     -0.0011  0.0013 -0.      0.0021  0.     -0.     -0.001
  0.0024  0.      0.      0.0018 -0.      0.     -0.0004  0.      0.0004
 -0.0037 -0.     -0.0001  0.0006 -0.0008 -0.0003 -0.     -0.     -0.
 -0.     -0.0009 -0.     -0.     -0.0029 -0.0006  0.0008 -0.0009  0.
  0.     -0.      0.      0.0001 -0.      0.     -0.0007  0.0001 -0.0008
  0.      0.0028 -0.0004 -0.      0.      0.     -0.     -0.0012 -0.
  0.      0.0039 -0.     -0.0012  0.     -0.0011 -0.0012 -0.0005  0.0025
  0.0023  0.0003  0.      0.0024 -0.     -0.     -0.0006  0.001  -0.
  0.0001  0.0018 -0.002  -0.      0.     -0.0007  0.0012 -0.     -0.0011
  0.     -0.      0.      0.0025  0.0022 -0.      0.0015  0.0007 -0.0001
 -0.      0.0008  0.     -0.0006  0.      0.     -0.0004  0.0017  0.
  0.      0.      0.0002  0.0032 -0.0007 -0.0009  0.     -0.0009  0.
 -0.     -0.      0.      0.     -0.0004 -0.0005  0.0009  0.0002 -0.0008
  0.0021 -0.0006  0.     -0.0003 -0.0009  0.0042 -0.0015  0.      0.0005
  0.0006  0.      0.0018  0.0005 -0.0007 -0.0014  0.0007  0.     -0.0027
 -0.0019 -0.0035 -0.0021  0.      0.0011  0.0004  0.003   0.      0.0015
  0.0023 -0.0005  0.0001  0.0026 -0.0008 -0.0008 -0.0018 -0.     -0.
  0.      0.      0.0004 -0.0011  0.      0.0001  0.0014 -0.0005  0.0008
```

```
-0.     -0.     -0.0002 -0.      0.     -0.      0.      0.     -0.0003
-0.0003  0.      0.      0.0004  0.      0.0028 -0.0038 -0.0007  0.0023
-0.     -0.0013 -0.0011 -0.      0.0011 -0.0021  0.0007 -0.0002 -0.
 0.0013 -0.0006 -0.      0.      0.      0.     -0.0004  0.0007  0.
-0.      0.      0.      0.      0.0005  0.0013 -0.0004 -0.0022 -0.0025
 0.0009 -0.0006  0.0003  0.     -0.0012  0.     -0.0013 -0.     -0.
-0.0003  0.0002  0.0002 -0.0008 -0.0003 -0.     -0.0072 -0.0024  0.0001
 0.0008 -0.0012  0.      0.0001 -0.001  -0.      0.0001  0.0001 -0.0008
-0.001   0.     -0.     -0.0001  0.001  -0.0034 -0.      0.0026 -0.
-0.0001  0.0003  0.0008  0.      0.     -0.0002 -0.     -0.011   0.
 0.      0.0007  0.     -0.0006 -0.0001 -0.      0.0012  0.      0.
 0.0008 -0.0007  0.      0.0007  0.0001 -0.     -0.     -0.     -0.0005
-0.0015  0.0006  0.0009  0.     -0.      0.0005 -0.     -0.     -0.0001
 0.0009  0.     -0.     -0.     -0.0006  0.      0.     -0.0021 -0.
 0.0007  0.     -0.001   0.0008  0.      0.0012  0.     -0.0021  0.0012
-0.0002  0.     -0.     -0.     -0.0013  0.     -0.     -0.0013 -0.0053
-0.      0.      0.0003  0.      0.      0.0008 -0.002   0.0029 -0.0016
-0.     -0.0024 -0.0011  0.     -0.      0.0015  0.0014 -0.002  -0.
-0.0005 -0.0006  0.0005 -0.0016  0.0013 -0.0007  0.0031 -0.0017  0.
 0.      0.0037  0.     -0.      0.0007  0.     -0.      0.0032 -0.0046
 0.0007 -0.0032 -0.0014  0.0013 -0.0003  0.0004 -0.003   0.001  -0.0001
 0.0003 -0.      0.      0.0016  0.     -0.0023 -0.0001  0.     -0.0015
 0.0005  0.      0.0018 -0.     -0.0012  0.0007  0.004   0.     -0.
-0.0027 -0.0018 -0.      0.      0.      0.0003 -0.     -0.0006 -0.
-0.001   0.     -0.      0.0013 -0.0007  0.     -0.0032  0.      0.
-0.0007 -0.      0.      0.     -0.0013 -0.      0.     -0.0013  0.0002
-0.      0.0019  0.      0.0022 -0.     -0.0025 -0.0022  0.     -0.0001
-0.0021 -0.     -0.0002  0.     -0.0019  0.0014  0.0004  0.0021 -0.0005
-0.0013  0.      0.      0.      0.     -0.001  -0.001   0.     -0.0001
-0.0008 -0.      0.0039 -0.     -0.0002 -0.001   0.     -0.      0.
 0.0021 -0.0006  0.0003  0.0003  0.0009  0.0019  0.0004  0.0017  0.
-0.     -0.0005 -0.0008  0.      0.0047 -0.      0.     -0.0007 -0.
-0.      0.      0.001   0.      0.0034 -0.0016 -0.0022 -0.     -0.
 0.      0.0005  0.     -0.0027  0.003  -0.0003 -0.      0.0018  0.
-0.0005  0.0003  0.002  -0.     -0.      0.     -0.     -0.     -0.
-0.0036 -0.      0.0004  0.      0.0011  0.0004 -0.     -0.005  -0.0006
 0.      0.0017 -0.      0.0013  0.0022 -0.     -0.0006  0.0003  0.0005
 0.0007  0.     -0.0003 -0.0021 -0.     -0.      0.     -0.0018 -0.
-0.     -0.0007 -0.0025  0.0008 -0.001  -0.001  -0.0004  0.0011 -0.0002
-0.0026  0.0024  0.0012 -0.      0.     -0.     -0.0019  0.0006  0.0011
-0.0012  0.0014  0.0016 -0.0015  0.      0.0042  0.0017  0.     -0.
-0.      0.      0.0037 -0.     -0.0036  0.0015  0.     -0.0013  0.0006
-0.     -0.0005  0.      0.      0.0024  0.0012  0.     -0.0002 -0.0014
 0.0009 -0.     -0.0005 -0.0011  0.0017  0.0014  0.0017  0.0006  0.0018
-0.     -0.     -0.003  -0.0025 -0.0001 -0.0014 -0.0009 -0.      0.
-0.0043 -0.0012 -0.0002  0.0001  0.      0.0032 -0.      0.0027  0.
-0.0013 -0.0009  0.0014 -0.0034  0.0002 -0.0004 -0.     -0.     -0.
 0.    ]
```

```
[0. 0. 0. … 0. 0. 0.]
```

[19]:

```
#Dual Problem- Part A question 4

w1 = cp.Variable(5000, nonneg = True)
w2 = cp.Variable(5000, nonneg = True)
w3 = cp.Variable(784, nonneg = True)
w4 = cp.Variable(784, nonneg = True)
lam = .025

objective = cp.Maximize(-cp.sum(w1))
constraints = [w3 + w4 == lam * np.ones(784)]
constraints += [(-1 *w1) @ y_train == 0]
constraints += [w3 - w4 == X_train.T @ (cp.multiply(w1, y_train))]
constraints += [w1 + w2 == (1/5000) * np.ones(5000)]
problem2 = cp.Problem(objective, constraints)
problem2.solve(solver = cp.CLARABEL)
print(w1.value)
print(w2.value)
print(w3.value)
print(w4.value)

#Question 5
```

```
[0. 0. 0. … 0. 0. 0.]
[0.0002 0.0002 0.0002 … 0.0002 0.0002 0.0002]
[0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125
 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125
 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125
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[161]:
```
#Part B Question 3

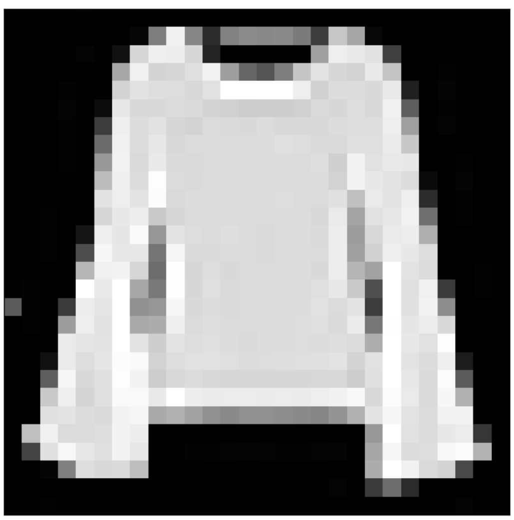
u = cp.Variable(784)
x = cp.Variable(784)

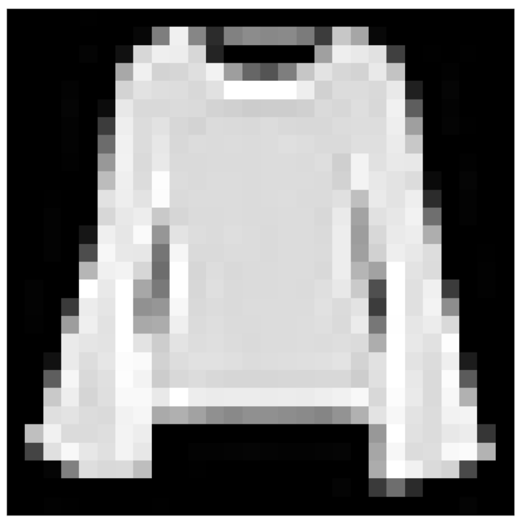
objective = cp.Minimize(cp.sum(u))
```

```python
constraints = [x - example_shirt <= u]
constraints += [-(x - example_shirt) <= u]
constraints += [ x @ asol[5] + bsol[5] <= 0]
constraints += [x >= 0, x <= 255]
problem4 = cp.Problem(objective, constraints)
problem4.solve()
xsol = x.value
visualize_images(xsol, example_shirt)
#print(xsol)
#print(example_shirt)
t = np.sum(np.abs(xsol - example_shirt))
print("l1 norm distance:", t) # l1-norm distance
count = 0
for i in range(0, 784):
    if( xsol[i] == example_shirt[i]):
        count = count + 1

print("pixel difference:", count) #pixels that are different
```

```
l1 norm distance: 91.17502362538634
pixel difference: 1
```





[ ]:

```python
```

[21]:

```python
#Part 3

# get shirt training and testing datasets
shirt_indices_train = np.where(y_train == 1)[0]
```

```
X_shirt_train = X_train[shirt_indices_train]
n_shirt_train = X_shirt_train.shape[0]
inspection_capacity_train = int(0.4 * n_shirt_train)

shirt_indices_test = np.where(y_test == 1)[0]
X_shirt_test = X_test[shirt_indices_test]
n_shirt_test = X_shirt_test.shape[0]
inspection_capacity_test = int(0.4 * n_shirt_test)
shirt_prob = []

#part a
for i in range(0, 2500):
    f = -(X_shirt_train[i] @ asol[5] + bsol[5])
    o = 1 / (1 + np.exp(-f))
    shirt_prob.append(o)
    #print(o)
print(X_shirt_train.shape)
print(X_shirt_test.shape)
print(len(shirt_prob))

#part b- "naive approach"

for j in range(0, 2500):
     if(shirt_prob[j] >= .8):
         print(shirt_prob[j], j)

print("cut-off")


for j in range(0, 2500):
     if(shirt_prob[j] >= .9):
         print(shirt_prob[j],j)

print("cut off")

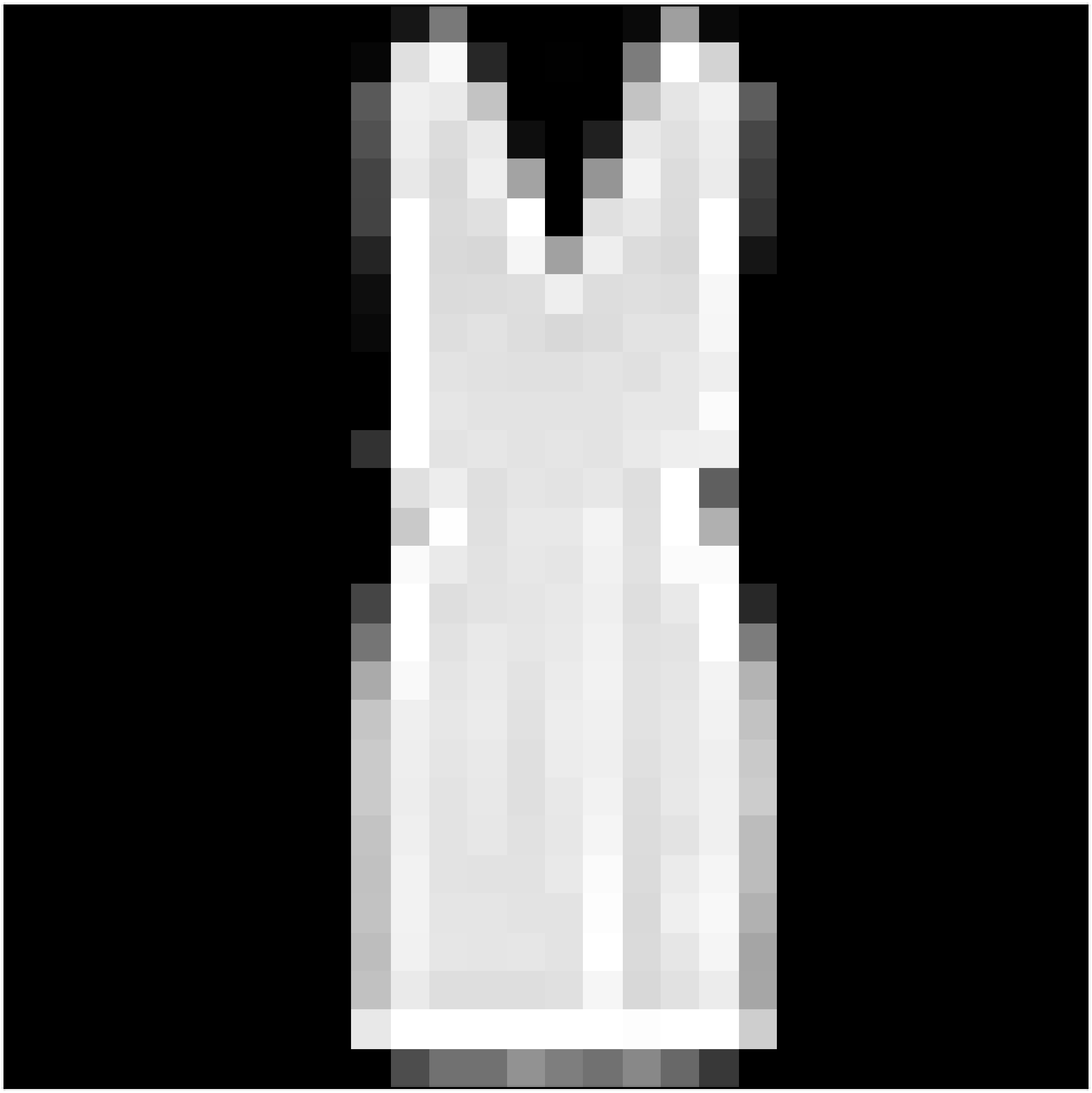
for j in range(0, 2500):
     if(shirt_prob[j] >= .95):
         print(shirt_prob[j],j)

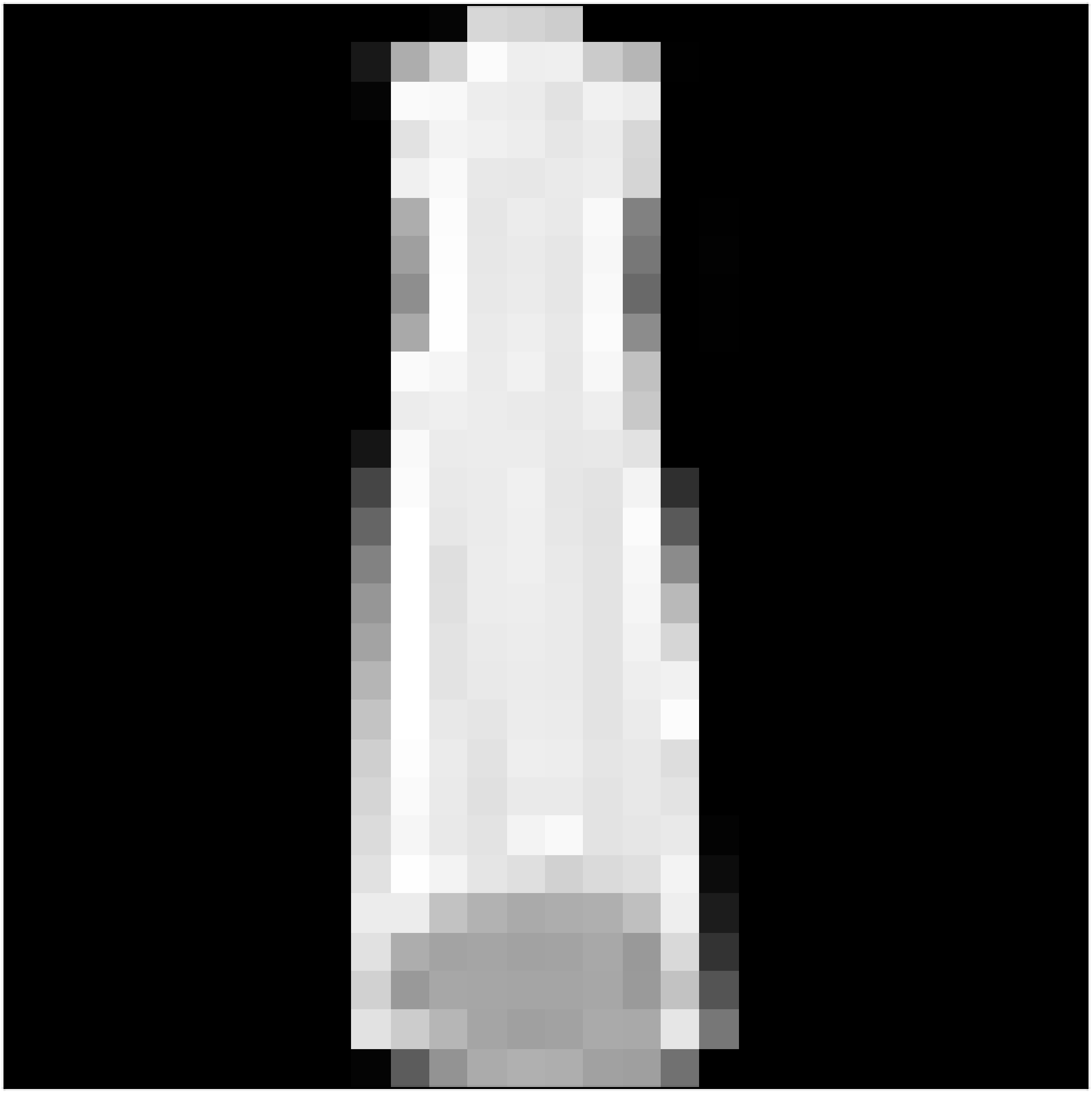
print("cut off")

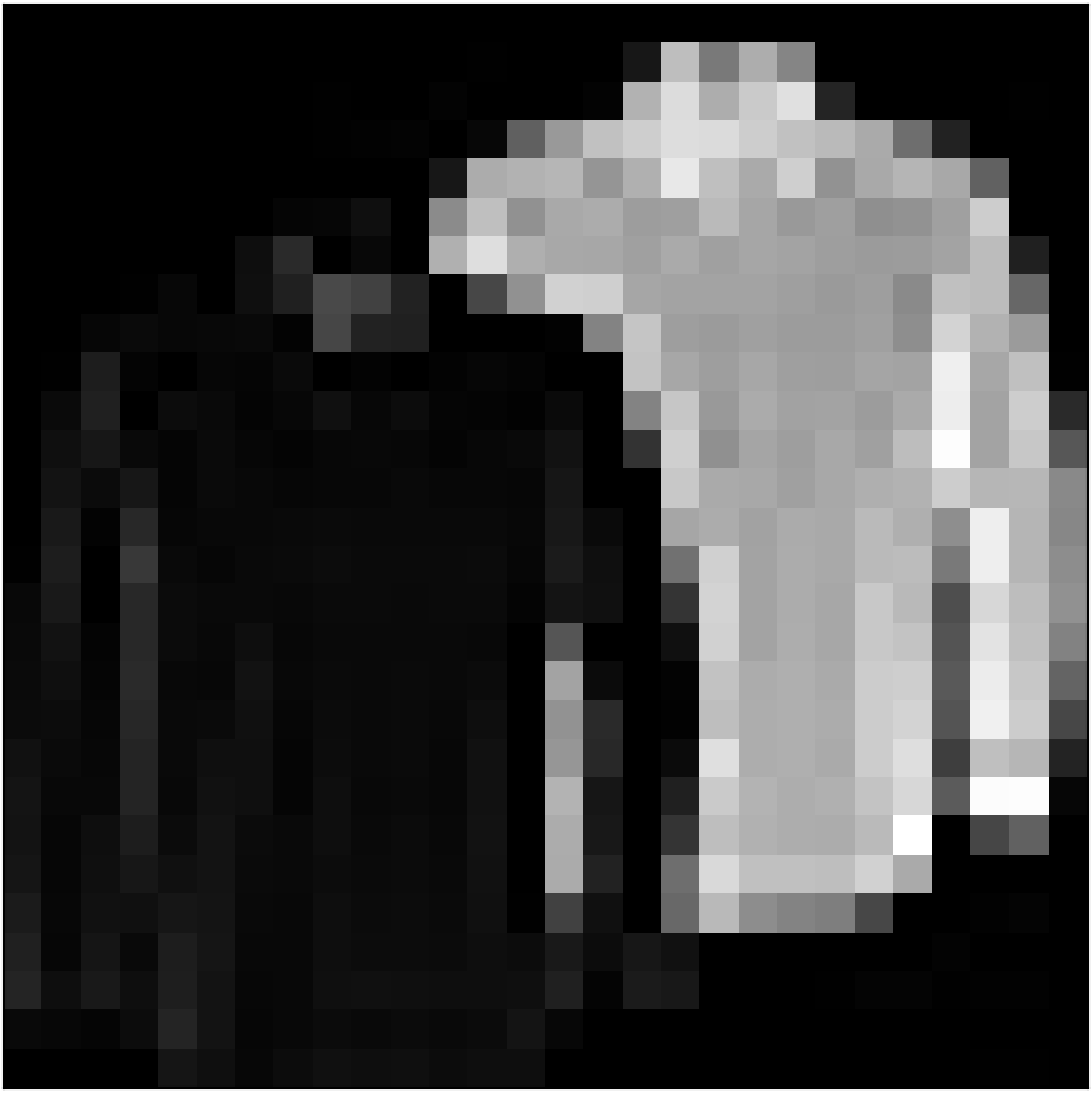
#shirt images
visualize_images(X_shirt_train[253])
visualize_images(X_shirt_train[914])
visualize_images(X_shirt_train[2366])
```

```
(2500, 784)
(500, 784)
2500
[0.9148] 165
[0.9972] 253
[0.8637] 345
[0.8014] 364
[0.8119] 448
[0.9092] 468
[0.8537] 503
[0.8988] 540
[0.8284] 597
[0.8768] 607
[0.8472] 664
[0.8357] 677
[0.9411] 692
[0.92] 863
[0.9822] 914
[0.8938] 954
[0.8045] 1011
[0.861] 1075
[0.8351] 1158
[0.8183] 1285
[0.9326] 1306
[0.8006] 1322
[0.8535] 1363
[0.8557] 1369
[0.9302] 1422
[0.9541] 1562
[0.8264] 1583
[0.804] 1609
[0.8926] 1783
[0.9443] 1815
[0.8378] 1877
[0.8324] 2271
[0.9779] 2366
[0.8394] 2393
[0.932] 2477
cut-off
[0.9148] 165
[0.9972] 253
[0.9092] 468
[0.9411] 692
[0.92] 863
[0.9822] 914
[0.9326] 1306
[0.9302] 1422
[0.9541] 1562
```

```
[0.9443] 1815
[0.9779] 2366
[0.932] 2477
cut off
[0.9972] 253
[0.9822] 914
[0.9541] 1562
[0.9779] 2366
cut off
```







[22]:
```
#3d
# training
c = 10
d = 20
s = 15
w = cp.Variable(2500, boolean = True)
objective = cp.Minimize( 10 * cp.sum(w) + d * cp.sum(cp.multiply((1-w),
 ↪shirt_prob))
                        - 15 * cp.sum(cp.multiply(w, shirt_prob)))
constraint = [ cp.sum(w) <=inspection_capacity_train]
problem5 = cp.Problem(objective, constraint)
problem5.solve(solver = cp.SCIPY)
wsol = w.value
```

```
print("expected cost train:", problem5.objective.value)
print(wsol)
```

```
/opt/conda/lib/python3.11/site-
packages/cvxpy/reductions/solvers/solving_chain.py:407: UserWarning: The problem
includes expressions that don't support CPP backend. Defaulting to the SCIPY
backend for canonicalization.
  warnings.warn(UserWarning(
```

```
expected cost train: 7482.488318449875
[0. 0. 0. … 1. 0. 0.]
```

[139]:
```
#3d
# testing
shirt_prob2 = []
for i in range(0, 500):
    f = -(X_shirt_test[i] @ asol[5] + bsol[5])
    o = 1 / (1 + np.exp(-f))
    shirt_prob2.append(o)

c = 10
d = 20
s = 15
w = cp.Variable(500, boolean = True)
objective = cp.Minimize( 10 * cp.sum(w) + d * cp.sum(cp.multiply((1-w),␣
 ↪shirt_prob2))
                        - 15 * cp.sum(cp.multiply(w, shirt_prob2)))
constraint = [ cp.sum(w) <=inspection_capacity_test]
problem5 = cp.Problem(objective, constraint)
problem5.solve(solver = cp.SCIPY)
print("expected cost test:", problem5.objective.value)
wsol = w.value
print(wsol)
```

```
1328.7581449199374
[1. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 1. 0. 0. 0. 1. 0. 1.
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