CS61B Lecture #14: Integers

Announcement:

- Programming contest SATURDAY! You can still sign up.

Today: Integer Types; Readings: *A Java Reference*, §6.3–4. *Head First Java*, Chapter 10.

Readings for Upcoming Topics: *Data Structures (Into Java)*, Chapter 1.

Integer Types and Literals

Type	Bits	Signed?	Literals
byte	8	Yes	
short	16	Yes	

char	16	No	`'a' // (char) 97`
			`'\n'  // newline ((char) 10)`
			`'\t'  // tab ((char) 8)`
			`'\\'  // backslash`
			`'A', '\101', '\u0041'  // == (char) 65`

int	32	Yes	`123`
			`0100  // Octal for 64`
			`0x3f, 0xffffffff // Hexadecimal 63, -1 (!)`

long	64	Yes	`123L, 01000L, 0x3fL` `1234567891011L`

- "N bits" means that there are 2^N integers in the domain of the type.

- If signed, range of values is $-2^{N-1} .. 2^{N-1} - 1$.

- If unsigned, only non-negative numbers, and range is $0 .. 2^N - 1$.

- Negative numerals are just negated (positive) literals.

- Use casting for **byte** and **short**: (byte) 12, (short) 2000.

Modular Arithmetic

- **Problem:** How do we handle overflow, such as occurs in `10000*10000*10000`?

- Some languages throw an exception (Ada), some give undefined results (C, C++)

- Java *defines* the result of any arithmetic operation or conversion on integer types to "wrap around"—*modular arithmetic.*

- That is, the "next number" after the largest in an integer type is the smallest (like "clock arithmetic").

- E.g., `(byte) 128 == (byte) (127+1) == (byte) -128`

- In general,

  - If the result of some arithmetic subexpression is supposed to have type T, an n-bit integer type,
  - then we compute the real (mathematical) value, x,
  - and yield a number, x', that is in the range of T, and that is equivalent to x modulo 2^n.
  - (That means that $x - x'$ is a multiple of 2^n.)

Modular Arithmetic II

- `(byte)` `(64*8)` yields 0, since $512 - 0 = 2 \cdot 2^8$.

- `(byte)` `(64*2)` and `(byte)` `(127+1)` yield -128, since $128 - (-128) = 1 \cdot 2^8$.

- `(byte)` `(345*6)` yields 22, since $2070 - 22 = 8 \cdot 2^8$.

- `(byte)` `(-30*13)` yields 122, since $-390 - 122 = -2 \cdot 2^8$.

- `(char)` `(-1)` yields $2^{16} - 1$, since $-1 - (2^{16} - 1) = -1 \cdot 2^{16}$.

- Natural definition for a machine that uses binary arithmetic:

Type **char**	Type **byte**
$0 = 0000000000000000_2$ $2^{16} - 1 = 1111111111111111_2$	$0 = 00000000_2$ $1 = 00000001_2$ $127 = 01111111_2$ $-128 = 10000000_2$ $-1 = 11111111_2$

- Terminology: rightmost (units) bit is *bit 0*, 2s bit is *bit 1*.

- Hence, changing bit n modifies value by 2^n; truncating on left to n bits computes modulo 2^n.

Negative numbers

- Why this representation for -1?

$$
\begin{array}{r|l}
1 & 00000001_2 \\
+ \ -1 & 11111111_2 \\
\hline
= \quad 0 & 1|00000000_2
\end{array}
$$

  Only 8 bits in a byte, so bit 8 falls off, leaving 0.

- The truncated bit is in the 2^8 place, so throwing it away gives an equal number modulo 2^8. All bits to the left of it are also divisible by 2^8.

- On unsigned types (**char**), arithmetic is the same, but we choose to represent only non-negative numbers modulo 2^{16}:

$$
\begin{array}{r|l}
1 & 0000000000000001_2 \\
+ \ 2^{16} - 1 & 1111111111111111_2 \\
\hline
= \ 2^{16} + 0 & 1|0000000000000000_2
\end{array}
$$

Conversion

- In general Java will silently convert from one type to another if this makes sense and no information is lost from value.

- Otherwise, cast explicitly, as in `(byte) x`.

- Hence, given

```
byte aByte; char aChar; short aShort; int anInt; long aLong;

// OK:
aShort = aByte; anInt = aByte; anInt = aShort; anInt = aChar;
aLong = anInt;

// Not OK, might lose information:
anInt = aLong; aByte = anInt; aChar = anInt; aShort = anInt;
aShort = aChar; aChar = aShort; aChar = aByte;

// OK by special dispensation:
aByte = 13;      // 13 is compile-time constant
aByte = 12+100 // 112 is compile-time constant
```

Promotion

- Arithmetic operations (+, *, ...) *promote* operands as needed.

- Promotion is just implicit conversion.

- For integer operations,

  – if any operand is **long**, promote both to **long**.

  – otherwise promote both to **int**.

- So,

```
aByte + 3 == (int) aByte + 3    // Type int
aLong + 3 == aLong + (long) 3   // Type long
'A' + 2 == (int) 'A' + 2        // Type int
aByte = aByte + 1               // ILLEGAL (why?)
```

- But fortunately,

```
aByte += 1;      // Defined as aByte = (byte) (aByte+1)
```

- Common example:

```
// Assume aChar is an upper-case letter
char lowerCaseChar = (char) ('a' + aChar - 'A'); // why cast?
```

Bit twiddling

- Java (and C, C++) allow for handling integer types as sequences of bits. No "conversion to bits" needed: they already are.

- Operations and their uses:

Mask	Set	Flip	Flip all
00101100	00101100	00101100	
& 10100111	| 10100111	^ 10100111	~ 10100111
00100100	10101111	10001011	01011000

- Shifting:

Left	Arithmetic Right	Logical Right
10101101 << 3	10101101 >> 3	10101100 >>> 3
01101000	11110101	00010101

- What is:
  $$(-1) >>> 29?$$
  $$x << n?$$
  $$x >> n?$$
  $$(x >>> 3) \ \& \ ((1<<5)-1)?$$

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- What is:

$$(-1) \texttt{ >>> } 29? \qquad = 7.$$
$$x \texttt{ << } n?$$
$$x \texttt{ >> } n?$$
$$(x \texttt{ >>> } 3) \texttt{ \& } ((1\texttt{<<}5)-1)?$$

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\begin{array}{l|l}
\text{(-1) >>> 29?} & = 7. \\
x \text{ << } n? & = x \cdot 2^n. \\
x \text{ >> } n? & \\
\text{(}x\text{ >>> 3) \& ((1<<5)-1)?} &
\end{array}
$$

.

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$\begin{aligned} \text{(-1) >>> 29?} \\ x \ \text{<<} \ n? \\ x \ \text{>>} \ n? \\ \text{(x >>> 3) \& ((1<<5)-1)?} \end{aligned}$ $\begin{aligned} &= 7. \\ &= x \cdot 2^n. \\ &= \lfloor x/2^n \rfloor \ \text{(i.e., rounded down).} \\ &\quad . \end{aligned}$

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x \text{ << } n? & = x \cdot 2^n. \\
x \text{ >> } n? & = \lfloor x/2^n \rfloor \text{ (i.e., rounded down).} \\
\text{(}x \text{ >>> 3) \& ((1<<5)-1)?} & \text{5-bit integer, bits 3–7 of } x.
\end{array}
$$