

Appendix : Performance Evaluation of State-Based Randomized Systems using Probabilistic Process Algebraic Tools

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1 APCTL formulas in MCL syntax

1.1 Bounded Retransmission Protocol

1.1.1 Property 1

```
prob
(((if not(prob
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where not(act = "BOT")})
is >= 1 end prob
or
prob
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where act = "BOT"}
is > 0 end prob) then false end if)).true or tau)*.
if not(prob
{?act: String
  ?any
  ?srep:nat
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?rrrep:nat
  ?any
  ?any
  ?any
  ?any
  ?recv:bool
  ?any
  ?any
  ?any
  where srep=1 and rrep=3 and recv and not(act="BOT")})
is >=1 end prob) then false end if
is >=? 0
end prob
```

1.1.2 Property 2

```
prob
(((if not(prob
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where not(act = "BOT")})
is >= 1 end prob
or
prob
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where act = "BOT"}
is > 0 end prob) then false end if)).true or tau)*.
if not(prob
{?act: String
  ?any
  ?srep:nat
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?any
  ?rrrep:nat
  ?any
  ?any
  ?any
  ?any
  ?recv:bool
  ?any
  ?any
  ?any
  where srep=3 and not(rrep=3) and recv and not(act="BOT")})
is >=1 end prob) then false end if
is >=? 0
end prob
```

1.1.3 Property 3

```
prob
(((if not(prob
```

[illegible]

1.1.4 Property 4

[illegible]

1.1.5 Property 5

```

prob
(((if not(prob
{act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where not(act = "BOT")})
is >= 1 end prob
or
prob
{act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where act = "BOT"})
is > 0 end prob) then false end if)).true or tau)*.
if not(prob

```

```
{?act: String
?s:nat
?srep:nat
?any
?i:nat
?any
?any
?any
?any
?any
?any
?any
?any
?any
?any
?any
?any
where s=5 and srep=1 and i>8 and not(act="BOT")}
is >=1 end prob) then false end if
is >=? 0
end prob
```

1.1.6 Property 6

```
prob  
(((if not(prob  
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any  
where not(act = "BOT")}  
is >= 1 end prob  
or  
prob  
{?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any  
where act = "BOT"}  
is > 0 end prob) then false end if)).true or tau)*.  
if not(prob  
{?act: String  
?any  
?srep:nat  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?recv:boolean  
?any  
?any  
?any  
where not(srep=0) and recv=false and not(act="BOT")}  
is >=1 end prob) then false end if  
is >=? 0  
end prob
```

1.2 Crowds Protocol

[illegible]

```
?observe0:int  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
where observe0 > 1 of int and not(act="BOT")}  
is >=1 end prob) then false end if  
is >=? 0  
end prob
```

1.3 Contract signing protocol

1.3.1 Property 1

[illegible]

```
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?any  
?knowB:bool  
?knowA:bool  
where not(knowA) and knowB and not(act="BOT")}  
is >=1 end prob) then false end if  
is >=? 0  
end prob
```

1.3.2 Property 2

[illegible]


```

?any
?any
?any
?any
?any
?any
?any
?any
?any
?any
?elected: bool
  where elected=true and not(act="BOT")}
is >=1 end prob) then false end if
is >=? 0
end prob

```

1.4.2 For the models having $N = 4$

```

prob
(((if not(prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where not(act = "BOT")} is >= 1 end prob
or
prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where act = "BOT"} is > 0 end prob) then false end if)).true or tau)*.
if not(prob {?act: String
?c: int
?s1: int
?u1: bool
?v1: int
?p1: int
?s2: int
?u2: bool
?v2: int
?p2: int
?s3: int
?u3: bool
?v3: int
?p3: int
?s4: int
?u4: bool
?v4: int
?p4: int
?elected: bool
  where elected=true and not(act="BOT")}
is >=1 end prob) then false end if
is >=? 0
end prob

```

1.4.3 For the models having $N = 5$

```

prob
(((if not(prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where not(act = "BOT")} is >= 1 end prob
or
prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any ?any
  where act = "BOT"} is > 0 end prob) then false end if)).true or tau)*.
if not(prob {?act: String
?c: int
?s1: int
?u1: bool
?v1: int
?p1: int
?s2: int
?u2: bool
?v2: int
?p2: int
?s3: int
?u3: bool
?v3: int
?p3: int
?s4: int
?u4: bool
?v4: int
?p4: int
?s5: int
?u5: bool
?v5: int
?p5: int
?elected: bool
  where elected=true and not(act="BOT")}
is >=1 end prob) then false end if

```



```
is >=? 0
end prob
```

1.4.4 For the models having $N = 6$

[illegible]

1.4.5 Bounded Until formula

This formula is for $N = 3$ and $L = 1$

```

prob
true{0 ... 8}.{"BOT"
?any
?s1: nat
?any
?any
?any
?s2: nat
?any
?any
?any
?s3: nat
?any
?any
?any
?any
?any
where s1=3 and s2=3 and s3=3}
is >= 0
end prob

```

This formula is for $N = 4$ and $L = 1$

```

prob
true{0 ... 10}.{"BOT"
?any
?s1: nat
?any
?any
?any
?s2: nat
?any
?any
?any

```

```

?s3: nat
?any
?any
?any
?s4: nat
?any
?any
?any
?any
where s1=3 and s2=3 and s3=3 and s4=3}
is >=? 0
end prob

```

Table 1: Table showing the upper bounds of the bounded until formula

N	K	L	Upper bound in PCTL	Upper bound in MCL
4	3	1	5	10
4	3	2	10	20
4	3	3	15	30
4	3	4	20	40
4	3	5	25	50
4	3	6	30	60
4	3	7	35	70
4	3	8	40	80
4	3	9	45	90
4	3	10	50	100
3	2	1	4	8
3	2	2	8	16
3	2	3	12	24
3	2	4	16	32
3	2	5	20	40
3	2	6	24	48
3	2	7	28	56
3	2	8	32	64
3	2	9	36	72
3	2	10	40	80

1.5 NAND multiplexing

```

prob
(((if not(prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any where not(act = "BOT")}) is >= 1 end prob
or
prob {?act: String ?any ?any ?any ?any ?any ?any ?any ?any where act = "BOT"} is > 0 end prob)
then false end if)).true or tau)*.
if not(prob {?act: String
?any
?any
?s:nat
?z:real
?any
?any
?any
?any
where s=4 and z/2 of real < 0.1 of real and not(act="BOT")})
is >=1 end prob) then false end if
is >=? 0
end prob

```

2 Tables and graphs of bounded until property of Case Study “Synchronous leader election protocol”

In this section, we compute the probability that a leader is elected within L rounds for $N = 3$, $N = 4$, and $N = 5$. For each value of N , we consider $K = 2, 3, 4, 5, 6$, and 8 . For every combination of N and

K , we vary L from 1 to 10.

$$\begin{aligned}
N = 3 & \quad P = ?[trueU \leq L * (N + 1)(s1 = 3 \ \& \ s2 = 3 \ \& \ s3 = 3)] \\
N = 4 & \quad P = ?[trueU \leq L * (N + 1)(s1 = 3 \ \& \ s2 = 3 \ \& \ s3 = 3 \ \& \ s4 = 3)] \\
N = 5 & \quad P = ?[trueU \leq L * (N + 1)(s1 = 3 \ \& \ s2 = 3 \ \& \ s3 = 3 \ \& \ s4 = 3 \ \& \ s5 = 3)]
\end{aligned}$$

In the graph, we have plotted the expected values with respect to L in the x-axis. **Models with value $N = 3$**

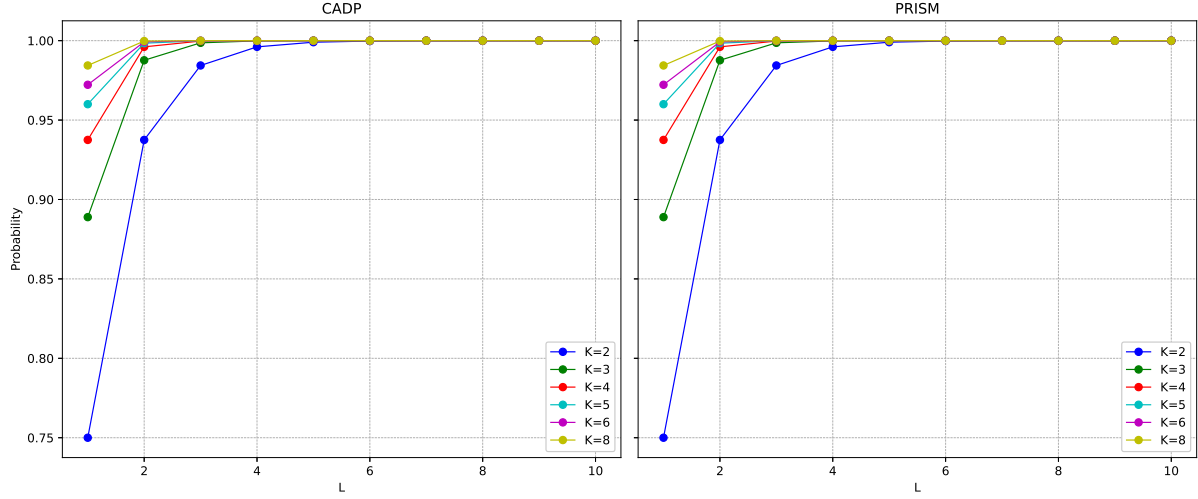


Figure 1: Table mapping the CADP and PRISM probabilities for model with parameter $N = 3$

Table 2: Probability values by CADP and PRISM for $N = 3$

N	K	L	CADP probability value	PRISM probability value
3	2	1	0.75	0.75
3	2	2	0.9375	0.9375
3	2	3	0.984375	0.984375
3	2	4	0.996094	0.99609375
3	2	5	0.999023	0.9990234375
3	2	6	0.999756	0.999755859375
3	2	7	0.999939	0.99993896484375
3	2	8	0.999985	0.9999847412109375
3	2	9	0.999996	0.9999961853027344
3	2	10	0.999999	0.9999990463256836
3	3	1	0.888889	0.8888888888888884
3	3	2	0.987654	0.9876543209876535
3	3	3	0.998628	0.9986282578875164
3	3	4	0.999848	0.9998475842097233
3	3	5	0.999983	0.9999830649121907
3	3	6	0.999998	0.9999981183235761
3	3	7	1	0.9999997909248411
3	3	8	1	0.999999976769426
3	3	9	1	0.9999999974188245
3	3	10	1	0.9999999997132021
3	4	1	0.9375	0.9375
3	4	2	0.996094	0.99609375
3	4	3	0.999756	0.999755859375
3	4	4	0.999985	0.9999847412109375
3	4	5	0.999999	0.9999990463256836
3	4	6	1	0.9999999403953552
3	4	7	1	0.9999999962747097
3	4	8	1	0.9999999997671694
3	4	9	1	0.9999999999854481
3	4	10	1	0.999999999990905
3	5	1	0.96	0.9600000000000007
3	5	2	0.9984	0.9984000000000008
3	5	3	0.999936	0.9999360000000008
3	5	4	0.999997	0.999997440000001
3	5	5	1	0.9999998976000007
3	5	6	1	0.9999999959040008
3	5	7	1	0.9999999998361608
3	5	8	1	0.999999999934472
3	5	9	1	0.999999999997388
3	5	10	1	0.999999999999903
3	6	1	0.972222	0.972222222222251
3	6	2	0.999228	0.9992283950617316
3	6	3	0.999979	0.9999785665294953
3	6	4	0.999999	0.9999994046258223
3	6	5	1	0.9999999834618314
3	6	6	1	0.9999999995406093
3	6	7	1	0.999999999987242
3	6	8	1	0.999999999996486
3	6	9	1	0.999999999999931
3	6	10	1	1.0000000000000029
3	8	1	0.984375	0.984375
3	8	2	0.999756	0.999755859375
3	8	3	0.999996	0.9999961853027344
3	8	4	1	0.9999999403953552
3	8	5	1	0.999999990686774
3	8	6	1	0.9999999999854481
3	8	7	1	0.999999999997726
3	8	8	1	0.999999999999964
3	8	9	1	1.0
3	8	10	1	1.0

Models with value $N = 4$

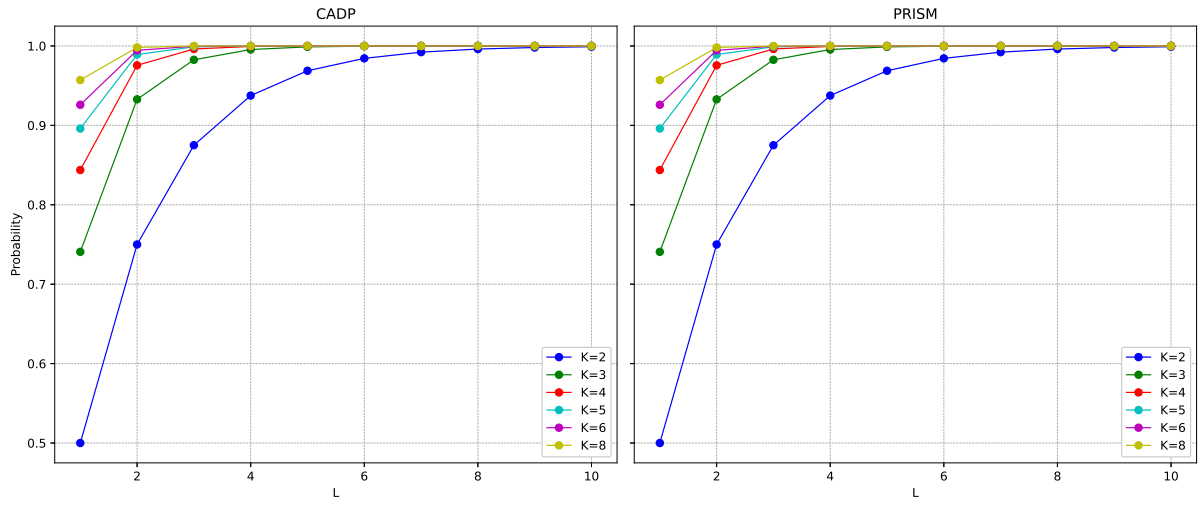


Figure 2: Table mapping the CADP and PRISM probabilities for the model with parameter $N = 4$

Table 3: Probability values by CADP and PRISM for $N = 4$

N	K	L	CADP probability value	PRISM probability value
4	2	1	0.5	0.5
4	2	2	0.75	0.75
4	2	3	0.875	0.875
4	2	4	0.9375	0.9375
4	2	5	0.96875	0.96875
4	2	6	0.984375	0.984375
4	2	7	0.992188	0.9921875
4	2	8	0.996094	0.99609375
4	2	9	0.998047	0.998046875
4	2	10	0.999023	0.9990234375
4	3	1	0.740741	0.7407407407407418
4	3	2	0.932785	0.9327846364883418
4	3	3	0.982574	0.9825737946451257
4	3	4	0.995482	0.995482094907996
4	3	5	0.998829	0.9988286912724443
4	3	6	0.999696	0.9996963273669313
4	3	7	0.999921	0.9999212700580946
4	3	8	0.99998	0.9999795885335816
4	3	9	0.999995	0.999994708138338
4	3	10	0.999999	0.9999986280358656
4	4	1	0.84375	0.84375
4	4	2	0.975586	0.9755859375
4	4	3	0.996185	0.996185302734375
4	4	4	0.999404	0.9994039535522461
4	4	5	0.999907	0.9999068677425385
4	4	6	0.999985	0.9999854480847716
4	4	7	0.999998	0.9999977262632456
4	4	8	1	0.9999996447286321
4	4	9	1	0.9999999444888488
4	4	10	1	0.9999999913263826
4	5	1	0.896	0.8960000000000092
4	5	2	0.989184	0.9891840000000127
4	5	3	0.998875	0.9988751360000127
4	5	4	0.999883	0.9998830141440133
4	5	5	0.999988	0.9999878334709892
4	5	6	0.999999	0.9999987346809942
4	5	7	1	0.9999998684068346
4	5	8	1	0.9999999863143206
4	5	9	1	0.9999999985766995
4	5	10	1	0.9999999998519886
4	6	1	0.925926	0.9259259259258992
4	6	2	0.994513	0.9945130315500353
4	6	3	0.999594	0.9995935578925635
4	6	4	0.99997	0.9999698931772002
4	6	5	0.999998	0.999997769864947
4	6	6	1	0.9999998348047807
4	6	7	1	0.9999999877632894
4	6	8	1	0.9999999990935462
4	6	9	1	0.999999999328276
4	6	10	1	0.999999999949966
4	8	1	0.957031	0.95703125
4	8	2	0.998154	0.9981536865234375
4	8	3	0.999921	0.999920666217804
4	8	4	0.999997	0.9999965911265463
4	8	5	1	0.9999998535249688
4	8	6	1	0.999999993706151
4	8	7	1	0.999999997295635
4	8	8	1	0.999999999883775
4	8	9	1	0.999999999994967
4	8	10	1	0.999999999999802

Models with value $N = 5$

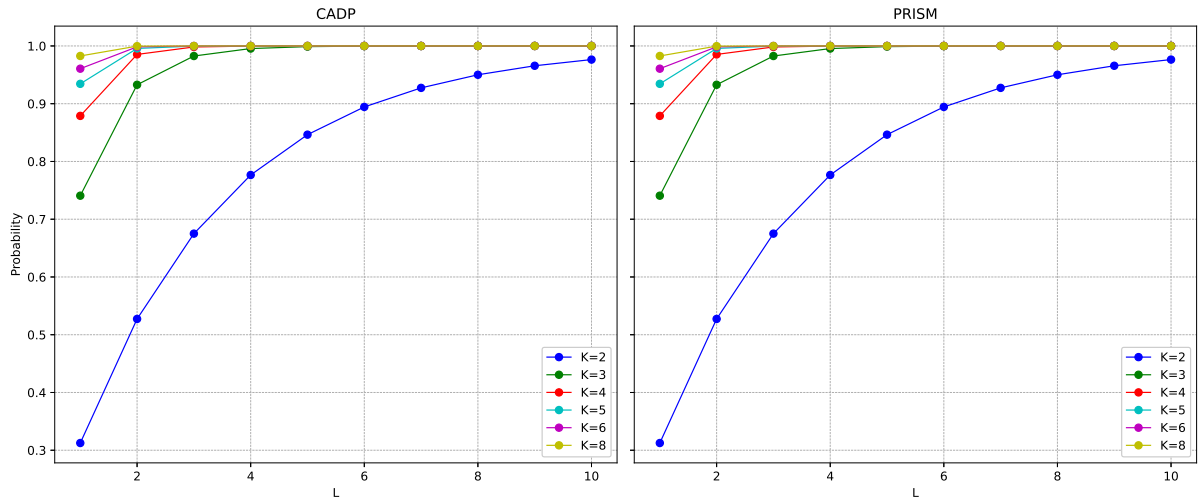


Figure 3: Table mapping the CADP and PRISM probabilities for the model with parameter $N = 5$

Table 4: Probability values by CADP and PRISM for $N = 5$

N	K	L	CADP probability value	PRISM probability value
5	2	1	0.3125	0.3125
5	2	2	0.527344	0.52734375
5	2	3	0.675049	0.675048828125
5	2	4	0.776596	0.7765960693359375
5	2	5	0.84641	0.846409797668457
5	2	6	0.894407	0.8944067358970642
5	2	7	0.927405	0.9274046309292316
5	2	8	0.950091	0.9500906837638468
5	2	9	0.965687	0.9656873450876446
5	2	10	0.97641	0.9764100497477557
5	3	1	0.740741	0.7407407407407387
5	3	2	0.932786	0.9327846364883379
5	3	3	0.982575	0.9825737946451223
5	3	4	0.995483	0.9954820949079933
5	3	5	0.99883	0.9988286912724397
5	3	6	0.999698	0.9996963273669253
5	3	7	0.999922	0.999921270058091
5	3	8	0.999981	0.9999795885335758
5	3	9	0.999996	0.9999947081383331
5	3	10	1	0.999998628035862
5	4	1	0.878906	0.87890625
5	4	2	0.985336	0.9853363037109375
5	4	3	0.998224	0.9982243180274963
5	4	4	0.999784	0.9997849760111421
5	4	5	0.999973	0.9999739619388492
5	4	6	0.999996	0.9999968469535325
5	4	7	0.999999	0.9999996181857798
5	4	8	0.999999	0.9999999537646866
5	4	9	0.999999	0.9999999944011939
5	4	10	0.999999	0.999999993220174
5	5	1	0.9344	0.9343999999999674
5	5	2	0.995697	0.9956966399999704
5	5	3	0.999718	0.9997176995839668
5	5	4	0.999981	0.9999814810926732
5	5	5	0.999999	0.9999987851596481
5	5	6	1	0.9999999203064366
5	5	7	1	0.999999994772068
5	5	8	1	0.9999999996570184
5	5	9	1	0.9999999999774722
5	5	10	1	0.9999999999984861
5	6	1	0.960648	0.9606481481480117
5	6	2	0.998451	0.9984514317556756
5	6	3	0.999939	0.9999390609717145
5	6	4	0.999998	0.9999976019362344
5	6	5	1	0.9999999056315987
5	6	6	1	0.9999999962862788
5	6	7	1	0.9999999998537179
5	6	8	1	0.9999999999941
5	6	9	1	0.999999999996296
5	6	10	1	0.999999999998415
5	8	1	0.982667	0.982666015625
5	8	2	0.9997	0.9996995329856873
5	8	3	0.999996	0.9999947917094687
5	8	4	1	0.9999999097195733
5	8	5	1	0.9999999984350856
5	8	6	1	0.999999999728808
5	8	7	1	0.99999999999543
5	8	8	1	0.99999999999997
5	8	9	1	1
5	8	10	1	1