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ME3011

Design Project Report

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Abstract

In machine design, every machine element must be calculated in a systematic matter. In this course, students are provided with essential skills to formulate almost every dimension manually, thus further improving their engineering skills before engaging the high-energy, fast-paced workforce.

When a machine element is being developed, it must satisfy some key engineering specifications such as being able to operate under designated lifespan, low cost and high efficiency. Other aspects are less important but also determined the overall design of the element include compactness, noise emission, appearance, etc.

To optimize the process of machine design, the general principles are considered as follows:

1. Identify the working principle and workload of the machine.
2. Formulate the overall working principle to satisfy the problem. Proposing feasible solutions and evaluating them to find the optimal design specifications.
3. Find force and moment diagram exerting on machine parts and characteristics of the workload.
4. Choose appropriate materials to make use of their properties and improve efficiency as well as reliability of individual elements.
5. Calculate dynamics, strength, safety factor, etc. to specify dimensions.
6. Design machine structure, parts to satisfy working condition and assembly.
7. Create presentation, instruction manual and maintenance.

In this report, I will design a fairly simple system to provide a concrete example of finishing all the tasks above.

Contents

List of Figures

List of Tables

Chapter 1

Design Problem

Nomenclature

C_a	number of shift daily, shifts	P	design power of the mixing tank, kW
K_{ng}	working days/year, days	T_1	working torque 1, $N \cdot m$
L	service life, years	T_2	working torque 2, $N \cdot m$
n	rotational velocity of the mixing tank, rpm	t_1	working time 1, s
		t_2	working time 2, s

1.1 Problem

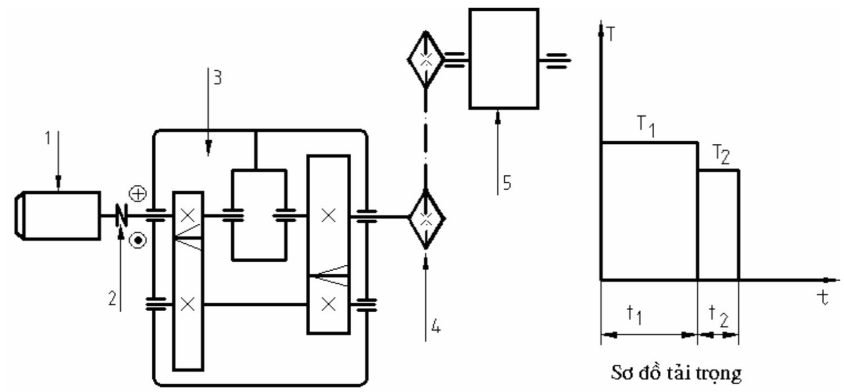

Figure 1.1: Working principle diagram and workload of the mixing machine: 1) electric motor, 2) elastic coupling, 3) two-stage coaxial helical speed reducer, 4) roller chain drive, 5) mixing tank (one-directional, light duty, operate 1 shift, 8 hours each)

1.2 Mixing machine parameters

From the parameters given in the document, we have:

$P = 7$ (kW)	$t_1 = 15$ (s)
$n = 65$ (rpm)	$t_2 = 11$ (s)
$L = 8$ (years)	$T_1 = T$ ($N \cdot m$)
$K_{ng} = 260$ (days)	$T_2 = 0.7T$ ($N \cdot m$)
$Ca = 1$ (shifts)	

1.3 Requirements

- 01 report.
- 01 assembly drawing.
- 01 detailed drawing.

1.4 Design problem

1. Decide the working power of the electric motor and transmission ratio of the system.
2. Calculate and design machine elements:
    (a) Calculate system drives (belt, chain or gear).
    (b) Calculate the elements in speed reducers (gears, lead screws).
    (c) Draw and calculate force diagram exerting on the transmission elements.
    (d) Calculate, design shafts and keys.
    (e) Choose bearings and couplings.
    (f) Choose machine bodies, fasteners and other elements.
3. Choose assembly tolerance.
4. Bibliography

Chapter 2

Choose Motor

Nomenclature

n_{sh}	rotational speed of shaft, rpm	u_{sys}	transmission ratio of the system
P_{mo}	calculated motor power to drive the system, kW	T_{mo}	motor torque, $\mathrm{N \cdot mm}$
P_{sh}	operating power of shaft, kW	T_{sh}	shaft torque, $\mathrm{N \cdot mm}$
P_w	operating power of the belt conveyor given a workload, kW	η_b	bearing efficiency
u_1	transmission ratio of quick stage	η_c	coupling efficiency
u_2	transmission ratio of slow stage	η_{ch}	chain drive efficiency
u_{ch}	transmission ratio of chain drive	η_{hg}	helical gear efficiency
u_h	transmission ratio of speed reducer	η_{sys}	efficiency of the system
		$_1$	subscript for shaft 1
		$_2$	subscript for shaft 2
		$_3$	subscript for shaft 3

Known parameters From Chapter 1, we know that:
$P = 7\,\mathrm{kW}$, $n = 65\,\mathrm{rpm}$
$T_1 = T$, $T_2 = 0.7T$
$t_1 = 15\,\mathrm{s}$, $t_2 = 11\,\mathrm{s}$

2.1 Choose motor for the mixing tank

The choice of motor will affect the entire system, so it is necessary to pick the right one.

Calculate system overall efficiency From Table 2.3 [1]:

- 1 elastic coupling which connects the motor and the speed reducer. $\eta_c = 1$
- 4 sealed rolling bearings. 3 of which belong to the speed reducer and the last one is used for the shaft of the mixing tank. $\eta_b = 0.99$
- 2 sealed pairs of helical gear drive which connect the shafts inside the speed reducer. $\eta_{hg} = 0.97$
- 1 sealed roller chain drive connecting the speed reduce and the mixing tank. $\eta_{ch} = 0.96$

Aggregate all efficiencies yields:

$$\eta_{sys} = \eta_c \eta_b^4 \eta_{hg}^2 \eta_{ch} = 1 \times 0.99^4 \times 0.97^2 \times 0.96 = 0.87$$

Calculate required power for operation The power P from Chapter 1 applies for systems with single loading input. In case of varying load each cycle, the equivalent power is calculated using Equation 2.13 [1]:

$$P_w = P\sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} = 7 \times \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15 + 11}} = 6.2\,(\mathrm{kW})$$

$$P_{mo} = \frac{P_w}{\eta_{sys}} = \frac{6.2}{0.87} = 7.14\,(\mathrm{kW})$$

Calculate n_{mo} Using Table 2.4 [1]:
$u_{ch} = 4, u_h = 10$
$u_{sys} = u_{ch} u_h = 40$
$n_{mo} = u_{sys} n = 2600$ (rpm)

Choose motor To work normally, the maximum operating power of the chosen motor must be no smaller than P_{mo}. In similar fashion, its rotational speed must also be no smaller than estimated n_{mo}.

Thus, from Table P1.3 [1], we choose motor 4A112M2Y3 which operates at 7.5 kW and 2922 rpm.
$\Rightarrow P_{mo} = 7.5\,(\mathrm{kW}), n_{mo} = 2922\,(\mathrm{rpm})$
Recalculating u_{sys} with the new P_{mo} and n_{mo}, we obtain:
$u_{sys} = \frac{n_{mo}}{n} = 44.95$
Assuming $u_h = const$:
$u_{ch} = \frac{u_{sys}}{u_h} = 4.5$

2.2 Calculate power, rotational speed and torque

Let us denote P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2 and P_{sh3}, n_{sh3} and T_{sh3} will be the transmitted parameters onto shaft 3. These notations will be used throughout the next chapters.

2.2.1 Power

$P_{ch} = \frac{P_w}{\eta_b} = 6.3\,(\mathrm{kW})$
$P_{sh3} = \frac{P_{ch}}{\eta_{ch}} = 6.52\,(\mathrm{kW})$
$P_{sh2} = \frac{P_{sh3}}{\eta_b \eta_{hg}} = 6.79\,(\mathrm{kW})$
$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} = 7.07\,(\mathrm{kW})$

2.2.2 Rotational speed

To minimize the speed reducer weight, we can use the following formula, see Equation 3.12 [1]:

$$u_1 = 0.7332 u_h^{0.6438}$$

Then,
$u_1 = 3.23$ and $u_2 = u_h / u_1 = 3.1$
$n_{sh1} = n_{mo} = 2922$ (rpm)
$n_{sh2} = n_{sh1} / u_1 = 905.02$ (rpm)
$n_{sh3} = n_{sh2} / u_2 = 292.20$ (rpm)

2.2.3 Torque

$T_{mo} = 9.55 \times 10^6 \times P_{mo}/n_{mo} = 23350.04$ (N · mm)
$T_{sh1} = 9.55 \times 10^6 \times P_{sh1}/n_{sh1} = 23106.95$ (N · mm)
$T_{sh2} = 9.55 \times 10^6 \times P_{sh2}/n_{sh2} = 71672.45$ (N · mm)
$T_{sh3} = 9.55 \times 10^6 \times P_{sh3}/n_{sh3} = 213175.22$ (N · mm)

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2	Shaft 3
P (kW)	7.5	7.07	6.79	6.52
u	-	1	3.23	3.1
n (rpm)	2922	2922	905.02	292.20
T (N · mm)	23350.04	23116.54	71672.45	213175.22

Table 2.1: Output specifications

Chapter 3

Chain Drive Design

3.1 Nomenclature

$[i]$	permissible impact times per second
$[s]$	permissible safety factor
$[P]$	permissible power, kW
$[\sigma_H]$	permissible contact stress, MPa
A	cross sectional area of chain hinge, mm^2
a	real center distance, mm
a_i	estimated center distance, mm
a_{max}	maximum center distance, mm
a_{min}	minimum center distance, mm
B	width between inner link plate, mm
d	chordal diameter, mm
d_a	addendum diameter, mm
d_f	dedendum diameter, mm
d_l	roller diameter, mm
d_O	pin diameter, mm
E	modulus of elasticity, MPa
F_0	sagging force, N
F_1	tight side tension force, N
F_2	slack side tension force, N
F_r	force on the shaft, N
F_t	effective peripheral force, N
F_v	centrifugal force, N
F_{vd}	contact force, N
i	impact times per second
K_d	weight distribution factor
k	overall factor
k_a	center distance and chain's length factor
k_{bt}	lubrication factor
k_c	rating factor
k_d	dynamic load factor
k_{dc}	chain tension factor
k_f	loosing factor
k_n	coefficient of rotational speed
k_r	number of tooth factor
k_x	chain weight factor
k_z	coefficient of number of teeth

k_0	arrangement of drive factor	v	instantaneous velocity along the chain, m/s
n	angular rotational speed, rpm	x	chain length in pitches, the number of links
n_{01}	experimental angular rotational speed, rpm	x_c	an even number of links
P_t	calculated power, kW	z	number of teeth of a sprocket
p	pitch, mm	z_{max}	maximum number of teeth of the driven sprocket
p_{max}	permissible sprocket pitch, mm		
Q	permissible load, N	σ_H	contact stress, MPa
q	mass per unit length, kg/m	$_1$	subscript for driving sprocket
s	safety factor	$_2$	subscript for driven sprocket

Known parameters From Chapter 1, we know that:
The chain type is roller.
$n_{sh3} = 292.2$ rpm.
$P_{ch} = 6.3$ kW, $u_{ch} = 4.5$.

3.2 Find p

The driving sprocket is connected to shaft 3, $n_1 = n_{sh3} = 292.2$ (rpm).

3.2.1 Calculate z

Since z_1 and z_2 is preferably an odd number, Equation 5.1 [1]:
$z_1 = 29 - 2u_{ch} = 21$
$z_2 = u_{ch}z_1 = 95 \leq 120$

3.2.2 Calculate $[P]$

Since 200 is the nearest to $n_{ch} = 292.2$ rpm, we choose $n_{01} = 200$ rpm, which is one of the experimental angular rotational speed values. Then, we can obtain the value $[P]$, Equation 5.3 [1]:

$$P_t = P_{ch}k_0k_ak_{dc}k_{bt}k_dk_ck_zk_n \leq [P]$$

where $k_z = 25/z_1 = 1.32$ and $k_n = n_{01}/n_1 = 1.02$.

Assuming good operational condition, $k_0 = k_a = k_{dc} = k_{bt} = 1$, $k_d = 1.25$, $k_c = 1$, see Table 5.6 [1]. Calculation yields $P_t = 6.38$ (kW) $\leq [P]$. Inspecting Table 5.5 [1] at column $n_{01} = 200$ rpm, choose the closet value $[P] = 11$ kW.

3.2.3 Determine p

Knowing $[P]$, we have $p = 25.4$ mm, Table 5.5. Consequently, $d_c = 7.95$ mm, $B = 22.61$ mm. Consulting Table 5.8, the pitch is indeed suitable.

3.3 Find a, x_c, and i

3.3.1 Find x_c

$a_{min} = 30p = 762$ (mm), $a_{max} = 50p = 1270$ (mm). Limiting the range of choice for a in $[a_{min}, a_{max}]$, we can approximate $a_i = 1000$ mm and find x_c:

$$x = \frac{2a_i}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a_i} = 140.26$$

Then, round x up to the nearest even number gives $x_c = 142$.

3.3.2 Find a

Using x_c to find the correct center distance, see Equation 5.13 [1]. In addition, it is recommended to loosing the chain an amount of $0.002 \div 0.004a$, which explains the coefficient 0.097 in the formula below:

$$a = \frac{0.097p}{4}\left[x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\left(\frac{z_2 - z_1}{\pi}\right)^2}\right] = 1019.99\ (\text{mm})$$

3.3.3 Find i

The permissible impact frequency is $[i] = 30$, Table 5.9 [1]. Calculating i gives:

$$i = \frac{z_1 n_1}{15x} = 2.92 < [i]$$

3.4 Strength of chain drive

In order to operate safely, the chain drive's safety factor must satisfy the following condition:

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \geq [s]$$

Rotational speed of the smaller sprocket is determined using the formula below:

$$v_1 = \frac{n_1 p z_1}{60000} = 2.6\ (\text{m/s})$$

Find k_d: Assuming moderate workload, choose $k_d = 1.2$.

Find F_t**,** F_v **and** F_0: Knowing p, it is easy to look up the values $Q = 56700$ N and $q = 2.6$ kg/m from Table 5.2 [1]:
$F_t = 10^3 P_{ch}/v_1 = 2410.48$ (N)
$F_v = qv_1^2 = 17.54$ (N)
$F_0 = 9.81 \times 10^{-3} k_f q a = 156.11$ (N)

Find k_f: Let the chain drive be parallel to the ground, we get $k_f = 6$.

Find $[s]$: The limit $[s] = 8.71$ is found using interpolation, Table 5.10 [1].

Replacing all the variables gives:

$$s = 18.49 \geq 8.71$$

which satisfies the condition.

3.5 Determine sprocket specifications and output force on the shaft

The following condition must be met, Equation 5.18 [1]:

$$\sigma_H = 0.47\sqrt{\frac{k_r(F_t k_d + F_{vd})E}{AK_d}} \leq [\sigma_H]$$

Since the chain drive only has one strand, $K_d = 1$.

Find $[\sigma_H]$ Quenched 45 steel is the material of use for the chain drive, which has HB210, $[\sigma_H] = 600$ (MPa) and $E = 2.1 \times 10^5$ (MPa), see Table 5.11 [1].

Find F_{vd} For 1-strand chain, $F_{vd} = 13 \times 10^{-7} n_1 p^3 = 6.22$ (N)

Find k_r Since z_1 is used to estimate k_r, $k_r = 0.47$.

Find E Assuming the sprockets and chain are made up from the same material (steel), $E = 2.1 \times 10^5$ MPa

Find A From the given parameters and value p, the area $A = 180\,\text{mm}^2$, Table 5.12 [1].

Knowing k_d and F_t, we get the result:

$$\sigma_H = 591.29\ (\text{MPa}) \leq 600\,\text{MPa}$$

which is satisfactory.

3.6 Force on shaft

Applying the following equations, see p.87 [1]:
$F_2 = F_0 + F_v = 173.65$ (N)
$F_1 = F_t + F_2 = 2584.13$ (N)
Choose $k_x = 1.15$ to obtain F_r, Equation 5.20 [1]:
$F_r = k_x F_t = 2772.05$ (N)

3.7 Other parameters

$d_1 = p/\sin\left(\frac{\pi}{z_1}\right) = 170.42$ (mm)
$d_2 = p/\sin\left(\frac{\pi}{z_2}\right) = 768.22$ (mm)
$d_{a1} = p\left(0.5 + \cot\frac{180}{z_1}\right) \approx 181.22$ (mm)
$d_{a2} = p\left(0.5 + \cot\frac{180}{z_2}\right) \approx 780.50$ (mm)
Look up to find $d_l = 15.88$ (mm), see Table 5.2 [1]:
$d_{f1} = d_1 - 2(0.502 d_l + 0.05) \approx 154.36$ (mm)
$d_{f2} = d_2 - 2(0.502 d_l + 0.05) \approx 752.16$ (mm)
In summary, we have the following table:

	driving	driven
$[P]$ (kW)	11	
a (mm)	1019.99	
B (mm)	22.61	
d (mm)	170.92	768.22
d_a (mm)	181.22	780.50
d_f (mm)	154.36	752.16
d_l (mm)	15.88	
d_O (mm)	7.95	
i	2.92	
p (mm)	25.4	
Q (N)	56700	
u_{ch}	4.5	
v (m/s)	2.6	
z	21	95

Table 3.1: Chain drive specifications

Chapter 4

Gearbox Design (Helix gears)

Nomenclature

$[\sigma]$	permissible stress, MPa
$[\sigma]_{max}$	permissible stress due to overloading, MPa
AG	accuracy grade of gear
a	center distance, mm
b	face width, mm
c	gear meshing rate
d	pitch circle, mm
d_a	addendum diameter, mm
d_b	base diameter, mm
d_f	deddendum diameter, mm
H	surface roughness, HB
K_d	coefficient of gear material
K	overall load factor
K_C	load placement factor
K_L	aging factor
K_v	dynamic load factor at meshing area
K_α	load distribution factor on gear teeth
K_β	load distribution factor on top land
K_{qt}	overloading factor

m_t	traverse module, mm	z_v	virtual number of teeth
m	root of fatigue curve in stress test	z_ε	meshing condition factor
m_n	normal module, mm	α	normal pressure angle. In TCVN 1065-71, $\alpha = 20°$
N_E	working cycle of equivalent tensile stress	α_t	traverse pressure angle, °
N_O	working cycle of bearing stress	ε_α	traverse contact ratio
S_F	safety factor of bending stress	ε_β	face contact ratio
S_H	safety factor of contact stress	β	helix angle, °
v	rotational velocity, m/s	β_b	base circle helix angle, °
Y_F	tooth shape factor	ψ_{ba}	width to shaft distance ratio
Y_R	surface roughness factor	ψ_{bd}	face width factor
Y_s	stress concentration factor	σ_b	ultimate strength, MPa
Y_β	helix angle factor	σ_{ch}	yield limit, MPa
Y_ε	contact ratio factor	σ^o_{lim}	permissible stress corresponding to working cycle, MPa
Z_R	surface roughness factor of the working's area	σ_{max}	stress due to overloading, MPa
Z_v	speed factor	$_1$	subscript for the pinion
z_H	contact surface's shape factor	$_2$	subscript for the driven gear
z_M	material's mechanical properties factor	$_F$	subscript relating to bending stress
z_{min}	minimum number of teeth corresponding to β	$_H$	subscript relating to contact stress
		$_q$	subscript for the quick transmission stage
		$_s$	subscript for the slow transmission stage
		$_w$	subscript for the value after correction

Known parameters From Chapter 1 and 2, we know that:
$L = 8$ years, $K_{ng} = 260$ days, $Ca = 1$ shift
$T_1 = T$, $T_2 = 0.7T$, $t_1 = 15$ s, $t_2 = 11$ s
$n_{sh1} = 2922$ rpm, $n_{sh2} = 905.02$ rpm, $n_{sh3} = 292.20$ rpm
$u_h = 10$, $u_1 = u_q = 3.23$, $u_2 = u_s = 3.1$

This chapter will increase readability by calculating both stages at the same time with the first one being quick stage and the latter is slow stage.

4.1 Choose material

Because all gears are the same in material and working hours except for their angular rotational speed, this section applies for both pairs.

The material of choice for the 2 pair of gears is steel 40X. The specifications are HB250, $\sigma_b = 850$ MPa, $\sigma_{ch} = 550$ MPa, see Table 6.1 [1].

From Table 6.2 [1], $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$.

Therefore, they have the same properties except for their surface roughness H, since $H_2 = H_1 - 10 \div 15$.

For the pinion, $H_1 = \text{HB}250 \Rightarrow \sigma^o_{Hlim1} = 570$ MPa, $\sigma^o_{Flim1} = 450$ MPa.

For the driven gear, $H_2 = \text{HB}240 \Rightarrow \sigma^o_{Hlim2} = 550$ MPa, $\sigma^o_{Flim2} = 432$ MPa.

In this part, distinguishing between 2 stages is unnecessary since the variables are material-dependent, which in this case all the gears have identical choice of material. Therefore, unless otherwise specified, a single subscript 1 or 2 indicates the variable applies for both stages

4.2 Calculate $[\sigma_H]$ and $[\sigma_F]$

The permissible stresses are calculated as follows, see Equation 6.1 and 6.2 [1]:

$$[\sigma_H] = \frac{\sigma^o_{Hlim}}{S_H} Z_R Z_v K_{xH} K_{HL}$$

$$[\sigma_F] = \frac{\sigma^o_{Flim}}{S_F} Y_R Y_s K_{xF} K_{FL}$$

Calculate working cycle of bearing stress The stress is found using the formula below, see Equation 6.5 [1]:
$N_{HO1} = 30H_1^{2.4} = 17067789.40$ (cycles).
$N_{HO2} = 30H_2^{2.4} = 15474913.67$ (cycles).

Calculate working cycle of equivalent tensile stress Since $H_1, H_2 \leq$ HB350, $m_H = 6$, $m_F = 6$. Also, both pairs of gears are meshed indefinitely, which makes $c = 1$. From working condition, we calculate:
$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times CaK_{ng}L = 16640$ (hours)
Find N_{HE} and N_{FE} using equation 6.7 and 6.8 [1]:

$$N_{HE1q} = 60n_{sh1}cL_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] = 2.11 \times 10^9 \text{ (cycles)}$$

$$N_{HE2q} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] = 0.65 \times 10^9 \text{ (cycles)}$$

$$N_{FE1q} = 60n_{sh1}cL_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] = 1.83 \times 10^9 \text{ (cycles)}$$

$$N_{FE2q} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] = 0.57 \times 10^9 \text{ (cycles)}$$

$$N_{HE1s} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] = 0.65 \times 10^9 \text{ (cycles)}$$

$$N_{HE2s} = 60n_{sh3}cL_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] = 0.21 \times 10^9 \text{ (cycles)}$$

$$N_{FE1s} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] = 0.57 \times 10^9 \text{ (cycles)}$$

$$N_{FE2s} = 60n_{sh3}cL_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] = 0.18 \times 10^9 \text{ (cycles)}$$

Calculate aging factor For steel, $N_{FO1} = N_{FO2} = 4 \times 10^6$ (MPa). Equation 6.3 and 6.4 [1] gives (if the factors are smaller than 1, round them up to 1, see p.94 [1]):
$K_{HL1q} = \sqrt[m_H]{N_{HO1}/N_{HE1q}} = 0.45 < 1 \Rightarrow K_{HL1q} = 1$
$K_{HL2q} = \sqrt[m_H]{N_{HO2}/N_{HE2q}} = 0.54 < 1 \Rightarrow K_{HL2q} = 1$
$K_{FL1q} = \sqrt[m_F]{N_{FO1}/N_{FE1q}} = 0.36 < 1 \Rightarrow K_{FL1q} = 1$
$K_{FL2q} = \sqrt[m_F]{N_{FO2}/N_{FE2q}} = 0.44 < 1 \Rightarrow K_{FL2q} = 1$
$K_{HL1s} = \sqrt[m_H]{N_{HO1}/N_{HE1s}} = 0.54 < 1 \Rightarrow K_{HL1s} = 1$
$K_{HL2s} = \sqrt[m_H]{N_{HO2}/N_{HE2s}} = 0.65 < 1 \Rightarrow K_{HL2s} = 1$
$K_{FL1s} = \sqrt[m_F]{N_{FO1}/N_{FE1s}} = 0.44 < 1 \Rightarrow K_{FL1s} = 1$
$K_{FL2s} = \sqrt[m_F]{N_{FO2}/N_{FE2s}} = 0.53 < 1 \Rightarrow K_{FL2s} = 1$

Calculate $[\sigma_H]$, $[\sigma_{F1}]$, $[\sigma_{F2}]$ Since the motor works in one direction, $K_{FC} = 1$, which means all K factors are equal to 1 and we can safely skip them.

For initial estimation, assume $Z_R Z_v K_{xH} = 1$ and $Y_R Y_s K_{xF} = 1$, we obtain the permissible stresses (notice they are equal in both quick and slow transmission stages):

$[\sigma_{H1}] = \sigma^o_{Hlim1}/S_H = 518.18$ (MPa)
$[\sigma_{H2}] = \sigma^o_{Hlim2}/S_H = 500$ (MPa)
$[\sigma_{F1}] = \sigma^o_{Flim1}/S_F = 257.14$ (MPa)
$[\sigma_{F2}] = \sigma^o_{Flim2}/S_F = 246.86$ (MPa)

The mean permissible contact stress must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. In this case, it is $1.25[\sigma_{H2}]$ or 625 MPa:

$$[\sigma_H] = \frac{1}{2}\left([\sigma_{H1}] + [\sigma_{H2}]\right) = 509.09\ \text{(MPa)} \le 625$$

which satisfy the condition.

In case of overloading, the permissible contact and bending stresses are calculated as follows:
$[\sigma_H]_{max} = 2.8\sigma_{ch} = 1540$ (MPa)
$[\sigma_F]_{max} = 0.8\sigma_{ch} = 440$ (MPa)

4.3 Determine basic specifications of the Transmission system

4.3.1 Determine basic parameters

Figure 1.1 shows both pairs are helical, which gives $K_a = 43$, Table 6.5 [1]. Also, the entire speed reducer has asymmetrical design, resulting in $\psi_{ba} = 0.4$, Table 6.6 [1]. This value is then used in Equation 6.16 [1] to find ψ_{bd}:
$\psi_{bdq} = 0.53\psi_{ba}(u_q + 1) = 0.90$
$\psi_{bds} = 0.53\psi_{ba}(u_s + 1) = 0.87$

Using interpolation, we approximate the factors, Table 6.7 [1]:
$K_{H\beta q} = 1.06$, $K_{F\beta q} = 1.14$
$K_{H\beta s} = 1.09$, $K_{F\beta s} = 1.19$
Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a using Equation 6.15a [1]. Also, we round up the center distance to the nearest multiple of 5 for small production, p.99 [1]:

$$a_q = K_a(u_q + 1)\sqrt[3]{\frac{T_{sh1}K_{H\beta q}}{[\sigma_H]^2 u_q \psi_{ba}}} = 76.06\ \text{(mm)} \Rightarrow a_q = 85\ \text{mm}$$

$$a_s = K_a(u_s + 1)\sqrt[3]{\frac{T_{sh2}K_{H\beta s}}{[\sigma_H]^2 u_s \psi_{ba}}} = 110.00\ \text{(mm)} \Rightarrow a_s = 120\ \text{mm}$$

4.3.2 Determine gear meshing parameters

In this section, we will find all necessary specifications of a gear. Due to complexity and maintenance, the undercutting process will be omitted throughout the calculations, that is, it is not factored in any variables of the gear drives.

Find m Using Equation 6.17 [1] and Table 6.8 [1], we determine m for each pair of gears:
$m_{tq} = (0.01 \div 0.02)a_q = 0.85 \div 1.7$ (mm) $\Rightarrow m_{tq} = 1.5$ mm
$m_{ts} = (0.01 \div 0.02)a_s = 1.2 \div 2.4$ (mm) $\Rightarrow m_{ts} = 2$ mm

Find z_1, z_2, b Arbitrarily choose $\beta = 20°$ in the range $8 \div 20°$. Combining Equation 6.18 and 6.20 [1] to calculate z_1. Then, find z_2 and b.

$$z_{1q} = \frac{2a_q\cos\beta}{m_{tq}(u_q + 1)} = 25.18 \Rightarrow z_{1q} = 26$$

$z_{2q} = u_q z_{1q} = 83.95 \Rightarrow z_{2q} = 84$
$b_q = \psi_{ba}a_q = 34.00$ (mm)

$$z_{1s} = \frac{2a_s\cos\beta}{m_{ts}(u_s + 1)} = 27.52 \Rightarrow z_{1s} = 28$$

$z_{2s} = u_s z_{1s} = 86.72 \Rightarrow z_{2s} = 87$
$b_s = \psi_{ba} a_s = 48.00\,(\text{mm})$

Correct β The helix angles are corrected to compensate for rounding center distances and number of teeth, Equation 6.32 [1]:

$$\beta_{wq} = \frac{m_{tq}(z_{1q} + z_{2q})}{2a_q} = 13.93^\circ$$
$$\beta_{ws} = \frac{m_{ts}(z_{1s} + z_{2s})}{2a_s} = 16.6^\circ$$

4.3.3 Basic parameters

For quick stage transmission:

$$d_{1q} = \frac{m_{tq} z_{1q}}{\cos\beta_{wq}} = 40.18\,(\text{mm})$$
$$d_{2q} = \frac{m_{tq} z_{2q}}{\cos\beta_{wq}} = 129.82\,(\text{mm})$$
$$d_{a1q} = d_{1q} + 2m_{tq} = 43.18\,(\text{mm})$$
$$d_{a2q} = d_{2q} + 2m_{tq} = 132.82\,(\text{mm})$$
$$d_{f1q} = d_{1q} - 2.5m_{tq} = 36.43\,(\text{mm})$$
$$d_{f2q} = d_{2q} - 2.5m_{tq} = 126.07\,(\text{mm})$$
$$d_{b1q} = d_{1q}\cos\alpha = 37.76\,(\text{mm})$$
$$d_{b2q} = d_{2q}\cos\alpha = 121.99\,(\text{mm})$$
$$\alpha_{tq} = \arctan\frac{\tan\alpha}{\cos\beta_{wq}} = 20.56^\circ$$
$$v_{1q} = \frac{\pi d_{1q} n_{sh1}}{60000} = 6.15\,(\text{m/s})$$

For slow stage transmission:

$$d_{1s} = \frac{m_{ts} z_{1s}}{\cos\beta_{ws}} = 58.43\,(\text{mm})$$
$$d_{2s} = \frac{m_{ts} z_{2s}}{\cos\beta_{ws}} = 181.57\,(\text{mm})$$
$$d_{a1s} = d_{1s} + 2m_{ts} = 62.43\,(\text{mm})$$
$$d_{a2s} = d_{2s} + 2m_{ts} = 185.57\,(\text{mm})$$
$$d_{f1s} = d_{1s} - 2.5m_{ts} = 53.43\,(\text{mm})$$
$$d_{f2s} = d_{2s} - 2.5m_{ts} = 176.57\,(\text{mm})$$
$$d_{b1s} = d_{1s}\cos\alpha = 54.91\,(\text{mm})$$
$$d_{b2s} = d_{2s}\cos\alpha = 170.62\,(\text{mm})$$
$$\alpha_{ts} = \arctan\frac{\tan\alpha}{\cos\beta_{ws}} = 20.80^\circ$$
$$v_{1s} = \frac{\pi d_{1s} n_{sh2}}{60000} = 2.77\,(\text{m/s})$$

4.4 Stress analysis

4.4.1 Correct $[\sigma_H]$, $[\sigma_{F1}]$ and $[\sigma_{F2}]$

In reality, the factors of Equation 6.1 and 6.2 [1] do not equal 1. As a result, we will try to approximate them given the values calculated above:

$$[\sigma_{Hw}] = [\sigma_H] Z_R Z_V K_{xH}$$
$$[\sigma_{Fw}] = [\sigma_F] Y_R Y_S K_{xF}$$

Assuming smooth surface condition, $Z_R = 1$.
All gears have low surface hardness:
$Z_{vq} = 0.85 v_{1q}^{0.1} = 1.02$
$Z_{vs} = 0.85 v_{1s}^{0.1} = 0.94$
All gears have small addendum, $K_{xH} = 1$.
The gears are properly polished, which means $Y_R = 1.2$
$Y_{sq} = 1.08 - 0.0695\ln(m_{tq}) = 1.05$
$Y_{ss} = 1.08 - 0.0695\ln(m_{ts}) = 1.03$
Since $d_{a1}, d_{a2} \leq 400\,(\text{mm})$, $K_{xF} = 1$. Then, multiplying all variables yields:

$[\sigma_{Hwq}] = 518.90\,(\text{MPa})$ $[\sigma_{Hws}] = 479.12\,(\text{MPa})$
$[\sigma_{Fw1q}] = 324.56\,(\text{MPa})$ $[\sigma_{Fw1s}] = 318.39\,(\text{MPa})$
$[\sigma_{Fw2q}] = 311.58\,(\text{MPa})$ $[\sigma_{Fw2s}] = 305.66\,(\text{MPa})$

4.4.2 Contact stress analysis

The contact stress applied on a gear surface must satisfy Equation 6.33 [1].

Find z_M According to Table 6.5 [1], $z_M = 274$.

Find z_H Applying Equation 6.34 [1] and 6.35 [1]:

$$\beta_{bq} = \arctan\left(\cos\alpha_{tq}\tan\beta_{wq}\right) = 13.08^\circ \Rightarrow z_{Hq} = \sqrt{2\frac{\cos\beta_{bq}}{\sin(2\alpha_{tq})}} = 1.72$$

$$\beta_{bs} = \arctan\left(\cos\alpha_{ts}\tan\beta_{ws}\right) = 15.57^\circ \Rightarrow z_{Hs} = \sqrt{2\frac{\cos\beta_{bs}}{\sin(2\alpha_{ts})}} = 1.70$$

Find z_ε Obtaining z_ε through calculations:

$$\varepsilon_{\alpha q} = \frac{\sqrt{d_{a1q}^2 - d_{b1q}^2} + \sqrt{d_{a2q}^2 - d_{b2q}^2} - 2a_q\sin\alpha_{tq}}{2\pi m_{tq}\dfrac{\cos\alpha_{tq}}{\cos\beta_q}} = 1.52$$

$$\varepsilon_{\beta q} = b_q\frac{\sin\beta_q}{m_{tq}\pi} > 1 \Rightarrow z_{\varepsilon q} = \varepsilon_{\alpha q}^{-0.5} = 0.81$$

$$\varepsilon_{\alpha s} = \frac{\sqrt{d_{a1s}^2 - d_{b1s}^2} + \sqrt{d_{a2s}^2 - d_{b2s}^2} - 2a_s\sin\alpha_{ts}}{2\pi m_{ts}\dfrac{\cos\alpha_{ts}}{\cos\beta_s}} = 1.43$$

$$\varepsilon_{\beta s} = b_s\frac{\sin\beta_s}{m_{ts}\pi} > 1 \Rightarrow z_{\varepsilon s} = \varepsilon_{\alpha s}^{-0.5} = 0.84$$

Find K_H and K_F We find K_H, K_F using Equation 6.39 and 6.45 [1]:

From Table 6.13 [1]:

$v_{1q} \leq 10\,(\mathrm{m/s}) \Rightarrow \mathrm{AG}_q = 8$

$v_{1s} \leq 4\,(\mathrm{m/s}) \Rightarrow \mathrm{AG}_s = 9$

From Table P2.3 [1], using interpolation, we approximate:

$K_{Hvq} = 1.06$, $K_{Fvq} = 1.17$

$K_{Hvs} = 1.04$, $K_{Fvs} = 1.10$

From Table 6.14 [1], using interpolation, we approximate:

$K_{H\alpha q} = 1.1$, $K_{F\alpha q} = 1.29$

$K_{H\alpha s} = 1.13$, $K_{F\alpha s} = 1.37$

Knowing the results of $K_{H\beta}$ from previous section, multiply all the values to obtain K_H:

$\Rightarrow K_{Hq} = 1.24$, $K_{Fq} = 1.73$

$\Rightarrow K_{Hs} = 1.28$, $K_{Fs} = 1.79$

Find σ_H After calculating z_M, z_H, z_ε, K_H, we get the following result:

For quick stage with transmission ratio u_q and input torque T_{sh1}:

$$\sigma_{Hq} = z_M z_{Hq} z_{\varepsilon q}\sqrt{2T_{sh1}K_{Hq}\frac{u_q+1}{b_q u_q d_{1q}^2}} \leq [\sigma_{Hwq}]$$

$$\sigma_{Hq} = 447.07\,(\mathrm{MPa}) \leq 518.90$$

For slow stage with transmission ratio u_s and input torque T_{sh2}:

$$\sigma_{Hs} = z_M z_{Hs} z_{\varepsilon s}\sqrt{2T_{sh3}K_{Hs}\frac{u_s+1}{b_s u_s d_{1s}^2}} \leq [\sigma_{Hws}]$$

$$\sigma_{Hs} = 476.18\,(\mathrm{MPa}) \leq 479.12$$

4.4.3 Bending stress analysis

For safety reasons, Equation 6.43 and 6.44 [1] must be met for both pairs of gears.

Find Y_ε Using ε_α calculated in the previous section, we find Y_ε:
$Y_{\varepsilon q} = \varepsilon_{\alpha q}^{-1} = 0.66$
$Y_{\varepsilon s} = \varepsilon_{\alpha s}^{-1} = 0.70$

Find Y_β The value of Y_β is calculated using the equation on p.108 [1]:
$Y_{\beta q} = 1 - \dfrac{\beta_{wq}}{140} = 0.90$
$Y_{\beta s} = 1 - \dfrac{\beta_{ws}}{140} = 0.88$

Find Y_F Using the formula $z_v = z\cos^{-3}(\beta_w)$ and Table 6.18 [1]:
$z_{v1q} = z_{1q}\cos^{-3}(\beta_{wq}) = 28.44 \Rightarrow Y_{F1q} = 3.83$
$z_{v2q} = z_{2q}\cos^{-3}(\beta_{wq}) = 91.87 \Rightarrow Y_{F2q} = 3.60$
$z_{v1s} = z_{1s}\cos^{-3}(\beta_{ws}) = 31.81 \Rightarrow Y_{F1s} = 3.78$
$z_{v2s} = z_{2s}\cos^{-3}(\beta_{ws}) = 98.85 \Rightarrow Y_{F2s} = 3.60$

Find K_F The value of K_F has already been found in the previous section.

Find σ_F Replacing the normal module in Equation 6.43 [1] with $m_n = m_t\cos\beta_w$, substituting all the values yields:

For quick stage with input torque T_{sh1}:

$$\begin{aligned}
\sigma_{F1q} &= 2\frac{T_{sh1}K_{Fq}Y_{\varepsilon q}Y_{\beta q}Y_{F1q}}{b_q d_{1q} m_{tq}\cos\beta_{wq}} &\leq [\sigma_{Fw1q}] \\
\sigma_{F2q} &= \frac{\sigma_{F1q}Y_{F2q}}{Y_{F1q}} &\leq [\sigma_{Fw2q}]
\end{aligned}$$

gives the results:

$$\begin{aligned}
\sigma_{F1q} &= 91.48\ (\text{MPa}) \leq 324.56\,\text{MPa} \\
\sigma_{F2q} &= 86.05\ (\text{MPa}) \leq 311.58\,\text{MPa}
\end{aligned}$$

For slow stage with input torque T_{sh2}:

$$\begin{aligned}
\sigma_{F1s} &= 2\frac{T_{sh2}K_{Fs}Y_{\varepsilon s}Y_{\beta s}Y_{F1s}}{b_s d_{1s} m_{ts}\cos\beta_{ws}} &\leq [\sigma_{Fw1s}] \\
\sigma_{F2s} &= \frac{\sigma_{F1s}Y_{F2s}}{Y_{F1s}} &\leq [\sigma_{Fw2s}]
\end{aligned}$$

gives the results:

$$\begin{aligned}
\sigma_{F1s} &= 111.83\ (\text{MPa}) \leq 318.39\,\text{MPa} \\
\sigma_{F2s} &= 106.47\ (\text{MPa}) \leq 305.66\,\text{MPa}
\end{aligned}$$

which is well below the yielding strength.

4.4.4 Overloading analysis

Inspecting Table P1.3 [1], the motor we chose in Chapter 2 has $K_{qt} = 2.2$. Using the values of $[\sigma_H]_{max}$ and $[\sigma_F]_{max}$ calculated in previous section combined with Equation 6.48 and 6.49 [1], we are able to verify the stresses are below overloading limits.

For quick stage transmission:

$$\begin{aligned}
\sigma_{Hmaxq} &= \sigma_{Hq}\sqrt{K_{qt}} = 663.11\ (\text{MPa}) \leq 1540\,\text{MPa} \\
\sigma_{F2maxq} &= \sigma_{F2q}K_{qt} = 189.31\ (\text{MPa}) \leq 440\,\text{MPa} \\
\sigma_{F1maxs} &= \sigma_{F1s}K_{qt} = 246.03\ (\text{MPa}) \leq 440\,\text{MPa}
\end{aligned}$$

For slow stage transmission:

$$\sigma_{Hmaxs} = \sigma_{Hs}\sqrt{K_{qt}} = 706.29\,(\text{MPa}) \le 1540\,\text{MPa}$$
$$\sigma_{F1maxq} = \sigma_{F1q}K_{qt} = 201.25\,(\text{MPa}) \le 440\,\text{MPa}$$
$$\sigma_{F2maxs} = \sigma_{F2s}K_{qt} = 234.24\,(\text{MPa}) \le 440\,\text{MPa}$$

which satisfy the conditions. In summary, we have the following table:

	pinion	driving gear
a_w (mm)	100	
b (mm)	50	
m (mm)	1.5	
d_w (mm)	33.33	166.67
d_a (mm)	37.23	168.77
d_f (mm)	30.48	162.02
d_b (mm)	31.32	156.62
u_{hg}	5	
v (m/s)	5	
z	21	105
α_{tw} (°)	20.65	
β_w (°)	19.09	

Table 4.1: Gearbox specifications

Chapter 5

Shaft Design

Nomenclature

$[s]$	permissible safety factor
$[\sigma]$	permissible static strength, MPa
$[\tau]$	permissible torsion, MPa
a_w	shaft distance, mm
b_O	rolling bearing width, mm
cb	role of gear on the shaft (active or passive)
cq	rotational direction of the shaft
d	base shaft diameter, mm
d_w	gear diameter, mm
F_a	axial force, N
F_r	radial force, N
F_t	tangential force, N
F	applied force, N
h_n	distance between bearing lid and bolt, mm
hr	tooth direction
K_x	surface tension concentration factor
K_y	diminish factor
K_σ	combined influence factor in tension
K_τ	combined influence factor in shear
$\tilde{k}_1$	distance between elements, mm
$\tilde{k}_2$	distance between bearing surface and inner walls of the gearbox, mm

$\tilde{k}_3$	distance between element surface and bearing lid, mm
k_σ	fatigue stress concentration factor in tension
k_τ	fatigue stress concentration factor in shear
l	length (general), mm
l_m	hub length (general), mm
M	moment at the cross section, N · mm
M_e	equivalent moment, N · mm
l_m	hub diameter, mm
q	standardized coefficient of shaft diameter
R	reaction force, N
r	shoulder fillet radius, mm
$\bar{r}$	position of applied force on the shaft, mm
S	length defined by table (6.1), mm
s	calculated safety factor
s_σ	safety factor in tensile stress
s_τ	safety factor in shear stress
W	section modulus, mm^3
W_O	polar section modulus, mm^3
α_{tw}	traverse meshing angle, °
β	helix angle, °
ψ_σ	mean stress influence factor
ψ_τ	mean shear influence factor
σ_{-1}	endurance limit at stress ratio of -1, MPa
σ_a	tensile stress amplitude, MPa
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
σ_m	mean tensile stress, MPa
σ_{td}	static strength, MPa
τ_{-1}	endurance limit at shear ratio of -1, MPa
τ_a	shear stress amplitude, MPa
τ_m	mean shear stress, MPa
$_1$	subscript for shaft 1
$_2$	subscript for shaft 2
$_3$	subscript for shaft 3
$_{max}$	subscript for maximum value
$_{sh1}$	subscript for shaft 1
$_{sh2}$	subscript for shaft 2
$_x$	subscript for x-axis
$_y$	subscript for y-axis
$_z$	subscript for z-axis

Known parameters From Chapter 1, 2, 3 and 4, we know that:
T_{sh1} $n_{sh3} = 292.2\,\mathrm{rpm}$.
$P_{ch} = 6.3\,\mathrm{kW}, u_{ch} = 4.5$.

5.1 Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows: $S \leq 100\,(\mathrm{mm})$, HB260, $\sigma_b = 850\,(\mathrm{MPa})$, $\sigma_{ch} = 650\,(\mathrm{MPa})$.

5.2 Transmission Design

5.2.1 Load on shafts

The subscript convention of the book will be used in this chapter, see p.186 [1]. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for counting the number of machine elements on the subsequent shaft:

- On shaft 1, the motor is labeled 1 and the pinion is labeled 2.
- On shaft 2, the driven gear is labeled 1 and the pinion is labeled 2.
- On shaft 3, the driven gear is labeled 1 and the driving sprocket is labeled 2.

Therefore, we obtain:
$\bar{r}_{12} = -d_{12}/2 =$ (mm), $\mathrm{hr}_{12} = -1$, $\mathrm{cb}_{12} = +1$, $\mathrm{cq}_{12} = -1$
$\bar{r}_{21} = +d_{21}/2 =$ (mm), $\mathrm{hr}_{21} = +1$, $\mathrm{cb}_{21} = -1$, $\mathrm{cq}_{21} = +1$
$\bar{r}_{22} = +d_{22}/2 =$ (mm), $\mathrm{hr}_{22} = +1$, $\mathrm{cb}_{22} = +1$, $\mathrm{cq}_{22} = +1$
$\bar{r}_{31} = +d_{31}/2 =$ (mm), $\mathrm{hr}_{31} = -1$, $\mathrm{cb}_{31} = -1$, $\mathrm{cq}_{31} = -1$

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: , $\beta_w = 13.59°$, $d_{w12} \approx 41.67$ (mm)

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 2402.28 \text{ (N)} \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha}{\cos\beta_w} \approx 925.46 \text{ (N)} \\ F_{a12} = F_{a21} = F_{t12}\tan\beta_w \approx 580.75 \text{ (N)} \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\mathrm{cq}_1\mathrm{cb}_{12}F_{t12} \approx -2402.28 \text{ (N)} \\ F_{y12} = -\dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\dfrac{\tan\alpha}{\cos\beta_w}F_{t12} \approx 925.46 \text{ (N)} \\ F_{z12} = \mathrm{cq}_1\mathrm{cb}_{12}\mathrm{hr}_{12}F_{t12}\tan\beta_w \approx 580.75 \text{ (N)} \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\mathrm{cq}_2\mathrm{cb}_{21}F_{t21} \approx 2402.28 \text{ (N)} \\ F_{y21} = -\dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\dfrac{\tan\alpha_{tw}}{\cos\beta_w}F_{t21} \approx -925.46 \text{ (N)} \\ F_{z21} = \mathrm{cq}_2\mathrm{cb}_{21}\mathrm{hr}_{21}F_{t21}\tan\beta_w \approx -580.75 \text{ (N)} \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2678.96$ (N) (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22}\cos 210° \approx -2320.05 \text{ (N)} \\ F_{y22} = F_{r22}\sin 210° \approx -1339.48 \text{ (N)} \end{cases}$$

5.2.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques T_{sh1} and T_{sh2}, respectively, $[\tau_1] = 15$ (MPa) and $[\tau_2] = 30$ (MPa). Using equation (10.9), we can approximate the base shaft diameters d_1 and d_2:

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 25.55 \text{ (mm)}$$
$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 34.1 \text{ (mm)}$$

Recall that our motor is 4A160M2Y3, inspecting table P1.7 we obtain the motor's output shaft diameter is 42 (mm). According to the recommendations on p.189, we limit the chosen range of $d_1 \geq (0.8 \div 1.2) \times 42$ (mm). For d_2, the chosen range must be around $(0.3 \div 0.35) \times a_w$ (mm). Thus, $d_1 = 35$ (mm), $d_2 = 40$ (mm). Consulting table (10.2) gives $b_{O1} \approx 21$ (mm) and $b_{O2} \approx 23$ (mm)

5.2.3 Identify the distance between bearings and applied forces

In this section, we will find all the parameters in Figure 5.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.
Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 = 45$ (mm), $l_{m23} = l_{m22} = 1.5d_2 = 52.5$ (mm), where l_{m22} is the chain hub.

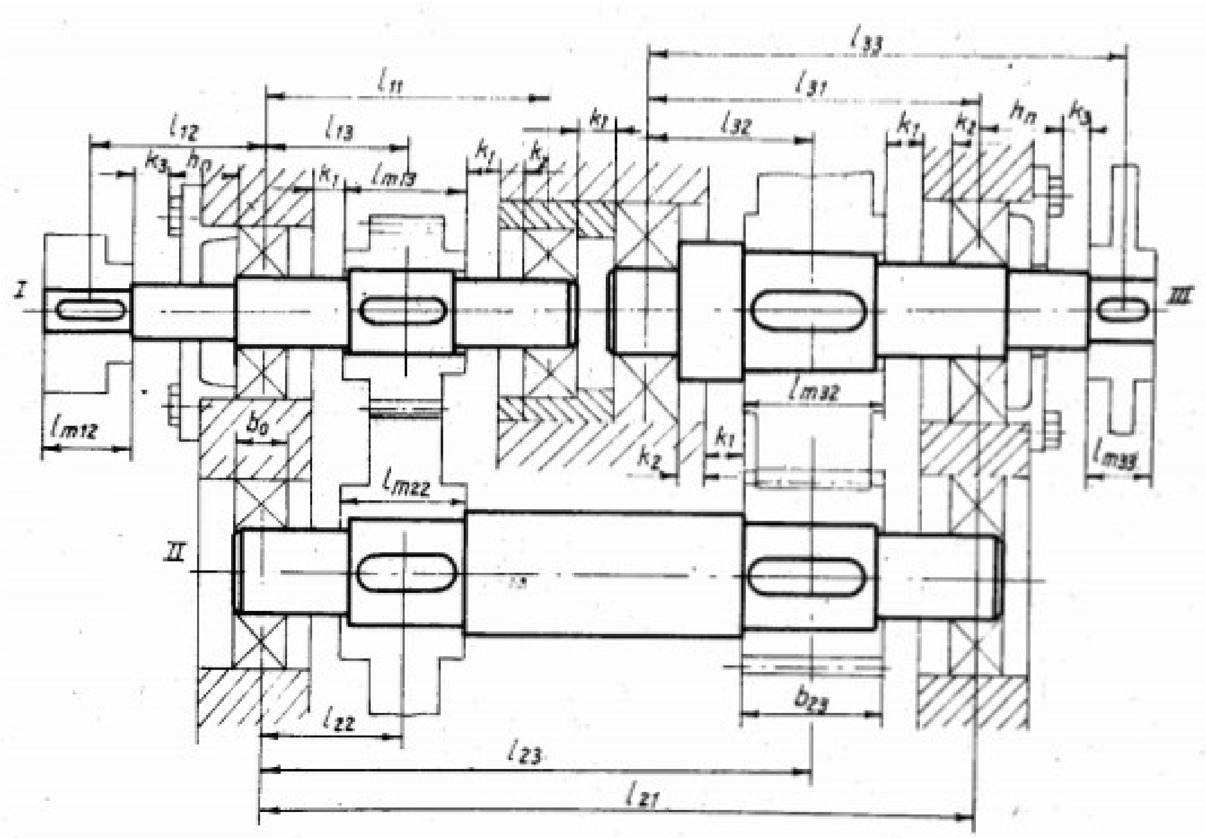

Figure 5.1: Shaft design and its dimensions

From table (10.3), we choose $\tilde{k}_1 = 10$ (mm), $\tilde{k}_2 = 8$ (mm), $\tilde{k}_3 = 15$ (mm), $h_n = 18$ (mm). This parameters apply for both shafts in the system.

Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:

On shaft 1:

$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + \tilde{k}_3 + h_n\right] = -69.75$ (mm)

$l_{13} = 0.5(l_{m13} + b_{O1}) + \tilde{k}_1 + \tilde{k}_2 = 54.75$ (mm)

$l_{11} = 2l_{13} = 109.5$ (mm)

On shaft 2:

$l_{22} = -l_{c22} = -\left[0.5(l_{m22} + b_{O2}) + \tilde{k}_3 + h_n\right] = -74.5$ (mm)

$l_{23} = 0.5(l_{m23} + b_{O2}) + \tilde{k}_1 + \tilde{k}_2 = 59.5$ (mm)

$l_{21} = 2l_{23} = 119$ (mm)

5.2.4 Determine shaft diameters and lengths

Find reaction forces From the diagram, we solve for the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F_i} &= 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} &= 0 \end{cases}$$

we obtain the results:

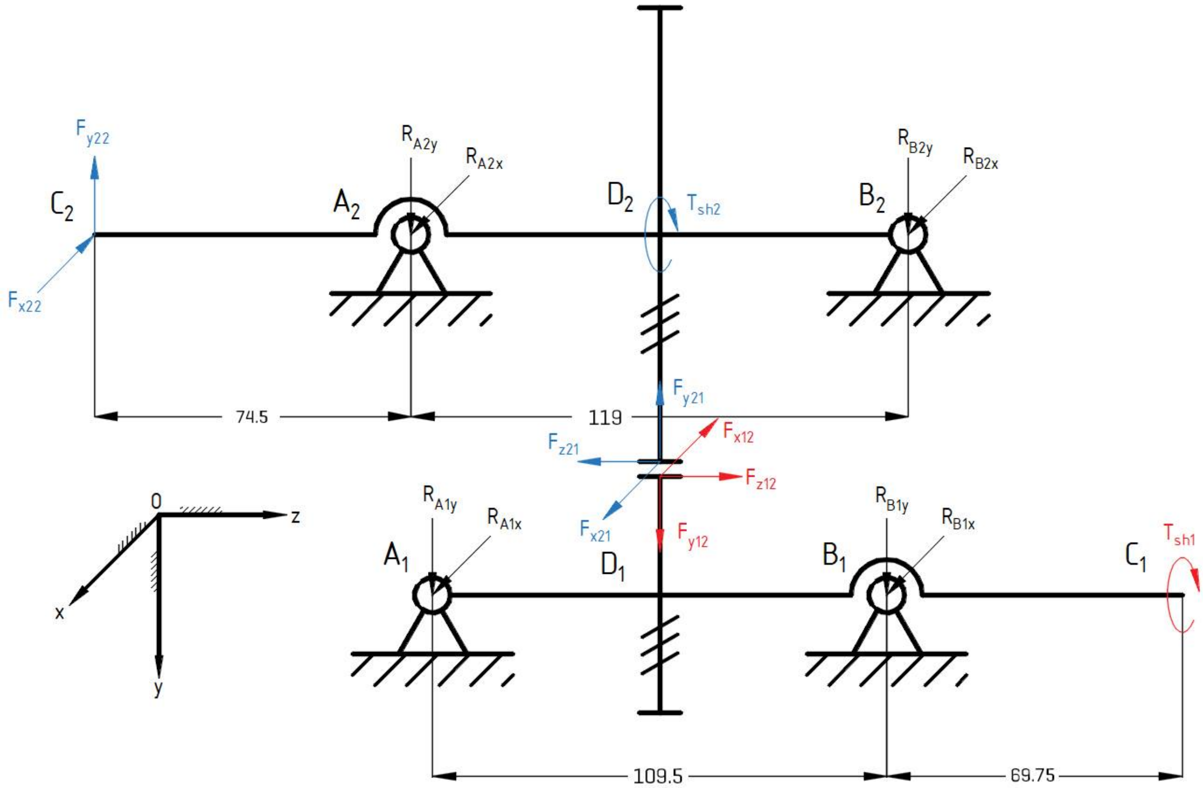

Figure 5.2: Force analysis of 2 shafts

$$\begin{cases} R_{A1x} \approx 1201.14\,(\mathrm{N}) \\ R_{A1y} \approx -352.24\,(\mathrm{N}) \\ R_{B1x} \approx 1201.14\,(\mathrm{N}) \\ R_{B1y} \approx -573.22\,(\mathrm{N}) \end{cases} \qquad \begin{cases} R_{A2x} \approx 943.15\,(\mathrm{N}) \\ R_{A2y} \approx 3668.4\,(\mathrm{N}) \\ R_{B2x} \approx -2005.96\,(\mathrm{N}) \\ R_{B2y} \approx -422.89\,(\mathrm{N}) \end{cases}$$

The total bending moments at 8 critical cross sections are also calculated (we use the formula (10.15) to derive $M = \sqrt{M_x^2 + M_y^2}$ at each section):

$$\begin{cases} M_{A1} \approx 0\,(\mathrm{N \cdot mm}) \\ M_{D1}^- \approx 68531.85\,(\mathrm{N \cdot mm}) \\ M_{D1}^+ \approx 72867.4\,(\mathrm{N \cdot mm}) \\ M_{B1} \approx 0\,(\mathrm{N \cdot mm}) \\ M_{C1} \approx 0\,(\mathrm{N \cdot mm}) \end{cases} \qquad \begin{cases} M_{C2} \approx 0\,(\mathrm{N \cdot mm}) \\ M_{A2} \approx 191545.76\,(\mathrm{N \cdot mm}) \\ M_{D2}^- \approx 146910\,(\mathrm{N \cdot mm}) \\ M_{D2}^+ \approx 121977.78\,(\mathrm{N \cdot mm}) \\ M_{B2} \approx 0\,(\mathrm{N \cdot mm}) \end{cases}$$

Draw bending moment - torque diagrams Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

Find equivalent moments Knowing T_{sh1} and T_{sh2}, we calculate equivalent moment M_e at the 8 cross sections specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

$$\begin{cases} M_{eA1} \approx 0\,(\mathrm{N \cdot mm}) \\ M_{eD1}^- \approx 81087.5\,(\mathrm{N \cdot mm}) \\ M_{eD1}^+ \approx 84783.4\,(\mathrm{N \cdot mm}) \\ M_{eB1} \approx 43342.46\,(\mathrm{N \cdot mm}) \\ M_{eC1} \approx 43342.46\,(\mathrm{N \cdot mm}) \end{cases} \qquad \begin{cases} M_{eC2} \approx 205963.35\,(\mathrm{N \cdot mm}) \\ M_{eA2} \approx 281266.2\,(\mathrm{N \cdot mm}) \\ M_{eD2}^- \approx 252989.03\,(\mathrm{N \cdot mm}) \\ M_{eD2}^+ \approx 239373.1\,(\mathrm{N \cdot mm}) \\ M_{eB2} \approx 0\,(\mathrm{N \cdot mm}) \end{cases}$$

Find permissible stress $[\sigma_1]$ and $[\sigma_2]$ are determined by table (10.5). Since we use quenched 45X steel, $[\sigma_1] = 67$ (MPa) and $[\sigma_2] = 64$ (MPa) ($[\sigma_2]$ is achieved using interpolation).

Find standardized diameters at specific locations on the shaft Having M_e and $[\sigma]$, the next step is to estimate specific diameter at the key points mentioned above using equation (10.17) on p.194, which only applies for rigid shafts:

$$d \geq \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\begin{cases} d_{A1} \approx 0\,(\text{mm}) \\ d_{D1} \approx 23.66\,(\text{mm}) \\ d_{B1} \approx 18.92\,(\text{mm}) \\ d_{C1} \approx 18.92\,(\text{mm}) \end{cases} \qquad \begin{cases} d_{C2} \approx 32.32\,(\text{mm}) \\ d_{A2} \approx 35.86\,(\text{mm}) \\ d_{D2} \approx 34.61\,(\text{mm}) \\ d_{B2} \approx 0\,(\text{mm}) \end{cases}$$

Through rough calculations, we will choose the diameters according to standards given on p.195 (one applies for bearings while the other is used for the remaining machine elements):

$$\begin{cases} d_{A1} = 35\,(\text{mm}) \\ d_{D1} = 24\,(\text{mm}) \\ d_{B1} = 35\,(\text{mm}) \\ d_{C1} = 19\,(\text{mm}) \end{cases} \qquad \begin{cases} d_{C2} = 34\,(\text{mm}) \\ d_{A2} = 40\,(\text{mm}) \\ d_{D2} = 36\,(\text{mm}) \\ d_{B2} = 40\,(\text{mm}) \end{cases}$$

5.3 Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

where
$$s_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$$
$$s_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$$

Assuming the surfaces are smooth, properly ground and quenched by high frequency voltage, we obtain $K_x = 1$ from table (10.8) and $K_y = 1.4$ from table (10.9), where $[\sigma_b] = 850$ (MPa) is the property of quenched 45X steel.

Find σ_{-1}, τ_{-1} Using formulas on p.196:
$\sigma_{-1} = 0.35[\sigma_b] + 120 \approx 417.5$ (MPa)
$\tau_{-1} \approx 0.58\sigma_{-1} \approx 242.15$ (MPa)

Find $\sigma_a, \tau_a, \sigma_m, \tau_m$ For this part, we divide into 3 key points:

1. For rotating shaft, $\sigma_m = 0, \sigma_a = \dfrac{\sqrt{M_x^2 + M_y^2}}{W}$ (equation (10.22)), where M_x and M_y are at the cross section of interest.
2. By design, the shafts only rotate in one direction, thus $\tau_m = \tau_a = \dfrac{T_{sh}}{2W_O}$ (equation (10.23)).
3. We also assume the shafts have circular cross section, which makes $W = \dfrac{\pi d^3}{32}$ and $W_O = \dfrac{\pi d^3}{16}$ according to table (10.6), where d is the diameter of a cross section of the shaft.

The table below shows the results after calculation: Since $\sigma_b = 850$ (MPa) for both shafts, $\psi_\sigma = 0.1$ and $\psi_\tau = 0.05$

	d (mm)	W (mm^3)	W_O (mm^3)	σ_m (MPa)	σ_a (MPa)	τ_m (MPa)	τ_a (MPa)
A_1	20	785.4	1570.8	0	0	15.93	15.93
D_1	24	1357.17	2714.34	0	49.2	9.22	9.22
B_1	20	785.4	1570.8	0	0	15.93	15.93
C_1	19	673.38	1346.76	0	0	18.58	18.58
C_2	32	3216.99	6433.98	0	0	18.48	18.48
A_2	40	6283.19	12566.37	0	29.74	9.46	9.46
D_2	34	3858.66	7717.32	0	36.67	15.41	15.41
B_2	35	4209.24	8418.49	0	0	14.13	14.13

Table 5.1: Calculated variables for $\sigma_a, \tau_a, \sigma_m, \tau_m$

Find K_σ, K_τ We calculate K_σ using formula:

$$K_\sigma = \left(\frac{k_\sigma}{\varepsilon_\sigma} + K_x - 1\right) K_y^{-1}$$

and K_τ with:

$$K_\tau = \left(\frac{k_\tau}{\varepsilon_\tau} + K_x - 1\right) K_y^{-1}$$

Table (10.10), (10.11) and (10.13) are examined to find $\dfrac{k_\sigma}{\varepsilon_\sigma}$ ratio. Given $[\sigma_H] = 850$ (MPa) base shaft diameters d_1 and d_2 are compared to the diameters at critical locations A, B, C, D. If the base shaft is smaller, table (10.10) and (10.11) are used. If it is larger, we will use table (10.13) instead; the concentration stress factor in this case is demonstrated in the figure:

Final calculation is provided in the table:

	d (mm)	r	k_σ	k_τ	ε_σ	ε_τ	$\frac{k_\sigma}{\varepsilon_\sigma}$	$\frac{k_\tau}{\varepsilon_\tau}$	K_x	K_y	K_σ	K_τ
A_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
D_1	24	0.48	3	1.95	0.81	0.85	3.7	2.29	1	1.4	2.65	1.64
B_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
C_1	19	0.38	3	1.95	0.84	0.89	3.57	2.19	1	1.4	2.55	1.57
C_2	32	0.64	3	1.95	0.76	0.80	3.95	2.44	1	1.4	2.82	1.74
A_2	40	-	-	-	-	-	3.34	2.46	1	1.4	2.39	1.76
D_2	34	0.68	3	1.95	0.74	0.80	4	2.44	1	1.4	2.86	1.75
B_2	35	-	-	-	-	-	3.3	2.44	1	1.4	2.36	1.74

Table 5.2: Calculated variables in K_σ and K_τ

Find s_σ, s_τ and s Combining the results altogether, we obtain the following table:
Since the smallest safety factor is at the cross section D_1, which has the value of $3.14 > [s] = 1.5 \div 2.5$, we can neglect rigidity analysis according to the conclusion on p.195.

	s_σ	s_τ	s
A_1	$\gg s_\tau$	9.41	9.41
D_1	3.21	16	3.14
B_1	$\gg s_\tau$	9.41	9.41
C_1	$\gg s_\tau$	8.07	8.07
C_2	$\gg s_\tau$	7.32	7.32
A_2	5.88	14	5.43
D_2	3.99	8.77	3.63
B_2	$\gg s_\tau$	9.56	9.56

Table 5.3: Safety factor at critical cross sections

5.4 Static Strength Analysis

Along with fatigue strength, static strength is also considered and every shaft must satisfy the following condition at critical cross sections (equation (10.27)):

$$\sigma_e = \sqrt{\left(\frac{M_{max}}{0.1d^3}\right)^2 + 3\left(\frac{T_{max}}{0.2d^3}\right)^2} \leq [\sigma]$$

where M_{max}, T_{max} are the largest bending moment and torque at the cross section, respectively. Let $[\sigma] \approx 0.8\sigma_{ch} = 520$ (MPa), the results are in the table below:

	A_1	D_1	B_1	C_1	C_2	A_2	D_2	B_2
σ_e (MPa)	54.18	57.59	54.18	63.19	62.86	43.45	63.58	48.04

Table 5.4: Calculated static strength at critical cross sections

which satisfy the given condition.

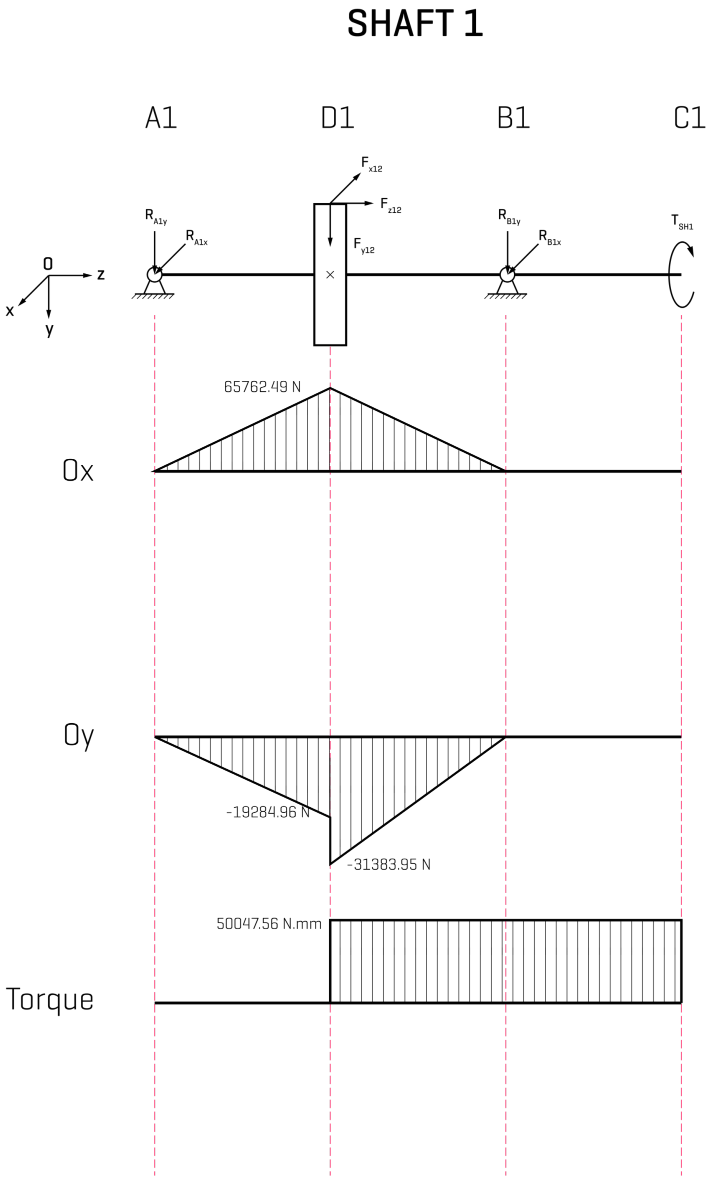

Figure 5.3: Bending moment-torque diagram of shaft 1

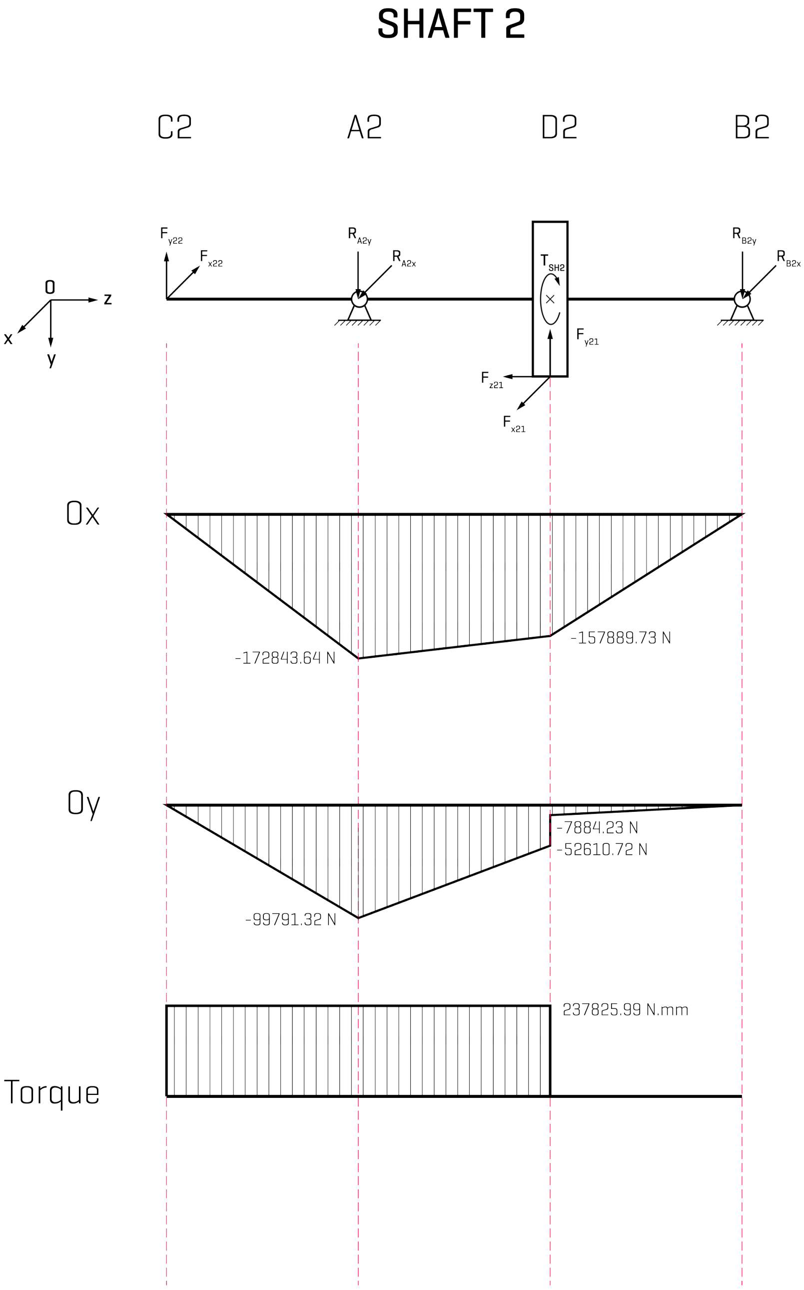

Figure 5.4: Bending moment-torque diagram of shaft 2

References

[1] Chat Trinh and Uyen Van Le. *Thiet Ke He Dan Dong Co Khi*. 6th ed. Vol. 1. Vietnam Education Publishing House Limited, 2006.