



HCM UNIVERSITY OF TECHNOLOGY
FLUID MECHANICS
CI2003

Assignment

Submitted To:

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Fluid Mechanics

Department

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Contents

1	Properties of Fluids	4
2	Pressure and Fluid Statics	9
2.1	Formulas	9
2.2	Problems	10
3	Fluid Kinematics - Bernoulli and Energy Equations	15
3.1	15
3.2	17
3.3	17

List of Tables

List of Figures

2.1	Differential manometer	10
2.2	Rotating rectangular valve	11
2.3	Jar under water	12
2.4	Manometer with jar	13

Chapter 1

Properties of Fluids

Q1.1

A steel vessel of 1% increase in volume when the pressure is increased by 70 MPa. At standard condition (pressure $P = 101.3$ KPa), the vessel is filled with $m = 450$ kg of water ($\rho = 1000$ kg/m³). Given bulk modulus of elasticity, $\kappa = 2.06 \times 10^9$ Pa. Compute the mass of water to add into the vessel to increase the pressure to 70 MPa.

Ans:

$$V = \frac{m}{\rho} = \frac{450}{1000} = 0.45 \text{ (m}^3\text{)}$$

$$\kappa = -V \frac{dP}{dV}$$

$$\begin{aligned} \Rightarrow dm &= \rho dV = -\rho V \frac{dP}{\kappa} \\ &= -1000 \times 0.45 \times \frac{101.3 \times 10^3 - 70 \times 10^6}{2.06 \times 10^9} = 15.27 \text{ (kg)} \end{aligned}$$

Q1.2

Determine the change in volume of $V_i = 3$ m³ of air when the pressure increases from $P_i = 100$ kPa to $P_f = 500$ kPa. Air is at $T = 23$ °C (assume ideal gas)

Ans:

Assume isothermal condition: $P_i V_i = P_f V_f$

$$\Rightarrow \Delta V = V_f - V_i = \frac{P_i V_i}{P_f} - V_i = \frac{100 \times 3}{500} - 3 = -2.4 \text{ (m}^3\text{)}$$

Q1.3

They compress the air into a vessel having volume, $V_1 = 0.3 \text{ m}^3$ under pressure $P_1 = 100 \text{ at}$. After a period of leakage, the air pressure in the vessel is lowered to $P_2 = 90 \text{ at}$. Regardless of the deformation of the vessel, determine the volume of air that is leaked during that period (corresponding to the atmospheric pressure, 1 atm), if the constant temperature and atmospheric pressure are considered to be at 1 at.

Ans:

For ideal gas, $\kappa = P_2 = -V_1 \frac{dP}{dV}$

$$\Rightarrow dV = -\frac{V_1}{P_2} dP = -\frac{0.3}{90} (90 - 100 - 1) = -0.0367 \text{ (m}^3\text{)}$$

Q1.4

A diameter piston $d = 50 \text{ mm}$ moves evenly in a cylinder $D = 50.1 \text{ mm}$. Determine the decrease in force acting on the piston (as a percentage) when the speed decreases by 5%.

Ans:

Assume constant speed: $\frac{\Delta V}{V_1} = -5\%$

$$\text{We also have } \tau = \frac{F}{A} = \mu \frac{du}{dy} = \mu \frac{V}{l} \Rightarrow \frac{F_1}{V_1} = \frac{F_2}{V_2} = \mu \frac{A}{l}$$

$$\text{From the relation: } \frac{\Delta F}{F_1} = \frac{\Delta V}{V_1} = -5\%$$

Q1.5

A machine axis having diameter, $D = 75 \text{ mm}$, with uniform movement with $V = 0.1 \text{ m/s}$ under the force, $F = 100 \text{ N}$. The lubricating layer thickness, l in

the bearing is $l = 0.07$ mm. Length of bearing $L = 200$ mm. Determine oil dynamic viscosity.

Ans:

$$A = (D + 2l)\pi \times L = (75 + 2 \times 0.07)\pi \times 200 = 47211.85 \text{ (mm}^3\text{)} = 4.72 \times 10^{-5} \text{ (m}^3\text{)}$$

$$\tau = \frac{F}{A} = \mu \frac{V}{l} \Rightarrow \mu = \frac{F l}{A V} = \frac{100}{4.72 \times 10^{-5}} \times \frac{0.07 \times 10^{-3}}{0.1} = 1.483 \text{ (kg/m} \cdot \text{s)}$$

Q1.6

A thin layer of Newton liquid with specific weight γ , dynamic viscosity μ and thickness t flows on a plane inclined at an angle α . The upper surface is exposed to the air. Assuming no friction between liquid and air. Find the expression of $u(y)$. Can u consider as a linear function of y ?

Ans:

The force exerting on the center of gravity of the liquid is

$$F = \gamma V \sin \alpha$$

Both contact surfaces

Q1.9

Determine the frictional force at the inner wall of a water supply pipe segment at $T = 20^\circ\text{C}$, radius $R = 80$ mm = 0.08 m, $L = 10$ m. The velocity at the points on the pipe cross-section varies according to the following:

$$u(r) = 0.5 \left(1 - \frac{r^2}{R^2} \right)$$

where r is the radius of considered point.

Ans:

From table at $T = 20^\circ\text{C} \Rightarrow \mu = 1.002 \times 10^{-3} \text{ (kg/m} \cdot \text{s)}$

$$A = 2\pi RL = 2\pi \times 0.08 \times 10 = 5.03 \text{ (m}^2\text{)}$$

$$F = \tau A = \mu \frac{du}{dr} A = \mu \frac{(-2)R}{R^2} A$$

$$= 1.002 \times 10^{-3} \times \frac{(-2) \times 0.08}{0.08^2} \times 5.03 = -0.126 \text{ (N)}$$

Q1.10

Determine the gauge pressure inside a water drop of diameter $d = 2 \text{ mm} = 0.002 \text{ m}$. The temperature of water is $T = 25^\circ\text{C}$.

Ans:

From table at $T = 25^\circ\text{C} \Rightarrow \sigma_s = 0.072 \text{ (N/m)}$

$$P_g = \Delta P_{\text{droplet}} = \frac{2\sigma_s}{d/2} = \frac{2 \times 0.072}{0.002/2} = 144 \text{ (Pa)}$$

Q1.11

A gas has a molar mass of $R = 32 \text{ kg/mol}$ under a pressure condition of $P = 5 \text{ at} = 490332.5 \text{ Pa}$, a temperature of $T = 30^\circ\text{C}$

1. Determine the gas density.
2. Determine the density of this gas if $P = \text{const}$, while temperature drops to $T_f = 15^\circ\text{C}$.
3. Determine the density of this gas if holding $T = \text{const}$, while the pressure drops to $P_f = 2 \text{ at}$.

Ans:

$$1. P = \rho RT \Rightarrow \rho = \frac{P}{RT} = \frac{32}{490332.5 \times (30 + 273)} = 0.215 \times 10^{-6} \text{ (kg/m}^3\text{)}$$

2. adiabatic condition

$$\rho T = \rho_f T_f \Rightarrow \rho_f = \frac{\rho T}{T_f} = \frac{0.215 \times 10^{-6} \times (30 + 273)}{15 + 273} = 0.226 \times 10^{-6} \text{ (kg/m}^3\text{)}$$

3. isothermal condition

$$\frac{P}{\rho} = \frac{P_f}{\rho_f} \Rightarrow \rho_f = \frac{\rho P_f}{P} = \frac{0.215 \times 10^{-6} \times 2}{5} = 0.086 \times 10^{-6} \text{ (kg/m}^3\text{)}$$

Q1.12

A liquid is compressed in a cylinder, the water initially has a volume of $V_o = 41 \text{ at}$ normal pressure, $P_o = 1 \text{ at} = 98066.5 \text{ Pa}$. The pressure in the cylinder increases to $p_1 = 6 \text{ at}$, the water volume decreases by 1 cm^3 .

1. Compute the bulk modulus of elasticity of water.
2. If the pressure in the cylinder increases to 20 at, calculate the volume of water V_f in the cylinder.
3. Calculate the pressure in the cylinder, if the volume of the water is reduced by 0.1%.

Ans:

$$1. \kappa = -V_o \frac{dP}{dV} = -4 \times 10^{-3} \times \frac{6 \times 98066.5}{(-1) \times 10^{-6}} = 2.36 \times 10^9 \text{ (Pa)}$$

2. Cylinder increase pressure to 20 at

$$\Rightarrow dV = V_f - 4 \times 10^{-3} = \frac{-20}{6} = -3.33 \text{ (cm}^3\text{)} = -3.33 \times 10^{-3} \text{ (l)}$$

$$\Rightarrow V_f = 3.997 \text{ (l)}$$

$$3. dP = P_f - P_o = -\kappa \times \frac{dV}{V_o} = -2.36 \times 10^9 \times \frac{(-0.1)}{100} = 2.36 \times 10^6 \text{ (Pa)}$$

$$\Rightarrow P_f = 2.458 \text{ (MPa)}$$

Q1.13

The air moving through a narrow tube into a water tank forms a stream of bubbles $d = 3 \text{ mm}$ in diameter. Calculate the difference between air pressure in the narrowed section and surrounding water pressure. Give the surface tension of water $\sigma_s = 0.0728 \text{ N/m}$.

Ans:

Chapter 2

Pressure and Fluid Statics

2.1 Formulas

Calculating magnitude of resultant force F_R :

$$F_R = (P_0 + \rho g y_C \sin \theta) A = P_C A$$

where P_0 is the pressure at the liquid surface (often is P_{atm} , which can be ignored)

P_C is the pressure taken at the centroid of the rigid body surface

$h_c = y_C \sin \theta$ is the vertical distance of the rigid body's centroid from the liquid surface

θ is the angle of the rigid body with respect to the liquid surface

For horizontal plate:

$$F_R = (P_0 + \rho g h) ab$$

For vertical plane (s is the upper vertical distance, b is the length of the plate):

$$F_R = (P_0 + \rho g (s + b/2)) ab$$

The location at which F_R acts on the body surface is:

$$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin \theta)] A}$$

where y_C is the centroid of the rigid body

$I_{xx,C}$ is the second moment of area about the x -axis passing through the centroid of the rigid body (normal to the surface itself)

For rectangular plates, it can also be written as:

$$y_P = s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin \theta)]}$$

The vertical distance from liquid surface $h_P = y_P \sin \theta$

2.2 Problems

Q2.1

A differential manometer consists of a U-shaped pipe of diameter d , connecting two cylinders of diameter D , the instrument being filled with two insoluble liquids of specific weight γ_1 and γ_2 . When the pressure difference, $\Delta p = p_1 - p_2 = 0$, the interface between two liquids is at position 0 on the scale.

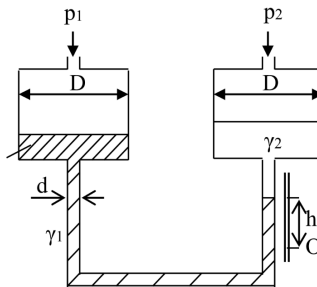


Figure 2.1: Differential manometer

1. Determine the relationship between Δp and the displacement of the interface between the two fluids, h . Given $d = 5 \text{ mm}$, $D = 50 \text{ mm}$, $\gamma_1 = 8530 \text{ N/m}^3$, $\gamma_2 = 8140 \text{ N/m}^3$, $h = 280 \text{ mm}$.
2. With the given Δp , how many times h will decrease, if $d = D = 5 \text{ mm}$.

Ans:

Let h_1, h_2 be the height of the liquid from O on the left and right pipe,

respectively.

The relation between P_1 and P_2 is described according to the formula:

$$\frac{D^2}{d^2} p_1 = p_2 \frac{D^2}{d^2} + \gamma_2 (h_2 - h) - \gamma_1 (h_1 - h)$$

Rearranging the equation yields:

$$h = \frac{\frac{D^2}{d^2} \Delta p + \gamma_1 h_1 - \gamma_2 h_2}{\gamma_1 - \gamma_2}$$

Q2.2

A rectangular valve length b rotates about the horizontal axis at point A. Neglecting valve thickness, determine the minimum weight G of the gate based on the parameters h_1, h_2, h_3, ρ, b and g such that the system is balanced. Use Figure 2.2 to solve the problem.

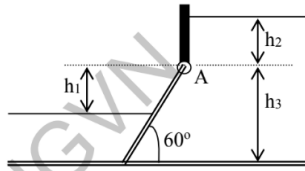


Figure 2.2: Rotating rectangular valve

Ans:

Weight of the triangular block of water: $W = \rho g V = \frac{1}{2\sqrt{3}} \rho g h_3^2$

The horizontal force acting on vertical plane is: $F_x = \rho g \left(h_2 + \frac{h_3}{2} \right) h_3$

The vertical force acting on horizontal plane is: $F_y = \rho g (h_2 + h_3) \frac{h_3}{\sqrt{3}}$

Projecting the forces onto x, y axes yields:

$$F_H = F_x = \frac{g h_3 \rho (2h_2 + h_3)}{2}$$

$$F_V = F_y - W = \frac{\sqrt{3} g h_3 \rho (2h_2 + h_3)}{6}$$

Thus,

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{F_x^2 + (F_y - W)^2} = \frac{1}{\sqrt{3}} g h_3 \rho (2h_2 + h_3)$$

$$\tan \theta = \frac{F_V}{F_H} = \frac{\sqrt{3}}{3} \Rightarrow \theta = 30^\circ$$

$$s = \frac{h_2}{\sqrt{3}}, y_C = h_1$$

$$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin \theta)]A} = \frac{\sqrt{3}h_3(3h_2 + 4h_3)}{9(h_2 + h_3)}$$

The system is at equilibrium when and only when:

$$M_{A\cup} = M_{A\cap}$$

$$\Rightarrow \frac{b}{2} G \cos 60^\circ = y_P F_R$$

$$\Rightarrow G = \frac{2}{\sqrt{3}} g h_3 \rho (2h_2 + h_3)$$

Q2.3

An empty cylindrical jar of diameter $d = 5 \text{ cm} = 0.05 \text{ m}$, length $L = 10 \text{ cm} = 0.1 \text{ m}$ is placed in the water. Determine the weight of the jar so that it reaches equilibrium below the depth $h = 1 \text{ m}$. Ignore the thickness of the wall of jar. Given $p_a = p_{\text{water @ } 10 \text{ m}} = 98.1 \times 10^3 \text{ (Pa)}$.

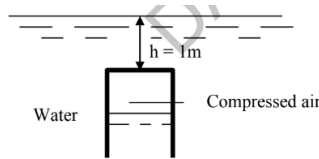


Figure 2.3: Jar under water

Ans:

The center of gravity of the jar is located at:

$$h_C = h + \frac{L}{2} = 1.05 \text{ (m)}$$

The weight of the jar:

$$W = F_B = \rho g h_C \frac{\pi d^2}{4} = 20.225 \text{ (N)}$$

Q2.4

A rectangular valve AB is inclined to the horizontal plane an angle α , having width b , the depths of A and B are h_2 and h_3 respectively, the pressure on the water surface in the tank is p_o . The water level in the manometer tube is higher than the water level in the jar, h_1 (see Figure 2.4). Let $b = 4 \text{ m}$, $h_1 = 2 \text{ m}$, $h_2 = 1 \text{ m}$, $h_3 = 3 \text{ m}$, $\alpha = 45^\circ$, $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$.

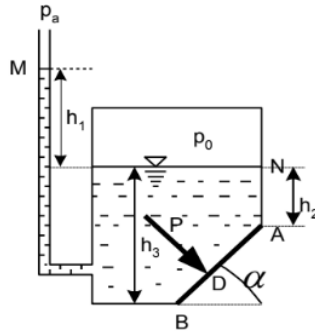


Figure 2.4: Manometer with jar

1. Compute gauge pressure p_o , p_A , p_B .
2. Compute the force by water acting on valve AB.
3. Determine the center of pressure D (compute BD).
4. Compute the minimum force required F , acting at B to remain the valve closed.

Ans:

1. $p_o = \rho g h_1 \Rightarrow p_o = 19.62 \text{ (kPa)}$
 $p_A = \rho g (h_1 + h_2) \Rightarrow p_A = 29.43 \text{ (kPa)}$
 $p_B = \rho g (h_1 + h_3) \Rightarrow p_B = 49.05 \text{ (kPa)}$
2. Weight of the triangular block of water:
 $W = \rho g (h_3 - h_2)^2 b = 156.96 \text{ (kN)}$
 The horizontal force acting on vertical plane is:
 $F_x = \rho g (h_1 + \frac{h_3 - h_2}{2})(h_3 - h_2)b = 235.44 \text{ (kN)}$
 The vertical force acting on horizontal plane is:
 $F_y = \rho g (h_1 + h_2)(h_3 - h_2)b = 235.44 \text{ (kN)}$
 Projecting the forces onto x, y axes yields:

$$F_H = F_x = 235.44 \text{ (kN)}$$

$$F_V = F_y + W = 392.4 \text{ (kN)}$$

Thus,

$$F_R = \sqrt{F_H^2 + F_V^2} = 457.61 \text{ (kN)}$$

$$\tan \theta = \frac{F_V}{F_H} = 1.667 \Rightarrow \theta = 59.04^\circ$$

$$3. \quad BD = \frac{h_3 - h_2}{\cos \alpha} = 2.83 \text{ (m)}$$

Chapter 3

Fluid Kinematics - Bernoulli and Energy Equations

3.1

Fluid flow with the velocity described by Eulerian Method as follows:

$$u = 3t; \quad v = xz; \quad w = ty^2$$

1. Is this flow steady or unsteady ?
2. Prove that the flow satisfies the continuity equation of incompressible fluid ?
3. Determine the acceleration of fluid particle ?
4. Find the streamline at time $t = 1$ and passing the origin $O(0, 0)$.

Ans:

1. The flow $V = 3t\hat{i} + xz\hat{j} + ty^2\hat{k}$ depends on time ($u = 3t, w = ty^2$), thus it is unsteady.

$$\begin{aligned}
2. \quad \nabla \cdot \mathbf{V} &= 0 \\
&\Leftrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \\
&\Leftrightarrow \frac{\partial(3t)}{\partial x} + \frac{\partial(xz)}{\partial y} + \frac{\partial(ty^2)}{\partial z} = 0 \text{ (True)}
\end{aligned}$$

The flow satisfies the continuity equation of incompressible fluid.

$$\begin{aligned}
3. \quad \mathbf{a} &= a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} + a_z \hat{\mathbf{k}} \\
a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 3 \\
a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = 3zt + xy^2t \\
a_z &= \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = y^2 + 2xyzt \\
&\Rightarrow \mathbf{a} = 3\hat{\mathbf{i}} + (3zt + xy^2t)\hat{\mathbf{j}} + (y^2 + 2xyzt)\hat{\mathbf{k}}
\end{aligned}$$

4. Since we have zero initial condition, it is concluded that $C = 0$:

$$\begin{aligned}
\text{On } xy\text{-plane:} \quad \frac{dy}{dx} &= \frac{v}{u} = \frac{xz}{3t} \\
\text{At time } t &= 1 \text{ (s):} \\
y &= \int \frac{xz}{3} dx = \frac{x^2 z}{6}
\end{aligned}$$

$$\begin{aligned}
\text{On } yz\text{-plane:} \quad \frac{dz}{dy} &= \frac{w}{v} = \frac{ty^2}{xz} \\
\text{At time } t &= 1 \text{ (s):} \\
z &= \int \frac{y^2}{xz} dy = \frac{y^3}{3xz}
\end{aligned}$$

$$\begin{aligned}
\text{On } xz\text{-plane:} \quad \frac{dz}{dx} &= \frac{w}{u} = \frac{ty^2}{3t} = \frac{y^2}{3} \\
\text{At time } t &= 1 \text{ (s):} \\
z &= \int \frac{y^2}{3} dx = \frac{xy^2}{3}
\end{aligned}$$

3.2

3.3

Plane fluid flow with the velocity described as follows:

$$u = \frac{-y}{b^2}; \quad v = \frac{x}{a^2}$$

1. Prove that the flow satisfies the continuity equation of incompressible fluid ?
2. Find the streamlines.

Ans:

$$\begin{aligned} 1. \quad \nabla \cdot \mathbf{V} &= 0 \\ \Leftrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\ \Leftrightarrow \frac{\partial(\frac{-y}{b^2})}{\partial x} + \frac{\partial(\frac{x}{a^2})}{\partial y} + \frac{\partial(0)}{\partial z} &= 0 \text{ (True)} \end{aligned}$$

The flow satisfies the continuity equation of incompressible fluid.

2.

On xy -plane: $\frac{dy}{dx} = \frac{v}{u} = \frac{xz}{3t}$

At time $t = 1$ (s):

$$y = \int \frac{xz}{3} dx = \frac{x^2 z}{6}$$

On yz -plane: $\frac{dz}{dy} = \frac{w}{v} = \frac{ty^2}{xz}$

At time $t = 1$ (s):

$$z = \int \frac{y^2}{xz} dy = \frac{y^3}{3xz}$$

On xz -plane: $\frac{dz}{dx} = \frac{w}{u} = \frac{ty^2}{3t} = \frac{y^2}{3}$

At time $t = 1$ (s):

$$z = \int \frac{y^2}{3} dx = \frac{xy^2}{3}$$