HCM UNIVERSITY OF TECHNOLOGY

MACHINE ELEMENTS

ME2007

Project Report

Submitted To:
Phan Dinh Huan
Asst. Professor
Faculty of Mechanical
Engineering

Submitted By :
Nguyen Quy Khoi
1852158
CC02
HK192

August 13, 2020

Contents

List of Tables

List of Figures

Design Problem

D_{bc}	pulley diameter, mm
F_t	tangential force, N
L	service life, years
T	working torque, $\text{N} \cdot \text{mm}$
t	working time, s
v_{bc}	conveyor belt speed, m/s
δ_u	error of speed ratio, %

Given the mechanical transmission system in figure 0.1, determine the specifications for each machine element.

1. Electric motor
2. Elastic coupling
3. Gearbox
4. Chain drive
5. Belt conveyor

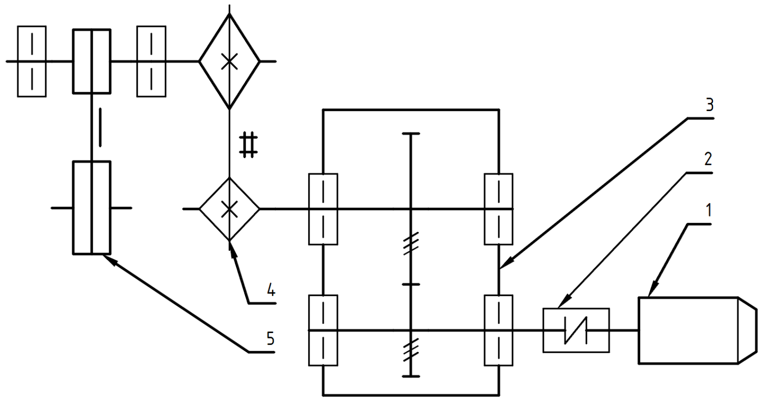


Fig. 0.1 Mechanical transmission system of a belt conveyor

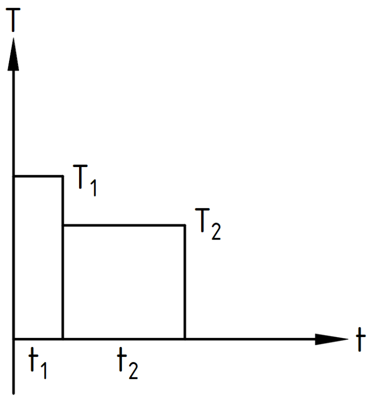


Fig. 0.2 Input load diagram

Design parameters

The chosen parameters are given in column 8:

- $F_t = 4500\,(\mathrm{N})$

- $v_{bc} = 3.05$ (m/s)
- $D_{bc} = 500$ (mm)
- $L = 4$ (years)
- $T_1 = T$ (N · mm), $t_1 = 12$ (s)
- $T_2 = 0.7T$ (N · mm), $t_2 = 60$ (s)
- $\delta_u \leq \pm 5\%$

The machine works in one direction for 300 days, 2 shifts per day, 8 hours each and the load is low. Also, for the rest of this report, we will consider shaft 1 is the one connected to the electric motor, while shaft 2 is the connection between the chain drive and gearbox.

Chapter 1
Motor Design

1.1 Nomenclature

n_{bc}	rotational speed of belt conveyor, rpm
n_{sh}	rotational speed of shaft, rpm
P_m	maximum operating power of belt conveyor, kW
P_{motor}	calculated motor power to drive the system, kW
P_{sh}	operating power of shaft, kW
P_w	operating power of the belt conveyor given a workload, kW
u_{ch}	transmission ratio of chain drive
u_{hg}	transmission ratio of helical gear
u_{sys}	transmission ratio of the system
T_{motor}	motor torque, $\text{N} \cdot \text{mm}$
T_{sh}	shaft torque, $\text{N} \cdot \text{mm}$
δ_u	relative error of u_{sys}
η_b	bearing efficiency
η_c	coupling efficiency
η_{ch}	chain drive efficiency
η_{hg}	helical gear efficiency
η_{sys}	efficiency of the system
$_1$	shaft 1
$_2$	shaft 2

1.2 Calculate η_{sys}

From table (2.3):
$\eta_c = 1$
$\eta_b = 0.99$
$\eta_{hg} = 0.96$
$\eta_{ch} = 0.95$
$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$

1.3 Calculate P_{motor}

$P_m = \dfrac{F_t v_{bc}}{1000} \approx 13.73$ (kW)
From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$

$P_{motor} = \dfrac{P_w}{\eta_{sys}} \approx 11.76$ (kW) $< P_m$

1.4 Calculate n_{motor}

$n_{bc} = \dfrac{6 \times 10^4 v}{\pi D_{bc}} \approx 116.5$ (rpm)
$u_{ch} = 5$ (table (2.4))
$u_{hg} = 5$ (table (2.4))
$u_{sys} = u_{ch} u_{hg} = 25$
$n_{motor} = u_{sys} n_{bc} \approx 2912.54$ (rpm)

1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated P_{motor} and P_m. Since $P_{motor} < P_m$ for our case, the minimum operating power of choice is P_m. In similar fashion, its rotational speed must also be no smaller than estimated n_{motor}.
Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm
$\Rightarrow P_{motor} = 18.5\,\text{kW}, n_{motor} = 2930$ (rpm)
Recalculating u_{sys} with the new P_{motor} and n_{motor}, we obtain:
$u_{sys} = \dfrac{n_{motor}}{n_{bc}} \approx 25.15$
Assuming $u_{hg} = const$:
$u_{ch} = \dfrac{u_{sys}}{u_{hg}} \approx 5.03$

Relative error of transmission ratio

The ratio is calculated as follows:

$$\delta_u = \frac{|25.15 - 25|}{25} \approx 0.6\% \leq 5\%$$

1.6 Calculate power, rotational speed and torque

Let us denote P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

1.6.1 Power

$$P_{ch} = P_m \approx 13.73\ (\text{kW})$$
$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59\ (\text{kW})$$
$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35\ (\text{kW})$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930\ (\text{rpm})$$
$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586\ (\text{rpm})$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63\ (\text{N} \cdot \text{mm})$$
$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56\ (\text{N} \cdot \text{mm})$$
$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99\ (\text{N} \cdot \text{mm})$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	18.5	15.35	14.59
u	5		5.03
n (rpm)	2930	2930	586
T (N · mm)	60298.63	50047.56	237825.99

Table 1.1 System overall specifications

Chapter 2

Chain Drive Design

2.1 Nomenclature

$[i]$	permissible impact times per second
$[s]$	permissible safety factor
$[P]$	permissible power, kW
$[\sigma_H]$	permissible contact stress, MPa
A	cross sectional area of chain hinge, mm^2
a	center distance, mm
a_{max}	maximum center distance, mm
a_{min}	minimum center distance, mm
B	width between inner link plate, mm
d	chordal diameter, mm
d_a	addendum diameter, mm
d_f	dedendum diameter, mm
d_l	roller diameter, mm
d_O	pin diameter, mm
E	modulus of elasticity, MPa
F_0	sagging force, N
F_1	tight side tension force, N
F_2	slack side tension force, N
F_r	force on the shaft, N
F_t	effective peripheral force, N
F_v	centrifugal force, N
F_{vd}	contact force, N
i	impact times per second
K_d	weight distribution factor on each
k	overall factor
k_0	arrangement of drive factor

k_a	center distance and chain's length factor
k_{bt}	lubrication factor
k_c	rating factor
k_d	dynamic load factor
k_{dc}	chain tension factor
k_f	loosing factor
k_n	coefficient of rotational speed
k_r	number of tooth factor
k_x	chain weight factor
k_z	coefficient of number of teeth
n_{01}	experimental rotational speed, rpm
n	sprocket rotational speed, rpm
n_{ch}	rotational speed of a sprocket, rpm
P_t	calculated power, kW
p	pitch, mm
p_{max}	permissible sprocket pitch,
Q	permissible load, N
q	mass per meter of chain, kg
s	safety factor
v	instantaneous velocity along
x	chain length in pitches, the n
x_c	an even number of links
z	number of teeth of a sprocke
z_{max}	maximum number of tee sprocket
σ_H	contact stress, MPa
$[\sigma_H]$	permissible contact stress, M
$_1$	subscript for driving sprocke
$_2$	subscript for driven sprocke

2.2 Find p

Since the driving sprocket is connected to shaft 1, $n_1 = n_{sh2} = 586$ (rpm).

Find z

Since z_1 and z_2 is preferably an odd number according to p.80:
$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$
$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$
Because $z_1 \geq 15$, we use table (5.8) and interpolation to approximate p_{max}. Therefore, $p_{max} \approx 33.58$ (mm).

Find k

Since $n_{ch} = 586 \approx 600$ (rpm), choose $n_{01} = 600$ (rpm), which is obtained from table (5.5). Then, we calculate k_z and k_n.
$k_z = \dfrac{25}{z_1} \approx 1.32$, $k_n = \dfrac{n_{01}}{n_{ch}} \approx 1.02$
Specifying the chain drive's working condition and ultizing table (5.6) , we find out that $k_0 = k_a = k_{dc} = k_{bt} = 1$, $k_d = 1.25$, $k_c = 1.3$.
$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$

Find p

From table (5.5):
$P_t = P_{ch} k k_z k_n \approx 30.05$ (kW) ≤ 42 (kW) $\Rightarrow [P] = 42$ (kW)
Using the table, we also get the other parameters:
$p = 31.75$ (mm), $d_O = 9.55$ (mm), $B = 27.46$ (mm), $d_l = 19.05$ (mm)
$d_1 = \dfrac{p}{\sin \dfrac{180°}{z_1}} \approx 192.9$ (mm), $d_2 = \dfrac{p}{\sin \dfrac{180°}{z_2}} \approx 980.49$ (mm)
Having $p = 31.75$ (mm) $\leq p_{max} \approx 33.58$ (mm), we can safely choose the number of chains as 1, which is in agreement with the given figure. Hence, from table (5.2), we obtain the parameters in the section for 1 strand chain drive:
$Q = 56.7 \times 10^3$ (N), $q = 2.6$ (kg/m)
By comparison to the conditions in the sub-table, the choice of B is satisfactory.

2.3 Find a, x_c, and i

Find x_c

$a_{min} = 30p = 952.5$ (mm), $a_{max} = 50p = 1587.5$ (mm). Limiting the range of choice for a in $[a_{min}, a_{max}]$, we can approximate $a = 1000$ (mm).
$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$

Find a

From equation (5.13) , we calculate a again with x_c:
$a = \frac{p}{4}\left(x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\frac{(z_2 - z_1)^2}{\pi^2}}\right) - 0.003a \approx 998.98$ (mm)

Find i

From table (5.9):
$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$

2.4 Strength of chain drive

2.4.1 Safety factor analysis

In order to operate safely, the chain drive's safety factor must satisfy the following condition:

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \geq [s]$$

For moderate workload, choose $k_d = 1.2$. Let the chain drive be angled 30° with respect to ground, we obtain $k_f = 4$.
$v_1 = \dfrac{n_{ch} p z_1}{6 \times 10^4} \approx 5.89$ (m/s)

Find F_t, F_v, F_0

We also need to calculate F_t, F_v and F_0:
$F_t = \dfrac{10^3 P_{ch}}{v_1} \approx 2329.53$ (N)
$F_v = q v_1^2 \approx 90.25$ (N)
$F_0 = 9.81 \times 10^{-3} k_f q a \approx 101.92$ (N)

Validate s

This value must be larger than the permissible safety factor to operate properly. From equation (5.15):
$s = \dfrac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2$, where $[s]$ is chosen from table (5.10).

2.4.2 Contact stress analysis

The following condition must be met:

$$\sigma_H = 0.47 \sqrt{\frac{k_r (F_t k_d + F_{vd}) E}{A K_d}} \leq [\sigma_H]$$

Since the chain drive only has one strand, $K_d = 1$.

Find $[\sigma_H]$

From table (5.11), quenched 45 steel is the material of use for the chain drive, which has HB210, $[\sigma_H] = 600$ (MPa) and $E = 2.1 \times 10^5$ (MPa).

Find F_{vd}

For 1-strand chain, $F_{vd} = 13 \times 10^{-7} n_1 p^3 \approx 24.38$ (N)

Find k_r

Based on given data on p.87, we estimate k_r from z, which is $k_r \approx 0.47$

Find A

According to table (5.12), $A = 262$ (mm)
Combining with $k_d = 1.2$, $F_t \approx 2329.53$ (N), we get the result:

$$\sigma \approx 494.32 \text{ (MPa)} \leq [\sigma_H] = 600 \text{ (MPa)}$$

which is satisfactory.

2.5 Force on shaft

From p.87:
$F_2 = F_0 + F_v \approx 192.17$ (N)
$F_1 = F_t + F_2 \approx 2521.7$ (N)
Choose $k_x = 1.15$ and follow equation (5.20) :
$F_r = k_x F_t \approx 2678.96$ (N)

2.6 Other parameters

$$d_{a1} = p\left(0.5 + \cot\frac{180}{z_1}\right) \approx 206.14\ (\text{mm})$$
$$d_{a2} = p\left(0.5 + \cot\frac{180}{z_2}\right) \approx 995.85\ (\text{mm})$$

Knowing that $d_l = 19.05$ (mm) from previous sections:
$d_{f1} = d_1 - 2(0.502d_l + 0.05) \approx 173.67$ (mm)
$d_{f2} = d_2 - 2(0.502d_l + 0.05) \approx 961.26$ (mm)
In summary, we have the following table:

	driving	driven
$[P]$ (kW)	42	
a (mm)	998.98	
B (mm)	27.46	
d (mm)	192.9	980.49
d_a (mm)	206.14	995.85
d_f (mm)	173.67	961.26
d_l (mm)	19.05	
d_O (mm)	9.55	
i	6	
p (mm)	31.75	
Q (N)	56700	
u_{ch}	5.03	
v (m/s)	5.89	
z	19	97

Table 2.1 Chain drive specifications

Chapter 3

Gearbox Design (Helix gears)

3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa
$[\sigma_H]_{max}$	permissible contact stress due to overload, MPa
$[\sigma_F]$	permissible bending stress, MPa
$[\sigma_F]_{max}$	permissible bending stress due to overload, MPa
AG	accuracy grade of gear
a	center distance, mm
b	face width, mm
c	gear meshing rate
d	pitch circle, mm
d_a	addendum diameter, mm
d_b	base diameter, mm
d_f	deddendum diameter, mm
F_a	axial force, N
F_r	radial force, N
F_t	tangential force, N
H	surface roughness, HB
K_d	coefficient of gear material
K_F	load factor from bending stre
K_{FC}	load placement factor
K_{FL}	aging factor due to bending st
K_{Fv}	factor of dynamic load from meshing area
$K_{F\alpha}$	factor of load distribution fro on gear teeth
$K_{F\beta}$	factor of load distribution fro on top land
K_H	load factor of contact stress

K_{HL} aging factor due to contact stress
K_{Hv} factor of dynamic load from contact stress at meshing area
$K_{H\alpha}$ factor of load distribution from contact stress on gear teeth
$K_{H\beta}$ factor of load distribution from contact stress on top land
k_x a coefficient
k_y a coefficient
m traverse module, mm
m_F root of fatigue curve in bending stress test
m_H root of fatigue curve in contact stress test
m_n normal module, mm
N_{FE} working cycle of equivalent tensile stress corresponding to $[\sigma_F]$
N_{FO} working cycle of bearing stress corresponding to $[\sigma_F]$
N_{HE} working cycle of equivalent tensile stress corresponding to $[\sigma_H]$
N_{HO} working cycle of bearing stress corresponding to $[\sigma_H]$
S specific length, mm
S_F safety factor of bending stress
S_H safety factor of contact stress
T input torque, $\mathrm{N \cdot mm}$
v rotational velocity, $\mathrm{m/s}$
x gear correction factor
Y_F tooth shape factor
Y_R surface roughness factor of
Y_s sensitivity to stress concent
Y_β helix angle factor
Y_ε contact ratio factor
y center displacement factor
Z_R surface roughness factor of
Z_v speed factor
z_H contact surface's shape fact
z_M material's mechanical prop
z_{min} minimum number of teeth
z_v virtual number of teeth
z_ε meshing condition factor
α normal pressure angle, f standard (TCVN 1065-71),
α_t traverse pressure angle, °
ε_α traverse contact ratio
ε_β face contact ratio
β helix angle, °

β_b base circle helix angle, °
ψ_{ba} width to shaft distance ratio
ψ_{bd} face width factor
σ_b ultimate strength, MPa
σ_{ch} yield limit, MPa
σ^o_{Flim} permissible bending stress corresponding to working cycle, MPa
σ^o_{Hlim} permissible contact stress co working cycle, MPa
$_1$ subscript for pinion
$_2$ subscript for driven gear
$_w$ subscript for variable value afte

3.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with $S \leq 100$ (mm), HB250, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 550$ (MPa).
Table (6.2) also gives $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$
Therefore, they have the same properties except for their surface roughness H. The reasoning is given on p.91, where $H_2 = H_1 - 10 \div 15$
For the pinion, $H_1 = \text{HB250} \Rightarrow \sigma^o_{Hlim1} = 570$ (MPa), $\sigma^o_{Flim1} = 450$ (MPa)
For the driven gear, $H_2 = \text{HB240} \Rightarrow \sigma^o_{Hlim2} = 550$ (MPa), $\sigma^o_{Flim2} = 432$ (MPa)

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

3.3.1 Working cycle of bearing stress

Using equation (6.5) :
$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6$ (cycles)
$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6$ (cycles)

3.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq$ HB350, $m_H = 6$, $m_F = 6$.
Both gears meshed indefinitely, thus $c = 1$.
From working condition, we calculate:

$$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times 2\left(\frac{\text{shifts}}{\text{day}}\right) \times 300\left(\frac{\text{days}}{\text{year}}\right) \times 4\,(\text{years}) = 19200\,(\text{hours})$$

Applying equation (6.7) and T_1, T_2, t_1, t_2 in the initial parameters:

$$N_{HE1} = 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] \approx 1.53\times10^9\,(\text{cycles})$$

$$N_{HE2} = 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}\right] \approx 0.31\times10^9\,(\text{cycles})$$

$$N_{FE1} = 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] \approx 0.89\times10^9\,(\text{cycles})$$

$$N_{FE2} = 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2}\right] \approx 0.18\times10^9\,(\text{cycles})$$

3.3.3 Aging factor

For steel, $N_{FO1} = N_{FO2} = 4 \times 10^6$ (MPa). Applying equations (6.3) and (6.4) yield (if $K_{HL}, K_{FL} < 1$, $K_{HL} = 1$ and $K_{FL} = 1$ according to the properties given on p.94):

$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 0.47 < 1 \Rightarrow K_{HL1} = 1$
$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 0.61 < 1 \Rightarrow K_{HL2} = 1$
$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 0.41 < 1 \Rightarrow K_{FL1} = 1$
$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 0.53 < 1 \Rightarrow K_{FL2} = 1$

3.3.4 Calculate $[\sigma_H]$, $[\sigma_{F1}]$, $[\sigma_{F2}]$

Since the motor works in one direction, $K_{FC} = 1$. In ideal conditions, we assume $Z_R Z_V K_{xH} = 1$ and $Y_R Y_s K_{xF} = 1$ according to p.92:
$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_{H1} \approx 518.18$ (MPa)
$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_{H2} \approx 500$ (MPa)
$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC1} K_{FL1}/S_{F1} \approx 257.14$ (MPa)
$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC2} K_{FL2}/S_{F2} \approx 246.86$ (MPa)
The mean permissible contact stress must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller:
$[\sigma_H] = \frac{1}{2}\left([\sigma_{H1}] + [\sigma_{H2}]\right) \approx 509.09 \text{ (MPa)} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H2}]$
The permissible contact stress and bending stress due to overload are calculated as follows:
$[\sigma_H]_{max} = 2.8\sigma_{ch} = 1540$ (MPa)
$[\sigma_F]_{max} = 0.8\sigma_{ch} = 440$ (MPa)

3.4 Transmission Design

3.4.1 Determine basic parameters

Examine table (6.5) gives $K_a = 43$
Assuming symmetrical design, table (6.6) also gives $\psi_{ba} = 0.5$
$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$
From table (6.7) , using interpolation, we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$
Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a using equation (6.15a) gives:
$a = K_a(u_{hg} + 1)\sqrt[3]{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} \approx 113.7$ (mm)
According to SEV229-75 standard, we choose $a_w = 125$ mm

3.4.2 Determine gear meshing parameters

Find m

Applying equation (6.17) and choose m from table (6.8) :
$m = (0.01 \div 0.02)a_w = 1.25 \div 2.5$ (mm) $\Rightarrow m = 1.5$ (mm)

Find z_1, z_2, b_w

Let $\beta = 14°$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1. From the result, z_1 is rounded up to the nearest odd number (preferably a prime number).
$z_1 = \dfrac{2a_w \cos\beta}{m(u_{hg} + 1)} \approx 26.95 \Rightarrow z_1 = 27$
$z_2 = u_{hg} z_1 = 135$
$\Rightarrow b = \psi_{ba} a_w = 62.5$ (mm)

Correct β

There are 2 approaches for correction involving the change of either α or β. Because altering α leads to many other corrections (d_1, d_2 and a_w), β will be used instead.
Since z_1 is rounded, we must find β to obtain the correct angle, ensuring that $\beta \in (8°, 20°)$. Using equation (6.32):
$\beta_w = \arccos \dfrac{m(z_1 + z_2)}{2a_w} \approx 13.59°$

Find x_1, x_2

To find x_1 and x_2, we will follow the calculation scheme provided in p.103. Since $\beta_w \approx 13.59° \in (10, 15]$, $z_{min} = 11$, which leads to z_1 satisfying condition $z_1 \geq z_{min} + 2 > 10$, according to table (6.9). Combined with

$u_{hg} = 5 \geq 3.5$, we obtain $x_1 = 0.3$, $x_2 = -0.3$, disregarding the calculation of y.

3.4.3 Basic parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos\beta} \approx 41.67 \text{ (mm)}$$
$$d_2 = d_{w2} = \frac{mz_2}{\cos\beta} \approx 208.33 \text{ (mm)}$$
$$d_{a1} = d_1 + 2(1 + x_1)m \approx 45.57 \text{ (mm)}$$
$$d_{a2} = d_2 + 2(1 + x_2)m \approx 210.43 \text{ (mm)}$$
$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 38.82 \text{ (mm)}$$
$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 203.68 \text{ (mm)}$$
$$d_{b1} = d_1 \cos\alpha \approx 39.15 \text{ (mm)}$$
$$d_{b2} = d_2 \cos\alpha \approx 195.77 \text{ (mm)}$$
$$\alpha_t = \alpha_{tw} = \arctan\frac{\tan\alpha}{\cos\beta_w} \approx 20.53$$
$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 6.39 \text{ (m/s)}$$

3.4.4 Find $[\sigma_{Hw}]$, $[\sigma_{Fw1}]$ and $[\sigma_{Fw2}]$

In this section, we will try to approximate these parameters based on the factors Z_R, Z_V, K_{xH} and Y_R, Y_s, K_{xF} to substitute to equation (6.1) and (6.2):

$$[\sigma_{Hw}] = [\sigma_H] Z_R Z_V K_{xH}$$
$$[\sigma_{Fw}] = [\sigma_F] Y_R Y_S K_{xF}$$

Assuming smooth surface condition, $Z_R = 1$.
$Z_V = 0.85v^{0.1} \approx 1.02$ with $H \leq 350$.
In case of $v > 5$ (m/s), $K_{xH} = 1$.
The pair of gears are properly polished, which makes $Y_R = 1.1$
$Y_s = 1.08 - 0.0695\ln(m) \approx 1.05$
Since $d_{a1}, d_{a2} \leq 400$ (mm), $K_{xF} = 1$, which leads to:
$[\sigma_{Hw}] = 520.93$ (MPa)
$[\sigma_{Fw1}] = 297.51$ (MPa)
$[\sigma_{Fw2}] = 285.61$ (MPa)

3.4.5 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\varepsilon \sqrt{2T_{sh1} K_H \frac{u_{hg}+1}{b u_{hg} d_{w1}^2}} \leq [\sigma_{Hw}]$$

Find z_M

$z_M = 274$, according to table (6.5)

Find z_H

$\beta_b = \arctan(\cos\alpha_t \tan\beta_w) \approx 12.76° \Rightarrow z_H = \sqrt{2\frac{\cos\beta_b}{\sin(2\alpha_{tw})}} \approx 1.72$

Find z_ε

Obtaining z_ε through calculations:

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_{tw}}{2\pi m \dfrac{\cos\alpha_t}{\cos\beta_w}} \approx 1.41$$

$$\varepsilon_\beta = b\frac{\sin\beta_w}{m\pi} \approx 3.12 > 1 \Rightarrow z_\varepsilon = \varepsilon_\alpha^{-0.5} \approx 0.86$$

Find K_H

We find K_H using equation $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$
From table (6.13), $v \leq 10$ (m/s) $\Rightarrow$ AG = 8

From table (P2.3), using interpolation, we approximate:
$K_{Hv} \approx 1.07$, $K_{Fv} \approx 1.18$
From table (6.14), using interpolation, we approximate:
$K_{H\alpha} \approx 1.1$, $K_{F\alpha} \approx 1.29$
$\Rightarrow K_H \approx 1.3$

Find σ_H

After calculating z_M, z_H, z_ε, K_H, we get the following result:

$$\sigma_H \approx 477.51\ (\text{MPa}) \leq [\sigma_{Hw}] \approx 509.09\ (\text{MPa})$$

3.4.6 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\begin{aligned} \sigma_{F1} &= 2\frac{T_{sh1} K_F Y_\varepsilon Y_\beta Y_{F1}}{b d_{w1} m_n} && \leq [\sigma_{Fw1}] \\ \sigma_{F2} &= \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} && \leq [\sigma_{Fw2}] \end{aligned}$$

Find Y_ε

Knowing that $\varepsilon_\alpha \approx 1.41$, we can calculate $Y_\varepsilon = \varepsilon_\alpha^{-1} \approx 0.71$

Find Y_β

$Y_\beta = 1 - \dfrac{\beta_w}{140} \approx 0.9$

Find Y_F

Using formula $z_v = z\cos^{-3}(\beta_w)$ and table (6.18):
$z_{v1} = z_1 \cos^{-3}(\beta_w) \approx 29.4 \Rightarrow Y_{F1} \approx 3.54$
$z_{v2} = z_2 \cos^{-3}(\beta_w) \approx 147.01 \Rightarrow Y_{F2} \approx 3.63$

Find K_F

Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:
$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.91$

Find σ_F

Since $m_n = m\cos\beta_w \approx 1.46$, substituting all the values, we find out that:

$$\sigma_{F1} \approx 114.11\ \text{(MPa)} \leq [\sigma_{Fw1}] \approx 297.51\ \text{(MPa)}$$
$$\sigma_{F2} \approx 117.01\ \text{(MPa)} \leq [\sigma_{Fw2}] \approx 285.61\ \text{(MPa)}$$

3.4.7 Force on shafts

$F_t = \dfrac{2T_{sh1}}{d_{w1}} \approx 2402.28$ (N)
$F_r = F_t \tan\alpha_{tw} \approx 899.55$ (N)
$F_a = F_t \tan\beta_w \approx 580.75$ (N)
In summary, we have the following table:

	pinion	driving gear
H (HB)	250	240
$[\sigma_F]$ (MPa)	257.14	246.86
$[\sigma_H]$ (MPa)	509.09	
$[\sigma_H]_{max}$ (MPa)	1540	
$[\sigma_F]_{max}$ (MPa)	440	
a_w (mm)	100	
b (mm)	50	
m (mm)	1.5	
d_w (mm)	33.33	166.67
d_a (mm)	37.23	168.77
d_f (mm)	30.48	162.02
d_b (mm)	31.32	156.62
u_{hg}	5	
v (m/s)	5	
x (mm)	0.3	-0.3
z	21	105
α_{tw} ($^\circ$)	20.65	
β_w ($^\circ$)	19.09	

Table 3.1 Gearbox specifications

Chapter 4
Shaft Design

4.1 Nomenclature

- $[s]$ permissible safety factor
- $[\sigma]$ permissible static strength, MPa
- $[\tau]$ permissible torsion, MPa
- a_w shaft distance, mm
- b_O rolling bearing width, mm
- cb role of gear on the shaft (active or passive)
- cq rotational direction of the shaft
- d base shaft diameter, mm
- d_w gear diameter, mm
- F_a axial force, N
- F_r radial force, N
- F_t tangential force, N
- F applied force, N
- h_n distance between bearing lid and bo
- hr tooth direction
- K_x surface tension concentration factor
- K_y diminish factor
- K_σ combined influence factor in tensio
- K_τ combined influence factor in shear
- $\tilde{k}_1$ distance between elements, mm
- $\tilde{k}_2$ distance between bearing surface walls of the gearbox, mm

$\tilde{k}_3$ distance between element surface and bearing lid, mm
k_σ fatigue stress concentration factor in tension
k_τ fatigue stress concentration factor in shear
l length (general), mm
l_m hub length (general), mm
M moment at the cross section, $\mathrm{N \cdot mm}$
M_e equivalent moment, $\mathrm{N \cdot mm}$
l_m hub diameter, mm
q standardized coefficient of shaft diameter
R reaction force, N
r shoulder fillet radius, mm
$\bar{r}$ position of applied force on the shaft, mm
S length defined by table (6.1), mm
s calculated safety factor
s_σ safety factor in tensile stress
s_τ safety factor in shear stress
T torque at the cross section, $\mathrm{N \cdot mm}$
W section modulus, $\mathrm{mm^3}$
W_O polar section modulus, $\mathrm{mm^3}$
α_{tw} traverse meshing angle, °
β helix angle, °
ψ_σ mean stress influence factor
ψ_τ mean shear influence factor
σ_{-1} endurance limit at stress ratio
σ_a tensile stress amplitude, MPa
σ_b ultimate strength, MPa
σ_{ch} yield limit, MPa
σ_m mean tensile stress, MPa
σ_{td} static strength, MPa
τ_{-1} endurance limit at shear ratio
τ_a shear stress amplitude, MPa
τ_m mean shear stress, MPa
$_1$ subscript for shaft 1
$_2$ subscript for shaft 2
$_{max}$ subscript for maximum value
$_{sh1}$ subscript for shaft 1
$_{sh2}$ subscript for shaft 2
$_x$ subscript for x-axis
$_y$ subscript for y-axis
$_z$ subscript for z-axis

4.2 Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows: $S \leq 100$ (mm), HB260, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 650$ (MPa).

4.3 Transmission Design

4.3.1 Load on shafts

4.3.1.1 Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.
On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:
$\bar{r}_{12} = -d_{w12}/2 \approx -20.83$ (mm), $\mathrm{hr}_{12} = +1$, $\mathrm{cb}_{12} = +1$, $\mathrm{cq}_1 = +1$
$\bar{r}_{21} = +d_{w21}/2 \approx +104.17$ (mm), $\mathrm{hr}_{21} = -1$, $\mathrm{cb}_{21} = -1$, $\mathrm{cq}_2 = -1$

Find magnitude of F_t, F_r, F_a

Using the results from the previous chapter: , $\beta_w = 13.59°$, $d_{w12} \approx 41.67$ (mm)

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 2402.28\ (\mathrm{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha}{\cos\beta_w} \approx 925.46\ (\mathrm{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta_w \approx 580.75\ (\mathrm{N}) \end{cases}$$

Find direction of F_t, F_r, F_a

Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\mathrm{cq}_1\mathrm{cb}_{12}F_{t12} \approx -2402.28\,(\mathrm{N}) \\ F_{y12} = -\dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\dfrac{\tan\alpha}{\cos\beta_w}F_{t12} \approx 925.46\,(\mathrm{N}) \\ F_{z12} = \mathrm{cq}_1\mathrm{cb}_{12}\mathrm{hr}_{12}F_{t12}\tan\beta_w \approx 580.75\,(\mathrm{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\mathrm{cq}_2\mathrm{cb}_{21}F_{t21} \approx 2402.28\,(\mathrm{N}) \\ F_{y21} = -\dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\dfrac{\tan\alpha_{tw}}{\cos\beta_w}F_{t21} \approx -925.46\,(\mathrm{N}) \\ F_{z21} = \mathrm{cq}_2\mathrm{cb}_{21}\mathrm{hr}_{21}F_{t21}\tan\beta_w \approx -580.75\,(\mathrm{N}) \end{cases}$$

4.3.1.2 Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2678.96$ (N) (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22}\cos 210^\circ \approx -2320.05\,(\mathrm{N}) \\ F_{y22} = F_{r22}\sin 210^\circ \approx -1339.48\,(\mathrm{N}) \end{cases}$$

4.3.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques T_{sh1} and T_{sh2}, respectively, $[\tau_1] = 15$ (MPa) and $[\tau_2] = 30$ (MPa). Using equation (10.9), we can approximate the base shaft diameters d_1 and d_2:

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 25.55\,(\mathrm{mm})$$
$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 34.1\,(\mathrm{mm})$$

Recall that our motor is 4A160M2Y3, inspecting table P1.7 we obtain the motor's output shaft diameter is 42 (mm). According to the recommendations on p.189, we limit the chosen range of $d_1 \geq (0.8 \div 1.2) \times 42\,(\text{mm})$. For d_2, the chosen range must be around $(0.3 \div 0.35) \times a_w\,(\text{mm})$. Thus, $d_1 = 35\,(\text{mm})$, $d_2 = 40\,(\text{mm})$. Consulting table (10.2) gives $b_{O1} \approx 21\,(\text{mm})$ and $b_{O2} \approx 23\,(\text{mm})$

4.3.3 *Identify the distance between bearings and applied forces*

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.
Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 = 45\,(\text{mm})$, $l_{m23} = l_{m22} = 1.5d_2 = 52.5\,(\text{mm})$, where l_{m22} is the chain hub.
From table (10.3), we choose $\tilde{k}_1 = 10\,(\text{mm})$, $\tilde{k}_2 = 8\,(\text{mm})$, $\tilde{k}_3 = 15\,(\text{mm})$, $h_n = 18\,(\text{mm})$. This parameters apply for both shafts in the system.
Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:
On shaft 1:
$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + \tilde{k}_3 + h_n\right] = -69.75\,(\text{mm})$
$l_{13} = 0.5(l_{m13} + b_{O1}) + \tilde{k}_1 + \tilde{k}_2 = 54.75\,(\text{mm})$
$l_{11} = 2l_{13} = 109.5\,(\text{mm})$
On shaft 2:
$l_{22} = -l_{c22} = -\left[0.5(l_{m22} + b_{O2}) + \tilde{k}_3 + h_n\right] = -74.5\,(\text{mm})$
$l_{23} = 0.5(l_{m23} + b_{O2}) + \tilde{k}_1 + \tilde{k}_2 = 59.5\,(\text{mm})$
$l_{21} = 2l_{23} = 119\,(\text{mm})$

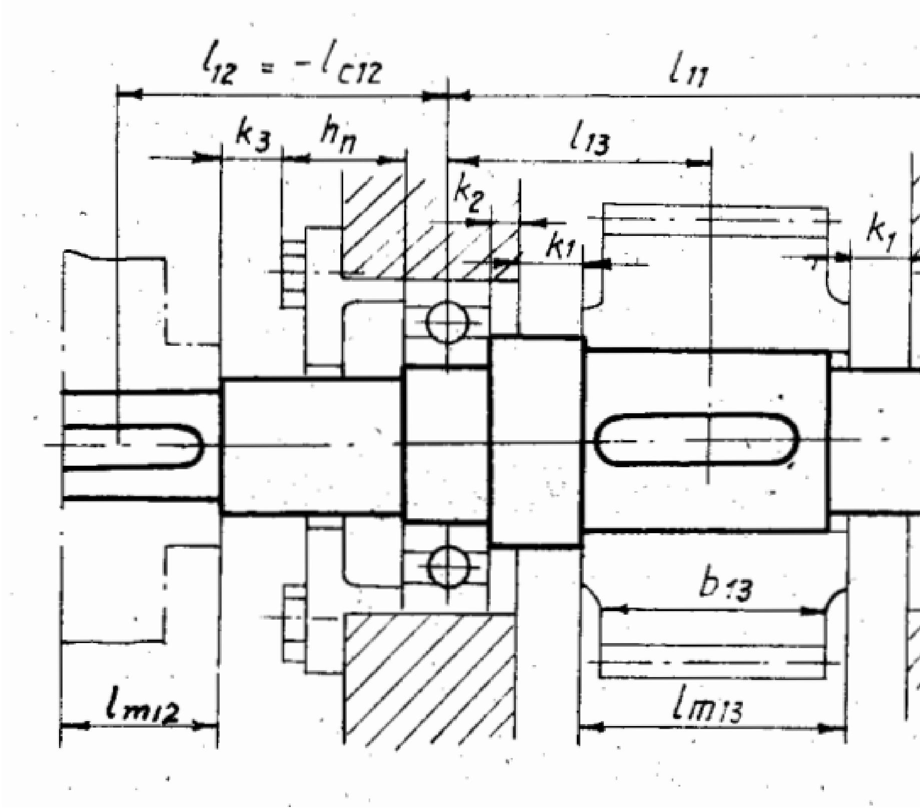


Fig. 4.1 Shaft design and its dimensions

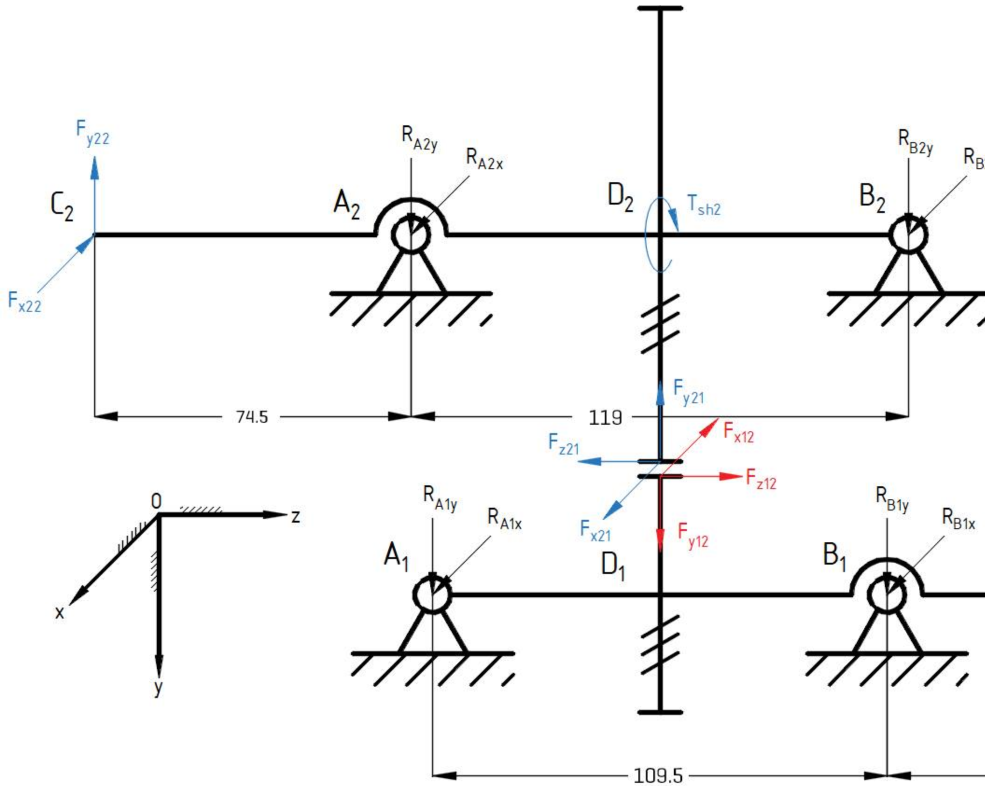


Fig. 4.2 Force analysis of 2 shafts

4.3.4 Determine shaft diameters and lengths

Find reaction forces

From the diagram, we solve for the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F_i} & = 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} & = 0 \end{cases}$$

we obtain the results:

$$\begin{cases} R_{A1x} \approx 1201.14 \text{ (N)} \\ R_{A1y} \approx -352.24 \text{ (N)} \\ R_{B1x} \approx 1201.14 \text{ (N)} \\ R_{B1y} \approx -573.22 \text{ (N)} \end{cases} \qquad \begin{cases} R_{A2x} \approx 943.15 \text{ (N)} \\ R_{A2y} \approx 3668.4 \text{ (N)} \\ R_{B2x} \approx -2005.96 \text{ (N)} \\ R_{B2y} \approx -422.89 \text{ (N)} \end{cases}$$

The total bending moments at 8 critical cross sections are also calculated (we use the formula (10.15) to derive $M = \sqrt{M_x^2 + M_y^2}$ at each section):

$$\begin{cases} M_{A1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{D1}^{-} \approx 68531.85 \text{ (N} \cdot \text{mm)} \\ M_{D1}^{+} \approx 72867.4 \text{ (N} \cdot \text{mm)} \\ M_{B1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{C1} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases} \qquad \begin{cases} M_{C2} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{A2} \approx 191545.76 \text{ (N} \cdot \text{mm)} \\ M_{D2}^{-} \approx 146910 \text{ (N} \cdot \text{mm)} \\ M_{D2}^{+} \approx 121977.78 \text{ (N} \cdot \text{mm)} \\ M_{B2} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases}$$

Draw bending moment - torque diagrams

Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

Find equivalent moments

Knowing T_{sh1} and T_{sh2}, we calculate equivalent moment M_e at the 8 cross sections specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

$$\begin{cases} M_{eA1} \approx 0\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eD1}^{-} \approx 81087.5\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eD1}^{+} \approx 84783.4\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eB1} \approx 43342.46\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eC1} \approx 43342.46\,(\mathrm{N}\cdot\mathrm{mm}) \end{cases} \qquad \begin{cases} M_{eC2} \approx 205963.35\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eA2} \approx 281266.2\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eD2}^{-} \approx 252989.03\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eD2}^{+} \approx 239373.1\,(\mathrm{N}\cdot\mathrm{mm}) \\ M_{eB2} \approx 0\,(\mathrm{N}\cdot\mathrm{mm}) \end{cases}$$

Find permissible stress

$[\sigma_1]$ and $[\sigma_2]$ are determined by table (10.5). Since we use quenched 45X steel, $[\sigma_1] = 67$ (MPa) and $[\sigma_2] = 64$ (MPa) ($[\sigma_2]$ is achieved using interpolation).

Find standardized diameters at specific locations on the shaft

Having M_e and $[\sigma]$, the next step is to estimate specific diameter at the key points mentioned above using equation (10.17) on p.194, which only applies for rigid shafts:

$$d \geq \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\begin{cases} d_{A1} \approx 0\,(\mathrm{mm}) \\ d_{D1} \approx 23.66\,(\mathrm{mm}) \\ d_{B1} \approx 18.92\,(\mathrm{mm}) \\ d_{C1} \approx 18.92\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} \approx 32.32\,(\mathrm{mm}) \\ d_{A2} \approx 35.86\,(\mathrm{mm}) \\ d_{D2} \approx 34.61\,(\mathrm{mm}) \\ d_{B2} \approx 0\,(\mathrm{mm}) \end{cases}$$

Through rough calculations, we will choose the diameters according to standards given on p.195 (one applies for bearings while the other is used for the remaining machine elements):

$$\begin{cases} d_{A1} = 35\,(\mathrm{mm}) \\ d_{D1} = 24\,(\mathrm{mm}) \\ d_{B1} = 35\,(\mathrm{mm}) \\ d_{C1} = 19\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} = 34\,(\mathrm{mm}) \\ d_{A2} = 40\,(\mathrm{mm}) \\ d_{D2} = 36\,(\mathrm{mm}) \\ d_{B2} = 40\,(\mathrm{mm}) \end{cases}$$

4.4 Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

where $s_\sigma = \dfrac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$

$s_\tau = \dfrac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$

Assuming the surfaces are smooth, properly ground and quenched by high frequency voltage, we obtain $K_x = 1$ from table (10.8) and $K_y = 1.4$ from table (10.9), where $[\sigma_b] = 850$ (MPa) is the property of quenched 45X steel.

Find σ_{-1}, τ_{-1}

Using formulas on p.196:
$\sigma_{-1} = 0.35[\sigma_b] + 120 \approx 417.5$ (MPa)
$\tau_{-1} \approx 0.58\sigma_{-1} \approx 242.15$ (MPa)

Find $\sigma_a, \tau_a, \sigma_m, \tau_m$

For this part, we divide into 3 key points:

1. For rotating shaft, $\sigma_m = 0, \sigma_a = \dfrac{\sqrt{M_x^2 + M_y^2}}{W}$ (equation (10.22)), where M_x and M_y are at the cross section of interest.
2. By design, the shafts only rotate in one direction, thus $\tau_m = \tau_a = \dfrac{T_{sh}}{2W_O}$ (equation (10.23)).

3. We also assume the shafts have circular cross section, which makes $W = \dfrac{\pi d^3}{32}$ and $W_O = \dfrac{\pi d^3}{16}$ according to table (10.6), where d is the diameter of a cross section of the shaft.

The table below shows the results after calculation: Since $\sigma_b = 850$ (MPa)

	d (mm)	W (mm^3)	W_O (mm^3)	σ_m (MPa)	σ_a (MPa)	τ_m (MPa)	τ_a (MPa)
A_1	20	785.4	1570.8	0	0	15.93	15.93
D_1	24	1357.17	2714.34	0	49.2	9.22	9.22
B_1	20	785.4	1570.8	0	0	15.93	15.93
C_1	19	673.38	1346.76	0	0	18.58	18.58
C_2	32	3216.99	6433.98	0	0	18.48	18.48
A_2	40	6283.19	12566.37	0	29.74	9.46	9.46
D_2	34	3858.66	7717.32	0	36.67	15.41	15.41
B_2	35	4209.24	8418.49	0	0	14.13	14.13

Table 4.1 Calculated variables for $\sigma_a, \tau_a, \sigma_m, \tau_m$

for both shafts, $\psi_\sigma = 0.1$ and $\psi_\tau = 0.05$

Find K_σ, K_τ

We calculate K_σ using formula:

$$K_\sigma = \left(\frac{k_\sigma}{\varepsilon_\sigma} + K_x - 1\right) K_y^{-1}$$

and K_τ with:

$$K_\tau = \left(\frac{k_\tau}{\varepsilon_\tau} + K_x - 1\right) K_y^{-1}$$

Table (10.10), (10.11) and (10.13) are examined to find $\frac{k_\sigma}{\varepsilon_\sigma}$ ratio. Given $[\sigma_H] = 850\,(\mathrm{MPa})$ base shaft diameters d_1 and d_2 are compared to the diameters at critical locations A, B, C, D. If the base shaft is smaller, table (10.10) and (10.11) are used. If it is larger, we will use table (10.13) instead; the concentration stress factor in this case is demonstrated in the figure:

Final calculation is provided in the table:

	d (mm)	r	k_σ	k_τ	ε_σ	ε_τ	$\frac{k_\sigma}{\varepsilon_\sigma}$	$\frac{k_\tau}{\varepsilon_\tau}$	K_x	K_y	K_σ	K_τ
A_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
D_1	24	0.48	3	1.95	0.81	0.85	3.7	2.29	1	1.4	2.65	1.64
B_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
C_1	19	0.38	3	1.95	0.84	0.89	3.57	2.19	1	1.4	2.55	1.57
C_2	32	0.64	3	1.95	0.76	0.80	3.95	2.44	1	1.4	2.82	1.74
A_2	40	-	-	-	-	-	3.34	2.46	1	1.4	2.39	1.76
D_2	34	0.68	3	1.95	0.74	0.80	4	2.44	1	1.4	2.86	1.75
B_2	35	-	-	-	-	-	3.3	2.44	1	1.4	2.36	1.74

Table 4.2 Calculated variables in K_σ and K_τ

Find s_σ, s_τ and s

Combining the results altogether, we obtain the following table:
Since the smallest safety factor is at the cross section D_1, which has the value of $3.14 > [s] = 1.5 \div 2.5$, we can neglect rigidity analysis according to the conclusion on p.195.

	s_σ	s_τ	s
A_1	$\gg s_\tau$	9.41	9.41
D_1	3.21	16	3.14
B_1	$\gg s_\tau$	9.41	9.41
C_1	$\gg s_\tau$	8.07	8.07
C_2	$\gg s_\tau$	7.32	7.32
A_2	5.88	14	5.43
D_2	3.99	8.77	3.63
B_2	$\gg s_\tau$	9.56	9.56

Table 4.3 Safety factor at critical cross sections

4.5 Static Strength Analysis

Along with fatigue strength, static strength is also considered and every shaft must satisfy the following condition at critical cross sections (equation (10.27)):

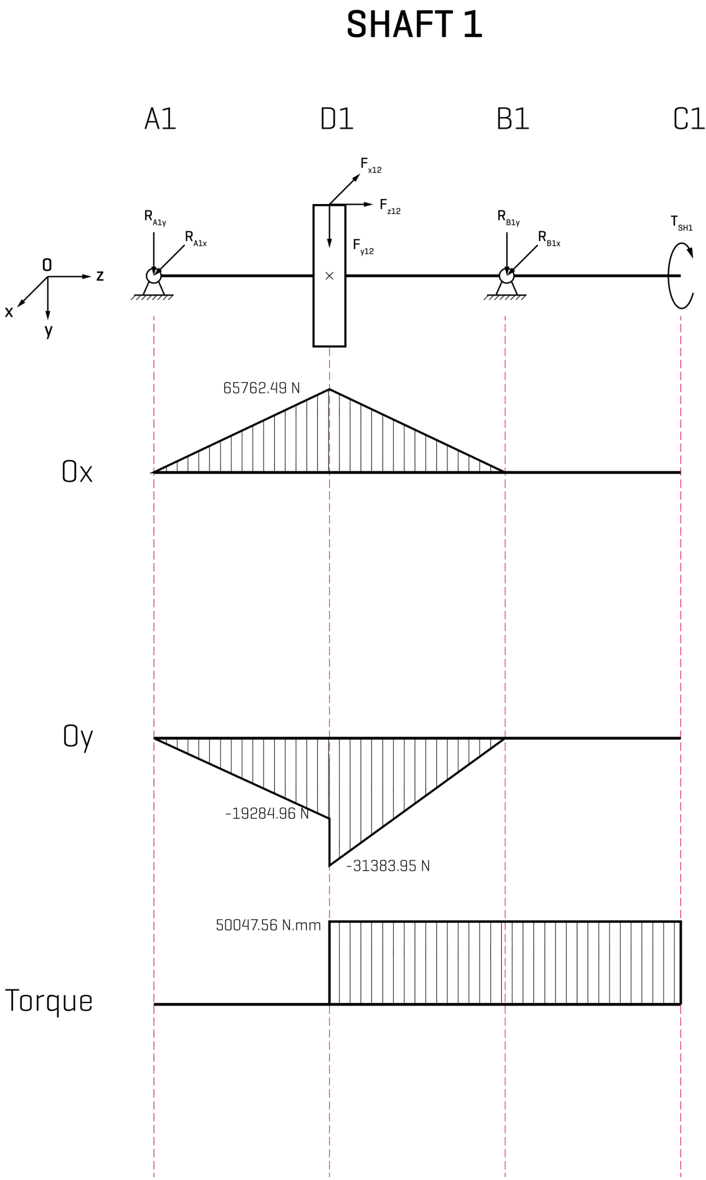
$$\sigma_e = \sqrt{\left(\frac{M_{max}}{0.1d^3}\right)^2 + 3\left(\frac{T_{max}}{0.2d^3}\right)^2} \leq [\sigma]$$

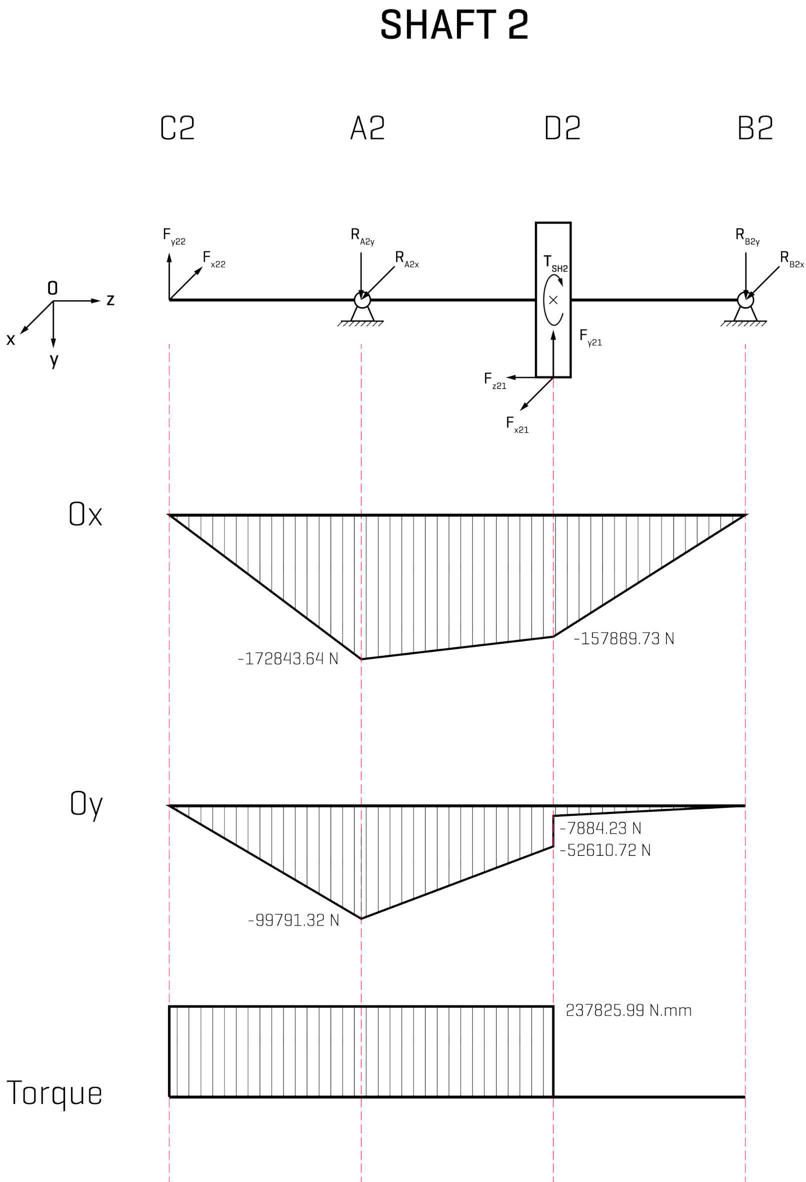
where M_{max}, T_{max} are the largest bending moment and torque at the cross section, respectively. Let $[\sigma] \approx 0.8\sigma_{ch} = 520$ (MPa), the results are in the table below:

	A_1	D_1	B_1	C_1	C_2	A_2	D_2	B_2
σ_e (MPa)	54.18	57.59	54.18	63.19	62.86	43.45	63.58	48.04

Table 4.4 Calculated static strength at critical cross sections

which satisfy the given condition.


SHAFT 1
A1
D1
B1
C1
F x12
F z12
F y12
R A1y
R A1x
R B1y
R B1x
T SH1
O
z
x
y
65762.49 N
Ox
Oy
-19284.96 N
-31383.95 N
50047.56 N.mm
Torque


SHAFT 2
C2
A2
D2
B2
F_{y22}
F_{x22}
R_{A2y}
R_{A2x}
T_{SH2}
R_{B2y}
R_{B2x}
O
z
x
y
F_{y21}
F_{z21}
F_{x21}
Ox
-172843.64 N
-157889.73 N
Oy
-7884.23 N
-52610.72 N
-99791.32 N
237825.99 N.mm
Torque

Chapter 5

Bearing Design

5.1 Nomenclature

- b_O rolling bearing width, mm
- C standardized dynamic load rating, N
- C_d basic dynamic load rating, N
- d diameter, mm
- F_a axial force, kN
- F_r radial force, kN
- F_t tangential force, kN
- F applied force, kN
- h_n distance between bearing lid and bolt, mm
- hr tooth direction
- k_d temperature factor
- k_t load condition factor
- L rated life in million revolutions, m
- L_h rated life in hours, h
- l length (general), mm
- l_m hub length (general), mm
- M moment, $N \cdot mm$
- M_e equivalent moment, $N \cdot mm$
- M_{max} maximum moment at the cross sect
- m load-life exponent
- Q equivalent dynamic load, kN
- q standardized coefficient of shaft di
- R reaction force, N

T torque at the cross section, $\text{N} \cdot \text{mm}$
X dynamic radial load factor
Y dynamic axial load factor
α contact angle, $^\circ$
$_{sh1}$ subscript for shaft 1
$_{sh2}$ subscript for shaft 2
$_x$ subscript for x-axis
$_y$ subscript for y-axis
$_z$ subscript for z-axis

5.2 Choose bearing type

As for the types, we will examine $\dfrac{F_a}{F_r}$ at A_1, B_1, A_2 and B_2 in the 2 shafts from the previous chapter, where F_a is the output axial force $|F_{z12}| = |F_{z21}| \approx 580.75\,(\text{N})$; F_r is the magnitude of combined reaction force $\sqrt{R_x^2 + R_y^2}$ from the shaft onto the bearing, which is essentially a radial load. The larger ratio in a shaft will be our ratio of choice to determine the bearing type.
Taking our results from the chapter 4:

$$\begin{cases} F_{rA1} = \sqrt{R_{A1x}^2 + R_{A1y}^2} \approx 1.2\,(\text{kN}) \\ F_{rB1} = \sqrt{R_{B1x}^2 + R_{B1y}^2} \approx 1.33\,(\text{kN}) \\ F_{aA1} = |F_{z12}| \approx 0.58\,(\text{kN}) \\ F_{aB1} = |F_{z12}| \approx 0.58\,(\text{kN}) \end{cases} \qquad \begin{cases} F_{rA2} = \sqrt{R_{A2x}^2 + R_{A2y}^2} \approx 3.79 \\ F_{rB2} = \sqrt{R_{B2x}^2 + R_{B2y}^2} \approx 2.05 \\ F_{aA2} = |F_{z21}| \approx 0.58\,(\text{kN}) \\ F_{aB2} = |F_{z21}| \approx 0.58\,(\text{kN}) \end{cases}$$

yields

$$\begin{cases} \dfrac{F_{aA1}}{F_{rA1}} \approx 0.46 \\ \dfrac{F_{aB1}}{F_{rB1}} \approx 0.44 \end{cases} \qquad \begin{cases} \dfrac{F_{aA2}}{F_{rA2}} \approx 0.15 \\ \dfrac{F_{aB2}}{F_{rB2}} \approx 0.28 \end{cases}$$

Since $0.46 > 0.3$ and $0.28 \le 0.3$, the pair of bearings on shaft 1 is single-row angular contact ball bearings with $\alpha_{sh1} = 12^\circ$ and the remaining pair is single-row deep-groove bearings ($\alpha_{sh2} = 0^\circ$); AG = 0 according to

the recommendations on p.212 and p.213.
We also have dimensions at the cross sections A_1, B_1, A_2, B_2 from the previous chapter:

$$\begin{cases} b_{O1} = 21\,(\text{mm}) \\ d_{A1} = 35\,(\text{mm}) \\ d_{B1} = 35\,(\text{mm}) \end{cases} \qquad \begin{cases} b_{O2} = 23\,(\text{mm}) \\ d_{A2} = 40\,(\text{mm}) \\ d_{B2} = 40\,(\text{mm}) \end{cases}$$

From these parameters, we will look up the tables at the end of the text. The pair of single-row angular contact ball bearings of choice is 46307, which is suitable for shaft 1 and $C_{o1} = 25.2\,(\text{kN})$. On shaft 2, the pair of single-row deep-grove bearings are type 308, where $C_{o2} = 21.7\,(\text{kN})$.

5.3 Bearing dimensions

5.3.1 Calculate basic dynamic load rating

$$C_d = Q_e \sqrt[m]{L}$$

5.3.1.1 Find equivalent dynamic load

Since we only use ball bearings, the following formula applies:

$$Q = (XVF_r + YF_{at})k_t k_d$$

Since the inner ring rotates, $V = 1$ and the $\dfrac{F_a}{VF_r} = \dfrac{F_a}{F_r}$, meaning that the ratios in the section above will be used to examine X and Y.
The design problem also does not give any further information about operating temperature, which gives $k_t = 1$. In addition, we get $k_d = 1$ from table (11.3) based on the machine's condition (low load and power rating).

Find the ratio $\dfrac{iF_a}{C_o}$

This ratio is calculated and applied for 2 shafts ($i = 1$ for single-row bearings in our case):
For shaft 1, $\dfrac{F_a}{C_{o1}} \approx 0.027$
For shaft 2, $\dfrac{F_a}{C_{o2}} \approx 0.023$

Compare with e

From the previous section, $\alpha_{sh1} = 12°, \alpha_{sh2} = 0°$. Inspecting table (11.4) and by interpolation, $e_{sh1} \approx 0.33$, $e_{sh2} \approx 0.22$. These values are then compared to $\left|\dfrac{F_a}{VF_r}\right|$ to look up the correct column.

Find X, Y

Table (11.3) and interpolation are used in finding these values:

For shaft 1, $\left|\dfrac{F_a}{VF_r}\right| = \left|\dfrac{F_{z12}}{VR_{A1y}}\right| \approx 0.46 > e_1 \Rightarrow X_1 = 0.56, Y_1 = 2.1.$
For shaft 2, $\left|\dfrac{F_a}{VF_r}\right| = \left|\dfrac{F_{z21}}{VR_{B2y}}\right| \approx 0.28 > e_2 \Rightarrow X_2 \approx 0.45, Y_2 \approx 1.64.$

Find F_{at}

For shaft 1, additional radial forces are also applied to the pair of angular contact ball bearings. From table (11.5), the first arrangement is used in the gearbox. Therefore, $\mathbf{F_{sA1}} \upharpoonleft\!\upharpoonright \mathbf{F_{z12}}$ and $\mathbf{F_{sB1}} \upharpoonleft\!\downharpoonright \mathbf{F_{z12}}$ (the direction of $\mathbf{F_{z12}}$ can be found at Figure 4.2) Following the sign convention on p.218 and

combining with equation (11.8), (11.10), (11.11a) and (11.11b): At cross section A_1:
$F_{sB1} = e_1 F_{rB1} \approx 0.43$ (kN)
$\sum F_{aA1} = F_{sB1} - F_{z12} \approx -0.15$ (kN)
At cross section B_1:
$F_{sA1} = e_1 F_{rA1} \approx 0.41$ (kN)
$\sum F_{aB1} = F_{sA1} + F_{z12} \approx 0.99$ (kN)
From equation (11.11a) and (11.11b):
$\sum F_{aA1} \leq F_{sA1} \Rightarrow F_{atA1} = F_{sA1} \approx 0.41$ (kN)
$\sum F_{aB1} > F_{sB1} \Rightarrow F_{atB1} = \sum F_{aB1} \approx 0.99$ (kN)

In contrast, shaft 2 does not have such additional forces since $\alpha_{sh2} = 0°$. Therefore, $F_{atA2} = F_{atB2} = |F_{z21}| = 0.58$ (kN).

Find Q_1, Q_2

Recall that the forces are in absolute values, the parameters are substituted to the formula to obtain:
$Q_{A1} \approx 1.56$ (kN)
$Q_{B1} \approx 2.82$ (kN)
$Q_{A2} \approx 2.66$ (kN)
$Q_{B2} \approx 1.87$ (kN)
We will compare these values and choose the larger load (according to the recommendation on p.219):
$Q_{A1} < Q_{B1} \Rightarrow Q_1 = Q_{B1} \approx 2.82$ (kN)
$Q_{A2} > Q_{B2} \Rightarrow Q_2 = Q_{A2} \approx 2.66$ (kN)

Find Q_1

Modifying equation (11.12), we obtain the equivalent load on 2 shafts (assuming the bearings are ball type):

$$Q_e = Q\sqrt[3]{\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2}}$$

For shaft 1, $Q_{e1} \approx 2.17$ (kN)
For shaft 2, $Q_{e2} \approx 1.88$ (kN)

5.3.1.2 Verifying condition for C_d

Find L

Equation (11.2) is rearranged to calculate L:

$$L = L_h 60 n_{sh} \times 10^6$$

The transmission system works for 19200 (hours) (the calculation has already been done in chapter 3), which gives:
$L_1 \approx 3375.36$ (million rev)
$L_2 \approx 675.07$ (million rev)

Find C_d

Combining the results and letting $m = 3$ (ball bearings are used in this case) yield:
$C_{d1} = 32.49$ (kN)
$C_{d2} = 16.53$ (kN)

Compare with C

From table (P2.15), we obtain the ratios using interpolation (knowing n_{sh1} and n_{sh2} and $L_h = 19200$ (hours)):
$C_1 \approx 14.96 Q_{e1} \approx 32.39$ (kN) $< C_{d1}$
$C_2 \approx 8.75 Q_{e2} \approx 16.47$ (kN) $< C_{d2}$

Since both shafts do not satisfy the condition, we will reduce the rated life L_h of the bearings in half based on the recommendation on p.220 and repeat the process of verification. The results are:
$C_1 \approx 26.84\,(\text{kN}) \geq C_{d1} \approx 25.79\,(\text{kN})$
$C_2 \approx 13.66\,(\text{kN}) \geq C_{d2} \approx 13.12\,(\text{kN})$

5.3.2 Calculate static load rating

The bearings are non-rotating, thus we will verify its static load rating using condition (11.18) on p.221:

$$\begin{cases} Q_t = X_O F_r + Y_O F_{at} \\ Q_t = F_r \end{cases} \leq C_O$$

For shaft 1, $X_O = 0.5, Y_O = 0.47$.
$Q_{t1} \approx 0.76\,(\text{kN}) < C_{o1}$
For shaft 2, $X_O = 0.6, Y_O = 0.5$.
$Q_{t2} \approx 0.64\,(\text{kN}) < C_{o2}$