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ME3011

Design Project Report

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I am also thankful for my friends and classmates. They are the readers that give me an objective view on how writing style affects the readability of the entire report.

Abstract

In machine design, every machine element must be calculated in a systematic matter. In this course, students are provided with essential skills to formulate almost every dimension manually, thus further improving their engineering skills before engaging the high-energy, fast-paced workforce.

When a machine element is being developed, it must satisfy some key engineering specifications such as being able to operate under designated lifespan, low cost and high efficiency. Other aspects are less important but also determined the overall design of the element include compactness, noise emission, appearance, etc.

To optimize the process of machine design, the general principles are considered as follows:

1. Identify the working principle and workload of the machine.
2. Formulate the overall working principle to satisfy the problem. Proposing feasible solutions and evaluating them to find the optimal design specifications.
3. Find force and moment diagram exerting on machine parts and characteristics of the workload.
4. Choose appropriate materials to make use of their properties and improve efficiency as well as reliability of individual elements.
5. Calculate dynamics, strength, safety factor, etc. to specify dimensions.
6. Design machine structure, parts to satisfy working condition and assembly.
7. Create presentation, instruction manual and maintenance.

In this design project, a simple system is examined to deliver a concrete example close to real world application. Although recommendations from reference books are taken into consideration, the numerical values provided are oftentimes obsolete and subjective. Thus, experience-based numbers are consulted according to the supervisor's point of view and the writer's preference.

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Design Problem

Nomenclature

C_a	number of shift daily, shifts	P	design power of the mixing tank, kW
K_{ng}	working days/year, days	T_1	working torque 1, $N \cdot m$
L	service life, years	T_2	working torque 2, $N \cdot m$
n	rotational velocity of the mixing tank, rpm	t_1	working time 1, s
		t_2	working time 2, s

I Problem

The problem is downloaded from E-learning website, designated number 8, see Figure 1.

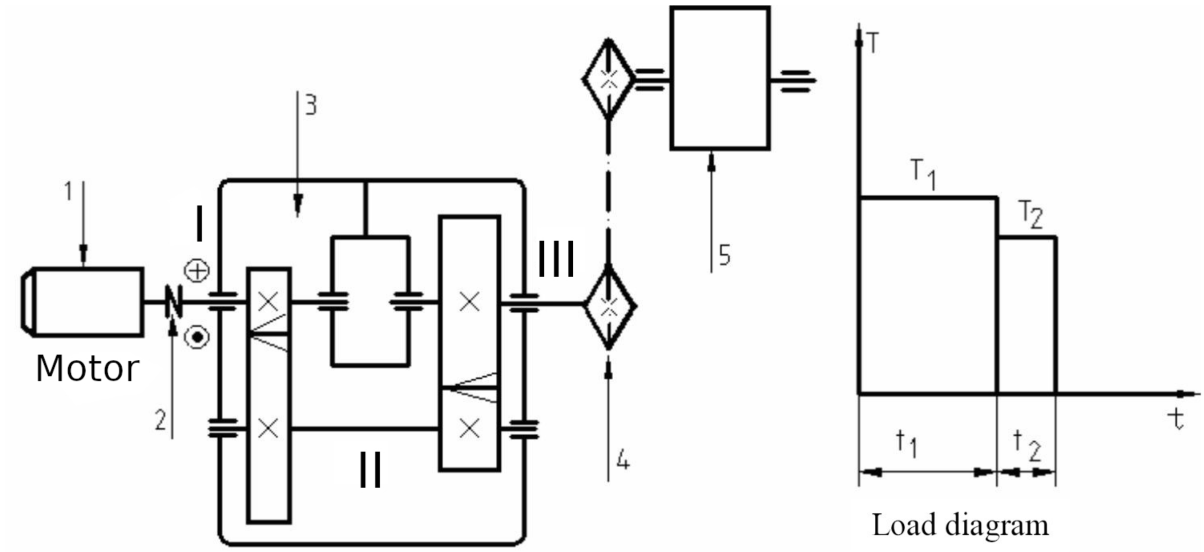


Figure 1 Working principle diagram and workload of the mixing machine: 1) electric motor, 2) elastic coupling, 3) two-stage coaxial helical speed reducer, 4) roller chain drive, 5) mixing tank (one-directional, light duty, operate 1 shift, 8 hours each)

II Mixing machine parameters

From the parameters given in the document, we have:

$P = 7$ (kW)	$t_1 = 15$ (s)
$n = 65$ (rpm)	$t_2 = 11$ (s)
$L = 8$ (years)	$T_1 = T$ ($N \cdot m$)
$K_{ng} = 260$ (days)	$T_2 = 0.7T$ ($N \cdot m$)
$Ca = 1$ (shifts)	

III Requirements

- 01 report.
- 01 assembly drawing.
- 01 detailed drawing.

IV Design problem

1. Decide the working power of the electric motor and transmission ratio of the system.
2. Calculate and design machine elements:
   (a) Calculate system drives (belt, chain or gear).
   (b) Calculate the elements in speed reducers (gears, lead screws).
   (c) Draw and calculate force diagram exerting on the transmission elements.
   (d) Calculate, design shafts and keys.
   (e) Choose bearings and couplings.
   (f) Choose machine bodies, fasteners and other elements.
3. Choose assembly tolerance.
4. Bibliography

Chapter 1

Choose Motor

I Motor selection for the mixing tank

1.1 Calculate system overall efficiency η_{sys}

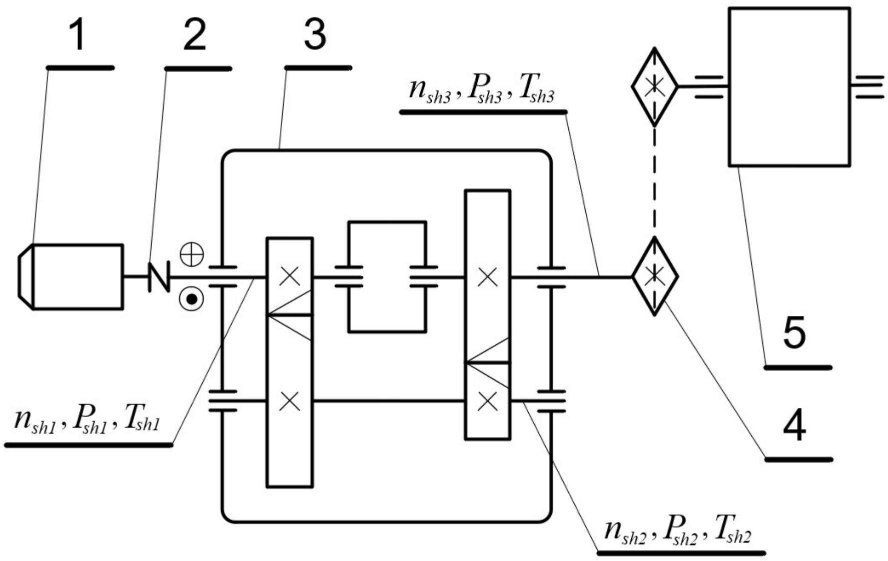


Figure 1.1 Working principle diagram with annotation for shafts

From Figure 1.1, the efficiency η_{sys} of the system is calculated using:

$$\eta_{sys} = \eta_c \eta_b^4 \eta_{hg}^2 \eta_{ch} = 0.99 \times 0.99^4 \times 0.98^2 \times 0.95 = 0.87$$

where

- $\eta_c = 0.99$ is the flexible coupling efficiency, Table 2.3 [9]. The coupling connects the motor and the speed reducer. In principle, it is designed to transmit torque smoothly while permitting some axial, radial and angular misalignment, typically ±3° [6]. Therefore, the power loss from motor shaft to shaft 1 should be included since it eventually transfers into sound and heat radiation.

- $\eta_b = 0.99$ is the bearings efficiency, Table 2.3 [9]. Housing is provided to 4 rolling bearings, 3 of which are in the speed reducer and the last one is used for the shaft of the mixing tank. During calculations, it is safer to assume the lowest value for better reliability. Therefore, the lowest efficiency is taken.

- $\eta_{hg} = 0.98$ is the helical gear efficiency, Table 2.3 [9]. In the speed reducer are 2 sealed pairs of helical gear drives. One pair connects shaft 1 and shaft 2, the other connects shaft 2 and 3. A common rule of thumb for spur, helical, and bevel gear meshes is to assume each mesh, including gears and supporting bearings, incurs a 2 percent power loss [2].

- $\eta_{ch} = 0.95$ is the chain drive efficiency, Table 2.3 [9]. The chain is protected via housing and provides connection from the speed reducer to the mixing tank. Similar to the approach from rolling bearing efficiency selection, the efficiency of the chain is chose at the lowest value.

1.2 Calculate required power P_{mo} for operation

The power P from design problem is the operating power of the mixing tank. In case of varying load each cycle, the equivalent power P_{mo} is calculated using Equation 2.13 [9]:

$$P_w = P\sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} = 7 \times \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15 + 11}} = 6.2\ (\mathrm{kW})$$

$$P_{mo} = \frac{P_w}{\eta_{sys}} = \frac{6.2}{0.87} = 7.14\ (\mathrm{kW})$$

where

- P_w is the operating power of the mixing tank given the workload, kW.

- P, T_1, T_2, t_1, t_2 are given in the design problem; η_{sys} is given in the previous section.

1.3 Choose motor

There are 2 common speed values for an induction motor in Vietnam which uses line frequency of 50 Hz:

1. Motor with 2 poles at nominal speed 3000 rpm, which is portable and cheap. However, high system ratio could result in additional expenditure in other machine elements.

2. Motor with 4 poles at nominal speed 1500 rpm, which is large and more costly. However, low speed ratio should be adequate since power transmission is the goal of the system, which could in principle, reduce the size of other elements. The choice is this motor type will affect u_h and u_{ch} in the next part.

Calculate working speed n_{mo} The selection of system speed ratio should be close to $1500/n = 1500/65 = 23.08$. The working speed n_{mo} is calculated as:

$$n_{mo} = u_{sys} n = 22.4 \times 65 = 1456\ (\mathrm{rpm})$$

where

- u_{sys} is the speed ratio of the system. The formula for u_{sys} is:

  $$u_{sys} = u_h u_{ch} = 8 \times 2.8 = 22.4$$

  where

  - $u_h = 8$ is the speed ratio of the speed reducer, which is a 2-level transmission, spur gear type, Table 2.4 [9]. The transmission ratio u_h of the speed reducer should be kept at minimum since in general, it is more costly (material, machining, maintenance, etc.) to manufacture than other mechanical drives such as belt drive and chain drive. The choice of this transmission ratio will be explained clearly in the next section.
  - $u_{ch} = 2.8$ is the speed ratio of the chain drive, roller type, Table 2.4 [9]. This mechanical drive provides the remaining factor to get close to the desired ratio.
- n is given in the design problem.

Choose motor The power of the motor should be around $P_{mo} = 7.14$ kW. Thus, from Table P1.3 [9], we choose motor 4A132S4Y3 operating at 7.5 kW maximum and 1455 rpm. As a result, the new value for n_{mo} is $n_{mo} = 1455$ rpm, which is not much different from 1456 rpm. Recalculating u_{sys} with the new n_{mo}:

$$u_{sys} = n_{mo}/n = 1455/65 = 22.38$$

Retaining the speed ratio of the speed reducer (i.e. let $u_h = const = 8$), the new speed ratio of the chain drive is then:

$$u_{ch} = u_{sys}/u_h = 22.46/8 = 2.83$$

II Power, rotational speed and torque of the system

Let P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} are the transmitted parameters onto shaft 2 and P_{sh3}, n_{sh3} and T_{sh3} are used for shaft 3. The numbering is specified in Figure 1.1. Unless otherwise stated, these notations will be used throughout the next chapters.

2.1 Calculate speed of the chain drive and the shafts

The entire system is described followed by calculation as follows:
Chain drive power P_{ch} is affected by the bearings on the shaft of the mixing tank:

$$P_{ch} = \frac{P_w}{\eta_b} = \frac{6.2}{0.99} = 6.26 \text{ (kW)}$$

Shaft 3 power P_{sh3} is affected by the chain drive:

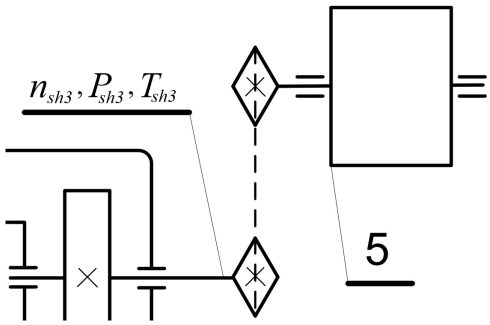
$$P_{sh3} = \frac{P_{ch}}{\eta_{ch}} = \frac{6.26}{0.95} = 6.52 \text{ (kW)}$$

Shaft 2 power P_{sh2} is affected by the bearings and gear drives on shaft 3:

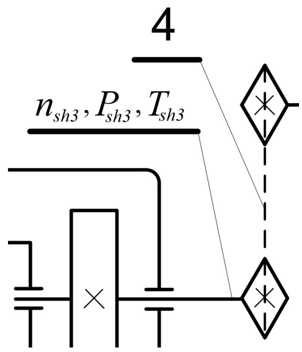
$$P_{sh2} = \frac{P_{sh3}}{\eta_b \eta_{hg}} = \frac{6.52}{0.99 \times 0.98} = 6.79 \text{ (kW)}$$

Shaft 1 power P_{sh1} is affected by the bearings and gear drives on shaft 2:

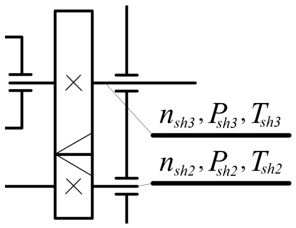
$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} = \frac{6.79}{0.99 \times 0.98} = 7.07 \text{ (kW)}$$



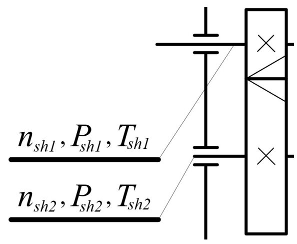
(a) Connection between the mixing tank shaft and chain drive: 1 pair of bearings



(b) Connection between shaft 3 and chain drive: 1 chain drive



(c) Connection between shaft 3 and shaft 2: 1 pair of bearings, 1 spur gear



(d) Connection between shaft 2 and shaft 1: 1 pair of bearings, 1 spur gear

Figure 1.2 Machine elements distribution between shafts and mechanical drives of the system

2.2 Calculate speed of the shafts

The design goal of the speed reducer is to lubricate both driven gears equally although there is a size disadvantage. Also, oil supply at the pitch line is desirable for moderate varying load [6]. Therefore, the speed ratio of each pair of gears is calculated using Equation 3.12 [9]:

$$u_1 = u_2 = \sqrt{u_h} = \sqrt{8} = 2.83$$

where

- u_1 is the speed ratio of the gear drive attaches to shaft 1 and shaft 2.
- u_2 is the speed ratio of the gear drive attaches to shaft 2 and shaft 3.

The value u_h is chosen deliberately as non-repeating decimal in order to create a *hunting tooth gear set*. A *hunting tooth ratio* is the ratio where the greatest common divisor of the number of teeth in the pinion and driven gear is 1. Under the same material and surface finishing condition, a greater improvement in the surface roughness was observed in the gears used in the hunting gear ratio compared to non-hunting counterpart even though fatigue damage might be presented. This is especially true for non-finishing soft gears with Brinell hardness number below HB300 [4].

Then,
the speed n_{sh1} from motor to shaft 1:

$$n_{sh1} = n_{mo} = 1455 \text{ (rpm)}$$

the speed n_{sh2} from shaft 1 to shaft 2:

$$n_{sh2} = n_{sh1}/u_1 = 1455/2.83 = 514.42 \text{ (rpm)}$$

the speed n_{sh3} from shaft 2 to shaft 3:

$$n_{sh3} = n_{sh2}/u_2 = 514.42/2.83 = 181.88 \text{ (rpm)}$$

2.3 Calculate torque of the motor and the shafts

Subsequently, the torque is calculated as follows:

$$T_{mo} = 9.55 \times 10^6 \times P_{mo}/n_{mo} = 9.55 \times 10^6 \times 7.14/1455 = 46892.66 \text{ (N} \cdot \text{mm)}$$
$$T_{sh1} = 9.55 \times 10^6 \times P_{sh1}/n_{sh1} = 9.55 \times 10^6 \times 7.07/1455 = 46423.73 \text{ (N} \cdot \text{mm)}$$
$$T_{sh2} = 9.55 \times 10^6 \times P_{sh2}/n_{sh2} = 9.55 \times 10^6 \times 6.79/514.42 = 126093.30 \text{ (N} \cdot \text{mm)}$$
$$T_{sh3} = 9.55 \times 10^6 \times P_{sh3}/n_{sh3} = 9.55 \times 10^6 \times 6.52/181.88 = 342486.86 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

Table 1.1 Output specification for 3 shafts and motor 4A132S4Y3

	Motor	Shaft 1	Shaft 2	Shaft 3
n (rpm)	1455	1455	514.42	181.88
P (kW)	7.14	7.07	6.79	6.52
T (N · mm)	46892.66	46423.73	126093.30	342486.86
u	-	1	2.83	2.83

Chapter 2

Chain Drive Design

I Determination of chain drive pitch

For power transmission purpose, three modes of failure are considered [6]:

1. fatigue of the link plate due to replication of tension in the tight side of the chain.
2. impact of the rollers as they engage the sprocket teeth.
3. galling between the pins of each link and the bushing on the pins.

Figure 2.1 Link plate failure occurs possibly due to overloading

To prevent failure in the chain's service life, one of the methods to find the right pitch is applying strength criterion which is derived into Equation 5.3 [9] as permissible power $[P]$. In this chapter, unless otherwise stated, subscript $_1$ is the driving sprocket attached to shaft 3, and $_2$ is the driven sprocket attached to the shaft of the mixing tank.

1.1 Calculate number of teeth of the sprocket z

The number of teeth z determines the how stable is the sprocket speed, which relates to the impact intensity and service life. Therefore, z_1 should be larger than 19 teeth and z_2 should be smaller than 120 teeth, p.80 [9]. Similar to gear drives, the wear life in general is improved for hunting tooth ratio [3]:

$$z_1 = 29 - 2u_{ch} = 29 - 2 \times 2.8 = 25 \geq 19$$

$$z_2 = u_{ch} z_1 = 2.8 \times 25 = 71 \leq 120$$

where

- z is the number of teeth in a sprocket.
- u_{ch} is given from previous chapter.

1.2 Calculate permissible power $[P]$ and find pitch p

An experiment is conducted to find the optimal pitch given the permissible power and angular rotational speed. A roller chain drive having 25 teeth on the driving sprocket is tested in 8 different cases of n_{01} in somewhat similar conditions with our design purpose, p.80 [9]. In this problem, $z_1 = 25$; $n_1 = n_{sh1} = 181.88\,\text{rpm}$ is the smaller sprocket speed, which is close to $n_{01} = 200\,\text{rpm}$. and . The value of calculated power P_t is:

$$\begin{aligned} P_t = P_{ch} k k_z k_n = P_{ch} k_0 k_a k_{dc} k_d k_c k_{bt} k_z k_n \\ = 6.26 \times 1 \times 1 \times 1 \times 1.5 \times 1 \times 1.3 \times 1 \times 1.1 = 10.33\,(\text{kW}) \leq [P] \end{aligned}$$

where

- $k_0 = 1$ is the arrangement of drive factor, Table 5.6 [9]. The centerline between 2 sprockets is parallel with the ground.
- k_a is the center distance factor, Table 5.6 [9]. The choice is in the recommended range $a = (30 \div 50)p$, which is similar to the experiment.
- $k_{dc} = 1$ is the chain tension factor, Table 5.6 [9]. The center distance is modifiable through displacing the sprockets.
- $k_d = 1.5$ dynamic load factor, Table 5.6 [9]. The impact load is moderate.
- $k_c = 1$ is the rating factor, Table 5.6 [9]. The system works for 1 shift of 8 hours.
- $k_{bt} = 1.3$ is the lubrication factor, 5.6 [9]. Moderate lubrication quality is applied in dusty condition.
- k_z is the teeth ratio, which is used to compare the actual number of teeth with the one in experiment. It is

  $$k_z = 25/z_1 = 25/25 = 1$$

- k_n is the speed ratio, which is used to compare the actual speed with the one in experiment. Since $n_{01} = 200\,\text{rpm}$ is closet to $n_1 = 181.88\,\text{rpm}$, the ratio is

  $$k_n = n_{01}/n_1 = 200/181.88 = 1.1$$

- P_{ch} is given in Chapter 1.

which satisfies the condition.

From P_t and n_{01}, the permissible power is $[P] = 11\,\text{kW}$, Table 5.5 [9]. Also from the table, the pitch is $p = 25.4\,\text{mm}$, bearing pin diameter $d_c = 7.95\,\text{mm}$, inner plate width $B = 22.61\,\text{mm}$. Because minimizing damage from impact onto the drive is essential, Table 5.8 [9] is consulted. In this case, the pitch is indeed suitable.

II Basic parameters of the chain drive

2.1 Find x_c

The recommended value for center distance is in the range $[a_{min}, a_{max}]$, where $a_{min} = 30p = 30\times25.4 = 762.00$ (mm), $a_{max} = 50p = 50\times25.4 = 1270.00$ (mm) for reducing the effect from chain weight. Since the working area for the system is not given, the lowest value is chosen, $a = 762.00$ mm to approximate x_c. Then, the value is rounded up to the nearest even number:

$$\begin{aligned} x_c &= \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \\ &= \frac{2\times762.00}{25.4} + \frac{25+71}{2} + \frac{(71-25)^2\times25.4}{4\pi^2\times762.00} \\ &= 109.79 = 112 \end{aligned}$$

2.2 Find center distance a

Equation 5.13 [9] calculates the center distance a between the sprockets. However, it is also recommended to loose the chain an amount of $0.002 \div 0.004a$ for tension reduction. Choosing $0.003a$ for the averaged value, Equation 5.13 [9] is multiplied by an additional amount of 0.997. Therefore, the center distance is

$$\begin{aligned} a &= \frac{0.997p}{4}\left[x_c - \frac{z_2+z_1}{2} + \sqrt{\left(x_c - \frac{z_2+z_1}{2}\right)^2 - 2\left(\frac{z_2-z_1}{\pi}\right)^2}\right] \\ &= \frac{0.997\times25.4}{4}\left[112 - \frac{71+25}{2} + \sqrt{\left(112 - \frac{71+25}{2}\right)^2 - 2\left(\frac{71-25}{\pi}\right)^2}\right] \\ &= 788.57 \text{ (mm)} \end{aligned}$$

where all the parameters are identified.

2.3 Find other parameters

The values below are necessary for modeling the chain drive:

$$\begin{aligned} d_1 &= p/\sin(180°/z_1) = 25.4/\sin(180°/25) = 202.66 \text{ (mm)} \\ d_2 &= p/\sin(180°/z_2) = 25.4/\sin(180°/71) = 574.23 \text{ (mm)} \\ d_{a1} &= p\,[0.5 + \cot(180°/z_1)] = 25.4\,[0.5 + \cot(180°/25)] = 213.76 \text{ (mm)} \\ d_{a2} &= p\,[0.5 + \cot(180°/z_2)] = 25.4\,[0.5 + \cot(180°/71)] = 586.37 \text{ (mm)} \\ d_{f1} &= d_1 - 2(0.502d_l + 0.05) = 202.66 - 2(0.502\times15.88 + 0.05) = 186.60 \text{ (mm)} \\ d_{f2} &= d_2 - 2(0.502d_l + 0.05) = 574.23 - 2(0.502\times15.88 + 0.05) = 558.17 \text{ (mm)} \end{aligned}$$

where

- $d_l = 15.88$ mm is the roller diameter, Table 5.2 [9].
- d is the chordal diameter, mm.

- d_a is the addendum diameter, mm.
- d_f is the deddendum diameter, mm.
- p, z_1, z_2 are given in previous sections.

III Strength of chain drive

3.1 Impact frequency analysis

After determining the center distance, it is necessary to validate the impact frequency limit. The condition for impact frequency i is:

$$i = \frac{z_1 n_1}{15 x_c} = \frac{25 \times 181.88}{15 \times 112} = 2.71 < [i] = 25$$

where

- $[i] = 25$ is the permissible impact frequency, Table 5.9 [9].
- z_1, n_1, x_c are given in previous sections.

which satisfies the condition.

3.2 Safety factor analysis

Overloading often occurs at the beginning of operation or due to large load, which could damage the chain drive. In order to operate safely, the chain drive's safety factor s is considered:

$$s = \frac{Q}{k_d F_t + F_0 + F_v} = \frac{56700}{1.2 \times 3253.04 + 120.68 + 9.63} = 19.23 \geq [s] = 7.66$$

where

- $Q = 56700\,\text{kN}$ is the permissible load, Table 5.2 [9].
- $k_d = 1.2$ is the dynamic load factor, p.86 [9]. The workload is moderate since When turning on the machine, it is around 1.5 times the amount of the nominal load.
- F_t is the effective peripheral force, N. The force is

  $$F_t = 1000 P_{ch}/v_1 = 1000 \times 6.26/1.92 = 3253.04\ (\text{N})$$

  where v_1 is the instantaneous speed along the chain, m/s. The speed is

  $$v_1 = n_1 p z_1/60000 = 181.88 \times 25.4 \times 25/60000 = 1.92\ (\text{m/s})$$

  where all parameters are identified.
- F_v is the centrifugal force, N. The force is

  $$F_v = q v_1^2 = 2.6 \times 1.92^2 = 9.63\ (\text{N})$$

  where $q = 2.6\,\text{kg/m}$ is the mass per unit length, Table 5.2 [9].

- k_d is given in previous section.
- F_0 is sagging force, N. The force is

$$F_0 = 9.81 \times 10^{-3} k_f q a = 9.81 \times 10^{-3} \times 6 \times 2.6 \times 788.57 = 120.68 \text{ (N)}$$

  where

  - $k_f = 6$ is the sagging factor, Table 5.2 [9]. The chain drive is parallel to the ground which is mentioned previously.
  - q, a are given in previous sections

- $[s] = 7.66$ is the permissible safety factor, Table 5.10 [9].

which satisfies the condition.

3.3 Contact stress analysis

For validating contact stress σ_H, the following condition must be met, Equation 5.18 [9]:

$$\begin{aligned}\sigma_H &= 0.47\sqrt{\frac{k_r(F_t k_d + F_{vd})E}{AK_d}} \\ &= 0.47\sqrt{\frac{0.42 \times (3253.04 \times 1.5 + 6.22) \times 148975.16}{180 \times 1}} = 476.25 \text{ (MPa)} \\ &\leq [\sigma_H] = 500 \text{ MPa}\end{aligned}$$

where

- $[\sigma_H] = 500$ MPa is the permissible contact stress, Table 5.18 [9]. From the table, several conditions have to be met. Since $z_2 = 71 > 30$ and $v_1 = 1.92 \text{ mm} < 5 \text{ mm}$. Thus the suitable material for the sprockets is quenched 45 steel. Another reason supporting the choice is that a considerable amount of power transmission is shared from the speed reducer.
- $k_r = 0.42$ is the tooth factor. p.87 [9]. The value is estimated from z_1.
- F_{vd} is the contact force, N. The force is

$$F_{vd} = 13 \times 10^{-7} n_1 p^3 = 13 \times 10^{-7} \times 181.88 \times 25.4^3 = 6.22 \text{ (N)}$$

  where all the parameters are identified.

- E is the equivalent modulus of elasticity, MPa. It is calculated as

$$E = \frac{2E_1E_2}{E_1 + E_2} = \frac{2 \times 205000 \times 117000}{205000 + 117000} = 148975.16 \text{ (MPa)}$$

  where

  - $E_1 = 205000$ MPa is the modulus of elasticity of the sprockets. The value is found on an online research paper [5].

  - $E_1 = 117000\,\text{MPa}$ is the modulus of elasticity of the chain, whose material is cast iron ASTM-A48 No. 30A [6]. It is chosen to keep σ_H below $[\sigma_H]$.

- $A = 180\,\text{mm}^2$ is the cross sectional area of the chain hinge, Table 5.12 [9]. The area depends on the pitch value.
- $K_d = 1$ is the number of strands factor. The chain drive only has one strand.
- F_t, k_d are found in previous sections.

which satisfies the condition.

IV Force on shaft

Apply the following equations, p.87 [9]:

$$F_2 = F_0 + F_v = 120.68 + 9.63 = 130.31\ (\text{N})$$
$$F_1 = F_t + F_2 = 3253.04 + 307.55 = 3383.35\ (\text{N})$$
$$F_r = k_x F_t = 1.15 \times 3253.04 = 3741.00\ (\text{N})$$

where

- $k_x = 1.15$ is the chain weight factor, p.88 [9]. The chain drive is parallel to the ground.
- F_1 is the tight side tension, N.
- F_2 is the slack side tension, N.
- F_r is the force on the shaft, N.
- F_0, F_v, F_t are found in previous sections.

In summary, we have the following table:

Table 2.1 Chain drive final specification. The chain is placed parallel to the ground

	Chain drive	Driving sprocket	Driven sprocket
Material	Cast iron ASTM-A48 No. 30A	quenched 45 steel	quenched 45 steel
a (mm)	788.57	-	-
B (mm)	22.61	-	-
d (mm)	-	202.66	574.23
d_a (mm)	-	213.76	586.37
d_c (mm)	7.95	-	-
d_f (mm)	-	186.60	558.17
d_l (mm)	15.88	-	-
p (mm)	25.4	-	-
z	-	25	71

Chapter 3

Gear Design (Stage One)

I Material selection

For small scale machines, type I material, which is often quenched and has low hardness (HB $\leq$ 350), is sufficient. Precise manufacture is possible albeit low permissible stresses. In this design problem, the gear drive uses quenched 45 steel, whose specification is displayed on Table 6.1 [9]:

- Brinell hardness HB205
- ultimate strength $\sigma_b = 600\,\text{MPa}$
- yield limit $\sigma_{ch} = 340\,\text{MPa}$

II Stress estimation

From working condition, the service life in hours are found:

$$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times CaK_{ng}L = 8 \times 1 \times 260 \times 8 = 16640\ (\text{hours})$$

where all parameters are given in the design problem.

Since many variables are different in both gears, it should be convenient to introduce 4 subscripts for the parameters. Unless otherwise stated,

- subscript $_1$ denotes the pinion attached to shaft 1.
- subscript $_2$ denotes the driven gear attached to shaft 2.
- subscript $_F$ denotes bending stress.
- subscript $_H$ denotes contact stress.

2.1 Estimate the permissible contact stress $[\sigma_H]$ and bending stress $[\sigma_F]$

The permissible stresses are initially estimated using Equation 6.1a and 6.2a [9]. The exact calculation will be considered in the next section:

$$[\sigma_H] = \frac{\sigma^o_{Hlim}}{S_H} K_{HL}$$

$$[\sigma_F] = \frac{\sigma^o_{Flim}}{S_F} K_{FC} K_{FL}$$

Find permissible stress corresponding to a working cycle σ^o_{lim} The value depends on hardness. The pinion also should be harder than the driving gear HB10 ÷ 15. For quenched 45 steel, the formulas below are used, Table 6.2 [9]:

$$\sigma^o_{Hlim} = 2\text{HB} + 70$$

$$\sigma^o_{Flim} = 1.8\text{HB}$$

For the pinion with H_1 = HB205:

$$\sigma^o_{Hlim1} = 2 \times 205 + 70 = 480\ (\text{MPa})$$

$$\sigma^o_{Flim1} = 1.8 \times 205 = 369\ (\text{MPa})$$

For the driven gear with $H_2 = H_1 - 15 = 205 - 15 =$ HB190:

$$\sigma^o_{Hlim2} = 2 \times 190 + 70 = 450\ (\text{MPa})$$

$$\sigma^o_{Flim2} = 1.8 \times 190 = 342\ (\text{MPa})$$

Find safety factor S The safety factors $S_H = 1.1$, $S_F = 1.75$ depend on material type, Table 6.2 [9].

Find load placement factor K_{FC} $K_{FC} = 1$ for unidirectional operation (i.e. load placement is in one way).

Find aging factor K_L The factor is determined by Equation 6.3 and 6.4 [9] for contact stress and bending stress, respectively. If calculation yields a value smaller than 1, it is then rounded up to 1 instead, p.94 [9]:

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} = \sqrt[6]{10600601.81/1.05 \times 10^9} = 0.46 < 1 \Rightarrow K_{HL1} = 1$$
$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} = \sqrt[6]{8833440.68/0.37 \times 10^9} = 0.54 < 1 \Rightarrow K_{HL2} = 1$$
$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} = \sqrt[6]{4000000/0.91 \times 10^9} = 0.40 < 1 \Rightarrow K_{FL1} = 1$$
$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} = \sqrt[6]{4000000/0.32 \times 10^9} = 0.48 < 1 \Rightarrow K_{FL2} = 1$$

where

- m is the root of fatigue curve in the stress test of a material. Since $H_1, H_2 \leq 350$, $m_H = 6$ and $m_F = 6$.

- N_O is the working cycle of equivalent bearing stress. Using Equation 6.5 [9]:

$$N_{HO1} = 30H_1^{2.4} = 30 \times 205^{2.4} = 10600601.81\ (\text{cycles})$$
$$N_{HO2} = 30H_2^{2.4} = 30 \times 190^{2.4} = 8833440.68\ (\text{cycles})$$

  where all parameters are given in previous sections.

  For bending stress, $N_{FO1} = N_{FO2} = 4000000$ (MPa) since both gears use steel material.

- N_E is the working cycle of equivalent tensile stress. From Equation 6.7 and 6.8 [9]:

$$
\begin{aligned}
N_{HE1} &= 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 1455 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^3 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^3 \frac{11}{15+11} \right] \\
&= 1.05 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{HE2} &= 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 514.42 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.37 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{FE1} &= 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 1455 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.91 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{FE2} &= 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 514.42 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.32 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

where

- $c = 1$ is the gear meshing rate. The value 1 indicates the gears are meshed indefinitely.
- T_1, T_2, t_1, t_2 are given in the design problem.
- n_{sh1}, n_{sh2} are given in Chapter 1.
- L_h, m_F are given in previous sections.

Calculate preliminary values of $[\sigma_H]$, $[\sigma_{F1}]$, $[\sigma_{F2}]$ Replacing all the values gives:

$$
\begin{aligned}
[\sigma_{H1}] &= \sigma^o_{Hlim1} K_{HL1}/S_H = 480 \times 1/1.1 = 436.36 \text{ (MPa)} \\
[\sigma_{H2}] &= \sigma^o_{Hlim2} K_{HL2}/S_H = 450 \times 1/1.1 = 409.09 \text{ (MPa)} \\
[\sigma_{F1}] &= \sigma^o_{Flim1} K_{FC} K_{FL1}/S_F = 369 \times 1 \times 1/1.75 = 210.86 \text{ (MPa)} \\
[\sigma_{F2}] &= \sigma^o_{Flim2} K_{FC} K_{FL2}/S_F = 342 \times 1 \times 1/1.75 = 195.43 \text{ (MPa)}
\end{aligned}
$$

The permissible contact stress of the gear drive $[\sigma_H]$ must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. In this case, it is $1.25[\sigma_{H2}]$:

$$[\sigma_H] = \frac{[\sigma_{H1}] + [\sigma_{H2}]}{2} = \frac{436.36 + 409.09}{2} = 422.73\,(\text{MPa})$$
$$\leq 1.25[\sigma_{H2}] = 1.25 \times 409.09 = 511.36\,(\text{MPa})$$

2.2 Find permissible contact stress $[\sigma_H]_{max}$ and bending stress $[\sigma_F]_{max}$ due to overload

Preventing overload failure is necessary. For quenched material, the permissible overload contact stress $[\sigma_H]_{max}$ is

$$[\sigma_H]_{max} = 2.8\sigma_{ch} = 2.8 \times 340 = 952.00\,(\text{MPa})$$

For material with hardness smaller than HB350, the permissible overload bending stress $[\sigma_F]_{max}$ is

$$[\sigma_F]_{max} = 0.8\sigma_{ch} = 0.8 \times 340 = 272.00\,(\text{MPa})$$

III Basic specifications of the transmission system

Undercutting process is omitted due to complexity and maintenance. That is, it is not factored in any variables of the gear drive.

3.1 Determine basic parameters

Find center distance a

For involute gears, the center distance a is estimated using Equation 6.15a [9] and rounded up to a multiple of 5 (small production type, p.99 [9]):

$$a = K_a(u+1)\sqrt[3]{\frac{TK_{H\beta}}{[\sigma_H]^2 u\psi_{ba}}} = 43 \times (2.83 + 1) \times \sqrt[3]{\frac{46423.73 \times 1.03}{422.73^2 \times 2.83 \times 0.30}}$$
$$= 120.14\,(\text{mm}) \Rightarrow a = 160\,\text{mm}$$

where

- $K_a = 43$ is the material factor, Table 6.5 [9]. The helical gear drive made of steel (both gears) corresponds to the value in the table.
- $u = u_1 = 2.83$ is the speed ratio of the gear drive, which is given in Chapter 1.
- $T = T_{sh1} = 46423.73\,\text{N}\cdot\text{mm}$ is the torque exerted on the pinion, which is transmitted from the motor to shaft 1.
- $K_{H\beta} = 1.03$ is the load distribution factor on gear teeth, Table 6.7 [9]. In the table, the gear placement in the speed reducer is similar to diagram 5
- $\psi_{ba} = 0.30$ is the width to shaft distance ratio, Table 6.6 [9]. The pinion is asymmetrical about the bearings and both gears surface have hardness smaller than HB350. The value is obtained after extensive calculation and changing parameters, which includes hardness,

center distance and traverse module. In addition, the remaining gear drive contributes to ψ_{ba}, eventually reducing the center distance further, saving material cost.

- $[\sigma_H]$ is given in previous section.

Find face width ratio ψ_{bd}

ψ_{bd} is the face width factor. Using Equation 6.16 [9]:

$$\psi_{bd} = 0.53\psi_{ba}(u + 1) = 0.53 \times 0.30 \times (2.83 + 1) = 0.69$$

where all parameters are identified. The calculated value satisfies even with the minimum permissible factor $\psi_{bdmax} = 1$, Table 6.6 [9].

Find traverse module m_t

Using Equation 6.17 and Table 6.8 [9], the traverse module m is

$$m_t = (0.01 \div 0.02)a = (0.01 \div 0.02) \times 160 = 1.6 \div 3.2\,(\text{mm}) \Rightarrow m_t = 2.00\,\text{mm}$$

The choice is based on calculation to satisfy strength condition, as mentioned in choosing ψ_{ba}.

Find number of teeth z

Through calculation, helix angle $\beta = 20°$ should be preferred (permissible value is $8 \div 20°$). Using Equation 6.31 and 6.20 [9], the pinion's number of teeth z_1 is

$$z_1 = \frac{2a\cos\beta}{m_t(u+1)} = \frac{2 \times 160\cos 20°}{2.00 \times (2.83 + 1)} = 40.17 \Rightarrow z_1 = 41$$

$$z_2 = uz_1 = 2.83 \times 41 = 115.97 \Rightarrow z_2 = 116$$

where all parameters are given in previous sections. The number of teeth selection follows the *hunting tooth ratio* [4] as mentioned in Chapter 1. The ratio yields $z_2/z_1 = 116/41 = 2.829$, which is essentially not much different from the intended value $u = 2.83$.

Correct β

The helix angles are corrected to compensate for rounding center distance and number of teeth, Equation 6.32 [9]:

$$\beta = \arccos\frac{m_t(z_2 + z_1)}{2a} = \arccos\frac{2.00 \times (116 + 41)}{2 \times 160} = 11.11°$$

3.2 Other parameters

The values below are necessary for modeling the gear drive:

$$b = \psi_{ba}a = 0.3 \times 160 = 48.00\,(\text{mm})$$

$$d_1 = m_t z_1/\cos\beta = 2.00 \times 41/\cos 9.07° = 83.57\,(\text{mm})$$

$$d_2 = m_t z_2/\cos\beta = 2.00 \times 116/\cos 9.07^\circ = 236.43 \text{ (mm)}$$
$$d_{a1} = d_1 + 2m_t = 83.57 + 2 \times 2.00 = 87.57 \text{ (mm)}$$
$$d_{a2} = d_2 + 2m_t = 236.43 + 2 \times 2.00 = 240.43 \text{ (mm)}$$
$$d_{b1} = d_1 \cos\alpha_n = 83.57 \times \cos 20^\circ = 78.53 \text{ (mm)}$$
$$d_{b2} = d_2 \cos\alpha_n = 236.43 \times \cos 20^\circ = 222.17 \text{ (mm)}$$
$$d_{f1} = d_1 - 2.5m_t = 83.57 - 2.5 \times 2.00 = 78.57 \text{ (mm)}$$
$$d_{f2} = d_2 - 2.5m_t = 236.43 - 2.5 \times 2.00 = 231.43 \text{ (mm)}$$
$$\alpha_t = \arctan(\tan\alpha/\cos\beta) = \arctan(\tan 20^\circ/\cos 11.11^\circ) = 20.35^\circ$$
$$v_1 = \pi d_1 n_{sh1}/60000 = \pi \times 83.57 \times 1455/60000 = 6.37 \text{ (m/s)}$$

where

- b is the face width, mm.
- d is the pitch circle diameter, mm.
- d_a is the addendum diameter, mm.
- d_b is the projection diameter, mm.
- d_f is the dedendum diameter, mm.
- α_t is the traverse pressure angle,°.
- $\alpha_n = 20^\circ$ is the normal pressure angle following TCVN 1065-71 standard of Vietnam.
- v is the average speed of the pinion, m/s.

IV Stress analysis

Tolerance grade affects the factors through stress analysis. Since $v_1 = 6.37$ m/s ≤ 10 m/s and the gear drive is helical gear, the grade is 8.

4.1 Correct the permissible contact stress $[\sigma_H]$ and bending stresses $[\sigma_{F1}]$, $[\sigma_{F2}]$

After preliminary estimation, the factors $Z_R Z_v K_{xH}$ and $Y_R Y_s K_{xF}$ are included to increase accuracy of permissible stress values, Equation 6.1 and 6.2 [9]:

$$[\sigma_H]' = [\sigma_H] Z_R Z_v K_{xH} = 422.73 \times 1 \times 1.02 \times 1 = 432.38 \text{ MPa}$$
$$[\sigma_{F1}]' = [\sigma_{F1}] Y_R Y_s K_{xF} = 210.86 \times 1 \times 1.03 \times 1 = 217.57 \text{ MPa}$$
$$[\sigma_{F2}]' = [\sigma_{F2}] Y_R Y_s K_{xF} = 195.43 \times 1 \times 1.03 \times 1 = 201.65 \text{ MPa}$$

where

- $Z_R = 1$ is the surface roughness factor of the working's area, corresponding to roughness deviation less than 1.25 μm. Hence, the gear drive is gone through honing process.

- Z_v is the speed factor. Since surface hardness of the gear drive is less than HB350, the formula is

$$Z_v = 0.85 v_1^{0.1} = 0.79 \times 6.37^{0.1} = 1.02$$

- $K_x = 1$ is the size factor. Since both gears have $d_{a1}, d_{a2} \leq 400\,\text{mm}$, $K_{xH} = K_{xF} = 1$.

- $Y_R = 1$ is the surface roughness factor. The gears are unpolished for economical purpose.

- Y_s is the stress concentration factor. Using the equation on p.92 [9], the factor is

$$Y_s = 1.08 - 0.0695 \ln(m_t) = 1.08 - 0.0695 \ln(2.00) = 1.03$$

- other parameters are given in previous sections.

4.2 Contact stress analysis

The contact stress applied on a gear surface must satisfy Equation 6.33 [9]:

$$\begin{aligned}\sigma_H &= z_M z_H z_\varepsilon \sqrt{2TK_H \frac{u+1}{bud_1^2}} \\ &= 274 \times 1.74 \times 0.79 \times \sqrt{2 \times 46423.73 \times 1.21 \times \frac{2.83+1}{48.00 \times 2.83 \times 83.57^2}} \\ &= 251.85\,(\text{MPa}) \leq [\sigma_H]' = 432.38\,\text{MPa}\end{aligned}$$

where

- $z_M = 274$ is the material's mechanical properties factor, Table 6.5 [9]. The value is chosen since both gears are steel material and helical type.

- z_H is the contact surface's shape factor. Using Equation 6.37 [9], the factor is

$$z_H = \sqrt{\frac{2\cos\beta_b}{\sin(2\alpha_t)}} = \sqrt{\frac{2 \times \cos 10.43^\circ}{\sin(2 \times 20.35^\circ)}} = 1.74$$

  where β_b is the base circle helix angle. Using Equation 6.35 [9], the angle is

$$\beta_b = \arctan(\cos\alpha_t \tan\beta) = \arctan(\cos 20.35^\circ \times \tan 11.11^\circ) = 10.43^\circ$$

- z_ε is meshing condition factor. Depending on the value of face contact ratio ε_β, 1 in 3 Equation 6.36a, 6.36b, 6.36c [9] is used. Using Equation 6.37 [9], the ratio is

$$\varepsilon_\beta = b\frac{\sin\beta}{m_t\pi} = 48.00 \times \frac{\sin 11.11^\circ}{2.00 \times \pi} = 1.20$$

  Since $\varepsilon_\beta > 1$, Equation 6.36c [9] is used:

$$z_\varepsilon = \varepsilon_\alpha^{-0.5} = 1.61^{-0.5} = 0.79$$

where ε_α is the traverse contact ratio. Using Equation 6.38a [9], the factor is

$$\begin{aligned}\varepsilon_\alpha &= \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a\sin\alpha_t}{2\pi m_t \cos\alpha_t/\cos\beta}\\ &= \frac{\sqrt{87.57^2 - 78.53^2} + \sqrt{240.43^2 - 222.17^2} - 2\times 160\times\sin 20.35°}{2\pi\times 2.00\times\cos 20.35°/\cos 11.11°}\\ &= 1.61\end{aligned}$$

where all parameters are identified in previous sections.

- $T = T_{sh1} = 46423.73\,\mathrm{N\cdot mm}$ is the torque exerted on the pinion.
- K_H is the overall factor. Using Equation 6.39 [9], the factor is

$$K_H = K_{H\beta}K_{Hv}K_{H\alpha} = 1.03\times 1.06\times 1.10 = 1.21$$

where

  - $K_{H\beta} = 1.03$ is the load distribution factor on gear teeth, Table 6.7 [9]. The value is given previous section.
  - $K_{Hv} = 1.06$ is the dynamic load factor at meshing area, Table P2.3 [9]. The value corresponds to helical gear and tolerance grade 8.
  - $K_{H\alpha} = 1.10$ is the load distribution factor on top land, Table 6.14 [9]. The value corresponds to the speed $v_1 = 6.37\,\mathrm{m/s}$ and tolerance grade 8.

4.3 Bending stress analysis

For safety reasons, the gear drive must follow Equation 6.43 and 6.44 [9]. The bending stress σ_{F1} on the pinion and σ_{F2} on the driving gear are

$$\begin{aligned}\sigma_{F1} = \frac{2TK_F Y_\varepsilon Y_\beta Y_{F1}}{bd_1 m_t\cos\beta}[\sigma_{F1}] &= \frac{2\times 46423.73\times 1.66\times 0.62\times 0.92\times 3.68}{48.00\times 83.57\times 2.00\times\cos 11.11°}\\ &= 41.09\ (\mathrm{MPa})\ \le\ [\sigma_{F1}]' = 217.57\,\mathrm{MPa}\end{aligned}$$

$$\sigma_{F2} = \sigma_{F1}Y_{F2}/Y_{F1} = 41.09\times 3.60/3.68 = 40.17\ (\mathrm{MPa})\ \le\ [\sigma_{F2}]' = 201.65\,\mathrm{MPa}$$

where

- $T = T_{sh1} = 46423.73\,\mathrm{N\cdot mm}$ is the torque exerted on the pinion. In this case, the source is the motor.
- K_F is the overall factor. Using Equation 6.39 [9], the factor is

$$K_F = K_{F\beta}K_{Fv}K_{F\alpha} = 1.08\times 1.18\times 1.30 = 1.66$$

where

  - $K_{F\beta} = 1.08$ is the load distribution factor on gear teeth, Table 6.7 [9]. In the table, the gear placement in the speed reducer is similar to diagram 5.

  - $K_{Fv} = 1.18$ is the dynamic load factor at meshing area, Table P2.3 [9], corresponding to helical gear and tolerance grade 8.
  - $K_{F\alpha} = 1.30$ is the load distribution factor on top land, Table 6.14 [9]. The value corresponds to the speed $v_1 = 6.37\,\text{m/s}$ and tolerance grade 8.

- Y_ε is the contact ratio factor. Using equation on p.108 [9], the factor is

$$Y_\varepsilon = 1/\varepsilon_\alpha = 1/1.61 = 0.62$$

  where ε_α is given in previous section.

- Y_β is the helix angle factor. Using equation on p.108 [9], the factor is

$$Y_\beta = 1 - \beta/140^\circ = 1 - 11.11^\circ/140^\circ = 0.92$$

- Y_F is the tooth shape factor, Table 6.18 [9]. The factor depends on the value of z_v, which is the virtual number of teeth. Using equation on p.108 [9] and the table, Y_{F1} and Y_{F2} are

$$z_{v1} = z_1 \cos^{-3}(\beta) = 41 \times \cos^{-3}(11.11^\circ) = 43.40 \Rightarrow Y_{F1} = 3.68$$

$$z_{v2} = z_2 \cos^{-3}(\beta) = 116 \times \cos^{-3}(11.11^\circ) = 122.78 \Rightarrow Y_{F2} = 3.60$$

- b, d_1, m_t, β are given in previous sections.

4.4 Overload analysis

From the load diagram, the overload factor K_{qt} is

$$\begin{aligned} K_{qt} &= \left[\frac{(T_1/T)^2\, t_1 + (T_2/T)^2\, t_2}{t_1 + t_2}\right]^{-1/2} \\ &= \left[\frac{(T/T)^2 \times 15 + (0.7T/T)^2 \times 11}{15 + 11}\right]^{-1/2} \\ &= 1.13 \end{aligned}$$

Using the values of $[\sigma_H]_{max}$ and $[\sigma_F]_{max}$ calculated in previous section combined with Equation 6.48 and 6.49 [9], it is possible to verify the stresses are below overload limits. Replacing all the variables gives

$$\sigma_{Hmax} = \sigma_H \sqrt{K_{qt}} = 251.85 \times \sqrt{1.13} = 267.63\,\text{(MPa)} \leq 952.00\,\text{MPa}$$

$$\sigma_{F1max} = \sigma_{F1} K_{qt} = 41.09 \times 1.13 = 41.09\,\text{(MPa)} \leq 272.00\,\text{MPa}$$

$$\sigma_{F2max} = \sigma_{F2} K_{qt} = 40.17 \times 1.13 = 45.36\,\text{(MPa)} \leq 272.00\,\text{MPa}$$

In summary, the table is obtained

Table 3.1 Gear drive final specification

	Gear drive	Pinion	Driven gear
Material	-	quenched 45 steel	quenched 45 steel
a (mm)	160	-	-
b (mm)	-	48.00	48.00
d (mm)	-	83.57	236.43
d_a (mm)	-	87.57	240.43
d_f (mm)	-	78.57	231.43
m (mm)	2.00	-	-
α_t (°)	20.35	-	-
β (°)	11.11	-	-
z	-	41	116

Chapter 4

Gear Design (Stage Two)

I Choose material

For small scale machines, type I material, which is often quenched and has low hardness (HB $\leq$ 350), is sufficient. Precise manufacture is possible albeit low permissible stresses. In this design problem, the gear drive uses quenched 45 steel, whose specification is displayed on Table 6.1 [9]:

- Brinell hardness HB205
- ultimate strength $\sigma_b = 600\,\text{MPa}$
- yield limit $\sigma_{ch} = 340\,\text{MPa}$

II Stress estimation

From working condition, the service life in hours are

$$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times CaK_{ng}L = 8 \times 1 \times 260 \times 8 = 16640\ (\text{hours})$$

where all parameters are given in the design problem.

Since many variables are different in both gears, it should be convenient to introduce 4 subscripts for the parameters. Unless otherwise stated,

- subscript $_1$ denotes the pinion attached to shaft 2.
- subscript $_2$ denotes the driven gear attached to shaft 3.
- subscript $_F$ denotes bending stress.
- subscript $_H$ denotes contact stress.

2.1 Estimate the permissible contact stress $[\sigma_H]$ and bending stress $[\sigma_F]$

Preliminary estimation of the permissible stresses uses Equation 6.1a and 6.2a [9]. The exact calculation is considered in the next section:

$$[\sigma_H] = \frac{\sigma^o_{Hlim}}{S_H}K_{HL}$$

$$[\sigma_F] = \frac{\sigma^o_{Flim}}{S_F}K_{FC}K_{FL}$$

Find permissible stress corresponding to a working cycle σ^o_{lim} The value depends on hardness. The pinion also should be harder than the driving gear HB10 ÷ 15. For quenched 45 steel, the formulas below are used, Table 6.2 [9]:

$$\sigma^o_{Hlim} = 2\text{HB} + 70$$

$$\sigma^o_{Flim} = 1.8\text{HB}$$

For the pinion with H_1 = HB205:

$$\sigma^o_{Hlim1} = 2 \times 205 + 70 = 480 \text{ (MPa)}$$

$$\sigma^o_{Flim1} = 1.8 \times 205 = 369 \text{ (MPa)}$$

For the driven gear with $H_2 = H_1 - 15 = 205 - 15 =$ HB190:

$$\sigma^o_{Hlim2} = 2 \times 190 + 70 = 450 \text{ (MPa)}$$

$$\sigma^o_{Flim2} = 1.8 \times 190 = 342 \text{ (MPa)}$$

Find safety factor S The safety factors $S_H = 1.1$, $S_F = 1.75$ depend on material type, Table 6.2 [9].

Find load placement factor K_{FC} $K_{FC} = 1$ for unidirectional operation (i.e. load placement is in one way).

Find aging factor K_L The factor is determined by Equation 6.3 and 6.4 [9] for contact stress and bending stress, respectively. If calculation yields a value smaller than 1, it is then rounded up to 1 instead, p.94 [9]:

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} = \sqrt[6]{10600601.81/0.37 \times 10^9} = 0.46 < 1 \Rightarrow K_{HL1} = 1$$
$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} = \sqrt[6]{8833440.68/0.13 \times 10^9} = 0.54 < 1 \Rightarrow K_{HL2} = 1$$
$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} = \sqrt[6]{4000000/0.32 \times 10^9} = 0.40 < 1 \Rightarrow K_{FL1} = 1$$
$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} = \sqrt[6]{4000000/0.11 \times 10^9} = 0.48 < 1 \Rightarrow K_{FL2} = 1$$

where

- m is the root of fatigue curve in the stress test of a material. Since $H_1, H_2 \leq 350$, $m_H = 6$ and $m_F = 6$.

- N_O is the working cycle of equivalent bearing stress. Using Equation 6.5 [9]:

$$N_{HO1} = 30H_1^{2.4} = 30 \times 205^{2.4} = 10600601.81 \text{ (cycles)}$$
$$N_{HO2} = 30H_2^{2.4} = 30 \times 190^{2.4} = 8833440.68 \text{ (cycles)}$$

  where all parameters are given in previous sections.

  For bending stress, $N_{FO1} = N_{FO2} = 4000000$ (MPa) since both gears use steel material.

- N_E is the working cycle of equivalent tensile stress. From Equation 6.7 and 6.8 [9]:

$$
\begin{aligned}
N_{HE1} &= 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 514.42 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^3 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^3 \frac{11}{15+11} \right] \\
&= 0.37 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{HE2} &= 60 n_{sh3} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 181.88 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.13 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{FE1} &= 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 514.42 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.32 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

$$
\begin{aligned}
N_{FE2} &= 60 n_{sh3} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right] \\
&= 60 \times 181.88 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] \\
&= 0.11 \times 10^9 \text{ (cycles)}
\end{aligned}
$$

where

- $c = 1$ is the gear meshing rate. The value 1 indicates the gears are meshed indefinitely.
- T_1, T_2, t_1, t_2 are given in the design problem.
- n_{sh2}, n_{sh3} are given in Chapter 1.
- L_h, m_F are given in previous sections.

Calculate preliminary values of $[\sigma_H]$, $[\sigma_{F1}]$, $[\sigma_{F2}]$ Replacing all the values gives:

$$
\begin{aligned}
[\sigma_{H1}] &= \sigma^o_{Hlim1} K_{HL1} / S_H = 480 \times 1/1.1 = 436.36 \text{ (MPa)} \\
[\sigma_{H2}] &= \sigma^o_{Hlim2} K_{HL2} / S_H = 450 \times 1/1.1 = 409.09 \text{ (MPa)} \\
[\sigma_{F1}] &= \sigma^o_{Flim1} K_{FC} K_{FL1} / S_F = 369 \times 1 \times 1/1.75 = 210.86 \text{ (MPa)} \\
[\sigma_{F2}] &= \sigma^o_{Flim2} K_{FC} K_{FL2} / S_F = 342 \times 1 \times 1/1.75 = 195.43 \text{ (MPa)}
\end{aligned}
$$

The permissible contact stress of the gear drive $[\sigma_H]$ must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. In this case, it is $1.25[\sigma_{H2}]$:

$$[\sigma_H] = \frac{[\sigma_{H1}] + [\sigma_{H2}]}{2} = \frac{436.36 + 409.09}{2} = 422.73\,(\mathrm{MPa})$$
$$\leq 1.25[\sigma_{H2}] = 1.25 \times 409.09 = 511.36\,(\mathrm{MPa})$$

2.2 Find permissible contact stress $[\sigma_H]_{max}$ and bending stress $[\sigma_F]_{max}$ due to overload

Preventing overload failure is necessary. For quenched material, the permissible overload contact stress $[\sigma_H]_{max}$ is

$$[\sigma_H]_{max} = 2.8\sigma_{ch} = 2.8 \times 340 = 952.00\,(\mathrm{MPa})$$

For material with hardness smaller than HB350, the permissible overload bending stress $[\sigma_F]_{max}$ is

$$[\sigma_F]_{max} = 0.8\sigma_{ch} = 0.8 \times 340 = 272.00\,(\mathrm{MPa})$$

III Determine basic specifications of the transmission system

Undercutting process is omitted due to complexity and maintenance. That is, it is not factored in any variables of the gear drive.

3.1 Determine basic parameters

Find center distance a

For involute gears, the center distance a is estimated using Equation 6.15a [9] and rounded up to a multiple of 5 (small production type, p.99 [9]):

$$a = K_a(u+1)\sqrt[3]{\frac{TK_{H\beta}}{[\sigma_H]^2 u\psi_{ba}}} = 43 \times (2.83 + 1) \times \sqrt[3]{\frac{126093.30 \times 1.08}{422.73^2 \times 2.83 \times 0.39}}$$
$$= 145.47\,(\mathrm{mm}) \Rightarrow a = 160\,\mathrm{mm}$$

where

- $K_a = 43$ is the material factor, Table 6.5 [9]. The helical gear drive made of steel (both gears) corresponds to the value in the table.
- $u = u_1 = 2.83$ is the speed ratio of the gear drive, which is given in Chapter 1.
- $T = T_{sh2} = 126093.30\,\mathrm{N \cdot mm}$ is the torque exerted on the pinion, which is transmitted from the motor to shaft 1.
- $K_{H\beta} = 1.08$ is the load distribution factor on gear teeth, Table 6.7 [9]. In the table, the gear placement in the speed reducer is similar to diagram 4
- $\psi_{ba} = 0.39$ is the width to shaft distance ratio, Table 6.6 [9]. The pinion is asymmetrical about the bearings and both gears surface have hardness smaller than HB350. The value is obtained after extensive calculation and changing parameters, which includes hardness,

center distance and traverse module. In addition, the remaining gear drive contributes to ψ_{ba}, eventually reducing the center distance further, saving material cost.

- $[\sigma_H]$ is given in previous section.

Find face width ratio ψ_{bd}

ψ_{bd} is the face width factor. Using Equation 6.16 [9]:

$$\psi_{bd} = 0.53\psi_{ba}(u + 1) = 0.53 \times 0.39 \times (2.83 + 1) = 0.80$$

where all parameters are identified. The calculated value satisfies even with the minimum permissible factor $\psi_{bdmax} = 1$, Table 6.6 [9].

Find traverse module m_t

Using Equation 6.17 and Table 6.8 [9], the traverse module m is

$$m_t = (0.01 \div 0.02)a = (0.01 \div 0.02) \times 160 = 1.6 \div 3.2\,(\text{mm}) \Rightarrow m_t = 3.00\,\text{mm}$$

The choice is based on calculation to satisfy strength condition, as mentioned in choosing ψ_{ba}.

Find number of teeth z

Through calculation, helix angle $\beta = 20°$ should be preferred (permissible value is $8 \div 20°$). Using Equation 6.31 and 6.20 [9], the pinion's number of teeth z_1 is

$$z_1 = \frac{2a\cos\beta}{m_t(u+1)} = \frac{2 \times 160\cos 20°}{3.00 \times (2.83 + 1)} = 26.18 \Rightarrow z_1 = 27$$

$$z_2 = uz_1 = 2.83 \times 27 = 76.37 \Rightarrow z_2 = 77$$

where all parameters are given in previous sections. The number of teeth selection follows the *hunting tooth ratio* [4] as mentioned in Chapter 1. The ratio yields $z_2/z_1 = 77/27 = 2.852$, which is essentially not much different from the intended value $u = 2.83$.

Correct β

The helix angles are corrected to compensate for rounding center distance and number of teeth, Equation 6.32 [9]:

$$\beta = \arccos\frac{m_t(z_2 + z_1)}{2a} = \arccos\frac{3.00 \times (77 + 27)}{2 \times 160} = 12.84°$$

3.2 Other parameters

The values below are necessary for modeling the gear drive:

$$b = \psi_{ba}a = 0.39 \times 160 = 62\,(\text{mm})$$

$$d_1 = m_t z_1/\cos\beta = 3.00 \times 27/\cos 9.07° = 83.08\,(\text{mm})$$

$$d_2 = m_t z_2/\cos\beta = 3.00 \times 77/\cos 9.07^\circ = 236.92\ \text{(mm)}$$
$$d_{a1} = d_1 + 2m_t = 83.08 + 2 \times 3.00 = 89.08\ \text{(mm)}$$
$$d_{a2} = d_2 + 2m_t = 236.92 + 2 \times 3.00 = 242.92\ \text{(mm)}$$
$$d_{b1} = d_1 \cos\alpha_n = 83.08 \times \cos 20^\circ = 78.07\ \text{(mm)}$$
$$d_{b2} = d_2 \cos\alpha_n = 236.92 \times \cos 20^\circ = 222.63\ \text{(mm)}$$
$$d_{f1} = d_1 - 2.5m_t = 83.08 - 2.5 \times 3.00 = 75.58\ \text{(mm)}$$
$$d_{f2} = d_2 - 2.5m_t = 236.92 - 2.5 \times 3.00 = 229.42\ \text{(mm)}$$
$$\alpha_t = \arctan(\tan\alpha_n/\cos\beta) = \arctan(\tan 20^\circ/\cos 12.84^\circ) = 20.47^\circ$$
$$v_1 = \pi d_1 n_{sh2}/60000 = \pi \times 83.08 \times 514.42/60000 = 2.24\ \text{(m/s)}$$

where

- b is the face width, mm.
- d is the pitch circle diameter, mm.
- d_a is the addendum diameter, mm.
- d_b is the projection diameter, mm.
- d_f is the dedendum diameter, mm.
- α_t is the traverse pressure angle,°.
- $\alpha_n = 20^\circ$ is the normal pressure angle following TCVN 1065-71 standard of Vietnam.
- v is the average speed of the pinion, m/s.

IV Stress analysis

Tolerance grade affects the factors through stress analysis. Since $v_1 = 2.24\,\text{m/s} \leq 4\,\text{m/s}$ and the gear drive is helical gear, the grade is 9.

4.1 Correct the permissible contact stress $[\sigma_H]$ and bending stresses $[\sigma_{F1}]$, $[\sigma_{F2}]$

After preliminary estimation, the factors $Z_R Z_v K_{xH}$ and $Y_R Y_s K_{xF}$ are included to increase accuracy of permissible stress values, Equation 6.1 and 6.2 [9]:

$$[\sigma_H]' = [\sigma_H] Z_R Z_v K_{xH} = 422.73 \times 1 \times 0.92 \times 1 = 389.46\,\text{MPa}$$
$$[\sigma_{F1}]' = [\sigma_{F1}] Y_R Y_s K_{xF} = 210.86 \times 1 \times 1.00 \times 1 = 211.63\,\text{MPa}$$
$$[\sigma_{F2}]' = [\sigma_{F2}] Y_R Y_s K_{xF} = 195.43 \times 1 \times 1.00 \times 1 = 196.14\,\text{MPa}$$

where

- $Z_R = 1$ is the surface roughness factor of the working's area, corresponding to roughness deviation less than 1.25 μm. Hence, the gear drive is gone through honing process.

- Z_v is the speed factor. Since surface hardness of the gear drive is less than HB350, the formula is

$$Z_v = 0.85v_1^{0.1} = 0.80 \times 2.24^{0.1} = 0.92$$

- $K_x = 1$ is the size factor. Since both gears have $d_{a1}, d_{a2} \leq 400\,\text{mm}$, $K_{xH} = K_{xF} = 1$.

- $Y_R = 1$ is the surface roughness factor. The gears are unpolished for economical purpose.

- Y_s is the stress concentration factor. Using the equation on p.92 [9], the factor is

$$Y_s = 1.08 - 0.0695\ln(m_t) = 1.08 - 0.0695\ln(3.00) = 1.00$$

- other parameters are given in previous sections.

4.2 Contact stress analysis

The contact stress applied on a gear surface must satisfy Equation 6.33 [9]:

$$\begin{aligned}\sigma_H &= z_M z_H z_\varepsilon \sqrt{2TK_H \frac{u+1}{bud_1^2}} \\ &= 274 \times 1.73 \times 0.80 \times \sqrt{2 \times 126093.30 \times 1.26 \times \frac{2.83+1}{62 \times 2.83 \times 83.08^2}} \\ &= 379.07\,(\text{MPa}) \leq [\sigma_H]' = 389.46\,\text{MPa}\end{aligned}$$

where

- $z_M = 274$ is the material's mechanical properties factor, Table 6.5 [9]. The value is chosen since both gears are steel material and helical type.

- z_H is the contact surface's shape factor. Using Equation 2.24 [9], the factor is

$$z_H = \sqrt{\frac{2\cos\beta_b}{\sin(2\alpha_t)}} = \sqrt{\frac{2 \times \cos 12.05^\circ}{\sin(2 \times 20.47^\circ)}} = 1.73$$

  where β_b is the base circle helix angle. Using Equation 6.35 [9], the angle is

$$\beta_b = \arctan(\cos\alpha_t \tan\beta) = \arctan(\cos 20.47^\circ \times \tan 12.84^\circ) = 12.05^\circ$$

- z_ε is meshing condition factor. Depending on the value of face contact ratio ε_β, 1 in 3 Equation 6.36a, 6.36b, 6.36c [9] is used. Using Equation 2.24 [9], the ratio is

$$\varepsilon_\beta = b\frac{\sin\beta}{m_t\pi} = 62 \times \frac{\sin 12.84^\circ}{3.00 \times \pi} = 1.47$$

  Since $\varepsilon_\beta > 1$, Equation 6.36c [9] is used:

$$z_\varepsilon = \varepsilon_\alpha^{-0.5} = 1.56^{-0.5} = 0.80$$

where ε_α is the traverse contact ratio. Using Equation 6.38a [9], the factor is

$$\begin{aligned}\varepsilon_\alpha &= \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a\sin\alpha_t}{2\pi m_t \cos\alpha_t / \cos\beta} \\ &= \frac{\sqrt{89.08^2 - 78.07^2} + \sqrt{242.92^2 - 222.63^2} - 2 \times 160 \times \sin 20.47^\circ}{2\pi \times 3.00 \times \cos 20.47^\circ / \cos 12.84^\circ} \\ &= 1.56\end{aligned}$$

where all parameters are identified in previous sections.

- $T = T_{sh2} = 126093.30\,\mathrm{N \cdot mm}$ is the torque exerted on the pinion.
- K_H is the overall factor. Using Equation 6.39 [9], the factor is

$$K_H = K_{H\beta} K_{Hv} K_{H\alpha} = 1.08 \times 1.03 \times 1.13 = 1.26$$

  where

  - $K_{H\beta} = 1.08$ is the load distribution factor on gear teeth, Table 6.7 [9]. The value is given previous section.
  - $K_{Hv} = 1.03$ is the dynamic load factor at meshing area, Table P2.3 [9]. The value corresponds to helical gear and tolerance grade 9.
  - $K_{H\alpha} = 1.13$ is the load distribution factor on top land, Table 6.14 [9]. The value corresponds to the speed $v_1 = 2.24\,\mathrm{m/s}$ and tolerance grade 9.

4.3 Bending stress analysis

For safety reasons, the gear drive must follow Equation 6.43 and 6.44 [9]. The bending stress σ_{F1} on the pinion and σ_{F2} on the driving gear are

$$\begin{aligned}\sigma_{F1} = \frac{2TK_F Y_\varepsilon Y_\beta Y_{F1}}{bd_1 m_t \cos\beta}[\sigma_{F1}] &= \frac{2 \times 126093.30 \times 1.73 \times 0.64 \times 0.91 \times 3.82}{62 \times 83.08 \times 3.00 \cos 12.84^\circ} \\ &= 63.96\,(\mathrm{MPa}) \leq [\sigma_{F1}]' = 211.63\,\mathrm{MPa}\end{aligned}$$

$$\sigma_{F2} = \sigma_{F1} Y_{F2} / Y_{F1} = 63.96 \times 3.61/3.82 = 60.46\,(\mathrm{MPa}) \leq [\sigma_{F2}]' = 196.14\,\mathrm{MPa}$$

where

- $T = T_{sh2} = 126093.30\,\mathrm{N \cdot mm}$ is the torque exerted on the pinion. In this case, the source is the motor.
- K_F is the overall factor. Using Equation 6.39 [9], the factor is

$$K_F = K_{F\beta} K_{Fv} K_{F\alpha} = 1.17 \times 1.08 \times 1.37 = 1.73$$

  where

  - $K_{F\beta} = 1.17$ is the load distribution factor on gear teeth, Table 6.7 [9]. In the table, the gear placement in the speed reducer is similar to diagram 4.

  - $K_{Fv} = 1.08$ is the dynamic load factor at meshing area, Table P2.3 [9], corresponding to helical gear and tolerance grade 9.
  - $K_{F\alpha} = 1.37$ is the load distribution factor on top land, Table 6.14 [9], corresponding to the speed $v_1 = 2.24\,\text{m/s}$ and tolerance grade 9.

- Y_ε is the contact ratio factor. Using equation on p.108 [9], the factor is

$$Y_\varepsilon = 1/\varepsilon_\alpha = 1/1.56 = 0.64$$

  where ε_α is given in previous section.

- Y_β is the helix angle factor. Using equation on p.108 [9], the factor is

$$Y_\beta = 1 - \beta/140^\circ = 1 - 12.84^\circ/140^\circ = 0.91$$

- Y_F is the tooth shape factor, Table 6.18 [9]. The factor depends on the value of z_v, which is the virtual number of teeth. Using equation on p.108 [9] and the table, Y_{F1} and Y_{F2} are

$$z_{v1} = z_1 \cos^{-3}(\beta) = 27 \times \cos^{-3}(12.84^\circ) = 29.13 \Rightarrow Y_{F1} = 3.82$$

$$z_{v2} = z_2 \cos^{-3}(\beta) = 77 \times \cos^{-3}(12.84^\circ) = 83.08 \Rightarrow Y_{F2} = 3.61$$

- b, d_1, m_t, β are given in previous sections.

4.4 Overload analysis

From the load diagram, we determine the overload factor K_{qt}:

$$\begin{aligned} K_{qt} &= \left[\frac{(T_1/T)^2\, t_1 + (T_2/T)^2\, t_2}{t_1 + t_2} \right]^{-1/2} \\ &= \left[\frac{(T/T)^2 \times 15 + (0.7T/T)^2 \times 11}{15 + 11} \right]^{-1/2} \\ &= 1.13 \end{aligned}$$

Using the values of $[\sigma_H]_{max}$ and $[\sigma_F]_{max}$ calculated in previous section combined with Equation 6.48 and 6.49 [9], it is possible to verify the stresses are below overload limits. Replacing all the variables gives

$$\begin{aligned} \sigma_{Hmax} &= \sigma_H \sqrt{K_{qt}} = 379.07 \times \sqrt{1.13} = 402.82\,(\text{MPa}) \leq 952.00\,\text{MPa} \\ \sigma_{F1max} &= \sigma_{F1} K_{qt} = 63.96 \times 1.13 = 72.23\,(\text{MPa}) \leq 272.00\,\text{MPa} \\ \sigma_{F2max} &= \sigma_{F2} K_{qt} = 60.46 \times 1.13 = 68.27\,(\text{MPa}) \leq 272.00\,\text{MPa} \end{aligned}$$

In summary, the table is obtained

Table 4.1 Gear drive final specification

	Gear drive	Pinion	Driven gear
Material	-	quenched 45 steel	quenched 45 steel
a (mm)	160	-	-
b (mm)	-	62	62
d (mm)	-	83.08	236.92
d_a (mm)	-	89.08	242.92
d_f (mm)	-	75.58	229.42
m (mm)	3.00	-	-
α_t (°)	20.47	-	-
β (°)	12.84	-	-
z	-	41	116

Chapter 5

Flex Coupling Design

In coupling design, SKF Flex Couplings is used. The catalog provided on https://www.skfptp.com/Publications/Publications# introduces procedure to select a suitable product. Thus, coupling selection shall follow the instruction on p.60 [8]

Service factor The purpose of the mixing tank is not given in design project. Therefore, assuming it is used as a muller mixer, the service factor is 1.5, Table 10, p.88 [8].

Design power The motor power is $P_{mo} = 7.14\,\text{kW}$, Table 1.1.

Coupling size The motor speed is $n_{mo} = 1455\,\text{rpm}$, Table 1.1. Consulting Table 1, p.61 [8], the nearest specification is speed $n = 1440\,\text{rpm}$ and power rating $P = 9.95\,\text{kW}$. This results in the coupling size of 50. Also from the table, the nominal torque and max torque are also satisfactory ($T_{sh} = 46892.66\,\text{N} \cdot \text{mm} < 66\,\text{N} \cdot \text{m}$).

Bore size Straight hub mounted coupling is used since the shaft is not tapered. Thus, SKF Flex flange type B size 50 (PHE F50RSBFLG) is selected, p.65 [8]. From the table on the page, the maximum bore diameter is 38 mm, which is equal to the motor shaft diameter.

In summary, the flex coupling dimensions and basic physical quantities are specified in the catalog [8] and product detail report [**bibid**].

The diameter of the shaft used is $d_c = 28\,\text{mm}$. The reason is due to repetitive design and analysis of the resulting gearbox.

Therefore, the force F_{11} exerted on the shaft by the coupling is:

$$F_{11} = \frac{2T_{sh1}}{d_c} = \frac{2 \times 46423.73}{28} = 3316\,(\text{N})$$

Chapter 6

Shaft Design

I Choose material

For moderate load, carbon steel AISI 1035 [1] is selected to design the shafts:

$$\text{HB}183, \sigma_b = 585\,\text{MPa}, \sigma_{ch} = 370\,\text{MPa}$$

II Loads on shafts

In this chapter, a subscript convention shall be used for clarity. The rule of the convention is as follows:

- a variable describing the mechanical drive's property on the shaft uses 2 numeric subscripts. The first subscript is the ordinal number of shafts. The second subscript is the ordinal number of mechanical drives (e.g. motor, gear, chain, belt).
- if a variable is of the shaft's property, it uses 1 numeric subscript.
- the location of a force on the shaft is specified with 1 capitalized letter after the numeric subscripts. On each shaft of this design problem, the letters A, B, C, D represent 4 critical sections from left to right.
- for a force vector, x, y, z are its algebraic values on x, y, z-axis, respectively.
- at critical sections with a moment jump, 2 superscipts are used. The left side is $^-$ while the right side is $^+$.

Therefore,

- On shaft 1: the motor is labeled 1, the pinion is labeled 2.
- On shaft 2: the driving gear is labeled 1, the pinion is labeled 2.
- On shaft 3: the driving gear is labeled 1, the chain is labeled 2.

Applying this rule in force computation yields:

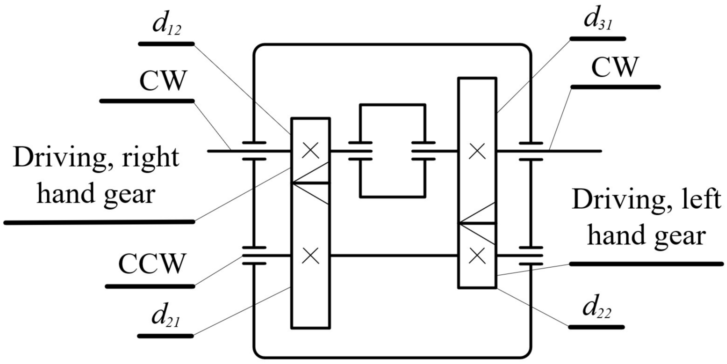


Figure 6.1 Detailed description of the speed reducer

Force from pinion 1:

$$
\begin{aligned}
F_{12x} &= c_{b12} c_{q12} \frac{2T_{sh1}}{d_{12}} = (+1) \times (-1) \times \frac{2 \times 46423.73}{83.57} = -1111.06 \text{ (N)} \\
F_{12y} &= -\frac{2T_{sh1}}{d_{12}} \frac{\tan \alpha_n}{\cos \beta_{12}} = -\frac{2 \times 46423.73}{83.57} \times \frac{\tan 20^\circ}{\cos 11.11^\circ} = -412.12 \text{ (N)} \\
F_{12z} &= c_{b12} c_{q12} h_{r12} \frac{2T_{sh1}}{d_{12}} \tan \beta_{12} \\
&= (+1) \times (-1) \times (+1) \times \frac{2 \times 46423.73}{83.57} \times \tan 11.11^\circ \\
&= -218.24 \text{ (N)}
\end{aligned}
$$

Force from driven gear 1:

$$
\begin{aligned}
F_{21x} &= -F_{12x} = 1111.06 \text{ (N)} \\
F_{21y} &= -F_{12y} = 412.12 \text{ (N)} \\
F_{21z} &= -F_{12z} = 218.24 \text{ (N)}
\end{aligned}
$$

Force from pinion 2:

$$
\begin{aligned}
F_{22x} &= c_{b22} c_{q22} \frac{2T_{sh2}}{d_{22}} = (+1) \times (+1) \times \frac{2 \times 126093.30}{83.08} = 1117.61 \text{ (N)} \\
F_{22y} &= -\frac{2T_{sh2}}{d_{22}} \frac{\tan \alpha_n}{\cos \beta_{22}} = -\frac{2 \times 126093.30}{83.08} \times \frac{\tan 20^\circ}{\cos 12.84^\circ} = -1133.19 \text{ (N)} \\
F_{22z} &= c_{b22} c_{q22} h_{r22} \frac{2T_{sh2}}{d_{22}} \tan \beta_{22} \\
&= (+1) \times (+1) \times (-1) \times \frac{2 \times 126093.30}{83.08} \times \tan 12.84^\circ \\
&= -691.82 \text{ (N)}
\end{aligned}
$$

Force from driven gear 2:

$$
\begin{aligned}
F_{31x} &= -F_{22x} = -1117.61 \text{ (N)} \\
F_{31y} &= -F_{22y} = 1133.19 \text{ (N)} \\
F_{31z} &= -F_{22z} = 691.82 \text{ (N)}
\end{aligned}
$$

where

- F is the force exerting on the shaft, N.

- c_q is the rotation direction of the shaft from the shaft end. For counterclockwise rotations (CCW), $c_q = +1$; for clockwise rotations (CW), $c_q = -1$. In this case, shaft 1 is assumed to rotate clockwise.
- c_b is the state of the gear. For driving gears, $c_b = +1$; for driven gears, $c_b = -1$.
- h_r is the tooth direction. For right hand gears, $h_r = +1$; for left hand gears, $h_r = -1$.
- β is the helix angle given in **Table 3.1** and **4.1**,°.
- d is the diameter of the gear, mm. From **Table 3.1** and **4.1**,

$$
\begin{aligned}
d_{12} &= 83.57\,\text{mm},\ d_{21} = 236.43\,\text{mm} \\
d_{22} &= 83.08\,\text{mm},\ d_{31} = 236.92\,\text{mm}
\end{aligned}
$$

- T_{sh1}, T_{sh2} are given in **Table 1.1**.

Let α be the angle of the chain drive with respect to ground. From **Table 2.1**'s description, the chain is parallel to ground, which makes $\alpha = 0°$.

$$
\begin{aligned}
F_{32x} &= F_{r32}\cos(\alpha) = 3741\cos(0) = 3741\ (\text{N}) \\
F_{32y} &= F_{r32}\sin(\alpha) = 3741\sin(0) = 0\ (\text{N})
\end{aligned}
$$

Similarly, the force derived from coupling is the same direction with F_{32}. Thus, $F_{11} = F_{11x} = 3315.98$ (N)

III Determination of shaft diameters at critical sections

3.1 Preliminary estimation of shafts diameter

For input shaft, it is recommended to choose permissible torsion stress a small value $[\tau]$ for the input shaft and a large value for the output shaft. From Equation 10.9 [9], the minimum nominal diameter of 3 shafts are

$$
\begin{aligned}
d_1 &\geq \sqrt[3]{\frac{5T_{sh1}}{[\tau_1]}} = \sqrt[3]{\frac{5 \times 46423.73}{20}} = 22.64\ (\text{mm}) \\
d_2 &\geq \sqrt[3]{\frac{5T_{sh2}}{[\tau_2]}} = \sqrt[3]{\frac{5 \times 126093.30}{25}} = 29.33\ (\text{mm}) \\
d_3 &\geq \sqrt[3]{\frac{5T_{sh3}}{[\tau_3]}} = \sqrt[3]{\frac{5 \times 342486.86}{30}} = 38.5\ (\text{mm})
\end{aligned}
$$

where

- d_1, d_2, d_3 are the diameters of shaft 1, 2, 3 respectively, mm.
- $T_{sh1}, T_{sh2}, T_{sh3}$ are given in **Table 1.1**.

Machining the gears integrally with the shaft is better suitable for precision special machines and production in large quantities. In this design, the scale is smaller and non-specialized. The gears are mounted on separate shafts, with the torque transmitted from the shaft to the gear through a key. The benefit of this setup is positive means of transmitting the torque while allowing easy assembly and disassembly [6].

Since the input shaft is connected with the motor shaft by a flexible coupling, the difference between 2 diameters must be within 20% of the motor diameter d_{mo}. Inspecting Table P1.7 for motor 4A132S4Y3, $d_{mo} = 42.00$ mm. Combining with Table 10.2 [9] to obtain the nominal diameter of 3 shafts and the corresponding bearings width:

$$d_1 = 30\,\text{mm}, d_2 = 30\,\text{mm}, d_3 = 40\,\text{mm}$$
$$b_{O1} = 19\,\text{mm}, b_{O2} = 19\,\text{mm}, b_{O3} = 23\,\text{mm}$$

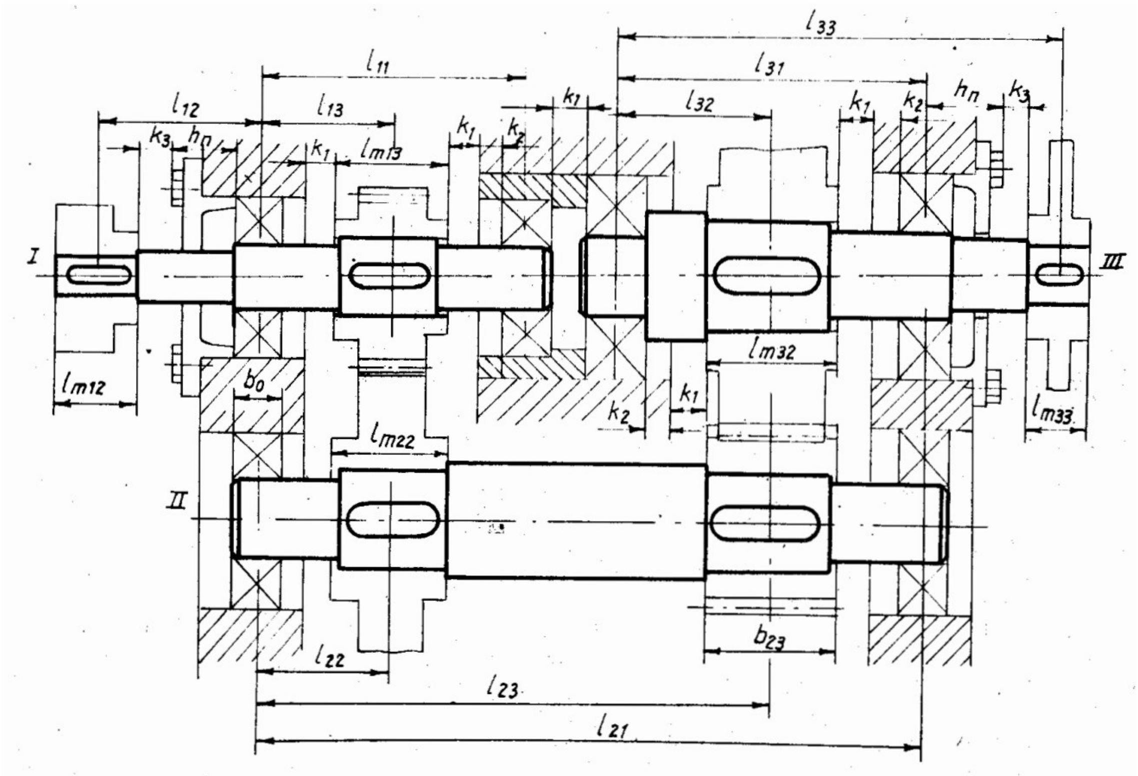


Figure 6.2 Dimensions and placement of 3 shafts in the speed reducer

The gear widths are $b_{12} = b_{21} = 48.00$ mm and $b_{22} = b_{31} = 62$ mm, **Table 3.1** and **4.1**. These values will be used to determine the minimum hub lengths $l_{m13}, l_{m22}, l_{m32}$ (i.e. the hub length equals to shaft length at the gear mating section). Table 10.3 [9] gives formulas to determine the necessary lengths in **Figure 6.2**. From design perspective, it is desirable to minimize the size of the speed reducer, thereby reducing material cost, bending moment and deflection of the shaft. This leads to the selection of hub lengths l_m shown below (refer to **Figure 6.2** for locating these dimensions):

$$l_{m12} = 1.40d_1 = 1.40 \times 30 = 42\ (\text{mm})$$
$$l_{m13} = b_1 = 48\,\text{mm}$$
$$l_{m22} = 1.20d_2 = 1.20 \times 30 = 42\ (\text{mm}) \Rightarrow l_{m13} = 48.00\,\text{mm}$$
$$l_{m32} = b_2 = 62\,\text{mm}$$
$$l_{m33} = 1.20d_3 = 1.20 \times 40 = 54\ (\text{mm}) \Rightarrow l_{m33} = 67.50\,\text{mm}$$

Equal spacing is better to reduce measuring time. Therefore, the lengths k_1, k_2, k_3, h_n are $k_1 = 5$ mm, $k_2 = 5$ mm, $k_3 = 5$ mm, $h_n = 15$ mm, where k_1 is the distance from the inner surface of the speed reducer to the nearer side of the mechanical drive and between mechanical drives, mm; k_2 is the distance from the inner surface of the bearings to the inner surface of the speed reducer, mm; k_3 is the distance from the bearing housing unit to the nearer side of the mechanical drive, mm; h_n is the vertical distance between the bearing housing unit and the screw head, mm. Then, the lengths in Table 10.4 [9] for two-stage coaxial helical speed reducers are calculated as follows:

On shaft 1:

$$
\begin{aligned}
l_{12} &= 0.5(l_{m12} + b_{O1}) + k_3 + h_n \\
&= 0.5(49 + 21) + 5 + 5 = 55.00\,(\text{mm}) \\
l_{13} &= 0.5(l_{m13} + b_{O1}) + k_1 + k_2 \\
&= 0.5(48.00 + 21) + 8 + 5 = 44.50\,(\text{mm}) \\
l_{11} &= 2l_{13} = 2 \times 44.50 = 89.00\,(\text{mm})
\end{aligned}
$$

On shaft 3:

$$
\begin{aligned}
l_{32} &= 0.5(l_{m32} + b_{O3}) + k_1 + k_2 \\
&= 0.5(62 + 25) + 5 + 5 = 54.00\,(\text{mm}) \\
l_{31} &= 2l_{32} = 2 \times 54.00 = 108.00\,(\text{mm}) \\
l_{33} &= l_{31} + 0.5(l_{m33} + b_{O3}) + k_3 + h_n \\
&= 108.00 + 0.5(67.50 + 25) + 5 + 15 = 174.25\,(\text{mm}) \Rightarrow l_{33} = 175\,\text{mm}
\end{aligned}
$$

On shaft 2:

$$
\begin{aligned}
l_{22} &= 0.5(l_{m22} + b_{O2}) + k_1 + k_2 \\
&= 0.5(48.00 + 21) + 5 + 5 = 44.50\,(\text{mm}) \\
l_{23} &= l_{11} + l_{32} + k_1 + 0.5(b_{O1} + b_{O3}) \\
&= 89.00 + 54.00 + 5 + 0.5(21 + 25) = 171.00\,(\text{mm}) \\
l_{21} &= l_{23} + l_{32} = 171.00 + 54.00 = 225.00\,(\text{mm})
\end{aligned}
$$

3.2 Find reaction forces

The speed reducer operates at constant speed. Therefore, the equilibrium conditions are

$$
\begin{cases}
\sum_i \mathbf{F_i} = 0 \\
\sum_i \mathbf{r_i} \times \mathbf{F_i} = 0
\end{cases}
$$

The equations above are applied to 3 pairs of bearings, each of which is mounted on 1 shaft. Projecting the forces onto 2 planes Oxy and Oyz yields 6 sets of matrices in the form of $AX + B = 0$ where X is the 2×1 matrix whose unit is N. The solutions are shown in **Table 6.1**:

From the reaction forces, the shear force diagrams for 3 shafts are created but will not be shown. From the shear force diagrams, the bending moment diagrams for 3 shafts are visualized as demonstrated in Fi**gure 6.3**, **6.4** and **6.5**.

After computing all bending moments at critical sections, Equation 10.17 [9] is used to estimate minimum diameter d_{est} at a cross section corresponding to the magnitude of moments

Table 6.1 Tabularized equations for reaction forces in matrix form

A	X (N)	B	$X = A^{-1}(-B)$
$\begin{bmatrix} 1 & 1 \\ l_{12} & l_{12}+l_{11} \end{bmatrix}$	$\begin{bmatrix} F_{1Bx} \\ F_{1Dx} \end{bmatrix}$	$\begin{bmatrix} F_{12x} \\ (l_{12}+l_{13})F_{12x}+0.5d_{12}F_{12z} \end{bmatrix}$	$\begin{bmatrix} -555.53 \\ -103.6 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 \\ l_{12} & l_{12}+l_{11} \end{bmatrix}$	$\begin{bmatrix} F_{1By} \\ F_{1Dy} \end{bmatrix}$	$\begin{bmatrix} F_{12y} \\ (l_{12}+l_{13})F_{12y} \end{bmatrix}$	$\begin{bmatrix} -555.53 \\ -308.52 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 \\ 0 & l_{21} \end{bmatrix}$	$\begin{bmatrix} F_{2Ax} \\ F_{2Dx} \end{bmatrix}$	$\begin{bmatrix} F_{21x}+F_{22x} \\ l_{22}F_{21x}+l_{23}F_{22x}+0.5d_{21}F_{21z}+0.5d_{22}F_{22z} \end{bmatrix}$	$\begin{bmatrix} 1159.54 \\ 71.7 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 \\ 0 & l_{21} \end{bmatrix}$	$\begin{bmatrix} F_{2Ay} \\ F_{2Dy} \end{bmatrix}$	$\begin{bmatrix} F_{21y}+F_{22y} \\ l_{22}F_{21y}+l_{23}F_{22y} \end{bmatrix}$	$\begin{bmatrix} 1069.12 \\ -792.77 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 \\ 0 & l_{31} \end{bmatrix}$	$\begin{bmatrix} F_{3Ax} \\ F_{3Cx} \end{bmatrix}$	$\begin{bmatrix} F_{31x}+F_{32x} \\ l_{32}F_{31x}+l_{33}F_{32x}+0.5d_{31}F_{31z} \end{bmatrix}$	$\begin{bmatrix} 1762 \\ -192.23 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 \\ 0 & l_{31} \end{bmatrix}$	$\begin{bmatrix} F_{3Ay} \\ F_{3Cy} \end{bmatrix}$	$\begin{bmatrix} F_{31y}+F_{32y} \\ l_{32}F_{31y}+l_{33}F_{32y} \end{bmatrix}$	$\begin{bmatrix} -6620.6 \\ 1325.42 \end{bmatrix}$

there. The values are summarized in col. 6, **Table 6.2**:

$$d_{est} = \sqrt[3]{\frac{10M_e}{[\sigma]}}$$

where

- M_e is the equivalent bending moment including the input torque. It is calculated using Equation 10.16, [9]. The values of M_e at each critical cross section are shown in col. 5, **Table 6.2**:

  $$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}}$$

  where

  – M_x is the bending moment on Oxz plane, calculated above.
  – M_y is the bending moment on Oyz plane, calculated above.
  – T_{sh} is the input torque from the motor, N · mm. The values are given in **Table 1.1**.

- $[\sigma]$ is the permissible stress of shaft material, Table 10.5 [9]. In this design project, all 3 shafts are fabricated from the same material (carbon steel AISI 1035) yielding at $\sigma_b = 586$ MPa (ultimate strength).

Table 6.2 Equivalent moment M_e and cross section diameters of 3 shafts

Sect.	M_x (N · mm)	M_y (N · mm)	T_{sh} (N · mm)	M_e (N · mm)	d_{est} (mm)	d (mm)
1A	0	0	46423.73	40204.13	20.12	32
1B	-5698.14	-30554.04	46423.73	50817.22	21.76	35
1C^-	35307.65	79996.02	46423.73	96241.19	26.92	42
1C^+	44426.31	79996.02	0	91504.43	26.47	42
1D	0	0	0	0	0	35
2A	0	0	0	0	0	45
2B^-	-18339.27	-49441.98	0	52733.66	23.3	54
2B^+	-44138.39	-49441.98	126093.3	127739.37	31.29	54
2C^-	149637.15	-240553.02	126093.3	303614.35	41.76	54
2C^+	178374.12	-240553.02	0	299471.34	41.57	54
2D	0	0	0	0	0	45
3A	0	0	0	0	0	60
3B^-	-61192.27	60350.85	0	85946.03	26.75	69
3B^+	-143145.85	60350.85	342486.86	334822.19	42.09	69
3C	0	-654674.15	342486.86	718728.86	54.3	60
3D	0	0	342486.86	296602.32	40.43	52

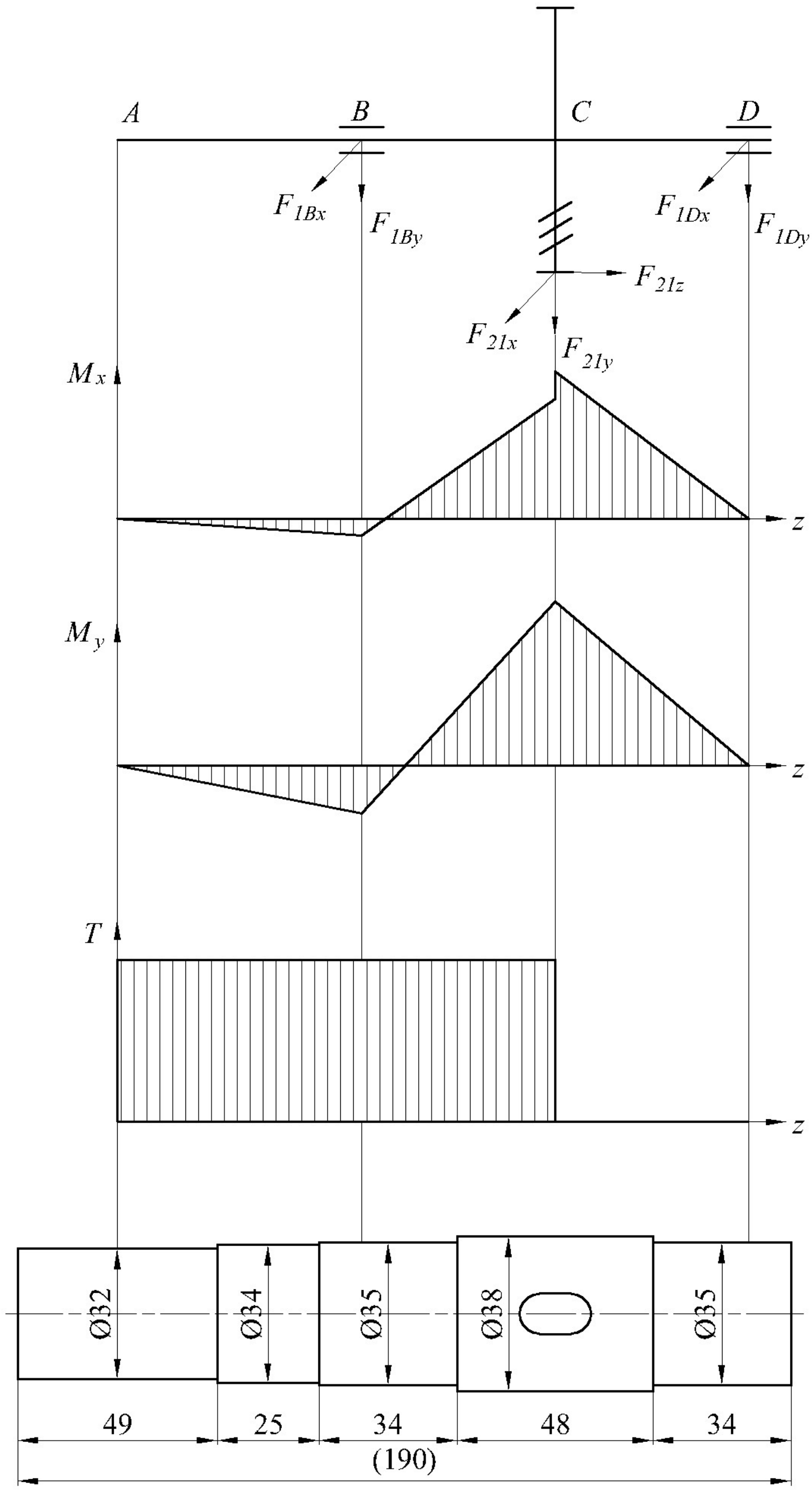


Figure 6.3 Bending moment-torque diagram and section view of shaft 1

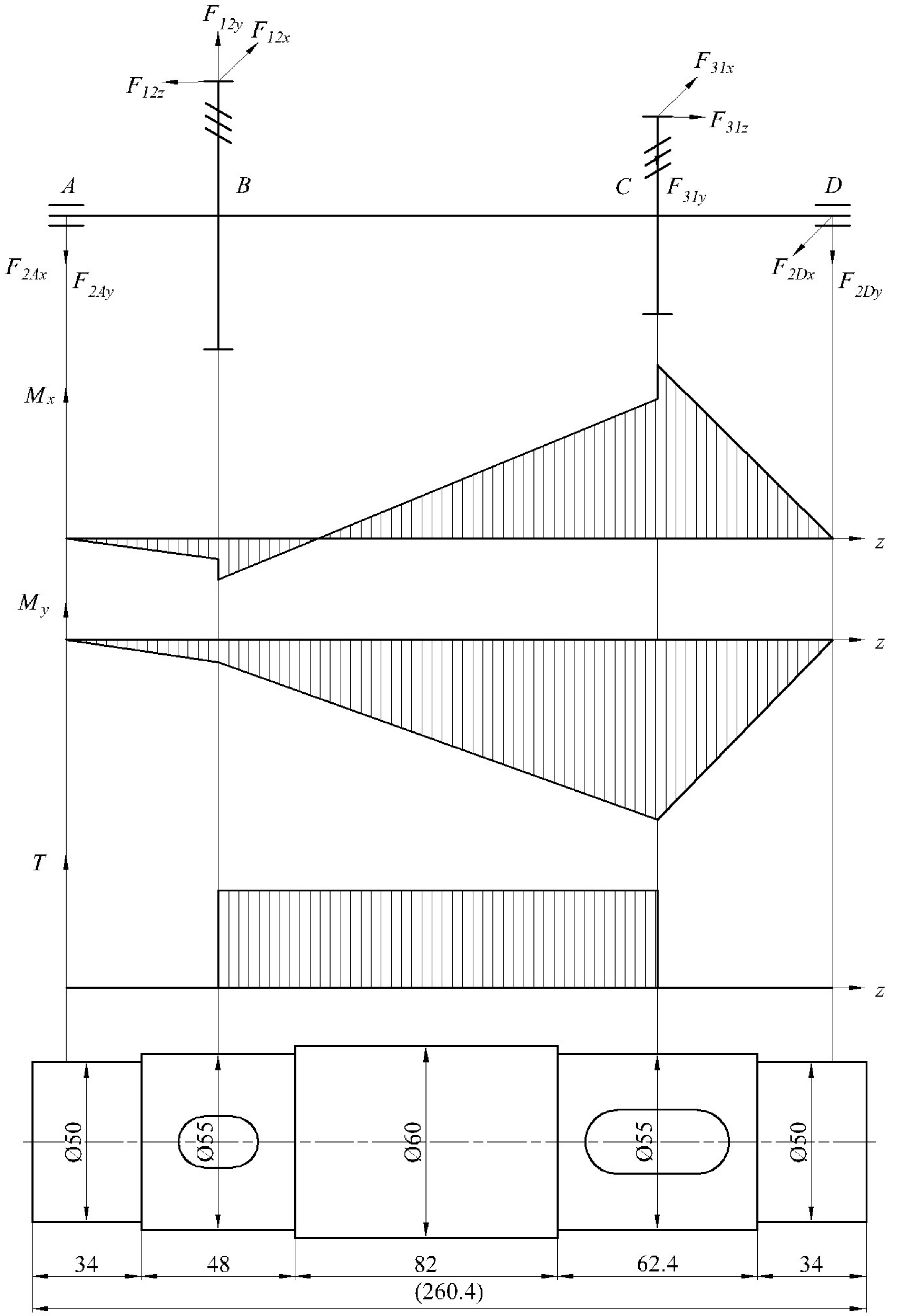


Figure 6.4 Bending moment-torque diagram and section view of shaft 2

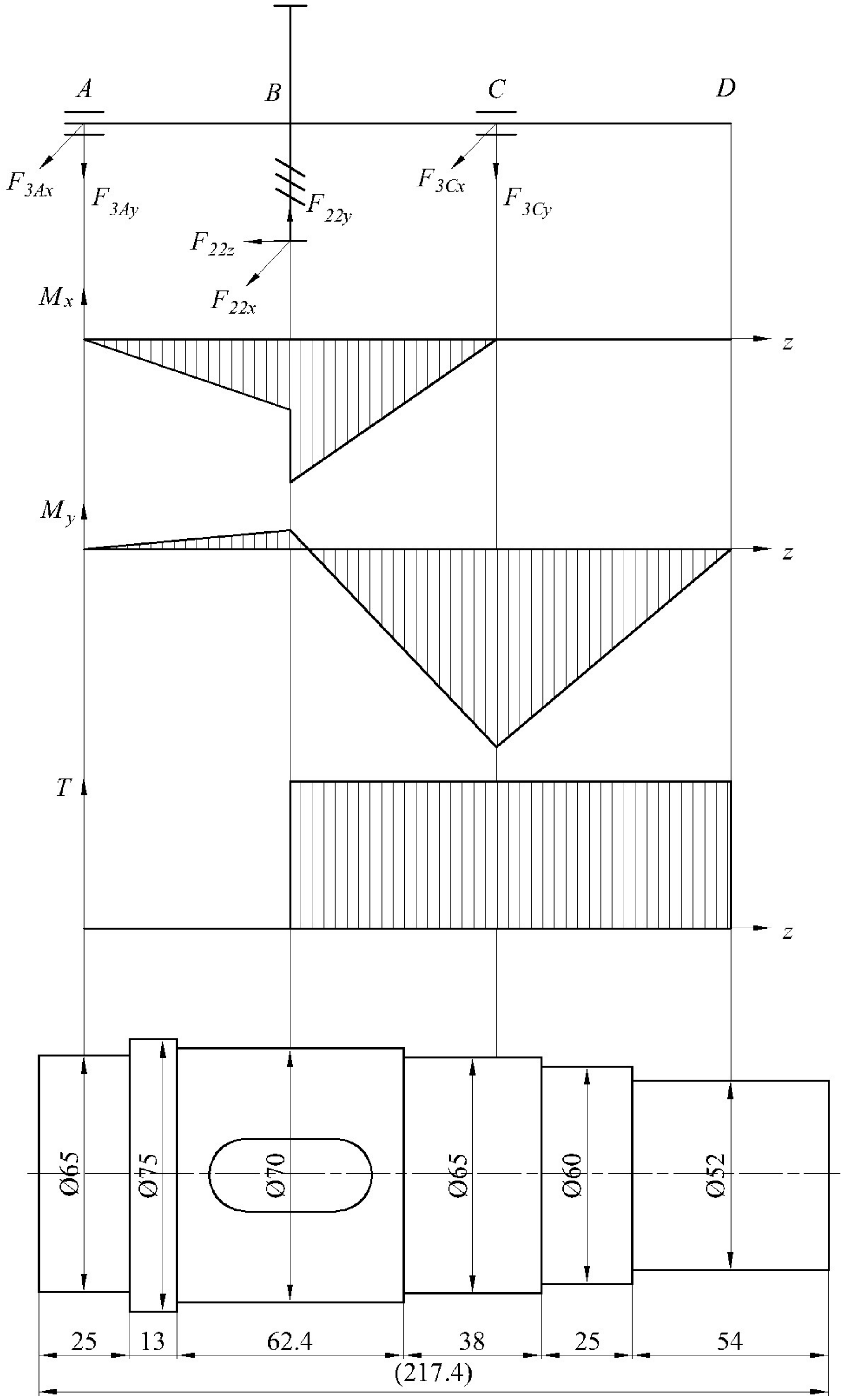


Figure 6.5 Bending moment-torque diagram and section view of shaft 3

IV Design of keys and keyways

4.1 Calculate key dimensions

Fixation of mechanical drives for power transmission purpose requires keys. For simple applications, parallel key is good enough. The dimensions of parallel keys are specified in **Figure 6.6**.

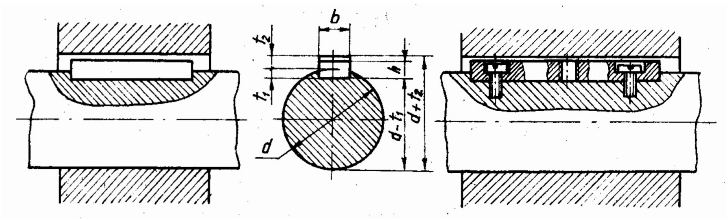


Figure 6.6 Key dimensions according to Vietnam Standard TCVN 2261-77

Choosing key dimension according to standard yields (Table 9.1a [9]):

Table 6.3 Selection of key dimensions (all units are in mm)

Sect.	d	b	h	t_1	t_2	l
1A	32	10	8	5	3.3	32
1C	42	10	8	5	3.3	32
2B	54	16	10	6	4.3	32
2C	54	16	10	6	4.3	45
3B	69	18	11	7	4.4	45
3D	52	16	10	6	4.3	35

After determining the dimensions, the edge of these keys are chamferred 0.4×0.4 mm to reduce stress concentration factor accumulated in sharp corners. For this reason, the edge of the keyways are machined into fillet radius $r = 0.4$ mm.

4.2 Stress analysis of key

The stress analysis of keys are determined by Equation 9.1 and 9.2 [9]:

$$\sigma_d = 2T/[(dl(h - t_1))] \leq [\sigma_d]$$
$$\tau_c = 2T/(dlb) \leq [\tau_c]$$

where

- σ_d is the compressive stress exerting on the key, MPa.
- T is the applied torque onto the shaft. The torque T_{sh1}, T_{sh2}, T_{sh3} exert on shaft 1, 2, 3, respectively. The numerical values are given in **Table 1.1**.
- d is the diameter of shaft at the cross section, mm. The value is given in **Table 6.2**.
- l, h, t_1, b are key dimensions. The values are provided in **Table 6.3**.

- $[\sigma_d]$ is the permissible compressive stress, MPa. The value is provided in Table 9.5 **??**. The gear is made of carbon steel, load condition is light impact and the assembly is fixed. This configuration corresponds to $[\sigma_d] = 100$ MPa
- τ_c is the torsion stress exerting on the key, MPa.
- $[\tau_c]$ is the permissible torsion stress exerting on the key, MPa. For light impact load, $[\tau_c] = 40$ MPa.

The results are summarized in **Table 6.4**.

Table 6.4 Stress analysis of keys

Sect.	T	d	l	h	t_1	b	σ_d	τ_c
$1A$	46423.73	32	32	8	5	10	30.22	9.07
$1C$	46423.73	42	32	8	5	10	23.03	5.76
$2B$	126093.3	54	32	10	6	16	36.49	9.12
$2C$	126093.3	54	45	10	6	16	25.95	6.49
$3B$	342486.86	69	45	11	7	18	41.36	12.26
$3D$	342486.86	52	35	10	6	16	70.57	23.52

V Strength of the shafts

5.1 Fatigue Strength Analysis

For each critical section, the fatigue strength safety factor there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

where

- $[s]$ is the permissible fatigue strength at a cross section. If $[s] > 2.5$, deflection analysis can be ignored.
- s_σ is the tensile stress safety factor at a cross section. It is computed using Inequation 10.20 [9]:

  $$s_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$$

  where

  - σ_{-1} is the permissible bending fatigue stress in a cycle, MPa. For carbon steel, it is estimated using equation:

    $$\sigma_{-1} = 0.436\sigma_b$$

    where σ_b is the yielding stress of material, MPa. All 3 shafts use the same material, which is C35 steel. Therefore, and $\sigma_{-1} = 0.436 \times 585 = 255.06$ MPa.

- K_σ is the overall factor regarding tension stress at a cross section. It is computed using Equation 10.25 [9]:

$$K_\sigma = \frac{k_\sigma/\varepsilon_\sigma + K_x - 1}{K_y}$$

where

  * k_σ is the stress concentration factor due to bending stress, Table 10.13 [9]. The factor depends on geometry of shoulder fillets. Although various techniques have been developed (undercut, inserted ring, undercut to simulate a ring, relief groove, etc.), for simplicity, corner radius method is used only.
    It is recommended that the fillet radius be as large as possible to minimize k_σ, However, the design of the gear, bearing, or other element affects the radius that can be used, p.517 [6]. Therefore, at cross sections where diameter changes, the fillet radius r is computed using the ratio $r/d = 0.5(D - d)$, where D is the larger diameter and d is the smaller diameter. In areas where parts are mounted to the shaft, the ratio is $r/d \leq 0.02$. The reason is to avoid collision between the part and the shaft if the part is not filleted around its edge. In case of shaft 1, those parts are coupling and bearings; for shaft 2, they are bearings; for shaft 3, they are bearings and driving sprocket of the chain drive.
  * ε_σ is the size factor corresponding to bending stress, Table 10.10 [9]. The factor depends on the choice of material, which in this case is carbon steel for all 3 shafts.
  * K_x is the stress concentration factor due to surface finishing, Table 10.8 [9]. For all 3 shafts, the surface is lathed roughly for assembly and cost reduction purpose. Thus, $K_x = 1.2$
  * K_y is the strengthening factor. It is unnecessary to harden the surface since the diameters of all 3 shafts are sufficiently larger than their minimal values, col. 6 and 7, **Table 6.2**. $K_y = 1$

- σ_a is the amplitude of bending stress in a cycle. For rotating shaft, which are all 3 shafts, it is computed using Equation 10.22 [9]:

$$\sigma_a = \sqrt{M_x^2 + M_y^2}/W$$

where M_x is the moment about x-axis; M_y is the moment about y-axis; W is the shaft volume on which the moment exerts. According to Table 10.6 [9], shaft 1 and 3 have 1 keyseat each, thus using the formula:

$$W = \frac{\pi d^3}{32} - \frac{bt_1(d - t_1)^2}{2d}$$

Shaft 2 has 2 keyseats, thus using the formula:

$$W = \frac{\pi d^3}{32} - \frac{bt_1(d - t_1)^2}{d}$$

where d is the diameter at the calculating cross section; b, t_1 is the key dimensions, given in the previous section.

- ψ_σ is the mean bending stress factor affecting on fatigue strength, Table 10.7 [9].

Inspecting the table with the known ultimate strength $\sigma_b = 586\,\text{MPa}$, $\psi_\sigma = 0.05$.

- σ_m is the mean bending stress in a cycle. For rotating shaft, $\sigma_m = 0$.

- s_τ is the torsion stress safety factor at a cross section. It is computed using Inequation 10.21 [9]:

$$s_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$$

where

- τ_{-1} is the permissible torsion fatigue stress in a cycle, MPa. For carbon steel, it is estimated using equation:

$$\tau_{-1} = 0.58\sigma_{-1}$$

where σ_{-1} is the permissible bending fatigue stress in a cycle, which was calculated above. Again, material selection is the same for all 3 shafts. Therefore, $\tau_{-1} = 0.58 \times 255.06 = 147.93$ (MPa).

- K_τ is the overall factor regarding tension stress at a cross section. It is computed using Equation 10.25 [9]:

$$K_\tau = \frac{k_\tau/\varepsilon_\tau + K_x - 1}{K_y}$$

where

* k_τ is the stress concentration factor due to torsion stress, Table 10.13 [9]. The choice of k_τ is similar to k_σ and is dependent on shoulder fillets.
* ε_τ is the size factor corresponding to torsion stress, Table 10.10 [9]. The factor depends on the choice of material, which in this case is carbon steel for all 3 shafts.
* K_x is the stress concentration factor due to surface finishing, Table 10.8 [9]. For all 3 shafts, the surface is lathed roughly for assembly and cost reduction purpose.
* K_y is the strengthening factor. It is unnecessary to harden the surface since the diameters of all 3 shafts are sufficiently larger than their minimal values, col. 6 and 7, **Table 6.2**.

- τ_a is the amplitude of torsion stress in a cycle. For rotating shaft, which are all 3 shafts, it is computed using Equation 10.23 [9]:

$$\tau_a = \sqrt{M_x^2 + M_y^2}/W_o$$

where M_x is the moment about x-axis; M_y is the moment about y-axis; W is the shaft volume on which the moment exerts. According to Table 10.6 [9], shaft 1 and 3 have 1 keyseat each, thus using the formula:

$$W_o = \frac{\pi d^3}{16} - \frac{b t_1 (d - t_1)^2}{2d}$$

Shaft 2 has 2 keyseats, thus using the formula:

$$W_o = \frac{\pi d^3}{16} - \frac{bt_1(d-t_1)^2}{d}$$

where d is the diameter at the calculating cross section; b, t_1 is the key dimensions, given in the previous section.

- ψ_τ is the mean torsion stress factor affecting on fatigue strength, Table 10.7 [9]. Inspecting the table with the known ultimate strength $\sigma_b = 586\,\text{MPa}$, $\psi_\tau = 0$.
- τ_m is the mean torsion stress in a cycle. For unidirectional rotating shafts (i.e. the motor only rotates in 1 direction), $\tau_m = \tau_a = \sqrt{M_x^2 + M_y^2}/W_o$.

• $[s]$ is the overall safety factor. For negligible deflection, $[s] \approx 2.5$.

Aggregating all results with the support from software (e.g. Microsoft Excel, Python), the following tables are generated:

For convenience, the common values for computing s_σ are mentioned here:

$$\sigma_{-1} = 255.06\,\text{MPa}, K_x = 1.2, K_y = 1, \psi_\sigma = 0.05, \sigma_m = 0$$

Table 6.5 Computation of tensile stress safety factor at critical cross sections s_σ

Sect.	d	b	t_1	r	r/d	k_σ	ε_σ	K_σ	$\lvert M_x\rvert$	$\lvert M_y\rvert$	W	σ_a	s_σ	
1A	32	10	5	0.5	0.5	0.02	2.61	0.87	3.05	0	0	2647.46	0	∞
1B	35	0	0	0.5	0.5	0.01	2.64	0.86	3.12	5698.14	30554.04	4209.24	7.38	11.09
1C	42	10	5	0.5	0.5	0.01	2.70	0.84	3.27	44426.31	79996.02	6295.72	14.53	5.37
1D	35	0	0	0.5	0.5	0.01	2.64	0.86	3.12	0	0	4209.24	0	∞
2A	45	0	0	0.5	0.5	0.01	2.72	0.83	3.34	0	0	11362.99	0	∞
2B	54	16	6	0.5	0.5	0.01	2.77	0.80	3.52	44138.39	49441.98	11362.99	5.83	12.42
2C	54	16	6	0.5	0.5	0.01	2.77	0.80	3.52	178374.12	240553.02	12142.99	26.35	2.75
2D	45	0	0	0.5	0.5	0.01	2.72	0.83	3.34	0	0	8946.18	0	∞
3A	60	0	0	0.5	0.5	0.01	2.79	0.79	3.62	0	0	21205.75	0	∞
3B	69	18	7	0.5	0.5	0.01	2.82	0.76	3.76	143145.85	60350.85	28741.56	5.4	12.56
3C	60	0	0	1.5	1.5	0.03	2.42	0.79	3.14	0	654674.15	21205.75	30.87	2.63
3D	52	16	6	0.5	0.5	0.01	2.76	0.81	3.49	0	0	11850.93	0	∞

For convenience, the common values for computing s_τ are mentioned here:

$$\tau_{-1} = 147.93\,\text{MPa}, K_x = 1.2, K_y = 1, \psi_\tau = 0$$

Table 6.6 Computation of torsion stress safety factor at critical cross sections s_τ and the overall safety factor s

Sect.	d	b	t_1	r	r/d	k_τ	ε_τ	K_τ	$\lvert M_x\rvert$	$\lvert M_y\rvert$	W_o	τ_m	τ_a	s_τ	s
1A	32	10	5	0.5	0.02	1.87	0.80	2.38	0	0	5864.45	3.96	3.96	15.7	-
1B	35	0	0	0.5	0.01	1.89	0.80	2.43	5698.14	30554.04	8418.49	2.76	2.76	22.06	9.91
1C	42	10	5	0.5	0.01	1.92	0.78	2.54	44426.31	79996.02	13569.29	1.71	1.71	34.1	5.3
1D	35	0	0	0.5	0.01	1.89	0.80	2.43	0	0	8418.49	0	0	∞	-
2A	45	0	0	0.5	0.01	1.93	0.77	2.57	0	0	17892.35	0	0	∞	-
2B	54	16	6	0.5	0.01	1.90	0.75	2.66	44138.39	49441.98	26821.98	2.35	2.35	23.65	11
2C	54	16	6	0.5	0.01	1.90	0.75	2.66	178374.12	240553.02	26821.98	2.35	2.35	23.65	2.73
2D	45	0	0	0.5	0.01	1.93	0.77	2.57	0	0	17892.35	0	0	∞	-
3A	60	0	0	0.5	0.01	1.98	0.74	2.71	0	0	42411.50	0	0	∞	-
3B	69	18	7	0.5	0.01	1.99	0.73	2.78	143145.85	60350.85	60992.85	2.81	2.81	18.94	10.47
3C	60	0	0	1.5	0.03	1.75	0.74	2.41	0	654674.15	42411.50	4.04	4.04	15.21	2.59
3D	52	16	6	0.5	0.01	1.96	0.76	2.64	0	0	25655.09	6.67	6.67	8.38	-

5.2 Static Strength Analysis

Along with fatigue strength, static strength is also considered. Every shaft must satisfy the condition below, which is a different representation of Equation 10.27 [9]:

$$\sigma_e = \sqrt{\left(\frac{\sqrt{M_x^2 + M_y^2}}{0.1d^3}\right)^2 + 3\left(\frac{T_{sh}}{0.2d^3}\right)^2} \leq [\sigma]$$

where

- $\sqrt{M_x^2 + M_y^2}$ is the largest bending moment at the cross section, $\mathrm{N} \cdot \mathrm{mm}$; M_x, M_y are calculated in **Table 6.2**.
- T_{sh} is the torsion moment at a cross section, $\mathrm{N} \cdot \mathrm{mm}$.
- $[\sigma]$ is the permissible static strength of the shaft at a cross section, MPa. It is computed using Equation 10.30 [9]:

  $$[\sigma] = 0.8\sigma_{ch} = 0.8 \times 370 = 296\,(\mathrm{MPa})$$

  where σ_{ch} is the yield strength of the material of the shaft. Since 3 shafts use the same material. The material is carbon steel AISI 1035 yielding at $\sigma_{ch} = 370\,\mathrm{MPa}$ [1].

Table 6.7 Equivalent moment M_e and cross section diameters of 3 shafts

Sect.	d	$\lvert M_x \rvert$	$\lvert M_y \rvert$	T_{sh}	σ_e
1A	32	0	0	46423.73	12.27
1B	35	5698.14	30554.04	46423.73	11.85
1C	42	44426.31	79996.02	46423.73	13.49
1D	35	0	0	0	0
2A	45	0	0	0	0
2B	54	44138.39	49441.98	126093.3	8.11
2C	54	178374.12	240553.02	126093.3	20.24
2D	45	0	0	0	0
3A	60	0	0	0	0
3B	69	143145.85	60350.85	342486.86	10.19
3C	60	0	654674.15	342486.86	33.27
3D	52	0	0	342486.86	21.09

Chapter 7

Bearing Design

I Choose bearing type

The reaction forces of bearings were calculated in Chapter 6. Since bearings selection is dependent on the ratio F_a/F_r, the list below identifies the loads on each shaft:

- On shaft 1, the radial loads are F_{1Bx}, F_{1By}, F_{1Dx}, F_{1Dy}; the axial load is F_{21z}.
- On shaft 2, the radial loads are F_{2Ax}, F_{2Ay}, F_{2Dx}, F_{2Dy}; the axial load is $F_{12z} + F_{31z}$.
- On shaft 3, the radial loads are F_{3Ax}, F_{3Ay}, F_{3Cx}, F_{3Cy}; the axial load is F_{22z}.

Since shaft is simply supported beam (i.e. a shaft has pinned support at 1 end and roller support at the other end), the left side bearing carries both radial and axial forces while the right side bearing only carries radial force. **Table 7.1** summarizes the calculation of those loads (refer to **Table 6.1** for numerical values of the loads):

Table 7.1 Calculation table for limiting ratio F_a/F_r

Sect.	F_x (kN)	F_y (kN)	F_z (kN)	$F_r = \sqrt{F_x^2 + F_y^2}$ (kN)	$F_a = \lvert F_z \rvert$ (kN)	F_a/F_r
1B	-0.56	-0.1	0.22	0.57	0.22	0.39
1D	-0.56	-0.31	0	0.64	0	-
2A	1.16	0.07	0.47	1.16	0.47	0.41
2D	1.07	-0.79	0	1.33	0	-
3A	1.76	-0.19	-0.69	1.77	0.69	0.39
3C	-6.62	1.33	0	6.75	0	-

The ratio $F_a/F_r \geq 0.3$ indicates the axial loads on 3 shafts are significant. Therefore, angular contact ball bearings, single row is selected. However, if the bearings are used, misalignment should not exist as recommended by the comparison table on p.72 [7].

The shafts are short in length, which is suitable for adjusted bearing arrangements since thermal expansion is small to axially displacing the bearings. **Figure 7.1** shows similar design and arrangement of the shafts in the speed reducer.

It is worth to mention that the actual radial load F_r and axial load F_a depend on both the bearing type and bearing arrangement. What computed in **Table 7.1** was the preliminary calculation to determine the suitable bearing type given the loads. The 'real' F_r and F_a shall be computed in the next section.

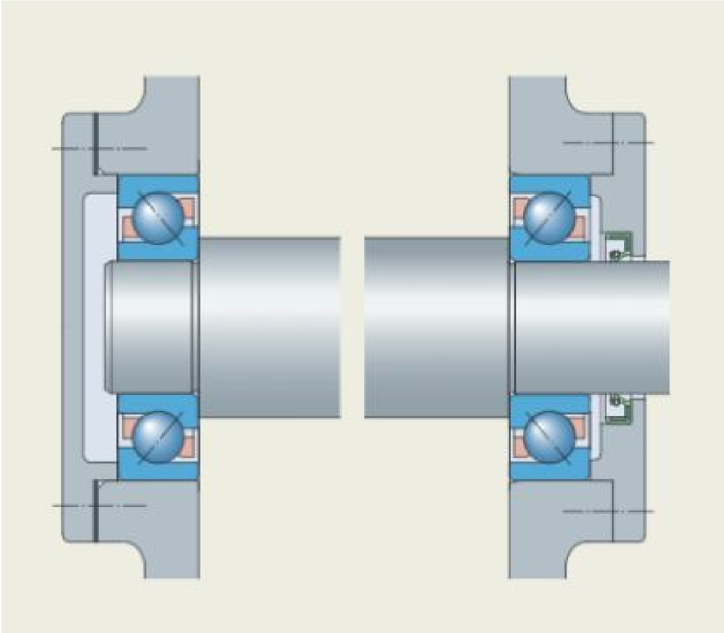


Figure 7.1 Adjusted bearing arrangement, angular contact ball bearings arranged face-to-face. It requires proper adjustment of clearance or preload during mounting, p.76 [7]

Also from the previous chapter, the bore diameter d and available space b are also computed:

$$d_{1B} = d_{1D} = 35\,\text{mm}, b_{O1} = 21\,\text{mm}$$
$$d_{2A} = d_{2D} = 45\,\text{mm}, b_{O2} = 21\,\text{mm}$$
$$d_{3A} = d_{3C} = 60\,\text{mm}, b_{O3} = 25\,\text{mm}$$

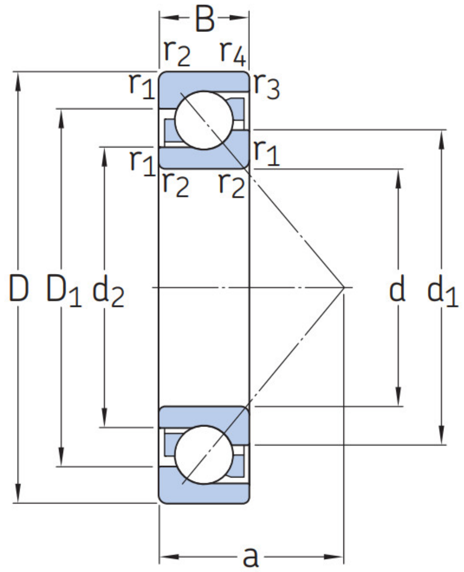
From the specifications above, the bearings are selected from standard single row angular contact ball bearings product table, p.406 [7]:

Table 7.2 Compact specification table of selected bearings according to SKF [7]

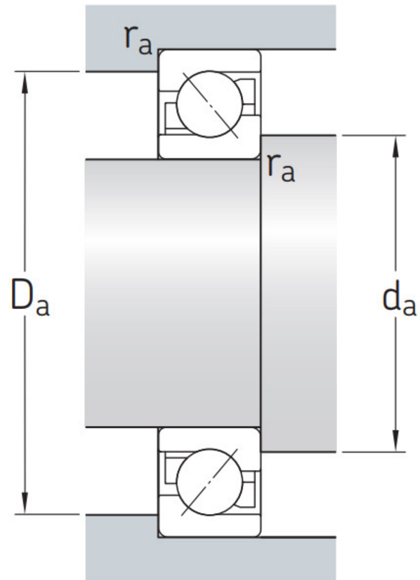
Specifications	Bearings 1	Bearings 2	Bearings 3
SKF designation	7207 BE-2RZP	7210 BE-2RZP	7212 BEP
d (mm)	30	45	60
D (mm)	72	90	110
B (mm)	17	20	22
C (kN)	29.1	37.7	57.2
C_0 (kN)	19	28.5	45.5
P_u (kN)	0.815	1.22	1.93
d_a (mm) min.	42	57	69
d_a (mm) max.	49	65	-
D_a (mm) max.	65	83	101
r_a (mm) max.	1	1	1.5
k_r	0.095	0.095	0.095

where

- D, B, d_a, D_a, r_a are the dimensions, specified in **Figure 7.2**.



(a) Front view of single row angular contact ball bearings



(b) Important dimensions in assembly of shaft and single row angular contact ball bearings in a housing unit

Figure 7.2 Important dimensions in assembly of single row angular contact ball bearings [7]

- C is the basic dynamic load rating, kN.
- C_0 is the basic static load rating, kN.
- P_u is the fatigue load limit, kN.
- A, k_r are the calculation factors.

Referring to **Table 6.5** and **6.6**, the fillet radius and shoulder diameter are in accordance with the limits provided in **Table 7.2**.

II SKF rating life

SKF uses a modified life factor in accordance with ISO 281 to optimize prediction of bearing life, p.89 [7]. In terms of operating hours, the SKF rating life L_{nmh} (unit is h) is

$$L_{nmh} = \left(\frac{10^6}{60n}\right) a_1 a_{SKF} \left(\frac{C}{P}\right)^p$$

where

- n is the speed of the shaft on which the bearing is mounted, rpm. The value is provided in **Table 1.1**.
- a_1 is the life adjustment factor for reliability, Table 3, p.90 [7]. The bearings are evaluated at 90% reliability for common machinery. Therefore, $a_1 = 1$.

- a_{SKF} is the life modification factor, a_{SKF}. The value relies on many other factors, as shown in Diagram 9, p.96 [7].

  In the diagram, the horizontal value is $\eta_c \frac{P_u}{P}$, where η_c is the contamination factor, provided in Table 6, p.105 [7] ($\eta_c = 0.5$ for normal cleanliness, chosen at worst case); P_u is the fatigue load limit, which was given in **Table 7.2**; P is the equivalent dynamic bearing load, which shall be mentioned below.

  The curves in the diagram correspond to a value of κ, where κ is the lubrication condition (viscosity ratio) of the bearing. The value κ is calculated using equation on p.102 [7]:

  $$\kappa = \frac{\nu}{\nu_1}$$

  where

  - ν is the actual operating viscosity of the oil or grease based oil, mm^2/s. The value depends on operating temperature, as given in Diagram 13, p.100 [7]. Industrial machines usually operate at 40°C, however the viscosity will be considered at 80°C in worst case. The mineral oil is ISO VG 46, which is a high quality industrial oil commonly used for mechanical lubrication. Therefore, the viscosity is $\nu = 30\,\mathrm{mm}^2/\mathrm{s}$.
  - ν_1 is the rated viscosity, function of the mean bearing diameter and rotational speed, mm^2/s. The value is evaluated as shown in Diagram 14, p.101 [7]. The variable in the horizontal axis is bearing mean diameter d_m, mm. It is calculated using bore diameter d and outer diameter D, p.102 [7] (all of which have been mentioned above). On the page, additional considerations are also mentioned. If the factor $nd_m \leq 10000\,\mathrm{mm/min}$, EP (extreme pressure) and AW (anti-wear) additives are needed to reduce wear. If $nd_m \geq 500,000\,\mathrm{mm/min}$ for $d_m < 200\,\mathrm{mm}$ or $nd_m \geq 400,000\,\mathrm{mm/min}$ for $d_m > 200\,\mathrm{mm}$, operating temperature must be given more attention. Since the factors in 3 shafts are outside these conditions, it is therefore omitted.

Table 7.3 Calculation table for lubrication condition κ

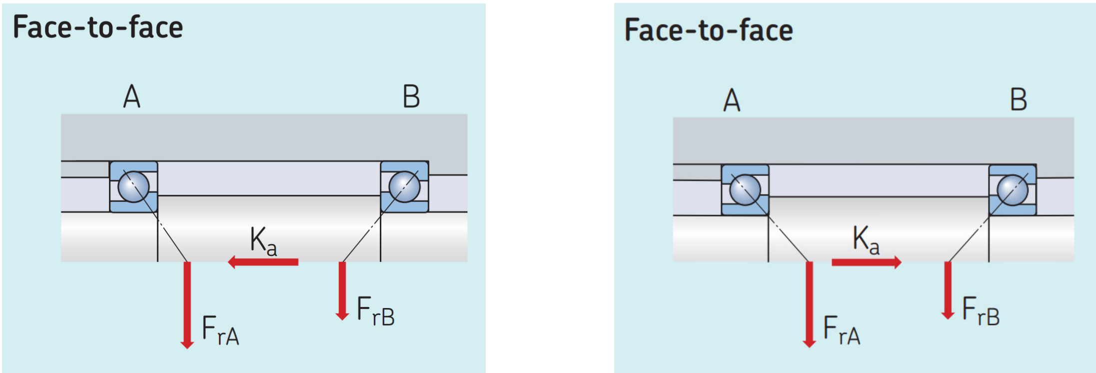
Shaft	d	D	$d_m = 0.5(d+D)$	n (rpm)	nd_m (mm/min)	ν_1 (mm^2/s)	κ
1	35	72	53.5	1455	77842.5	15	2.33
2	45	90	67.5	514.42	34723.36	30	1.17
3	60	110	90	181.88	16368.75	48	0.73

To summarize the results, **Table 7.4** is provided below:

- C is the basic dynamic load rating, kN. The value is provided in **Table 7.2**.
- P is the equivalent dynamic bearing load, kN. It is calculated using the formula on p.398 [7] multiplied by the gear factor 1.05 (pitch and form errors $< 0.02\,\mathrm{mm}$, p.93 [7]). For the necessary factors X, Y, e, Table 10 on p.400 [7] is used for bearings pairs arranged face-to-face:

Table 7.4 Calculation table life modification factor a_{SKF}

Sect.	P_u (kN)	P (kN)	$\eta_c \frac{P_u}{P}$	κ	a_{SKF}
$1B$	0.81	0.88	0.46	0.87	15
$1D$	0.81	1.08	0.38	0.87	10
$2A$	1.22	1.81	0.34	0.43	1.1
$2D$	1.22	2.26	0.27	0.43	0.9
$3A$	1.93	2.76	0.35	0.27	0.22
$3C$	1.93	7.59	0.13	0.27	0.2



(a) K_a points to the left **(b)** K_a points to the right

Figure 7.3 Face-to-face force distribution of single row angular contact ball bearings [7]

$$\begin{cases} P = 1.05(F_r + 0.55F_a), F_a/F_r \leq 1.14 \\ P = 1.05(0.57F_r + 0.93F_a), F_a/F_r > 1.14 \end{cases}$$

where

- F_a is the axial force, N. The value is computed according to Table 11, p.401 [7]. All 3 bearings are B-series as designated on p.404 [7], therefore they have 40° contact angle. This value corresponds to factor $R = 0.88$. Following the provided conditions and the direction of F_z (all units are in kN), **Table 7.5** is obtained:

Table 7.5 Compact specification table of selected bearings according to SKF [7]

Shaft	F_{rA}	F_{rB}	$K_a = \|F_z\|$	$R(F_{rB} - F_{rA})$	F_{aA}	F_{aB}
1	0.57	0.64	0.22 (**Fig. 7.3a**)	0.06 (case 1b)	0.5	0.72
2	1.16	1.33	0.47 (**Fig. 7.3a**)	0.15 (case 1b)	1.02	1.5
3	1.77	6.75	0.69 (**Fig. 7.3b**)	4.38 (case 2c)	1.56	0.87

where F_{rA} and F_{rB} are the radial forces of the left and right bearings, respectively.

- F_r is the radial force, kN. The value is provided in **Table 7.2**. It is required that F_r

must be larger than the minimum radial load F_{rm}, which is calculated as

$$F_{rm} = k_r \left(\frac{\nu n}{1000}\right)^{2/3} (\frac{d_m}{100})^2$$

where

* k_r is a calculation factor, which is given in **Table 7.2**.
* ν is the actual operating viscosity of the oil, which was mentioned above.
* n is the shaft speed, which was mentioned above.
* d_m is the bearing mean diameter, mm. The value is computed in Table

Table 7.6 Condition check for radial load for bearing pairs arranged face-to-face

Shaft	k_r	d	D	$d_m = 0.5(d + D)$	F_{rA}	F_{rB}	F_{rm}	Condition $F_r \geq F_{rm}$
1	0.095	35	72	53.5	0.57	0.64	0.34	Satisfied
2	0.095	45	90	67.5	1.16	1.33	0.27	Satisfied
3	0.095	60	110	90	1.77	6.75	0.24	Satisfied

In total, the equivalent dynamic bearing load on each bearing is shown in **Table 7.7** (the units are in kN except for ratio F_a/F_r, which is dimensionless):

Table 7.7 Calculation table for equivalent dynamic bearing load P

Sect.	F_r	F_a	$F_a/F_r < e$	P (kN)
1B	0.57	0.5	0.88<1.14	0.88
1D	0.64	0.72	1.13<1.14	1.08
2A	1.16	1.02	0.88<1.14	1.81
2D	1.33	1.5	1.12<1.14	2.26
3A	1.77	1.56	0.88<1.14	2.76
3C	6.75	0.87	0.13<1.14	7.59

- p is the exponent of the life equation. For ball bearings, $p = 3$.

In summary, the rating life L_{nmh} is computed in **Table 7.8**. According to calculation, under specified environmental conditions (normal cleanliness, working at 80°C and 90% reliability), the bearings are guaranteed to operate for at least $12,000$ hours. This is the typical life span in machines for use 8 hours a day, but not always fully utilized, Table 1, p.88 [7].

SKF recommends when $\kappa < 1$, EP/AW additives should be used, p.102 [7]. However, it is always possible to obtain higher lubrication condition κ if better viscosity oil is used (ISO VG 63). Also, the speeds of 3 shafts are not too low (> 10 (rpm)) that static loading analysis is needed. Therefore, the selected SKF bearings are still valid.

The proposed bearings are listed as follows (specifications are provided in SKF bearings catalog [7]):

- Shaft 1: 7207 BE-2RZP

- Shaft 2: 7210 BE-2RZP
- Shaft 3: 7212 BEP

Table 7.8 Calculation table for SKF rating life L_{nmh} in hours

Sect.	n (rpm)	a_{SKF}	C (kN)	P (kN)	L_{nmh} (h)
1B	1455	15	29.1	0.88	$6,201,499.81$
1D	1455	10	29.1	1.08	$2,238,026.65$
2A	514.42	1.1	37.7	1.81	$321,910.25$
2D	514.42	0.9	37.7	2.26	$135,097.87$
3A	181.88	0.22	66.3	2.76	$278,897.56$
3C	181.88	0.2	66.3	7.59	$12,211.88$

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