

# Machine Elements Report

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- $F_t$  tangential force, N
- $v$  conveyor belt speed, m/s
- $D$  pulley diameter, mm
- $L$  service life, year
- $T$  working torque, N · mm
- $t$  working time, s
- $\delta_u$  error of speed ratio, %

# Chapter 1

## Motor Design

### 1.1 Nomenclature

$\eta_c$	coupling efficiency	$n_{bc}$	rotational speed of belt conveyor, rpm
$\eta_b$	bearing efficiency	$n_{sh}$	rotational speed of shaft, rpm
$\eta_{hg}$	helical gear efficiency	$u_{hg}$	transmission ratio of helical gear
$\eta_{ch}$	chain drive efficiency	$u_{ch}$	transmission ratio of chain drive
$\eta_{sys}$	efficiency of the system	$u_{sys}$	transmission ratio of the system
$P_m$	maximum operating power of belt conveyor, kW	$T_{motor}$	motor torque, N · mm
$P_w$	opearting power of belt conveyor given a workload, kW	$T_{sh}$	shaft torque, N · mm
$P_{motor}$	calculated motor power to drive the system, kW		
$P_{sh}$	operating power of shaft, kW		

### 1.2 Calculate $\eta_{sys}$

From table 2.3 :

$$\eta_c = 1, \eta_b = 0.99, \eta_{hg} = 0.96, \eta_{ch} = 0.95$$

$$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$$

### 1.3 Calculate $P_{motor}$

$$P_m = \frac{F_t v}{1000} \approx 13.73 \text{ (kW)}$$

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$
$$P_{motor} = \frac{P_w}{\eta_{sys}} \approx 11.76 \text{ (kW)}$$

### 1.4 Calculate $n_{motor}$

$$n_{bc} = \frac{6 \times 10^4 v}{\pi D} \approx 116.5 \text{ (rpm)}$$

$$u_{ch} = 5, u_{hg} = 5 \text{ (table 2.4)}$$

$$u_{sys} = u_{ch} u_{hg} = 25$$

$$n_{motor} = u_{sys} n_{bc} \approx 2912.54 \text{ (rpm)}$$

### 1.5 Choose motor

The operating power and rotational speed of the chosen motor must be larger than estimated  $P_{motor}$  and  $n_{motor}$ , respectively. Thus, from table P1.3, we choose motor 4A160S2Y3 operating at 15 kW and 2930 rpm

$$\Rightarrow P_{motor} = 15 \text{ kW}, n_{motor} = 2930 \text{ (rpm)}$$

Recalculating  $u_{sys}$  with the new  $P_{motor}$  and  $n_{motor}$ , we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming  $u_{hg} = \text{const}$ :

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

## 1.6 Calculate power, rotational speed and torque of the motor and 2 shafts

### 1.6.1 Power

$$P_{ch} = P_w \approx 10.41 \text{ (kW)}$$

$$P_{sh2} = \frac{P_{ch}}{\eta_{ch}} \approx 10.96 \text{ (kW)}$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 11.53 \text{ (kW)}$$

$$P_{motor} = \frac{P_{sh1}}{\eta_b \eta_c} \approx 11.64 \text{ (kW)}$$

### 1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$

$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

### 1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 37950.46 \text{ (N} \cdot \text{mm)}$$

$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 37570.93 \text{ (N} \cdot \text{mm)}$$

$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 178537.08 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
$P$ (kW)	11.64	11.527	10.96
$u$	5	5.03	
$n$ (rpm)	2930	2930	586
$T$ (N · mm)	37950.44	37570.93	178537.08

Table 1.1: System properties

# Chapter 2

## Chain Drive Design

### 2.1 Nomenclature

$z_1$	number of teeth on the driving sprocket	$q$	mass per meter of chain, kg/m
		$v_1$	driving sprocket speed, m/s
$z_2$	number of teeth on the driven sprocket	$F_t$	tangential force on shaft, N
		$F_v$	centrifugal force, N
$z_{max}$	maximum number of teeth on the driven sprocket	$F_0$	tension from passive chain part, N
		$F_r$	force on shaft, N
$[P]$	permissible power, kW	$n_{ch}$	rotational speed of chain drive, rpm
$p$	sprocket pitch, mm		
$d_1$	driving sprocket diameter, mm	$n_{01}$	experimental rotational speed, rpm
$d_2$	driven sprocket diameter, mm		
$d_c$	pin diameter, mm	$k_z$	coefficient of number of teeth
$B$	bush length, mm	$k_n$	coefficient of rotational speed
$Q$	permissible load, N	$k$	overall factor
$a$	center distance, mm	$k_0$	arrangement of drive factor
$a_{min}$	minimum center distance, mm	$k_a$	center distance and chain's length factor
$a_{max}$	maximum center distance, mm		



$x$	number of links	$k_{dc}$	chain tension factor
$x_c$	an even number of links	$k_{bt}$	lubrication factor
$i$	impact times per second	$k_d$	dynamic loads factor
$[i]$	permissible impact times per second	$k_c$	rating factor
		$k_f$	loosing factor
$s$	safety factor	$k_x$	chain weight factor
$[s]$	permissible safety factor		

## 2.2 Find $p$

$$n_{ch} = n_{sh2} = 586 \text{ (rpm)}$$

**Find  $z$**  Since  $z_1$  and  $z_2$  is preferably an odd number (p.80):

$$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$$

$$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$$

**Find  $k$**  Since  $n_{ch} = 586 \approx 600 \text{ (rpm)}$ , choose  $n_{01} = 600 \text{ (rpm)}$ , which is obtained from table 5.5. Then, we calculate  $k_z$  and  $k_n$

$$k_z = \frac{25}{z_1} \approx 1.32, k_n = \frac{n_{01}}{n_{ch}} \approx 1.02$$

Specifying the chain drive's working condition and ultizing table 5.6 , we find out that  $k_0 = k_a = k_{dc} = k_{bt} = 1, k_d = 1.25, k_c = 1.3$

$$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$$

**Find  $p$**  From table 5.5:

$$[P] = P_{ch} k k_z k_n \approx 22.78 \text{ (kW)} \leq 25.7 \text{ (kW)} \Rightarrow [P] = 25.7 \text{ (kW)}$$

Using the table, we also get the other parameters:

$$p = 25.4 \text{ (mm)}, d_c = 7.95 \text{ (mm)}, B = 22.61 \text{ (mm)},$$

$$d_1 = \frac{P}{\sin \frac{\pi}{z_1}} \approx 154.32 \text{ (mm)}, d_2 = \frac{P}{\sin \frac{\pi}{z_2}} \approx 784.39 \text{ (mm)}$$

## 2.3 Find $a$ , $x_c$ and $i$

**Find  $x_c$**   $a_{min} = 30p = 762$  (mm),  $a_{max} = 50p = 1270$  (mm). Therefore, we can approximate  $a = 800$  (mm)

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 123.71 \Rightarrow x_c = 124$$

**Find  $a$**  From equation (5.13), recalculating  $a$  with  $x_c$ :

$$a = \frac{p}{4} \left( x_c - \frac{z_2 + z_1}{2} + \sqrt{\left( x_c - \frac{z_2 + z_1}{2} \right)^2 - 2 \frac{(z_2 - z_1)^2}{\pi^2}} \right) - 0.003 \cdot 800 \approx 771.66 \text{ (mm)}$$

**Find  $i$**  From table 5.9:

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 30$$

## 2.4 Strength of chain drive

Choose  $k_d = 1.2$ ,  $k_f = 6$

Given  $p$  from previous calculations,  $Q$  and  $q$  are obtained from table 5.2 :

$$Q = 56.7 \times 10^3 \text{ (N)}, q = 2.6 \text{ (kg/m)}$$

$$v_1 = \frac{n_{ch} p z_1}{6 \times 10^4} \approx 4.71 \text{ (m/s)}$$

**Find  $F_t$ ,  $F_v$ ,  $F_0$**  We also need to calculate  $F_t$ ,  $F_v$  and  $F_0$

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2208.07 \text{ (N)}$$

$$F_v = q v_1^2 \approx 57.76 \text{ (N)}$$

$$F_0 = 9.81 \times 10^{-3} k_f q a \approx 118.09 \text{ (N)}$$

**Validate  $s$**  From equation (5.15) :

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \approx 20.07 \geq [s] = 10.3, \text{ where } [s] \text{ is chosen from table 5.10}$$

## 2.5 Force on shaft

Choose  $k_x = 1.15$  and follow equation (5.20) :

$$F_r = k_x F_t \approx 2539.28 \text{ (N)}$$

In summary, we have the following table:

$[P]$ (kW)	25.7
$n$ (rpm)	586
$u_{ch}$	5.03
$z_1$	19
$z_2$	97
$p$ (mm)	25.4
$d_1$ (mm)	154.32
$d_2$ (mm)	784.39
$d_c$ (mm)	7.95
$B$ (mm)	22.61
$x_c$	124
$a$ (mm)	771.66
$i$	6

Table 2.1: Table of chain drive specifications

# Chapter 3

## Gearbox Design (Helix gears)

### 3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, $MPa$	$K_{HL}$	aging factor due to contact stress
$[\sigma_F]$	permissible bending stress, $MPa$	$K_{FL}$	aging factor due to bending stress
$\sigma_{Hlim}^o$	permissible contact stress corresponding to working cycle, $MPa$	$K_{FC}$	load placement factor
		$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth
$\sigma_{Flim}^o$	permissible bending stress corresponding to working cycle, $MPa$	$K_{H\beta}$	factor of load distribution from contact stress on top land
		$K_{Hv}$	factor of dynamic load from contact stress at meshing area
$\sigma_b$	ultimate strength, $MPa$		
$\sigma_{ch}$	yield limit, $MPa$	$K_H$	load factor from contact stress
$H$	surface roughness, HB	$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
$S$	length, $mm$		
$S_H$	safety factor of contact stress	$K_{F\beta}$	factor of load distribution from bending stress on top land
$S_F$	safety factor of bending stress		
$N_{HO}$	working cycle of bearing stress corresponding to $[\sigma_H]$	$K_{Fv}$	factor of dynamic load from bending stress at meshing area
		$K_F$	load factor from bending stress

$N_{HE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$	$K_d$	coefficient of gear material
$N_{FO}$	working cycle of bearing stress corresponding to $[\sigma_F]$	$Y_\epsilon$	meshing factor
$N_{FE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$	$Y_\beta$	helix angle factor
$AG$	accuracy grade of gear	$Y_F$	tooth shape factor
$z_M$	material's mechanical properties factor	$c$	gear meshing rate
$z_H$	contact surface's shape factor	$a_w$	center distance, $mm$
$z_\epsilon$	meshing condition factor	$b_w$	face width, $mm$
$z_{min}$	minimum number of teeth corresponding to $\beta$	$d$	pitch circle diameter, $mm$
$z_v$	equivalent number of teeth	$d_w$	rolling circle diameter, $mm$
$\epsilon_\alpha$	horizontal meshing condition factor	$d_a$	addendum diameter, $mm$
$\epsilon_\beta$	vertical meshing condition factor	$d_f$	deddendum diameter, $mm$
$\alpha$	base profile angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20^\circ$	$d_b$	base diameter, $mm$
$\alpha_t$	profile angle of a gear tooth, $^\circ$	$m_H$	root of fatigue curve in contact stress test
$\alpha_{tw}$	meshing profile angle, $^\circ$	$m_F$	root of fatigue curve in bending stress test
$\beta$	helix angle, $^\circ$	$m$	traverse module, $mm$
$\beta_b$	helix angle at base circle, $^\circ$	$m_n$	normal module, $mm$
$\psi_{ba}$	width to shaft distance ratio	$v$	rotational velocity, $m/s$
$\psi_{bd}$	width to pinion diameter ratio	$x$	gear correction factor
		$y$	center displacement factor
		1	subscript for driving gear
		2	subscript for driven gear

## 3.2 Choose material

From table 6.1 , the material of choice for both gears is steel 40X with  $S \leq 100$  mm, HB250,  $\sigma_b = 850$  MPa,  $\sigma_{ch} = 550$  MPa.

Table 6.2 also gives  $\sigma_{Hlim}^o = 2HB + 70$ ,  $S_H = 1.1$ ,  $\sigma_{Flim}^o = 1.8HB$ ,  $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness  $H$ .

For the driving gear,  $H_1 = HB250 \Rightarrow \sigma_{Hlim1}^o = 570$  MPa,  $\sigma_{Flim1}^o = 450$  MPa

For the driven gear,  $H_2 = HB240 \Rightarrow \sigma_{Hlim2}^o = 550$  MPa,  $\sigma_{Flim2}^o = 432$  MPa

## 3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

### 3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6 \text{ cycles}$$

$$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6 \text{ cycles}$$

### 3.3.2 Working cycle of equivalent tensile stress

Since  $H_1, H_2 \leq HB350$ ,  $m_H = 6$ ,  $m_F = 6$ .

Both gears meshed indefinitely, thus  $c = 1$ .

Applying equation (6.7) and  $T_1, T_2, t_1, t_2$  from the initial parameters:

$$N_{HE1} = 60c \left[ \left( \frac{T_1}{T} \right)^3 n_1 t_1 + \left( \frac{T_2}{T} \right)^3 n_2 t_2 \right] \approx 5.73 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c \left[ \left( \frac{T_1}{T} \right)^3 n_1 t_1 + \left( \frac{T_2}{T} \right)^3 n_2 t_2 \right] \approx 1.15 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c \left[ \left( \frac{T_1}{T} \right)^{m_F} n_1 t_1 + \left( \frac{T_2}{T} \right)^{m_F} n_2 t_2 \right] \approx 3.35 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c \left[ \left( \frac{T_1}{T} \right)^{m_F} n_1 t_1 + \left( \frac{T_2}{T} \right)^{m_F} n_2 t_2 \right] \approx 0.67 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

### 3.3.3 Aging factor

For steel,  $N_{FO} = 4 \times 10^6$  MPa. Applying equation (6.3) and (6.4) yield:

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$$

$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$$

$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$$

$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$$

### 3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction,  $K_{FC} = 1$

$$[\sigma_{H1}] = \sigma_{Hlim1}^o K_{HL1}/S_{H1} \approx 621.61 \text{ MPa}$$

$$[\sigma_{H2}] = \sigma_{Hlim2}^o K_{HL2}/S_{H2} \approx 771.63 \text{ MPa}$$

$$[\sigma_{F1}] = \sigma_{Flim1}^o K_{FC1} K_{FL1}/S_{F1} \approx 264.85 \text{ MPa}$$

$$[\sigma_{F2}] = \sigma_{Flim2}^o K_{FC2} K_{FL2}/S_{F2} \approx 332.48 \text{ MPa}$$

$$[\sigma_H]_{max} = \frac{1}{2} ([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62 \text{ MPa} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$$

Permissible bending stress during overload:

$$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440 \text{ MPa}$$

## 3.4 Transmission Design

### 3.4.1 Determine basic parameters

Examine table 6.5 gives  $K_a = 43$

Assuming symmetrical design, table 6.6 also gives  $\psi_{ba} = 0.5$

$$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$$

From table 6.7 , using interpolation we approximate  $K_{H\beta} \approx 1.108$ ,  $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate  $a_w$  using equation (6.15a) before following SEV229-75 standard:

$$a_w = K_a(u_{hg} + 1)^3 \sqrt{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} = 83.84 \text{ mm}$$

### 3.4.2 Determine gear meshing parameters

**Find  $m$**  Applying equation (6.17) and choose  $m$  from table (6.8) :

$$m = (0.01 \div 0.02)a_w = (0.84 \div 1.68) \text{ mm} \Rightarrow m = 1.5 \text{ mm}$$

**Find  $z_1, z_2, a_w$**  We have  $\beta = \alpha = 20^\circ$ . Combining equation (6.18) and (6.20), we come up with the formula to calculate  $z_1$ . From the result,  $z_1$  is rounded to the nearest odd number (preferably a prime number).

$$z_1 = \frac{2a_w \cos \beta}{m(u+1)} \approx 17.51 \approx 17$$

$$z_2 = u_{hg} z_1 = 85$$

According to SEV229-75 standard, we choose  $a_w = 80 \text{ mm}$

$$\Rightarrow b_w = \psi_{ba} a_w = 40 \text{ mm}$$

**Find  $x_1, x_2$**  Let  $\beta = 20^\circ$ ,  $z_{min} = 15$ . Knowing that  $y = \frac{a_w}{m} - \frac{z_1 + z_2}{2} = 0$ , we conclude  $z_1$  must not be smaller than 17 as mentioned by table 6.9 . Hence, there is no need for correction ( $x_1 = x_2 = 0$ ) and  $z_1 = 17$  satisfy the condition.

**Find  $\alpha_{tw}$**  Since  $y = 0 \Rightarrow \alpha_{tw} = \alpha_t = \tan^{-1} \frac{\tan \alpha}{\cos \beta} \approx 21.17^\circ$  (p.105)

### 3.4.3 Other parameters

$$\alpha_t = \tan^{-1} \frac{\tan \alpha}{\cos \beta} \approx 21.17^\circ$$

$$d_1 = \frac{m z_1}{\cos \beta} \approx 27.14 \text{ mm}$$

$$d_2 = \frac{m z_2}{\cos \beta} \approx 135.68 \text{ mm}$$

$$d_{a1} = d_1 + 2m \approx 30.14 \text{ mm}$$

$$d_{a2} = d_2 + 2m \approx 138.68 \text{ mm}$$

$$d_{f1} = d_1 - 2.5m \approx 23.39 \text{ mm}$$

$$d_{f2} = d_2 - 2.5m \approx 131.93 \text{ mm}$$

$$d_{b1} = d_1 \cos \alpha \approx 25.3 \text{ mm}$$

$$d_{b2} = d_2 \cos \alpha \approx 126.52 \text{ mm}$$

$$d_{w1} = d_1 \approx 27.14 \text{ mm}$$

$$d_{w2} = d_2 \approx 135.68 \text{ mm}$$

$$v = \frac{\pi d_1 n_1}{6 \times 10^4} \approx 4.16 \text{ m/s}$$



### 3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1} K_H \frac{u_{hg} + 1}{b_w u_{hg} d_{w1}^2}} \leq [\sigma_H] \quad (3.1)$$

**Find  $z_M$**   $z_M = 274$ , according to table 6.5

**Find  $z_H$**  Since correction is unused in our calculation:

$$\beta_b = \arctan(\cos \alpha_t \tan \beta) \approx 18.75^\circ \Rightarrow z_H = \sqrt{2 \frac{\cos \beta_b}{\sin(2\alpha_t)}} \approx 1.68$$

**Find  $z_\epsilon$**  Obtaining  $z_\epsilon$  through calculations:

$$\epsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin \alpha_t}{2\pi m \frac{\cos \alpha_t}{\cos \beta}} \approx 1.64$$

$$\epsilon_\beta = b_w \frac{\sin \beta}{m\pi} \approx 2.9 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.78$$

**Find  $K_H$**  We find  $K_H$  using equation  $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table 6.13,  $v \leq 10 \text{ m/s} \Rightarrow \text{AG} = 8$

From table P2.3, using interpolation, we approximate:

$$K_{Hv} \approx 1.0417, K_{Fv} \approx 1.1145$$

From table 6.14, using interpolation, we approximate:

$$K_{H\alpha} \approx 1.0766, K_{F\alpha} \approx 1.253$$

$$\Rightarrow K_H \approx 1.24$$

**Find  $\sigma_H$**  After calculating  $z_M, z_H, z_\epsilon, K_H$ , we get the following result:

$$\sigma_H \approx 699.12 \text{ MPa} \leq [\sigma_H] \approx 696.62 \text{ MPa}$$

Since  $\sigma_H$  and  $[\sigma_H]$  are almost equal to each other, i.e.  $\|\sigma_H - [\sigma_H]\| < 4\%$ , the assumed parameters are appropriate.

### 3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2 \frac{T_{sh1} K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} \leq [\sigma_{F1}] \quad (3.2)$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{F2}] \quad (3.3)$$

**Find  $Y_\epsilon$**  Knowing that  $\epsilon_\alpha \approx 1.64$ , we can calculate  $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.61$

**Find  $Y_\beta$**  Since  $\beta = 20^\circ \Rightarrow Y_\beta = 1 - \frac{\beta}{140} \approx 0.86$

**Find  $Y_F$**  Using formula  $z_v = z \cos^{-3}(\beta)$  and table 6.18:

$$z_{v1} = z_1 \cos^{-3}(\beta) \approx 20.49 \Rightarrow Y_{F1} \approx 4.06$$

$$z_{v2} = z_2 \cos^{-3}(\beta) \approx 102.44 \Rightarrow Y_{F2} \approx 3.6$$

**Find  $K_F$**  Using  $K_{F\beta}$ ,  $K_{F\alpha}$ ,  $K_{Fv}$  calculated from the sections above, we derive:

$$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.75$$

**Find  $\sigma_F$**  Substituting all the values, we find out that:

$$\sigma_{F1} \approx 182.39 \text{ MPa} \leq [\sigma_{F1}] \approx 264.85 \text{ MPa}$$

$$\sigma_{F2} \approx 161.69 \text{ MPa} \leq [\sigma_{F2}] \approx 332.48 \text{ MPa}$$

The calculated results are appropriate.

Through calculations, there is no correction needed, i.e.  $y = 0$ . Thus, the specifications will not include corrections.

In summary, we have the following table:

	pinion	driving gear
$H$ HB	250	240
$[\sigma_H]$ MPa	621.61	771.63
$[\sigma_F]$ MPa	264.85	332.48
$[\sigma_H]$ MPa	696.62	
$\sigma_F$ MPa	182.39	161.69
$\sigma_H$ MPa	699.12	
$\alpha_{tw}^\circ$	21.17	
$\beta^\circ$	20	
$a_w$ mm	80	
$b_w$ mm	40	
$m$ mm	1.5	
$z$	17	85
$d$ mm	27.14	135.68
$d_a$ mm	30.14	138.68
$d_f$ mm	23.39	131.93
$d_b$ mm	25.3	126.52

Table 3.1: Gearbox specifications



# Chapter 4

## Shaft Design

### 4.1 Nomenclature

$[\tau]$	permissible torsion, MPa	$q$	standardized coefficient of shaft diameter
$r$	position of applied force on the shaft, mm	$b_O$	rolling bearing width, mm
$hr$	tooth direction	$l_m$	hub diameter, mm
$cb$	role of gear on the shaft (active or passive)	$k_1$	distance between elements, mm
$cq$	rotational direction of the shaft	$k_2$	distance between bearing surface and inner walls of the gearbox, mm
$\sigma_b$	ultimate strength, MPa	$k_3$	distance between element surface and bearing lid, mm
$\sigma_{ch}$	yield limit, MPa	$h_n$	distance between bearing lid and bolt, mm
$S$	safety factor	$T$	torque on shaft
$F_x$	applied force, N	$\alpha_{tw}$	meshing profile angle, °
$F_t$	tangential force, N	$\beta$	helix angle, °
$F_r$	radial force, N	$_1$	subscript for shaft 1
$F_a$	axial force, N	$_2$	subscript for shaft 2
$a_w$	shaft distance, mm	$_x$	subscript for x-axis
$d$	shaft diameter, mm	$_y$	subscript for y-axis
$d_w$	gear diameter, mm	$_z$	subscript for z-axis

## 4.2 Choose material

For moderate load, we will use quenched steel 40X to design the shafts. From table 6.1, the specifications are as follows:  $S \leq 100$  (mm), HB260,  $\sigma_b = 850$  (MPa),  $\sigma_{ch} = 550$  (MPa).

## 4.3 Transmission Design

### 4.3.1 Load on shafts

#### Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements. On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$$r_{12} = d_{w12}/2 \approx 13.57 \text{ (mm)}, hr_{12} = +1, cb_{12} = +1, cq_1 = +1$$

$$r_{21} = d_{w21}/2 \approx 67.84 \text{ (mm)}, hr_{21} = +1, cb_{21} = -1, cq_2 = -1$$

**Find magnitude of  $F_t, F_r, F_a$**  Using the results from the previous chapter:  $\alpha_{tw} \approx 21.17^\circ, \beta = 20^\circ, d_{w12} \approx 27.14$  (mm)

$$\left\{ \begin{array}{l} F_{t12} = F_{t21} = \frac{2T_{sh1}}{d_{w12}} \approx 2769.03 \text{ (N)} \\ F_{r12} = F_{r21} = \frac{F_{t12} \tan \alpha_{tw}}{\cos \beta} \approx 1141.36 \text{ (N)} \\ F_{a12} = F_{a21} = F_{t12} \tan \beta \approx 1007.84 \text{ (N)} \end{array} \right.$$

**Find direction of  $F_t, F_r, F_a$**  Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \frac{r_{12}}{|r_{12}|} c q_1 c b_{12} F_{t12} \approx 2769.03 \text{ (N)} \\ F_{y12} = -\frac{r_{12}}{|r_{12}|} \frac{\tan \alpha_{tw}}{\cos \beta} F_{t12} \approx -1141.36 \text{ (N)} \\ F_{z12} = c q_1 c b_{12} h r_{12} F_{t12} \tan \beta \approx 1007.84 \text{ (N)} \end{cases}$$

$$\begin{cases} F_{x21} = \frac{r_{21}}{|r_{21}|} c q_2 c b_{21} F_{t21} \approx 2769.03 \text{ (N)} \\ F_{y21} = -\frac{r_{21}}{|r_{21}|} \frac{\tan \alpha_{tw}}{\cos \beta} F_{t21} \approx -1141.36 \text{ (N)} \\ F_{z21} = c q_2 c b_{21} h r_{21} F_{t21} \tan \beta \approx 1007.84 \text{ (N)} \end{cases}$$

### Applied forces from Chain drives

Assuming the angle between x-axis and  $F_r$  is  $150^\circ$  and  $F_r \approx 2539.28 \text{ (N)}$  (chapter 2), we get the direction of  $F_r$  on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 150^\circ \approx -2199.08 \text{ (N)} \\ F_{y22} = F_{r22} \sin 150^\circ \approx 1269.64 \text{ (N)} \end{cases}$$

### 4.3.2 Preliminary calculations

Since shaft 1 receives input torque  $T_{sh1}$  and shaft 2 produces output torque  $T_{sh2}$ ,  $[\tau_1] = 15 \text{ (MPa)}$  and  $[\tau_2] = 30 \text{ (MPa)}$ . Using equation (10.9), we can approximate  $d_1$  and  $d_2$ :

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 23.22 \text{ (mm)} \Rightarrow d_1 = 25 \text{ (mm)}$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 30.99 \text{ (mm)} \Rightarrow d_2 = 35 \text{ (mm)}$$

### 4.3.3 Identify the distance between bearings and applied forces

From table (10.2), we can estimate  $b_O$ . On shaft 1,  $b_{O1} = 15 \text{ (mm)}$ . On shaft 2,  $b_{O2} = 21 \text{ (mm)}$ . Using equation (10.10),  $l_{m12} = 1.5d_1 \approx 34.83 \text{ (mm)}$ ,  $l_{m21} = l_{m22} = 1.5d_2 \approx 46.48 \text{ (mm)}$ .

From table (10.3), we choose  $k_1 = 10$  (mm),  $k_2 = 8$  (mm),  $k_3 = 15$  (mm),  $h_n = 18$  (mm). These parameters apply for both shafts in the system.

Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the formulas below are used:

$$l_{22} = -l_{c22} = -[0.5(l_{m22} + b_{O2}) + k_3 + h_n]$$

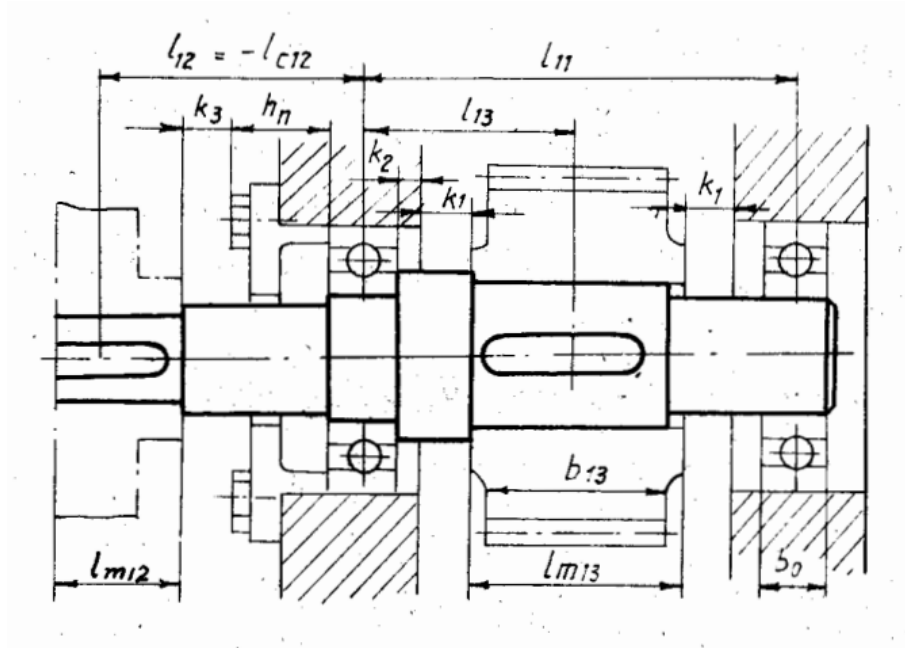


Figure 4.1: Shaft design and its dimensions