



HCM UNIVERSITY OF TECHNOLOGY

MACHINE ELEMENTS

ME2007

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# Project Report

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July 23, 2020

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## Design Problem

$D_{bc}$  pulley diameter, mm

$F_t$  tangential force, N

$L$  service life, years

$T$  working torque, N · mm

$t$  working time, s

$v_{bc}$  conveyor belt speed, m/s

$\delta_u$  error of speed ratio, %

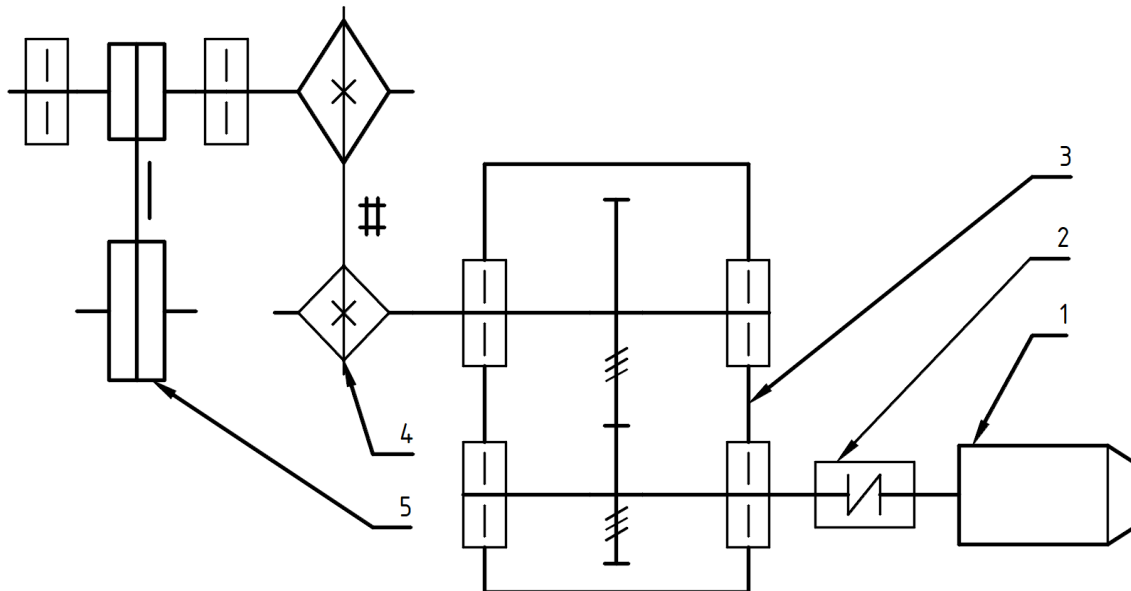


Figure 1: Mechanical transmission system of a belt conveyor

Given the mechanical transmission system in figure 1, determine the specifications for each machine element.

1. Electric motor
2. Elastic coupling
3. Gearbox
4. Chain drive
5. Belt conveyor

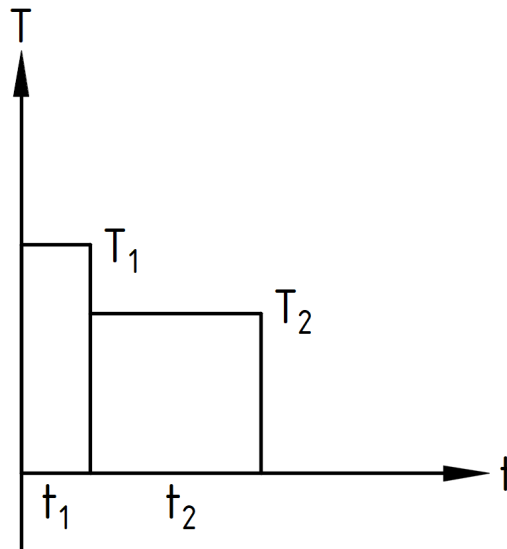


Figure 2: Input load diagram

**Design parameters** The chosen parameters are given in column 8:

- $F_t = 4500 \text{ (N)}$
- $v_{bc} = 3.05 \text{ (m/s)}$
- $D_{bc} = 500 \text{ (mm)}$
- $L = 4 \text{ (years)}$
- $T_1 = T \text{ (N} \cdot \text{mm)}, t_1 = 12 \text{ (s)}$
- $T_2 = 0.7T \text{ (N} \cdot \text{mm)}, t_2 = 60 \text{ (s)}$
- $\delta_u \leq \pm 5\%$

The machine works in one direction for 300 days, 2 shifts per day, 8 hours each and the load is low. Also, for the rest of this report, we will consider shaft 1 is the one connected to the electric motor, while shaft 2 is the connection between the chain drive and gearbox.



# Chapter 1

## Motor Design

### 1.1 Nomenclature

$n_{bc}$	rotational speed of belt conveyor, rpm	$u_{hg}$	transmission ratio of helical gear
$n_{sh}$	rotational speed of shaft, rpm	$u_{sys}$	transmission ratio of the system
$P_m$	maximum operating power of belt conveyor, kW	$T_{motor}$	motor torque, N · mm
$P_{motor}$	calculated motor power to drive the system, kW	$T_{sh}$	shaft torque, N · mm
$P_{sh}$	operating power of shaft, kW	$\delta_u$	relative error of $u_{sys}$
$P_w$	operating power of the belt conveyor given a workload, kW	$\eta_b$	bearing efficiency
		$\eta_c$	coupling efficiency
		$\eta_{ch}$	chain drive efficiency
		$\eta_{hg}$	helical gear efficiency
$u_{ch}$	transmission ratio of chain drive	$\eta_{sys}$	efficiency of the system
		1	shaft 1
		2	shaft 2

## 1.2 Calculate $\eta_{sys}$

From table (2.3):

$$\eta_c = 1$$

$$\eta_b = 0.99$$

$$\eta_{hg} = 0.96$$

$$\eta_{ch} = 0.95$$

$$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$$

## 1.3 Calculate $P_{motor}$

$$P_m = \frac{F_t v_{bc}}{1000} \approx 13.73 \text{ (kW)}$$

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$

$$P_{motor} = \frac{P_w}{\eta_{sys}} \approx 11.76 \text{ (kW)} < P_m$$

## 1.4 Calculate $n_{motor}$

$$n_{bc} = \frac{6 \times 10^4 v}{\pi D_{bc}} \approx 116.5 \text{ (rpm)}$$

$$u_{ch} = 5 \text{ (table (2.4))}$$

$$u_{hg} = 5 \text{ (table (2.4))}$$

$$u_{sys} = u_{ch} u_{hg} = 25$$

$$n_{motor} = u_{sys} n_{bc} \approx 2912.54 \text{ (rpm)}$$

## 1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated  $P_{motor}$  and  $P_m$ . Since  $P_{motor} < P_m$  for our case, the minimum operating power of choice is  $P_m$ . In similar fashion, its rotational speed must also be no smaller than estimated  $n_{motor}$ .

Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$$\Rightarrow P_{motor} = 18.5 \text{ kW}, n_{motor} = 2930 \text{ (rpm)}$$

Recalculating  $u_{sys}$  with the new  $P_{motor}$  and  $n_{motor}$ , we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming  $u_{hg} = const$ :

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

**Relative error of transmission ratio** The ratio is calculated as follows:

$$\delta_u = \frac{|25.15 - 25|}{25} \approx 0.6\% \leq 5\%$$

## 1.6 Calculate power, rotational speed and torque

Let us denote  $P_{sh1}$ ,  $n_{sh1}$  and  $T_{sh1}$  be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly,  $P_{sh2}$ ,  $n_{sh2}$  and  $T_{sh2}$  will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

### 1.6.1 Power

$$P_{ch} = P_m \approx 13.73 \text{ (kW)}$$

$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59 \text{ (kW)}$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35 \text{ (kW)}$$

### 1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$

$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

### 1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63 \text{ (N} \cdot \text{mm)}$$

$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56 \text{ (N} \cdot \text{mm)}$$

$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
$P$ (kW)	18.5	15.35	14.59
$u$	5	5.03	
$n$ (rpm)	2930	2930	586
$T$ (N · mm)	60298.63	50047.56	237825.99

Table 1.1: System overall specifications

# Chapter 2

## Chain Drive Design

### 2.1 Nomenclature

$[i]$	permissible impact times per second	$d_l$	roller diameter, mm
		$d_o$	pin diameter, mm
$[s]$	permissible safety factor	$E$	modulus of elasticity, MPa
$[P]$	permissible power, kW	$F_0$	sagging force, N
$[\sigma_H]$	permissible contact stress, MPa	$F_1$	tight side tension force, N
$A$	cross sectional area of chain hinge, mm <sup>2</sup>	$F_2$	slack side tension force, N
		$F_r$	force on the shaft, N
$a$	center distance, mm	$F_t$	effective peripheral force, N
$a_{max}$	maximum center distance, mm	$F_v$	centrifugal force, N
$a_{min}$	minimum center distance, mm	$F_{vd}$	contact force, N
$B$	width between inner link plate, mm	$i$	impact times per second
		$K_d$	weight distribution factor on each strand
$d$	chordal diameter, mm	$k$	overall factor
$d_a$	addendum diameter, mm	$k_0$	arrangement of drive factor
$d_f$	dedendum diameter, mm		

$k_a$	center distance and chain's length factor	$p$	pitch, mm
$k_{bt}$	lubrication factor	$p_{max}$	permissible sprocket pitch, mm
$k_c$	rating factor	$Q$	permissible load, N
$k_d$	dynamic load factor	$q$	mass per meter of chain, kg/m
$k_{dc}$	chain tension factor	$s$	safety factor
$k_f$	loosing factor	$v$	instantaneous velocity along the chain, m/s
$k_n$	coefficient of rotational speed	$x$	chain length in pitches, the number of links
$k_r$	number of tooth factor	$x_c$	an even number of links
$k_x$	chain weight factor	$z$	number of teeth of a sprocket
$k_z$	coefficient of number of teeth	$z_{max}$	maximum number of teeth of the driven sprocket
$n_{01}$	experimental rotational speed, rpm	$\sigma_H$	contact stress, MPa
$n$	sprocket rotational speed, rpm	$[\sigma_H]$	permissible contact stress, MPa
$n_{ch}$	rotational speed of a sprocket, rpm	1	subscript for driving sprocket
$P_t$	calculated power, kW	2	subscript for driven sprocket

## 2.2 Find $p$

Since the driving sprocket is connected to shaft 1,  $n_1 = n_{sh2} = 586$  (rpm).

**Find  $z$**  Since  $z_1$  and  $z_2$  is preferably an odd number according to p.80:

$$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$$

$$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$$

Because  $z_1 \geq 15$ , we use table (5.8) and interpolation to approximate  $p_{max}$ .

Therefore,  $p_{max} \approx 33.58$  (mm).

**Find  $k$**  Since  $n_{ch} = 586 \approx 600$  (rpm), choose  $n_{01} = 600$  (rpm), which is obtained from table (5.5). Then, we calculate  $k_z$  and  $k_n$ .

$$k_z = \frac{25}{z_1} \approx 1.32, k_n = \frac{n_{01}}{n_{ch}} \approx 1.02$$

Specifying the chain drive's working condition and ultizing table (5.6) , we find out that  $k_0 = k_a = k_{dc} = k_{bt} = 1$ ,  $k_d = 1.25$ ,  $k_c = 1.3$ .

$$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$$

**Find  $p$**  From table (5.5):

$$P_t = P_{ch} k k_z k_n \approx 30.05 \text{ (kW)} \leq 42 \text{ (kW)} \Rightarrow [P] = 42 \text{ (kW)}$$

Using the table, we also get the other parameters:

$$p = 31.75 \text{ (mm)}, d_O = 9.55 \text{ (mm)}, B = 27.46 \text{ (mm)}, d_l = 19.05 \text{ (mm)}$$

$$d_1 = \frac{p}{\sin \frac{180^\circ}{z_1}} \approx 192.9 \text{ (mm)}, d_2 = \frac{p}{\sin \frac{180^\circ}{z_2}} \approx 980.49 \text{ (mm)}$$

Having  $p = 31.75 \text{ (mm)} \leq p_{\max} \approx 33.58 \text{ (mm)}$ , we can safely choose the number of chains as 1, which is in agreement with the given figure. Hence, from table (5.2), we obtain the parameters in the section for 1 strand chain drive:

$$Q = 56.7 \times 10^3 \text{ (N)}, q = 2.6 \text{ (kg/m)}$$

By comparison to the conditions in the sub-table, the choice of  $B$  is satisfactory.

## 2.3 Find $a$ , $x_c$ , and $i$

**Find  $x_c$**   $a_{\min} = 30p = 952.5 \text{ (mm)}$ ,  $a_{\max} = 50p = 1587.5 \text{ (mm)}$ . Limiting the range of choice for  $a$  in  $[a_{\min}, a_{\max}]$ , we can approximate  $a = 1000 \text{ (mm)}$ .

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$$

**Find  $a$**  From equation (5.13) , we calculate  $a$  again with  $x_c$ :

$$a = \frac{p}{4} \left( x_c - \frac{z_2 + z_1}{2} + \sqrt{\left( x_c - \frac{z_2 + z_1}{2} \right)^2 - 2 \frac{(z_2 - z_1)^2}{\pi^2}} \right) - 0.003a \approx 998.98 \text{ (mm)}$$

**Find  $i$**  From table (5.9):

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$$

## 2.4 Strength of chain drive

### 2.4.1 Safety factor analysis

In order to operate safely, the chain drive's safety factor must satisfy the following condition:

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \geq [s]$$

For moderate workload, choose  $k_d = 1.2$ . Let the chain drive be angled  $30^\circ$  with respect to ground, we obtain  $k_f = 4$ .

$$v_1 = \frac{n_{ch} P z_1}{6 \times 10^4} \approx 5.89 \text{ (m/s)}$$

**Find  $F_t, F_v, F_0$**  We also need to calculate  $F_t, F_v$  and  $F_0$ :

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2329.53 \text{ (N)}$$

$$F_v = q v_1^2 \approx 90.25 \text{ (N)}$$

$$F_0 = 9.81 \times 10^{-3} k_f q a \approx 101.92 \text{ (N)}$$

**Validate  $s$**  This value must be larger than the permissible safety factor to operate properly. From equation (5.15):

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2, \text{ where } [s] \text{ is chosen from table (5.10).}$$

### 2.4.2 Contact stress analysis

The following condition must be met:

$$\sigma_H = 0.47 \sqrt{\frac{k_r (F_t k_d + F_{vd}) E}{A K_d}} \leq [\sigma_H]$$



Since the chain drive only has one strand,  $K_d = 1$ .

**Find  $[\sigma_H]$**  From table (5.11), quenched 45 steel is the material of use for the chain drive, which has HB210,  $[\sigma_H] = 600$  (MPa) and  $E = 2.1 \times 10^5$  (MPa).

**Find  $F_{vd}$**  For 1-strand chain,  $F_{vd} = 13 \times 10^{-7} n_1 p^3 \approx 24.38$  (N)

**Find  $k_r$**  Based on given data on p.87, we estimate  $k_r$  from  $z$ , which is  $k_r \approx 0.47$

**Find  $A$**  According to table (5.12),  $A = 262$  (mm)

Combining with  $k_d = 1.2$ ,  $F_t \approx 2329.53$  (N), we get the result:

$$\sigma \approx 494.32 \text{ (MPa)} \leq [\sigma_H] = 600 \text{ (MPa)}$$

which is satisfactory.

## 2.5 Force on shaft

From p.87:

$$F_2 = F_0 + F_v \approx 192.17 \text{ (N)}$$

$$F_1 = F_t + F_2 \approx 2521.7 \text{ (N)}$$

Choose  $k_x = 1.15$  and follow equation (5.20) :

$$F_r = k_x F_t \approx 2678.96 \text{ (N)}$$

## 2.6 Other parameters

$$d_{a1} = p \left( 0.5 + \cot \frac{180}{z_1} \right) \approx 206.14 \text{ (mm)}$$

$$d_{a2} = p \left( 0.5 + \cot \frac{180}{z_2} \right) \approx 995.85 \text{ (mm)}$$

Knowing that  $d_l = 19.05$  (mm) from previous sections:

$$d_{f1} = d_1 - 2(0.502d_l + 0.05) \approx 173.67 \text{ (mm)}$$

$$d_{f2} = d_2 - 2(0.502d_l + 0.05) \approx 961.26 \text{ (mm)}$$

In summary, we have the following table:

	driving	driven
$[P]$ (kW)	42	
$a$ (mm)	998.98	
$B$ (mm)	27.46	
$d$ (mm)	192.9	980.49
$d_a$ (mm)	206.14	995.85
$d_f$ (mm)	173.67	961.26
$d_l$ (mm)	19.05	
$d_O$ (mm)	9.55	
$i$	6	
$p$ (mm)	31.75	
$Q$ (N)	56700	
$u_{ch}$	5.03	
$v$ (m/s)	5.89	
$z$	19	97

Table 2.1: Chain drive specifications

# Chapter 3

## Gearbox Design (Helix gears)

### 3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa	$F_r$	radial force, N
$[\sigma_H]_{max}$	permissible contact stress due to overload, MPa	$F_t$	tangential force, N
		$H$	surface roughness, HB
$[\sigma_F]$	permissible bending stress, MPa	$K_d$	coefficient of gear material
$[\sigma_F]_{max}$	permissible bending stress due to overload, MPa	$K_F$	load factor from bending stress
		$K_{FC}$	load placement factor
AG	accuracy grade of gear	$K_{FL}$	aging factor due to bending stress
$a$	center distance, mm		
$b$	face width, mm	$K_{Fv}$	factor of dynamic load from bending stress at meshing area
$c$	gear meshing rate		
$d$	pitch circle, mm	$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
$d_a$	addendum diameter, mm		
$d_b$	base diameter, mm	$K_{F\beta}$	factor of load distribution from bending stress on top land
$d_f$	deddendum diameter, mm		
$F_a$	axial force, N	$K_H$	load factor of contact stress

$K_{HL}$	aging factor due to contact stress	$T$	input torque, N · mm
$K_{Hv}$	factor of dynamic load from contact stress at meshing area	$v$	rotational velocity, m/s
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth	$x$	gear correction factor
$K_{H\beta}$	factor of load distribution from contact stress on top land	$Y_F$	tooth shape factor
		$Y_R$	surface roughness factor of the gear's face
$k_x$	a coefficient	$Y_s$	sensitivity to stress concentration factor
$k_y$	a coefficient	$Y_\beta$	helix angle factor
$m$	transverse module, mm	$Y_\varepsilon$	contact ratio factor
$m_F$	root of fatigue curve in bending stress test	$y$	center displacement factor
$m_H$	root of fatigue curve in contact stress test	$Z_R$	surface roughness factor of the working's area
$m_n$	normal module, mm	$Z_v$	speed factor
$N_{FE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$	$z_H$	contact surface's shape factor
		$z_M$	material's mechanical properties factor
$N_{FO}$	working cycle of bearing stress corresponding to $[\sigma_F]$	$z_{min}$	minimum number of teeth corresponding to $\beta$
$N_{HE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$	$z_v$	virtual number of teeth
$N_{HO}$	working cycle of bearing stress corresponding to $[\sigma_H]$	$z_\varepsilon$	meshing condition factor
$S$	specific length, mm	$\alpha$	normal pressure angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20^\circ$
$S_F$	safety factor of bending stress	$\alpha_t$	traverse pressure angle, $^\circ$
$S_H$	safety factor of contact stress	$\varepsilon_\alpha$	traverse contact ratio
		$\varepsilon_\beta$	face contact ratio
		$\beta$	helix angle, $^\circ$

$\beta_b$	base circle helix angle, °	$\sigma_{Hlim}^o$	permissible contact stress
$\psi_{ba}$	width to shaft distance ratio		corresponding to working cycle,
$\psi_{bd}$	face width factor		MPa
$\sigma_b$	ultimate strength, MPa	1	subscript for pinion
$\sigma_{ch}$	yield limit, MPa	2	subscript for driven gear
$\sigma_{Flim}^o$	permissible bending stress	$w$	subscript for variable value after
	corresponding to working cycle,		correction
	MPa		

## 3.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with  $S \leq 100$  (mm), HB250,  $\sigma_b = 850$  (MPa),  $\sigma_{ch} = 550$  (MPa).

Table (6.2) also gives  $\sigma_{Hlim}^o = 2HB + 70$ ,  $S_H = 1.1$ ,  $\sigma_{Flim}^o = 1.8HB$ ,  $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness  $H$ .

The reasoning is given on p.91, where  $H_2 = H_1 - 10 \div 15$

For the pinion,  $H_1 = HB250 \Rightarrow \sigma_{Hlim1}^o = 570$  (MPa),  $\sigma_{Flim1}^o = 450$  (MPa)

For the driven gear,  $H_2 = HB240 \Rightarrow \sigma_{Hlim2}^o = 550$  (MPa),  $\sigma_{Flim2}^o = 432$  (MPa)

## 3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

### 3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6 \text{ (cycles)}$$

$$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6 \text{ (cycles)}$$

### 3.3.2 Working cycle of equivalent tensile stress

Since  $H_1, H_2 \leq \text{HB350}$ ,  $m_H = 6$ ,  $m_F = 6$ .

Both gears meshed indefinitely, thus  $c = 1$ .

From working condition, we calculate:

$$L_h = 8 \left( \frac{\text{hours}}{\text{shift}} \right) \times 2 \left( \frac{\text{shifts}}{\text{day}} \right) \times 300 \left( \frac{\text{days}}{\text{year}} \right) \times 4 (\text{years}) = 19200 (\text{hours})$$

Applying equation (6.7) and  $T_1, T_2, t_1, t_2$  in the initial parameters:

$$N_{HE1} = 60n_{sh1}cL_h \left[ \left( \frac{T_1}{T} \right)^3 \frac{t_1}{t_1 + t_2} + \left( \frac{T_2}{T} \right)^3 \frac{t_2}{t_1 + t_2} \right] \approx 1.53 \times 10^9 (\text{cycles})$$

$$N_{HE2} = 60n_{sh2}cL_h \left[ \left( \frac{T_1}{T} \right)^3 \frac{t_1}{t_1 + t_2} + \left( \frac{T_2}{T} \right)^3 \frac{t_2}{t_1 + t_2} \right] \approx 0.31 \times 10^9 (\text{cycles})$$

$$N_{FE1} = 60n_{sh1}cL_h \left[ \left( \frac{T_1}{T} \right)^{m_F} \frac{t_1}{t_1 + t_2} + \left( \frac{T_2}{T} \right)^{m_F} \frac{t_2}{t_1 + t_2} \right] \approx 0.89 \times 10^9 (\text{cycles})$$

$$N_{FE2} = 60n_{sh2}cL_h \left[ \left( \frac{T_1}{T} \right)^{m_F} \frac{t_1}{t_1 + t_2} + \left( \frac{T_2}{T} \right)^{m_F} \frac{t_2}{t_1 + t_2} \right] \approx 0.18 \times 10^9 (\text{cycles})$$

### 3.3.3 Aging factor

For steel,  $N_{FO1} = N_{FO2} = 4 \times 10^6$  (MPa). Applying equations (6.3) and (6.4) yield (if  $K_{HL}, K_{FL} < 1$ ,  $K_{HL} = 1$  and  $K_{FL} = 1$  according to the properties given on p.94):

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 0.47 < 1 \Rightarrow K_{HL1} = 1$$

$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 0.61 < 1 \Rightarrow K_{HL2} = 1$$

$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 0.41 < 1 \Rightarrow K_{FL1} = 1$$

$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 0.53 < 1 \Rightarrow K_{FL2} = 1$$

### 3.3.4 Calculate $[\sigma_H]$ , $[\sigma_{F1}]$ , $[\sigma_{F2}]$

Since the motor works in one direction,  $K_{FC} = 1$ . In ideal conditions, we assume

$Z_R Z_V K_{xH} = 1$  and  $Y_R Y_s K_{xF} = 1$  according to p.92:

$$[\sigma_{H1}] = \sigma_{Hlim1}^o K_{HL1} / S_{H1} \approx 518.18 (\text{MPa})$$

$$[\sigma_{H2}] = \sigma_{Hlim2}^o K_{HL2} / S_{H2} \approx 500 (\text{MPa})$$

$$[\sigma_{F1}] = \sigma_{Flim1}^o K_{FC1} K_{FL1} / S_{F1} \approx 257.14 \text{ (MPa)}$$

$$[\sigma_{F2}] = \sigma_{Flim2}^o K_{FC2} K_{FL2} / S_{F2} \approx 246.86 \text{ (MPa)}$$

The mean permissible contact stress must be lower than 1.25 times of either

$[\sigma_{H1}]$  or  $[\sigma_{H2}]$ , whichever is smaller:

$$[\sigma_H] = \frac{1}{2} ([\sigma_{H1}] + [\sigma_{H2}]) \approx 509.09 \text{ (MPa)} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H2}]$$

The permissible contact stress and bending stress due to overload are calculated as follows:

$$[\sigma_H]_{max} = 2.8\sigma_{ch} = 1540 \text{ (MPa)}$$

$$[\sigma_F]_{max} = 0.8\sigma_{ch} = 440 \text{ (MPa)}$$

## 3.4 Transmission Design

### 3.4.1 Determine basic parameters

Examine table (6.5) gives  $K_a = 43$

Assuming symmetrical design, table (6.6) also gives  $\psi_{ba} = 0.5$

$$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$$

From table (6.7) , using interpolation, we approximate  $K_{H\beta} \approx 1.108$ ,  $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate  $a$  using equation (6.15a) gives:

$$a = K_a(u_{hg} + 1) \sqrt[3]{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} \approx 113.7 \text{ (mm)}$$

According to SEV229-75 standard, we choose  $a_w = 125 \text{ mm}$

### 3.4.2 Determine gear meshing parameters

**Find  $m$**  Applying equation (6.17) and choose  $m$  from table (6.8) :

$$m = (0.01 \div 0.02)a_w = 1.25 \div 2.5 \text{ (mm)} \Rightarrow m = 1.5 \text{ (mm)}$$

**Find**  $z_1, z_2, b_w$  Let  $\beta = 14^\circ$ . Combining equation (6.18) and (6.20), we come up with the formula to calculate  $z_1$ . From the result,  $z_1$  is rounded up to the nearest odd number (preferably a prime number).

$$z_1 = \frac{2a_w \cos \beta}{m(u_{hg} + 1)} \approx 26.95 \Rightarrow z_1 = 27$$

$$z_2 = u_{hg} z_1 = 135$$

$$\Rightarrow b = \psi_{ba} a_w = 62.5 \text{ (mm)}$$

**Correct**  $\beta$  There are 2 approaches for correction involving the change of either  $\alpha$  or  $\beta$ . Because altering  $\alpha$  leads to many other corrections ( $d_1, d_2$  and  $a_w$ ),  $\beta$  will be used instead.

Since  $z_1$  is rounded, we must find  $\beta$  to obtain the correct angle, ensuring that  $\beta \in (8^\circ, 20^\circ)$ . Using equation (6.32):

$$\beta_w = \arccos \frac{m(z_1 + z_2)}{2a_w} \approx 13.59^\circ$$

**Find**  $x_1, x_2$  To find  $x_1$  and  $x_2$ , we will follow the calculation scheme provided in p.103. Since  $\beta_w \approx 13.59^\circ \in (10, 15]$ ,  $z_{min} = 11$ , which leads to  $z_1$  satisfying condition  $z_1 \geq z_{min} + 2 > 10$ , according to table (6.9). Combined with  $u_{hg} = 5 \geq 3.5$ , we obtain  $x_1 = 0.3, x_2 = -0.3$ , disregarding the calculation of  $y$ .

### 3.4.3 Basic parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos \beta} \approx 41.67 \text{ (mm)}$$

$$d_2 = d_{w2} = \frac{mz_2}{\cos \beta} \approx 208.33 \text{ (mm)}$$

$$d_{a1} = d_1 + 2(1 + x_1)m \approx 45.57 \text{ (mm)}$$

$$d_{a2} = d_2 + 2(1 + x_2)m \approx 210.43 \text{ (mm)}$$

$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 38.82 \text{ (mm)}$$

$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 203.68 \text{ (mm)}$$

$$d_{b1} = d_1 \cos \alpha \approx 39.15 \text{ (mm)}$$

$$d_{b2} = d_2 \cos \alpha \approx 195.77 \text{ (mm)}$$

$$\alpha_t = \alpha_{tw} = \arctan \frac{\tan \alpha}{\cos \beta_w} \approx 20.53^\circ$$

$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 6.39 \text{ (m/s)}$$



### 3.4.4 Find $[\sigma_{Hw}]$ , $[\sigma_{Fw1}]$ and $[\sigma_{Fw2}]$

In this section, we will try to approximate these parameters based on the factors  $Z_R$ ,  $Z_V$ ,  $K_{xH}$  and  $Y_R$ ,  $Y_S$ ,  $K_{xF}$  to substitute to equation (6.1) and (6.2):

$$[\sigma_{Hw}] = [\sigma_H] Z_R Z_V K_{xH}$$

$$[\sigma_{Fw}] = [\sigma_F] Y_R Y_S K_{xF}$$

Assuming smooth surface condition,  $Z_R = 1$ .

$$Z_V = 0.85v^{0.1} \approx 1.02 \text{ with } H \leq 350.$$

In case of  $v > 5$  (m/s),  $K_{xH} = 1$ .

The pair of gears are properly polished, which makes  $Y_R = 1.1$

$$Y_S = 1.08 - 0.0695 \ln(m) \approx 1.05$$

Since  $d_{a1}, d_{a2} \leq 400$  (mm),  $K_{xF} = 1$ , which leads to:

$$[\sigma_{Hw}] = 520.93 \text{ (MPa)}$$

$$[\sigma_{Fw1}] = 297.51 \text{ (MPa)}$$

$$[\sigma_{Fw2}] = 285.61 \text{ (MPa)}$$

### 3.4.5 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\varepsilon \sqrt{2T_{sh1} K_H \frac{u_{hg} + 1}{b u_{hg} d_{w1}^2}} \leq [\sigma_{Hw}]$$

**Find  $z_M$**   $z_M = 274$ , according to table (6.5)

**Find  $z_H$**   $\beta_b = \arctan(\cos \alpha_t \tan \beta_w) \approx 12.76^\circ \Rightarrow z_H = \sqrt{2 \frac{\cos \beta_b}{\sin(2\alpha_{tw})}} \approx 1.72$

**Find  $z_\varepsilon$**  Obtaining  $z_\varepsilon$  through calculations:

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin \alpha_{tw}}{2\pi m \frac{\cos \alpha_t}{\cos \beta_w}} \approx 1.41$$

$$\varepsilon_\beta = b \frac{\sin \beta_w}{m\pi} \approx 3.12 > 1 \Rightarrow z_\varepsilon = \varepsilon_\alpha^{-0.5} \approx 0.86$$

**Find  $K_H$**  We find  $K_H$  using equation  $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table (6.13),  $v \leq 10$  (m/s)  $\Rightarrow$  AG = 8

From table (P2.3), using interpolation, we approximate:

$$K_{Hv} \approx 1.07, K_{Fv} \approx 1.18$$

From table (6.14), using interpolation, we approximate:

$$K_{H\alpha} \approx 1.1, K_{F\alpha} \approx 1.29$$

$$\Rightarrow K_H \approx 1.3$$

**Find  $\sigma_H$**  After calculating  $z_M, z_H, z_\varepsilon, K_H$ , we get the following result:

$$\sigma_H \approx 477.51 \text{ (MPa)} \leq [\sigma_{Hw}] \approx 509.09 \text{ (MPa)}$$

### 3.4.6 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2 \frac{T_{sh1} K_F Y_\varepsilon Y_\beta Y_{F1}}{b d_{w1} m_n} \leq [\sigma_{Fw1}]$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{Fw2}]$$

**Find  $Y_\varepsilon$**  Knowing that  $\varepsilon_\alpha \approx 1.41$ , we can calculate  $Y_\varepsilon = \varepsilon_\alpha^{-1} \approx 0.71$

**Find  $Y_\beta$**   $Y_\beta = 1 - \frac{\beta_w}{140} \approx 0.9$

**Find  $Y_F$**  Using formula  $z_v = z \cos^{-3}(\beta_w)$  and table (6.18):

$$z_{v1} = z_1 \cos^{-3}(\beta_w) \approx 29.4 \Rightarrow Y_{F1} \approx 3.54$$

$$z_{v2} = z_2 \cos^{-3}(\beta_w) \approx 147.01 \Rightarrow Y_{F2} \approx 3.63$$

**Find  $K_F$**  Using  $K_{F\beta}$ ,  $K_{F\alpha}$ ,  $K_{Fv}$  calculated from the sections above, we derive:

$$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.91$$

**Find  $\sigma_F$**  Since  $m_n = m \cos \beta_w \approx 1.46$ , substituting all the values, we find out that:

$$\sigma_{F1} \approx 114.11 \text{ (MPa)} \leq [\sigma_{Fw1}] \approx 297.51 \text{ (MPa)}$$

$$\sigma_{F2} \approx 117.01 \text{ (MPa)} \leq [\sigma_{Fw2}] \approx 285.61 \text{ (MPa)}$$

### 3.4.7 Force on shafts

$$F_t = \frac{2T_{sh1}}{d_{w1}} \approx 2402.28 \text{ (N)}$$

$$F_r = F_t \tan \alpha_{tw} \approx 899.55 \text{ (N)}$$

$$F_a = F_t \tan \beta_w \approx 580.75 \text{ (N)}$$

In summary, we have the following table:

	pinion	driving gear
$H$ (HB)	250	240
$[\sigma_F]$ (MPa)	257.14	246.86
$[\sigma_H]$ (MPa)	509.09	
$[\sigma_H]_{max}$ (MPa)	1540	
$[\sigma_F]_{max}$ (MPa)	440	
$a_w$ (mm)	100	
$b$ (mm)	50	
$m$ (mm)	1.5	
$d_w$ (mm)	33.33	166.67
$d_a$ (mm)	37.23	168.77
$d_f$ (mm)	30.48	162.02
$d_b$ (mm)	31.32	156.62
$u_{hg}$	5	
$v$ (m/s)	5	
$x$ (mm)	0.3	-0.3
$z$	21	105
$\alpha_{tw}$ ( $^\circ$ )	20.65	
$\beta_w$ ( $^\circ$ )	19.09	

Table 3.1: Gearbox specifications

# Chapter 4

## Shaft Design

### 4.1 Nomenclature

$[s]$	permissible safety factor	$h_n$	distance between bearing lid and bolt, mm
$[\sigma]$	permissible static strength, MPa	hr	tooth direction
$[\tau]$	permissible torsion, MPa	$K_x$	surface tension concentration factor
$a_w$	shaft distance, mm	$K_y$	diminish factor
$b_O$	rolling bearing width, mm	$K_\sigma$	combined influence factor in tension
cb	role of gear on the shaft (active or passive)	$K_\tau$	combined influence factor in shear
cq	rotational direction of the shaft	$\tilde{k}_1$	distance between elements, mm
$d$	base shaft diameter, mm	$\tilde{k}_2$	distance between bearing surface and inner walls of the gearbox, mm
$d_w$	gear diameter, mm		
$F_a$	axial force, N		
$F_r$	radial force, N		
$F_t$	tangential force, N		
$F$	applied force, N		

$\tilde{k}_3$	distance between element surface and bearing lid, mm	$W$	section modulus, mm <sup>3</sup>
$k_\sigma$	fatigue stress concentration factor in tension	$W_O$	polar section modulus, mm <sup>3</sup>
$k_\tau$	fatigue stress concentration factor in shear	$\alpha_{tw}$	traverse meshing angle, °
		$\beta$	helix angle, °
		$\psi_\sigma$	mean stress influence factor
		$\psi_\tau$	mean shear influence factor
$l$	length (general), mm	$\sigma_{-1}$	endurance limit at stress ratio of -1, MPa
$l_m$	hub length (general), mm		
$M$	moment at the cross section, N · mm	$\sigma_a$	tensile stress amplitude, MPa
		$\sigma_b$	ultimate strength, MPa
$M_e$	equivalent moment, N · mm	$\sigma_{ch}$	yield limit, MPa
$l_m$	hub diameter, mm	$\sigma_m$	mean tensile stress, MPa
$q$	standardized coefficient of shaft diameter	$\sigma_{td}$	static strength, MPa
		$\tau_{-1}$	endurance limit at shear ratio of -1, MPa
$R$	reaction force, N		
$r$	shoulder fillet radius, mm	$\tau_a$	shear stress amplitude, MPa
$\bar{r}$	position of applied force on the shaft, mm	$\tau_m$	mean shear stress, MPa
		1	subscript for shaft 1
$S$	length defined by table (6.1), mm	2	subscript for shaft 2
		$max$	subscript for maximum value
$s$	calculated safety factor	$sh1$	subscript for shaft 1
$s_\sigma$	safety factor in tensile stress	$sh2$	subscript for shaft 2
$s_\tau$	safety factor in shear stress	$x$	subscript for x-axis
$T$	torque at the cross section, N · mm	$y$	subscript for y-axis
		$z$	subscript for z-axis

## 4.2 Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows:  $S \leq 100$  (mm), HB260,  $\sigma_b = 850$  (MPa),  $\sigma_{ch} = 650$  (MPa).

## 4.3 Transmission Design

### 4.3.1 Load on shafts

#### Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.

On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$$\bar{r}_{12} = -d_{w12}/2 \approx -20.83 \text{ (mm)}, \text{hr}_{12} = +1, \text{cb}_{12} = +1, \text{cq}_1 = +1$$

$$\bar{r}_{21} = +d_{w21}/2 \approx +104.17 \text{ (mm)}, \text{hr}_{21} = -1, \text{cb}_{21} = -1, \text{cq}_2 = -1$$

**Find magnitude of  $F_t$ ,  $F_r$ ,  $F_a$**  Using the results from the previous chapter: ,  $\beta_w = 13.59^\circ$ ,  $d_{w12} \approx 41.67$  (mm)

$$\begin{cases} F_{t12} = F_{t21} = \frac{2T_{sh1}}{d_{w12}} \approx 2402.28 \text{ (N)} \\ F_{r12} = F_{r21} = \frac{F_{t12} \tan \alpha}{\cos \beta_w} \approx 925.46 \text{ (N)} \\ F_{a12} = F_{a21} = F_{t12} \tan \beta_w \approx 580.75 \text{ (N)} \end{cases}$$

**Find direction of  $F_t$ ,  $F_r$ ,  $F_a$**  Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \frac{\bar{r}_{12}}{|\bar{r}_{12}|} c_{q1} c_{b12} F_{t12} \approx -2402.28 \text{ (N)} \\ F_{y12} = -\frac{\bar{r}_{12}}{|\bar{r}_{12}|} \frac{\tan \alpha}{\cos \beta_w} F_{t12} \approx 925.46 \text{ (N)} \\ F_{z12} = c_{q1} c_{b12} h_{r12} F_{t12} \tan \beta_w \approx 580.75 \text{ (N)} \end{cases}$$

$$\begin{cases} F_{x21} = \frac{\bar{r}_{21}}{|\bar{r}_{21}|} c_{q2} c_{b21} F_{t21} \approx 2402.28 \text{ (N)} \\ F_{y21} = -\frac{\bar{r}_{21}}{|\bar{r}_{21}|} \frac{\tan \alpha_{tw}}{\cos \beta_w} F_{t21} \approx -925.46 \text{ (N)} \\ F_{z21} = c_{q2} c_{b21} h_{r21} F_{t21} \tan \beta_w \approx -580.75 \text{ (N)} \end{cases}$$

### Applied forces from Chain drives

Assuming the angle between x-axis and  $F_r$  is  $210^\circ$  and  $F_r \approx 2678.96 \text{ (N)}$  (chapter 2), we get the direction of  $F_r$  on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 210^\circ \approx -2320.05 \text{ (N)} \\ F_{y22} = F_{r22} \sin 210^\circ \approx -1339.48 \text{ (N)} \end{cases}$$

### 4.3.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques  $T_{sh1}$  and  $T_{sh2}$ , respectively,  $[\tau_1] = 15 \text{ (MPa)}$  and  $[\tau_2] = 30 \text{ (MPa)}$ . Using equation (10.9), we can approximate the base shaft diameters  $d_1$  and  $d_2$ :

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 25.55 \text{ (mm)}$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 34.1 \text{ (mm)}$$

Recall that our motor is 4A160M2Y3, inspecting table P1.7 we obtain the motor's output shaft diameter is 42 (mm). According to the recommendations on p.189, we limit the chosen range of  $d_1 \geq (0.8 \div 1.2) \times 42 \text{ (mm)}$ . For  $d_2$ , the chosen range



must be around  $(0.3 \div 0.35) \times a_w$  (mm). Thus,  $d_1 = 35$  (mm),  $d_2 = 40$  (mm). Consulting table (10.2) gives  $b_{O1} \approx 21$  (mm) and  $b_{O2} \approx 23$  (mm)

### 4.3.3 Identify the distance between bearings and applied forces

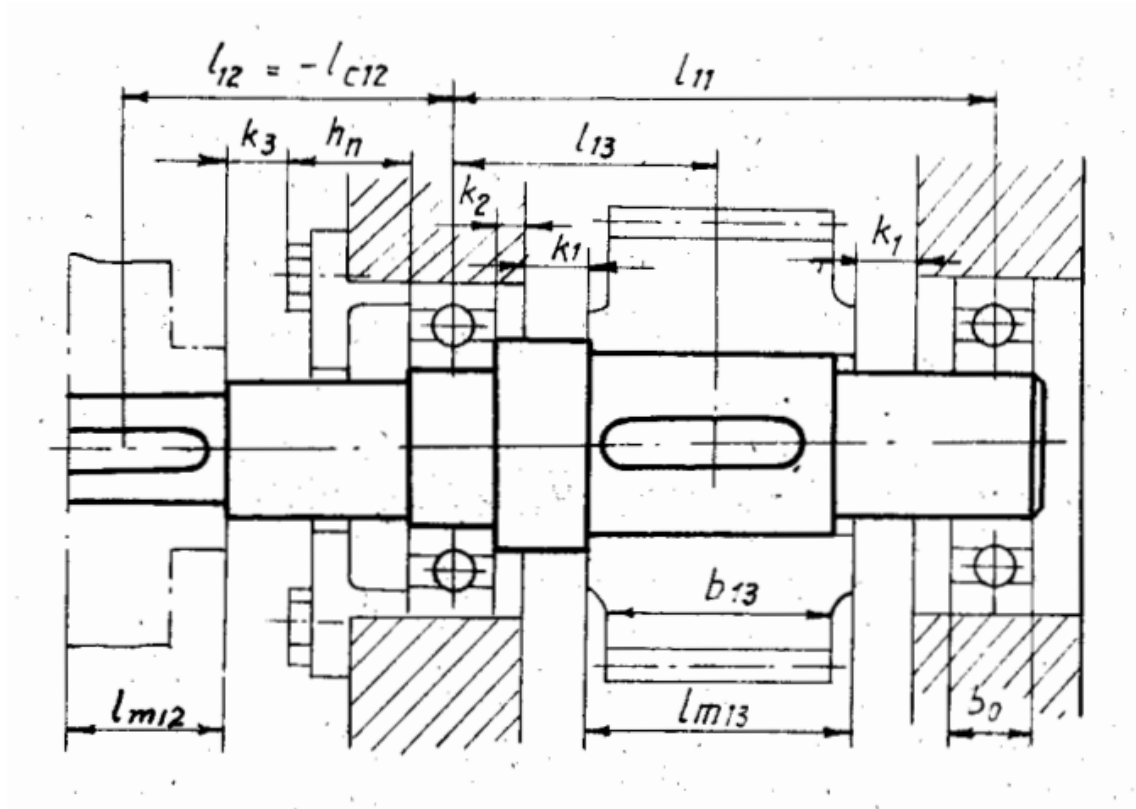


Figure 4.1: Shaft design and its dimensions

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

Using equation (10.10), the gear hubs are  $l_{m13} = l_{m12} = 1.5d_1 = 45$  (mm),  $l_{m23} = l_{m22} = 1.5d_2 = 52.5$  (mm), where  $l_{m22}$  is the chain hub.

From table (10.3), we choose  $\tilde{k}_1 = 10$  (mm),  $\tilde{k}_2 = 8$  (mm),  $\tilde{k}_3 = 15$  (mm),  $h_n = 18$  (mm). This parameters apply for both shafts in the system.

Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:

On shaft 1:

$$l_{12} = -l_{c12} = -[0.5(l_{m12} + b_{O1}) + \tilde{k}_3 + h_n] = -69.75 \text{ (mm)}$$

$$l_{13} = 0.5(l_{m13} + b_{O1}) + \tilde{k}_1 + \tilde{k}_2 = 54.75 \text{ (mm)}$$

$$l_{11} = 2l_{13} = 109.5 \text{ (mm)}$$

On shaft 2:

$$l_{22} = -l_{c22} = -[0.5(l_{m22} + b_{O2}) + \tilde{k}_3 + h_n] = -74.5 \text{ (mm)}$$

$$l_{23} = 0.5(l_{m23} + b_{O2}) + \tilde{k}_1 + \tilde{k}_2 = 59.5 \text{ (mm)}$$

$$l_{21} = 2l_{23} = 119 \text{ (mm)}$$

#### 4.3.4 Determine shaft diameters and lengths

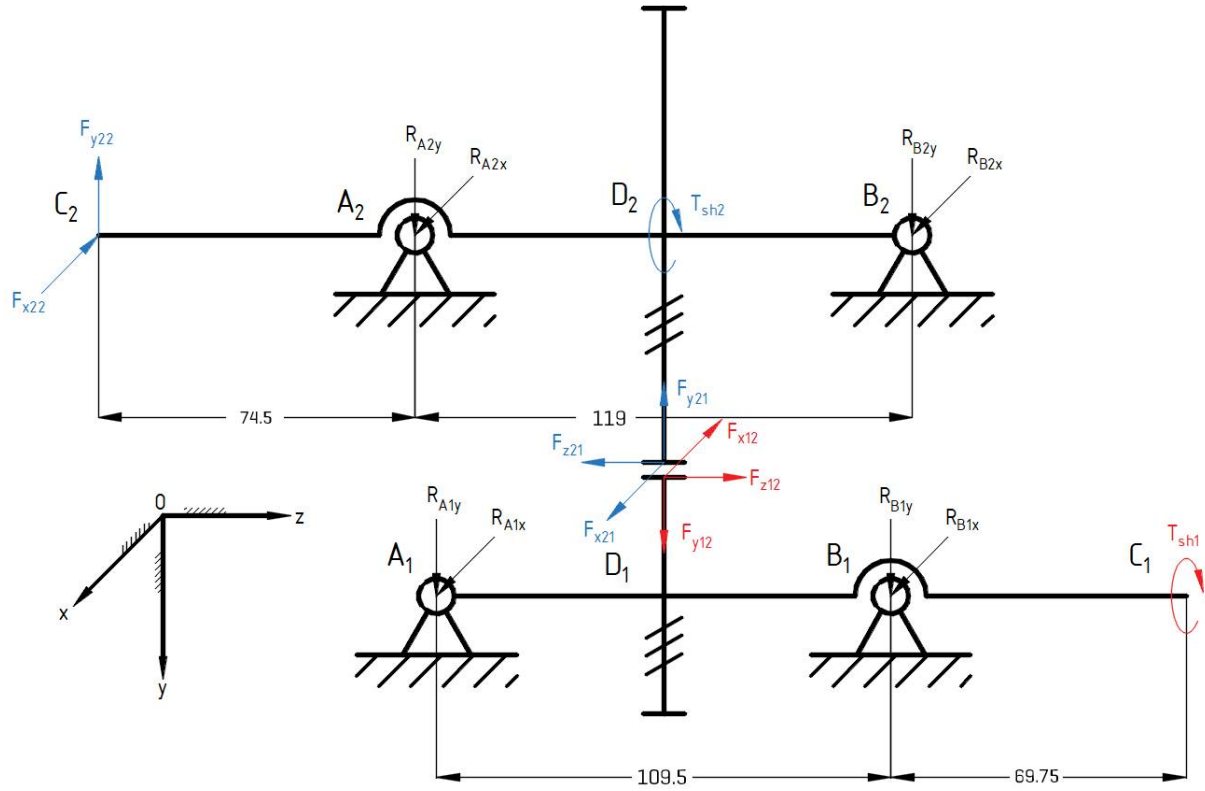


Figure 4.2: Force analysis of 2 shafts

**Find reaction forces** From the diagram, we solve for the reaction forces at  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$ , which are  $R_{A1x}$ ,  $R_{A1y}$ ,  $R_{B1x}$ ,  $R_{B1y}$ ,  $R_{A2x}$ ,  $R_{A2y}$ ,  $R_{B2x}$ ,  $R_{B2y}$ . Using

equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F}_i = 0 \\ \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0 \end{cases}$$

we obtain the results:

$$\begin{cases} R_{A1x} \approx 1201.14 \text{ (N)} \\ R_{A1y} \approx -352.24 \text{ (N)} \\ R_{B1x} \approx 1201.14 \text{ (N)} \\ R_{B1y} \approx -573.22 \text{ (N)} \end{cases} \quad \begin{cases} R_{A2x} \approx 943.15 \text{ (N)} \\ R_{A2y} \approx 3668.4 \text{ (N)} \\ R_{B2x} \approx -2005.96 \text{ (N)} \\ R_{B2y} \approx -422.89 \text{ (N)} \end{cases}$$

The total bending moments at 8 critical cross sections are also calculated (we use the formula (10.15) to derive  $M = \sqrt{M_x^2 + M_y^2}$  at each section):

$$\begin{cases} M_{A1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{D1}^- \approx 68531.85 \text{ (N} \cdot \text{mm)} \\ M_{D1}^+ \approx 72867.4 \text{ (N} \cdot \text{mm)} \\ M_{B1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{C1} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases} \quad \begin{cases} M_{C2} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{A2} \approx 191545.76 \text{ (N} \cdot \text{mm)} \\ M_{D2}^- \approx 146910 \text{ (N} \cdot \text{mm)} \\ M_{D2}^+ \approx 121977.78 \text{ (N} \cdot \text{mm)} \\ M_{B2} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases}$$

**Draw bending moment - torque diagrams** Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

**Find equivalent moments** Knowing  $T_{sh1}$  and  $T_{sh2}$ , we calculate equivalent moment  $M_e$  at the 8 cross sections specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

$$\left\{ \begin{array}{l} M_{eA1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{eD1}^- \approx 81087.5 \text{ (N} \cdot \text{mm)} \\ M_{eD1}^+ \approx 84783.4 \text{ (N} \cdot \text{mm)} \\ M_{eB1} \approx 43342.46 \text{ (N} \cdot \text{mm)} \\ M_{eC1} \approx 43342.46 \text{ (N} \cdot \text{mm)} \end{array} \right. \quad \left\{ \begin{array}{l} M_{eC2} \approx 205963.35 \text{ (N} \cdot \text{mm)} \\ M_{eA2} \approx 281266.2 \text{ (N} \cdot \text{mm)} \\ M_{eD2}^- \approx 252989.03 \text{ (N} \cdot \text{mm)} \\ M_{eD2}^+ \approx 239373.1 \text{ (N} \cdot \text{mm)} \\ M_{eB2} \approx 0 \text{ (N} \cdot \text{mm)} \end{array} \right.$$

**Find permissible stress**  $[\sigma_1]$  and  $[\sigma_2]$  are determined by table (10.5). Since we use quenched 45X steel,  $[\sigma_1] = 67 \text{ (MPa)}$  and  $[\sigma_2] = 64 \text{ (MPa)}$  ( $[\sigma_2]$  is achieved using interpolation).

**Find standardized diameters at specific locations on the shaft** Having  $M_e$  and  $[\sigma]$ , the next step is to estimate specific diameter at the key points mentioned above using equation (10.17) on p.194, which only applies for rigid shafts:

$$d \geq \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\left\{ \begin{array}{l} d_{A1} \approx 0 \text{ (mm)} \\ d_{D1} \approx 23.66 \text{ (mm)} \\ d_{B1} \approx 18.92 \text{ (mm)} \\ d_{C1} \approx 18.92 \text{ (mm)} \end{array} \right. \quad \left\{ \begin{array}{l} d_{C2} \approx 32.32 \text{ (mm)} \\ d_{A2} \approx 35.86 \text{ (mm)} \\ d_{D2} \approx 34.61 \text{ (mm)} \\ d_{B2} \approx 0 \text{ (mm)} \end{array} \right.$$

Through rough calculations, we will choose the diameters according to standards given on p.195 (one applies for bearings while the other is used for the remaining machine elements):

$$\left\{ \begin{array}{l} d_{A1} = 35 \text{ (mm)} \\ d_{D1} = 24 \text{ (mm)} \\ d_{B1} = 35 \text{ (mm)} \\ d_{C1} = 19 \text{ (mm)} \end{array} \right. \quad \left\{ \begin{array}{l} d_{C2} = 34 \text{ (mm)} \\ d_{A2} = 40 \text{ (mm)} \\ d_{D2} = 36 \text{ (mm)} \\ d_{B2} = 40 \text{ (mm)} \end{array} \right.$$

## 4.4 Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

$$\text{where } s_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$$

$$s_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$$

Assuming the surfaces are smooth, properly ground and quenched by high frequency voltage, we obtain  $K_x = 1$  from table (10.8) and  $K_y = 1.4$  from table (10.9), where  $[\sigma_b] = 850$  (MPa) is the property of quenched 45X steel.

**Find**  $\sigma_{-1}, \tau_{-1}$  Using formulas on p.196:

$$\sigma_{-1} = 0.35[\sigma_b] + 120 \approx 417.5 \text{ (MPa)}$$

$$\tau_{-1} \approx 0.58\sigma_{-1} \approx 242.15 \text{ (MPa)}$$

**Find**  $\sigma_a, \tau_a, \sigma_m, \tau_m$  For this part, we divide into 3 key points:

1. For rotating shaft,  $\sigma_m = 0, \sigma_a = \frac{\sqrt{M_x^2 + M_y^2}}{W}$  (equation (10.22)), where  $M_x$  and  $M_y$  are at the cross section of interest.
2. By design, the shafts only rotate in one direction, thus  $\tau_m = \tau_a = \frac{T_{sh}}{2W_O}$  (equation (10.23)).
3. We also assume the shafts have circular cross section, which makes  $W = \frac{\pi d^3}{32}$  and  $W_O = \frac{\pi d^3}{16}$  according to table (10.6), where  $d$  is the diameter of a cross section of the shaft.

The table below shows the results after calculation: Since  $\sigma_b = 850$  (MPa) for both shafts,  $\psi_\sigma = 0.1$  and  $\psi_\tau = 0.05$

	$d$ (mm)	$W$ (mm <sup>3</sup> )	$W_O$ (mm <sup>3</sup> )	$\sigma_m$ (MPa)	$\sigma_a$ (MPa)	$\tau_m$ (MPa)	$\tau_a$ (MPa)
$A_1$	20	785.4	1570.8	0	0	15.93	15.93
$D_1$	24	1357.17	2714.34	0	49.2	9.22	9.22
$B_1$	20	785.4	1570.8	0	0	15.93	15.93
$C_1$	19	673.38	1346.76	0	0	18.58	18.58
$C_2$	32	3216.99	6433.98	0	0	18.48	18.48
$A_2$	40	6283.19	12566.37	0	29.74	9.46	9.46
$D_2$	34	3858.66	7717.32	0	36.67	15.41	15.41
$B_2$	35	4209.24	8418.49	0	0	14.13	14.13

Table 4.1: Calculated variables for  $\sigma_a, \tau_a, \sigma_m, \tau_m$ 

**Find  $K_\sigma, K_\tau$**  We calculate  $K_\sigma$  using formula:

$$K_\sigma = \left( \frac{k_\sigma}{\varepsilon_\sigma} + K_x - 1 \right) K_y^{-1}$$

and  $K_\tau$  with:

$$K_\tau = \left( \frac{k_\tau}{\varepsilon_\tau} + K_x - 1 \right) K_y^{-1}$$

Table (10.10), (10.11) and (10.13) are examined to find  $\frac{k_\sigma}{\varepsilon_\sigma}$  ratio. Given  $[\sigma_H] = 850$  (MPa) base shaft diameters  $d_1$  and  $d_2$  are compared to the diameters at critical locations  $A, B, C, D$ . If the base shaft is smaller, table (10.10) and (10.11) are used. If it is larger, we will use table (10.13) instead; the concentration stress factor in this case is demonstrated in the figure:

Final calculation is provided in the table:

**Find  $s_\sigma, s_\tau$  and  $s$**  Combining the results altogether, we obtain the following table:

Since the smallest safety factor is at the cross section  $D_1$ , which has the value of  $3.14 > [s] = 1.5 \div 2.5$ , we can neglect rigidity analysis according to the conclusion on p.195.

	$d$ (mm)	$r$	$k_\sigma$	$k_\tau$	$\varepsilon_\sigma$	$\varepsilon_\tau$	$\frac{k_\sigma}{\varepsilon_\sigma}$	$\frac{k_\tau}{\varepsilon_\tau}$	$K_x$	$K_y$	$K_\sigma$	$K_\tau$
$A_1$	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
$D_1$	24	0.48	3	1.95	0.81	0.85	3.7	2.29	1	1.4	2.65	1.64
$B_1$	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
$C_1$	19	0.38	3	1.95	0.84	0.89	3.57	2.19	1	1.4	2.55	1.57
$C_2$	32	0.64	3	1.95	0.76	0.80	3.95	2.44	1	1.4	2.82	1.74
$A_2$	40	-	-	-	-	-	3.34	2.46	1	1.4	2.39	1.76
$D_2$	34	0.68	3	1.95	0.74	0.80	4	2.44	1	1.4	2.86	1.75
$B_2$	35	-	-	-	-	-	3.3	2.44	1	1.4	2.36	1.74

Table 4.2: Calculated variables in  $K_\sigma$  and  $K_\tau$ 

	$s_\sigma$	$s_\tau$	$s$
$A_1$	$\gg s_\tau$	9.41	9.41
$D_1$	3.21	16	3.14
$B_1$	$\gg s_\tau$	9.41	9.41
$C_1$	$\gg s_\tau$	8.07	8.07
$C_2$	$\gg s_\tau$	7.32	7.32
$A_2$	5.88	14	5.43
$D_2$	3.99	8.77	3.63
$B_2$	$\gg s_\tau$	9.56	9.56

Table 4.3: Safety factor at critical cross sections

## 4.5 Static Strength Analysis

Along with fatigue strength, static strength is also considered and every shaft must satisfy the following condition at critical cross sections (equation (10.27)):

$$\sigma_e = \sqrt{\left(\frac{M_{max}}{0.1d^3}\right)^2 + 3\left(\frac{T_{max}}{0.2d^3}\right)^2} \leq [\sigma]$$

where  $M_{max}$ ,  $T_{max}$  are the largest bending moment and torque at the cross section, respectively. Let  $[\sigma] \approx 0.8\sigma_{ch} = 520$  (MPa), the results are in the table below:

	$A_1$	$D_1$	$B_1$	$C_1$	$C_2$	$A_2$	$D_2$	$B_2$
$\sigma_e$ (MPa)	54.18	57.59	54.18	63.19	62.86	43.45	63.58	48.04

Table 4.4: Calculated static strength at critical cross sections

which satisfy the given condition.



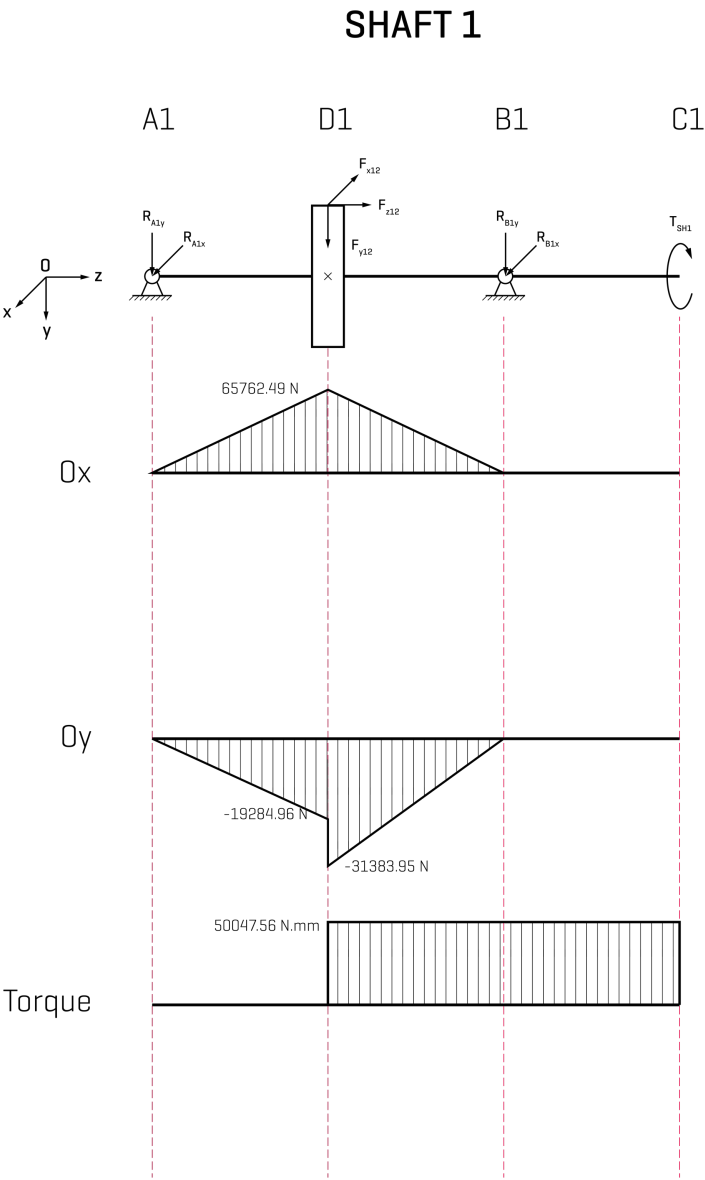


Figure 4.3: Bending moment-torque diagram of shaft 1

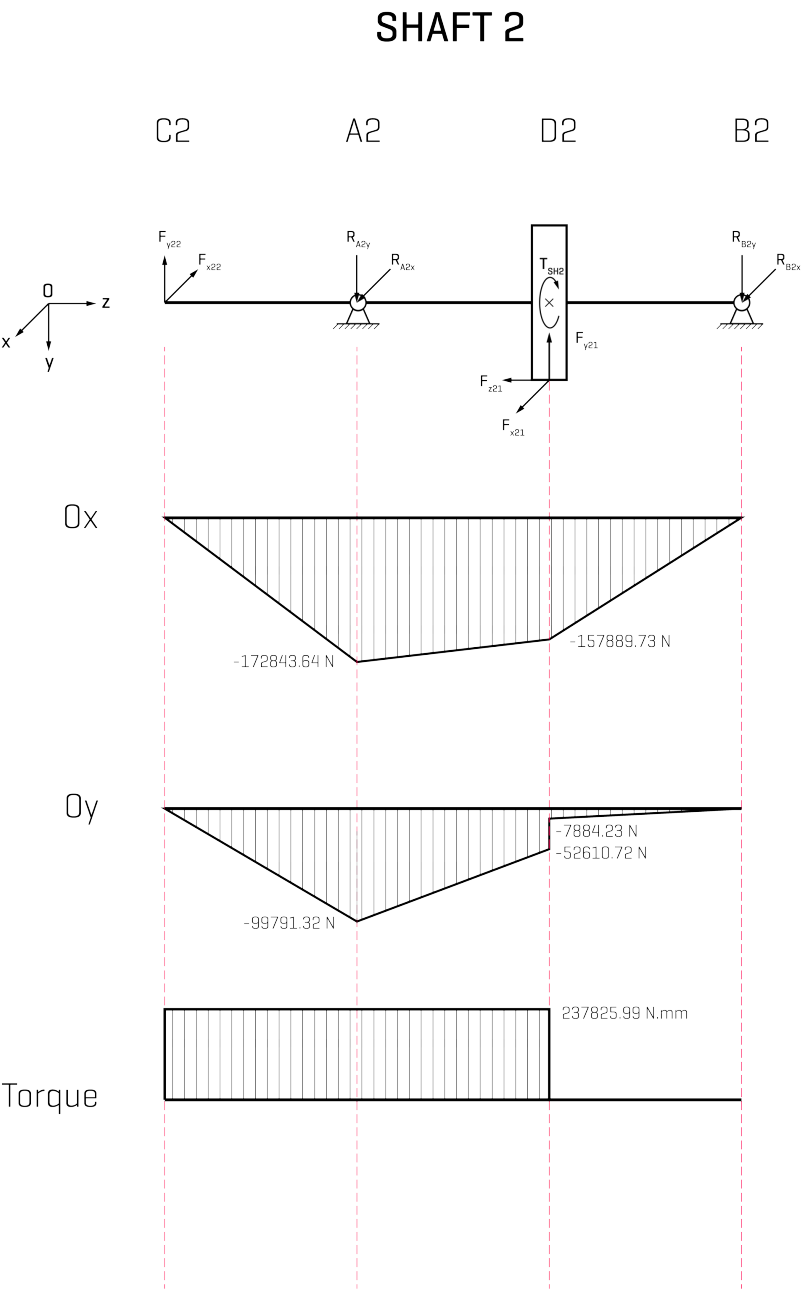


Figure 4.4: Bending moment-torque diagram of shaft 2

# Chapter 5

## Bearing Design

### 5.1 Nomenclature

$b_O$	rolling bearing width, mm	$L$	rated life in million revolutions, million rev
$C$	standardized dynamic load rating, N	$L_h$	rated life in hours, h
$C_d$	basic dynamic load rating, N	$l$	length (general), mm
$d$	diameter, mm	$l_m$	hub length (general), mm
$F_a$	axial force, kN	$M$	moment, N · mm
$F_r$	radial force, kN	$M_e$	equivalent moment, N · mm
$F_t$	tangential force, kN	$M_{max}$	maximum moment at the cross section, N · mm
$F$	applied force, kN	$m$	load-life exponent
$h_n$	distance between bearing lid and bolt, mm	$Q$	equivalent dynamic load, kN
$hr$	tooth direction	$q$	standardized coefficient of shaft diameter
$k_d$	temperature factor	$R$	reaction force, N
$k_t$	load condition factor		

$T$	torque at the cross section,	$_{sh1}$	subscript for shaft 1
	N · mm	$_{sh2}$	subscript for shaft 2
$X$	dynamic radial load factor	$_x$	subscript for x-axis
$Y$	dynamic axial load factor	$_y$	subscript for y-axis
$\alpha$	contact angle, °	$_z$	subscript for z-axis

## 5.2 Choose bearing type

As for the types, we will examine  $\frac{F_a}{F_r}$  at  $A_1, B_1, A_2$  and  $B_2$  in the 2 shafts from the previous chapter, where  $F_a$  is the output axial force  $|F_{z12}| = |F_{z21}| \approx 580.75$  (N);  $F_r$  is the magnitude of combined reaction force  $\sqrt{R_x^2 + R_y^2}$  from the shaft onto the bearing, which is essentially a radial load. The larger ratio in a shaft will be our ratio of choice to determine the bearing type.

Taking our results from the chapter 4:

$$\left\{ \begin{array}{l} F_{rA1} = \sqrt{R_{A1x}^2 + R_{A1y}^2} \approx 1.2 \text{ (kN)} \\ F_{rB1} = \sqrt{R_{B1x}^2 + R_{B1y}^2} \approx 1.33 \text{ (kN)} \\ F_{aA1} = |F_{z12}| \approx 0.58 \text{ (kN)} \\ F_{aB1} = |F_{z12}| \approx 0.58 \text{ (kN)} \end{array} \right. \quad \left\{ \begin{array}{l} F_{rA2} = \sqrt{R_{A2x}^2 + R_{A2y}^2} \approx 3.79 \text{ (kN)} \\ F_{rB2} = \sqrt{R_{B2x}^2 + R_{B2y}^2} \approx 2.05 \text{ (kN)} \\ F_{aA2} = |F_{z21}| \approx 0.58 \text{ (kN)} \\ F_{aB2} = |F_{z21}| \approx 0.58 \text{ (kN)} \end{array} \right.$$

yields

$$\left\{ \begin{array}{l} \frac{F_{aA1}}{F_{rA1}} \approx 0.46 \\ \frac{F_{aB1}}{F_{rB1}} \approx 0.44 \end{array} \right. \quad \left\{ \begin{array}{l} \frac{F_{aA2}}{F_{rA2}} \approx 0.15 \\ \frac{F_{aB2}}{F_{rB2}} \approx 0.28 \end{array} \right.$$

Since  $0.46 > 0.3$  and  $0.28 \leq 0.3$ , the pair of bearings on shaft 1 is single-row angular contact ball bearings with  $\alpha_{sh1} = 12^\circ$  and the remaining pair is single-row deep-groove bearings ( $\alpha_{sh2} = 0^\circ$ ); AG = 0 according to the recommendations on p.212 and p.213.

We also have dimensions at the cross sections  $A_1, B_1, A_2, B_2$  from the previous

chapter:

$$\left\{ \begin{array}{l} b_{O1} = 21 \text{ (mm)} \\ d_{A1} = 35 \text{ (mm)} \\ d_{B1} = 35 \text{ (mm)} \end{array} \right. \quad \left\{ \begin{array}{l} b_{O2} = 23 \text{ (mm)} \\ d_{A2} = 40 \text{ (mm)} \\ d_{B2} = 40 \text{ (mm)} \end{array} \right.$$

From these parameters, we will look up the tables at the end of the text. The pair of single-row angular contact ball bearings of choice is 46307, which is suitable for shaft 1 and  $C_{o1} = 25.2$  (kN). On shaft 2, the pair of single-row deep-groove bearings are type 308, where  $C_{o2} = 21.7$  (kN).

## 5.3 Bearing dimensions

### 5.3.1 Calculate basic dynamic load rating

$$C_d = Q_e \sqrt[n]{L}$$

#### Find equivalent dynamic load

Since we only use ball bearings, the following formula applies:

$$Q = (XVF_r + YF_{at})k_tk_d$$

Since the inner ring rotates,  $V = 1$  and the  $\frac{F_a}{VF_r} = \frac{F_a}{F_r}$ , meaning that the ratios in the section above will be used to examine  $X$  and  $Y$ .

The design problem also does not give any further information about operating temperature, which gives  $k_t = 1$ . In addition, we get  $k_d = 1$  from table (11.3) based on the machine's condition (low load and power rating).

**Find the ratio  $\frac{iF_a}{C_o}$**  This ratio is calculated and applied for 2 shafts ( $i = 1$  for single-row bearings in our case):

For shaft 1,  $\frac{F_a}{C_{o1}} \approx 0.027$

For shaft 2,  $\frac{F_a}{C_{o2}} \approx 0.023$

**Compare with  $e$**  From the previous section,  $\alpha_{sh1} = 12^\circ$ ,  $\alpha_{sh2} = 0^\circ$ . Inspecting table (11.4) and by interpolation,  $e_{sh1} \approx 0.33$ ,  $e_{sh2} \approx 0.22$ . These values are then compared to  $\left| \frac{F_a}{VF_r} \right|$  to look up the correct column.

**Find  $X, Y$**  Table (11.3) and interpolation are used in finding these values:

For shaft 1,  $\left| \frac{F_a}{VF_r} \right| = \left| \frac{F_{z12}}{VR_{A1y}} \right| \approx 0.46 > e_1 \Rightarrow X_1 = 0.56, Y_1 = 2.1$ .

For shaft 2,  $\left| \frac{F_a}{VF_r} \right| = \left| \frac{F_{z21}}{VR_{B2y}} \right| \approx 0.28 > e_2 \Rightarrow X_2 \approx 0.45, Y_2 \approx 1.64$ .

**Find  $F_{at}$**  For shaft 1, additional radial forces are also applied to the pair of angular contact ball bearings. From table (11.5), the first arrangement is used in the gearbox. Therefore,  $\mathbf{F}_{sA1} \uparrow \uparrow \mathbf{F}_{z12}$  and  $\mathbf{F}_{sB1} \uparrow \downarrow \mathbf{F}_{z12}$  (the direction of  $\mathbf{F}_{z12}$  can be found at Figure 4.2) Following the sign convention on p.218 and combining with equation (11.8), (11.10), (11.11a) and (11.11b): At cross section  $A_1$ :

$$F_{sB1} = e_1 F_{rB1} \approx 0.43 \text{ (kN)}$$

$$\sum F_{aA1} = F_{sB1} - F_{z12} \approx -0.15 \text{ (kN)}$$

At cross section  $B_1$ :

$$F_{sA1} = e_1 F_{rA1} \approx 0.41 \text{ (kN)}$$

$$\sum F_{aB1} = F_{sA1} + F_{z12} \approx 0.99 \text{ (kN)}$$

From equation (11.11a) and (11.11b):

$$\sum F_{aA1} \leq F_{sA1} \Rightarrow F_{atA1} = F_{sA1} \approx 0.41 \text{ (kN)}$$

$$\sum F_{aB1} > F_{sB1} \Rightarrow F_{atB1} = \sum F_{aB1} \approx 0.99 \text{ (kN)}$$

In contrast, shaft 2 does not have such additional forces since  $\alpha_{sh2} = 0^\circ$ .

Therefore,  $F_{atA2} = F_{atB2} = |F_{z21}| = 0.58 \text{ (kN)}$ .

**Find  $Q_1, Q_2$**  Recall that the forces are in absolute values, the parameters are substituted to the formula to obtain:

$$Q_{A1} \approx 1.56 \text{ (kN)}$$

$$Q_{B1} \approx 2.82 \text{ (kN)}$$

$$Q_{A2} \approx 2.66 \text{ (kN)}$$

$$Q_{B2} \approx 1.87 \text{ (kN)}$$

We will compare these values and choose the larger load (according to the recommendation on p.219):

$$Q_{A1} < Q_{B1} \Rightarrow Q_1 = Q_{B1} \approx 2.82 \text{ (kN)}$$

$$Q_{A2} > Q_{B2} \Rightarrow Q_2 = Q_{A2} \approx 2.66 \text{ (kN)}$$

**Find  $Q_1$**  Modifying equation (11.12), we obtain the equivalent load on 2 shafts (assuming the bearings are ball type):

$$Q_e = Q \sqrt[3]{\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1 + t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1 + t_2}}$$

$$\text{For shaft 1, } Q_{e1} \approx 2.17 \text{ (kN)}$$

$$\text{For shaft 2, } Q_{e2} \approx 1.88 \text{ (kN)}$$

**Verifying condition for  $C_d$**

**Find  $L$**  Equation (11.2) is rearranged to calculate  $L$ :

$$L = L_h 60 n_{sh} \times 10^6$$

The transmission system works for 19200 (hours) (the calculation has already been done in chapter 3), which gives:

$$L_1 \approx 3375.36 \text{ (million rev)}$$

$$L_2 \approx 675.07 \text{ (million rev)}$$

**Find  $C_d$**  Combining the results and letting  $m = 3$  (ball bearings are used in this case) yield:

$$C_{d1} = 32.49 \text{ (kN)}$$

$$C_{d2} = 16.53 \text{ (kN)}$$

**Compare with  $C$**  From table (P2.15), we obtain the ratios using interpolation (knowing  $n_{sh1}$  and  $n_{sh2}$  and  $L_h = 19200$  (hours)):

$$C_1 \approx 14.96 Q_{e1} \approx 32.39 \text{ (kN)} < C_{d1}$$

$$C_2 \approx 8.75 Q_{e2} \approx 16.47 \text{ (kN)} < C_{d2}$$

Since both shafts do not satisfy the condition, we will reduce the rated life  $L_h$  of the bearings in half based on the recommendation on p.220 and repeat the process of verification. The results are:

$$C_1 \approx 26.84 \text{ (kN)} \geq C_{d1} \approx 25.79 \text{ (kN)}$$

$$C_2 \approx 13.66 \text{ (kN)} \geq C_{d2} \approx 13.12 \text{ (kN)}$$

### 5.3.2 Calculate static load rating

The bearings are non-rotating, thus we will verify its static load rating using condition (11.18) on p.221:

$$\begin{cases} Q_t = X_O F_r + Y_O F_{at} \\ Q_t = F_r \end{cases} \leq C_O$$

For shaft 1,  $X_O = 0.5, Y_O = 0.47$ .

$$Q_{t1} \approx 0.76 \text{ (kN)} < C_{o1}$$

For shaft 2,  $X_O = 0.6, Y_O = 0.5$ .

$$Q_{t2} \approx 0.64 \text{ (kN)} < C_{o2}$$