HCM University of Technology

Machine Elements

ME2007

Project Report

Submitted To:

Phan Dinh Huan

Asst. Professor

Faculty of Mechanical Engineering

Submitted By :

Nguyen Quy Khoi

1852158

CC02

HK192

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Design Problem

D_{bc}	pulley diameter, mm
F_t	tangential force, N
L	service life, years
T	working torque, $\mathrm{N \cdot mm}$
t	working time, s
v_{bc}	conveyor belt speed, $\mathrm{m/s}$
δ_u	error of speed ratio, %

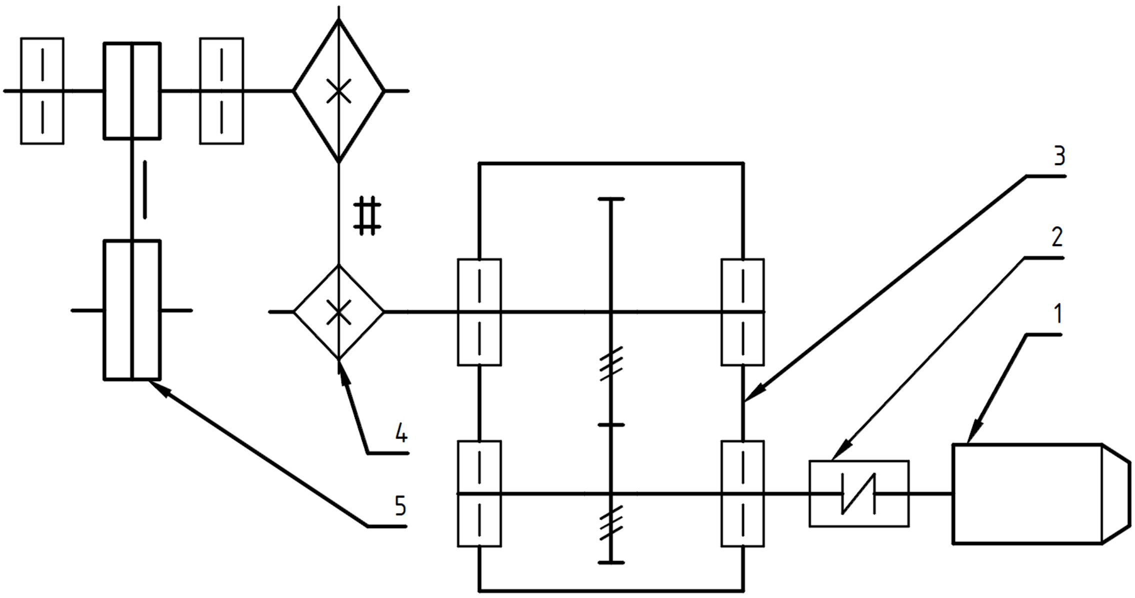

Figure 1: Mechanical transmission system of a belt conveyor

Given the mechanical transmission system in figure 1, determine the specifications for each machine element.

1. Electric motor
2. Elastic coupling
3. Gearbox
4. Chain drive

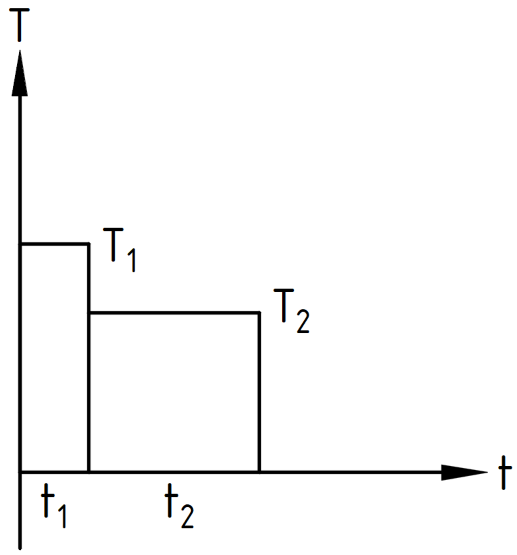

Figure 2: Input load diagram

5. Belt conveyor

Design parameters The chosen parameters are given in column 8:

- $F_t = 4500\,(\mathrm{N})$
- $v_{bc} = 3.05\,(\mathrm{m/s})$
- $D_{bc} = 500\,(\mathrm{mm})$
- $L = 4\,(\text{years})$
- $T_1 = T\,(\mathrm{N \cdot mm})$, $t_1 = 12\,(\mathrm{s})$
- $T_2 = 0.7T\,(\mathrm{N \cdot mm})$, $t_2 = 60\,(\mathrm{s})$
- $\delta_u \leq \pm 5\%$

The machine works in one direction for 300 days, 2 shifts per day, 8 hours each and the load is low. Also, for the rest of this report, we will consider shaft 1 is the one connected to the electric motor, while shaft 2 is the connection between the chain drive and gearbox.

Chapter 1

Motor Design

1.1 Nomenclature

n_{bc}	rotational speed of belt conveyor, rpm
n_{sh}	rotational speed of shaft, rpm
P_m	maximum operating power of belt conveyor, kW
P_{motor}	calculated motor power to drive the system, kW
P_{sh}	operating power of shaft, kW
P_w	operating power of the belt conveyor given a workload, kW
u_{ch}	transmission ratio of chain drive
u_{hg}	transmission ratio of helical gear
u_{sys}	transmission ratio of the system
T_{motor}	motor torque, $N \cdot mm$
T_{sh}	shaft torque, $N \cdot mm$
η_b	bearing efficiency
η_c	coupling efficiency
η_{ch}	chain drive efficiency
η_{hg}	helical gear efficiency
η_{sys}	efficiency of the system
$_1$	shaft 1
$_2$	shaft 2

1.2 Calculate η_{sys}

From table (2.3):

$\eta_c = 1$

$\eta_b = 0.99$

$\eta_{hg} = 0.96$

$\eta_{ch} = 0.95$

$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$

1.3 Calculate P_{motor}

$P_m = \dfrac{F_t v_{bc}}{1000} \approx 13.73\,(\text{kW})$

From equation (2.13) :

$P_w = P_m \sqrt{\dfrac{\left(\dfrac{T_1}{T}\right)^2 t_1 + \left(\dfrac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41\,(\text{kW})$

$P_{motor} = \dfrac{P_w}{\eta_{sys}} \approx 11.76\,(\text{kW}) < P_m$

1.4 Calculate n_{motor}

$n_{bc} = \dfrac{6 \times 10^4 v}{\pi D_{bc}} \approx 116.5\,(\text{rpm})$

$u_{ch} = 5$ (table (2.4))

$u_{hg} = 5$ (table (2.4))

$u_{sys} = u_{ch} u_{hg} = 25$

$n_{motor} = u_{sys} n_{bc} \approx 2912.54\,(\text{rpm})$

1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated P_{motor} and P_m. Since $P_{motor} < P_m$ for our case, the minimum operating power of choice is P_m. In similar fashion, its rotational speed must also be no smaller than estimated n_{motor}.

Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$\Rightarrow P_{motor} = 18.5\,\text{kW}, n_{motor} = 2930\,(\text{rpm})$

Recalculating u_{sys} with the new P_{motor} and n_{motor}, we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming $u_{hg} = const$:

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

1.6 Calculate power, rotational speed and torque

Let us denote P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

1.6.1 Power

$$P_{ch} = P_m \approx 13.73\,(\text{kW})$$

$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59\,(\text{kW})$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35\,(\text{kW})$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930\,(\text{rpm})$$
$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586\,(\text{rpm})$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63\,(\text{N} \cdot \text{mm})$$
$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56\,(\text{N} \cdot \text{mm})$$
$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99\,(\text{N} \cdot \text{mm})$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	18.5	15.35	14.59
u	5	5.03	
n (rpm)	2930	2930	586
T (N · mm)	60298.63	50047.56	237825.99

Table 1.1: System overall specifications

Chapter 2

Chain Drive Design

2.1 Nomenclature

$[i]$	permissible impact times per second
$[s]$	permissible safety factor
$[P]$	permissible power, kW
a	center distance, mm
a_{max}	maximum center distance, mm
a_{min}	minimum center distance, mm
B	bush length, mm
d	driving sprocket diameter, mm
d_c	pin diameter, mm
F_0	sagging force, N
F_1	tight side tension force, N
F_2	slack side tension force, N
F_r	force on the shaft, N
F_t	effective peripheral force, N
F_v	centrifugal force, N
i	impact times per second
k	overall factor
k_0	arrangement of drive factor
k_a	center distance and chain's length factor
k_{bt}	lubrication factor
k_c	rating factor
k_d	dynamic loads factor
k_{dc}	chain tension factor
k_f	loosing factor
k_n	coefficient of rotational speed
k_x	chain weight factor
k_z	coefficient of number of teeth

n_{01}	experimental rotational speed, rpm	v	instantaneous velocity along the chain, m/s
n_{ch}	rotational speed of a sprocket, rpm	x	chain length in pitches, the number of links
P_t	calculated power, kW	x_c	an even number of links
p	pitch, mm	z	number of teeth of a sprocket
p_{max}	permissible sprocket pitch, mm	z_{max}	maximum number of teeth of the driven sprocket
Q	permissible load, N		
q	mass per meter of chain, kg/m	$_1$	subscript for driving sprocket
s	safety factor	$_2$	subscript for driven sprocket

2.2 Find p

Since the driving sprocket is connected to shaft 1, $n_1 = n_{sh2} = 586\,(\text{rpm})$.

Find z Since z_1 and z_2 is preferably an odd number according to p.80:

$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$

$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$

Because $z_1 \geq 15$, we use table (5.8) and interpolation to approximate p_{max}. Therefore, $p_{max} \approx 33.58\,(\text{mm})$.

Find k Since $n_{ch} = 586 \approx 600\,(\text{rpm})$, choose $n_{01} = 600\,(\text{rpm})$, which is obtained from table (5.5). Then, we calculate k_z and k_n.

$k_z = \dfrac{25}{z_1} \approx 1.32$, $k_n = \dfrac{n_{01}}{n_{ch}} \approx 1.02$

Specifying the chain drive's working condition and ultizing table (5.6) , we find out that $k_0 = k_a = k_{dc} = k_{bt} = 1$, $k_d = 1.25$, $k_c = 1.3$.

$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$

Find p From table (5.5):

$P_t = P_{ch}kk_zk_n \approx 30.05\,(\text{kW}) \leq 42\,(\text{kW}) \Rightarrow [P] = 42\,(\text{kW})$

Using the table, we also get the other parameters:

$p = 31.75\,(\text{mm})$, $d_c = 9.55\,(\text{mm})$, $B = 27.46\,(\text{mm})$,

$d_1 = \dfrac{p}{\sin\dfrac{180^\circ}{z_1}} \approx 192.9\,(\text{mm})$, $d_2 = \dfrac{p}{\sin\dfrac{180^\circ}{z_2}} \approx 980.49\,(\text{mm})$

Having $p = 31.75\,(\text{mm}) \leq \text{p}_{\text{max}} \approx 33.58\,(\text{mm})$, we can safely choose the number of chains as 1, which is in agreement with the given figure. Hence, from table (5.2), we obtain the parameters in the section for 1 strand chain drive:

$Q = 56.7 \times 10^3\,(\text{N})$, $q = 2.6\,(\text{kg/m})$

By comparison to the conditions in the sub-table, the choice of B is satisfactory.

2.3 Find a, x_c, and i

Find x_c $a_{min} = 30p = 952.5\,(\text{mm})$, $a_{max} = 50p = 1587.5\,(\text{mm})$. Limiting the range of choice for a in $[a_{min}, a_{max}]$, we can approximate $a = 1000\,(\text{mm})$.

$x = \dfrac{2a}{p} + \dfrac{z_1 + z_2}{2} + \dfrac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$

Find a From equation (5.13) , we calculate a again with x_c:

$a = \dfrac{p}{4}\left(x_c - \dfrac{z_2 + z_1}{2} + \sqrt{\left(x_c - \dfrac{z_2 + z_1}{2}\right)^2 - 2\dfrac{(z_2 - z_1)^2}{\pi^2}}\right) - 0.003a \approx 998.98\,(\text{mm})$

Find i From table (5.9):

$i = \dfrac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$

2.4 Strength of chain drive

For moderate workload, choose $k_d = 1.2$. Let the chain drive be angled 30° with respect to ground, we obtain $k_f = 4$.
$v_1 = \dfrac{n_{ch} p z_1}{6 \times 10^4} \approx 5.89\,(\mathrm{m/s})$

Find F_t, F_v, F_0 We also need to calculate F_t, F_v and F_0:
$F_t = \dfrac{10^3 P_{ch}}{v_1} \approx 2329.53\,(\mathrm{N})$
$F_v = q v_1^2 \approx 90.25\,(\mathrm{N})$
$F_0 = 9.81 \times 10^{-3} k_f q a \approx 101.92\,(\mathrm{N})$

Validate s This value must be larger than the permissible safety factor to operate properly. From equation (5.15):
$s = \dfrac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2$, where $[s]$ is chosen from table (5.10).

2.5 Force on shaft

From p.87:

$F_2 = F_0 + F_v \approx 192.17\,(\mathrm{N})$

$F_1 = F_t + F_2 \approx 2521.7\,(\mathrm{N})$

Choose $k_x = 1.15$ and follow equation (5.20) :

$F_r = k_x F_t \approx 2678.96\,(\mathrm{N})$

In summary, we have the following table:

	driving	driven
$[P]$ (kW)	42	
Q (N)	56700	
p (mm)	31.75	
i	6	
a (mm)	998.98	
z	19	97
d (mm)	192.9	980.49
d_c (mm)	9.55	
B (mm)	27.46	
v (m/s)	5.01	
u_{ch}	5	

Table 2.1: Chain drive specifications

Chapter 3

Gearbox Design (Helix gears)

3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa
$[\sigma_F]$	permissible bending stress, MPa
$[\sigma_H]_{max}$	permissible contact stress due to overload, MPa
$[\sigma_F]_{max}$	permissible bending stress due to overload, MPa
AG	accuracy grade of gear
a	center distance, *mm*
b	face width, *mm*
c	gear meshing rate
d	pitch circle, *mm*
d_a	addendum diameter, *mm*
d_b	base diameter, *mm*
d_f	deddendum diameter, *mm*
F_a	axial force, N
F_r	radial force, N
F_t	tangential force, N
H	surface roughness, HB
K_d	coefficient of gear material
K_F	load factor from bending stress
K_{FC}	load placement factor
K_{FL}	aging factor due to bending stress
K_{Fv}	factor of dynamic load from bending stress at meshing area
$K_{F\alpha}$	factor ofload distribution from bending stress on gear teeth
$K_{F\beta}$	factor of load distribution from bending stress on top land

K_H	load factor of contact stress
K_{HL}	aging factor due to contact stress
K_{Hv}	factor of dynamic load from contact stress at meshing area
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth
$K_{H\beta}$	factor of load distribution from contact stress on top land
k_x	a coefficient
k_y	a coefficient
m	traverse module, mm
m_F	root of fatigue curve in bending stress test
m_H	root of fatigue curve in contact stress test
m_n	normal module, mm
N_{FE}	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$
N_{FO}	working cycle of bearing stress corresponding to $[\sigma_F]$
N_{HE}	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$
N_{HO}	working cycle of bearing stress corresponding to $[\sigma_H]$
S	length, mm
S_F	safety factor of bending stress
S_H	safety factor of contact stress
v	rotational velocity, m/s
x	gear correction factor
Y_F	tooth shape factor
Y_β	helix angle factor
Y_ϵ	contact ratio factor
y	center displacement factor
z_H	contact surface's shape factor
z_M	material's mechanical properties factor
z_{min}	minimum number of teeth corresponding to β
z_v	virtual number of teeth
z_ϵ	meshing condition factor
α	normal pressure angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20°$
α_t	traverse pressure angle, °
ϵ_α	traverse contact ratio
ϵ_β	face contact ratio
β	helix angle, °
β_b	base circle helix angle, °
ψ_{ba}	width to shaft distance ratio
ψ_{bd}	face width factor
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa

σ^o_{Flim}	permissible bending stress corresponding to working cycle, MPa	$_1$	subscript for pinion
σ^o_{Hlim}	permissible contact stress corresponding to working cycle, MPa	$_2$	subscript for driven gear
		$_w$	subscript for variable value after correction

3.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with $S \leq 100\,(\text{mm})$, HB250, $\sigma_b = 850\,(\text{MPa})$, $\sigma_{ch} = 550\,(\text{MPa})$.
Table (6.2) also gives $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$
Therefore, they have the same properties except for their surface roughness H.
For the pinion, $H_1 = \text{HB}250 \Rightarrow \sigma^o_{Hlim1} = 570\,(\text{MPa})$, $\sigma^o_{Flim1} = 450\,(\text{MPa})$
For the driven gear, $H_2 = \text{HB}240 \Rightarrow \sigma^o_{Hlim2} = 550\,(\text{MPa})$, $\sigma^o_{Flim2} = 432\,(\text{MPa})$

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

3.3.1 Working cycle of bearing stress

Using equation (6.5) :
$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6\,(\text{cycles})$
$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6\,(\text{cycles})$

3.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq$ HB350, $m_H = 6$, $m_F = 6$.

Both gears meshed indefinitely, thus $c = 1$.

Applying equation (6.7) and T_1, T_2, t_1, t_2 in the initial parameters:

$$N_{HE1} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 5.73 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 1.15 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 3.35 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 0.67 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

3.3.3 Aging factor

For steel, $N_{FO1} = N_{FO2} = 4 \times 10^6$ (MPa). Applying equations (6.3) and (6.4) yield:

$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$

$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$

$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$

$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$

3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction, $K_{FC} = 1$

$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_{H1} \approx 621.61$ (MPa)

$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_{H2} \approx 771.63$ (MPa)

$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC1} K_{FL1}/S_{F1} \approx 264.85$ (MPa)

$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC2} K_{FL2}/S_{F2} \approx 332.48$ (MPa)

The permissible contact stress due to overload must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. For permissible bending stress, it is equal to either $[\sigma_{F1}]$ or $[\sigma_{F2}]$, whichever is larger:

$[\sigma_H]_{max} = \dfrac{1}{2}([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62\,(\text{MPa}) \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$

$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440\,(\text{MPa})$

3.4 Transmission Design

3.4.1 Determine basic parameters

Examine table (6.5) gives $K_a = 43$

Assuming symmetrical design, table (6.6) also gives $\psi_{ba} = 0.5$

$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$

From table (6.7) , using interpolation, we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a using equation (6.15a) gives:

$a = K_a(u_{hg} + 1)\sqrt[3]{\dfrac{T_{sh1}K_{H\beta}}{[\sigma_H]^2 u_{hg}\psi_{ba}}} \approx 91.94\,(\text{mm})$

According to SEV229-75 standard, we choose $a_w = 100\,\text{mm}$

3.4.2 Determine gear meshing parameters

Find m Applying equation (6.17) and choose m from table (6.8) :

$m = (0.01 \div 0.02)a_w \approx (0.92 \div 1.84)\,(\text{mm}) \Rightarrow m = 1.5\,(\text{mm})$

Find z_1**,** z_2**,** b_w Let $\beta = 15°$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1. From the result, z_1 is rounded up to the nearest odd number (preferably a prime number).

$z_1 = \dfrac{2a_w \cos\beta}{m(u_{hg}+1)} \approx 19.73 \Rightarrow z_1 = 21$

$z_2 = u_{hg} z_1 = 105$

$\Rightarrow b_w = \psi_{ba} a_w = 50\,(\text{mm})$

Recalculate β There are 2 approaches for correction involving the change of either α or β. Because altering α leads to many other corrections (d_1, d_2 and a), β will be used instead.

Since z_1 is rounded, we must find β to obtain the correct angle, ensuring that $\beta \in (8°, 20°)$. Using equation (6.32):

$\beta = \arccos \dfrac{m(z_1+z_2)}{2a_w} \approx 19.09°$

Find x_1, x_2 To find x_1 and x_2, we will follow the calculation scheme provided in p.103. Since $\beta \approx 19.09° \in (17, 21]$, $z_{min} = 15$, which leads to z_1 satisfying condition $z_1 \geq z_{min} + 2 > 10$, according to table (6.9). Combined with $u_{hg} = 5 \geq 3.5$, we obtain $x_1 = 0.3$, $x_2 = -0.3$, disregarding the calculation of y.

3.4.3 Other parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos\beta} \approx 33.33\,(\text{mm})$$

$$d_2 = d_{w2} = \frac{mz_2}{\cos\beta} \approx 166.67\,(\text{mm})$$

$$d_{a1} = d_1 + 2(1+x_1)m \approx 37.23\,(\text{mm})$$

$$d_{a2} = d_2 + 2(1+x_2)m \approx 168.77\,(\text{mm})$$

$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 30.48\,(\text{mm})$$

$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 162.02\,(\text{mm})$$

$$d_{b1} = d_1 \cos\alpha \approx 31.32\,(\text{mm})$$

$$d_{b2} = d_2 \cos\alpha \approx 156.62\,(\text{mm})$$

$$\alpha_t = \alpha_{tw} = \arctan\frac{\tan\alpha}{\cos\beta} \approx 20.65°$$

$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 5\,(\text{m/s})$$

3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1}K_H \frac{u_{hg}+1}{b_w u_{hg} d_{w1}^2}} \leq [\sigma_H]$$

Find z_M $z_M = 274$, according to table (6.5)

Find z_H $\beta_b = \arctan(\cos\alpha_t \tan\beta) \approx 17.94° \Rightarrow z_H = \sqrt{2\frac{\cos\beta_b}{\sin(2\alpha_t)}} \approx 1.7$

Find z_ϵ Obtaining z_ϵ through calculations:

$$\epsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_t}{2\pi m \frac{\cos\alpha_t}{\cos\beta}} \approx 1.34$$

$$\epsilon_\beta = b_w \frac{\sin\beta}{m\pi} \approx 3.47 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.86$$

Find K_H We find K_H using equation $K_H = K_{H\beta}K_{H\alpha}K_{Hv}$

From table (6.13), $v \leq 6\,(\mathrm{m/s}) \Rightarrow \mathrm{AG} = 8$

From table (P2.3), using interpolation, we approximate:

$K_{Hv} \approx 1.05$, $K_{Fv} \approx 1.14$

From table (6.14), using interpolation, we approximate:

$K_{H\alpha} \approx 1.09$, $K_{F\alpha} \approx 1.27$

$\Rightarrow K_H \approx 1.27$

Find σ_H After calculating z_M, z_H, z_ϵ, K_H, we get the following result:

$$\sigma_H \approx 663.86\,\mathrm{MPa} \leq [\sigma_H] \approx 696.62\,\mathrm{MPa}$$

3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\begin{aligned} \sigma_{F1} &= 2\frac{T_{sh1}K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} &\leq [\sigma_{F1}] \\ \sigma_{F2} &= \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} &\leq [\sigma_{F2}] \end{aligned}$$

Find Y_ϵ Knowing that $\epsilon_\alpha \approx 1.64$, we can calculate $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.75$

Find Y_β $Y_\beta = 1 - \dfrac{\beta}{140} \approx 0.86$

Find Y_F Using formula $z_v = z\cos^{-3}(\beta)$ and table (6.18):
$z_{v1} = z_1 \cos^{-3}(\beta) \approx 24.88 \Rightarrow Y_{F1} \approx 3.6$
$z_{v2} = z_2 \cos^{-3}(\beta) \approx 124.42 \Rightarrow Y_{F2} \approx 3.64$

Find K_F Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:
$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.82$

Find σ_F Since $m_n = m\cos\beta \approx 1.42$, substituting all the values, we find out that:

$$\sigma_{F1} \approx 179.07\,(\text{MPa}) \leq [\sigma_{F1}] \approx 264.85\,(\text{MPa})$$

$$\sigma_{F2} \approx 181.06\,(\text{MPa}) \leq [\sigma_{F2}] \approx 332.48\,(\text{MPa})$$

3.4.6 Force on shafts

$$F_t = \frac{2T_{sh1}}{d_{w1}} \approx 3003.15\,(\text{N})$$
$$F_r = F_t \tan\alpha_{tw} \approx 1131.8\,(\text{N})$$
$$F_a = F_t \tan\beta \approx 1039.35\,(\text{N})$$

In summary, we have the following table:

	pinion	driving gear
H (HB)	250	240
$[\sigma_H]$ (MPa)	621.61	771.63
$[\sigma_F]$ (MPa)	264.85	332.48
$[\sigma_H]_{max}$ (MPa)	696.62	
$[\sigma_F]_{max}$ (MPa)	440	
σ_H (MPa)	621.61	771.63
σ_F (MPa)	179.07	181.06
σ_H (MPa)	663.86	
α_{tw} (°)	20.65	
β (°)	19.09	
a_w (mm)	100	
b_w (mm)	50	
m (mm)	1.5	
z	21	105
d_w (mm)	33.33	166.67
d_a (mm)	37.23	168.77
d_f (mm)	30.48	162.02
d_b (mm)	31.32	156.62
v (m/s)	5	
u_{hg}	5	

Table 3.1: Gearbox specifications

Chapter 4

Shaft Design

4.1 Nomenclature

$[s]$	permissible safety factor
$[\tau]$	permissible torsion, MPa
a_w	shaft distance, mm
b_O	rolling bearing width, mm
cb	role of gear on the shaft (active or passive)
cq	rotational direction of the shaft
d	base shaft diameter, mm
d_w	gear diameter, mm
F_a	axial force, N
F_r	radial force, N
F_t	tangential force, N
F_x	applied force, N
h_n	distance between bearing lid and bolt, mm
hr	tooth direction
α_{tw}	traverse meshing angle, $^\circ$
β	helix angle, $^\circ$
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
$_1$	subscript for shaft 1
$_2$	subscript for shaft 2

K_σ combined influence factor

k_1 distance between elements, mm

k_2 distance between bearing surface and inner walls of the gearbox, mm

k_3 distance between element surface and bearing lid, mm

l length (general), mm

l_m hub length (general), mm

M moment, $\mathrm{N \cdot mm}$

M_e equivalent moment, $\mathrm{N \cdot mm}$

l_m hub diameter, mm

q standardized coefficient of shaft diameter

R reaction force, N

r position of applied force on the shaft, mm

S length defined by table (6.1), mm

s calculated safety factor

s_σ safety factor in tensile stress

s_τ safety factor in shear stress

T torque on shaft

$_{sh1}$ subscript for shaft 1

$_{sh2}$ subscript for shaft 2

$_x$ subscript for x-axis

$_y$ subscript for y-axis

$_z$ subscript for z-axis

4.2 Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows: $S \leq 100\,(\text{mm})$, HB260, $\sigma_b = 850\,(\text{MPa})$, $\sigma_{ch} = 650\,(\text{MPa})$.

4.3 Transmission Design

4.3.1 Load on shafts

Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.
On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:
$r_{12} = -d_{w12}/2 \approx -16.67\,(\text{mm})$, $hr_{12} = +1$, $cb_{12} = +1$, $cq_1 = +1$
$r_{21} = +d_{w21}/2 \approx +83.34\,(\text{mm})$, $hr_{21} = -1$, $cb_{21} = -1$, $cq_2 = -1$

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: $\alpha_{tw} \approx 20.65°$, $\beta = 19.09°$, $d_{w12} \approx 33.33\,(\text{mm})$

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 3003.15\,(\text{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha_{tw}}{\cos\beta} \approx 1197.69\,(\text{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta \approx 1039.35\,(\text{N}) \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{r_{12}}{|r_{12}|} cq_1 cb_{12} F_{t12} \approx -3003.15\,(\text{N}) \\ F_{y12} = -\dfrac{r_{12}}{|r_{12}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t12} \approx 1197.67\,(\text{N}) \\ F_{z12} = cq_1 cb_{12} hr_{12} F_{t12} \tan\beta \approx 1039.35\,(\text{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{r_{21}}{|r_{21}|} cq_2 cb_{21} F_{t21} \approx 3003.15\,(\text{N}) \\ F_{y21} = -\dfrac{r_{21}}{|r_{21}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t21} \approx -1197.67\,(\text{N}) \\ F_{z21} = cq_2 cb_{21} hr_{21} F_{t21} \tan\beta \approx -1039.35\,(\text{N}) \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2678.96\,(\text{N})$ (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22}\cos 210^\circ \approx -1339.48\,(\text{N}) \\ F_{y22} = F_{r22}\sin 210^\circ \approx -2320.05\,(\text{N}) \end{cases}$$

4.3.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques T_{sh1} and T_{sh2}, respectively, $[\tau_1] = 15\,(\text{MPa})$ and $[\tau_2] = 30\,(\text{MPa})$. Using equation (10.9), we can approximate d_1 and d_2. Then, from table (10.2), d_1 and d_2 are chosen accordingly along with b_{O1} and b_{O2}:

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 25.55\,(\text{mm}) \Rightarrow d_1 = 30\,(\text{mm}),\ b_{O1} = 19\,(\text{mm})$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 34.1\,(\text{mm}) \Rightarrow d_2 = 35\,(\text{mm}),\ b_{O2} = 21\,(\text{mm})$$

4.3.3 Identify the distance between bearings and applied forces

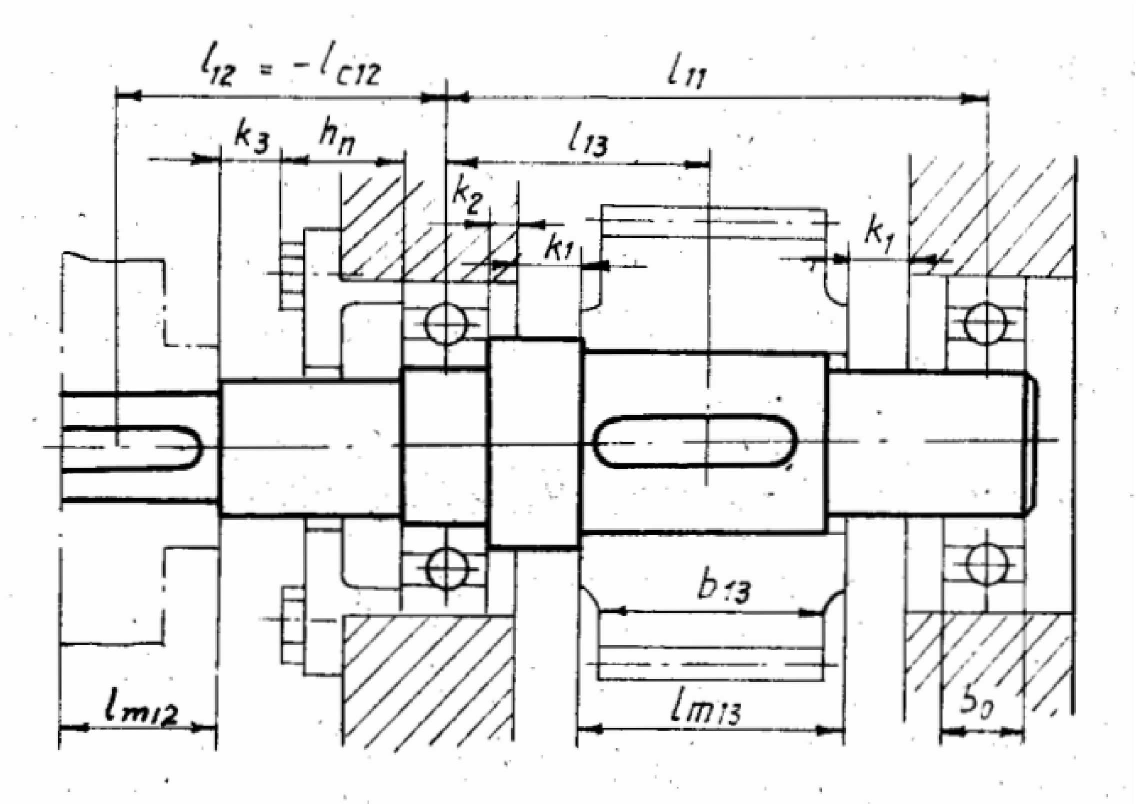

Figure 4.1: Shaft design and its dimensions

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 = 45\,(\text{mm})$, $l_{m23} = l_{m22} = 1.5d_2 = 52.5\,(\text{mm})$, where l_{m22} is the chain hub.

From table (10.3), we choose $k_1 = 10\,(\text{mm})$, $k_2 = 8\,(\text{mm})$, $k_3 = 15\,(\text{mm})$, $h_n = 18\,(\text{mm})$. This parameters apply for both shafts in the system.

Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:

On shaft 1:

$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + k_3 + h_n\right] = -65\,(\text{mm})$

$l_{13} = 0.5(l_{m13} + b_{O1}) + k_1 + k_2 = 50\,(\text{mm})$

$l_{11} = 2l_{13} = 100\,(\text{mm})$

On shaft 2:

$l_{22} = -l_{c22} = -[0.5(l_{m22} + b_{O2}) + k_3 + h_n] = -69.75\,(\text{mm})$

$l_{23} = 0.5(l_{m23} + b_{O2}) + k_1 + k_2 = 54.75\,(\text{mm})$

$l_{21} = 2l_{23} = 109.5\,(\text{mm})$

4.3.4 Determine shaft diameters and lengths

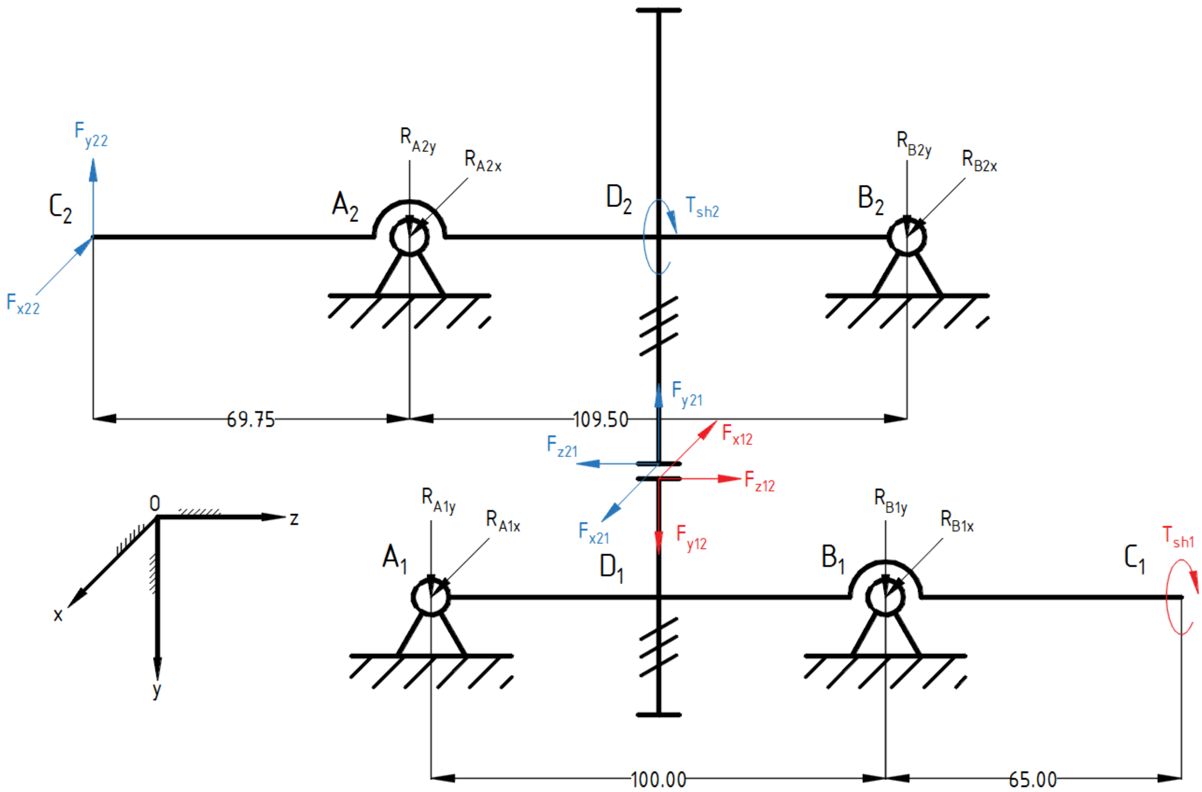

Figure 4.2: Force analysis of 2 shafts

Find reaction forces From the diagram, we solve for the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F_i} &= 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} &= 0 \end{cases}$$

we obtain the results:

$$\begin{cases} R_{A1x} \approx 1501.58\,(\text{N}) \\ R_{A1y} \approx -945.25\,(\text{N}) \\ R_{B1x} \approx 1501.58\,(\text{N}) \\ R_{B1y} \approx -252.42\,(\text{N}) \end{cases} \qquad \begin{cases} R_{A2x} \approx 691.13\,(\text{N}) \\ R_{A2y} \approx 2814.73\,(\text{N}) \\ R_{B2x} \approx -2354.81\,(\text{N}) \\ R_{B2y} \approx 702.99\,(\text{N}) \end{cases}$$

Draw bending moment - torque diagrams Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

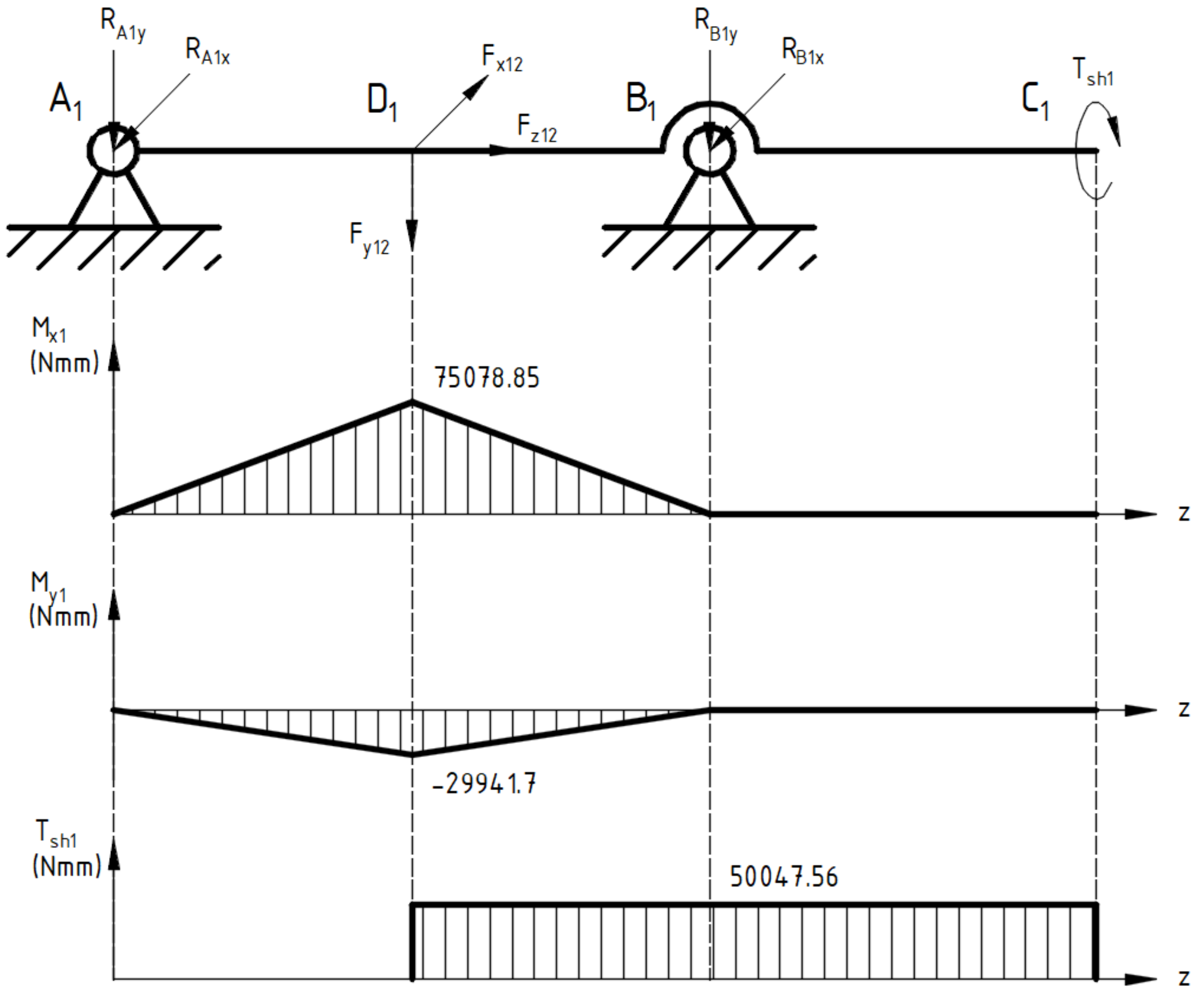

Figure 4.3: Bending moment-torque diagram of shaft 1

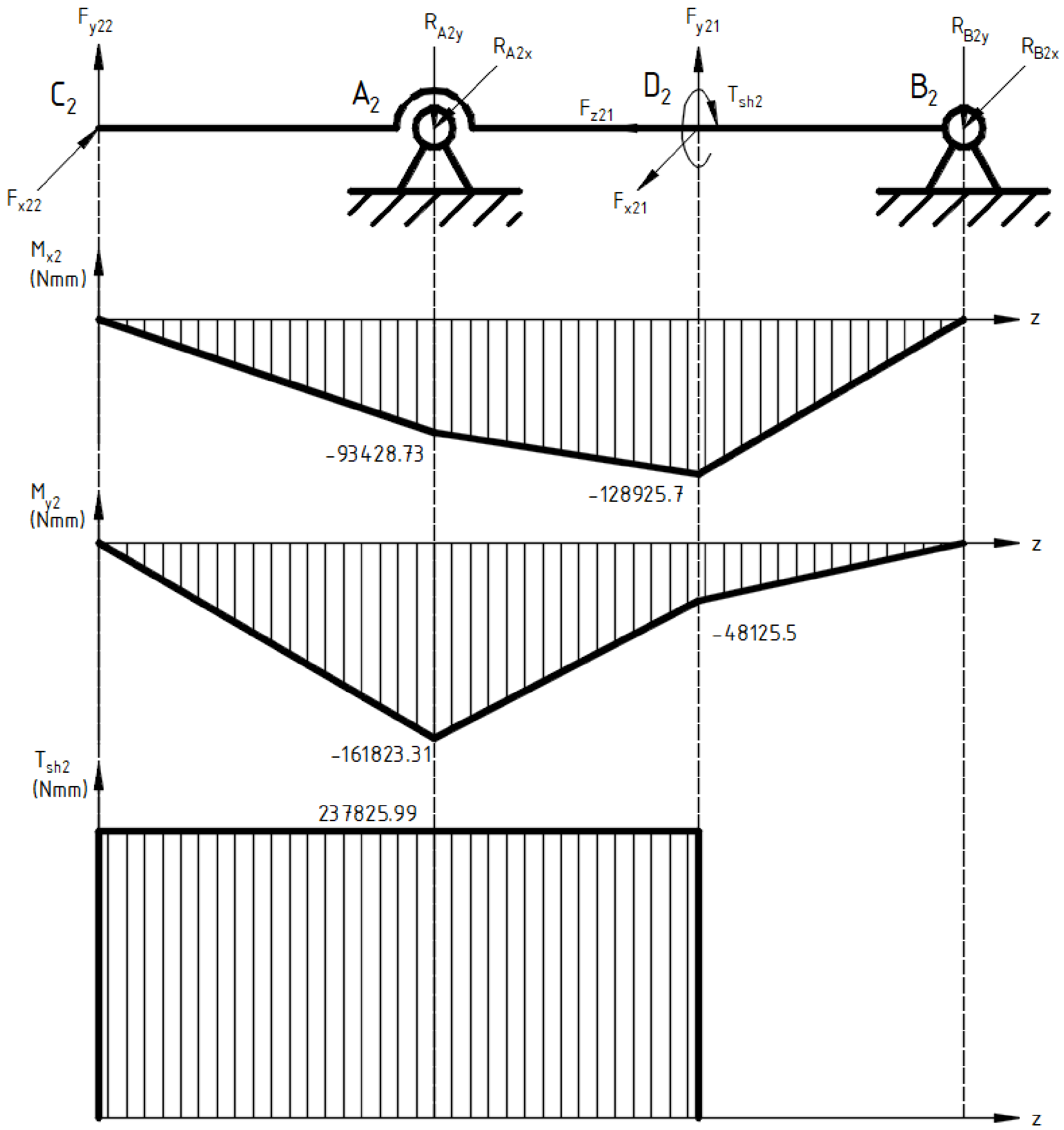

Figure 4.4: Bending moment-torque diagram of shaft 2

Find equivalent moments Knowing T_{sh1} and T_{sh2}, we calculate equivalent moment M_e at the 8 points specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

$$\begin{cases} M_{eA1} \approx 43342.4\,(\mathrm{N \cdot mm}) \\ M_{eD1} \approx 91716.45\,(\mathrm{N \cdot mm}) \\ M_{eB1} \approx 43342.46\,(\mathrm{N \cdot mm}) \\ M_{eC1} \approx 43342.46\,(\mathrm{N \cdot mm}) \end{cases} \qquad \begin{cases} M_{eC2} \approx 205963.35\,(\mathrm{N \cdot mm}) \\ M_{eA2} \approx 278094.61\,(\mathrm{N \cdot mm}) \\ M_{eD2} \approx 247707.09\,(\mathrm{N \cdot mm}) \\ M_{eB2} \approx 205963.35\,(\mathrm{N \cdot mm}) \end{cases}$$

Find permissible stress $[\sigma_1]$ and $[\sigma_2]$ are determined by table (10.5). Since we use quenched 45X steel, $[\sigma_1] = 67$ (MPa) and $[\sigma_2] = 64$ (MPa) ($[\sigma_2]$ is achieved using interpolation).

Find standardized diameters at specific locations on the shaft Having M_e and $[\sigma]$, the next step is to estimate specific diameter at the key points mentioned above using this formula, which only applies for rigid shafts:

$$d = \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\begin{cases} d_{A1} \approx 18.63\,(\mathrm{mm}) \\ d_{D1} \approx 23.92\,(\mathrm{mm}) \\ d_{B1} \approx 18.63\,(\mathrm{mm}) \\ d_{C1} \approx 18.63\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} \approx 31.81\,(\mathrm{mm}) \\ d_{A2} \approx 35.16\,(\mathrm{mm}) \\ d_{D2} \approx 33.83\,(\mathrm{mm}) \\ d_{B2} \approx 31.81\,(\mathrm{mm}) \end{cases}$$

After rough calculations, we will choose the diameters based on standards (one applies for bearings while the other is used for the remaining machine elements), which is given on p.195:

$$\begin{cases} d_{A1} = 20\,(\mathrm{mm}) \\ d_{D1} = 24\,(\mathrm{mm}) \\ d_{B1} = 20\,(\mathrm{mm}) \\ d_{C1} = 19\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} = 32\,(\mathrm{mm}) \\ d_{A2} = 40\,(\mathrm{mm}) \\ d_{D2} = 34\,(\mathrm{mm}) \\ d_{B2} = 35\,(\mathrm{mm}) \end{cases}$$

4.4 Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

Find K_σ