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ME3011

Design Project Report

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I am also thankful for my friends on reviewing material to fix the details of this report.

Abstract

In machine design, every machine element must be calculated in a systematic matter. In this course, students are provided with essential skills to formulate almost every dimension manually, thus further improving their engineering skills before engaging the high-energy, fast-paced workforce.

When a machine element is being developed, it must satisfy some key engineering specifications such as being able to operate under designated lifespan, low cost and high efficiency. Other aspects are less important but also determined the overall design of the element include compactness, noise emission, appearance, etc.

To optimize the process of machine design, the general principles are considered as follows:

1. Identify the working principle and workload of the machine.
2. Formulate the overall working principle to satisfy the problem. Proposing feasible solutions and evaluating them to find the optimal design specifications.
3. Find force and moment diagram exerting on machine parts and characteristics of the workload.
4. Choose appropriate materials to make use of their properties and improve efficiency as well as reliability of individual elements.
5. Calculate dynamics, strength, safety factor, etc. to specify dimensions.
6. Design machine structure, parts to satisfy working condition and assembly.
7. Create presentation, instruction manual and maintenance.

In this report, I will design a fairly simple system to provide a concrete example of finishing all the tasks above.

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Nomenclature

C_a	number of shift daily, shifts	P	design power of the mixing tank, kW
K_{ng}	working days/year, days	T_1	working torque 1, $\text{N} \cdot \text{m}$
L	service life, years	T_2	working torque 2, $\text{N} \cdot \text{m}$
n	rotational velocity of the mixing tank, rpm	t_1	working time 1, s
		t_2	working time 2, s

I Problem

The problem is downloaded from E-learning website, designated number 8, see Figure 1.

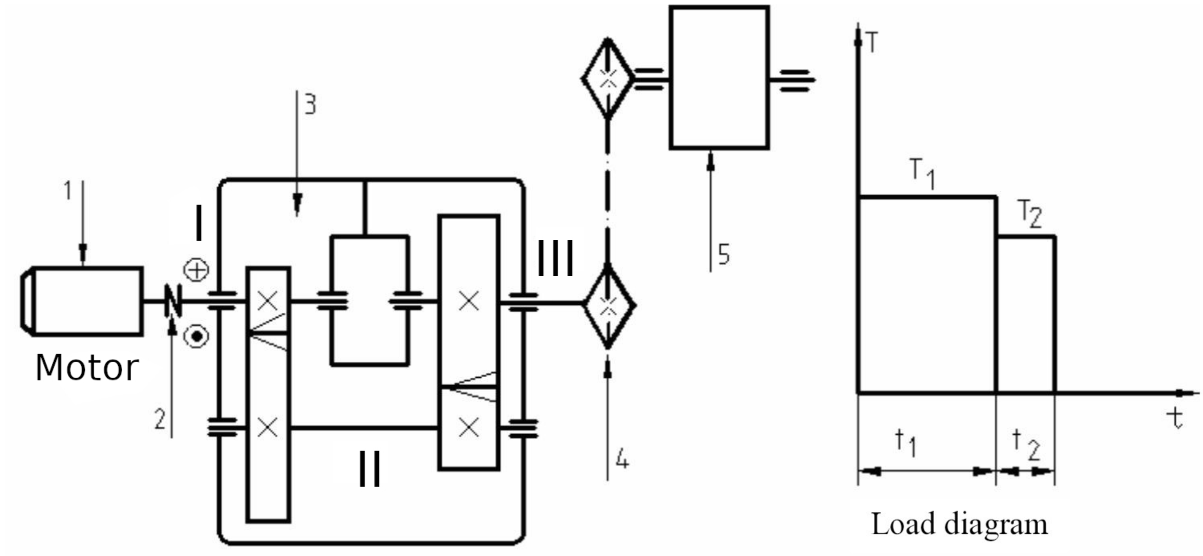


Figure 1: Working principle diagram and workload of the mixing machine: 1) electric motor, 2) elastic coupling, 3) two-stage coaxial helical speed reducer, 4) roller chain drive, 5) mixing tank (one-directional, light duty, operate 1 shift, 8 hours each)

II Mixing machine parameters

From the parameters given in the document, we have:

$P = 7$ (kW)	$t_1 = 15$ (s)
$n = 65$ (rpm)	$t_2 = 11$ (s)
$L = 8$ (years)	$T_1 = T$ ($\text{N} \cdot \text{m}$)
$K_{ng} = 260$ (days)	$T_2 = 0.7T$ ($\text{N} \cdot \text{m}$)
$Ca = 1$ (shifts)	

III Requirements

- 01 report.
- 01 assembly drawing.
- 01 detailed drawing.

IV Design problem

1. Decide the working power of the electric motor and transmission ratio of the system.
2. Calculate and design machine elements:
   (a) Calculate system drives (belt, chain or gear).
   (b) Calculate the elements in speed reducers (gears, lead screws).
   (c) Draw and calculate force diagram exerting on the transmission elements.
   (d) Calculate, design shafts and keys.
   (e) Choose bearings and couplings.
   (f) Choose machine bodies, fasteners and other elements.
3. Choose assembly tolerance.
4. Bibliography

Chapter 1

Choose Motor

I Motor selection for the mixing tank

1.1 Calculate system overall efficiency η_{sys}

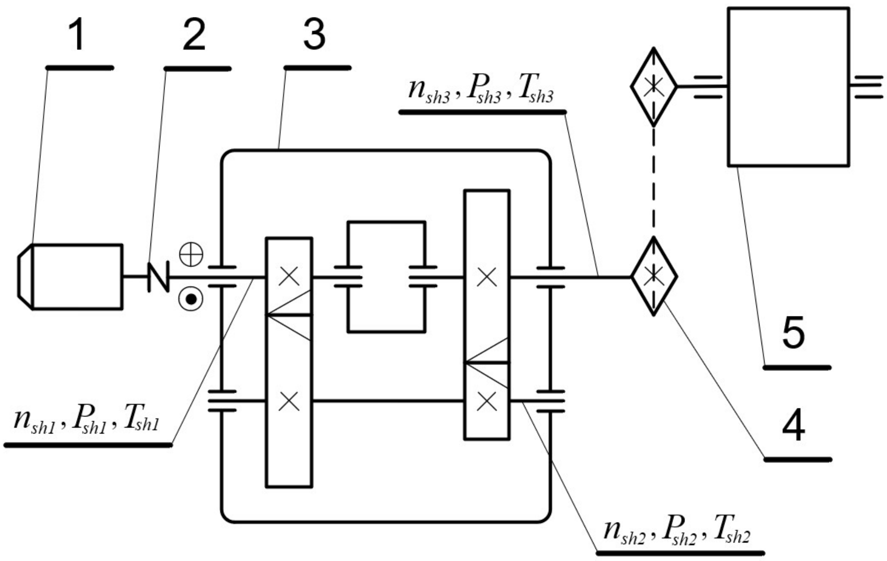


Figure 1.1: Working principle diagram with annotation for shafts

From Figure 1.1, the efficiency η_{sys} of the system is calculated using:

$$\eta_{sys} = \eta_c \eta_b^4 \eta_{hg}^2 \eta_{ch} = 0.99 \times 0.99^4 \times 0.98^2 \times 0.95 = 0.87$$

where

- $\eta_c = 0.99$ is the flexible coupling efficiency, Table 2.3 [6]. The coupling connects the motor and the speed reducer. In principle, it is designed to transmit torque smoothly while permitting some axial, radial and angular misalignment, typically ±3° [5]. Therefore, the power loss from motor shaft to shaft 1 should be included since it eventually transfers into sound and heat radiation.

- $\eta_b = 0.99$ is the bearings efficiency, Table 2.3 [6]. Housing is provided to 4 rolling bearings, 3 of which are in the speed reducer and the last one is used for the shaft of the mixing tank. During calculations, it is safer to assume the lowest value for better reliability. Therefore, the lowest efficiency is taken.

- $\eta_{hg} = 0.98$ is the helical gear efficiency, Table 2.3 [6]. In the speed reducer are 2 sealed pairs of helical gear drives. One pair connects shaft 1 and shaft 2, the other connects shaft 2 and 3. A common rule of thumb for spur, helical, and bevel gear meshes is to assume each mesh, including gears and supporting bearings, incurs a 2 percent power loss [1].
- $\eta_{ch} = 0.95$ is the chain drive efficiency, Table 2.3 [6]. The chain is protected via housing and provides connection from the speed reducer to the mixing tank. Similar to the approach from rolling bearing efficiency selection, the efficiency of the chain is chose at the lowest value.

1.2 Calculate required power P_{mo} for operation

The power P from design problem is the operating power of the mixing tank. In case of varying load each cycle, the equivalent power P_{mo} is calculated using Equation 2.13 [6]:

$$P_w = P\sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} = 7 \times \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15 + 11}} = 6.2\ \text{(kW)}$$

$$P_{mo} = \frac{P_w}{\eta_{sys}} = \frac{6.2}{0.87} = 7.14\ \text{(kW)}$$

where

- P_w is the operating power of the mixing tank given the workload, kW.
- P, T_1, T_2, t_1, t_2 are given in the design problem; η_{sys} is given in the previous section.

1.3 Choose motor

There are 2 common speed values for an induction motor in Vietnam which uses line frequency of 50 Hz:

1. Motor with 2 poles at nominal speed 3000 rpm, which is portable and cheap. However, high system ratio could result in additional expenditure in other machine elements.
2. Motor with 4 poles at nominal speed 1500 rpm, which is large and more costly. However, low speed ratio should be adequate since power transmission is the goal of the system, which could in principle, reduce the size of other elements. The choice is this motor type will affect u_h and u_{ch} in the next part.

Calculate working speed n_{mo} The selection of system speed ratio should be close to $1500/n = 1500/65 = 23.08$. The working speed n_{mo} is calculated as:

$$n_{mo} = u_{sys} n = 22.4 \times 65 = 1456\ \text{(rpm)}$$

where

- u_{sys} is the speed ratio of the system. The formula for u_{sys} is:

  $$u_{sys} = u_h u_{ch} = 8 \times 2.8 = 22.4$$

  where

  - $u_h = 8$ is the speed ratio of the speed reducer, which is a 2-level transmission, spur gear type, Table 2.4 [6]. The transmission ratio u_h of the speed reducer should be kept at minimum since in general, it is more costly (material, machining, maintenance, etc.) to manufacture than other mechanical drives such as belt drive and chain drive. The choice of this transmission ratio will be explained clearly in the next section.
  - $u_{ch} = 2.8$ is the speed ratio of the chain drive, roller type, Table 2.4 [6]. This mechanical drive provides the remaining factor to get close to the desired ratio.

- n is given in the design problem.

Choose motor The power of the motor should be around $P_{mo} = 7.14$ kW. Thus, from Table P1.3 [6], we choose motor 4A132S4Y3 operating at 7.5 kW maximum and 1455 rpm. As a result, the new value for n_{mo} is $n_{mo} = 1455$ rpm, which is not much different from 1456 rpm. Recalculating u_{sys} with the new n_{mo}:

$$u_{sys} = n_{mo}/n = 1455/65 = 22.38$$

Retaining the speed ratio of the speed reducer (i.e. let $u_h = const = 8$), the new speed ratio of the chain drive is then:

$$u_{ch} = u_{sys}/u_h = 22.46/8 = 2.83$$

II Power, rotational speed and torque of the system

Let P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} are the transmitted parameters onto shaft 2 and P_{sh3}, n_{sh3} and T_{sh3} are used for shaft 3. The numbering is specified in Figure 1.1. Unless otherwise stated, these notations will be used throughout the next chapters.

2.1 Calculate speed of the chain drive and the shafts

The entire system is described followed by calculation as follows:
Chain drive power P_{ch} is affected by the bearings on the shaft of the mixing tank:

$$P_{ch} = \frac{P_w}{\eta_b} = \frac{6.2}{0.99} = 6.26\ (\text{kW})$$

Shaft 3 power P_{sh3} is affected by the chain drive:

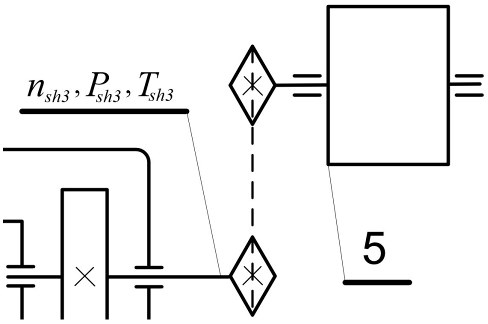
$$P_{sh3} = \frac{P_{ch}}{\eta_{ch}} = \frac{6.26}{0.95} = 6.52\ (\text{kW})$$

Shaft 2 power P_{sh2} is affected by the bearings and gear drives on shaft 3:

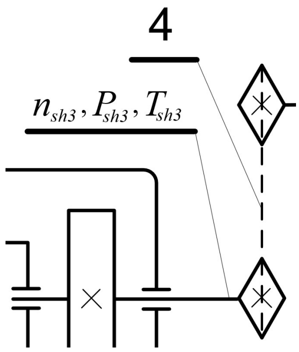
$$P_{sh2} = \frac{P_{sh3}}{\eta_b \eta_{hg}} = \frac{6.52}{0.99 \times 0.98} = 6.79\ (\text{kW})$$

Shaft 1 power P_{sh1} is affected by the bearings and gear drives on shaft 2:

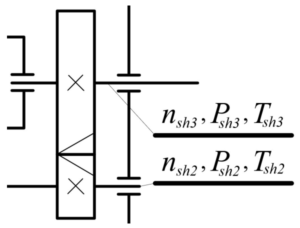
$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} = \frac{6.79}{0.99 \times 0.98} = 7.07\ (\text{kW})$$



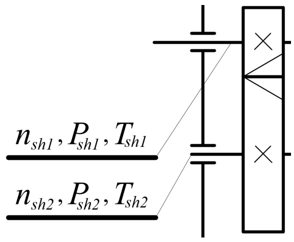
(**a**) Connection between the mixing tank shaft and chain drive: 1 pair of bearings



(**b**) Connection between shaft 3 and chain drive: 1 chain drive



(**c**) Connection between shaft 3 and shaft 2: 1 pair of bearings, 1 spur gear



(**d**) Connection between shaft 2 and shaft 1: 1 pair of bearings, 1 spur gear

Figure 1.2: Machine elements distribution between shafts and mechanical drives of the system

2.2 Calculate speed of the shafts

The design goal of the speed reducer is to lubricate both driven gears equally although there is a size disadvantage. Therefore, the speed ratio of each pair of gears is calculated using Equation 3.12 [6]:

$$u_1 = u_2 = \sqrt{u_h} = \sqrt{8} = 2.83$$

where

- u_1 is the speed ratio of the gear drive attaches to shaft 1 and shaft 2.
- u_2 is the speed ratio of the gear drive attaches to shaft 2 and shaft 3.

The value u_h is chosen deliberately as non-repeating decimal in order to create a *hunting tooth gear set*. A *hunting tooth ratio* is the ratio where the greatest common divisor of the number of teeth in the pinion and driven gear is 1. Under the same material and surface finishing condition, a greater improvement in the surface roughness was observed in the gears used in the hunting gear ratio compared to non-hunting counterpart even though fatigue damage might be presented. This is especially true for non-finishing soft gears with Brinell hardness number below HB300 [3].

Then,
the speed n_{sh1} from motor to shaft 1:

$$n_{sh1} = n_{mo} = 1455 \text{ (rpm)}$$

the speed n_{sh2} from shaft 1 to shaft 2:

$$n_{sh2} = n_{sh1}/u_1 = 1455/2.83 = 514.42 \text{ (rpm)}$$

the speed n_{sh3} from shaft 2 to shaft 3:

$$n_{sh3} = n_{sh2}/u_2 = 514.42/2.83 = 181.88 \text{ (rpm)}$$

2.3 Calculate torque of the motor and the shafts

Subsequently, the torque is calculated as follows:

$$T_{mo} = 9.55 \times 10^6 \times P_{mo}/n_{mo} = 9.55 \times 10^6 \times 7.14/1455 = 46892.66 \text{ (N} \cdot \text{mm)}$$
$$T_{sh1} = 9.55 \times 10^6 \times P_{sh1}/n_{sh1} = 9.55 \times 10^6 \times 7.07/1455 = 46423.73 \text{ (N} \cdot \text{mm)}$$
$$T_{sh2} = 9.55 \times 10^6 \times P_{sh2}/n_{sh2} = 9.55 \times 10^6 \times 6.79/514.42 = 126093.30 \text{ (N} \cdot \text{mm)}$$
$$T_{sh3} = 9.55 \times 10^6 \times P_{sh3}/n_{sh3} = 9.55 \times 10^6 \times 6.52/181.88 = 342486.86 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

Table 1.1: Output specification for the shafts and motor

	Motor	Shaft 1	Shaft 2	Shaft 3
n (rpm)	1455	1455	514.42	181.88
P (kW)	7.14	7.07	6.79	6.52
T (N · mm)	46892.66	46423.73	126093.30	342486.86
u	-	1	2.83	2.83

Chapter 2

Shaft Design

I Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows: $S \leq 100$ (mm), HB260, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 650$ (MPa).

II Transmission Design

2.1 Load on shafts

Applied forces from Gears

In this chapter, a subscript convention should be used for convenience and clarity. The rule of the convention is as follows:

- use 2 numeric subscripts in a variable.
- the first numeric subscript is the ordinal number of shafts.
- the second numeric subscript is the ordinal number of mechanical drives (e.g. motor, gear, chain, belt).
- for a force vector, x, y, z are its algebraic values on x, y, z-axis, respectively.

Therefore,

- On shaft 1: the motor is labeled 1, the pinion is labeled 2.
- On shaft 2: the driving gear is labeled 1, the pinion is labeled 2.
- On shaft 3: the driving gear is labeled 1, the chain is labeled 2.

$$\bar{r}_{12} = -d_{w12}/2 = -20.83 \text{ (mm)}, \mathrm{hr}_{12} = +1, \mathrm{cb}_{12} = +1, \mathrm{cq}_1 = +1$$
$$\bar{r}_{21} = +d_{w21}/2 = +104.17 \text{ (mm)}, \mathrm{hr}_{21} = -1, \mathrm{cb}_{21} = -1, \mathrm{cq}_2 = -1$$
$$\bar{r}_{21} = +d_{w21}/2 = +104.17 \text{ (mm)}, \mathrm{hr}_{21} = -1, \mathrm{cb}_{21} = -1, \mathrm{cq}_2 = -1$$

where

- $\bar{r}$ is the position of the force exerted on the shaft, mm. The value is positive if
- hr is the tooth direction.

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: , $\beta_w = 13.59°$, $d_{w12} = 41.67$ (mm)

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} = 2402.28\,(\text{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha}{\cos\beta_w} = 925.46\,(\text{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta_w = 580.75\,(\text{N}) \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\mathrm{cq}_1\mathrm{cb}_{12}F_{t12} = -2402.28\,(\text{N}) \\ F_{y12} = -\dfrac{\bar{r}_{12}}{|\bar{r}_{12}|}\dfrac{\tan\alpha}{\cos\beta_w}F_{t12} = 925.46\,(\text{N}) \\ F_{z12} = \mathrm{cq}_1\mathrm{cb}_{12}\mathrm{hr}_{12}F_{t12}\tan\beta_w = 580.75\,(\text{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\mathrm{cq}_2\mathrm{cb}_{21}F_{t21} = 2402.28\,(\text{N}) \\ F_{y21} = -\dfrac{\bar{r}_{21}}{|\bar{r}_{21}|}\dfrac{\tan\alpha_{tw}}{\cos\beta_w}F_{t21} = -925.46\,(\text{N}) \\ F_{z21} = \mathrm{cq}_2\mathrm{cb}_{21}\mathrm{hr}_{21}F_{t21}\tan\beta_w = -580.75\,(\text{N}) \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r = 2678.96$ (N) (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22}\cos 210° = -2320.05\,(\text{N}) \\ F_{y22} = F_{r22}\sin 210° = -1339.48\,(\text{N}) \end{cases}$$

2.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques T_{sh1} and T_{sh2}, respectively, $[\tau_1] = 15$ (MPa) and $[\tau_2] = 30$ (MPa). Using equation (10.9), we can approximate the base shaft diameters d_1 and d_2:

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} = 25.55\,(\text{mm})$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} = 34.1\,(\text{mm})$$

Recall that our motor is 4A160M2Y3, inspecting table P1.7 we obtain the motor's output shaft diameter is 42 (mm). According to the recommendations on p.189, we limit the chosen range of $d_1 \geq (0.8 \div 1.2) \times 42$ (mm). For d_2, the chosen range must be around $(0.3 \div 0.35) \times a_w$ (mm). Thus, $d_1 = 35$ (mm), $d_2 = 40$ (mm). Consulting table (10.2) gives $b_{O1} = 21$ (mm) and $b_{O2} = 23$ (mm)

2.3 Identify the distance between bearings and applied forces

In this section, we will find all the parameters in Figure 2.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

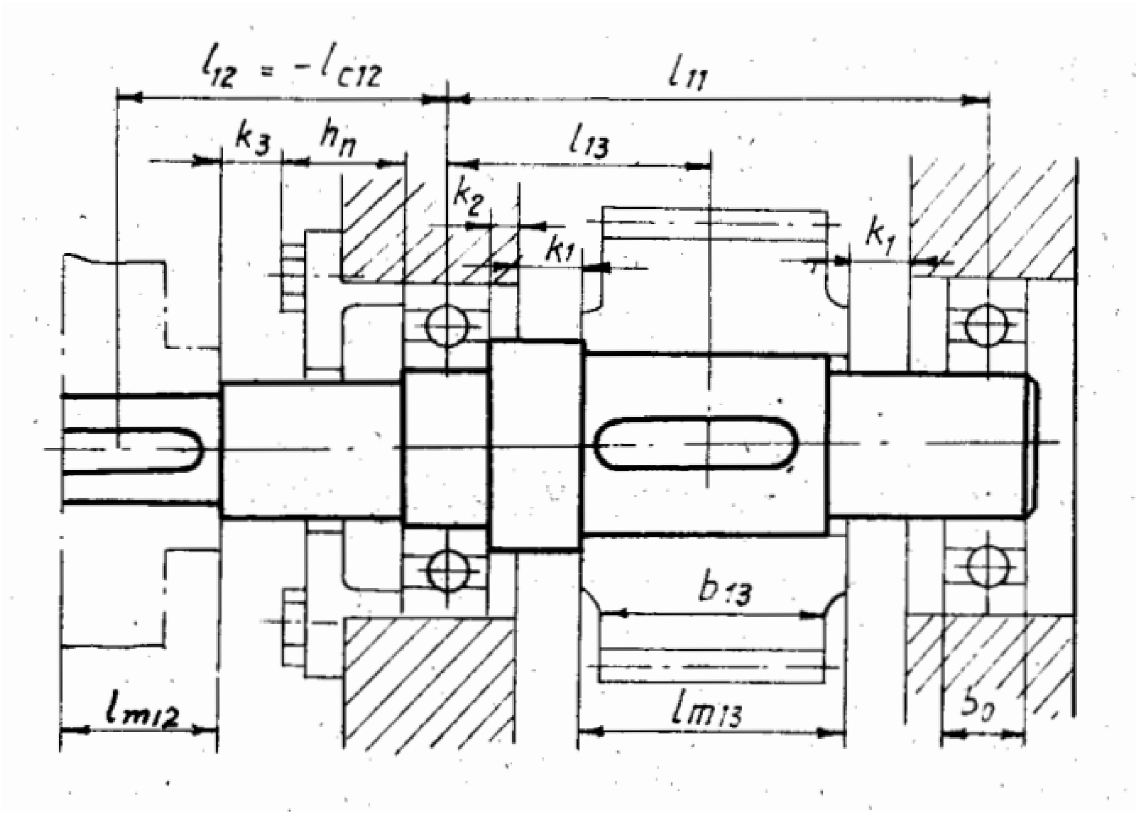


Figure 2.1: Shaft design and its dimensions

Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 = 45$ (mm), $l_{m23} = l_{m22} = 1.5d_2 = 52.5$ (mm), where l_{m22} is the chain hub.
From table (10.3), we choose $\tilde{k}_1 = 10$ (mm), $\tilde{k}_2 = 8$ (mm), $\tilde{k}_3 = 15$ (mm), $h_n = 18$ (mm). This parameters apply for both shafts in the system.
Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:
On shaft 1:
$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + \tilde{k}_3 + h_n\right] = -69.75$ (mm)
$l_{13} = 0.5(l_{m13} + b_{O1}) + \tilde{k}_1 + \tilde{k}_2 = 54.75$ (mm)
$l_{11} = 2l_{13} = 109.5$ (mm)
On shaft 2:
$l_{22} = -l_{c22} = -\left[0.5(l_{m22} + b_{O2}) + \tilde{k}_3 + h_n\right] = -74.5$ (mm)
$l_{23} = 0.5(l_{m23} + b_{O2}) + \tilde{k}_1 + \tilde{k}_2 = 59.5$ (mm)
$l_{21} = 2l_{23} = 119$ (mm)

2.4 Determine shaft diameters and lengths

Find reaction forces From the diagram, we solve for the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F_i} = 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} = 0 \end{cases}$$

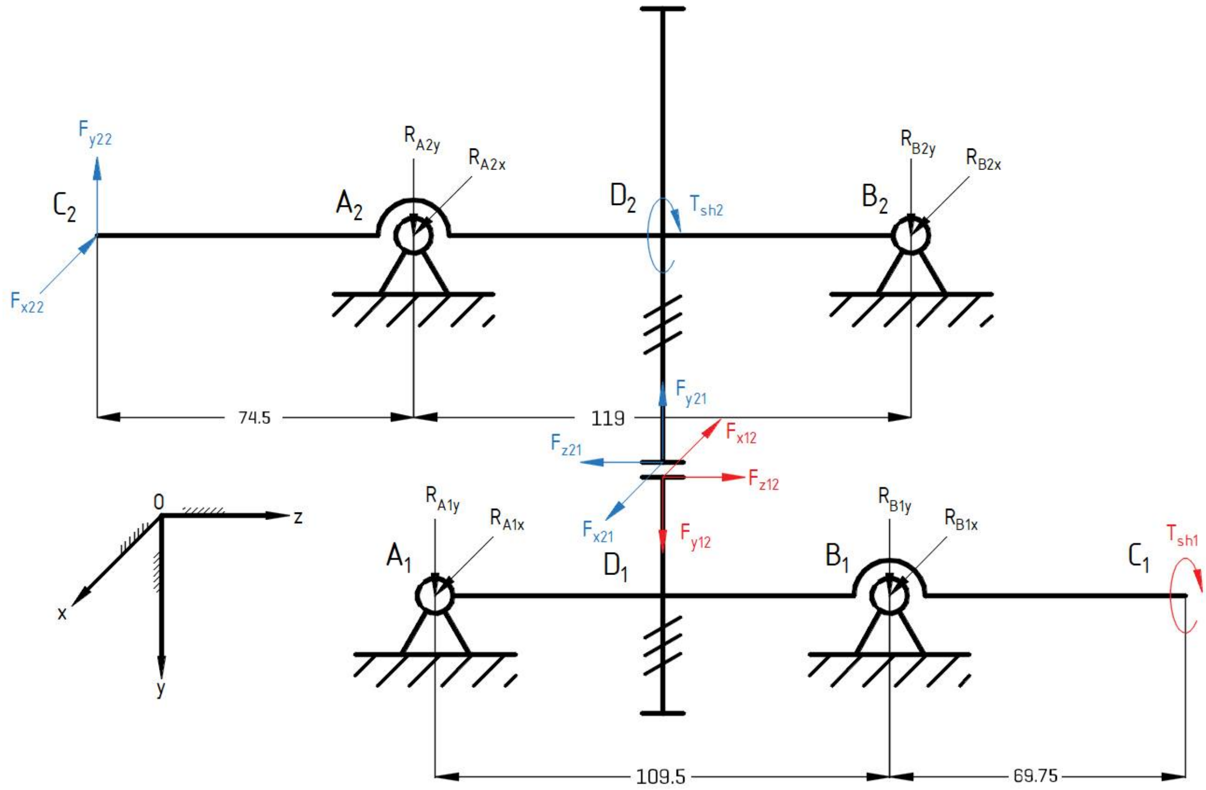


Figure 2.2: Force analysis of 2 shafts

we obtain the results:

$$\begin{cases} R_{A1x} = 1201.14\ (\text{N}) \\ R_{A1y} = -352.24\ (\text{N}) \\ R_{B1x} = 1201.14\ (\text{N}) \\ R_{B1y} = -573.22\ (\text{N}) \end{cases} \qquad \begin{cases} R_{A2x} = 943.15\ (\text{N}) \\ R_{A2y} = 3668.4\ (\text{N}) \\ R_{B2x} = -2005.96\ (\text{N}) \\ R_{B2y} = -422.89\ (\text{N}) \end{cases}$$

The total bending moments at 8 critical cross sections are also calculated (we use the formula (10.15) to derive $M = \sqrt{M_x^2 + M_y^2}$ at each section):

$$\begin{cases} M_{A1} = 0\ (\text{N}\cdot\text{mm}) \\ M_{D1}^- = 68531.85\ (\text{N}\cdot\text{mm}) \\ M_{D1}^+ = 72867.4\ (\text{N}\cdot\text{mm}) \\ M_{B1} = 0\ (\text{N}\cdot\text{mm}) \\ M_{C1} = 0\ (\text{N}\cdot\text{mm}) \end{cases} \qquad \begin{cases} M_{C2} = 0\ (\text{N}\cdot\text{mm}) \\ M_{A2} = 191545.76\ (\text{N}\cdot\text{mm}) \\ M_{D2}^- = 146910\ (\text{N}\cdot\text{mm}) \\ M_{D2}^+ = 121977.78\ (\text{N}\cdot\text{mm}) \\ M_{B2} = 0\ (\text{N}\cdot\text{mm}) \end{cases}$$

Draw bending moment - torque diagrams Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

Find equivalent moments Knowing T_{sh1} and T_{sh2}, we calculate equivalent moment M_e at the 8 cross sections specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

$$\begin{cases} M_{eA1} = 0\,(\mathrm{N\cdot mm}) \\ M^-_{eD1} = 81087.5\,(\mathrm{N\cdot mm}) \\ M^+_{eD1} = 84783.4\,(\mathrm{N\cdot mm}) \\ M_{eB1} = 43342.46\,(\mathrm{N\cdot mm}) \\ M_{eC1} = 43342.46\,(\mathrm{N\cdot mm}) \end{cases} \qquad \begin{cases} M_{eC2} = 205963.35\,(\mathrm{N\cdot mm}) \\ M_{eA2} = 281266.2\,(\mathrm{N\cdot mm}) \\ M^-_{eD2} = 252989.03\,(\mathrm{N\cdot mm}) \\ M^+_{eD2} = 239373.1\,(\mathrm{N\cdot mm}) \\ M_{eB2} = 0\,(\mathrm{N\cdot mm}) \end{cases}$$

Find permissible stress $[\sigma_1]$ and $[\sigma_2]$ are determined by table (10.5). Since we use quenched 45X steel, $[\sigma_1] = 67$ (MPa) and $[\sigma_2] = 64$ (MPa) ($[\sigma_2]$ is achieved using interpolation).

Find standardized diameters at specific locations on the shaft Having M_e and $[\sigma]$, the next step is to estimate specific diameter at the key points mentioned above using equation (10.17) on p.194, which only applies for rigid shafts:

$$d \geq \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\begin{cases} d_{A1} = 0\,(\mathrm{mm}) \\ d_{D1} = 23.66\,(\mathrm{mm}) \\ d_{B1} = 18.92\,(\mathrm{mm}) \\ d_{C1} = 18.92\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} = 32.32\,(\mathrm{mm}) \\ d_{A2} = 35.86\,(\mathrm{mm}) \\ d_{D2} = 34.61\,(\mathrm{mm}) \\ d_{B2} = 0\,(\mathrm{mm}) \end{cases}$$

Through rough calculations, we will choose the diameters according to standards given on p.195 (one applies for bearings while the other is used for the remaining machine elements):

$$\begin{cases} d_{A1} = 35\,(\mathrm{mm}) \\ d_{D1} = 24\,(\mathrm{mm}) \\ d_{B1} = 35\,(\mathrm{mm}) \\ d_{C1} = 19\,(\mathrm{mm}) \end{cases} \qquad \begin{cases} d_{C2} = 34\,(\mathrm{mm}) \\ d_{A2} = 40\,(\mathrm{mm}) \\ d_{D2} = 36\,(\mathrm{mm}) \\ d_{B2} = 40\,(\mathrm{mm}) \end{cases}$$

III Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

where $$s_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$$
$$s_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$$

Assuming the surfaces are smooth, properly ground and quenched by high frequency voltage, we obtain $K_x = 1$ from table (10.8) and $K_y = 1.4$ from table (10.9), where $[\sigma_b] = 850$ (MPa) is the property of quenched 45X steel.

Find σ_{-1}, τ_{-1} Using formulas on p.196:
$\sigma_{-1} = 0.35[\sigma_b] + 120 = 417.5$ (MPa)
$\tau_{-1} = 0.58\sigma_{-1} = 242.15$ (MPa)

Find $\sigma_a, \tau_a, \sigma_m, \tau_m$ For this part, we divide into 3 key points:

1. For rotating shaft, $\sigma_m = 0, \sigma_a = \frac{\sqrt{M_x^2 + M_y^2}}{W}$ (equation (10.22)), where M_x and M_y are at the cross section of interest.
2. By design, the shafts only rotate in one direction, thus $\tau_m = \tau_a = \frac{T_{sh}}{2W_O}$ (equation (10.23)).
3. We also assume the shafts have circular cross section, which makes $W = \frac{\pi d^3}{32}$ and $W_O = \frac{\pi d^3}{16}$ according to table (10.6), where d is the diameter of a cross section of the shaft.

The table below shows the results after calculation: Since $\sigma_b = 850\,(\text{MPa})$ for both shafts,

	d (mm)	W (mm^3)	W_O (mm^3)	σ_m (MPa)	σ_a (MPa)	τ_m (MPa)	τ_a (MPa)
A_1	20	785.4	1570.8	0	0	15.93	15.93
D_1	24	1357.17	2714.34	0	49.2	9.22	9.22
B_1	20	785.4	1570.8	0	0	15.93	15.93
C_1	19	673.38	1346.76	0	0	18.58	18.58
C_2	32	3216.99	6433.98	0	0	18.48	18.48
A_2	40	6283.19	12566.37	0	29.74	9.46	9.46
D_2	34	3858.66	7717.32	0	36.67	15.41	15.41
B_2	35	4209.24	8418.49	0	0	14.13	14.13
C_2	32	3216.99	6433.98	0	0	18.48	18.48
A_2	40	6283.19	12566.37	0	29.74	9.46	9.46
D_2	34	3858.66	7717.32	0	36.67	15.41	15.41
B_2	35	4209.24	8418.49	0	0	14.13	14.13

Table 2.1: Calculated variables for $\sigma_a, \tau_a, \sigma_m, \tau_m$

$\psi_\sigma = 0.1$ and $\psi_\tau = 0.05$

Find K_σ, K_τ We calculate K_σ using formula:

$$K_\sigma = \left(\frac{k_\sigma}{\varepsilon_\sigma} + K_x - 1\right) K_y^{-1}$$

and K_τ with:

$$K_\tau = \left(\frac{k_\tau}{\varepsilon_\tau} + K_x - 1\right) K_y^{-1}$$

Table (10.10), (10.11) and (10.13) are examined to find $\frac{k_\sigma}{\varepsilon_\sigma}$ ratio. Given $[\sigma_H] = 850\,(\text{MPa})$ base shaft diameters d_1 and d_2 are compared to the diameters at critical locations A, B, C, D. If the base shaft is smaller, table (10.10) and (10.11) are used. If it is larger, we will use table (10.13) instead; the concentration stress factor in this case is demonstrated in the figure:

Final calculation is provided in the table:

	d (mm)	r	k_σ	k_τ	ε_σ	ε_τ	$\frac{k_\sigma}{\varepsilon_\sigma}$	$\frac{k_\tau}{\varepsilon_\tau}$	K_x	K_y	K_σ	K_τ
A_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
D_1	24	0.48	3	1.95	0.81	0.85	3.7	2.29	1	1.4	2.65	1.64
B_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
C_1	19	0.38	3	1.95	0.84	0.89	3.57	2.19	1	1.4	2.55	1.57
C_2	32	0.64	3	1.95	0.76	0.80	3.95	2.44	1	1.4	2.82	1.74
A_2	40	-	-	-	-	-	3.34	2.46	1	1.4	2.39	1.76
D_2	34	0.68	3	1.95	0.74	0.80	4	2.44	1	1.4	2.86	1.75
B_2	35	-	-	-	-	-	3.3	2.44	1	1.4	2.36	1.74

Table 2.2: Calculated variables in K_σ and K_τ

Find s_σ, s_τ and s Combining the results altogether, we obtain the following table:

	s_σ	s_τ	s
A_1	$\gg s_\tau$	9.41	9.41
D_1	3.21	16	3.14
B_1	$\gg s_\tau$	9.41	9.41
C_1	$\gg s_\tau$	8.07	8.07
C_2	$\gg s_\tau$	7.32	7.32
A_2	5.88	14	5.43
D_2	3.99	8.77	3.63
B_2	$\gg s_\tau$	9.56	9.56

Table 2.3: Safety factor at critical cross sections

Since the smallest safety factor is at the cross section D_1, which has the value of $3.14 > [s] = 1.5 \div 2.5$, we can neglect rigidity analysis according to the conclusion on p.195.

IV Static Strength Analysis

Along with fatigue strength, static strength is also considered and every shaft must satisfy the following condition at critical cross sections (equation (10.27)):

$$\sigma_e = \sqrt{\left(\frac{M_{max}}{0.1d^3}\right)^2 + 3\left(\frac{T_{max}}{0.2d^3}\right)^2} \leq [\sigma]$$

where M_{max}, T_{max} are the largest bending moment and torque at the cross section, respectively. Let $[\sigma] = 0.8\sigma_{ch} = 520$ (MPa), the results are in the table below:

	A_1	D_1	B_1	C_1	C_2	A_2	D_2	B_2
σ_e (MPa)	54.18	57.59	54.18	63.19	62.86	43.45	63.58	48.04

Table 2.4: Calculated static strength at critical cross sections

which satisfy the given condition.

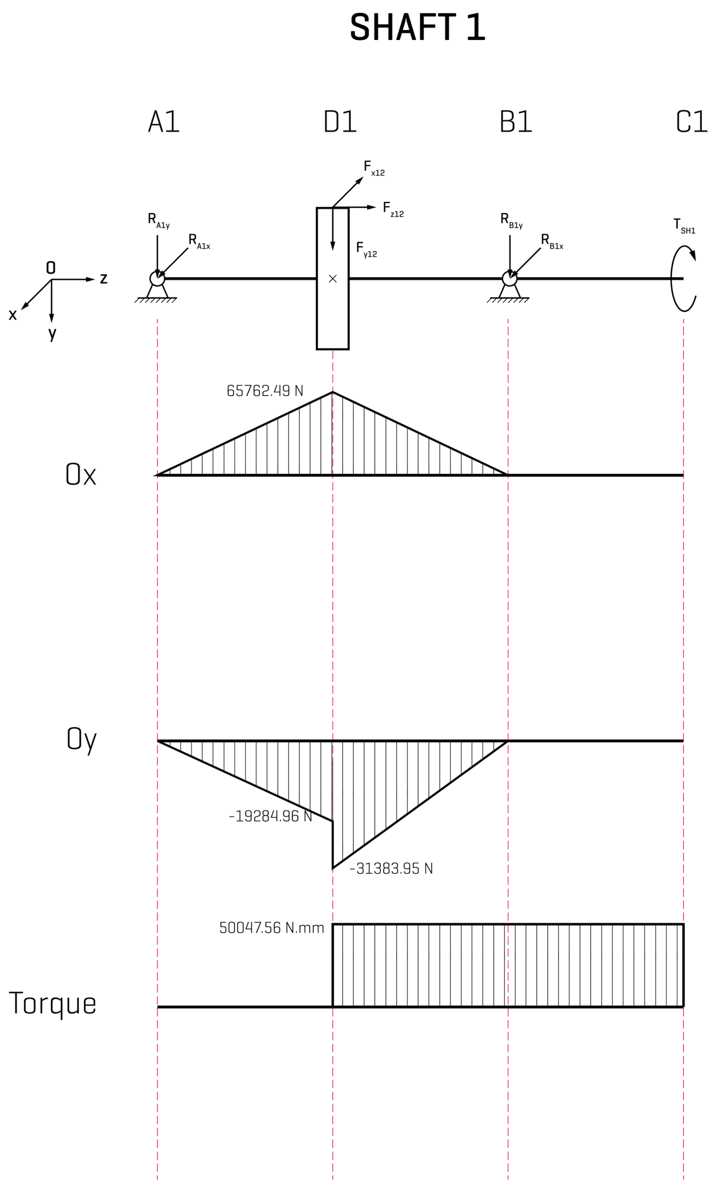


Figure 2.3: Bending moment-torque diagram of shaft 1

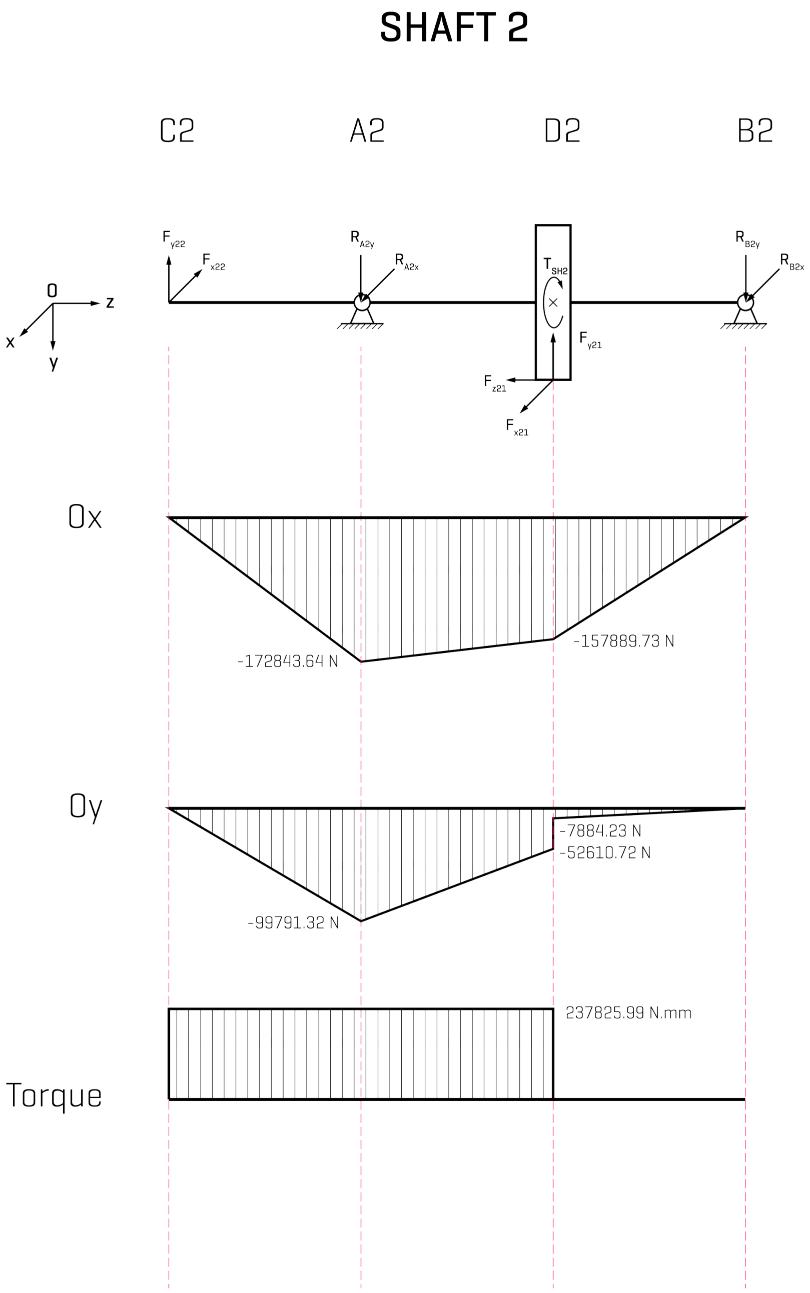


Figure 2.4: Bending moment-torque diagram of shaft 2

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