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# Project Report

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## Design Problem

$D_{bc}$  pulley diameter, mm

$F_t$  tangential force, N

$L$  service life, years

$T$  working torque, N · mm

$t$  working time, s

$v_{bc}$  conveyor belt speed, m/s

$\delta_u$  error of speed ratio, %

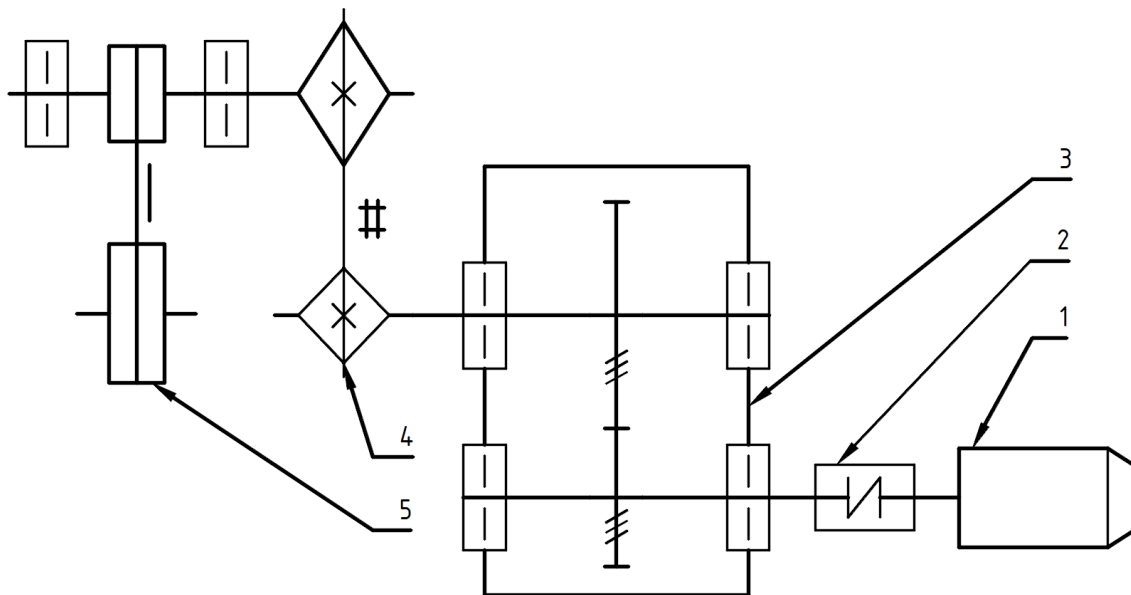


Figure 1: Mechanical transmission system of a belt conveyor

Given the mechanical transmission system in figure 1, determine the specifications for each machine element.

1. Electric motor
2. Elastic coupling
3. Gearbox
4. Chain drive

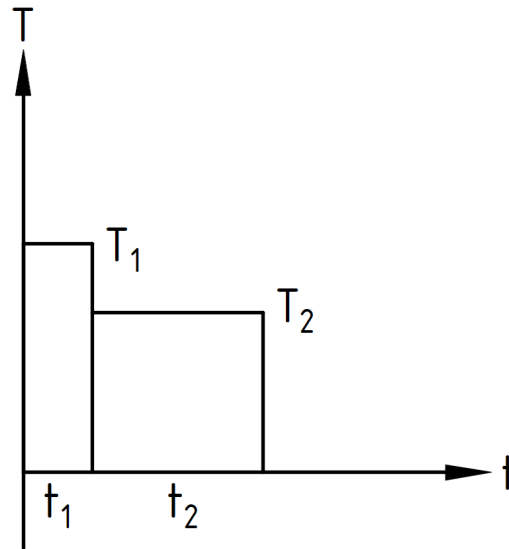


Figure 2: Input load diagram

## 5. Belt conveyor

**Design parameters** The chosen parameters are given in column 8:

- $F_t = 4500 \text{ (N)}$
- $v_{bc} = 3.05 \text{ (m/s)}$
- $D_{bc} = 500 \text{ (mm)}$
- $L = 4 \text{ (years)}$
- $T_1 = T \text{ (N} \cdot \text{mm)}, t_1 = 12 \text{ (s)}$
- $T_2 = 0.7T \text{ (N} \cdot \text{mm)}, t_2 = 60 \text{ (s)}$
- $\delta_u \leq \pm 5\%$

The machine works in one direction for 300 days, 2 shifts per day, 8 hours each and the load is low. Also, for the rest of this report, we will consider shaft 1 is the one connected to the electric motor, while shaft 2 is the connection between the chain drive and gearbox.

# Chapter 1

## Motor Design

### 1.1 Nomenclature

$n_{bc}$	rotational speed of belt conveyor, rpm	$u_{hg}$	transmission ratio of helical gear
$n_{sh}$	rotational speed of shaft, rpm	$u_{sys}$	transmission ratio of the system
$P_m$	maximum operating power of belt conveyor, kW	$T_{motor}$	motor torque, N · mm
$P_{motor}$	calculated motor power to drive the system, kW	$T_{sh}$	shaft torque, N · mm
$P_{sh}$	operating power of shaft, kW	$\eta_b$	bearing efficiency
$P_w$	operating power of the belt conveyor given a workload, kW	$\eta_c$	coupling efficiency
		$\eta_{ch}$	chain drive efficiency
		$\eta_{hg}$	helical gear efficiency
		$\eta_{sys}$	efficiency of the system
$u_{ch}$	transmission ratio of chain drive	1	shaft 1
		2	shaft 2

## 1.2 Calculate $\eta_{sys}$

From table (2.3):

$$\eta_c = 1$$

$$\eta_b = 0.99$$

$$\eta_{hg} = 0.96$$

$$\eta_{ch} = 0.95$$

$$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$$

## 1.3 Calculate $P_{motor}$

$$P_m = \frac{F_t v_{bc}}{1000} \approx 13.73 \text{ (kW)}$$

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$

$$P_{motor} = \frac{P_w}{\eta_{sys}} \approx 11.76 \text{ (kW)} < P_m$$

## 1.4 Calculate $n_{motor}$

$$n_{bc} = \frac{6 \times 10^4 v}{\pi D_{bc}} \approx 116.5 \text{ (rpm)}$$

$$u_{ch} = 5 \text{ (table (2.4))}$$

$$u_{hg} = 5 \text{ (table (2.4))}$$

$$u_{sys} = u_{ch} u_{hg} = 25$$

$$n_{motor} = u_{sys} n_{bc} \approx 2912.54 \text{ (rpm)}$$



## 1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated  $P_{motor}$  and  $P_m$ . Since  $P_{motor} < P_m$  for our case, the minimum operating power of choice is  $P_m$ . In similar fashion, its rotational speed must also be no smaller than estimated  $n_{motor}$ .

Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$$\Rightarrow P_{motor} = 18.5 \text{ kW}, n_{motor} = 2930 \text{ (rpm)}$$

Recalculating  $u_{sys}$  with the new  $P_{motor}$  and  $n_{motor}$ , we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming  $u_{hg} = \text{const}$ :

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

## 1.6 Calculate power, rotational speed and torque

Let us denote  $P_{sh1}$ ,  $n_{sh1}$  and  $T_{sh1}$  be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly,  $P_{sh2}$ ,  $n_{sh2}$  and  $T_{sh2}$  will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

### 1.6.1 Power

$$P_{ch} = P_m \approx 13.73 \text{ (kW)}$$

$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59 \text{ (kW)}$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35 \text{ (kW)}$$

## 1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$

$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

## 1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63 \text{ (N} \cdot \text{mm)}$$

$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56 \text{ (N} \cdot \text{mm)}$$

$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
$P$ (kW)	18.5	14.59	15.35
$u$	5	5.03	
$n$ (rpm)	2930	2930	586
$T$ (N · mm)	60298.63	50047.56	237825.99

Table 1.1: System overall specifications

# Chapter 2

## Chain Drive Design

### 2.1 Nomenclature

$[i]$	permissible impact times per second	$F_v$	centrifugal force, N
		$i$	impact times per second
$[s]$	permissible safety factor	$k$	overall factor
$[P]$	permissible power, kW	$k_0$	arrangement of drive factor
$a$	center distance, mm	$k_a$	center distance and chain's length factor
$a_{max}$	maximum center distance, mm		
$a_{min}$	minimum center distance, mm	$k_{bt}$	lubrication factor
$B$	bush length, mm	$k_c$	rating factor
$d$	driving sprocket diameter, mm	$k_d$	dynamic loads factor
$d_c$	pin diameter, mm	$k_{dc}$	chain tension factor
$F_0$	sagging force, N	$k_f$	loosing factor
$F_1$	tight side tension force, N	$k_n$	coefficient of rotational speed
$F_2$	slack side tension force, N	$k_x$	chain weight factor
$F_r$	force on the shaft, N	$k_z$	coefficient of number of teeth
$F_t$	effective peripheral force, N		

$n_{01}$	experimental rotational speed, rpm	$v$	instantaneous velocity along the chain, m/s
$n_{ch}$	rotational speed of a sprocket, rpm	$x$	chain length in pitches, the number of links
$P_t$	calculated power, kW	$x_c$	an even number of links
$p$	pitch, mm	$z$	number of teeth of a sprocket
$p_{max}$	permissible sprocket pitch, mm	$z_{max}$	maximum number of teeth of the driven sprocket
$Q$	permissible load, N		
$q$	mass per meter of chain, kg/m	1	subscript for driving sprocket
$s$	safety factor	2	subscript for driven sprocket

## 2.2 Find $p$

Since the driving sprocket is connected to shaft 1,  $n_1 = n_{sh2} = 586$  (rpm).

**Find  $z$**  Since  $z_1$  and  $z_2$  is preferably an odd number according to p.80:

$$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$$

$$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$$

Because  $z_1 \geq 15$ , we use table (5.8) and interpolation to approximate  $p_{max}$ .

Therefore,  $p_{max} \approx 33.58$  (mm).

**Find  $k$**  Since  $n_{ch} = 586 \approx 600$  (rpm), choose  $n_{01} = 600$  (rpm), which is obtained from table (5.5). Then, we calculate  $k_z$  and  $k_n$ .

$$k_z = \frac{25}{z_1} \approx 1.32, k_n = \frac{n_{01}}{n_{ch}} \approx 1.02$$

Specifying the chain drive's working condition and utilizing table (5.6), we find

$$\text{out that } k_0 = k_a = k_{dc} = k_{bt} = 1, k_d = 1.25, k_c = 1.3.$$

$$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$$

**Find  $p$**  From table (5.5):

$$P_t = P_{ch} k k_z k_n \approx 30.05 \text{ (kW)} \leq 42 \text{ (kW)} \Rightarrow [P] = 42 \text{ (kW)}$$

Using the table, we also get the other parameters:

$$p = 31.75 \text{ (mm)}, d_c = 9.55 \text{ (mm)}, B = 27.46 \text{ (mm)},$$

$$d_1 = \frac{p}{\sin \frac{z_1}{180^\circ}} \approx 192.9 \text{ (mm)}, d_2 = \frac{p}{\sin \frac{z_2}{180^\circ}} \approx 980.49 \text{ (mm)}$$

Having  $p = 31.75 \text{ (mm)} \leq p_{\max} \approx 33.58 \text{ (mm)}$ , we can safely choose the number of chains as 1, which is in agreement with the given figure. Hence, from table (5.2), we obtain the parameters in the section for 1 strand chain drive:

$$Q = 56.7 \times 10^3 \text{ (N)}, q = 2.6 \text{ (kg/m)}$$

By comparison to the conditions in the sub-table, the choice of  $B$  is satisfactory.

## 2.3 Find $a$ , $x_c$ , and $i$

**Find  $x_c$**   $a_{\min} = 30p = 952.5 \text{ (mm)}$ ,  $a_{\max} = 50p = 1587.5 \text{ (mm)}$ . Limiting the range of choice for  $a$  in  $[a_{\min}, a_{\max}]$ , we can approximate  $a = 1000 \text{ (mm)}$ .

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$$

**Find  $a$**  From equation (5.13), we calculate  $a$  again with  $x_c$ :

$$a = \frac{p}{4} \left( x_c - \frac{z_2 + z_1}{2} + \sqrt{\left( x_c - \frac{z_2 + z_1}{2} \right)^2 - 2 \frac{(z_2 - z_1)^2}{\pi^2}} \right) - 0.003a \approx 998.98 \text{ (mm)}$$

**Find  $i$**  From table (5.9):

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$$

## 2.4 Strength of chain drive

For moderate workload, choose  $k_d = 1.2$ . Let the chain drive be angled  $30^\circ$  with respect to ground, we obtain  $k_f = 4$ .

$$v_1 = \frac{n_{ch} P_{z1}}{6 \times 10^4} \approx 5.89 \text{ (m/s)}$$

**Find**  $F_t, F_v, F_0$  We also need to calculate  $F_t, F_v$  and  $F_0$ :

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2329.53 \text{ (N)}$$

$$F_v = q v_1^2 \approx 90.25 \text{ (N)}$$

$$F_0 = 9.81 \times 10^{-3} k_f q a \approx 101.92 \text{ (N)}$$

**Validate**  $s$  This value must be larger than the permissible safety factor to operate properly. From equation (5.15):

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2, \text{ where } [s] \text{ is chosen from table (5.10).}$$

## 2.5 Force on shaft

From p.87:

$$F_2 = F_0 + F_v \approx 192.17 \text{ (N)}$$

$$F_1 = F_t + F_2 \approx 2521.7 \text{ (N)}$$

Choose  $k_x = 1.15$  and follow equation (5.20) :

$$F_r = k_x F_t \approx 2678.96 \text{ (N)}$$

In summary, we have the following table:

	driving	driven
$[P]$ (kW)	42	
$Q$ (N)	56700	
$p$ (mm)	31.75	
$i$	6	
$a$ (mm)	998.98	
$z$	19	97
$d$ (mm)	192.9	980.49
$d_c$ (mm)	9.55	
$B$ (mm)	27.46	
$v$ (m/s)	5.01	
$u_{ch}$	5	

Table 2.1: Chain drive specifications

# Chapter 3

## Gearbox Design (Helix gears)

### 3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, $MPa$	$F_a$	axial force, N
$[\sigma_F]$	permissible bending stress, $MPa$	$F_r$	radial force, N
		$F_t$	tangential force, N
$[\sigma_H]_{max}$	permissible contact stress due to overload, $MPa$	$H$	surface roughness, HB
		$K_d$	coefficient of gear material
$[\sigma_F]_{max}$	permissible bending stress due to overload, $MPa$	$K_F$	load factor from bending stress
		$K_{FC}$	load placement factor
AG	accuracy grade of gear	$K_{FL}$	aging factor due to bending stress
$a$	center distance, $mm$		
$b$	face width, $mm$	$K_{Fv}$	factor of dynamic load from bending stress at meshing area
$c$	gear meshing rate		
$d$	pitch circle, $mm$	$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
$d_a$	addendum diameter, $mm$		
$d_b$	base diameter, $mm$	$K_{F\beta}$	factor of load distribution from bending stress on top land
$d_f$	deddendum diameter, $mm$		



$K_H$	load factor of contact stress	$S_H$	safety factor of contact stress
$K_{HL}$	aging factor due to contact stress	$v$	rotational velocity, $m/s$
$K_{Hv}$	factor of dynamic load from contact stress at meshing area	$x$	gear correction factor
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth	$Y_F$	tooth shape factor
$K_{H\beta}$	factor of load distribution from contact stress on top land	$Y_\beta$	helix angle factor
$k_x$	a coefficient	$Y_\epsilon$	contact ratio factor
$k_y$	a coefficient	$y$	center displacement factor
$m$	traverse module, $mm$	$z_H$	contact surface's shape factor
$m_F$	root of fatigue curve in bending stress test	$z_M$	material's mechanical properties factor
$m_H$	root of fatigue curve in contact stress test	$z_{min}$	minimum number of teeth corresponding to $\beta$
$m_n$	normal module, $mm$	$z_v$	virtual number of teeth
$N_{FE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$	$z_\epsilon$	meshing condition factor
$N_{FO}$	working cycle of bearing stress corresponding to $[\sigma_F]$	$\alpha$	normal pressure angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20^\circ$
$N_{HE}$	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$	$\alpha_t$	traverse pressure angle, $^\circ$
$N_{HO}$	working cycle of bearing stress corresponding to $[\sigma_H]$	$\epsilon_\alpha$	traverse contact ratio
$S$	length, $mm$	$\epsilon_\beta$	face contact ratio
$S_F$	safety factor of bending stress	$\beta$	helix angle, $^\circ$
		$\beta_b$	base circle helix angle, $^\circ$
		$\psi_{ba}$	width to shaft distance ratio
		$\psi_{bd}$	face width factor
		$\sigma_b$	ultimate strength, $MPa$
		$\sigma_{ch}$	yield limit, $MPa$

$\sigma_{Flim}^o$	permissible bending stress	1	subscript for pinion
	corresponding to working cycle,	2	subscript for driven gear
$MPa$		$w$	subscript for variable value after
$\sigma_{Hlim}^o$	permissible contact stress		correction
	corresponding to working cycle,		
$MPa$			

## 3.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with  $S \leq 100$  (mm), HB250,  $\sigma_b = 850$  (MPa),  $\sigma_{ch} = 550$  (MPa).

Table (6.2) also gives  $\sigma_{Hlim}^o = 2HB + 70$ ,  $S_H = 1.1$ ,  $\sigma_{Flim}^o = 1.8HB$ ,  $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness  $H$ .

For the pinion,  $H_1 = HB250 \Rightarrow \sigma_{Hlim1}^o = 570$  (MPa),  $\sigma_{Flim1}^o = 450$  (MPa)

For the driven gear,  $H_2 = HB240 \Rightarrow \sigma_{Hlim2}^o = 550$  (MPa),  $\sigma_{Flim2}^o = 432$  (MPa)

## 3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

### 3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6 \text{ (cycles)}$$

$$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6 \text{ (cycles)}$$

### 3.3.2 Working cycle of equivalent tensile stress

Since  $H_1, H_2 \leq \text{HB350}$ ,  $m_H = 6$ ,  $m_F = 6$ .

Both gears meshed indefinitely, thus  $c = 1$ .

Applying equation (6.7) and  $T_1, T_2, t_1, t_2$  in the initial parameters:

$$N_{HE1} = 60c \left[ \left( \frac{T_1}{T} \right)^3 n_1 t_1 + \left( \frac{T_2}{T} \right)^3 n_2 t_2 \right] \approx 5.73 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c \left[ \left( \frac{T_1}{T} \right)^3 n_1 t_1 + \left( \frac{T_2}{T} \right)^3 n_2 t_2 \right] \approx 1.15 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c \left[ \left( \frac{T_1}{T} \right)^{m_F} n_1 t_1 + \left( \frac{T_2}{T} \right)^{m_F} n_2 t_2 \right] \approx 3.35 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c \left[ \left( \frac{T_1}{T} \right)^{m_F} n_1 t_1 + \left( \frac{T_2}{T} \right)^{m_F} n_2 t_2 \right] \approx 0.67 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

### 3.3.3 Aging factor

For steel,  $N_{FO1} = N_{FO2} = 4 \times 10^6$  (MPa). Applying equations (6.3) and (6.4) yield:

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$$

$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$$

$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$$

$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$$

### 3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction,  $K_{FC} = 1$

$$[\sigma_{H1}] = \sigma_{Hlim1}^o K_{HL1}/S_{H1} \approx 621.61 \text{ (MPa)}$$

$$[\sigma_{H2}] = \sigma_{Hlim2}^o K_{HL2}/S_{H2} \approx 771.63 \text{ (MPa)}$$

$$[\sigma_{F1}] = \sigma_{Flim1}^o K_{FC1} K_{FL1}/S_{F1} \approx 264.85 \text{ (MPa)}$$

$$[\sigma_{F2}] = \sigma_{Flim2}^o K_{FC2} K_{FL2}/S_{F2} \approx 332.48 \text{ (MPa)}$$

The permissible contact stress due to overload must be lower than 1.25 times of either  $[\sigma_{H1}]$  or  $[\sigma_{H2}]$ , whichever is smaller. For permissible bending stress, it is equal to either  $[\sigma_{F1}]$  or  $[\sigma_{F2}]$ , whichever is larger:

$$[\sigma_H]_{max} = \frac{1}{2} ([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62 \text{ (MPa)} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$$

$$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440 \text{ (MPa)}$$

## 3.4 Transmission Design

### 3.4.1 Determine basic parameters

Examine table (6.5) gives  $K_a = 43$

Assuming symmetrical design, table (6.6) also gives  $\psi_{ba} = 0.5$

$$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$$

From table (6.7) , using interpolation, we approximate  $K_{H\beta} \approx 1.108$ ,  $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate  $a$  using equation (6.15a) gives:

$$a = K_a(u_{hg} + 1) \sqrt[3]{\frac{T_{sh1}K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} \approx 91.94 \text{ (mm)}$$

According to SEV229-75 standard, we choose  $a_w = 100 \text{ mm}$

### 3.4.2 Determine gear meshing parameters

**Find  $m$**  Applying equation (6.17) and choose  $m$  from table (6.8) :

$$m = (0.01 \div 0.02)a_w \approx (0.92 \div 1.84) \text{ (mm)} \Rightarrow m = 1.5 \text{ (mm)}$$

**Find  $z_1, z_2, b_w$**  Let  $\beta = 15^\circ$ . Combining equation (6.18) and (6.20), we come up with the formula to calculate  $z_1$ . From the result,  $z_1$  is rounded up to the

nearest odd number (preferably a prime number).

$$z_1 = \frac{2a_w \cos \beta}{m(u_{hg} + 1)} \approx 19.73 \Rightarrow z_1 = 21$$

$$z_2 = u_{hg} z_1 = 105$$

$$\Rightarrow b_w = \psi_{ba} a_w = 50 \text{ (mm)}$$

**Recalculate  $\beta$**  There are 2 approaches for correction involving the change of either  $\alpha$  or  $\beta$ . Because altering  $\alpha$  leads to many other corrections ( $d_1$ ,  $d_2$  and  $a$ ),  $\beta$  will be used instead.

Since  $z_1$  is rounded, we must find  $\beta$  to obtain the correct angle, ensuring that  $\beta \in (8^\circ, 20^\circ)$ . Using equation (6.32):

$$\beta = \arccos \frac{m(z_1 + z_2)}{2a_w} \approx 19.09^\circ$$

**Find  $x_1$ ,  $x_2$**  To find  $x_1$  and  $x_2$ , we will follow the calculation scheme provided in p.103. Since  $\beta \approx 19.09^\circ \in (17, 21]$ ,  $z_{min} = 15$ , which leads to  $z_1$  satisfying condition  $z_1 \geq z_{min} + 2 > 10$ , according to table (6.9). Combined with  $u_{hg} = 5 \geq 3.5$ , we obtain  $x_1 = 0.3$ ,  $x_2 = -0.3$ , disregarding the calculation of  $y$ .

### 3.4.3 Other parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos \beta} \approx 33.33 \text{ (mm)}$$

$$d_2 = d_{w2} = \frac{mz_2}{\cos \beta} \approx 166.67 \text{ (mm)}$$

$$d_{a1} = d_1 + 2(1 + x_1)m \approx 37.23 \text{ (mm)}$$

$$d_{a2} = d_2 + 2(1 + x_2)m \approx 168.77 \text{ (mm)}$$

$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 30.48 \text{ (mm)}$$

$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 162.02 \text{ (mm)}$$

$$d_{b1} = d_1 \cos \alpha \approx 31.32 \text{ (mm)}$$

$$d_{b2} = d_2 \cos \alpha \approx 156.62 \text{ (mm)}$$

$$\alpha_t = \alpha_{tw} = \arctan \frac{\tan \alpha}{\cos \beta} \approx 20.65^\circ$$

$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 5 \text{ (m/s)}$$

### 3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1} K_H \frac{u_{hg} + 1}{b_w u_{hg} d_{w1}^2}} \leq [\sigma_H]$$

**Find  $z_M$**   $z_M = 274$ , according to table (6.5)

**Find  $z_H$**   $\beta_b = \arctan(\cos \alpha_t \tan \beta) \approx 17.94^\circ \Rightarrow z_H = \sqrt{2 \frac{\cos \beta_b}{\sin(2\alpha_t)}} \approx 1.7$

**Find  $z_\epsilon$**  Obtaining  $z_\epsilon$  through calculations:

$$\epsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin \alpha_t}{2\pi m \frac{\cos \alpha_t}{\cos \beta}} \approx 1.34$$

$$\epsilon_\beta = b_w \frac{\sin \beta}{m\pi} \approx 3.47 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.86$$

**Find  $K_H$**  We find  $K_H$  using equation  $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table (6.13),  $v \leq 6$  (m/s)  $\Rightarrow AG = 8$

From table (P2.3), using interpolation, we approximate:

$$K_{Hv} \approx 1.05, K_{Fv} \approx 1.14$$

From table (6.14), using interpolation, we approximate:

$$K_{H\alpha} \approx 1.09, K_{F\alpha} \approx 1.27$$

$$\Rightarrow K_H \approx 1.27$$

**Find  $\sigma_H$**  After calculating  $z_M, z_H, z_\epsilon, K_H$ , we get the following result:

$$\sigma_H \approx 663.86 \text{ MPa} \leq [\sigma_H] \approx 696.62 \text{ MPa}$$

### 3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2 \frac{T_{sh1} K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} \leq [\sigma_{F1}]$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{F2}]$$

**Find  $Y_\epsilon$**  Knowing that  $\epsilon_\alpha \approx 1.64$ , we can calculate  $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.75$

**Find  $Y_\beta$**   $Y_\beta = 1 - \frac{\beta}{140} \approx 0.86$

**Find  $Y_F$**  Using formula  $z_v = z \cos^{-3}(\beta)$  and table (6.18):

$$z_{v1} = z_1 \cos^{-3}(\beta) \approx 24.88 \Rightarrow Y_{F1} \approx 3.6$$

$$z_{v2} = z_2 \cos^{-3}(\beta) \approx 124.42 \Rightarrow Y_{F2} \approx 3.64$$

**Find  $K_F$**  Using  $K_{F\beta}$ ,  $K_{F\alpha}$ ,  $K_{Fv}$  calculated from the sections above, we derive:

$$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.82$$

**Find  $\sigma_F$**  Since  $m_n = m \cos \beta \approx 1.42$ , substituting all the values, we find out that:

$$\sigma_{F1} \approx 179.07 \text{ (MPa)} \leq [\sigma_{F1}] \approx 264.85 \text{ (MPa)}$$

$$\sigma_{F2} \approx 181.06 \text{ (MPa)} \leq [\sigma_{F2}] \approx 332.48 \text{ (MPa)}$$

### 3.4.6 Force on shafts

$$F_t = \frac{2T_{sh1}}{d_1} \approx 3003.15 \text{ (N)}$$

$$F_r = F_t \tan \alpha_t \approx 1131.8 \text{ (N)}$$

$$F_a = F_t \tan \beta \approx 1039.35 \text{ (N)}$$

In summary, we have the following table:

	pinion	driving gear
$H$ (HB)	250	240
$[\sigma_H]$ (MPa)	621.61	771.63
$[\sigma_F]$ (MPa)	264.85	332.48
$[\sigma_H]_{max}$ (MPa)	696.62	
$[\sigma_F]_{max}$ (MPa)	440	
$\sigma_H$ (MPa)	621.61	771.63
$\sigma_F$ (MPa)	179.07	181.06
$\sigma_H$ (MPa)	663.86	
$\alpha_{tw}$ (°)	20.65	
$\beta$ (°)	19.09	
$a_w$ (mm)	100	
$b_w$ (mm)	50	
$m$ (mm)	1.5	
$z$	21	105
$d$ (mm)	33.33	166.67
$d_a$ (mm)	37.23	168.77
$d_f$ (mm)	30.48	162.02
$d_b$ (mm)	31.32	156.62
$v$ (m/s)	5	
$u_{hg}$	5	

Table 3.1: Gearbox specifications





# Chapter 4

## Shaft Design

### 4.1 Nomenclature

$[\tau]$	permissible torsion, MPa	$q$	standardized coefficient of shaft diameter
$r$	position of applied force on the shaft, mm	$b_O$	rolling bearing width, mm
$hr$	tooth direction	$l_m$	hub diameter, mm
$cb$	role of gear on the shaft (active or passive)	$k_1$	distance between elements, mm
$cq$	rotational direction of the shaft	$k_2$	distance between bearing surface and inner walls of the gearbox, mm
$\sigma_b$	ultimate strength, MPa	$k_3$	distance between element surface and bearing lid, mm
$\sigma_{ch}$	yield limit, MPa	$h_n$	distance between bearing lid and bolt, mm
$S$	safety factor	$T$	torque on shaft
$F_x$	applied force, N	$\alpha_{tw}$	meshing profile angle, °
$F_t$	tangential force, N	$\beta$	helix angle, °
$F_r$	radial force, N	$_1$	subscript for shaft 1
$F_a$	axial force, N	$_2$	subscript for shaft 2
$a_w$	shaft distance, mm		
$d$	shaft diameter, mm		
$d_w$	gear diameter, mm		
	subscript for gear axis		

## 4.2 Choose material

For moderate load, we will use quenched steel 40X to design the shafts. From table 6.1, the specifications are as follows:  $S \leq 100$  (mm), HB260,  $\sigma_b = 850$  (MPa),  $\sigma_{ch} = 550$  (MPa).

## 4.3 Tranmission Design

### 4.3.1 Load on shafts

#### Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.

On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$$r_{12} = -d_{w12}/2 \approx -13.57 \text{ (mm)}, hr_{12} = +1, cb_{12} = +1, cq_1 = +1$$

$$r_{21} = +d_{w21}/2 \approx +67.84 \text{ (mm)}, hr_{21} = -1, cb_{21} = -1, cq_2 = -1$$

**Find magnitude of  $F_t$ ,  $F_r$ ,  $F_a$**  Using the results from the previous chapter:

$$\alpha_{tw} \approx 21.17^\circ, \beta = 20^\circ, d_{w12} \approx 27.14 \text{ (mm)}$$

$$\begin{cases} F_{t12} = F_{t21} = \frac{2T_{sh1}}{d_{w12}} \approx 2769.03 \text{ (N)} \\ F_{r12} = F_{r21} = \frac{F_{t12} \tan \alpha_{tw}}{\cos \beta} \approx 1141.36 \text{ (N)} \\ F_{a12} = F_{a21} = F_{t12} \tan \beta \approx 1007.84 \text{ (N)} \end{cases}$$

**Find direction of  $F_t$ ,  $F_r$ ,  $F_a$**  Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \frac{r_{12}}{|r_{12}|} c q_1 c b_{12} F_{t12} \approx -2769.03 \text{ (N)} \\ F_{y12} = -\frac{r_{12}}{|r_{12}|} \frac{\tan \alpha_{tw}}{\cos \beta} F_{t12} \approx 1141.36 \text{ (N)} \\ F_{z12} = c q_1 c b_{12} h r_{12} F_{t12} \tan \beta \approx 1007.84 \text{ (N)} \end{cases}$$

$$\begin{cases} F_{x21} = \frac{r_{21}}{|r_{21}|} c q_2 c b_{21} F_{t21} \approx 2769.03 \text{ (N)} \\ F_{y21} = -\frac{r_{21}}{|r_{21}|} \frac{\tan \alpha_{tw}}{\cos \beta} F_{t21} \approx -1141.36 \text{ (N)} \\ F_{z21} = c q_2 c b_{21} h r_{21} F_{t21} \tan \beta \approx -1007.84 \text{ (N)} \end{cases}$$

### Applied forces from Chain drives

Assuming the angle between x-axis and  $F_r$  is  $210^\circ$  and  $F_r \approx 2539.28 \text{ (N)}$  (chapter 2), we get the direction of  $F_r$  on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 210^\circ \approx -1269.64 \text{ (N)} \\ F_{y22} = F_{r22} \sin 210^\circ \approx -2199.08 \text{ (N)} \end{cases}$$

### 4.3.2 Preliminary calculations

Since shaft 1 receives input torque  $T_{sh1}$  and shaft 2 produces output torque  $T_{sh2}$ ,  $[\tau_1] = 15 \text{ (MPa)}$  and  $[\tau_2] = 30 \text{ (MPa)}$ . Using equation (10.9), we can approximate  $d_1$  and  $d_2$ :

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 23.22 \text{ (mm)} \Rightarrow d_1 = 25 \text{ (mm)}$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 30.99 \text{ (mm)} \Rightarrow d_2 = 35 \text{ (mm)}$$

### 4.3.3 Identify the distance between bearings and applied forces

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

From table (10.2), we can estimate  $b_O$ . On shaft 1,  $b_{O1} = 15$  (mm). On shaft 2,  $b_{O2} = 21$  (mm). Using equation (10.10), the gear hubs are  $l_{m13} = l_{m12} = 1.5d_1 \approx 34.83$  (mm),  $l_{m23} = l_{m22} = 1.5d_2 \approx 46.48$  (mm) ( $l_{m22}$  is the chain hub)

From table (10.3), we choose  $k_1 = 10$  (mm),  $k_2 = 8$  (mm),  $k_3 = 15$  (mm),  $h_n = 18$  (mm). This parameters apply for both shafts in the system.

Table (10.4) introduce the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the formulas below are used:

On shaft 1:

$$l_{12} = -l_{c12} = -[0.5(l_{m12} + b_{O1}) + k_3 + h_n] \approx -57.92 \text{ (mm)}$$

$$l_{13} = 0.5(l_{m13} + b_{O1}) + k_1 + k_2 \approx 42.92 \text{ (mm)}$$

$$l_{11} = 2l_{13} \approx 85.83 \text{ (mm)}$$

On shaft 2:

$$l_{22} = -l_{c22} = -[0.5(l_{m22} + b_{O2}) + k_3 + h_n] \approx -66.74 \text{ (mm)}$$

$$l_{23} = 0.5(l_{m23} + b_{O2}) + k_1 + k_2 \approx 51.74 \text{ (mm)}$$

$$l_{21} = 2l_{23} \approx 103.48 \text{ (mm)}$$

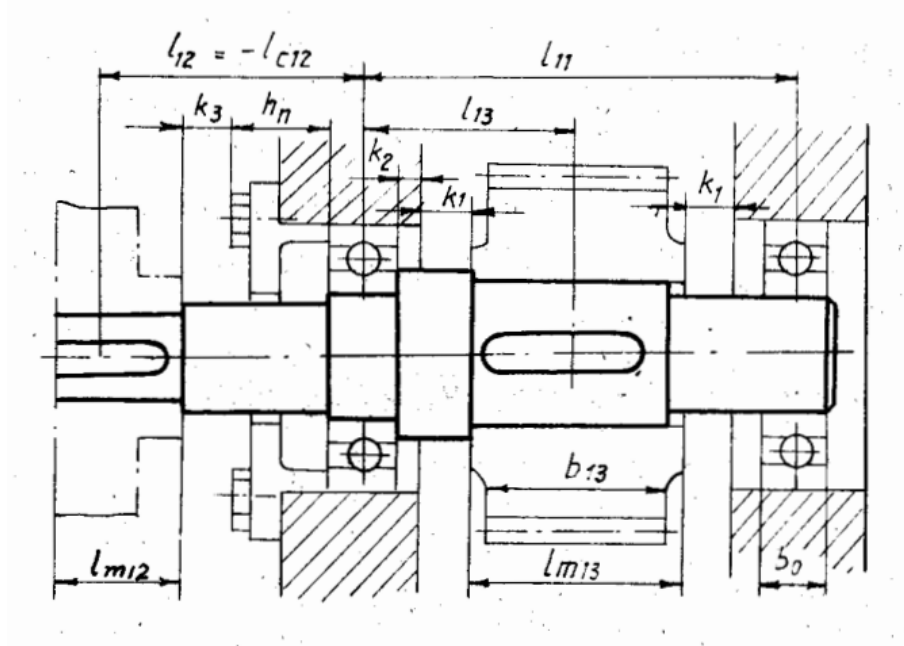


Figure 4.1: Shaft design and its dimensions

#### 4.3.4 Determine shaft diameters and lengths

From the diagram, we derive the reaction forces at  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$ , which are  $R_{A1x}$ ,  $R_{A1y}$ ,  $R_{B1x}$ ,  $R_{B1y}$ ,  $R_{A2x}$ ,  $R_{A2y}$ ,  $R_{B2x}$ ,  $R_{B2y}$ . Using equilibrium conditions:

$$\begin{cases} \sum_i \mathbf{F}_i = 0 \\ \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0 \end{cases}$$

We obtain the results as follows:

$$\begin{cases} R_{A1x} \approx 1384.51 \text{ (N)} \\ R_{A1y} \approx -570.68 \text{ (N)} \\ R_{B1x} \approx 1384.51 \text{ (N)} \\ R_{B1y} \approx -570.68 \text{ (N)} \end{cases} \quad \begin{cases} R_{A2x} \approx 703.98 \text{ (N)} \\ R_{A2y} \approx 4188.06 \text{ (N)} \\ R_{B2x} \approx -2203.37 \text{ (N)} \\ R_{B2y} \approx -847.62 \text{ (N)} \end{cases}$$

From the reaction forces, we can easily draw shear force-bending moment diagram for both shafts on 2 major planes ( $xOz$ ) and ( $yOz$ ).

From equation (10.15), we calculate the total bending moment at point  $C_2$ ,  $A_2$ ,

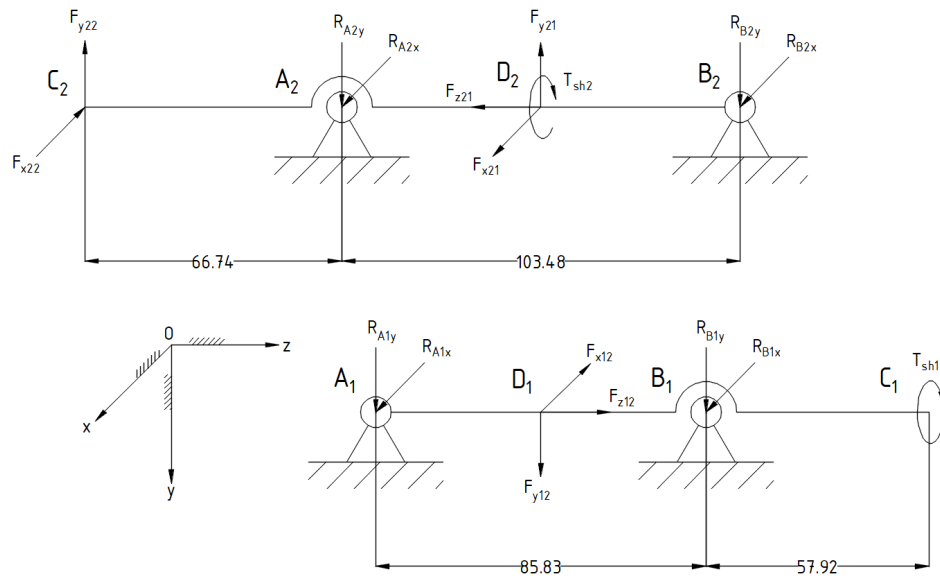


Figure 4.2: Force analysis of shafts

$D_2, B_2, A_1, D_1, B_1, C_1$

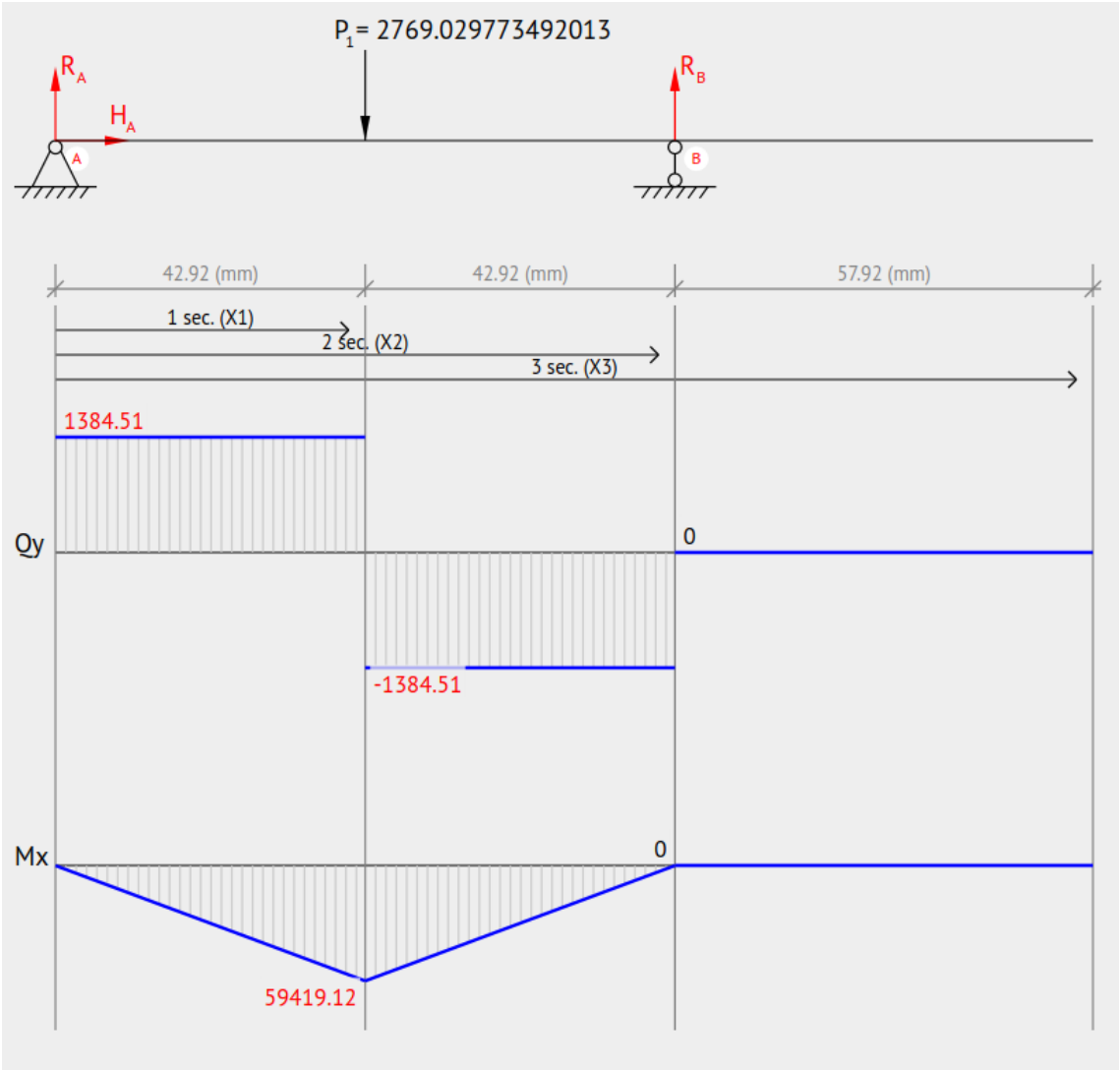


Figure 4.3: Shear force - Bending moment diagram on (xOz) of shaft 1



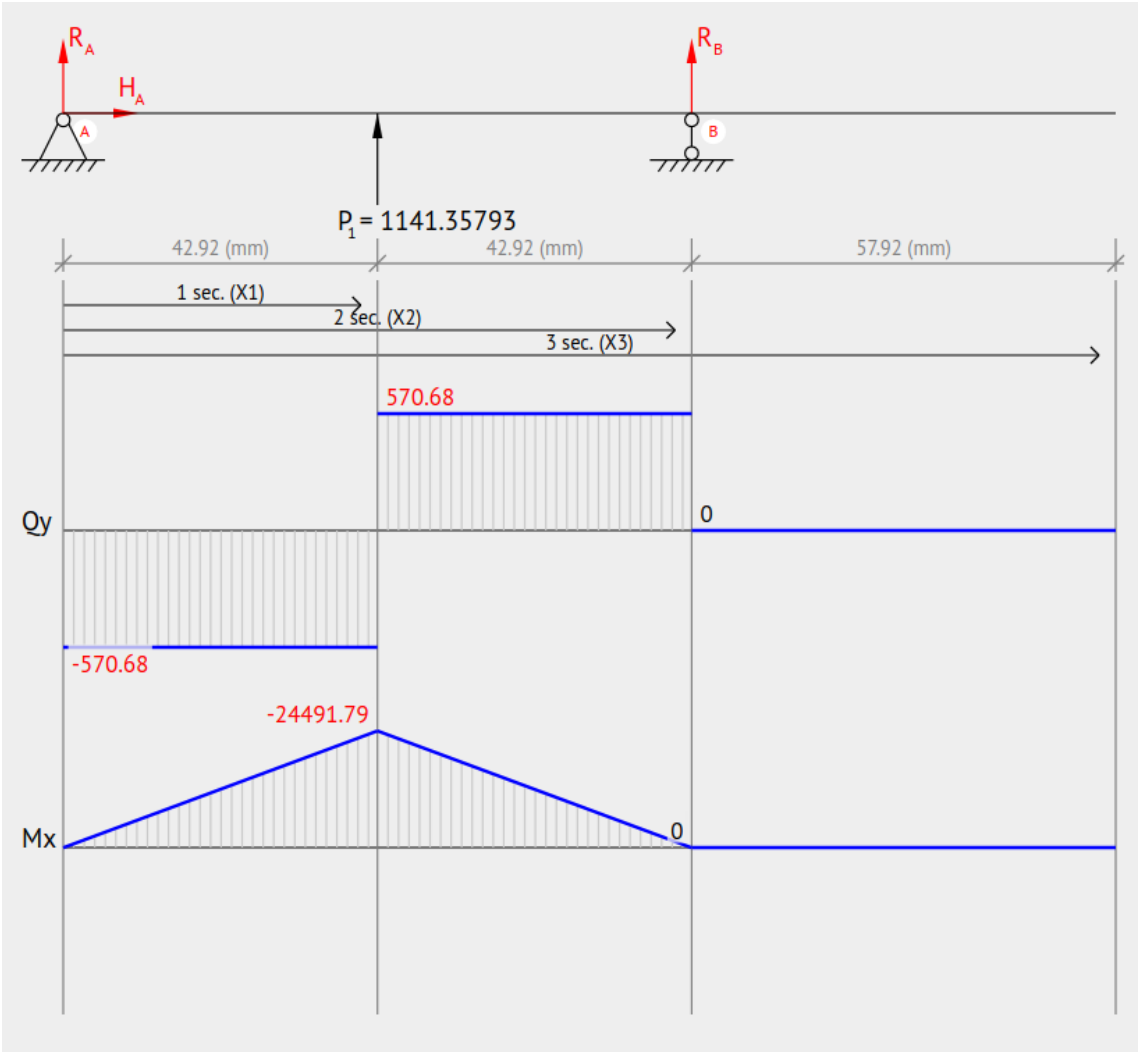


Figure 4.4: Shear force - bending moment diagram on (yOz) of shaft 1

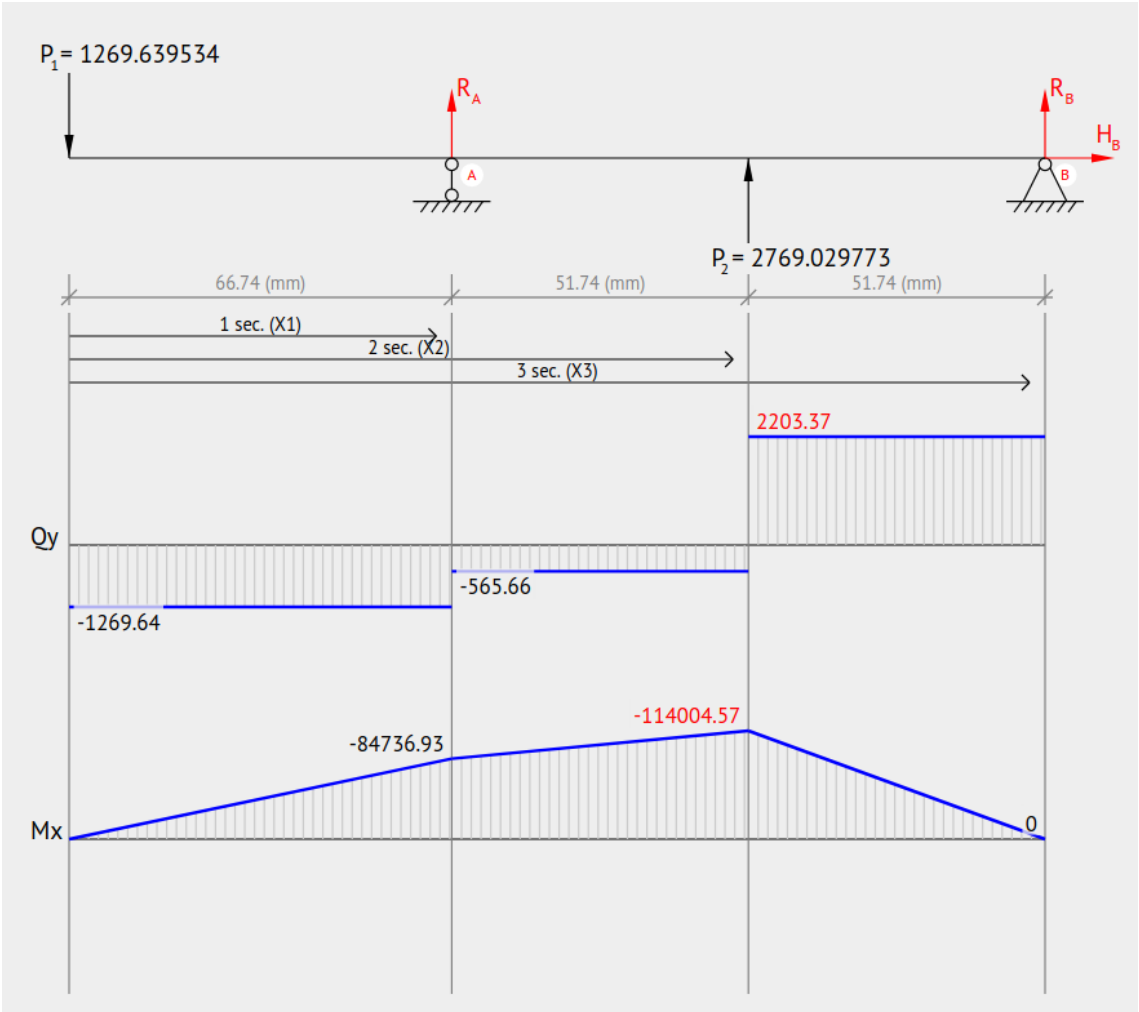


Figure 4.5: Shear force - Bending moment diagram on (xOz) of shaft 2

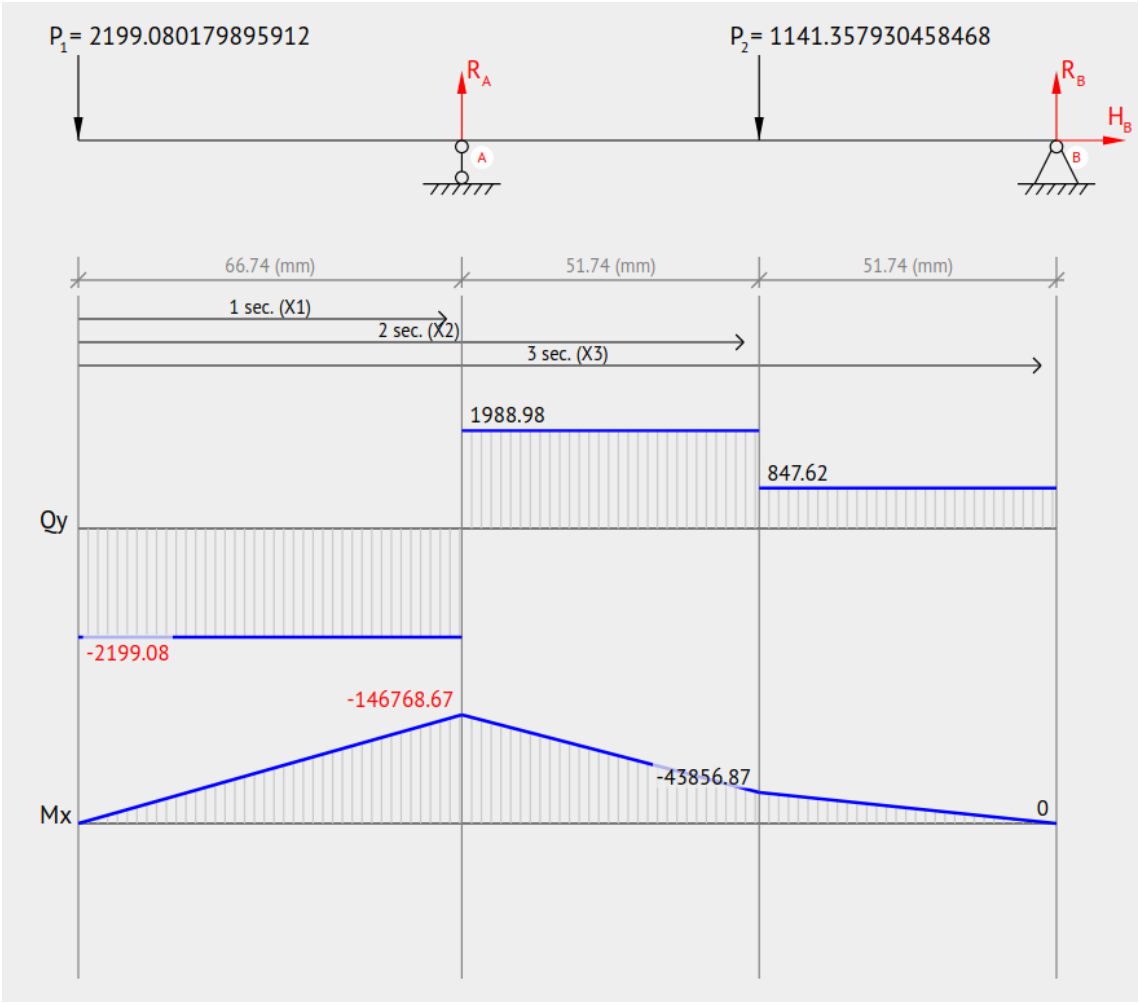


Figure 4.6: Shear force - Bending moment diagram on  $(yOz)$  of shaft 2