Machine Elements Report

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Contents

List of Tables

F_t tangential force, N
v conveyor belt speed, m/s
D pulley diameter, mm
L service life, year
T working torque, N · mm
t working time, s
δ_u error of speed ratio, %

Chapter 1

Motor Design

1.1 Nomenclature

n_{bc}	rotational speed of belt conveyor, rpm	u_{hg}	transmission ratio of helical gear
		u_{sys}	transmission ratio of the system
n_{sh}	rotational speed of shaft, rpm	T_{motor}	motor torque, $\mathrm{N} \cdot \mathrm{mm}$
P_m	maximum operating power of belt conveyor, kW	T_{sh}	shaft torque, $\mathrm{N} \cdot \mathrm{mm}$
		η_b	bearing efficiency
P_{motor}	calculated motor power to drive the system, kW	η_c	coupling efficiency
		η_{ch}	chain drive efficiency
P_{sh}	operating power of shaft, kW	η_{hg}	helical gear efficiency
P_w	opearting power of belt conveyor given a workload, kW	η_{sys}	efficiency of the system
		$_1$	shaft 1
u_{ch}	transmission ratio of chain drive	$_2$	shaft 2

1.2 Calculate η_{sys}

From table (2.3):

$\eta_c = 1$

$\eta_b = 0.99$

$\eta_{hg} = 0.96$

$\eta_{ch} = 0.95$

$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$

1.3 Calculate P_{motor}

$P_m = \dfrac{F_t v}{1000} \approx 13.73$ (kW)

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$

$P_{motor} = \dfrac{P_w}{\eta_{sys}} \approx 11.76$ (kW) $< P_m$

1.4 Calculate n_{motor}

$n_{bc} = \dfrac{6 \times 10^4 v}{\pi D} \approx 116.5$ (rpm)

$u_{ch} = 5$ (table (2.4))

$u_{hg} = 5$ (table (2.4))

$u_{sys} = u_{ch} u_{hg} = 25$

$n_{motor} = u_{sys} n_{bc} \approx 2912.54$ (rpm)

1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated P_{motor} and P_m. Since $P_{motor} < P_m$ for our case, the minimum operating power of choice is P_m. In similar fashion, its rotational speed must also be no smaller than estimated n_{motor}.

Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$\Rightarrow P_{motor} = 18.5$ kW, $n_{motor} = 2930$ (rpm)

Recalculating u_{sys} with the new P_{motor} and n_{motor}, we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming $u_{hg} = const$:

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

1.6 Calculate power, rotational speed and torque

Let us denote P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

1.6.1 Power

$$P_{ch} = P_m \approx 13.73 \text{ (kW)}$$

$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59 \text{ (kW)}$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35 \text{ (kW)}$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$

$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63 \text{ (N} \cdot \text{mm)}$$

$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56 \text{ (N} \cdot \text{mm)}$$

$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	18.5	14.59	15.35
u	5	5.03	
n (rpm)	2930	2930	586
T ($\mathrm{N} \cdot \mathrm{mm}$)	60298.63	50047.56	237825.99

Table 1.1: System properties

Chapter 2

Chain Drive Design

2.1 Nomenclature

$[i]$	permissible impact times per second
$[s]$	permissible safety factor
$[P]$	permissible power, kW
a	center distance, mm
a_{max}	maximum center distance, mm
a_{min}	minimum center distance, mm
B	bush length, mm
d	driving sprocket diameter, mm
d_c	pin diameter, mm
F_0	sagging force, N
F_1	tight side tension force, N
F_2	slack side tension force, N
F_r	force on the shaft, N
F_t	effective peripheral force, N
F_v	centrifugal force, N
i	impact times per second
k	overall factor
k_0	arrangement of drive factor
k_a	center distance and chain's length factor
k_{bt}	lubrication factor
k_c	rating factor
k_d	dynamic loads factor
k_{dc}	chain tension factor
k_f	loosing factor
k_n	coefficient of rotational speed
k_x	chain weight factor
k_z	coefficient of number of teeth
n_{01}	experimental rotational speed, rpm
n_{ch}	rotational speed of a sprocket, rpm
P_t	calculated power, kW
p	pitch, mm

p_{max}	permissible sprocket pitch, mm	x_c	an even number of links
Q	permissible load, N	z	number of teeth of a sprocket
q	mass per meter of chain, kg/m	z_{max}	maximum number of teeth of the driven sprocket
s	safety factor	$_1$	subscript for driving sprocket
v	instantaneous velocity along the chain, m/s	$_2$	subscript for driven sprocket
x	chain length in pitches, the number of links		

2.2 Find p

Since the driving sprocket is connected to shaft 1, $n_1 = n_{sh2} = 586$ (rpm).

Find z Since z_1 and z_2 is preferably an odd number according to p.80:

$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$

$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$

Because $z_1 \geq 15$, we use table (5.8) and interpolation to approximate p_{max}. Therefore, $p_{max} \approx 33.58$ (mm).

Find k Since $n_{ch} = 586 \approx 600$ (rpm), choose $n_{01} = 600$ (rpm), which is obtained from table (5.5). Then, we calculate k_z and k_n.

$k_z = \dfrac{25}{z_1} \approx 1.32$, $k_n = \dfrac{n_{01}}{n_{ch}} \approx 1.02$

Specifying the chain drive's working condition and ultizing table (5.6) , we find out that $k_0 = k_a = k_{dc} = k_{bt} = 1$, $k_d = 1.25$, $k_c = 1.3$.

$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$

Find p From table (5.5):

$P_t = P_{ch} k k_z k_n \approx 30.05$ (kW) ≤ 42 (kW) $\Rightarrow [P] = 42$ (kW)

Using the table, we also get the other parameters:

$p = 31.75$ (mm), $d_c = 9.55$ (mm), $B = 27.46$ (mm),

$d_1 = \dfrac{p}{\sin\dfrac{180°}{z_1}} \approx 192.9\,(\text{mm})$, $d_2 = \dfrac{p}{\sin\dfrac{180°}{z_2}} \approx 980.49\,(\text{mm})$

Having $p = 31.75\,(\text{mm}) \le \text{p}_{\text{max}} \approx 33.58\,(\text{mm})$, we can safely choose the number of chains as 1, which is in agreement with the given figure. Hence, from table (5.2), we obtain the parameters in the section for 1 strand chain drive:

$Q = 56.7 \times 10^3\,(\text{N})$, $q = 2.6\,(\text{kg/m})$

By comparison to the conditions in the sub-table, the choice of B is satisfactory.

2.3 Find a, x_c, and i

Find x_c $a_{min} = 30p = 952.5\,(\text{mm})$, $a_{max} = 50p = 1587.5\,(\text{mm})$. Limiting the range of choice for a in $[a_{min}, a_{max}]$, we can approximate $a = 1000\,(\text{mm})$.

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$$

Find a From equation (5.13) , we calculate a again with x_c:

$$a = \frac{p}{4}\left(x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\frac{(z_2 - z_1)^2}{\pi^2}}\right) - 0.003a \approx 998.98\,(\text{mm})$$

Find i From table (5.9):

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$$

2.4 Strength of chain drive

For moderate workload, choose $k_d = 1.2$. Let the chain drive be angled 30° with respect to ground, we obtain $k_f = 4$.

$$v_1 = \frac{n_{ch} p z_1}{6 \times 10^4} \approx 5.89\,(\text{m/s})$$

Find F_t, F_v, F_0 We also need to calculate F_t, F_v and F_0:

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2329.53\,(\text{N})$$

$F_v = qv_1^2 \approx 90.25$ (N)

$F_0 = 9.81 \times 10^{-3} k_f qa \approx 101.92$ (N)

Validate s This value must be larger than the permissible safety factor to operate properly. From equation (5.15):

$s = \dfrac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2$, where $[s]$ is chosen from table (5.10).

2.5 Force on shaft

From p.87:

$F_2 = F_0 + F_v \approx 192.17$ (N)

$F_1 = F_t + F_2 \approx 2521.7$ (N)

Choose $k_x = 1.15$ and follow equation (5.20) :

$F_r = k_x F_t \approx 2678.96$ (N)

In summary, we have the following table:

	driving	driven
$[P]$ (kW)	42	
Q (N)	56700	
p (mm)	31.75	
i	6	
a (mm)	998.98	
z	19	97
d (mm)	192.9	980.49
d_c (mm)	9.55	
B (mm)	27.46	
v (m/s)	5.01	
u_{ch}	5	

Table 2.1: Gearbox specifications

Chapter 3

Gearbox Design (Helix gears)

3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa
$[\sigma_F]$	permissible bending stress, MPa
$[\sigma_H]_{max}$	permissible contact stress due to overload, MPa
$[\sigma_F]_{max}$	permissible bending stress due to overload, MPa
AG	accuracy grade of gear
a	center distance, mm
b	face width, mm
c	gear meshing rate
d	pitch circle, mm
d_a	addendum diameter, mm
d_b	base diameter, mm
d_f	deddendum diameter, mm
F_a	axial force, N
F_r	radial force, N
F_t	tangential force, N
H	surface roughness, HB
K_d	coefficient of gear material
K_F	load factor from bending stress
K_{FC}	load placement factor
K_{FL}	aging factor due to bending stress
K_{Fv}	factor of dynamic load from bending stress at meshing area
$K_{F\alpha}$	factor ofload distribution from bending stress on gear teeth
$K_{F\beta}$	factor of load distribution from bending stress on top land
K_H	load factor of contact stress
K_{HL}	aging factor due to contact stress
K_{Hv}	factor of dynamic load from contact stress at meshing area
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth

$K_{H\beta}$	factor of load distribution from contact stress on top land
k_x	a coefficient
k_y	a coefficient
m	traverse module, mm
m_F	root of fatigue curve in bending stress test
m_H	root of fatigue curve in contact stress test
m_n	normal module, mm
N_{FE}	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$
N_{FO}	working cycle of bearing stress corresponding to $[\sigma_F]$
N_{HE}	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$
N_{HO}	working cycle of bearing stress corresponding to $[\sigma_H]$
S	length, mm
S_F	safety factor of bending stress
S_H	safety factor of contact stress
v	rotational velocity, m/s
x	gear correction factor
Y_F	tooth shape factor
Y_β	helix angle factor
Y_ϵ	contact ratio factor
y	center displacement factor
z_H	contact surface's shape factor
z_M	material's mechanical properties factor
z_{min}	minimum number of teeth corresponding to β
z_v	virtual number of teeth
z_ϵ	meshing condition factor
α	normal pressure angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20°$
α_t	traverse pressure angle, °
ϵ_α	traverse contact ratio
ϵ_β	face contact ratio
β	helix angle, °
β_b	base circle helix angle, °
ψ_{ba}	width to shaft distance ratio
ψ_{bd}	face width factor
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
σ^o_{Flim}	permissible bending stress corresponding to working cycle, MPa
σ^o_{Hlim}	permissible contact stress corresponding to working cycle, MPa
$_1$	subscript for pinion
$_2$	subscript for driven gear
$_w$	subscript for variable value after correction

3.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with $S \leq 100$ (mm), HB250, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 550$ (MPa).

Table (6.2) also gives $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness H.

For the pinion, $H_1 = \text{HB250} \Rightarrow \sigma^o_{Hlim1} = 570$ (MPa), $\sigma^o_{Flim1} = 450$ (MPa)

For the driven gear, $H_2 = \text{HB240} \Rightarrow \sigma^o_{Hlim2} = 550$ (MPa), $\sigma^o_{Flim2} = 432$ (MPa)

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6$ (cycles)

$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6$ (cycles)

3.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq \text{HB350}$, $m_H = 6$, $m_F = 6$.

Both gears meshed indefinitely, thus $c = 1$.

Applying equation (6.7) and T_1, T_2, t_1, t_2 in the initial parameters:

$$N_{HE1} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 5.73 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 1.15 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 3.35 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 0.67 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

3.3.3 Aging factor

For steel, $N_{FO1} = N_{FO2} = 4 \times 10^6$ (MPa). Applying equations (6.3) and (6.4) yield:

$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$

$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$

$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$

$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$

3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction, $K_{FC} = 1$

$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_{H1} \approx 621.61$ (MPa)

$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_{H2} \approx 771.63$ (MPa)

$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC1} K_{FL1}/S_{F1} \approx 264.85$ (MPa)

$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC2} K_{FL2}/S_{F2} \approx 332.48$ (MPa)

The permissible contact stress due to overload must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. For permissible bending stress, it is equal to either $[\sigma_{F1}]$ or $[\sigma_{F2}]$, whichever is larger:

$[\sigma_H]_{max} = \frac{1}{2}([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62 \text{ (MPa)} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$

$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440$ (MPa)

3.4 Transmission Design

3.4.1 Determine basic parameters

Examine table (6.5) gives $K_a = 43$

Assuming symmetrical design, table (6.6) also gives $\psi_{ba} = 0.5$

$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$

From table (6.7) , using interpolation, we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer

gearbox, we estimate a using equation (6.15a) gives:
$a = K_a(u_{hg} + 1)\sqrt[3]{\dfrac{T_{sh1}K_{H\beta}}{[\sigma_H]^2 u_{hg}\psi_{ba}}} \approx 91.94\,(\text{mm})$
According to SEV229-75 standard, we choose $a_w = 100\,\text{mm}$

3.4.2 Determine gear meshing parameters

Find m Applying equation (6.17) and choose m from table (6.8) :
$m = (0.01 \div 0.02)a_w \approx (0.92 \div 1.84)\,(\text{mm}) \Rightarrow m = 1.5\,(\text{mm})$

Find z_1**,** z_2**,** b_w Let $\beta = 15°$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1. From the result, z_1 is rounded up to the nearest odd number (preferably a prime number).
$z_1 = \dfrac{2a_w \cos\beta}{m(u_{hg} + 1)} \approx 19.73 \Rightarrow z_1 = 21$
$z_2 = u_{hg}z_1 = 105$
$\Rightarrow b_w = \psi_{ba}a_w = 50\,(\text{mm})$

Recalculate β There are 2 approaches for correction involving the change of either α or β. Because altering α leads to many other corrections (d_1, d_2 and a), β will be used instead.
Since z_1 is rounded, we must find β to obtain the correct angle, ensuring that $\beta \in (8°, 20°)$. Using equation (6.32):
$\beta = \arccos\dfrac{m(z_1 + z_2)}{2a_w} \approx 19.09°$

Find x_1**,** x_2 To find x_1 and x_2, we will follow the calculation scheme provided in p.103. Since $\beta \approx 19.09° \in (17, 21]$, $z_{min} = 15$, which leads to z_1 satisfying condition $z_1 \geq z_{min} + 2 > 10$, according to table (6.9). Combined with $u_{hg} = 5 \geq 3.5$, we obtain $x_1 = 0.3$, $x_2 = -0.3$, disregarding the calculation of y.

3.4.3 Other parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos\beta} \approx 33.33\,(\text{mm})$$
$$d_2 = d_{w2} = \frac{mz_2}{\cos\beta} \approx 166.67\,(\text{mm})$$
$$d_{a1} = d_1 + 2(1 + x_1)m \approx 37.23\,(\text{mm})$$
$$d_{a2} = d_2 + 2(1 + x_2)m \approx 168.77\,(\text{mm})$$
$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 30.48\,(\text{mm})$$
$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 162.02\,(\text{mm})$$
$$d_{b1} = d_1 \cos\alpha \approx 31.32\,(\text{mm})$$
$$d_{b2} = d_2 \cos\alpha \approx 156.62\,(\text{mm})$$
$$\alpha_t = \alpha_{tw} = \arctan\frac{\tan\alpha}{\cos\beta} \approx 20.65°$$
$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 5\,(\text{m/s})$$

3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1}K_H \frac{u_{hg} + 1}{b_w u_{hg} d_{w1}^2}} \le [\sigma_H] \tag{3.1}$$

Find z_M $z_M = 274$, according to table (6.5)

Find z_H $\beta_b = \arctan(\cos\alpha_t \tan\beta) \approx 17.94° \Rightarrow z_H = \sqrt{2\frac{\cos\beta_b}{\sin(2\alpha_t)}} \approx 1.7$

Find z_ϵ Obtaining z_ϵ through calculations:

$$\epsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_t}{2\pi m \frac{\cos\alpha_t}{\cos\beta}} \approx 1.34$$

$$\epsilon_\beta = b_w \frac{\sin\beta}{m\pi} \approx 3.47 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.86$$

Find K_H We find K_H using equation $K_H = K_{H\beta}K_{H\alpha}K_{Hv}$

From table (6.13), $v \le 6\,(\text{m/s}) \Rightarrow \text{AG} = 8$

From table (P2.3), using interpolation, we approximate:

$K_{Hv} \approx 1.05$, $K_{Fv} \approx 1.14$

From table (6.14), using interpolation, we approximate:

$K_{H\alpha} \approx 1.09$, $K_{F\alpha} \approx 1.27$

$\Rightarrow K_H \approx 1.27$

Find σ_H After calculating z_M, z_H, z_ϵ, K_H, we get the following result:

$$\sigma_H \approx 663.86\,\text{MPa} \leq [\sigma_H] \approx 696.62\,\text{MPa}$$

3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2\frac{T_{sh1}K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} \leq [\sigma_{F1}] \tag{3.2}$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{F2}] \tag{3.3}$$

Find Y_ϵ Knowing that $\epsilon_\alpha \approx 1.64$, we can calculate $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.75$

Find Y_β $Y_\beta = 1 - \dfrac{\beta}{140} \approx 0.86$

Find Y_F Using formula $z_v = z\cos^{-3}(\beta)$ and table (6.18):

$z_{v1} = z_1 \cos^{-3}(\beta) \approx 24.88 \Rightarrow Y_{F1} \approx 3.6$

$z_{v2} = z_2 \cos^{-3}(\beta) \approx 124.42 \Rightarrow Y_{F2} \approx 3.64$

Find K_F Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:

$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.82$

Find σ_F Since $m_n = m\cos\beta \approx 1.42$, substituting all the values, we find out that:

$$\sigma_{F1} \approx 179.07\,(\text{MPa}) \leq [\sigma_{F1}] \approx 264.85\,(\text{MPa})$$

$$\sigma_{F2} \approx 181.06\,(\text{MPa}) \leq [\sigma_{F2}] \approx 332.48\,(\text{MPa})$$

3.4.6 Force on shafts

$F_t = \dfrac{2T_{sh1}}{d_1} \approx 3003.15$ (N)

$F_r = F_t \tan \alpha_t \approx 1131.8$ (N)

$F_a = F_t \tan \beta \approx 1039.35$ (N)

In summary, we have the following table:

	pinion	driving gear
H (HB)	250	240
$[\sigma_H]$ (MPa)	621.61	771.63
$[\sigma_F]$ (MPa)	264.85	332.48
$[\sigma_H]_{max}$ (MPa)	696.62	
$[\sigma_F]_{max}$ (MPa)	440	
σ_H (MPa)	621.61	771.63
σ_F (MPa)	179.07	181.06
σ_H (MPa)	663.86	
α_{tw} (°)	20.65	
β (°)	19.09	
a_w (mm)	100	
b_w (mm)	50	
m (mm)	1.5	
z	21	105
d (mm)	33.33	166.67
d_a (mm)	37.23	168.77
d_f (mm)	30.48	162.02
d_b (mm)	31.32	156.62
v (m/s)	5	
u_{hg}	5	

Table 3.1: Gearbox specifications

Chapter 4

Shaft Design

4.1 Nomenclature

$[\tau]$ permissible torsion, MPa

r position of applied force on the shaft, mm

hr tooth direction

cb role of gear on the shaft (active or passive)

cq rotational direction of the shaft

σ_b ultimate strength, MPa

σ_{ch} yield limit, MPa

S safety factor

F_x applied force, N

F_t tangential force, N

F_r radial force, N

F_a axial force, N

a_w shaft distance, mm

d shaft diameter, mm

d_w gear diameter, mm

$_x$ subscript for x-axis

$_y$ subscript for y-axis

q standardized coefficient of shaft diameter

b_O rolling bearing width, mm

l_m hub diameter, mm

k_1 distance between elements, mm

k_2 distance between bearing surface and inner walls of the gearbox, mm

k_3 distance between element surface and bearing lid, mm

h_n distance between bearing lid and bolt, mm

T torque on shaft

α_{tw} meshing profile angle, °

β helix angle, °

$_1$ subscript for shaft 1

$_2$ subscript for shaft 2

4.2 Choose material

For moderate load, we will use quenched steel 40X to design the shafts. From table 6.1, the specifications are as follows: $S \leq 100\,(\text{mm})$, HB260, $\sigma_b = 850\,(\text{MPa})$, $\sigma_{ch} = 550\,(\text{MPa})$.

4.3 Tranmission Design

4.3.1 Load on shafts

Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.
On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:
$r_{12} = -d_{w12}/2 \approx -13.57\,(\text{mm})$, $hr_{12} = +1$, $cb_{12} = +1$, $cq_1 = +1$
$r_{21} = +d_{w21}/2 \approx +67.84\,(\text{mm})$, $hr_{21} = -1$, $cb_{21} = -1$, $cq_2 = -1$

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: $\alpha_{tw} \approx 21.17°$, $\beta = 20°$, $d_{w12} \approx 27.14\,(\text{mm})$

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 2769.03\,(\text{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha_{tw}}{\cos\beta} \approx 1141.36\,(\text{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta \approx 1007.84\,(\text{N}) \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{r_{12}}{|r_{12}|} cq_1 cb_{12} F_{t12} \approx -2769.03\,(\mathrm{N}) \\ F_{y12} = -\dfrac{r_{12}}{|r_{12}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t12} \approx 1141.36\,(\mathrm{N}) \\ F_{z12} = cq_1 cb_{12} hr_{12} F_{t12} \tan\beta \approx 1007.84\,(\mathrm{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{r_{21}}{|r_{21}|} cq_2 cb_{21} F_{t21} \approx 2769.03\,(\mathrm{N}) \\ F_{y21} = -\dfrac{r_{21}}{|r_{21}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t21} \approx -1141.36\,(\mathrm{N}) \\ F_{z21} = cq_2 cb_{21} hr_{21} F_{t21} \tan\beta \approx -1007.84\,(\mathrm{N}) \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2539.28\,(\mathrm{N})$ (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 210^\circ \approx -1269.64\,(\mathrm{N}) \\ F_{y22} = F_{r22} \sin 210^\circ \approx -2199.08\,(\mathrm{N}) \end{cases}$$

4.3.2 Preliminary calculations

Since shaft 1 receives input torque T_{sh1} and shaft 2 produces output torque T_{sh2}, $[\tau_1] = 15\,(\mathrm{MPa})$ and $[\tau_2] = 30\,(\mathrm{MPa})$. Using equation (10.9), we can approximate d_1 and d_2:

$d_1 \geq \sqrt[3]{\dfrac{T_{sh1}}{0.2[\tau_1]}} \approx 23.22\,(\mathrm{mm}) \Rightarrow d_1 = 25\,(\mathrm{mm})$

$d_2 \geq \sqrt[3]{\dfrac{T_{sh2}}{0.2[\tau_2]}} \approx 30.99\,(\mathrm{mm}) \Rightarrow d_2 = 35\,(\mathrm{mm})$

4.3.3 Identify the distance between bearings and applied forces

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

From table (10.2), we can estimate b_O. On shaft 1, $b_{O1} = 15\,(\mathrm{mm})$. On shaft 2,

$b_{O2} = 21$ (mm). Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 \approx 34.83$ (mm), $l_{m23} = l_{m22} = 1.5d_2 \approx 46.48$ (mm) (l_{m22} is the chain hub)

From table (10.3), we choose $k_1 = 10$ (mm), $k_2 = 8$ (mm), $k_3 = 15$ (mm), $h_n = 18$ (mm). This parameters apply for both shafts in the system.

Table (10.4) introduce the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the formulas below are used:

On shaft 1:

$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + k_3 + h_n\right] \approx -57.92$ (mm)

$l_{13} = 0.5(l_{m13} + b_{O1}) + k_1 + k_2 \approx 42.92$ (mm)

$l_{11} = 2l_{13} \approx 85.83$ (mm)

On shaft 2:

$l_{22} = -l_{c22} = -\left[0.5(l_{m22} + b_{O2}) + k_3 + h_n\right] \approx -66.74$ (mm)

$l_{23} = 0.5(l_{m23} + b_{O2}) + k_1 + k_2 \approx 51.74$ (mm)

$l_{21} = 2l_{23} \approx 103.48$ (mm)

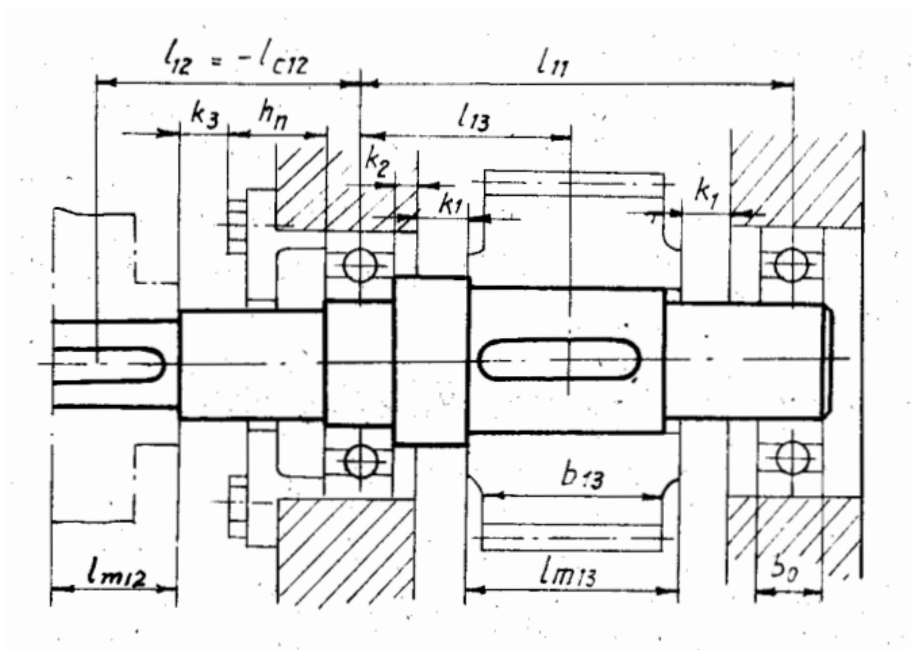

Figure 4.1: Shaft design and its dimensions

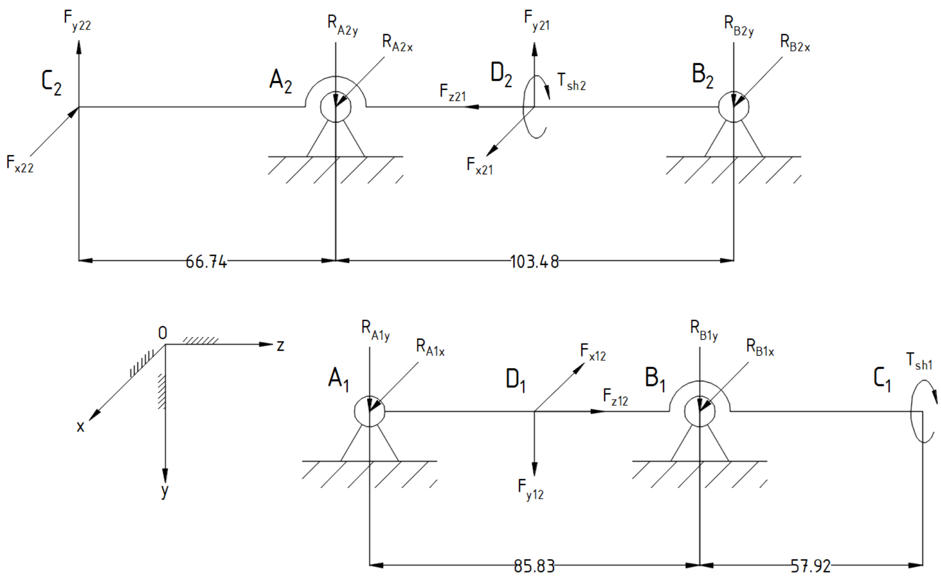

Figure 4.2: Force analysis of shafts

4.3.4 Determine shaft diameters and lengths

From the diagram, we derive the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions:

$$\begin{cases} \sum_i \mathbf{F_i} &= 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} &= 0 \end{cases}$$

We obtain the results as follows:

$$\begin{cases} R_{A1x} \approx 1384.51 \text{ (N)} \\ R_{A1y} \approx -570.68 \text{ (N)} \\ R_{B1x} \approx 1384.51 \text{ (N)} \\ R_{B1y} \approx -570.68 \text{ (N)} \end{cases} \qquad \begin{cases} R_{A2x} \approx 703.98 \text{ (N)} \\ R_{A2y} \approx 4188.06 \text{ (N)} \\ R_{B2x} \approx -2203.37 \text{ (N)} \\ R_{B2y} \approx -847.62 \text{ (N)} \end{cases}$$

From the reaction forces, we can easily draw shear force-bending moment diagram for both shafts on 2 major planes (xOz) and (yOz).

From equation (10.15), we calculate the total bending moment at point C_2, A_2, D_2, B_2, A_1, D_1, B_1, C_1

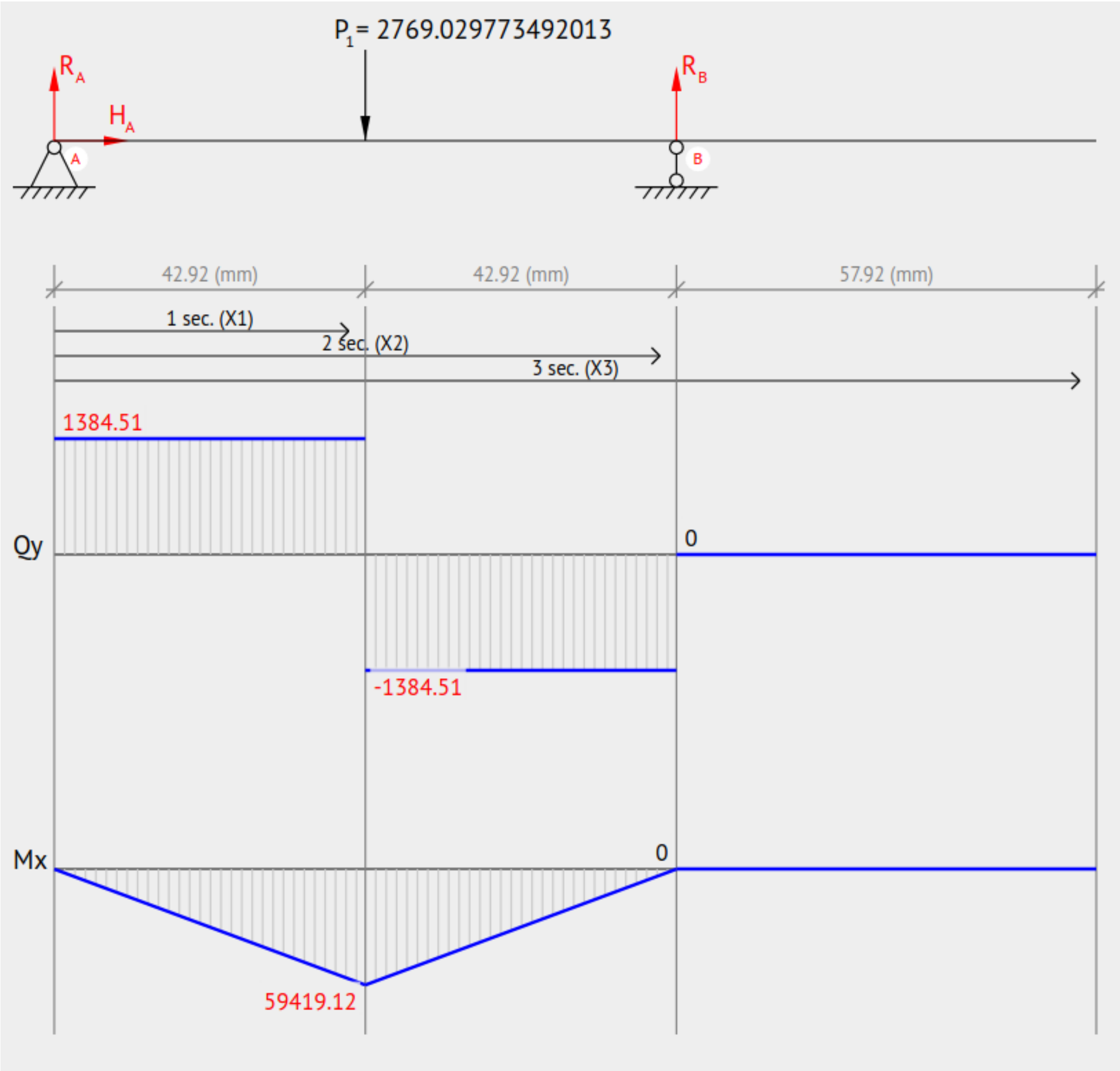

Figure 4.3: Shear force - Bending moment diagram on (xOz) of shaft 1

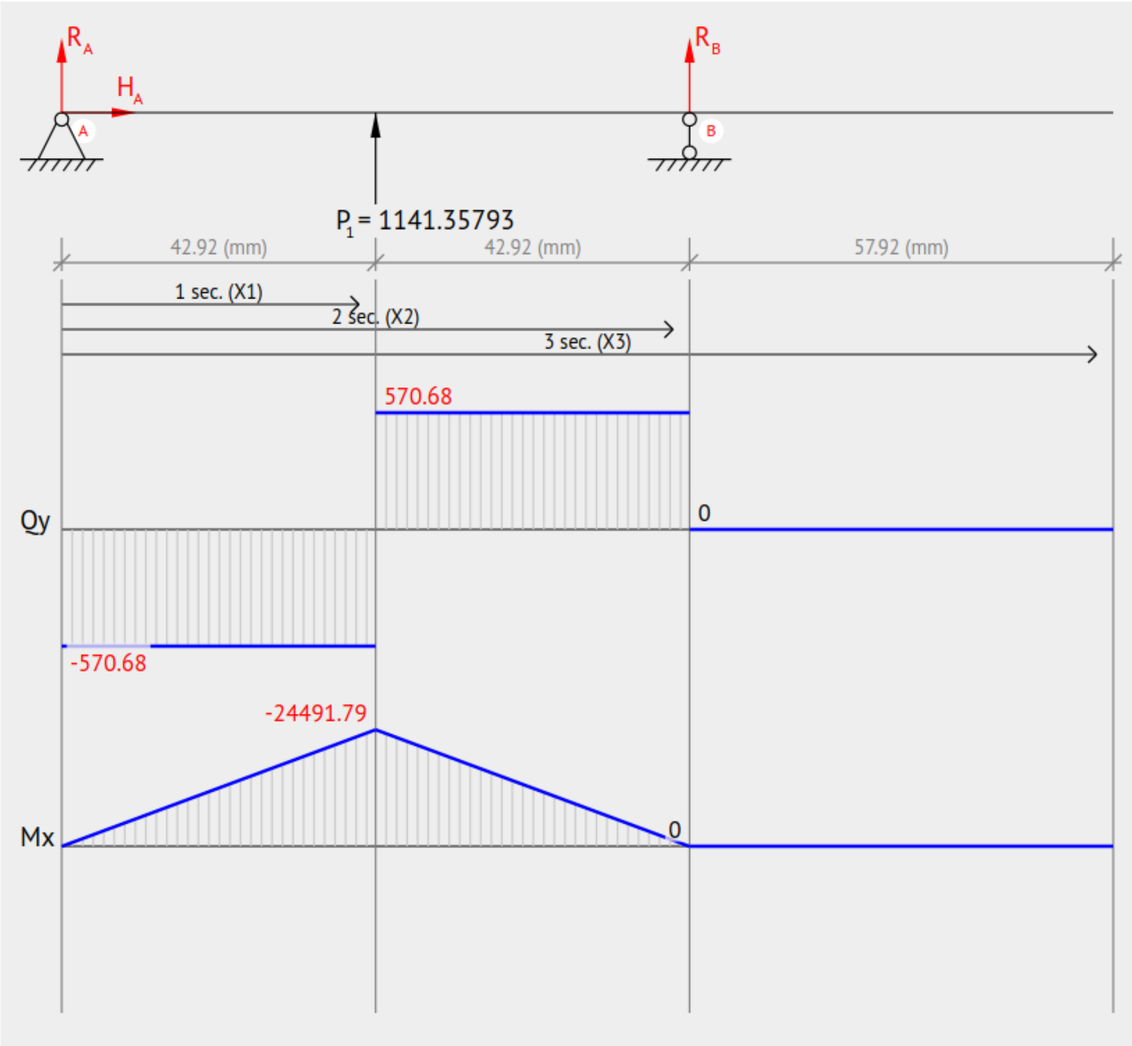

Figure 4.4: Shear force - bending moment diagram on (yOz) of shaft 1

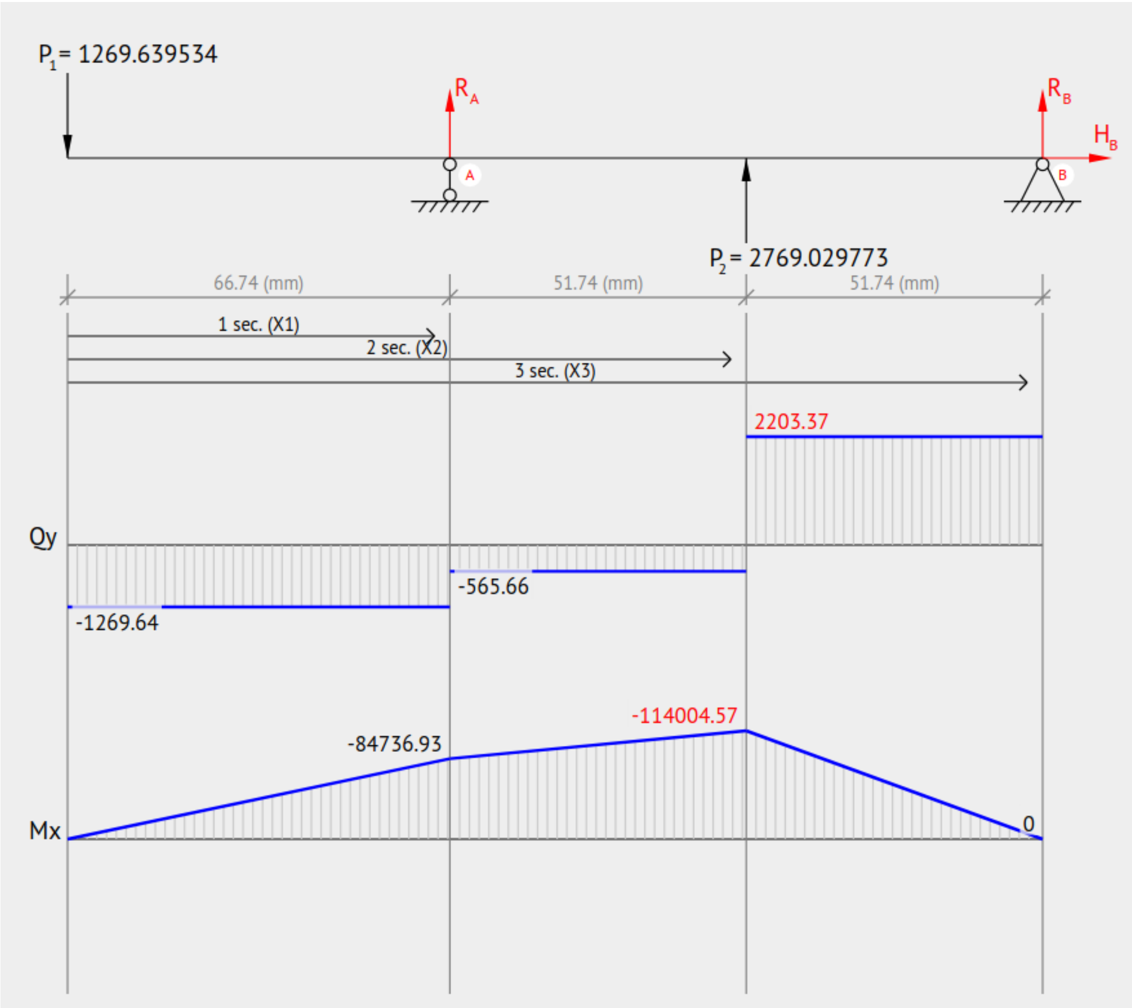

Figure 4.5: Shear force - Bending moment diagram on (xOz) of shaft 2

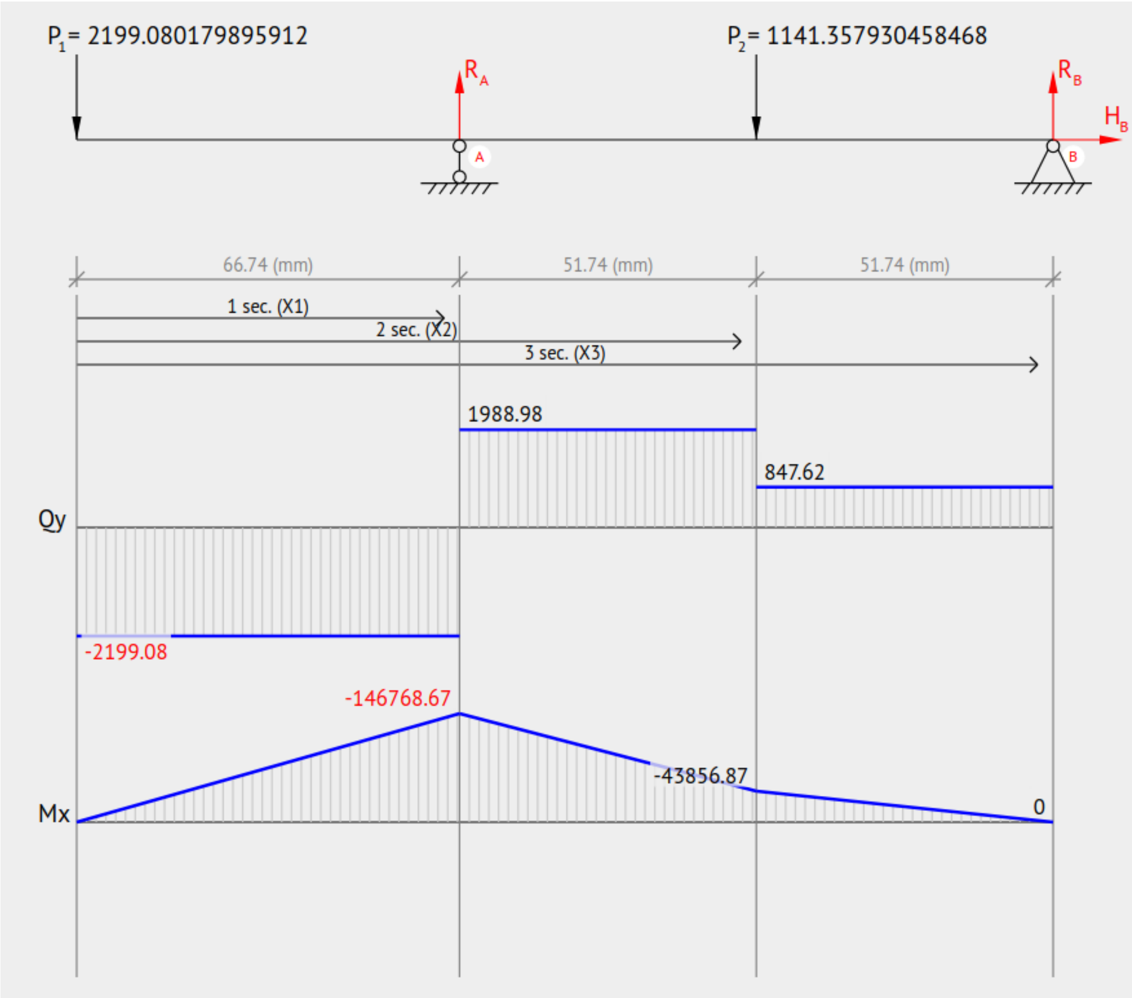

Figure 4.6: Shear force - Bending moment diagram on (yOz) of shaft 2