Machine Elements Report

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Contents

1 Motor Design **4**

1.1 Nomenclature 4

1.2 Calculate η_{sys} 4

1.3 Calculate P_{motor} 5

1.4 Calculate n_{motor} 5

1.5 Choose motor 5

1.6 Calculate power, rotational speed and torque of the motor and 2 shafts 6

1.6.1 Power 6

1.6.2 Rotational speed 6

1.6.3 Torque 6

2 Chain Drive Design **8**

2.1 Nomenclature 8

2.2 Find p 9

2.3 Find a, x_c and i 10

2.4 Strength of chain drive 10

2.5 Force on shaft 11

3 Gearbox Design (Helix gears) **12**

3.1 Nomenclature 12

3.2 Choose material 14

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$ 14

3.3.1 Working cycle of bearing stress 14
3.3.2 Working cycle of equivalent tensile stress 14
3.3.3 Aging factor 15
3.3.4 Calculate [σ_H] and [σ_F] 15
3.4 Transmission Design 15
3.4.1 Determine basic parameters 15
3.4.2 Determine gear meshing parameters 16
3.4.3 Other parameters 16
3.4.4 Contact stress analysis 17
3.4.5 Bending stress analysis 18

4 Shaft Design **20**
4.1 Nomenclature 20
4.2 Choose material 21
4.3 Tranmission Design 21
4.3.1 Load on shafts 21
4.3.2 Preliminary calculations 22
4.3.3 Identify the distance between bearings and applied forces . . 23
4.3.4 Determine shaft diameters and lengths 24

List of Tables

1.1 System properties 7

2.1 Table of chain drive specifications 11

3.1 Gearbox specifications 19

F_t tangential force, N
v conveyor belt speed, m/s
D pulley diameter, mm
L service life, year
T working torque, N · mm
t working time, s
δ_u error of speed ratio, %

Chapter 1

Motor Design

1.1 Nomenclature

η_c	coupling efficiency
η_b	bearing efficiency
η_{hg}	helical gear efficiency
η_{ch}	chain drive efficiency
η_{sys}	efficiency of the system
P_m	maximum operating power of belt conveyor, kW
P_w	opearting power of belt conveyor given a workload, kW
P_{motor}	calculated motor power to drive the system, kW
P_{sh}	operating power of shaft, kW
n_{bc}	rotational speed of belt conveyor, rpm
n_{sh}	rotational speed of shaft, rpm
u_{hg}	transmission ratio of helical gear
u_{ch}	transmission ratio of chain drive
u_{sys}	transmission ratio of the system
T_{motor}	motor torque, $\text{N} \cdot \text{mm}$
T_{sh}	shaft torque, $\text{N} \cdot \text{mm}$

1.2 Calculate η_{sys}

From table 2.3 :

$\eta_c = 1$, $\eta_b = 0.99$, $\eta_{hg} = 0.96$, $\eta_{ch} = 0.95$

$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$

1.3 Calculate P_{motor}

$P_m = \dfrac{F_t v}{1000} \approx 13.73\,(\mathrm{kW})$

From equation (2.13) :

$P_w = P_m \sqrt{\dfrac{\left(\dfrac{T_1}{T}\right)^2 t_1 + \left(\dfrac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41\,(\mathrm{kW})$

$P_{motor} = \dfrac{P_w}{\eta_{sys}} \approx 11.76\,(\mathrm{kW})$

1.4 Calculate n_{motor}

$n_{bc} = \dfrac{6 \times 10^4 v}{\pi D} \approx 116.5\,(\mathrm{rpm})$

$u_{ch} = 5$, $u_{hg} = 5$ (table 2.4)

$u_{sys} = u_{ch} u_{hg} = 25$

$n_{motor} = u_{sys} n_{bc} \approx 2912.54\,(\mathrm{rpm})$

1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated P_{motor} and P_m. Since $P_{motor} < P_m$ for our case, the minimum operating power of choice is P_m. In similar fashion, its rotational speed must also be no smaller than estimated n_{motor}.

Thus, from table P1.3, we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$\Rightarrow P_{motor} = 18.5\,\mathrm{kW}, n_{motor} = 2930\,(\mathrm{rpm})$

Recalculating u_{sys} with the new P_{motor} and n_{motor}, we obtain:

$u_{sys} = \dfrac{n_{motor}}{n_{bc}} \approx 25.15$

Assuming $u_{hg} = const$:

$u_{ch} = \dfrac{u_{sys}}{u_{hg}} \approx 5.03$

1.6 Calculate power, rotational speed and torque of the motor and 2 shafts

Let us denote P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

1.6.1 Power

$$P_{ch} = P_m \approx 13.72\,(\mathrm{kW})$$
$$P_{sh2} = \frac{P_{ch}}{\eta_{ch}} \approx 14.45\,(\mathrm{kW})$$
$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.2\,(\mathrm{kW})$$
$$P_{motor} = \frac{P_{sh1}}{\eta_b \eta_c} \approx 15.35\,(\mathrm{kW})$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930\,(\mathrm{rpm})$$
$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586\,(\mathrm{rpm})$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 50047.36\,(\mathrm{N} \cdot \mathrm{mm})$$
$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 49547.08\,(\mathrm{N} \cdot \mathrm{mm})$$
$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 235447.73\,(\mathrm{N} \cdot \mathrm{mm})$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	15.35	15.2	14.45
u	5	5.03	
n (rpm)	2930	2930	586
T ($\text{N} \cdot \text{mm}$)	50047.36	49547.08	235447.73

Table 1.1: System properties

Chapter 2

Chain Drive Design

2.1 Nomenclature

z	number of teeth on the driving sprocket	q	mass per meter of chain, kg/m
z_{max}	maximum number of teeth on the driven sprocket	v_1	driving sprocket speed, m/s
$[P]$	permissible power, kW	F_t	tangential force on shaft, N
p	sprocket pitch, mm	F_v	centrifugal force, N
p_{max}	permissible sprocket pitch, mm	F_0	tension from passive chain part, N
d	driving sprocket diameter, mm	F_r	force on shaft, N
d_c	pin diameter, mm	F_1	force from active side, N
B	bush length, mm	F_2	force from passive side, N
Q	permissible load, N	n_{ch}	rotational speed of chain drive, rpm
a	center distance, mm	n_{01}	experimental rotational speed, rpm
a_{min}	minimum center distance, mm	k_z	coefficient of number of teeth
a_{max}	maximum center distance, mm	k_n	coefficient of rotational speed
x	number of links	k	overall factor
x_c	an even number of links	k_0	arrangement of drive factor

i	impact times per second	k_a	center distance and chain's length factor
$[i]$	permissible impact times per second	k_{dc}	chain tension factor
s	safety factor	k_{bt}	lubrication factor
$[s]$	permissible safety factor	k_d	dynamic loads factor
$_1$	subscript for driving sprocket	k_c	rating factor
$_2$	subscript for driven sprocket	k_f	loosing factor
		k_x	chain weight factor

2.2 Find p

$n_1 = n_{sh2} = 586\,(\text{rpm})$

Find z Since z_1 and z_2 is preferably an odd number (p.80):

$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$

$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$

Because $z_1 \geq 15$, we will use table 5.8 and interpolation to approximate p_{max}. Therefore, $p_{max} \approx 33.5784\,(\text{mm})$

Find k Since $n_{ch} = 586 \approx 600\,(\text{rpm})$, choose $n_{01} = 600\,(\text{rpm})$, which is obtained from table 5.5. Then, we calculate k_z and k_n.

$k_z = \dfrac{25}{z_1} \approx 1.32,\ k_n = \dfrac{n_{01}}{n_{ch}} \approx 1.02$

Specifying the chain drive's working condition and ultizing table 5.6 , we find out that $k_0 = k_a = k_{dc} = k_{bt} = 1,\ k_d = 1.25,\ k_c = 1.3$

$\Rightarrow k = k_0k_ak_{dc}k_{bt}k_dk_c = 1.625$

Find p From table 5.5:

$[P] = P_{ch}kk_zk_n \approx 30.05\,(\text{kW}) \leq 42\,(\text{kW}) \Rightarrow [P] = 42\,(\text{kW})$

Using the table, we also get the other parameters:

$p = 31.75\,(\text{mm}),\ d_c = 9.55\,(\text{mm}),\ B = 27.46\,(\text{mm})$.

$$d_1 = \frac{p}{\sin\dfrac{\pi}{z_1}} \approx 192.9\,(\text{mm}),\ d_2 = \frac{p}{\sin\dfrac{\pi}{z_2}} \approx 980.49\,(\text{mm})$$

Having $p = 31.75\,(\text{mm}) \leq \mathrm{p_{max}} \approx 33.5784\,(\text{mm})$, we can safely choose the number of chains as 1. Hence, from table 5.2, we get the parameters in the sub-table 1:

$Q = 56.7 \times 10^3\,(\text{N}),\ q = 2.6\,(\text{kg/m})$

By comparison to the conditions in the sub-table, the choice of B is satisfactory.

2.3 Find a, x_c and i

Find x_c $a_{min} = 30p = 952.5\,(\text{mm})$, $a_{max} = 50p = 1587.5\,(\text{mm})$. Therefore, we can approximate $a = 1000\,(\text{mm})$

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 125.89 \Rightarrow x_c = 126$$

Find a From equation (5.13) , recalculating a with x_c:

$$a = \frac{p}{4}\left(x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\frac{(z_2 - z_1)^2}{\pi^2}}\right) - 0.003 \cdot 1000 \approx 998.98\,(\text{mm})$$

Find i From table 5.9:

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 25$$

2.4 Strength of chain drive

For moderate workload, choose $k_d = 1.2$. Let the chain drive be angled 30° with respect to ground, we obtain $k_f = 4$.

$$v_1 = \frac{n_{ch} p z_1}{6 \times 10^4} \approx 5.89\,(\text{m/s})$$

Find F_t, F_v, F_0 We also need to calculate F_t, F_v and F_0:

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2329.53\,(\text{N})$$

$$F_v = q v_1^2 \approx 90.25\,(\text{N})$$

$$F_0 = 9.81 \times 10^{-3} k_f q a \approx 101.92\,(\text{N})$$

Validate s From equation (5.15) :

$s = \dfrac{Q}{k_d F_t + F_0 + F_v} \approx 18.98 \geq [s] = 13.2$, where $[s]$ is chosen from table 5.10.

2.5 Force on shaft

From p.87:

$F_2 = F_0 + F_v \approx 192.17\,(\text{N})$

$F_1 = F_t + F_2 \approx 2521.7\,(\text{N})$

Choose $k_x = 1.15$ and follow equation (5.20) :

$F_r = k_x F_t \approx 2678.96\,(\text{N})$

In summary, we have the following table:

$[P]$ (kW)	42
n (rpm)	586
u_{ch}	5.03
z_1	19
z_2	97
p (mm)	31.75
d_1 (mm)	192.9
d_2 (mm)	980.49
d_c (mm)	9.55
B (mm)	27.46
x_c	126
a (mm)	998.98
i	6

Table 2.1: Table of chain drive specifications

Chapter 3

Gearbox Design (Helix gears)

3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa
$[\sigma_F]$	permissible bending stress, MPa
σ^o_{Hlim}	permissible contact stress corresponding to working cycle, MPa
σ^o_{Flim}	permissible bending stress corresponding to working cycle, MPa
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
H	surface roughness, HB
S	length, mm
S_H	safety factor of contact stress
S_F	safety factor of bending stress
N_{HO}	working cycle of bearing stress corresponding to $[\sigma_H]$
K_{HL}	aging factor due to contact stress
K_{FL}	aging factor due to bending stress
K_{FC}	load placement factor
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth
$K_{H\beta}$	factor of load distribution from contact stress on top land
K_{Hv}	factor of dynamic load from contact stress at meshing area
K_H	load factor from contact stress
$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
$K_{F\beta}$	factor of load distribution from bending stress on top land
K_{Fv}	factor of dynamic load from bending stress at meshing area
K_F	load factor from bending stress

N_{HE} working cycle of equivalent tensile stress corresponding to $[\sigma_H]$

N_{FO} working cycle of bearing stress corresponding to $[\sigma_F]$

N_{FE} working cycle of equivalent tensile stress corresponding to $[\sigma_F]$

AG accuracy grade of gear

z_M material's mechanical properties factor

z_H contact surface's shape factor

z_ϵ meshing condition factor

z_{min} minimum number of teeth corresponding to β

z_v equivalent number of teeth

ϵ_α horizontal meshing condition factor

ϵ_β vertical meshing condition factor

α base profile angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20°$

α_t profile angle of a gear tooth, °

α_{tw} meshing profile angle, °

β helix angle, °

β_b helix angle at base circle, °

ψ_{ba} width to shaft distance ratio

ψ_{bd} width to pinion diameter ratio

K_d coefficient of gear material

Y_ϵ meshing factor

Y_β helix angle factor

Y_F tooth shape factor

c gear meshing rate

a_w center distance, *mm*

b_w face width, *mm*

d pitch circle diameter, *mm*

d_w rolling circle diameter, *mm*

d_a addendum diameter, *mm*

d_f deddendum diameter, *mm*

d_b base diameter, *mm*

m_H root of fatigue curve in contact stress test

m_F root of fatigue curve in bending stress test

m traverse module, *mm*

m_n normal module, *mm*

v rotational velocity, m/s

x gear correction factor

y center displacement factor

$_1$ subscript for driving gear

$_2$ subscript for driven gear

3.2 Choose material

From table 6.1 , the material of choice for both gears is steel 40X with $S \leq 100\,\text{mm}$, HB250, $\sigma_b = 850\,\text{MPa}$, $\sigma_{ch} = 550\,\text{MPa}$.

Table 6.2 also gives $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness H.

For the driving gear, $H_1 = \text{HB250} \Rightarrow \sigma^o_{Hlim1} = 570\,\text{MPa}$, $\sigma^o_{Flim1} = 450\,\text{MPa}$

For the driven gear, $H_2 = \text{HB240} \Rightarrow \sigma^o_{Hlim2} = 550\,\text{MPa}$, $\sigma^o_{Flim2} = 432\,\text{MPa}$

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6$ cycles

$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6$ cycles

3.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq \text{HB350}$, $m_H = 6$, $m_F = 6$.

Both gears meshed indefinitely, thus $c = 1$.

Applying equation (6.7) and T_1, T_2, t_1, t_2 from the initial parameters:

$$N_{HE1} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 5.73 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 1.15 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 3.35 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 0.67 \times 10^6 \text{ cycles } (n_1 = n_2 = n_{sh2})$$

3.3.3 Aging factor

For steel, $N_FO = 4 \times 10^6$ MPa. Applying equation (6.3) and (6.4) yield:

$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$

$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$

$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$

$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$

3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction, $K_{FC} = 1$

$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_{H1} \approx 621.61$ MPa

$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_{H2} \approx 771.63$ MPa

$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC1} K_{FL1}/S_{F1} \approx 264.85$ MPa

$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC2} K_{FL2}/S_{F2} \approx 332.48$ MPa

$[\sigma_H]_{max} = \frac{1}{2}([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62\,\text{MPa} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$

Permissible bending stress during overload:

$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440$ MPa

3.4 Transmission Design

3.4.1 Determine basic parameters

Examine table 6.5 gives $K_a = 43$

Assuming symmetrical design, table 6.6 also gives $\psi_{ba} = 0.5$

$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$

From table 6.7 , using interpolation we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a_w using equation (6.15a) before following SEV229-75 standard:

$a_w = K_a(u_{hg} + 1)\sqrt[3]{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} = 83.84$ mm

3.4.2 Determine gear meshing parameters

Find m Applying equation (6.17) and choose m from table (6.8) :

$m = (0.01 \div 0.02)a_w = (0.84 \div 1.68)\,\text{mm} \Rightarrow m = 1.5\,\text{mm}$

Find z_1, z_2, a_w We have $\beta = \alpha = 20°$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1. From the result, z_1 is rounded to the nearest odd number (preferably a prime number).

$$z_1 = \frac{2a_w \cos\beta}{m(u+1)} \approx 17.51 \approx 17$$

$$z_2 = u_{hg} z_1 = 85$$

According to SEV229-75 standard, we choose $a_w = 80\,\text{mm}$

$\Rightarrow b_w = \psi_{ba} a_w = 40\,\text{mm}$

Find x_1, x_2 Let $\beta = 20°$, $z_{min} = 15$. Knowing that $y = \dfrac{a_w}{m} - \dfrac{z_1 + z_2}{2} = 0$, we conclude z_1 must not be smaller than 17 as mentioned by table 6.9 . Hence, there is no need for correction ($x_1 = x_2 = 0$) and $z_1 = 17$ satisfy the condition.

Find α_{tw} Since $y = 0 \Rightarrow \alpha_{tw} = \alpha_t = \tan^{-1}\dfrac{\tan\alpha}{\cos\beta} \approx 21.17°$ (p.105)

3.4.3 Other parameters

$$\alpha_t = \tan^{-1}\frac{\tan\alpha}{\cos\beta} \approx 21.17°$$

$$d_1 = \frac{mz_1}{\cos\beta} \approx 27.14\,\text{mm}$$

$$d_2 = \frac{mz_2}{\cos\beta} \approx 135.68\,\text{mm}$$

$$d_{a1} = d_1 + 2m \approx 30.14\,\text{mm}$$

$$d_{a2} = d_2 + 2m \approx 138.68\,\text{mm}$$

$$d_{f1} = d_1 - 2.5m \approx 23.39\,\text{mm}$$

$$d_{f2} = d_2 - 2.5m \approx 131.93\,\text{mm}$$

$$d_{b1} = d_1 \cos\alpha \approx 25.3\,\text{mm}$$

$$d_{b2} = d_2 \cos\alpha \approx 126.52\,\text{mm}$$

$$d_{w1} = d_1 \approx 27.14\,\text{mm}$$

$$d_{w2} = d_2 \approx 135.68\,\text{mm}$$

$$v = \frac{\pi d_1 n_1}{6 \times 10^4} \approx 4.16\,\text{m/s}$$

3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1} K_H \frac{u_{hg}+1}{b_w u_{hg} d_{w1}^2}} \leq [\sigma_H] \tag{3.1}$$

Find z_M $z_M = 274$, according to table 6.5

Find z_H Since correction is unused in our calculation:
$\beta_b = \arctan(\cos\alpha_t \tan\beta) \approx 18.75° \Rightarrow z_H = \sqrt{2\frac{\cos\beta_b}{\sin(2\alpha_t)}} \approx 1.68$

Find z_ϵ Obtaining z_ϵ through calculations:

$\epsilon_\alpha = \dfrac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_t}{2\pi m \dfrac{\cos\alpha_t}{\cos\beta}} \approx 1.64$

$\epsilon_\beta = b_w \dfrac{\sin\beta}{m\pi} \approx 2.9 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.78$

Find K_H We find K_H using equation $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table 6.13, $v \leq 10\,\mathrm{m/s} \Rightarrow \mathrm{AG} = 8$

From table P2.3, using interpolation, we approximate:

$K_{Hv} \approx 1.0417$, $K_{Fv} \approx 1.1145$

From table 6.14, using interpolation, we approximate:

$K_{H\alpha} \approx 1.0766$, $K_{F\alpha} \approx 1.253$

$\Rightarrow K_H \approx 1.24$

Find σ_H After calculating z_M, z_H, z_ϵ, K_H, we get the following result:

$$\sigma_H \approx 699.12\,\mathrm{MPa} \leq [\sigma_H] \approx 696.62\,\mathrm{MPa}$$

Since σ_H and $[\sigma_H]$ are almost equal to each other, i.e $\|\sigma_H - [\sigma_H]\| < 4\%$, the assumed parameters are appropriate.

3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2\frac{T_{sh1}K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} \leq [\sigma_{F1}] \tag{3.2}$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{F2}] \tag{3.3}$$

Find Y_ϵ Knowing that $\epsilon_\alpha \approx 1.64$, we can calculate $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.61$

Find Y_β Since $\beta = 20° \Rightarrow Y_\beta = 1 - \dfrac{\beta}{140} \approx 0.86$

Find Y_F Using formula $z_v = z\cos^{-3}(\beta)$ and table 6.18:

$z_{v1} = z_1 \cos^{-3}(\beta) \approx 20.49 \Rightarrow Y_{F1} \approx 4.06$

$z_{v2} = z_2 \cos^{-3}(\beta) \approx 102.44 \Rightarrow Y_{F2} \approx 3.6$

Find K_F Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:

$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.75$

Find σ_F Substituting all the values, we find out that:

$$\sigma_{F1} \approx 182.39\,\text{MPa} \leq [\sigma_{F1}] \approx 264.85\,\text{MPa}$$

$$\sigma_{F2} \approx 161.69\,\text{MPa} \leq [\sigma_{F2}] \approx 332.48\,\text{MPa}$$

The calculated results are appropriate.

Through calculations, there is no correction needed, i.e. $y = 0$. Thus, the specifications will not include corrections.

In summary, we have the following table:

	pinion	driving gear
H HB	250	240
$[\sigma_H]$ MPa	621.61	771.63
$[\sigma_F]$ MPa	264.85	332.48
$[\sigma_H]$ MPa	696.62	
σ_F MPa	182.39	161.69
σ_H MPa	699.12	
α_{tw} $^\circ$	21.17	
β $^\circ$	20	
a_w mm	80	
b_w mm	40	
m mm	1.5	
z	17	85
d mm	27.14	135.68
d_a mm	30.14	138.68
d_f mm	23.39	131.93
d_b mm	25.3	126.52

Table 3.1: Gearbox specifications

Chapter 4

Shaft Design

4.1 Nomenclature

$[\tau]$	permissible torsion, MPa
r	position of applied force on the shaft, mm
hr	tooth direction
cb	role of gear on the shaft (active or passive)
cq	rotational direction of the shaft
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
S	safety factor
F_x	applied force, N
F_t	tangential force, N
F_r	radial force, N
F_a	axial force, N
a_w	shaft distance, mm
d	shaft diameter, mm
d_w	gear diameter, mm
q	standardized coefficient of shaft diameter
b_O	rolling bearing width, mm
l_m	hub diameter, mm
k_1	distance between elements, mm
k_2	distance between bearing surface and inner walls of the gearbox, mm
k_3	distance between element surface and bearing lid, mm
h_n	distance between bearing lid and bolt, mm
T	torque on shaft
α_{tw}	meshing profile angle, °
β	helix angle, °
$_1$	subscript for shaft 1
$_2$	subscript for shaft 2

$_x$ subscript for x-axis

$_y$ subscript for y-axis

$_z$ subscript for z-axis

$_{sh1}$ subscript for shaft 1

$_{sh2}$ subscript for shaft 2

4.2 Choose material

For moderate load, we will use quenched steel 40X to design the shafts. From table 6.1, the specifications are as follows: $S \leq 100\,(\text{mm})$, HB260, $\sigma_b = 850\,(\text{MPa})$, $\sigma_{ch} = 550\,(\text{MPa})$.

4.3 Tranmission Design

4.3.1 Load on shafts

Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.

On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$r_{12} = -d_{w12}/2 \approx -13.57\,(\text{mm})$, $hr_{12} = +1$, $cb_{12} = +1$, $cq_1 = +1$

$r_{21} = +d_{w21}/2 \approx +67.84\,(\text{mm})$, $hr_{21} = -1$, $cb_{21} = -1$, $cq_2 = -1$

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: $\alpha_{tw} \approx 21.17°$, $\beta = 20°$, $d_{w12} \approx 27.14\,(\mathrm{mm})$

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 2769.03\,(\mathrm{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha_{tw}}{\cos\beta} \approx 1141.36\,(\mathrm{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta \approx 1007.84\,(\mathrm{N}) \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{r_{12}}{|r_{12}|} cq_1 cb_{12} F_{t12} \approx -2769.03\,(\mathrm{N}) \\ F_{y12} = -\dfrac{r_{12}}{|r_{12}|}\dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t12} \approx 1141.36\,(\mathrm{N}) \\ F_{z12} = cq_1 cb_{12} hr_{12} F_{t12}\tan\beta \approx 1007.84\,(\mathrm{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{r_{21}}{|r_{21}|} cq_2 cb_{21} F_{t21} \approx 2769.03\,(\mathrm{N}) \\ F_{y21} = -\dfrac{r_{21}}{|r_{21}|}\dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t21} \approx -1141.36\,(\mathrm{N}) \\ F_{z21} = cq_2 cb_{21} hr_{21} F_{t21}\tan\beta \approx -1007.84\,(\mathrm{N}) \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2539.28$ (N) (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22}\cos 210° \approx -1269.64\,(\mathrm{N}) \\ F_{y22} = F_{r22}\sin 210° \approx -2199.08\,(\mathrm{N}) \end{cases}$$

4.3.2 Preliminary calculations

Since shaft 1 receives input torque T_{sh1} and shaft 2 produces output torque T_{sh2}, $[\tau_1] = 15\,(\mathrm{MPa})$ and $[\tau_2] = 30\,(\mathrm{MPa})$. Using equation (10.9), we can approximate d_1 and d_2:

$d_1 \geq \sqrt[3]{\dfrac{T_{sh1}}{0.2[\tau_1]}} \approx 23.22\,(\mathrm{mm}) \Rightarrow d_1 = 25\,(\mathrm{mm})$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 30.99\,(\text{mm}) \Rightarrow d_2 = 35\,(\text{mm})$$

4.3.3 Identify the distance between bearings and applied forces

In this section, we will find all the parameters in Figure 4.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

From table (10.2), we can estimate b_O. On shaft 1, $b_{O1} = 15\,(\text{mm})$. On shaft 2, $b_{O2} = 21\,(\text{mm})$. Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 \approx 34.83\,(\text{mm})$, $l_{m23} = l_{m22} = 1.5d_2 \approx 46.48\,(\text{mm})$ (l_{m22} is the chain hub)

From table (10.3), we choose $k_1 = 10\,(\text{mm})$, $k_2 = 8\,(\text{mm})$, $k_3 = 15\,(\text{mm})$, $h_n = 18\,(\text{mm})$. This parameters apply for both shafts in the system.

Table (10.4) introduce the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the formulas below are used:

On shaft 1:

$l_{12} = -l_{c12} = -\left[0.5(l_{m12} + b_{O1}) + k_3 + h_n\right] \approx -57.92\,(\text{mm})$

$l_{13} = 0.5(l_{m13} + b_{O1}) + k_1 + k_2 \approx 42.92\,(\text{mm})$

$l_{11} = 2l_{13} \approx 85.83\,(\text{mm})$

On shaft 2:

$l_{22} = -l_{c22} = -\left[0.5(l_{m22} + b_{O2}) + k_3 + h_n\right] \approx -66.74\,(\text{mm})$

$l_{23} = 0.5(l_{m23} + b_{O2}) + k_1 + k_2 \approx 51.74\,(\text{mm})$

$l_{21} = 2l_{23} \approx 103.48\,(\text{mm})$

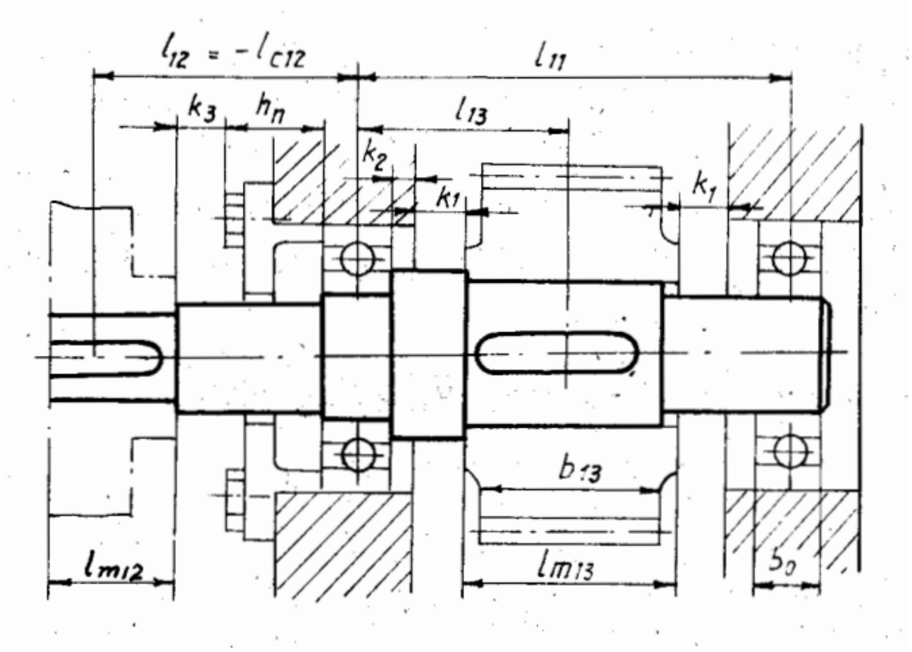

Figure 4.1: Shaft design and its dimensions

4.3.4 Determine shaft diameters and lengths

From the diagram, we derive the reaction forces at A_1, A_2, B_1, B_2, which are R_{A1x}, R_{A1y}, R_{B1x}, R_{B1y}, R_{A2x}, R_{A2y}, R_{B2x}, R_{B2y}. Using equilibrium conditions:

$$\begin{cases} \sum_i \mathbf{F_i} & = 0 \\ \sum_i \mathbf{r_i} \times \mathbf{F_i} & = 0 \end{cases}$$

We obtain the results as follows:

$$\begin{cases} R_{A1x} \approx 1384.51\,(\mathrm{N}) \\ R_{A1y} \approx -570.68\,(\mathrm{N}) \\ R_{B1x} \approx 1384.51\,(\mathrm{N}) \\ R_{B1y} \approx -570.68\,(\mathrm{N}) \end{cases} \qquad \begin{cases} R_{A2x} \approx 703.98\,(\mathrm{N}) \\ R_{A2y} \approx 4188.06\,(\mathrm{N}) \\ R_{B2x} \approx -2203.37\,(\mathrm{N}) \\ R_{B2y} \approx -847.62\,(\mathrm{N}) \end{cases}$$

From the reaction forces, we can easily draw shear force-bending moment diagram for both shafts on 2 major planes (xOz) and (yOz). Let us remind that the symbols of the figures below does not relate to any forces or moments mentioned in the nomenclature section. Hence, we will only focus on the form of the diagrams.

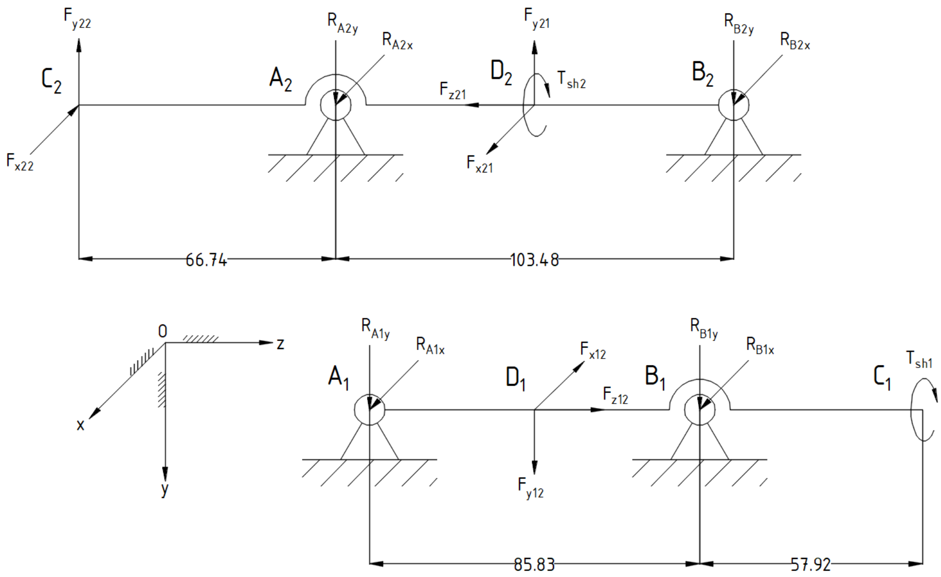

Figure 4.2: Force analysis of shafts

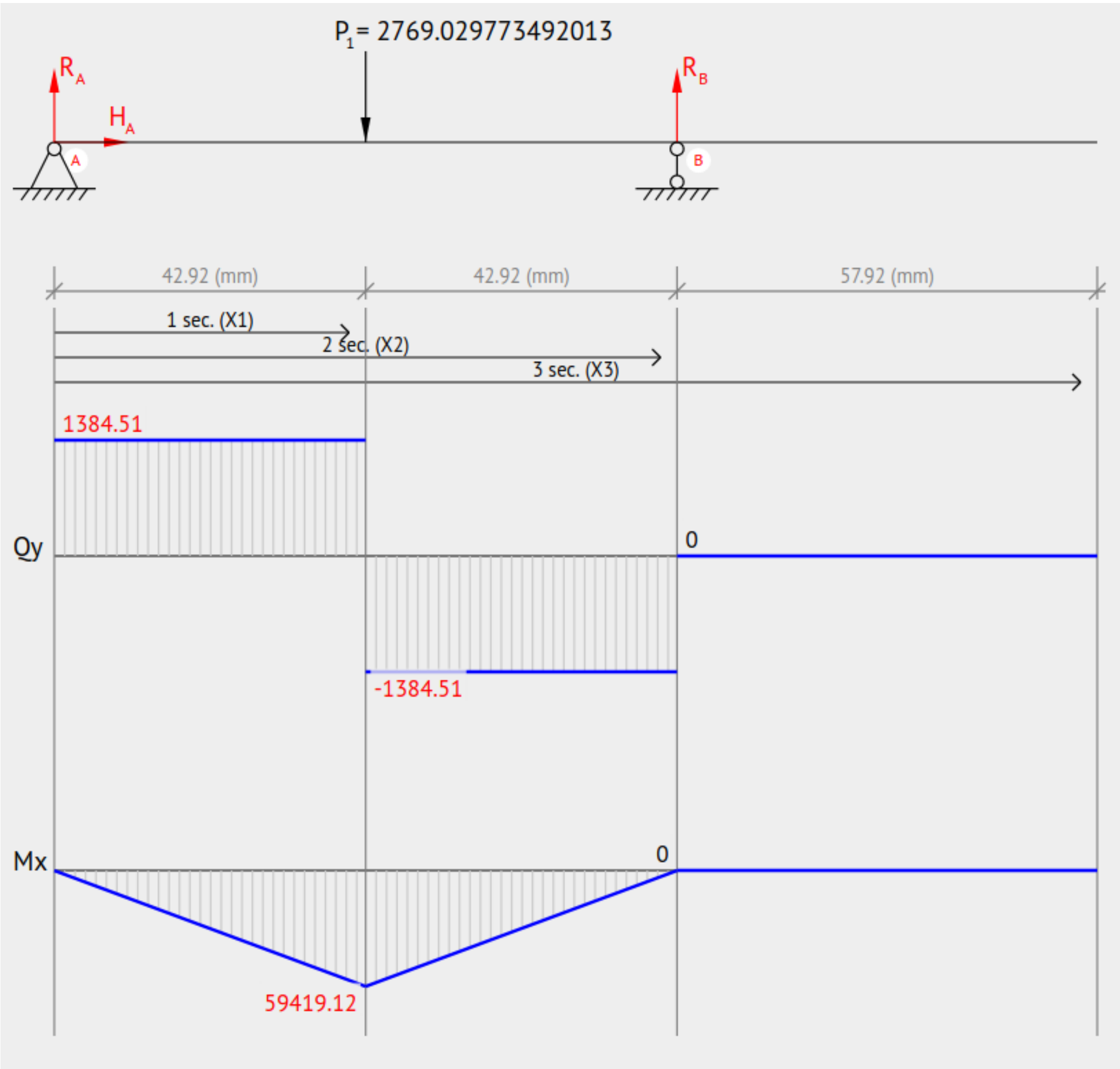

Figure 4.3: Shear force - Bending moment diagram on (xOz) of shaft 1

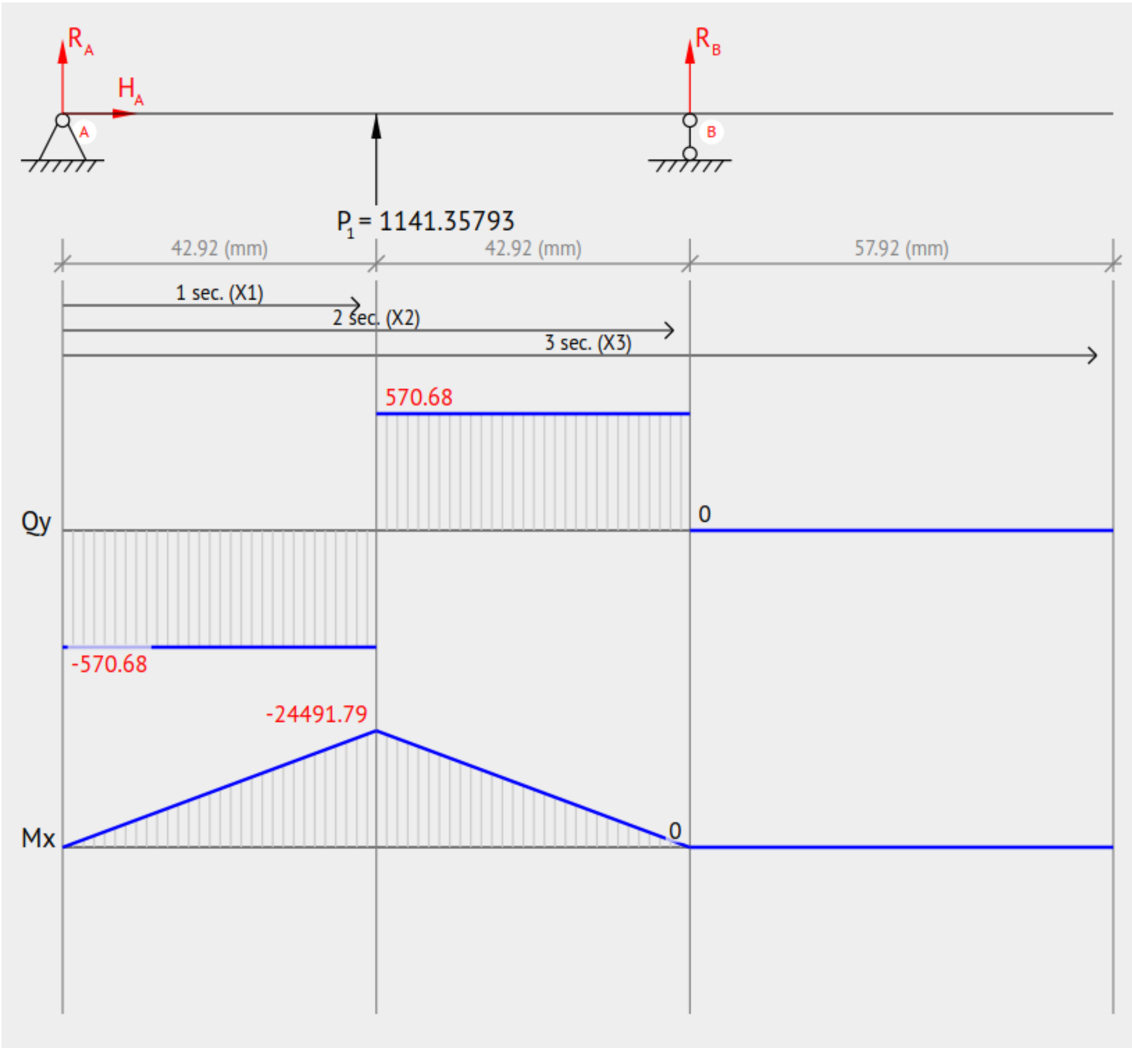

Figure 4.4: Shear force - bending moment diagram on (yOz) of shaft 1

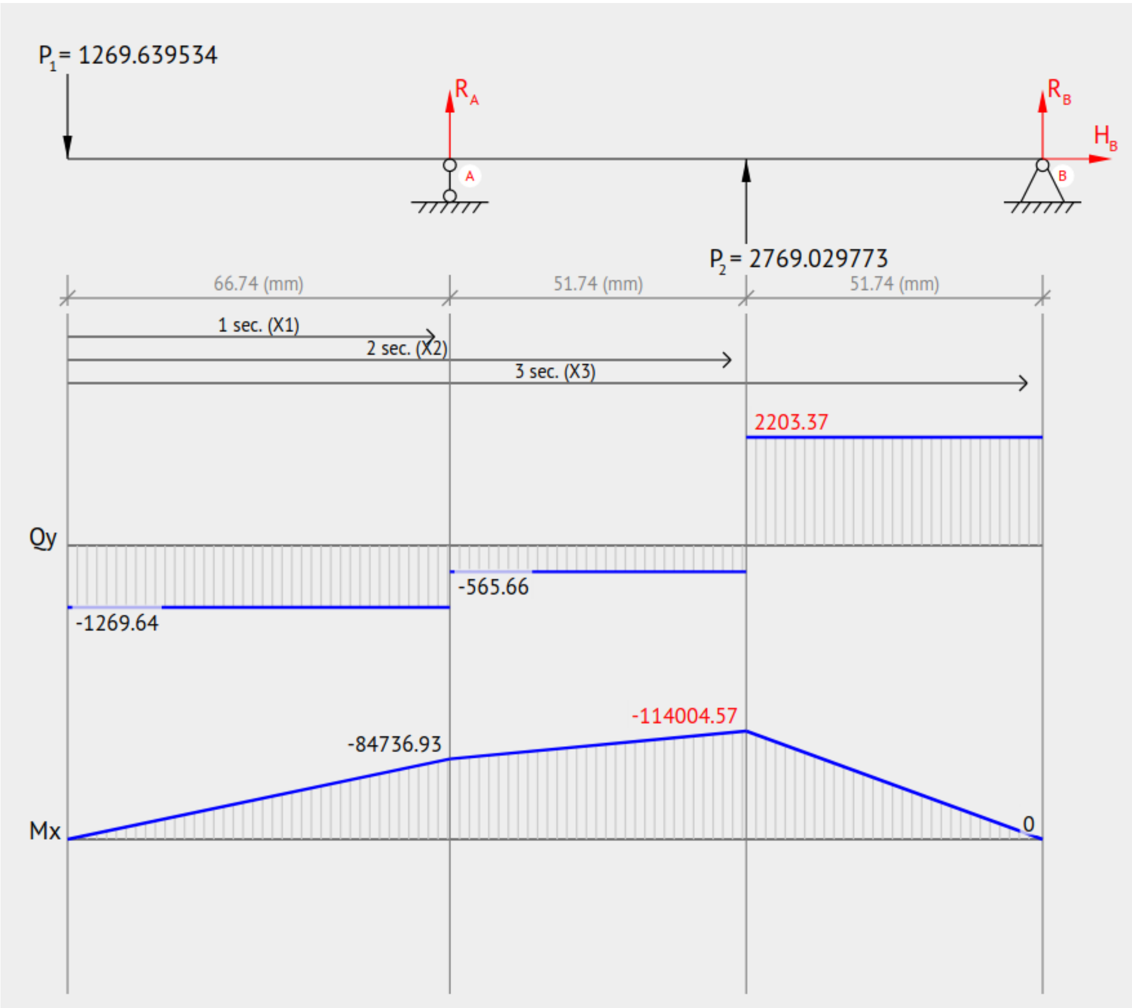

Figure 4.5: Shear force - Bending moment diagram on (xOz) of shaft 2

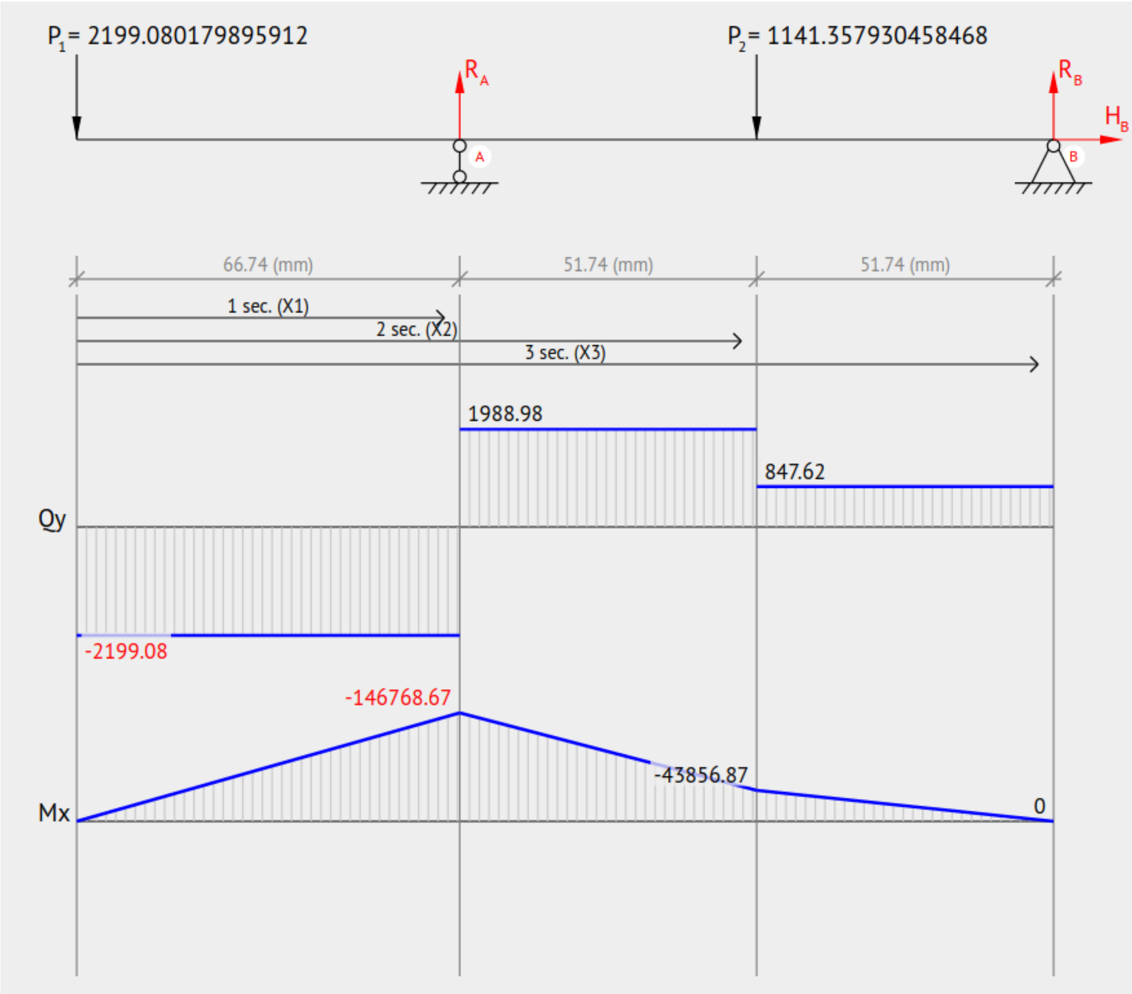

Figure 4.6: Shear force - Bending moment diagram on (yOz) of shaft 2