Machine Elements Report

June 23, 2020

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Chapter 1

Motor Design

1.1 Nomenclature

η_c	coupling efficiency
η_b	bearing efficiency
η_{hg}	helical gear efficiency
η_{ch}	chain drive efficiency
η_{sys}	efficiency of the system
P_m	maximum operating power of belt conveyor, kW
P_w	opearting power of belt conveyor given a workload, kW
P_{motor}	calculated motor power to drive the system, kW
P_{sh}	operating power of shaft, kW
n_{bc}	rotational speed of belt conveyor, rpm
n_{sh}	rotational speed of shaft, rpm
u_{hg}	transmission ratio of helical gear
u_{ch}	transmission ratio of chain drive
u_{sys}	transmission ratio of the system
T_{motor}	motor torque, $\mathrm{N \cdot mm}$
T_{sh}	shaft torque, $\mathrm{N \cdot mm}$

1.2 Calculate η_{sys}

From table 2.3 :

$\eta_c = 1, \eta_b = 0.99, \eta_{hg} = 0.96, \eta_{ch} = 0.95$

$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$

1.3 Calculate P_{motor}

$$P_m = \frac{F_t v}{1000} \approx 13.73\ (\text{kW})$$

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41\ (\text{kW})$$

$$P_{motor} = \frac{P_w}{\eta_{sys}} \approx 11.76\ (\text{kW})$$

1.4 Calculate n_{motor}

$$n_{bc} = \frac{6 \times 10^4 v}{\pi D} \approx 116.5\ (\text{rpm})$$

$u_{ch} = 5$, $u_{hg} = 5$ (table 2.4)

$$u_{sys} = u_{ch} u_{hg} = 25$$

$$n_{motor} = u_{sys} n_{bc} \approx 2912.54\ (\text{rpm})$$

1.5 Choose motor

The operating power and rotational speed of the chosen motor must be larger than estimated P_{motor} and n_{motor}, respectively. Thus, from table P1.3, we choose motor 4A160S2Y3 operating at 15 kW and 2930 rpm

$\Rightarrow P_{motor} = 15\,\text{kW}, n_{motor} = 2930\ (\text{rpm})$

Recalculating u_{sys} with the new P_{motor} and n_{motor}, we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming $u_{hg} = const$:

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

1.6 Calculate power, rotational speed and torque of the motor and 2 shafts

1.6.1 Power

$$P_{ch} = P_w \approx 10.41 \text{ (kW)}$$
$$P_{sh2} = \frac{P_{ch}}{\eta_{ch}} \approx 10.96 \text{ (kW)}$$
$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 11.53 \text{ (kW)}$$
$$P_{motor} = \frac{P_{sh1}}{\eta_b \eta_c} \approx 11.64 \text{ (kW)}$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$
$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 37950.46 \text{ (N} \cdot \text{mm)}$$
$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 37570.93 \text{ (N} \cdot \text{mm)}$$
$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 178537.08 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	11.64	11.527	10.96
u	5	5.03	
n (rpm)	2930	2930	586
T (N · mm)	37950.44	37570.93	178537.08

Table 1.1: System properties

Chapter 2

Chain Drive Design

2.1 Nomenclature

z_1	number of teeth on the driving sprocket
z_2	number of teeth on the driven sprocket
z_{max}	maximum number of teeth on the driven sprocket
$[P]$	permissible power, kW
p	sprocket pitch, mm
d_1	driving sprocket diameter, mm
d_2	driven sprocket diameter, mm
d_c	pin diameter, mm
B	bush length, mm
Q	permissible load, N
a	center distance, mm
a_{min}	minimum center distance, mm
a_{max}	maximum center distance, mm
q	mass per meter of chain, kg/m
v_1	driving sprocket speed, m/s
F_t	tangential force on shaft, N
F_v	centrifugal force, N
F_0	tension from passive chain part, N
F_r	force on shaft, N
n_{ch}	rotational speed of chain drive, rpm
n_{01}	experimental rotational speed, rpm
k_z	coefficient of number of teeth
k_n	coefficient of rotational speed
k	overall factor
k_0	arrangement of drive factor
k_a	center distance and chain's length factor

x	number of links	k_{dc}	chain tension factor
x_c	an even number of links	k_{bt}	lubrication factor
i	impact times per second	k_d	dynamic loads factor
$[i]$	permissible impact times per second	k_c	rating factor
		k_f	loosing factor
s	safety factor	k_x	chain weight factor
$[s]$	permissible safety factor		

2.2 Find p

$n_{ch} = n_{sh2} = 586$ (rpm)

Find z Since z_1 and z_2 is preferably an odd number (p.80):

$z_1 = 29 - 2u_{ch} = 18.94 \approx 19$

$z_2 = u_{ch}z_1 = 95.57 \approx 97 \leq z_{max} = 120$

Find k Since $n_{ch} = 586 \approx 600$ (rpm), choose $n_{01} = 600$ (rpm), which is obtained from table 5.5. Then, we calculate k_z and k_n

$k_z = \frac{25}{z_1} \approx 1.32$, $k_n = \frac{n_{01}}{n_{ch}} \approx 1.02$

Specifying the chain drive's working condition and ultizing table 5.6 , we find out that $k_0 = k_a = k_{dc} = k_{bt} = 1$, $k_d = 1.25$, $k_c = 1.3$

$\Rightarrow k = k_0 k_a k_{dc} k_{bt} k_d k_c = 1.625$

Find p From table 5.5:

$[P] = P_{ch} k k_z k_n \approx 22.78$ (kW) ≤ 25.7 (kW) $\Rightarrow [P] = 25.7$ (kW)

Using the table, we also get the other parameters:

$p = 25.4$ (mm), $d_c = 7.95$ (mm), $B = 22.61$ (mm),

$d_1 = \dfrac{p}{\sin \dfrac{\pi}{z_1}} \approx 154.32$ (mm), $d_2 = \dfrac{p}{\sin \dfrac{\pi}{z_2}} \approx 784.39$ (mm)

2.3 Find a, x_c and i

Find x_c $a_{min} = 30p = 762$ (mm), $a_{max} = 50p = 1270$ (mm). Therefore, we can approximate $a = 800$ (mm)

$$x = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \approx 123.71 \Rightarrow x_c = 124$$

Find a From equation (5.13) , recalculating a with x_c:

$$a = \frac{p}{4}\left(x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\frac{(z_2 - z_1)^2}{\pi^2}}\right) - 0.003 \cdot 800 \approx 771.66 \text{ (mm)}$$

Find i From table 5.9:

$$i = \frac{z_1 n_{sh2}}{15x} \approx 6 < [i] = 30$$

2.4 Strength of chain drive

Choose $k_d = 1.2$, $k_f = 6$

Given p from previous calculations, Q and q are obtained from table 5.2 :

$Q = 56.7 \times 10^3$ (N), $q = 2.6$ (kg/m)

$$v_1 = \frac{n_{ch} p z_1}{6 \times 10^4} \approx 4.71 \text{ (m/s)}$$

Find F_t, F_v, F_0 We also need to calculate F_t, F_v and F_0

$$F_t = \frac{10^3 P_{ch}}{v_1} \approx 2208.07 \text{ (N)}$$

$$F_v = q v_1^2 \approx 57.76 \text{ (N)}$$

$$F_0 = 9.81 \times 10^{-3} k_f q a \approx 118.09 \text{ (N)}$$

Validate s From equation (5.15) :

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \approx 20.07 \geq [s] = 10.3, \text{ where } [s] \text{ is chosen from table 5.10}$$

2.5 Force on shaft

Choose $k_x = 1.15$ and follow equation (5.20) :

$F_r = k_x F_t \approx 2539.28$ (N)

In su(mm)ary, we have the following table:

$[P]$ (kW)	25.7
n (rpm)	586
u_{ch}	5.03
z_1	19
z_2	97
p (mm)	25.4
d_1 (mm)	154.32
d_2 (mm)	784.39
d_c (mm)	7.95
B (mm)	22.61
x_c	124
a (mm)	771.66
i	6

Table 2.1: Table of chain drive specifications

Chapter 3

Gearbox Design (Helix gears)

3.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa
$[\sigma_F]$	permissible bending stress, MPa
σ^o_{Hlim}	permissible contact stress corresponding to working cycle, MPa
σ^o_{Flim}	permissible bending stress corresponding to working cycle, MPa
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
H	surface roughness, HB
S	length, mm
S_H	safety factor of contact stress
S_F	safety factor of bending stress
N_{HO}	working cycle of bearing stress corresponding to $[\sigma_H]$
K_{HL}	aging factor due to contact stress
K_{FL}	aging factor due to bending stress
K_{FC}	load placement factor
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth
$K_{H\beta}$	factor of load distribution from contact stress on top land
K_{Hv}	factor of dynamic load from contact stress at meshing area
K_H	load factor from contact stress
$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
$K_{F\beta}$	factor of load distribution from bending stress on top land
K_{Fv}	factor of dynamic load from bending stress at meshing area
K_F	load factor from bending stress

N_{HE} working cycle of equivalent tensile stress corresponding to $[\sigma_H]$

N_{FO} working cycle of bearing stress corresponding to $[\sigma_F]$

N_{FE} working cycle of equivalent tensile stress corresponding to $[\sigma_F]$

AG accuracy grade of gear

z_M material's mechanical properties factor

z_H contact surface's shape factor

z_ϵ meshing condition factor

z_{min} minimum number of teeth corresponding to β

z_v equivalent number of teeth

ϵ_α horizontal meshing condition factor

ϵ_β vertical meshing condition factor

α base profile angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20°$

α_t profile angle of a gear tooth, °

α_{tw} meshing profile angle, °

β helix angle, °

β_b helix angle at base circle, °

ψ_{ba} width to shaft distance ratio

ψ_{bd} width to pinion diameter ratio

K_d coefficient of gear material

Y_ϵ meshing factor

Y_β helix angle factor

Y_F tooth shape factor

c gear meshing rate

a_w center distance, mm

b_w face width, mm

d pitch circle diameter, mm

d_w rolling circle diameter, mm

d_a addendum diameter, mm

d_f deddendum diameter, mm

d_b base diameter, mm

m_H root of fatigue curve in contact stress test

m_F root of fatigue curve in bending stress test

m traverse module, mm

m_n normal module, mm

v rotational velocity, m/s

x gear correction factor

y center displacement factor

$_1$ subscript for driving gear

$_2$ subscript for driven gear

3.2 Choose material

From table 6.1 , the material of choice for both gears is steel 40X with $S \leq 100\,\text{mm}$, HB250, $\sigma_b = 850\,\text{MPa}$, $\sigma_{ch} = 550\,\text{MPa}$.

Table 6.2 also gives $\sigma^o_{Hlim} = 2\text{HB} + 70$, $S_H = 1.1$, $\sigma^o_{Flim} = 1.8\text{HB}$, $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness H.

For the driving gear, $H_1 = \text{HB}250 \Rightarrow \sigma^o_{Hlim1} = 570\,\text{MPa}$, $\sigma^o_{Flim1} = 450\,\text{MPa}$

For the driven gear, $H_2 = \text{HB}240 \Rightarrow \sigma^o_{Hlim2} = 550\,\text{MPa}$, $\sigma^o_{Flim2} = 432\,\text{MPa}$

3.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

3.3.1 Working cycle of bearing stress

Using equation (6.5) :

$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6\,\text{cycles}$

$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6\,\text{cycles}$

3.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq \text{HB}350$, $m_H = 6$, $m_F = 6$.

Both gears meshed indefinitely, thus $c = 1$.

Applying equation (6.7) and T_1, T_2, t_1, t_2 from the initial parameters:

$$N_{HE1} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 5.73 \times 10^6\,\text{cycles}\ (n_1 = n_2 = n_{sh1})$$

$$N_{HE2} = 60c\left[\left(\frac{T_1}{T}\right)^3 n_1t_1 + \left(\frac{T_2}{T}\right)^3 n_2t_2\right] \approx 1.15 \times 10^6\,\text{cycles}\ (n_1 = n_2 = n_{sh2})$$

$$N_{FE1} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 3.35 \times 10^6\,\text{cycles}\ (n_1 = n_2 = n_{sh1})$$

$$N_{FE2} = 60c\left[\left(\frac{T_1}{T}\right)^{m_F} n_1t_1 + \left(\frac{T_2}{T}\right)^{m_F} n_2t_2\right] \approx 0.67 \times 10^6\,\text{cycles}\ (n_1 = n_2 = n_{sh2})$$

3.3.3 Aging factor

For steel, $N_FO = 4 \times 10^6$ MPa. Applying equation (6.3) and (6.4) yield:

$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 1.2$

$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 1.54$

$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 1.03$

$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 1.35$

3.3.4 Calculate $[\sigma_H]$ and $[\sigma_F]$

Since the motor works in one direction, $K_{FC} = 1$

$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_{H1} \approx 621.61$ MPa

$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_{H2} \approx 771.63$ MPa

$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC1} K_{FL1}/S_{F1} \approx 264.85$ MPa

$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC2} K_{FL2}/S_{F2} \approx 332.48$ MPa

$[\sigma_H]_{max} = \frac{1}{2}([\sigma_{H1}] + [\sigma_{H2}]) \approx 696.62\,\text{MPa} \le 1.25[\sigma_H]_{min} = 1.25[\sigma_{H1}]$

Permissible bending stress during overload:

$[\sigma_F]_{max} = [\sigma_{F2}] = 0.8\sigma_{ch} \approx 440$ MPa

3.4 Transmission Design

3.4.1 Determine basic parameters

Examine table 6.5 gives $K_a = 43$

Assuming symmetrical design, table 6.6 also gives $\psi_{ba} = 0.5$

$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$

From table 6.7 , using interpolation we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a_w using equation (6.15a) before following SEV229-75 standard:

$$a_w = K_a(u_{hg} + 1)\sqrt[3]{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} = 83.84\,\text{mm}$$

3.4.2 Determine gear meshing parameters

Find m Applying equation (6.17) and choose m from table (6.8) :

$m = (0.01 \div 0.02)a_w = (0.84 \div 1.68)\,\text{mm} \Rightarrow m = 1.5\,\text{mm}$

Find z_1, z_2, a_w We have $\beta = \alpha = 20°$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1. From the result, z_1 is rounded to the nearest odd number (preferably a prime number).

$z_1 = \dfrac{2a_w \cos\beta}{m(u+1)} \approx 17.51 \approx 17$

$z_2 = u_{hg} z_1 = 85$

According to SEV229-75 standard, we choose $a_w = 80\,\text{mm}$

$\Rightarrow b_w = \psi_{ba} a_w = 40\,\text{mm}$

Find x_1, x_2 Let $\beta = 20°$, $z_{min} = 15$. Knowing that $y = \dfrac{a_w}{m} - \dfrac{z_1 + z_2}{2} = 0$, we conclude z_1 must not be smaller than 17 as mentioned by table 6.9 . Hence, there is no need for correction ($x_1 = x_2 = 0$) and $z_1 = 17$ satisfy the condition.

Find α_{tw} Since $y = 0 \Rightarrow \alpha_{tw} = \alpha_t = \tan^{-1}\dfrac{\tan\alpha}{\cos\beta} \approx 21.17°$ (p.105)

3.4.3 Other parameters

$\alpha_t = \tan^{-1}\dfrac{\tan\alpha}{\cos\beta} \approx 21.17°$

$d_1 = \dfrac{mz_1}{\cos\beta} \approx 27.14\,\text{mm}$

$d_2 = \dfrac{mz_2}{\cos\beta} \approx 135.68\,\text{mm}$

$d_{a1} = d_1 + 2m \approx 30.14\,\text{mm}$

$d_{a2} = d_2 + 2m \approx 138.68\,\text{mm}$

$d_{f1} = d_1 - 2.5m \approx 23.39\,\text{mm}$

$d_{f2} = d_2 - 2.5m \approx 131.93\,\text{mm}$

$d_{b1} = d_1 \cos\alpha \approx 25.3\,\text{mm}$

$d_{b2} = d_2 \cos\alpha \approx 126.52\,\text{mm}$

$d_{w1} = d_1 \approx 27.14\,\text{mm}$

$d_{w2} = d_2 \approx 135.68\,\text{mm}$

$v = \dfrac{\pi d_1 n_1}{6 \times 10^4} \approx 4.16\,\text{m/s}$

3.4.4 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\epsilon \sqrt{2T_{sh1} K_H \frac{u_{hg}+1}{b_w u_{hg} d_{w1}^2}} \leq [\sigma_H] \tag{3.1}$$

Find z_M $z_M = 274$, according to table 6.5

Find z_H Since correction is unused in our calculation:
$\beta_b = \arctan(\cos\alpha_t \tan\beta) \approx 18.75° \Rightarrow z_H = \sqrt{2\frac{\cos\beta_b}{\sin(2\alpha_t)}} \approx 1.68$

Find z_ϵ Obtaining z_ϵ through calculations:

$$\epsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_t}{2\pi m \frac{\cos\alpha_t}{\cos\beta}} \approx 1.64$$

$$\epsilon_\beta = b_w \frac{\sin\beta}{m\pi} \approx 2.9 > 1 \Rightarrow z_\epsilon = \epsilon_\alpha^{-0.5} \approx 0.78$$

Find K_H We find K_H using equation $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table 6.13, $v \leq 10\,\mathrm{m/s} \Rightarrow \mathrm{AG} = 8$

From table P2.3, using interpolation, we approximate:

$K_{Hv} \approx 1.0417$, $K_{Fv} \approx 1.1145$

From table 6.14, using interpolation, we approximate:

$K_{H\alpha} \approx 1.0766$, $K_{F\alpha} \approx 1.253$

$\Rightarrow K_H \approx 1.24$

Find σ_H After calculating z_M, z_H, z_ϵ, K_H, we get the following result:

$$\sigma_H \approx 699.12\,\mathrm{MPa} \leq [\sigma_H] \approx 696.62\,\mathrm{MPa}$$

Since σ_H and $[\sigma_H]$ are almost equal to each other, i.e $\|\sigma_H - [\sigma_H]\| < 4\%$, the assumed parameters are appropriate.

3.4.5 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2\frac{T_{sh1}K_F Y_\epsilon Y_\beta Y_{F1}}{b_w d_{w1} m_n} \leq [\sigma_{F1}] \tag{3.2}$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{F2}] \tag{3.3}$$

Find Y_ϵ Knowing that $\epsilon_\alpha \approx 1.64$, we can calculate $Y_\epsilon = \epsilon_\alpha^{-1} \approx 0.61$

Find Y_β Since $\beta = 20^\circ \Rightarrow Y_\beta = 1 - \dfrac{\beta}{140} \approx 0.86$

Find Y_F Using formula $z_v = z\cos^{-3}(\beta)$ and table 6.18:
$z_{v1} = z_1 \cos^{-3}(\beta) \approx 20.49 \Rightarrow Y_{F1} \approx 4.06$
$z_{v2} = z_2 \cos^{-3}(\beta) \approx 102.44 \Rightarrow Y_{F2} \approx 3.6$

Find K_F Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:
$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.75$

Find σ_F Substituting all the values, we find out that:

$$\sigma_{F1} \approx 182.39\,\text{MPa} \leq [\sigma_{F1}] \approx 264.85\,\text{MPa}$$

$$\sigma_{F2} \approx 161.69\,\text{MPa} \leq [\sigma_{F2}] \approx 332.48\,\text{MPa}$$

The calculated results are appropriate.
Through calculations, there is no correction needed, i.e. $y = 0$. Thus, the specifications will not include corrections.

In summary, we have the following table:

	pinion	driving gear
H HB	250	240
$[\sigma_H]$ MPa	621.61	771.63
$[\sigma_F]$ MPa	264.85	332.48
$[\sigma_H]$ MPa	696.62	
σ_F MPa	182.39	161.69
σ_H MPa	699.12	
α_{tw} °	21.17	
β °	20	
a_w mm	80	
b_w mm	40	
m mm	1.5	
z	17	85
d mm	27.14	135.68
d_a mm	30.14	138.68
d_f mm	23.39	131.93
d_b mm	25.3	126.52

Table 3.1: Gearbox specifications

Chapter 4

Shaft Design

4.1 Nomenclature

$[\tau]$	permissible torsion, MPa
r	position of applied force on the shaft, mm
hr	tooth direction
cb	role of gear on the shaft (active or passive)
cq	rotational direction of the shaft
σ_b	ultimate strength, MPa
σ_{ch}	yield limit, MPa
S	safety factor
F_x	applied force, N
F_t	tangential force, N
F_r	radial force, N
F_a	axial force, N
a_w	shaft distance, mm
d	shaft diameter, mm
d_w	gear diameter, mm
q	standardized coefficient of shaft diameter
b_O	rolling bearing width, mm
l_m	hub diameter, mm
T	torque on shaft
α_{tw}	meshing profile angle, °
β	helix angle, °
$_1$	subscript for shaft 1
$_2$	subscript for shaft 2
$_x$	subscript for x-axis
$_y$	subscript for y-axis
$_z$	subscript for z-axis
$_{sh1}$	subscript for shaft 1
$_{sh2}$	subscript for shaft 2

4.2 Choose material

For moderate load, we will use quenched steel 40X to design the shafts. From table 6.1, the specifications are as follows: $S \leq 100\,(\text{mm})$, HB260, $\sigma_b = 850\,(\text{MPa})$, $\sigma_{ch} = 550\,(\text{MPa})$.

4.3 Tranmission Design

4.3.1 Load on shafts

Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements. On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$r_{12} = d_{w12}/2 \approx 13.57\,(\text{mm})$, $hr_{12} = +1$, $cb_{12} = +1$, $cq_1 = +1$

$r_{21} = d_{w21}/2 \approx 67.84\,(\text{mm})$, $hr_{21} = +1$, $cb_{21} = -1$, $cq_2 = -1$

Find magnitude of F_t, F_r, F_a Using the results from the previous chapter: $\alpha_{tw} \approx 21.17°$, $\beta = 20°$, $d_{w12} \approx 27.14\,(\text{mm})$

$$\begin{cases} F_{t12} = F_{t21} = \dfrac{2T_{sh1}}{d_{w12}} \approx 2769.03\,(\text{N}) \\ F_{r12} = F_{r21} = \dfrac{F_{t12}\tan\alpha_{tw}}{\cos\beta} \approx 1141.36\,(\text{N}) \\ F_{a12} = F_{a21} = F_{t12}\tan\beta \approx 1007.84\,(\text{N}) \end{cases}$$

Find direction of F_t, F_r, F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \dfrac{r_{12}}{|r_{12}|} cq_1 cb_{12} F_{t12} \approx 2769.03\,(\text{N}) \\ F_{y12} = -\dfrac{r_{12}}{|r_{12}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t12} \approx -1141.36\,(\text{N}) \\ F_{z12} = cq_1 cb_{12} hr_{12} F_{t12} \tan\beta \approx 1007.84\,(\text{N}) \end{cases}$$

$$\begin{cases} F_{x21} = \dfrac{r_{21}}{|r_{21}|} cq_2 cb_{21} F_{t21} \approx 2769.03\,(\text{N}) \\ F_{y21} = -\dfrac{r_{21}}{|r_{21}|} \dfrac{\tan\alpha_{tw}}{\cos\beta} F_{t21} \approx -1141.36\,(\text{N}) \\ F_{z21} = cq_2 cb_{21} hr_{21} F_{t21} \tan\beta \approx 1007.84\,(\text{N}) \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 150° and $F_r \approx 2539.28\,(\text{N})$ (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 150^\circ \approx -2199.08\,(\text{N}) \\ F_{y22} = F_{r22} \sin 150^\circ \approx 1269.64\,(\text{N}) \end{cases}$$

4.3.2 Preliminary calculations

Since shaft 1 receives input torque T_{sh1} and shaft 2 is produces output torque T_{sh2}, $[\tau_1] = 15\,(\text{MPa})$ and $[\tau_2] = 30\,(\text{MPa})$. Using equation (10.9), we can approximate d_1 and d_2:

$d_1 \geq \sqrt[3]{\dfrac{T_{sh1}}{0.2[\tau_1]}} \approx 23.22\,(\text{mm}) \Rightarrow d_1 = 25\,(\text{mm})$

$d_2 \geq \sqrt[3]{\dfrac{T_{sh2}}{0.2[\tau_2]}} \approx 30.99\,(\text{mm}) \Rightarrow d_1 = 35\,(\text{mm})$

4.3.3 Identify the distance between bearings and applied forces

From table (10.2), we can estimate b_O. On shaft 1, $b_{O1} = 15\,(\text{mm})$. On shaft 2, $b_{O2} = 21\,(\text{mm})$. Using equation (10.10), $l_{m1} = 1.5 d_1 \approx 34.83\,(\text{mm})$, $l_{m2} = 1.5 d_2 \approx 46.48\,(\text{mm})$.

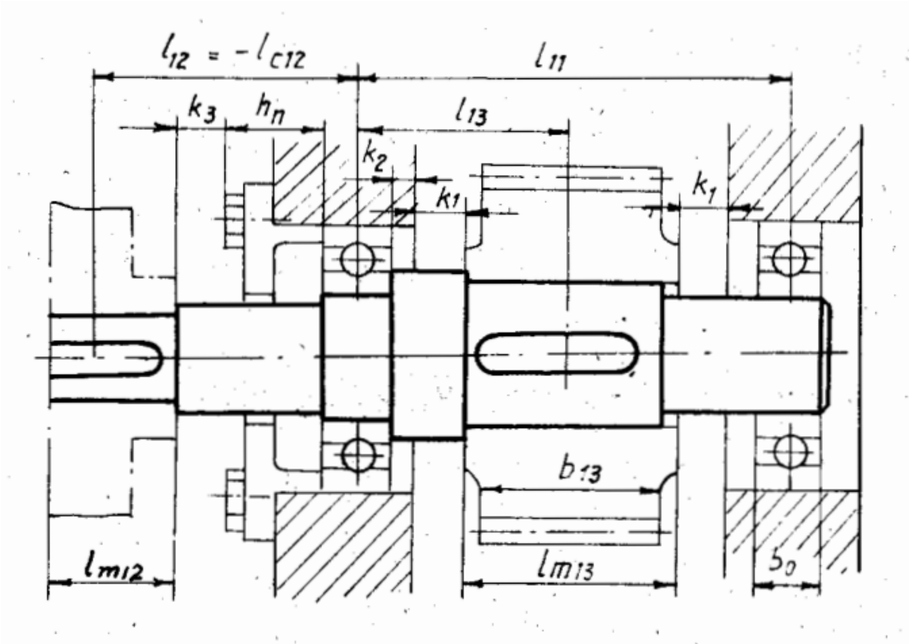

Figure 4.1: Shaft design and its dimensions