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ME3011

Design Project Report

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Abstract

In machine design, every machine element must be calculated in a systematic matter. In this course, students are provided with essential skills to formulate almost every dimension manually, thus further improving their engineering skills before engaging the high-energy, fast-paced workforce.

When a machine element is being developed, it must satisfy some key engineering specifications such as being able to operate under designated lifespan, low cost and high efficiency. Other aspects are less important but also determined the overall design of the element include compactness, noise emission, appearance, etc.

To optimize the process of machine design, the general principles are considered as follows:

1. Identify the working principle and workload of the machine.
2. Formulate the overall working principle to satisfy the problem. Proposing feasible solutions and evaluating them to find the optimal design specifications.
3. Find force and moment diagram exerting on machine parts and characteristics of the workload.
4. Choose appropriate materials to make use of their properties and improve efficiency as well as reliability of individual elements.
5. Calculate dynamics, strength, safety factor, etc. to specify dimensions.
6. Design machine structure, parts to satisfy working condition and assembly.
7. Create presentation, instruction manual and maintenance.

In this report, I will design a fairly simple system to provide a concrete example of finishing all the tasks above.

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Nomenclature

C_a	number of shift daily, shifts	P	design power of the mixing tank, kW
K_{ng}	working days/year, days	T_1	working torque 1, $\text{N} \cdot \text{m}$
L	service life, years	T_2	working torque 2, $\text{N} \cdot \text{m}$
n	rotational velocity of the mixing tank, rpm	t_1	working time 1, s
		t_2	working time 2, s

0.1 Problem

The problem is downloaded from E-learning website, designated number 8, see Figure 1.

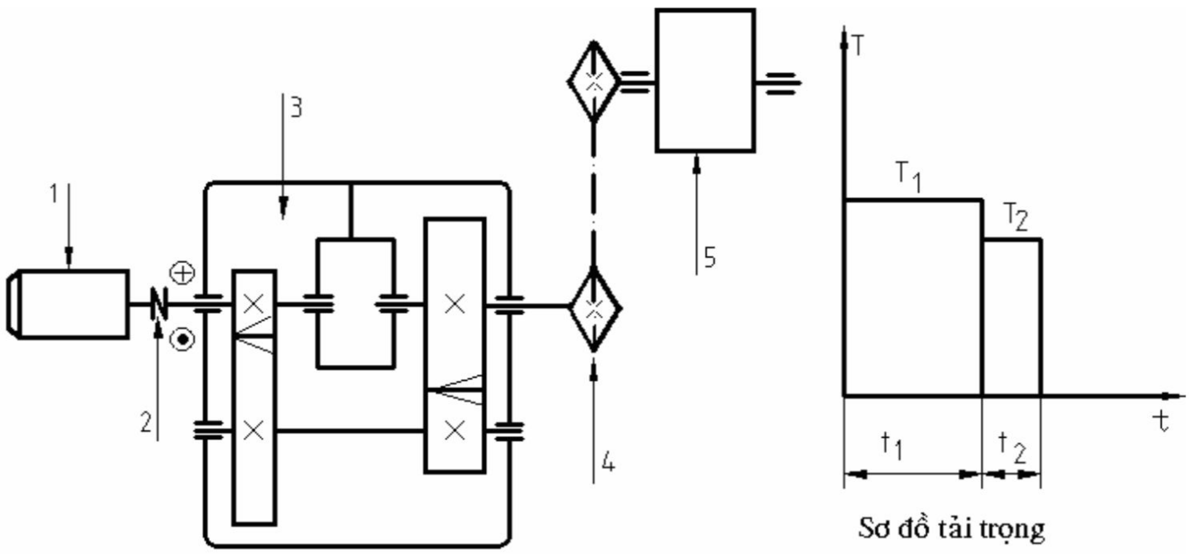

Figure 1: Working principle diagram and workload of the mixing machine: 1) electric motor, 2) elastic coupling, 3) two-stage coaxial helical speed reducer, 4) roller chain drive, 5) mixing tank (one-directional, light duty, operate 1 shift, 8 hours each)

0.2 Mixing machine parameters

From the parameters given in the document, we have:

$P = 7$ (kW)
$n = 65$ (rpm)
$L = 8$ (years)
$K_{ng} = 260$ (days)
$Ca = 1$ (shifts)
$t_1 = 15$ (s)
$t_2 = 11$ (s)
$T_1 = T$ $(\mathrm{N} \cdot \mathrm{m})$
$T_2 = 0.7T$ $(\mathrm{N} \cdot \mathrm{m})$

0.3 Requirements

- 01 report.
- 01 assembly drawing.
- 01 detailed drawing.

0.4 Design problem

1. Decide the working power of the electric motor and transmission ratio of the system.
2. Calculate and design machine elements:
   (a) Calculate system drives (belt, chain or gear).
   (b) Calculate the elements in speed reducers (gears, lead screws).
   (c) Draw and calculate force diagram exerting on the transmission elements.
   (d) Calculate, design shafts and keys.
   (e) Choose bearings and couplings.
   (f) Choose machine bodies, fasteners and other elements.
3. Choose assembly tolerance.
4. Bibliography

Chapter 1

Choose Motor

Nomenclature

n_{sh}	rotational speed of shaft, rpm	u_{sys}	transmission ratio of the system
P_{mo}	calculated motor power to drive the system, kW	T_{mo}	motor torque, $\mathrm{N \cdot mm}$
		T_{sh}	shaft torque, $\mathrm{N \cdot mm}$
P_{sh}	operating power of shaft, kW	η_b	bearing efficiency
P_w	operating power of the belt conveyor given a workload, kW	η_c	coupling efficiency
		η_{ch}	chain drive efficiency
u_1	transmission ratio of quick stage	η_{hg}	helical gear efficiency
u_2	transmission ratio of slow stage	η_{sys}	efficiency of the system
u_{ch}	transmission ratio of chain drive	$_1$	subscript for shaft 1
u_h	transmission ratio of speed reducer	$_2$	subscript for shaft 2
		$_3$	subscript for shaft 3

1.1 Choose motor for the mixing tank

The choice of motor will affect the entire system, so it is necessary to pick the right one.

Calculate system overall efficiency From Table 2.3 [3]:

- 1 elastic coupling which connects the motor and the speed reducer. $\eta_c = 1$
- 4 sealed rolling bearings. 3 of which belong to the speed reducer and the last one is used for the shaft of the mixing tank. $\eta_b = 0.99$
- 2 sealed pairs of helical gear drives which connect the shafts inside the speed reducer. $\eta_{hg} = 0.97$
- 1 sealed roller chain drive connecting the speed reduce and the mixing tank. $\eta_{ch} = 0.96$

Aggregate all efficiencies yields:

$$\eta_{sys} = \eta_c \eta_b^4 \eta_{hg}^2 \eta_{ch} = 1 \times 0.99^4 \times 0.97^2 \times 0.96 = 0.87$$

Calculate required power for operation The power P from Chapter 1 applies for systems with single loading input. In case of varying load each cycle, the equivalent power is calculated using Equation 2.13 [3]:

$$P_w = P\sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} = 7 \times \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15 + 11}} = 6.2\ (\text{kW})$$

$$P_{mo} = \frac{P_w}{\eta_{sys}} = \frac{6.2}{0.87} = 7.14\ (\text{kW})x$$

Calculate n_{mo} The purpose is to Using Table 2.4 [3]:

- 2-level transmission speed reducer, spur gear type. $u_h = 11$
- 1 chain drive, roller type. $u_{ch} = 2$

Multiplying all transmission ratio yields:
$u_{sys} = u_h u_{ch} = 11 \times 2 = 22$
$n_{mo} = u_{sys} n = 22 \times 65 = 1430$ (rpm)

Choose motor To work normally, the maximum operating power of the chosen motor must be no smaller than P_{mo}. In similar fashion, its rotational speed must also be no smaller than n_{mo}. Thus, from Table P1.3 [3], we choose motor 4A132S4Y3 which operates at 7.5 kW maximum and 1455 rpm, which makes $n_{mo} = 1455$ rpm.

Recalculating u_{sys} with the new n_{mo} derived from the chosen motor:
$u_{sys} = n_{mo}/n = 1455/65 = 22.38$

Retaining the transmission ratio of the speed reducer (i.e. let $u_h = const = 11$), the new transmission ratio of the chain drive is then:
$u_{ch} = u_{sys}/u_h = 22.38/11 = 2.03$

1.2 Calculate power, rotational speed and torque

Let P_{sh1}, n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2}, n_{sh2} and T_{sh2} are the transmitted parameters onto shaft 2 and P_{sh3}, n_{sh3} and T_{sh3} are used for shaft 3. Unless otherwise stated, these notations will be used throughout the next chapters.

Power The entire system is described followed by calculation as follows:
Chain drive power is affected by the bearings on the shaft of the mixing tank.

$P_{ch} = \frac{P_w}{\eta_b} = \frac{6.2}{0.99} = 6.26$ (kW)

Shaft 3 power is affected by the chain drive.

$P_{sh3} = \frac{P_{ch}}{\eta_{ch}} = \frac{6.26}{0.96} = 6.52$ (kW)

Shaft 2 power is affected by the bearings and gear drives on shaft 3.

$P_{sh2} = \frac{P_{sh3}}{\eta_b \eta_{hg}} = \frac{6.52}{0.99 \times 0.97} = 6.79$ (kW)

Shaft 1 power is affected by the bearings and gear drives on shaft 2.

$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} = \frac{6.79}{0.99 \times 0.97} = 7.07$ (kW)

Rotational speed The design goal of the speed reducer is to lubricate both driven gears equally, which has a size disadvantage. Therefore, the transmission ratio of each pair of gears is calculated using Equation 3.12 [3]:

$$u_1 = u_2 = \sqrt{u_h} = \sqrt{11} = 3.32$$

Then,
from motor to shaft 1: $n_{sh1} = n_{mo} = 1455$ (rpm)
from shaft 1 to shaft 2: $n_{sh2} = n_{sh1}/u_1 = 1455/3.32 = 438.70$ (rpm)
from shaft 2 to shaft 3: $n_{sh3} = n_{sh2}/u_2 = 438.70/3.32 = 132.27$ (rpm)

Torque Subsequently, the torque is calculated as follows:
$T_{mo} = 9.55 \times 10^6 \times P_{mo}/n_{mo} = 9.55 \times 10^6 \times 7.14/1455 = 46892.66$ (N · mm)
$T_{sh1} = 9.55 \times 10^6 \times P_{sh1}/n_{sh1} = 9.55 \times 10^6 \times 7.07/1455 = 46423.73$ (N · mm)
$T_{sh2} = 9.55 \times 10^6 \times P_{sh2}/n_{sh2} = 9.55 \times 10^6 \times 6.79/438.70 = 147857.49$ (N · mm)
$T_{sh3} = 9.55 \times 10^6 \times P_{sh3}/n_{sh3} = 9.55 \times 10^6 \times 6.52/132.27 = 470919.44$ (N · mm)

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2	Shaft 3
P (kW)	7.14	7.07	6.79	6.52
u	-	1	3.32	3.32
n (rpm)	1455	1455	438.70	132.27
T (N · mm)	46892.66	46423.73	147857.49	470919.44

Table 1.1: Output specifications

Chapter 2

Chain Drive Design

Nomenclature

$[i]$	permissible impact times per second
$[s]$	permissible safety factor
$[P]$	permissible power, kW
$[\sigma_H]$	permissible contact stress, MPa
A	cross sectional area of chain hinge, mm^2
a	real center distance, mm
a_i	estimated center distance, mm
a_{max}	maximum center distance, mm
a_{min}	minimum center distance, mm
B	width between inner link plate, mm
d	chordal diameter, mm
d_a	addendum diameter, mm
d_f	dedendum diameter, mm
d_l	roller diameter, mm
d_O	pin diameter, mm
E	modulus of elasticity, MPa
F_0	sagging force, N
F_1	tight side tension force, N
F_2	slack side tension force, N
F_r	force on the shaft, N
F_t	effective peripheral force, N
F_v	centrifugal force, N
F_{vd}	contact force, N
i	impact times per second
K_d	weight distribution factor
k	overall factor
k_a	center distance and chain's length factor
k_{bt}	lubrication factor
k_c	rating factor
k_d	dynamic load factor
k_{dc}	chain tension factor
k_f	loosing factor
k_n	coefficient of rotational speed
k_r	number of tooth factor
k_x	chain weight factor
k_z	coefficient of number of teeth

k_0	arrangement of drive factor	v	instantaneous velocity along the chain, m/s
n	angular rotational speed, rpm	x	chain length in pitches, the number of links
n_{01}	experimental angular rotational speed, rpm	x_c	an even number of links
P_t	calculated power, kW	z	number of teeth of a sprocket
p	pitch, mm	z_{max}	maximum number of teeth of the driven sprocket
p_{max}	permissible sprocket pitch, mm	σ_H	contact stress, MPa
Q	permissible load, N	$_1$	subscript for driving sprocket
q	mass per unit length, kg/m	$_2$	subscript for driven sprocket
s	safety factor		

Known parameters From Chapter 1, we know that:
The chain type is roller.
$n_{sh3} = 132.27\,\text{rpm}$
$P_{ch} = 6.26\,\text{kW}, u_{ch} = 4.5$

2.1 Find the chain drive pitch

Chain hinge wear is one of the main failure modes, which poses a risk of damaging the entire system. In this section, the method to find the right pitch is due to the strength criterion of the chain hinge, derived into Equation 5.3 [3]. The driving sprocket is connected to shaft 3, $n_1 = n_{sh3} = 132.27\,\text{rpm}$.

Calculate z The number of teeth determines the how stable is the rotational speed, which relates to the impact intensity and service life:
$z_1 = 29 - 2u_{ch} = 29 - 2 \times 2.03 = 25 \geq 19$ (rounded to the nearest odd number)
$z_2 = u_{ch} z_1 = 2.03 \times 25 = 51 \leq 120$ (Equation 5.1 [3])

Calculate $[P]$ An experiment is conducted to find the optimal pitch given the permissible power and angular rotational speed. A roller chain drive having 25 teeth on the driving sprocket is tested in 8 different cases of n_{01} in somewhat similar condition with our design purpose, p.80 [3]. In this problem, $z_1 = 25$; $n_1 = 132.27\,\text{rpm}$, which is close to $n_{01} = 200\,\text{rpm}$, which yields $k_z = 25/z_1 = 25/25 = 1$ and $k_n = n_{01}/n_1 = 200/132.27 = 1.51$.

Another step is to specify the working condition of the chain, Table 5.6:

- The centerline between 2 sprockets is parallel with the ground. $k_0 = 1$
- Center distance $a = (30 \div 50)p$, which is similar to the experiment. $k_a = 1$
- Center distance is modifiable through displacing the sprockets. $k_{dc} = 1$
- Moderate impact load. $k_d = 1.5$
- 1 shift. $k_c = 1$
- Dusty condition with moderate lubrication quality. $k_{bt} = 1.3$

Then, we can obtain the value $[P]$, Equation 5.3 [3]:

$$
\begin{aligned}
P_t &= P_{ch} k k_z k_n = P_{ch} k_0 k_a k_{dc} k_d k_c k_{bt} k_z k_n \\
&= 6.26 \times 1 \times 1 \times 1 \times 1.5 \times 1 \times 1.3 \times 1 \times 1.51 = 18.46\,(\text{kW}) \leq [P]
\end{aligned}
$$

Inspecting Table 5.5 [3] at column $n_{01} = 200\,\text{rpm}$, the closet value is $[P] = 19.3\,\text{kW}$. Knowing $[P]$, the pitch is $p = 31.75\,\text{mm}$, Table 5.5 [3]. Consequently, $d_c = 9.55\,\text{mm}$, $B = 27.46\,\text{mm}$. Because minimizing damage from impact onto the drive is essential, Table 5.8 [3] is consulted. In this case, the pitch is indeed suitable.

2.2 Determine basic parameters of the chain drive

2.2.1 Find number of links and center distance

The parameters are found:

Find x_c: The $a_{min} = 30p = 30 \times 31.75 = 952.50\,(\text{mm})$, $a_{max} = 50p = 50 \times 31.75 = 1587.50\,(\text{mm})$. Limiting the range of choice for a in $[a_{min}, a_{max}]$ to reduce the effect from chain weight, we can approximate $a = 1300\,\text{mm}$ and find x_c:

$$\begin{aligned} x &= \frac{2a}{p} + \frac{z_1 + z_2}{2} + \frac{(z_2 - z_1)^2 p}{4\pi^2 a} \\ &= \frac{2 \times 1300}{31.75} + \frac{25 + 51}{2} + \frac{(51 - 25)^2 \times 31.75}{4\pi^2 \times 1300} = 120.31 \end{aligned}$$

x is rounded up to the nearest even number. $x_c = 122$

Find a: Using x_c to find the correct center distance, Equation 5.13 [3]. In addition, it is recommended to loose the chain an amount of $0.002 \div 0.004a$ to reduce tension. Choosing the amount of $0.003a$, the coefficient 0.997 is included in the formula below:

$$\begin{aligned} a &= \frac{0.997p}{4}\left[x_c - \frac{z_2 + z_1}{2} + \sqrt{\left(x_c - \frac{z_2 + z_1}{2}\right)^2 - 2\left(\frac{z_2 - z_1}{\pi}\right)^2}\right] \\ &= \frac{0.997 \times 31.75}{4}\left[122 - \frac{51 + 25}{2} + \sqrt{\left(122 - \frac{51 + 25}{2}\right)^2 - 2\left(\frac{51 - 25}{\pi}\right)^2}\right] \\ &= 1019.99\,(\text{mm}) \end{aligned}$$

Find other parameters: The values below are necessary for modeling the chain drive (the last two values use $d_l = 19.05\,\text{mm}$ from Table 5.2 [3]):

$$d_1 = p/\sin\left(\frac{\pi}{z_1}\right) = 31.75/\sin\left(\frac{180}{25}\right) = 253.32\,(\text{mm})$$

$$d_2 = p/\sin\left(\frac{\pi}{z_2}\right) = 31.75/\sin\left(\frac{180}{51}\right) = 515.75\,(\text{mm})$$

$$d_{a1} = p\left(0.5 + \cot\frac{180}{z_1}\right) = 31.75\left(0.5 + \cot\frac{180}{25}\right) = 267.20\,(\text{mm})$$

$$d_{a2} = p\left(0.5 + \cot\frac{180}{z_2}\right) = 31.75\left(0.5 + \cot\frac{180}{51}\right) = 530.65\,(\text{mm})$$

$$d_{f1} = d_1 - 2(0.502d_l + 0.05) = 253.32 - 2(0.502 \times 19.05 + 0.05) = 234.08\,(\text{mm})$$

$$d_{f2} = d_2 - 2(0.502d_l + 0.05) = 515.75 - 2(0.502 \times 19.05 + 0.05) = 496.50\,(\text{mm})$$

2.3 Strength of chain drive

2.3.1 Impact frequency analysis

After determine the center distance, it is necessary to compare the impact frequency with its permissible value, which is $[i] = 25$, Table 5.9 [3]. Replacing all the variables gives:

$$i = \frac{z_1 n_1}{15x} = \frac{25 \times 132.27}{15 \times 120.31} = 1.83 < [i]$$

which satisfies the condition.

2.3.2 Safety factor analysis

Overloading often occurs at the beginning of operation or due to large load, which could damage the chain drive. In order to operate safely, the chain drive's safety factor must satisfy the following condition:

$$s = \frac{Q}{k_d F_t + F_0 + F_v} \geq [s]$$

Find v Calculated for other variables. The rotational speed of the smaller sprocket is determined using the formula:

$$v_1 = n_1 p z_1/60000 = 132.27 \times 31.75 \times 25/60000 = 2.6\ (\mathrm{m/s})$$

Find Q **and** q Table 5.2 [3] and $p = 31.75$ are used. $Q = 88500\,\mathrm{N}$ and $q = 3.8\,\mathrm{kg/m}$
Find k_d: The workload is moderate. When turning on the machine, it is around 1.5 times the amount of the nominal load. $k_d = 1.2$
Find k_f: The chain drive is parallel to the ground. $k_f = 6$
Find F_t**,** F_v **and** F_0 Substituting the values above to obtain the values:

$$F_t = 1000 P_{ch}/v_1 = 1000 \times 6.26/2.6 = 3578.34\ (\mathrm{N})$$

$$F_v = q v_1^2 = 3.8 \times 6.26^2 = 11.64\ (\mathrm{N})$$

$$F_0 = 9.81 \times 10^{-3} k_f q a = 9.81 \times 10^{-3} k_f q a = 295.92\ (\mathrm{N})$$

Find $[s]$: The limit is found using interpolation, Table 5.10 [3]. $[s] = 7.66$
Replacing all the variables gives:

$$s = \frac{88500}{1.2 \times 3578.34 + 295.92 + 11.64} = 19.23 \geq 7.66$$

which satisfies the condition.

2.3.3 Contact stress analysis

The following condition must be met, Equation 5.18 [3]:

$$\sigma_H = 0.47 \sqrt{\frac{k_r (F_t k_d + F_{vd}) E}{A K_d}} \leq [\sigma_H]$$

Find $[\sigma_H]$ The chain drive has $z_2 = 51 > 30$ and $v_1 = 2.6\,\text{mm} < 5\,\text{mm}$. Thus the suitable material is quenched 45 steel, which has $[\sigma_H] = 500\,\text{MPa}$, Table 5.11 [3] and $E_{sprocket} = 205000\,\text{MPa}$ [1].

Find K_d Since the chain drive only has one strand, $K_d = 1$.

Find F_{vd} For 1-strand chain, $F_{vd} = 13\times10^{-7} n_1 p^3 = 13\times10^{-7}\times132.27\times31.75^3 = 6.22\,\text{(N)}$

Find k_r Since z_1 is used to estimate k_r on p.87 [3], $k_r = 0.42$.

Find E Assuming the sprockets and chain are made up from the same material (steel). The chain is cast iron ASTM-A48 No. 30A, which has $E_{chain} = 117000\,\text{MPa}$ [2]. The equivalent modulus of elasticity is then:

$$E = \frac{2E_{sprocket}E_{chain}}{E_{sprocket} + E_{chain}} = \frac{2\times205000\times117000}{205000+117000} = 148975.16\,\text{(MPa)}$$

Find A The projected area of the hinge depends on the pitch $p = 31.75\,\text{mm}$, Table 5.12 [3]. $A = 262\,\text{mm}^2$.

The value $k_d = 1.5$ and $F_t = 3578.34\,\text{N}$ are found above. Replacing all the variables gives:

$$\sigma_H = 0.47\sqrt{\frac{0.42\times(3578.34\times1.5+6.22)\times148975.16}{262\times1}} = 476.25\,\text{(MPa)} \le 500\,\text{MPa}$$

which satisfies the condition.

2.4 Force on shaft

Apply the following equations, p.87 [3]:

$$F_2 = F_0 + F_v = 295.92 + 11.64 = 307.55\,\text{(N)}$$
$$F_1 = F_t + F_2 = 3578.34 + 307.55 = 3885.89\,\text{(N)}$$

The chain drive is parallel to the ground corresponding to $k_x = 1.15$. Apply Equation 5.20 [3]:

$$F_r = k_x F_t = 1.15\times3578.34 = 2772.05\,\text{(N)}$$

In summary, we have the following table:

	Chain drive	Driving sprocket	Driven sprocket
a (mm)	1323.02	-	-
B (mm)	27.46	-	-
d (mm)	-	253.32	515.75
d_a (mm)	-	267.20	530.65
d_c (mm)	9.55	-	-
d_f (mm)	-	234.08	496.50
d_l (mm)	19.05	-	-
z		25	51

Table 2.1: Chain drive specifications

Chapter 3

Gear Design (shaft 1-2)

Nomenclature

$[\sigma]$	permissible stress, MPa
$[\sigma]_{max}$	permissible stress due to overloading, MPa
AG	accuracy grade of gear
a	center distance, mm
b	face width, mm
c	gear meshing rate
d	pitch circle, mm
d_a	addendum diameter, mm
d_b	base diameter, mm
d_f	deddendum diameter, mm
H	surface roughness, HB
K_d	coefficient of gear material
K	overall load factor
K_C	load placement factor
K_L	aging factor
K_v	dynamic load factor at meshing area
K_α	load distribution factor on gear teeth
K_β	load distribution factor on top land
K_{qt}	overloading factor
m	root of fatigue curve in stress test
m_t	traverse module, mm
m_n	normal module, mm
N_E	working cycle of equivalent tensile stress
N_O	working cycle of bearing stress
S	safety factor
v	rotational velocity, m/s
Y_F	tooth shape factor
Y_R	surface roughness factor
Y_s	stress concentration factor
Y_β	helix angle factor
Y_ε	contact ratio factor
Z_R	surface roughness factor of the working's area
Z_v	speed factor
z_H	contact surface's shape factor
z_M	material's mechanical properties factor
z_v	virtual number of teeth
z_ε	meshing condition factor
α	normal pressure angle. In TCVN 1065-71, $\alpha = 20°$
α_t	traverse pressure angle, °
ε_α	traverse contact ratio
ε_β	face contact ratio
β	helix angle, °

β_b	base circle helix angle, °	σ_{max}	stress due to overloading, MPa
ψ_{ba}	width to shaft distance ratio	$_1$	subscript for the pinion
ψ_{bd}	face width factor	$_2$	subscript for the driven gear
σ_b	ultimate strength, MPa	$_F$	subscript relating to bending stress
σ_{ch}	yield limit, MPa	$_H$	subscript relating to contact stress
σ^o_{lim}	permissible stress corresponding to working cycle, MPa		

3.1 Choose material

For small scale machines, type I is sufficient. Materials of this type are often quenched and have low hardness value (HB $\leq$ 350), which make it easy to manufacture precisely. However, their permissible stresses are low. In this design problem, 45 steel is used to manufacture both gears. The specifications of the material are HB205, $\sigma_b = 600$ MPa, $\sigma_{ch} = 340$ MPa, Table 6.1 [3].

3.2 Estimate the permissible contact stress and bending stress

The permissible stresses are calculated as follows, Equation 6.1 and 6.2 [3]:

$$[\sigma_H] = \frac{\sigma^o_{Hlim}}{S_H} Z_R Z_v K_{xH} K_{HL}$$

$$[\sigma_F] = \frac{\sigma^o_{Flim}}{S_F} Y_R Y_s K_{xF} K_{FC} K_{FL}$$

Initial estimation of these values assumes $Z_R Z_v K_{xH} = 1$ and $Y_R Y_s K_{xF} = 1$. The factors will be accounted for in the next sections.

Find σ^o_{lim} The value depends on hardness. It is also recommended to have the hardness of the pinion slightly higher than the driving gear from 10 to 15 units. Using Table 6.2 [3], the formulas below are used for quenched 45 steel:
$\sigma^o_{Hlim} = 2\text{HB} + 70$
$\sigma^o_{Flim} = 1.8\text{HB}$

For the pinion with $H_1 = \text{HB}205$:
$\sigma^o_{Hlim1} = 2 \times 205 + 70 = 480$ (MPa)
$\sigma^o_{Flim1} = 1.8 \times 205 = 369$ (MPa).

For the driven gear with $H_2 = H_1 - 15 = 205 - 15 = \text{HB}190$:
$\sigma^o_{Hlim2} = 2 \times 190 + 70 = 450$ (MPa)
$\sigma^o_{Flim2} = 1.8 \times 190 = 342$ (MPa).

Find S The safety factors are found using Table 6.2 [3]. $S_H = 1.1$, $S_F = 1.75$

Find K_{FC} The motor works in one direction and the load is placed in one way. $K_{FC} = 1$

Find K_L The aging factor is determined by Equation 6.3 and 6.4 [3] for contact stress and bending stress, respectively. The surface hardness of the gears is smaller than 350, which makes $m_H = 6$, $m_F = 6$. Also, both pairs of gears are meshed indefinitely which makes $c = 1$. From working condition, the service life in hours are found:

$$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times CaK_{ng}L = 8 \times 1 \times 260 \times 8 = 16640 \text{ (hours)}$$

N_{HE} and N_{FE} are found using Equation 6.7 and 6.8 [3]:

$$N_{HE1} = 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right]$$
$$= 60 \times 1455 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^3 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^3 \frac{11}{15+11} \right] = 1.05 \times 10^9 \text{ (cycles)}$$

$$N_{HE2} = 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^3 \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^3 \frac{t_2}{t_1+t_2} \right]$$
$$= 60 \times 438.70 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] = 0.32 \times 10^9 \text{ (cycles)}$$

$$N_{FE1} = 60 n_{sh1} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right]$$
$$= 60 \times 1455 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] = 0.91 \times 10^9 \text{ (cycles)}$$

$$N_{FE2} = 60 n_{sh2} c L_h \left[\left(\frac{T_1}{T}\right)^{m_F} \frac{t_1}{t_1+t_2} + \left(\frac{T_2}{T}\right)^{m_F} \frac{t_2}{t_1+t_2} \right]$$
$$= 60 \times 438.70 \times 1 \times 16640 \left[\left(\frac{T}{T}\right)^6 \frac{15}{15+11} + \left(\frac{0.7T}{T}\right)^6 \frac{11}{15+11} \right] = 0.27 \times 10^9 \text{ (cycles)}$$

The working cycle of bearing stress is found using the formula below, Equation 6.5 [3]:
$N_{HO1} = 30 H_1^{2.4} = 30 \times 205^{2.4} = 10600601.81$ (cycles).
$N_{HO2} = 30 H_2^{2.4} = 30 \times 190^{2.4} = 8833440.68$ (cycles).

For steel, $N_{FO1} = N_{FO2} = 4000000$ (MPa). The factors K_L are rounded up to 1 if it is less than 1, p.94 [3]. Apply Equation 6.3 and 6.4 [3]:
$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} = \sqrt[6]{10600601.81/1.05 \times 10^9} = 0.46 < 1 \Rightarrow K_{HL1} = 1$
$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} = \sqrt[6]{8833440.68/0.32 \times 10^9} = 0.55 < 1 \Rightarrow K_{HL2} = 1$
$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} = \sqrt[6]{4000000/0.91 \times 10^9} = 0.40 < 1 \Rightarrow K_{FL1} = 1$
$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} = \sqrt[6]{4000000/0.27 \times 10^9} = 0.49 < 1 \Rightarrow K_{FL2} = 1$

Calculate $[\sigma_H]$**,** $[\sigma_{F1}]$**,** $[\sigma_{F2}]$ Replacing all the values:
$[\sigma_{H1}] = \sigma^o_{Hlim1} K_{HL1}/S_H = 480 \times 1/1.1 = 436.36$ (MPa)
$[\sigma_{H2}] = \sigma^o_{Hlim2} K_{HL2}/S_H = 450 \times 1/1.1 = 409.09$ (MPa)
$[\sigma_{F1}] = \sigma^o_{Flim1} K_{FC} K_{FL1}/S_F = 369 \times 1 \times 1/1.75 = 210.86$ (MPa)
$[\sigma_{F2}] = \sigma^o_{Flim2} K_{FC} K_{FL2}/S_F = 342 \times 1 \times 1/1.75 = 195.43$ (MPa)

The mean permissible contact stress must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. In this case, it is $1.25[\sigma_{H2}]$. Replacing all the variables gives:

$$[\sigma_H] = \frac{[\sigma_{H1}] + [\sigma_{H2}]}{2} = \frac{436.36 + 409.09}{2} = 422.73 \text{ (MPa)}$$
$$\leq 1.25[\sigma_{H2}] = 1.25 \times 409.09 = 511.36 \text{ (MPa)}$$

which satisfy the condition.

In case of overloading, the permissible contact and bending stresses are calculated as follows:
For quenched material: $[\sigma_H]_{max} = 2.8 \times 600 = 952$ (MPa)
For material with hardness smaller than HB350: $[\sigma_F]_{max} = 0.8\sigma_{ch} = 0.8 \times 340 = 272$ (MPa)

3.3 Determine basic specifications of the transmission system

3.3.1 Determine basic parameters

Figure 1 shows both pairs are helical, which gives $K_a = 43$, Table 6.5 [3]. Also, the pinion is symmetrical about the bearings and both gears surface have hardness smaller than HB350, resulting in $\psi_{ba} = 0.3$, Table 6.6 [3]. Then, ψ_{bd} is calculated using Equation 6.16 [3]:
$\psi_{bd} = 0.53\psi_{ba}(u+1) = 0.53 \times 0.3 \times (3.32+1) = 0.69$, which is smaller than the permissible ratio $\psi_{bdmax} = 1.2$, Table 6.6 [3].

The gear placement in the speed reducer is similar to diagram 5 of Table 6.7 [3]. $K_{H\beta} = 1.04$, $K_{F\beta} = 1.1$

Find a Since the gear drive only has involute gears, a is estimated using Equation 6.15a [3]. It is also rounded up to the nearest multiple of 5 (small production type, p.99 [3]):

$$a = K_a(u+1)\sqrt[3]{\frac{T_{sh1}K_{H\beta}}{[\sigma_H]^2 u\psi_{ba}}} = 43\times(3.32+1)\times\sqrt[3]{\frac{46423.73 \times 1.04}{422.73^2 \times 3.32 \times 0.3}} = 120.14 \text{ (mm)}$$

$\Rightarrow a = 180\,\text{mm}$

Due to complexity and maintenance, the undercutting process will be omitted throughout the calculations, that is, it is not factored in any variables of the gear drives.

Find m Using Equation 6.17 [3] and Table 6.8 [3], we determine m for each pair of gears:
$m_t = (0.01 \div 0.02)a = (0.01 \div 0.02) \times 180 = 1.8 \div 3.6$ (mm) $\Rightarrow m_t = 2\,\text{mm}$

Find z_1**,** z_2**,** b Arbitrarily choose $\beta = 20°$ (permissible value is $8 \div 20°$). Combining Equation 6.18 and 6.20 [3] to calculate z_1:

$$z_1 = \frac{2a\cos\beta}{m_t(u+1)} = \frac{2 \times 180 \cos 20°}{2 \times (3.32+1)} = 39.18 \Rightarrow z_1 = 40$$

Then, find z_2 and b:
$z_2 = uz_1 = 3.32 \times 40 = 132.66 \Rightarrow z_2 = 133$
$b = \psi_{ba}a = 0.3 \times 180 = 54$ (mm)

Correct β The helix angles are corrected to compensate for rounding center distances and number of teeth, Equation 6.32 [3]:

$$\beta = \arccos\frac{m_t(z_2+z_1)}{2a} = \arccos\frac{2 \times (133+40)}{2 \times 180} = 16.03°$$

3.3.2 Other parameters

$d_1 = m_t z_1/\cos\beta = 2 \times 40/\cos 16.03° = 83.24$ (mm)
$d_2 = m_t z_2/\cos\beta = 2 \times 133/\cos 16.03° = 276.76$ (mm)
$d_{a1} = d_1 + 2m_t = 83.24 + 2 \times 2 = 87.24$ (mm)
$d_{a2} = d_2 + 2m_t = 276.76 + 2 \times 2 = 280.76$ (mm)
$d_{f1} = d_1 - 2.5m_t = 83.24 - 2.5 \times 2 = 78.24$ (mm)

$d_{f2} = d_2 - 2.5m_t = 276.76 - 2.5 \times 2 = 271.76$ (mm)
$d_{b1} = d_1 \cos\alpha = 83.24 \times \cos 20° = 78.22$ (mm)
$d_{b2} = d_2 \cos\alpha = 276.76 \times \cos 20° = 260.07$ (mm)
$\alpha_t = \arctan(\tan\alpha / \cos\beta) = \arctan(\tan 20° / \cos 16.03°) = 20.74°$
$v_1 = \pi d_1 n_{sh1}/60000 = \pi \times 83.24 \times 1455/60000 = 6.34$ (m/s)

3.4 Stress analysis

3.4.1 Correct $[\sigma_H]$, $[\sigma_{F1}]$ and $[\sigma_{F2}]$

In reality, the factors $Z_R Z_v K_{xH}$ and $Y_R Y_s K_{xF}$ do not equal to 1. This part will correct them to obtain the correct permissible stresses.

- The roughness deviation is less than $1.25 \mu m$. $Z_R = 1$
- The surface hardness of the gear drive is less than HB350. $Z_v = 0.85 v_1^{0.1} = 0.85 \times 6.34^{0.1} = 1.02$
- The gears have $d_{a1}, d_{a2} \leq 700$ mm. $K_{xH} = 1$
- The gears are not polished. $Y_R = 1$
- $Y_s = 1.08 - 0.0695 \ln(m_t) = 1.03$
- $d_{a1}, d_{a2} \leq 400$ mm. $K_{xF} = 1$

Replacing all the variables gives:
$[\sigma_H] = 422.73 \times 1 \times 1.02 \times 1 = 432.21$ MPa
$[\sigma_{F1}] = 210.86 \times 1 \times 1.03 \times 1 = 217.57$ MPa
$[\sigma_{F2}] = 195.43 \times 1 \times 1.03 \times 1 = 201.65$ MPa

3.4.2 Contact stress analysis

The contact stress applied on a gear surface must satisfy Equation 6.33 [3]:

$$\sigma_H = z_M z_H z_\varepsilon \sqrt{2T_{sh1} K_H \frac{u+1}{bud_1^2}} \leq [\sigma_H]$$

Find z_M According to Table 6.5 [3], $z_M = 274$.

Find z_H Applying Equation 6.34 [3] and 6.35 [3]:

$\beta_b = \arctan(\cos\alpha_t \tan\beta) = \arctan(\cos 20.47° \tan 16.03°) = 15.04°$

$$z_H = \sqrt{\frac{2\cos\beta_b}{\sin(2\alpha_t)}} = \sqrt{\frac{2 \times \cos 15.04°}{\sin(2 \times 20.74°)}} = 1.71$$

Find z_ε Obtaining z_ε through calculations:

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a\sin\alpha_t}{2\pi m_t \cos\alpha_t / \cos\beta}$$
$$= \frac{\sqrt{87.24^2 - 78.22^2} + \sqrt{280.76^2 - 260.07^2} - 2 \times 180 \times \sin 20.74°}{2\pi \times 2 \times \cos 20.74° / \cos 16.03°} = 1.38$$

$\varepsilon_\beta = b\dfrac{\sin\beta}{m_t\pi} = 54 \times \dfrac{\sin 16.03°}{2\times\pi} = 2.37 > 1 \Rightarrow z_\varepsilon = \varepsilon_\alpha^{-0.5} = 1.38^{-0.5} = 0.85$

Find K_H and K_F We find K_H, K_F using Equation 6.39 and 6.45 [3].

The gear drive velocity determines the accuracy grade, Table 6.13 [3]. In case of helical gear, it is:
$v_1 = 6.34\,\text{m/s} \le 10\,\text{m/s} \Rightarrow \text{AG} = 8$

From Table P2.3 [3], using interpolation, we approximate:
$K_{Hv} = 1.04,\ K_{Fv} = 1.18$

From Table 6.14 [3], using interpolation, we approximate:
$K_{H\alpha} = 1.06,\ K_{F\alpha} = 1.3$

Knowing $K_{H\beta}$ and $K_{H\beta}$ from previous section, multiply all the values to obtain K_H and K_F:
$K_H = K_{H\beta}K_{Hv}K_{H\alpha} = 1.04 \times 1.1 \times 1.06 = 1.21$
$K_F = K_{F\beta}K_{Fv}K_{F\alpha} = 1.1 \times 1.18 \times 1.3 = 1.68$

Find σ_H Replacing all the variables gives:

$$\sigma_H = 274 \times 1.71 \times 1.55 \times \sqrt{2 \times 46423.73 \times 1.21 \times \frac{3.32+1}{54 \times 3.32 \times 83.24^2}} = 249.04\ (\text{MPa}) \le 432.21\,\text{MPa}$$

which satisfies the condition.

3.4.3 Bending stress analysis

For safety reasons, Equation 6.43 and 6.44 [3] must be met for both pairs of gears:

$$\begin{aligned}\sigma_{F1} &= \frac{2T_{sh1}K_FY_\varepsilon Y_\beta Y_{F1}}{bd_1m_t\cos\beta} &\le [\sigma_{F1}]\\ \sigma_{F2} &= \sigma_{F1}Y_{F2}/Y_{F1} &\le [\sigma_{F2}]\end{aligned}$$

Find Y_ε Using ε_α calculated in the previous section, $Y_\varepsilon = \varepsilon_\alpha^{-1} = 1.38^{-1} = 0.72$

Find Y_β The value of Y_β is calculated using the equation on p.108 [3]:
$Y_\beta = 1 - \beta/140° = 1 - 16.03°/140° = 0.89$

Find Y_F Using the formula $z_v = z\cos^{-3}(\beta)$ and Table 6.18 [3]:
$z_{v1} = z_1\cos^{-3}(\beta) = 40 \times \cos^{-3}(16.03°) = 45.05 \Rightarrow Y_{F1} = 3.67$
$z_{v2} = z_2\cos^{-3}(\beta) = 133 \times \cos^{-3}(16.03°) = 149.81 \Rightarrow Y_{F2} = 3.60$

Find K_F The value of K_F has already been found in the previous section. $K_F = 1.68$

Find σ_F The module in Equation 6.43 [3] is $m_n = m_t\cos\beta$. Replacing all the values gives:

$$\begin{aligned}\sigma_{F1} &= \frac{2 \times 46423.73 \times 1.68 \times 0.72 \times 0.89 \times 3.67}{54 \times 83.24 \times 2 \times \cos 16.03°} = 42.44\ (\text{MPa}) &\le 217.57\,\text{MPa}\\ \sigma_{F2} &= 42.44 \times 3.60/3.66 = 41.57\ (\text{MPa}) &\le 201.65\,\text{MPa}\end{aligned}$$

which satisfies the conditions.

3.4.4 Overloading analysis

From the load diagram, we determine the overloading factor:

$$K_{qt} = \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} = \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15 + 11}} = 1.13$$

Using the values of $[\sigma_H]_{max}$ and $[\sigma_F]_{max}$ calculated in previous section combined with Equation 6.48 and 6.49 [3], we are able to verify the stresses are below overloading limits. Replacing all the variables gives

$$\begin{aligned}
\sigma_{Hmax} &= \sigma_H \sqrt{K_{qt}} = 249.04 \times \sqrt{1.13} = 264.64\ (\text{MPa}) \leq 952.00\,\text{MPa} \\
\sigma_{F1max} &= \sigma_{F1s} K_{qt} = 42.44 \times 1.13 = 47.92\ (\text{MPa}) \leq 440\,\text{MPa} \\
\sigma_{F2max} &= \sigma_{F2} K_{qt} = 41.57 \times 1.13 = 46.95\ (\text{MPa}) \leq 272.00\,\text{MPa}
\end{aligned}$$

which satisfy the conditions.

In summary, we have the following table:

	Gear drive	Pinion	Driven gear
a (mm)	180	-	-
d (mm)	-	83.24	276.76
d_a (mm)	-	87.24	280.76
d_b (mm)	-	78.22	260.07
d_f (mm)	-	78.24	271.76
m (mm)	2	-	-
u	3.32	-	-
α_t (°)	20.74	-	-
β (°)	16.03	-	-

Table 3.1: Gear drive specifications. Material of choice is 45 steel

Chapter 4

Gear Design (shaft 2-3)

Nomenclature

$[\sigma]$	permissible stress, MPa
$[\sigma]_{max}$	permissible stress due to over-loading, MPa
AG	accuracy grade of gear
a	center distance, mm
b	face width, mm
c	gear meshing rate
d	pitch circle, mm
d_a	addendum diameter, mm
d_b	base diameter, mm
d_f	deddendum diameter, mm
H	surface roughness, HB
K_d	coefficient of gear material
K	overall load factor
K_C	load placement factor
K_L	aging factor
K_v	dynamic load factor at meshing area
K_α	load distribution factor on gear teeth
K_β	load distribution factor on top land
K_{qt}	overloading factor
m	root of fatigue curve in stress test
m_t	traverse module, mm
m_n	normal module, mm
N_E	working cycle of equivalent tensile stress
N_O	working cycle of bearing stress
S	safety factor
v	rotational velocity, m/s
Y_F	tooth shape factor
Y_R	surface roughness factor
Y_s	stress concentration factor
Y_β	helix angle factor
Y_ε	contact ratio factor
Z_R	surface roughness factor of the working's area
Z_v	speed factor
z_H	contact surface's shape factor
z_M	material's mechanical properties factor
z_v	virtual number of teeth
z_ε	meshing condition factor
α	normal pressure angle. In TCVN 1065-71, $\alpha = 20°$
α_t	traverse pressure angle, °
ε_α	traverse contact ratio
ε_β	face contact ratio
β	helix angle, °

β_b	base circle helix angle, °	σ_{max}	stress due to overloading, MPa
ψ_{ba}	width to shaft distance ratio	$_1$	subscript for the pinion
ψ_{bd}	face width factor	$_2$	subscript for the driven gear
σ_b	ultimate strength, MPa	$_F$	subscript relating to bending stress
σ_{ch}	yield limit, MPa	$_H$	subscript relating to contact stress
σ^o_{lim}	permissible stress corresponding to working cycle, MPa		

4.1 Choose material

For small scale machines, type I is sufficient. Materials of this type are often quenched and have low hardness value (HB $\leq$ 350), which make it easy to manufacture precisely. However, their permissible stresses are low. In this design problem, 45 steel is used to manufacture both gears. The specifications of the material are HB205, $\sigma_b = 600$ MPa, $\sigma_{ch} = 340$ MPa, Table 6.1 [3].

4.2 Estimate the permissible contact stress and bending stress

The permissible stresses are calculated as follows, Equation 6.1 and 6.2 [3]:

$$[\sigma_H] = \frac{\sigma^o_{Hlim}}{S_H} Z_R Z_v K_{xH} K_{HL}$$

$$[\sigma_F] = \frac{\sigma^o_{Flim}}{S_F} Y_R Y_s K_{xF} K_{FC} K_{FL}$$

Initial estimation of these values assumes $Z_R Z_v K_{xH} = 1$ and $Y_R Y_s K_{xF} = 1$. The factors will be accounted for in the next sections.

Find σ^o_{lim} The value depends on hardness. It is also recommended to have the hardness of the pinion slightly higher than the driving gear from 10 to 15 units. Using Table 6.2 [3], the formulas below are used for quenched 45 steel:
$\sigma^o_{Hlim} = 2\text{HB} + 70$
$\sigma^o_{Flim} = 1.8\text{HB}$

For the pinion with $H_1 = \text{HB}205$:
$\sigma^o_{Hlim1} = 2 \times 205 + 70 = 480$ (MPa)
$\sigma^o_{Flim1} = 1.8 \times 205 = 369$ (MPa).

For the driven gear with $H_2 = H_1 - 15 = 205 - 15 = \text{HB}190$:
$\sigma^o_{Hlim2} = 2 \times 190 + 70 = 450$ (MPa)
$\sigma^o_{Flim2} = 1.8 \times 190 = 342$ (MPa).

Find S The safety factors are found using Table 6.2 [3]. $S_H = 1.1$, $S_F = 1.75$

Find K_{FC} The motor works in one direction and the load is placed in one way. $K_{FC} = 1$

Find K_L The aging factor is determined by Equation 6.3 and 6.4 [3] for contact stress and bending stress, respectively. The surface hardness of the gears is smaller than 350, which makes $m_H = 6$, $m_F = 6$. Also, both pairs of gears are meshed indefinitely which makes $c = 1$. From working condition, the service life in hours are found:

$$L_h = 8\left(\frac{\text{hours}}{\text{shift}}\right) \times CaK_{ng}L = 8 \times 1 \times 260 \times 8 = 16640 \text{ (hours)}$$

N_{HE} and N_{FE} are found using Equation 6.7 and 6.8 [3]:

$$N_{HE1} = 60n_{sh1}cL_h\left[\left(\frac{T_1}{T}\right)^3\frac{t_1}{t_1+t_2}+\left(\frac{T_2}{T}\right)^3\frac{t_2}{t_1+t_2}\right]$$
$$= 60\times1455\times1\times16640\left[\left(\frac{T}{T}\right)^3\frac{15}{15+11}+\left(\frac{0.7T}{T}\right)^3\frac{11}{15+11}\right] = 1.05\times10^9 \text{ (cycles)}$$

$$N_{HE2} = 60n_{sh2}cL_h\left[\left(\frac{T_1}{T}\right)^3\frac{t_1}{t_1+t_2}+\left(\frac{T_2}{T}\right)^3\frac{t_2}{t_1+t_2}\right]$$
$$= 60\times438.70\times1\times16640\left[\left(\frac{T}{T}\right)^6\frac{15}{15+11}+\left(\frac{0.7T}{T}\right)^6\frac{11}{15+11}\right] = 0.32\times10^9 \text{ (cycles)}$$

$$N_{FE1} = 60n_{sh1}cL_h\left[\left(\frac{T_1}{T}\right)^{m_F}\frac{t_1}{t_1+t_2}+\left(\frac{T_2}{T}\right)^{m_F}\frac{t_2}{t_1+t_2}\right]$$
$$= 60\times1455\times1\times16640\left[\left(\frac{T}{T}\right)^6\frac{15}{15+11}+\left(\frac{0.7T}{T}\right)^6\frac{11}{15+11}\right] = 0.91\times10^9 \text{ (cycles)}$$

$$N_{FE2} = 60n_{sh2}cL_h\left[\left(\frac{T_1}{T}\right)^{m_F}\frac{t_1}{t_1+t_2}+\left(\frac{T_2}{T}\right)^{m_F}\frac{t_2}{t_1+t_2}\right]$$
$$= 60\times438.70\times1\times16640\left[\left(\frac{T}{T}\right)^6\frac{15}{15+11}+\left(\frac{0.7T}{T}\right)^6\frac{11}{15+11}\right] = 0.27\times10^9 \text{ (cycles)}$$

The working cycle of bearing stress is found using the formula below, Equation 6.5 [3]:
$N_{HO1} = 30H_1^{2.4} = 30\times205^{2.4} = 10600601.81$ (cycles).
$N_{HO2} = 30H_2^{2.4} = 30\times190^{2.4} = 8833440.68$ (cycles).

For steel, $N_{FO1} = N_{FO2} = 4000000$ (MPa). The factors K_L are rounded up to 1 if it is less than 1, p.94 [3]. Apply Equation 6.3 and 6.4 [3]:
$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} = \sqrt[6]{10600601.81/1.05\times10^9} = 0.46 < 1 \Rightarrow K_{HL1} = 1$
$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} = \sqrt[6]{8833440.68/0.32\times10^9} = 0.55 < 1 \Rightarrow K_{HL2} = 1$
$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} = \sqrt[6]{4000000/0.91\times10^9} = 0.40 < 1 \Rightarrow K_{FL1} = 1$
$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} = \sqrt[6]{4000000/0.27\times10^9} = 0.49 < 1 \Rightarrow K_{FL2} = 1$

Calculate $[\sigma_H]$**,** $[\sigma_{F1}]$**,** $[\sigma_{F2}]$ Replacing all the values:
$[\sigma_{H1}] = \sigma^o_{Hlim1}K_{HL1}/S_H = 480\times1/1.1 = 436.36$ (MPa)
$[\sigma_{H2}] = \sigma^o_{Hlim2}K_{HL2}/S_H = 450\times1/1.1 = 409.09$ (MPa)
$[\sigma_{F1}] = \sigma^o_{Flim1}K_{FC}K_{FL1}/S_F = 369\times1\times1/1.75 = 210.86$ (MPa)
$[\sigma_{F2}] = \sigma^o_{Flim2}K_{FC}K_{FL2}/S_F = 342\times1\times1/1.75 = 195.43$ (MPa)

The mean permissible contact stress must be lower than 1.25 times of either $[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller. In this case, it is $1.25[\sigma_{H2}]$. Replacing all the variables gives:

$$[\sigma_H] = \frac{[\sigma_{H1}]+[\sigma_{H2}]}{2} = \frac{436.36+409.09}{2} = 422.73 \text{ (MPa)}$$
$$\le 1.25[\sigma_{H2}] = 1.25\times409.09 = 511.36 \text{ (MPa)}$$

which satisfy the condition.

In case of overloading, the permissible contact and bending stresses are calculated as follows:
For quenched material: $[\sigma_H]_{max} = 2.8 \times 600 = 952$ (MPa)
For material with hardness smaller than HB350: $[\sigma_F]_{max} = 0.8\sigma_{ch} = 0.8 \times 340 = 272$ (MPa)

4.3 Determine basic specifications of the transmission system

4.3.1 Determine basic parameters

Figure 1 shows both pairs are helical, which gives $K_a = 43$, Table 6.5 [3]. Also, the pinion is symmetrical about the bearings and both gears surface have hardness smaller than HB350, resulting in $\psi_{ba} = 0.3$, Table 6.6 [3]. Then, ψ_{bd} is calculated using Equation 6.16 [3]:
$\psi_{bd} = 0.53\psi_{ba}(u+1) = 0.53 \times 0.3 \times (3.32+1) = 0.69$, which is smaller than the permissible ratio $\psi_{bdmax} = 1.2$, Table 6.6 [3].

The gear placement in the speed reducer is similar to diagram 5 of Table 6.7 [3]. $K_{H\beta} = 1.04$, $K_{F\beta} = 1.1$

Find a Since the gear drive only has involute gears, a is estimated using Equation 6.15a [3]. It is also rounded up to the nearest multiple of 5 (small production type, p.99 [3]):

$$a = K_a(u+1)\sqrt[3]{\frac{T_{sh1}K_{H\beta}}{[\sigma_H]^2 u\psi_{ba}}} = 43\times(3.32+1)\times\sqrt[3]{\frac{46423.73 \times 1.04}{422.73^2 \times 3.32 \times 0.3}} = 120.14 \text{ (mm)}$$

$\Rightarrow a = 180$ mm

Due to complexity and maintenance, the undercutting process will be omitted throughout the calculations, that is, it is not factored in any variables of the gear drives.

Find m Using Equation 6.17 [3] and Table 6.8 [3], we determine m for each pair of gears:
$m_t = (0.01 \div 0.02)a = (0.01 \div 0.02) \times 180 = 1.8 \div 3.6$ (mm) $\Rightarrow m_t = 2$ mm

Find z_1, z_2, b Arbitrarily choose $\beta = 20°$ (permissible value is $8 \div 20°$). Combining Equation 6.18 and 6.20 [3] to calculate z_1:

$$z_1 = \frac{2a\cos\beta}{m_t(u+1)} = \frac{2 \times 180 \cos 20°}{2 \times (3.32+1)} = 39.18 \Rightarrow z_1 = 40$$

Then, find z_2 and b:
$z_2 = uz_1 = 3.32 \times 40 = 132.66 \Rightarrow z_2 = 133$
$b = \psi_{ba} a = 0.3 \times 180 = 54$ (mm)

Correct β The helix angles are corrected to compensate for rounding center distances and number of teeth, Equation 6.32 [3]:

$$\beta = \arccos\frac{m_t(z_2+z_1)}{2a} = \arccos\frac{2 \times (133+40)}{2 \times 180} = 16.03°$$

4.3.2 Other parameters

$d_1 = m_t z_1/\cos\beta = 2 \times 40/\cos 16.03° = 83.24$ (mm)
$d_2 = m_t z_2/\cos\beta = 2 \times 133/\cos 16.03° = 276.76$ (mm)
$d_{a1} = d_1 + 2m_t = 83.24 + 2 \times 2 = 87.24$ (mm)
$d_{a2} = d_2 + 2m_t = 276.76 + 2 \times 2 = 280.76$ (mm)
$d_{f1} = d_1 - 2.5m_t = 83.24 - 2.5 \times 2 = 78.24$ (mm)

$d_{f2} = d_2 - 2.5m_t = 276.76 - 2.5 \times 2 = 271.76$ (mm)
$d_{b1} = d_1 \cos\alpha = 83.24 \times \cos 20° = 78.22$ (mm)
$d_{b2} = d_2 \cos\alpha = 276.76 \times \cos 20° = 260.07$ (mm)
$\alpha_t = \arctan(\tan\alpha/\cos\beta) = \arctan(\tan 20°/\cos 16.03°) = 20.74°$
$v_1 = \pi d_1 n_{sh1}/60000 = \pi \times 83.24 \times 1455/60000 = 6.34$ (m/s)

4.4 Stress analysis

4.4.1 Correct $[\sigma_H]$, $[\sigma_{F1}]$ and $[\sigma_{F2}]$

In reality, the factors $Z_R Z_v K_{xH}$ and $Y_R Y_s K_{xF}$ do not equal to 1. This part will correct them to obtain the correct permissible stresses.

- The roughness deviation is less than $1.25\mu m$. $Z_R = 1$
- The surface hardness of the gear drive is less than HB350. $Z_v = 0.85 v_1^{0.1} = 0.85 \times 6.34^{0.1} = 1.02$
- The gears have $d_{a1}, d_{a2} \leq 700$ mm. $K_{xH} = 1$
- The gears are not polished. $Y_R = 1$
- $Y_s = 1.08 - 0.0695 \ln(m_t) = 1.03$
- $d_{a1}, d_{a2} \leq 400$ mm. $K_{xF} = 1$

Replacing all the variables gives:
$[\sigma_H] = 422.73 \times 1 \times 1.02 \times 1 = 432.21$ MPa
$[\sigma_{F1}] = 210.86 \times 1 \times 1.03 \times 1 = 217.57$ MPa
$[\sigma_{F2}] = 195.43 \times 1 \times 1.03 \times 1 = 201.65$ MPa

4.4.2 Contact stress analysis

The contact stress applied on a gear surface must satisfy Equation 6.33 [3]:

$$\sigma_H = z_M z_H z_\varepsilon \sqrt{2T_{sh1} K_H \frac{u+1}{bud_1^2}} \leq [\sigma_H]$$

Find z_M According to Table 6.5 [3], $z_M = 274$.

Find z_H Applying Equation 6.34 [3] and 6.35 [3]:
$\beta_b = \arctan(\cos\alpha_t \tan\beta) = \arctan(\cos 20.47° \tan 16.03°) = 15.04°$
$z_H = \sqrt{\dfrac{2\cos\beta_b}{\sin(2\alpha_t)}} = \sqrt{\dfrac{2 \times \cos 15.04°}{\sin(2 \times 20.74°)}} = 1.71$

Find z_ε Obtaining z_ε through calculations:

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a\sin\alpha_t}{2\pi m_t \cos\alpha_t/\cos\beta}$$
$$= \frac{\sqrt{87.24^2 - 78.22^2} + \sqrt{280.76^2 - 260.07^2} - 2 \times 180 \times \sin 20.74°}{2\pi \times 2 \times \cos 20.74°/\cos 16.03°} = 1.38$$

$\varepsilon_\beta = b\dfrac{\sin\beta}{m_t\pi} = 54 \times \dfrac{\sin 16.03°}{2\times\pi} = 2.37 > 1 \Rightarrow z_\varepsilon = \varepsilon_\alpha^{-0.5} = 1.38^{-0.5} = 0.85$

Find K_H **and** K_F We find K_H, K_F using Equation 6.39 and 6.45 [3].

The gear drive velocity determines the accuracy grade, Table 6.13 [3]. In case of helical gear, it is:
$v_1 = 6.34\,\mathrm{m/s} \leq 10\,\mathrm{m/s} \Rightarrow \mathrm{AG} = 8$

From Table P2.3 [3], using interpolation, we approximate:
$K_{Hv} = 1.04$, $K_{Fv} = 1.18$

From Table 6.14 [3], using interpolation, we approximate:
$K_{H\alpha} = 1.06$, $K_{F\alpha} = 1.3$

Knowing $K_{H\beta}$ and $K_{H\beta}$ from previous section, multiply all the values to obtain K_H and K_F:
$K_H = K_{H\beta}K_{Hv}K_{H\alpha} = 1.04 \times 1.1 \times 1.06 = 1.21$
$K_F = K_{F\beta}K_{Fv}K_{F\alpha} = 1.1 \times 1.18 \times 1.3 = 1.68$

Find σ_H Replacing all the variables gives:

$$\sigma_H = 274 \times 1.71 \times 1.55 \times \sqrt{2 \times 46423.73 \times 1.21 \times \frac{3.32+1}{54 \times 3.32 \times 83.24^2}}$$
$$= 249.04\,(\mathrm{MPa}) \leq 432.21\,\mathrm{MPa}$$

which satisfies the condition.

4.4.3 Bending stress analysis

For safety reasons, Equation 6.43 and 6.44 [3] must be met for both pairs of gears:

$$\begin{aligned} \sigma_{F1} &= \frac{2T_{sh1}K_F Y_\varepsilon Y_\beta Y_{F1}}{bd_1 m_t \cos\beta} && \leq [\sigma_{F1}] \\ \sigma_{F2} &= \sigma_{F1}Y_{F2}/Y_{F1} && \leq [\sigma_{F2}] \end{aligned}$$

Find Y_ε Using ε_α calculated in the previous section, $Y_\varepsilon = \varepsilon_\alpha^{-1} = 1.38^{-1} = 0.72$

Find Y_β The value of Y_β is calculated using the equation on p.108 [3]:
$Y_\beta = 1 - \beta/140° = 1 - 16.03°/140° = 0.89$

Find Y_F Using the formula $z_v = z\cos^{-3}(\beta)$ and Table 6.18 [3]:
$z_{v1} = z_1\cos^{-3}(\beta) = 40 \times \cos^{-3}(16.03°) = 45.05 \Rightarrow Y_{F1} = 3.67$
$z_{v2} = z_2\cos^{-3}(\beta) = 133 \times \cos^{-3}(16.03°) = 149.81 \Rightarrow Y_{F2} = 3.60$

Find K_F The value of K_F has already been found in the previous section. $K_F = 1.68$

Find σ_F The module in Equation 6.43 [3] is $m_n = m_t\cos\beta$. Replacing all the values gives:

$$\begin{aligned} \sigma_{F1} &= \frac{2 \times 46423.73 \times 1.68 \times 0.72 \times 0.89 \times 3.67}{54 \times 83.24 \times 2 \times \cos 16.03°} = 42.44\,(\mathrm{MPa}) && \leq 217.57\,\mathrm{MPa} \\ \sigma_{F2} &= 42.44 \times 3.60/3.66 = 41.57\,(\mathrm{MPa}) && \leq 201.65\,\mathrm{MPa} \end{aligned}$$

which satisfies the conditions.

4.4.4 Overloading analysis

From the load diagram, we determine the overloading factor:

$$K_{qt} = \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1+t_2}} = \sqrt{\frac{\left(\frac{T}{T}\right)^2 \times 15 + \left(\frac{0.7T}{T}\right)^2 \times 11}{15+11}} = 1.13$$

Using the values of $[\sigma_H]_{max}$ and $[\sigma_F]_{max}$ calculated in previous section combined with Equation 6.48 and 6.49 [3], we are able to verify the stresses are below overloading limits. Replacing all the variables gives

$$\begin{aligned}\sigma_{Hmax} &= \sigma_H\sqrt{K_{qt}} = 249.04 \times \sqrt{1.13} = 264.64\ (\text{MPa}) \leq 952.00\,\text{MPa}\\ \sigma_{F1max} &= \sigma_{F1s}K_{qt} = 42.44 \times 1.13 = 47.92\ (\text{MPa}) \leq 440\,\text{MPa}\\ \sigma_{F2max} &= \sigma_{F2}K_{qt} = 41.57 \times 1.13 = 46.95\ (\text{MPa}) \leq 272.00\,\text{MPa}\end{aligned}$$

which satisfy the conditions.

In summary, we have the following table:

	Gear drive	Pinion	Driven gear
a (mm)	180	-	-
d (mm)	-	83.24	276.76
d_a (mm)	-	87.24	280.76
d_b (mm)	-	78.22	260.07
d_f (mm)	-	78.24	271.76
m (mm)	2	-	-
u	3.32	-	-
α_t (°)	20.74	-	-
β (°)	16.03	-	-

Table 4.1: Gear drive specifications. Material of choice is 45 steel

References

[1] Charles Mareau, Daniel Cuillerier, and Franck Morel. *Experimental and numerical study of the evolution of stored and dissipated energies in a medium carbon steel under cyclic loading C*. 2013.

[2] Robert L. Mott, Edward M. Vavrek, and Jyhwen Wang. *Machine elements in mechanical design*. 6th ed. Pearson, 2018.

[3] Chat Trinh and Uyen Van Le. *Thiet Ke He Dan Dong Co Khi*. 6th ed. Vol. 1. Vietnam Education Publishing House Limited, 2006.