HCM University of Technology

Fluid Mechanics

CI2003

Assignment

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Chapter 1

Properties of Fluids

Q1.1

A steel vessel of 1% increase in volume when the pressure is increased by 70 MPa. At standard condition (pressure $P = 101.3$ KPa), the vessel is filled with $m = 450$ kg of water ($\rho = 1000\,\mathrm{kg/m^3}$). Given bulk modulus of elasticity, $\kappa = 2.06 \times 10^9$ Pa. Compute the mass of water to add into the vessel to increase the pressure to 70 MPa.

Ans:

$$V = \frac{m}{\rho} = \frac{450}{1000} = 0.45\ (\mathrm{m^3})$$

$$\kappa = -V\frac{dP}{dV}$$

$$\Rightarrow dm = \rho dV = -\rho V\frac{dP}{\kappa}$$

$$= -1000 \times 0.45 \times \frac{101.3 \times 10^3 - 70 \times 10^6}{2.06 \times 10^9} = 15.27\ (\mathrm{kg})$$

Q1.2

Determine the change in volume of $V_i = 3\,\mathrm{m^3}$ of air when the pressure increases from $P_i = 100$ kPa to $P_f = 500$ kPa. Air is at $T = 23\,°\mathrm{C}$ (assume ideal gas)

Ans:
Assume isothermal condition: $P_i V_i = P_f V_f$

$$\Rightarrow \Delta V = V_f - V_i = \frac{P_i V_i}{P_f} - V_i = \frac{100 \times 3}{500} - 3 = -2.4\ (\mathrm{m}^3)$$

Q1.3

They compress the air into a vessel having volume, $V_1 = 0.3\ \mathrm{m}^3$ under pressure $P_1 = 100\,\mathrm{at}$. After a period of leakage, the air pressure in the vessel is lowered to $P_2 = 90\,\mathrm{at}$. Regardless of the deformation of the vessel, determine the volume of air that is leaked during that period (corresponding to the atmospheric pressure, 1 atm), if the constant temperature and atmospheric pressure are considered to be at 1 at.

Ans:
For ideal gas, $\kappa = P_2 = -V_1 \dfrac{dP}{d_V}$

$$\Rightarrow dV = -\frac{V_1}{P_2} dP = -\frac{0.3}{90}(90 - 100 - 1) = -0.0367\ (\mathrm{m}^3)$$

Q1.4

A diameter piston $d = 50\,\mathrm{mm}$ moves evenly in a cylinder $D = 50.1\,\mathrm{mm}$. Determine the decrease in force acting on the piston (as a percentage) when the speed decreases by 5%.

Ans:
Assume constant speed: $\dfrac{\Delta V}{V_1} = -5\%$

We also have $\tau = \dfrac{F}{A} = \mu \dfrac{du}{dy} = \mu \dfrac{V}{l} \Rightarrow \dfrac{F_1}{V_1} = \dfrac{F_2}{V_2} = \mu \dfrac{A}{l}$

From the relation: $\dfrac{\Delta F}{F_1} = \dfrac{\Delta V}{V_1} = -5\%$

Q1.5

A machine axis having diameter, $D = 75\,\mathrm{mm}$, with uniform movement with $V = 0.1\,\mathrm{m/s}$ under the force, $F = 100\,\mathrm{N}$. The lubricating layer thickness, l in the bearing is $l = 0.07\,\text{mm}$. Length of bearing $L = 200\,\text{mm}$. Determine oil dynamic viscosity.

Ans:
$A = (D + 2l)\pi \times L = (75 + 2 \times 0.07)\pi \times 200 = 47211.85\,(\text{mm}^3) = 4.72 \times 10^{-5}\,(\text{m}^3)$
$\tau = \dfrac{F}{A} = \mu\dfrac{V}{l} \Rightarrow \mu = \dfrac{F}{A}\dfrac{l}{V} = \dfrac{100}{4.72 \times 10^{-5}} \times \dfrac{0.07 \times 10^{-3}}{0.1} = 1.483\,(\text{kg/m} \cdot \text{s})$

Q1.6

A thin layer of Newton liquid with specific weight γ, dynamic viscosity μ and thickness t flows on a plane inclined at an angle α. The upper surface is exposed to the air. Assuming no friction between liquid and air. Find the expression of $u(y)$. Can u consider as a linear function of y?

Ans:
The force exerting on the center of gravity of the liquid is

$$F = \gamma V \sin\alpha$$

Both contact surfaces

Q1.9

Determine the frictional force at the inner wall of a water supply pipe segment at $T = 20\,^\circ\text{C}$, radius $R = 80\,\text{mm} = 0.08\,\text{m}$, $L = 10\,\text{m}$. The velocity at the points on the pipe cross-section varies according to the following:

$$u(r) = 0.5\left(1 - \frac{r^2}{R^2}\right)$$

where r is the radius of considered point.

Ans:
From table at $T = 20\,^\circ\text{C} \Rightarrow \mu = 1.002 \times 10^{-3}\,(\text{kg/m} \cdot \text{s})$
$A = 2\pi RL = 2\pi \times 0.08 \times 10 = 5.03\,(\text{m}^2)$
$F = \tau A = \mu\dfrac{du}{dr}A = \mu\dfrac{(-2)R}{R^2}A$
$= 1.002 \times 10^{-3} \times \dfrac{(-2) \times 0.08}{0.08^2} \times 5.03 = -0.126\,(\text{N})$

Q1.10

Determine the gauge pressure inside a water drop of diameter $d = 2\,\text{mm} = 0.002\,\text{m}$. The temperature of water is $T = 25\,°\text{C}$.

Ans:
From table at $T = 25\,°\text{C} \Rightarrow \sigma_s = 0.072\ (\text{N/m})$
$$P_g = \Delta P_{droplet} = \frac{2\sigma_s}{d/2} = \frac{2 \times 0.072}{0.002/2} = 144\ (\text{Pa})$$

Q1.11

A gas has a molar mass of $R = 32\,\text{kg/mol}$ under a pressure condition of $P = 5\,\text{at} = 490332.5\,\text{Pa}$, a temperature of $T = 30\,°\text{C}$

1. Determine the gas density.
2. Determine the density of this gas if $P = const$, while temperature drops to $T_f = 15\,°\text{C}$.
3. Determine the density of this gas if holding $T = const$, while the pressure drops to $P_f = 2\,\text{at}$.

Ans:

1. $P = \rho RT \Rightarrow \rho = \dfrac{P}{RT} = \dfrac{32}{490332.5 \times (30 + 273)} = 0.215{\times}10^{-6}\ (\text{kg/m}^3)$
2. adiabatic condition
   $\rho T = \rho_f T_f \Rightarrow \rho_f = \dfrac{\rho T}{T_f} = \dfrac{0.215 \times 10^{-6} \times (30 + 273)}{15 + 273} = 0.226 \times 10^{-6}\ (\text{kg/m}^3)$
3. isothermal condition
   $\dfrac{P}{\rho} = \dfrac{P_f}{\rho_f} \Rightarrow \rho_f = \dfrac{\rho P_f}{P} = \dfrac{0.215 \times 10^{-6} \times 2}{5} = 0.086 \times 10^{-6}\ (\text{kg/m}^3)$

Q1.12

A liquid is compressed in a cylinder, the water initially has a volume of $V_o = 4\,l$ at normal pressure, $P_o = 1\,\text{at} = 98066.5\,\text{Pa}$. The pressure in the cylinder increases to $p_1 = 6\,\text{at}$, the water volume decreases by $1\,\text{cm}^3$.

1. Compute the bulk modulus of elasticity of water.
2. If the pressure in the cylinder increases to 20 at, calculate the volume of water V_f in the cylinder.
3. Calculate the pressure in the cylinder, if the volume of the water is reduced by 0.1%.

Ans:

1. $\kappa = -V_o \dfrac{dP}{dV} = -4 \times 10^{-3} \times \dfrac{6 \times 98066.5}{(-1) \times 10^{-6}} = 2.36 \times 10^9 \text{ (Pa)}$
2. Cylinder increase pressure to 20 at
   $\Rightarrow dV = V_f - 4 \times 10^{-3} = \dfrac{-20}{6} = -3.33 \text{ (cm}^3) = -3.33 \times 10^{-3} \text{ (l)}$
   $\Rightarrow V_f = 3.997 \text{ (l)}$
3. $dP = P_f - P_o = -\kappa \times \dfrac{dV}{V_o} = -2.36 \times 10^9 \times \dfrac{(-0.1)}{100} = 2.36 \times 10^6 \text{ (Pa)}$
   $\Rightarrow P_f = 2.458 \text{ (MPa)}$

Q1.13

The air moving through a narrow tube into a water tank forms a stream of bubbles $d = 3$ mm in diameter. Calculate the difference between air pressure in the narrowed section and surrounding water pressure. Give the surface tension of water $\sigma_s = 0.0728$ N/m.

Ans:

Chapter 2

Pressure and Fluid Statics

2.1 Formulas

Calculating magnitude of resultant force F_R:

$$F_R = (P_0 + \rho g y_C \sin\theta)A = P_C A$$

where P_0 is the pressure at the liquid surface (often is P_{atm}, which can be ignored)
P_C is the pressure taken at the centroid of the rigid body surface
$h_c = y_C \sin\theta$ is the vertical distance of the rigid body's centroid from the liquid surface
θ is the angle of the rigid body with respect to the liquid surface

For horizontal plate:

$$F_R = (P_0 + \rho g h)ab$$

For vertical plane (s is the upper vertical distance, b is the length of the plate):

$$F_R = (P_0 + \rho g(s + b/2))ab$$

The location at which F_R acts on the body surface is:

$$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

where y_C is the centroid of the rigid body
$I_{xx,C}$ is the second moment of area about the x-axis passing through the centroid of the rigid body (normal to the surface itself)

For rectangular plates, it can also be written as:

$$y_P = s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]}$$

The vertical distance from liquid surface $h_P = y_P \sin\theta$

2.2 Problems

Q2.1

A differential manometer consists of a U-shaped pipe of diameter d, connecting two cylinders of diameter D, the instrument being filled with two insoluble liquids of specific weight γ_1 and γ_2. When the pressure difference, $\Delta p = p_1 - p_2 = 0$, the interface between two liquids is at position 0 on the scale.

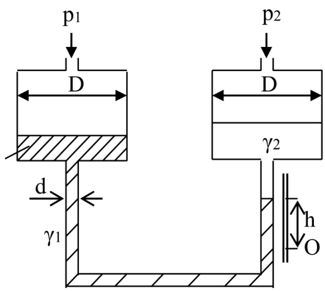

Figure 2.1: Differential manometer

1. Determine the relationship between Δp and the displacement of the interface between the two fluids, h. Given $d = 5\,\text{mm}$, $D = 50\,\text{mm}$, $\gamma_1 = 8530\,\text{N/m}^3$, $\gamma_2 = 8140\,\text{N/m}^3$, $h = 280\,\text{mm}$.
2. With the given Δp, how many times h will decrease, if $d = D = 5\,\text{mm}$.

Ans:

Let h_1, h_2 be the height of the liquid from O on the left and right pipe,

respectively.
The relation between P_1 and P_2 is described according to the formula:

$$\frac{D^2}{d^2}p_1 = p_2\frac{D^2}{d^2} + \gamma_2(h_2 - h) - \gamma_1(h_1 - h)$$

Rearranging the equation yields:

$$h = \frac{\frac{D^2}{d^2}\Delta p + \gamma_1 h_1 - \gamma_2 h_2}{\gamma_1 - \gamma_2}$$

Q2.2

A rectangular valve length b rotates about the horizontal axis at point A. Neglecting valve thickness, determine the minimum weight G of the gate based on the parameters $h1$, $h2$, $h3$, ρ, b and g such that the system is balanced. Use Figure 2.2 to solve the problem.

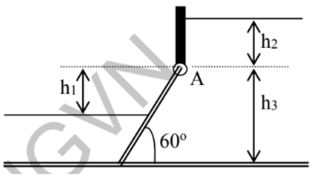

Figure 2.2: Rotating rectangular valve

Ans:
Weight of the triangular block of water: $W = \rho g V = \frac{1}{2\sqrt{3}}\rho g h_3^2$

The horizontal force acting on vertical plane is: $F_x = \rho g\left(h_2 + \frac{h_3}{2}\right)h_3$

The vertical force acting on horizontal plane is: $F_y = \rho g(h_2 + h_3)\frac{h_3}{\sqrt{3}}$

Projecting the forces onto x, y axes yields:

$$\begin{aligned} F_H = \quad & F_x = \frac{g h_3 \rho\,(2h_2 + h_3)}{2} \\ F_V = \quad & F_y - W = \frac{\sqrt{3} g h_3 \rho\,(2h_2 + h_3)}{6} \end{aligned}$$

Thus,

$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{F_x^2 + (F_y - W)^2} = \frac{1}{\sqrt{3}} g h_3 \rho (2h_2 + h_3)$

$\tan\theta = \frac{F_V}{F_H} = \frac{\sqrt{3}}{3} \Rightarrow \theta = 30°$

$s = \frac{h_2}{\sqrt{3}}, y_C = h_1$

$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin\theta)]A} = \frac{\sqrt{3} h_3 \,(3h_2 + 4h_3)}{9\,(h_2 + h_3)}$

The system is at equilibrium when and only when:

$$M_{A\circlearrowleft} = M_{A\circlearrowright}$$

$\Rightarrow \frac{b}{2} G \cos 60° = y_P F_R$

$\Rightarrow G = \frac{2}{\sqrt{3}} g h_3 \rho (2h_2 + h_3)$

Q2.3

An empty cylindrical jar of diameter $d = 5\,\text{cm} = 0.05\,\text{m}$, length $L = 10\,\text{cm} = 0.1\,\text{m}$ is placed in the water. Determine the weight of the jar so that it reaches equilibrium below the depth $h = 1\,\text{m}$. Ignore the thickness of the wall of jar. Given $p_a = p_{water\;@10\,\text{m}} = 98.1 \times 10^3$ (Pa).

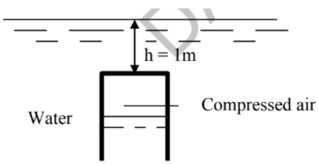

Figure 2.3: Jar under water

Ans:

The center of gravity of the jar is located at:

$$h_C = h + \frac{L}{2} = 1.05\,(\text{m})$$

The weight of the jar:

$$W = F_B = \rho g h_C \frac{\pi d^2}{4} = 20.225\ (\mathrm{N})$$

Q2.4

A rectangular valve AB is inclined to the horizontal plane an angle α, having width b, the depths of A and B are h_2 and h_3 respectively, the pressure on the water surface in the tank is p_o. The water level in the manometer tube is higher than the water level in the jar, h_1 (see Figure 2.4). Let $b = 4\,\mathrm{m}$, $h_1 = 2\,\mathrm{m}$, $h_2 = 1\,\mathrm{m}$, $h_3 = 3\,\mathrm{m}$, $\alpha = 45°$, $\rho = 1000\,\mathrm{kg/m^3}$, $g = 9.81\,\mathrm{m/s^2}$.

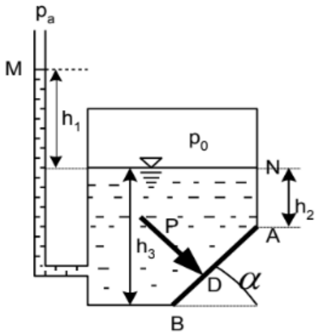

Figure 2.4: Manometer with jar

1. Compute gauge pressure p_o, p_A, p_B.
2. Compute the force by water acting on valve AB.
3. Determine the center of pressure D (compute BD).
4. Compute the minimum force required F, acting at B to remain the valve closed.

Ans:

1. $p_o = \rho g h_1 \Rightarrow p_o = 19.62$ (kPa)
   $p_A = \rho g(h_1 + h_2) \Rightarrow p_A = 29.43$ (kPa)
   $p_B = \rho g(h_1 + h_3) \Rightarrow p_B = 49.05$ (kPa)
2. Weight of the triangular block of water:
   $W = \rho g(h_3 - h_2)^2 b = 156.96$ (kN)
   The horizontal force acting on vertical plane is:
   $F_x = \rho g(h_1 + \dfrac{h_3 - h_2}{2})(h_3 - h_2)b = 235.44$ (kN)
   The vertical force acting on horizontal plane is:
   $F_y = \rho g(h_1 + h_2)(h_3 - h_2)b = 235.44$ (kN)
   Projecting the forces onto x, y axes yields:

   $$\begin{aligned} F_H &= F_x = 235.44 \text{ (kN)} \\ F_V &= F_y + W = 392.4 \text{ (kN)} \end{aligned}$$

   Thus,
   $F_R = \sqrt{F_H^2 + F_V^2} = 457.61$ (kN)
   $\tan\theta = \dfrac{F_V}{F_H} = 1.667 \Rightarrow \theta = 59.04°$
3. $BD = \dfrac{h_3 - h_2}{\cos\alpha} = 2.83$ (m)

Chapter 3

Fluid Kinematics - Bernoulli and Energy Equations

3.1

Fluid flow with the velocity described by Eulerian Method as follows:

$$u = 3t; \qquad v = xz; \qquad w = ty^2$$

1. Is this flow steady or unsteady ?
2. Prove that the flow satisfies the continuity equation of incompressible fluid ?
3. Determine the acceleration of fluid particle ?
4. Find the streamline at time $t = 1$ and passing the origin $O(0, 0)$.

Ans:

1. The flow $V = 3t\hat{\mathbf{i}} + xz\hat{\mathbf{j}} + ty^2\hat{\mathbf{k}}$ depends on time ($u = 3t, w = ty^2$), thus it is unsteady.

2. For steady continuous flow:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\Leftrightarrow \dfrac{\partial(3t)}{\partial x} + \dfrac{\partial(xz)}{\partial y} + \dfrac{\partial(ty^2)}{\partial z} = 0$ (True)
The flow satisfies the continuity equation of incompressible fluid.

3. $a = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$
$a_x = \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = 3$
$a_y = \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} = 3zt + xy^2t$
$a_z = \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} = y^2 + 2xyzt$
$\Rightarrow a = 3\hat{\mathbf{i}} + (3zt + xy^2t)\hat{\mathbf{j}} + (y^2 + 2xyzt)\hat{\mathbf{k}}$

4. Since we have zero initial condition, it is concluded that $C = 0$:

On $xy-$**plane:** $\dfrac{dy}{dx} = \dfrac{v}{u} = \dfrac{xz}{3t}$
At time $t = 1$ (s):
$y = \displaystyle\int \frac{xz}{3}dx = \frac{x^2z}{6}$

On $yz-$**plane:** $\dfrac{dz}{dy} = \dfrac{w}{v} = \dfrac{ty^2}{xz}$
At time $t = 1$ (s):
$z = \displaystyle\int \frac{y^2}{xz}dy = \frac{y^3}{3xz}$

On $xz-$**plane:** $\dfrac{dz}{dx} = \dfrac{w}{u} = \dfrac{ty^2}{3t} = \dfrac{y^2}{3}$
At time $t = 1$ (s):
$z = \displaystyle\int \frac{y^2}{3}dx = \frac{xy^2}{3}$

3.2

3.3

Plane fluid flow with the velocity described as follows:

$$u = \frac{-y}{b^2}; \qquad v = \frac{x}{a^2}$$

1. Prove that the flow satisfies the continuity equation of incompressible fluid ?
2. Find the streamlines.

Ans:

1. For steady continuous flow:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\Leftrightarrow \dfrac{\partial(\frac{-y}{b^2})}{\partial x} + \dfrac{\partial(\frac{x}{a^2})}{\partial y} + \dfrac{\partial(0)}{\partial z} = 0$ (True)
The flow satisfies the continuity equation of incompressible fluid.

2.

On $xy-$**plane:** $\dfrac{dy}{dx} = \dfrac{v}{u} = \dfrac{x/a^2}{-y/b^2}$
$\int ydy = -\dfrac{b^2}{a^2}\int xdx$
$y = \pm\sqrt{\dfrac{b^2}{a^2}x^2 + C_1}$

On $yz-$**plane:** $\dfrac{dz}{dy} = \dfrac{w}{v} = 0 \Rightarrow z = C_2$

On $xz-$**plane:** $\dfrac{dz}{dx} = \dfrac{w}{u} = 0 \Rightarrow z = C_3$

3.4

Given plane flow as follows:

$$u = e^x \sin y - x^2 y + \frac{y^3}{3}; \qquad v = e^x \cos y - y^2 x - \frac{x^3}{3} + 1;$$

Prove the flow to be irrotational and then the potential flow?

Ans:
Since the w component is zero, the vorticity of the flow must be zero:
$\vec{\zeta} \;=\; \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{\mathbf{k}} \;=\; \left[(e^x \cos y + y^2 - x^2) - (e^x \cos y - x^2 + y^2)\right]\hat{\mathbf{k}} \;=\; \vec{0},$
which is true.
The streamline equation is:
$u = \frac{\partial \Phi}{\partial x}$
$\Rightarrow \Phi = \int u dx = \int e^x \sin y - x^2 y + \frac{y^3}{3} dx = e^x \sin y - \frac{x^3 y}{3} + \frac{x y^3}{3} + C$

3.5

Given fluid flow field with the velocity described as follows:

$$\vec{V} = (3t)\hat{\mathbf{i}} + (xz)\hat{\mathbf{j}} + (ty^2)\hat{\mathbf{k}}$$

where x, y, z are in meters, t is in seconds and velocity is in 10^{-3} m/s.

1. Is this the incompressible fluid flow? Irrotational flow?
2. Compute velocity at point $M(0, 1, 1)$ at time $t = 10\,\text{s}$.
3. Compute acceleration at point $M(0, 1, 1)$ at time $t = 10\,\text{s}$.

Ans:

1. For steady continuous flow:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\Leftrightarrow \dfrac{\partial(3t)}{\partial x} + \dfrac{\partial(xz)}{\partial y} + \dfrac{\partial(ty^2)}{\partial z} = 0$ (True)
The flow satisfies the continuity equation of incompressible fluid.

For irrotational flow:

$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{\mathbf{k}} = \vec{0}$$

$\Leftrightarrow \vec{\zeta} = (2ty - x)\hat{\mathbf{i}} + (0-0)\hat{\mathbf{j}} + (z-0)\hat{\mathbf{k}} = (2ty - x)\hat{\mathbf{i}} + (z)\hat{\mathbf{k}} \neq \vec{0}$
Thus the flow is rotational.

2. $\vec{V}(0, 1, 1, 10) = 30\hat{\mathbf{i}} + 10\hat{\mathbf{k}}$

3. $\vec{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$
$a_x = \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = 3$
$a_y = \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} = 3zt + xy^2t = 30$
$a_z = \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} = y^2 + 2xyzt = 1$
$\Rightarrow \vec{a}(0, 1, 1, 10) = 3\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + \hat{\mathbf{k}}$

3.6

The steady 3-D flow with the velocity as follows:

$$u = x^2yz + 3y; \qquad v = -xy^2z + x^2y; \qquad w = ?$$

Determine the z-component velocity? Is the fluid flow rotational or irrotational?

Ans:
For steady continuous flow:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\Leftrightarrow \dfrac{\partial(x^2yz + 3y)}{\partial x} + \dfrac{\partial(-xy^2z + x^2y)}{\partial y} + \dfrac{\partial w}{\partial z} = 0$
$\Leftrightarrow (2xyz) + (-2xyz + x^2) + \dfrac{\partial w}{\partial z} = 0$

$\Leftrightarrow w = \int -x^2 dz = -x^2 z + C$
For irrotational flow:

$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{\mathbf{k}} = \vec{0}$$

$\Leftrightarrow \vec{\zeta} = (0 + xy^2)\hat{\mathbf{i}} + (x^2 y + 2xz)\hat{\mathbf{j}} + (-y^2 z + 2xy - x^2 z - 3)\hat{\mathbf{k}} \neq \vec{0}$
Thus the flow is rotational.

3.7

Given fluid flow field with the velocity described as follows:

$$u = 2x^2 - xy + z^2; \qquad v = x^2 - 4xy + y^2; \qquad w = -2xy - yz + y^2$$

1. Is this flow steady or unsteady?
2. Compute the rotation vector $\vec{\omega}$? Is the flow rotational or irrotational ?

Ans:

1. Since the flow is independent of time, it is steady.
2. For irrotational flow:

$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{\mathbf{k}} = \vec{0}$$

   $\Leftrightarrow \vec{\zeta} = (-2x + 2y - z)\hat{\mathbf{i}} + (2y + 2z)\hat{\mathbf{j}} + (3x - 4y)\hat{\mathbf{k}} \neq \vec{0}$
   Thus the flow is rotational.
   $\vec{\omega} = \frac{1}{2}\vec{\zeta} = (-x + y - 0.5z)\hat{\mathbf{i}} + (y + z)\hat{\mathbf{j}} + (1.5x - 2y)\hat{\mathbf{k}}$

3.8

Given incompressible fluid flow with the velocity described as follows:

$$u = 6xy; \qquad v = -3y^2$$

Find the streamline equation passing point $A(1, 1)$.

Ans:
$\dfrac{dy}{dx} = \dfrac{v}{u} = \dfrac{-3y^2}{6xy} = \dfrac{-y}{2x}$
$\Leftrightarrow \displaystyle\int \frac{dy}{y} = \frac{-1}{2} \int \frac{dx}{x}$
$\Leftrightarrow \ln(y) = \ln(x^{-1/2}) + C$
Replace $x = 1, y = 1 \Rightarrow C = 0$
$\Leftrightarrow y = x^{-1/2}$

3.9

A layer of oil flowing on a plane with the velocity distribution described by following rule:

$$\frac{u}{V} = A\frac{y}{d} + B\left(\frac{y}{d}\right)^3$$

where A, B are constants (see figure 3.1)

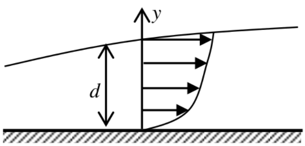

Figure 3.1: Velocity distribution of the fluid

At $y = d \Rightarrow u = V = 0.3\,\mathrm{m/s}$
At $y = \dfrac{d}{2} \Rightarrow \dfrac{u}{V} = \dfrac{11}{16}$
The specific gravity of oil $\delta = 0.8$ and the kinematic viscosity of oil $\nu = 4 \times 10^{-4}\,\mathrm{m^2/s}$ and the thickness of oil layer $d = 5\,\mathrm{mm}$. Compute the value of shear stress on the plane surface.

Ans:
We obtain the following system of equations:
$$\begin{cases} A & +B & = 1 \\ 0.5A & +0.125B & = 11/16 \end{cases} \Rightarrow \begin{cases} A & = 1.5 \\ B & = -0.5 \end{cases}$$
$\Leftrightarrow u = 90y - 1.2 \times 10^6 y^3$

$$\Leftrightarrow \frac{du}{dy} = 90 - 3.6 \times 10^6 y^2$$
$$\Leftrightarrow \tau = \mu \frac{du}{dy} = \nu(\delta \rho_{H_2O}) \frac{du}{dy} = 282.53\,\mathrm{N/m^2}$$

3.10

Plane flow between two walls having the gap, $B = 3$ m; velocity $u_{max} = 8$ m/s. The velocity profile obeys the parabolic rule $u = ax^2 + bx + c$. Give at two wall boundaries, velocities are zero (See figure 3.2). Determine the volume flow rate per unit thickness.

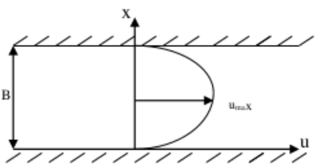

Figure 3.2: Velocity distribution of the fluid

Ans:
$a(0)^2 + b(0) + c = 0 \Rightarrow c = 0$

$$\begin{cases} 1.5^2 a & +1.5b & = 8 \\ 3^2 a & +3b & = 0 \end{cases} \Leftrightarrow \begin{cases} a & = \frac{-32}{9} \\ b & = \frac{32}{3} \end{cases}$$

$$\Rightarrow u = \frac{-32}{9}x^2 + \frac{32}{3}x$$

$$\frac{\dot{v}}{l} = \int_0^3 u dx \pi = 50.27\,\mathrm{m^2/s}$$

3.11

3.12

3.13

The pump draws water from a pool, splashes out into air as shown in figure 3.3. Diameter of suction and pushing pipe $D = 8\,\text{cm}$, the outlet is narrowed with diameter $d = 5\,\text{cm}$. Ignore the energy loss. Given $H_1 = 4\,\text{m}$, $H_2 = 8\,\text{m}$ and maximum gauge pressure measured after pump being $P_{gage} = 100\,\text{KPa}$.

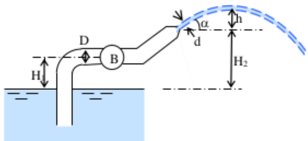

Figure 3.3: Pump model

1. Compute the velocity and discharge of water pushing out, into the air.
2. The height h if $\alpha = 30°$.

Ans:

1. Using Bernoulli equation:
   $\frac{P_{gage}}{\rho g} = \frac{V_1^2}{2g} + H_2$
   $\Rightarrow V_1 = 6.56\,\text{m/s}$
   $V_1 D^2 = V_2 d^2 \Rightarrow V_2 = 16.79\,\text{m/s}$
   $\dot{v} = V_2 A_c = V_2 \frac{\pi d^2}{4} = 0.033\,\text{m}^3/\text{s}$
2. $h = \frac{V_2^2}{2g} = 14.37\,\text{m}$

3.14

3.15

3.16

3.17

3.18

3.19