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MACHINE ELEMENTS

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Project Report

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Design Problem

D_{bc} pulley diameter, mm

F_t tangential force, N

L service life, years

T working torque, N · mm

t working time, s

v_{bc} conveyor belt speed, m/s

δ_u error of speed ratio, %

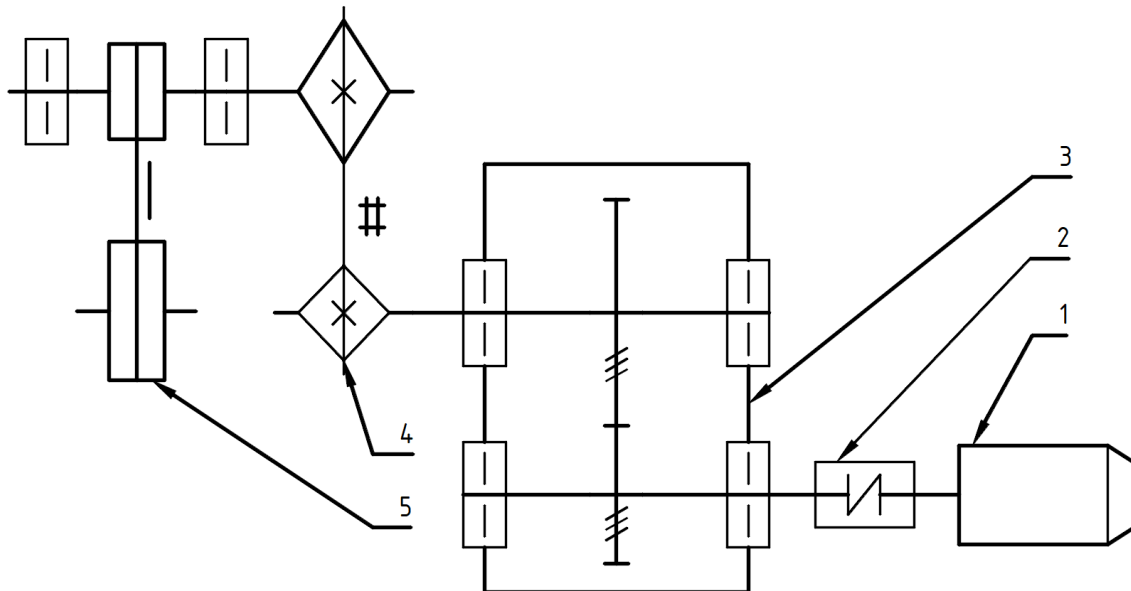


Figure 1: Mechanical transmission system of a belt conveyor

Given the mechanical transmission system in figure 1, determine the specifications for each machine element.

1. Electric motor
2. Elastic coupling
3. Gearbox
4. Chain drive
5. Belt conveyor

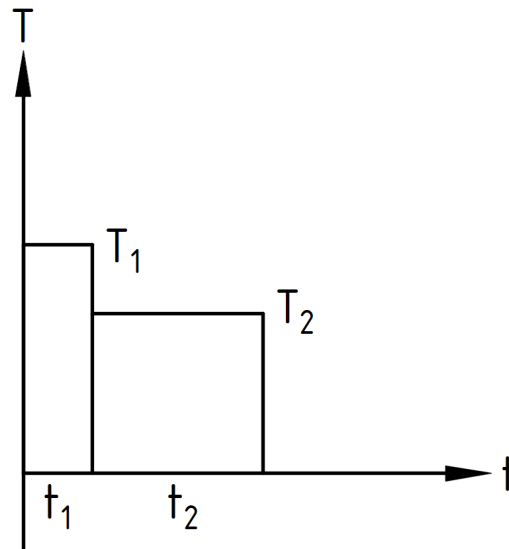


Figure 2: Input load diagram

Design parameters The chosen parameters are given in column 8:

- $F_t = 4500 \text{ (N)}$
- $v_{bc} = 3.05 \text{ (m/s)}$
- $D_{bc} = 500 \text{ (mm)}$
- $L = 4 \text{ (years)}$
- $T_1 = T \text{ (N} \cdot \text{mm)}, t_1 = 12 \text{ (s)}$
- $T_2 = 0.7T \text{ (N} \cdot \text{mm)}, t_2 = 60 \text{ (s)}$
- $\delta_u \leq \pm 5\%$

The machine works in one direction for 300 days, 2 shifts per day, 8 hours each and the load is low. Also, for the rest of this report, we will consider shaft 1 is the one connected to the electric motor, while shaft 2 is the connection between the chain drive and gearbox.

Chapter 1

Motor Design

1.1 Nomenclature

n_{bc}	rotational speed of belt conveyor, rpm	u_{hg}	transmission ratio of helical gear
n_{sh}	rotational speed of shaft, rpm	u_{sys}	transmission ratio of the system
P_m	maximum operating power of belt conveyor, kW	T_{motor}	motor torque, N · mm
P_{motor}	calculated motor power to drive the system, kW	T_{sh}	shaft torque, N · mm
P_{sh}	operating power of shaft, kW	δ_u	relative error of u_{sys}
P_w	operating power of the belt conveyor given a workload, kW	η_b	bearing efficiency
		η_c	coupling efficiency
		η_{ch}	chain drive efficiency
		η_{hg}	helical gear efficiency
u_{ch}	transmission ratio of chain drive	η_{sys}	efficiency of the system
		1	shaft 1
		2	shaft 2

1.2 Calculate η_{sys}

From table (2.3):

$$\eta_c = 1$$

$$\eta_b = 0.99$$

$$\eta_{hg} = 0.96$$

$$\eta_{ch} = 0.95$$

$$\eta_{sys} = \eta_c \eta_b^3 \eta_{hg} \eta_{ch} \approx 0.88$$

1.3 Calculate P_{motor}

$$P_m = \frac{F_t v_{bc}}{1000} \approx 13.73 \text{ (kW)}$$

From equation (2.13) :

$$P_w = P_m \sqrt{\frac{\left(\frac{T_1}{T}\right)^2 t_1 + \left(\frac{T_2}{T}\right)^2 t_2}{t_1 + t_2}} \approx 10.41 \text{ (kW)}$$

$$P_{motor} = \frac{P_w}{\eta_{sys}} \approx 11.76 \text{ (kW)} < P_m$$

1.4 Calculate n_{motor}

$$n_{bc} = \frac{6 \times 10^4 v}{\pi D_{bc}} \approx 116.5 \text{ (rpm)}$$

$$u_{ch} = 5 \text{ (table (2.4))}$$

$$u_{hg} = 5 \text{ (table (2.4))}$$

$$u_{sys} = u_{ch} u_{hg} = 25$$

$$n_{motor} = u_{sys} n_{bc} \approx 2912.54 \text{ (rpm)}$$

1.5 Choose motor

To work normally, the maximum operating power of the chosen motor must be no smaller than both estimated P_{motor} and P_m . Since $P_{motor} < P_m$ for our case, the minimum operating power of choice is P_m . In similar fashion, its rotational speed must also be no smaller than estimated n_{motor} .

Thus, from table (P1.3), we choose motor 4A160M2Y3 which operates at 18.5 kW and 2930 rpm

$$\Rightarrow P_{motor} = 18.5 \text{ kW}, n_{motor} = 2930 \text{ (rpm)}$$

Recalculating u_{sys} with the new P_{motor} and n_{motor} , we obtain:

$$u_{sys} = \frac{n_{motor}}{n_{bc}} \approx 25.15$$

Assuming $u_{hg} = const$:

$$u_{ch} = \frac{u_{sys}}{u_{hg}} \approx 5.03$$

Relative error of transmission ratio The ratio is calculated as follows:

$$\delta_u = \frac{|25.15 - 25|}{25} \approx 0.6\% \leq 5\%$$

1.6 Calculate power, rotational speed and torque

Let us denote P_{sh1} , n_{sh1} and T_{sh1} be the transmitted power, rotational speed and torque onto shaft 1, respectively. Similarly, P_{sh2} , n_{sh2} and T_{sh2} will be the transmitted parameters onto shaft 2. These notations will be used throughout the next chapters.

1.6.1 Power

$$P_{ch} = P_m \approx 13.73 \text{ (kW)}$$

$$P_{sh2} = \frac{P_{ch}}{\eta_b \eta_{ch}} \approx 14.59 \text{ (kW)}$$

$$P_{sh1} = \frac{P_{sh2}}{\eta_b \eta_{hg}} \approx 15.35 \text{ (kW)}$$

1.6.2 Rotational speed

$$n_{sh1} = n_{motor} = 2930 \text{ (rpm)}$$

$$n_{sh2} = \frac{n_{sh1}}{u_{hg}} = 586 \text{ (rpm)}$$

1.6.3 Torque

$$T_{motor} = 9.55 \times 10^6 \frac{P_{motor}}{n_{motor}} \approx 60298.63 \text{ (N} \cdot \text{mm)}$$

$$T_{sh1} = 9.55 \times 10^6 \frac{P_{sh1}}{n_{sh1}} \approx 50047.56 \text{ (N} \cdot \text{mm)}$$

$$T_{sh2} = 9.55 \times 10^6 \frac{P_{sh2}}{n_{sh2}} \approx 237825.99 \text{ (N} \cdot \text{mm)}$$

In summary, we obtain the following table:

	Motor	Shaft 1	Shaft 2
P (kW)	18.5	15.35	14.59
u	5	5.03	
n (rpm)	2930	2930	586
T (N · mm)	60298.63	50047.56	237825.99

Table 1.1: System overall specifications

Chapter 2

Gearbox Design (Helix gears)

2.1 Nomenclature

$[\sigma_H]$	permissible contact stress, MPa	F_r	radial force, N
$[\sigma_H]_{max}$	permissible contact stress due to overload, MPa	F_t	tangential force, N
		H	surface roughness, HB
$[\sigma_F]$	permissible bending stress, MPa	K_d	coefficient of gear material
$[\sigma_F]_{max}$	permissible bending stress due to overload, MPa	K_F	load factor from bending stress
		K_{FC}	load placement factor
AG	accuracy grade of gear	K_{FL}	aging factor due to bending stress
a	center distance, mm		
b	face width, mm	K_{Fv}	factor of dynamic load from bending stress at meshing area
c	gear meshing rate		
d	pitch circle, mm	$K_{F\alpha}$	factor of load distribution from bending stress on gear teeth
d_a	addendum diameter, mm		
d_b	base diameter, mm	$K_{F\beta}$	factor of load distribution from bending stress on top land
d_f	deddendum diameter, mm		
F_a	axial force, N	K_H	load factor of contact stress

K_{HL}	aging factor due to contact stress	T	input torque, N · mm
K_{Hv}	factor of dynamic load from contact stress at meshing area	v	rotational velocity, m/s
$K_{H\alpha}$	factor of load distribution from contact stress on gear teeth	x	gear correction factor
$K_{H\beta}$	factor of load distribution from contact stress on top land	Y_F	tooth shape factor
		Y_R	surface roughness factor of the gear's face
k_x	a coefficient	Y_s	sensitivity to stress concentration factor
k_y	a coefficient	Y_β	helix angle factor
m	transverse module, mm	Y_ε	contact ratio factor
m_F	root of fatigue curve in bending stress test	y	center displacement factor
m_H	root of fatigue curve in contact stress test	Z_R	surface roughness factor of the working's area
m_n	normal module, mm	Z_v	speed factor
N_{FE}	working cycle of equivalent tensile stress corresponding to $[\sigma_F]$	z_H	contact surface's shape factor
		z_M	material's mechanical properties factor
N_{FO}	working cycle of bearing stress corresponding to $[\sigma_F]$	z_{min}	minimum number of teeth corresponding to β
N_{HE}	working cycle of equivalent tensile stress corresponding to $[\sigma_H]$	z_v	virtual number of teeth
N_{HO}	working cycle of bearing stress corresponding to $[\sigma_H]$	z_ε	meshing condition factor
S	specific length, mm	α	normal pressure angle, following Vietnam standard (TCVN 1065-71), i.e. $\alpha = 20^\circ$
S_F	safety factor of bending stress	α_t	traverse pressure angle, $^\circ$
S_H	safety factor of contact stress	ε_α	traverse contact ratio
		ε_β	face contact ratio
		β	helix angle, $^\circ$

β_b	base circle helix angle, °	σ_{Hlim}^o	permissible contact stress
ψ_{ba}	width to shaft distance ratio		corresponding to working cycle,
ψ_{bd}	face width factor		MPa
σ_b	ultimate strength, MPa	1	subscript for pinion
σ_{ch}	yield limit, MPa	2	subscript for driven gear
σ_{Flim}^o	permissible bending stress corresponding to working cycle, MPa	w	subscript for variable value after correction

2.2 Choose material

From table (6.1) , the material of choice for both gears is steel 40X with $S \leq 100$ (mm), HB250, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 550$ (MPa).

Table (6.2) also gives $\sigma_{Hlim}^o = 2HB + 70$, $S_H = 1.1$, $\sigma_{Flim}^o = 1.8HB$, $S_F = 1.75$

Therefore, they have the same properties except for their surface roughness H .

The reasoning is given on p.91, where $H_2 = H_1 - 10 \div 15$

For the pinion, $H_1 = HB250 \Rightarrow \sigma_{Hlim1}^o = 570$ (MPa), $\sigma_{Flim1}^o = 450$ (MPa)

For the driven gear, $H_2 = HB240 \Rightarrow \sigma_{Hlim2}^o = 550$ (MPa), $\sigma_{Flim2}^o = 432$ (MPa)

2.3 Calculate $[\sigma_H]$ and $[\sigma_F]$

2.3.1 Working cycle of bearing stress

Using equation (6.5) :

$$N_{HO1} = 30H_1^{2.4} = 17.07 \times 10^6 \text{ (cycles)}$$

$$N_{HO2} = 30H_2^{2.4} = 15.4749 \times 10^6 \text{ (cycles)}$$

2.3.2 Working cycle of equivalent tensile stress

Since $H_1, H_2 \leq \text{HB350}$, $m_H = 6$, $m_F = 6$.

Both gears meshed indefinitely, thus $c = 1$.

From working condition, we calculate:

$$L_h = 8 \left(\frac{\text{hours}}{\text{shift}} \right) \times 2 \left(\frac{\text{shifts}}{\text{day}} \right) \times 300 \left(\frac{\text{days}}{\text{year}} \right) \times 4 (\text{years}) = 19200 (\text{hours})$$

Applying equation (6.7) and T_1, T_2, t_1, t_2 in the initial parameters:

$$N_{HE1} = 60n_{sh1}cL_h \left[\left(\frac{T_1}{T} \right)^3 \frac{t_1}{t_1 + t_2} + \left(\frac{T_2}{T} \right)^3 \frac{t_2}{t_1 + t_2} \right] \approx 1.53 \times 10^9 (\text{cycles})$$

$$N_{HE2} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T} \right)^3 \frac{t_1}{t_1 + t_2} + \left(\frac{T_2}{T} \right)^3 \frac{t_2}{t_1 + t_2} \right] \approx 0.31 \times 10^9 (\text{cycles})$$

$$N_{FE1} = 60n_{sh1}cL_h \left[\left(\frac{T_1}{T} \right)^{m_F} \frac{t_1}{t_1 + t_2} + \left(\frac{T_2}{T} \right)^{m_F} \frac{t_2}{t_1 + t_2} \right] \approx 0.89 \times 10^9 (\text{cycles})$$

$$N_{FE2} = 60n_{sh2}cL_h \left[\left(\frac{T_1}{T} \right)^{m_F} \frac{t_1}{t_1 + t_2} + \left(\frac{T_2}{T} \right)^{m_F} \frac{t_2}{t_1 + t_2} \right] \approx 0.18 \times 10^9 (\text{cycles})$$

2.3.3 Aging factor

For steel, $N_{FO1} = N_{FO2} = 4 \times 10^6$ (MPa). Applying equations (6.3) and (6.4) yield (if $K_{HL}, K_{FL} < 1$, $K_{HL} = 1$ and $K_{FL} = 1$ according to the properties given on p.94):

$$K_{HL1} = \sqrt[m_H]{N_{HO1}/N_{HE1}} \approx 0.47 < 1 \Rightarrow K_{HL1} = 1$$

$$K_{HL2} = \sqrt[m_H]{N_{HO2}/N_{HE2}} \approx 0.61 < 1 \Rightarrow K_{HL2} = 1$$

$$K_{FL1} = \sqrt[m_F]{N_{FO1}/N_{FE1}} \approx 0.41 < 1 \Rightarrow K_{FL1} = 1$$

$$K_{FL2} = \sqrt[m_F]{N_{FO2}/N_{FE2}} \approx 0.53 < 1 \Rightarrow K_{FL2} = 1$$

2.3.4 Calculate $[\sigma_H]$, $[\sigma_{F1}]$, $[\sigma_{F2}]$

Since the motor works in one direction, $K_{FC} = 1$. In ideal conditions, we assume

$Z_R Z_V K_{xH} = 1$ and $Y_R Y_s K_{xF} = 1$ according to p.92:

$$[\sigma_{H1}] = \sigma_{Hlim1}^o K_{HL1} / S_{H1} \approx 518.18 (\text{MPa})$$

$$[\sigma_{H2}] = \sigma_{Hlim2}^o K_{HL2} / S_{H2} \approx 500 (\text{MPa})$$

$$[\sigma_{F1}] = \sigma_{Flim1}^o K_{FC1} K_{FL1} / S_{F1} \approx 257.14 \text{ (MPa)}$$

$$[\sigma_{F2}] = \sigma_{Flim2}^o K_{FC2} K_{FL2} / S_{F2} \approx 246.86 \text{ (MPa)}$$

The mean permissible contact stress must be lower than 1.25 times of either

$[\sigma_{H1}]$ or $[\sigma_{H2}]$, whichever is smaller:

$$[\sigma_H] = \frac{1}{2} ([\sigma_{H1}] + [\sigma_{H2}]) \approx 509.09 \text{ (MPa)} \leq 1.25[\sigma_H]_{min} = 1.25[\sigma_{H2}]$$

The permissible contact stress and bending stress due to overload are calculated as follows:

$$[\sigma_H]_{max} = 2.8\sigma_{ch} = 1540 \text{ (MPa)}$$

$$[\sigma_F]_{max} = 0.8\sigma_{ch} = 440 \text{ (MPa)}$$

2.4 Transmission Design

2.4.1 Determine basic parameters

Examine table (6.5) gives $K_a = 43$

Assuming symmetrical design, table (6.6) also gives $\psi_{ba} = 0.5$

$$\Rightarrow \psi_{bd} = 0.53\psi_{ba}(u_{hg} + 1) = 1.59$$

From table (6.7) , using interpolation, we approximate $K_{H\beta} \approx 1.108$, $K_{F\beta} \approx 1.2558$

Since the gear system only consists of involute gears and it is also a speed reducer gearbox, we estimate a using equation (6.15a) gives:

$$a = K_a(u_{hg} + 1) \sqrt[3]{\frac{T_{sh1} K_{H\beta}}{[\sigma_H]^2 u_{hg} \psi_{ba}}} \approx 113.7 \text{ (mm)}$$

According to SEV229-75 standard, we choose $a_w = 125 \text{ mm}$

2.4.2 Determine gear meshing parameters

Find m Applying equation (6.17) and choose m from table (6.8) :

$$m = (0.01 \div 0.02)a_w = 1.25 \div 2.5 \text{ (mm)} \Rightarrow m = 1.5 \text{ (mm)}$$

Find z_1, z_2, b_w Let $\beta = 14^\circ$. Combining equation (6.18) and (6.20), we come up with the formula to calculate z_1 . From the result, z_1 is rounded up to the nearest odd number (preferably a prime number).

$$z_1 = \frac{2a_w \cos \beta}{m(u_{hg} + 1)} \approx 26.95 \Rightarrow z_1 = 27$$

$$z_2 = u_{hg} z_1 = 135$$

$$\Rightarrow b = \psi_{ba} a_w = 62.5 \text{ (mm)}$$

Correct β There are 2 approaches for correction involving the change of either α or β . Because altering α leads to many other corrections (d_1, d_2 and a_w), β will be used instead.

Since z_1 is rounded, we must find β to obtain the correct angle, ensuring that $\beta \in (8^\circ, 20^\circ)$. Using equation (6.32):

$$\beta_w = \arccos \frac{m(z_1 + z_2)}{2a_w} \approx 13.59^\circ$$

Find x_1, x_2 To find x_1 and x_2 , we will follow the calculation scheme provided in p.103. Since $\beta_w \approx 13.59^\circ \in (10, 15]$, $z_{min} = 11$, which leads to z_1 satisfying condition $z_1 \geq z_{min} + 2 > 10$, according to table (6.9). Combined with $u_{hg} = 5 \geq 3.5$, we obtain $x_1 = 0.3, x_2 = -0.3$, disregarding the calculation of y .

2.4.3 Basic parameters

$$d_1 = d_{w1} = \frac{mz_1}{\cos \beta} \approx 41.67 \text{ (mm)}$$

$$d_2 = d_{w2} = \frac{mz_2}{\cos \beta} \approx 208.33 \text{ (mm)}$$

$$d_{a1} = d_1 + 2(1 + x_1)m \approx 45.57 \text{ (mm)}$$

$$d_{a2} = d_2 + 2(1 + x_2)m \approx 210.43 \text{ (mm)}$$

$$d_{f1} = d_1 - (2.5 - 2x_1)m \approx 38.82 \text{ (mm)}$$

$$d_{f2} = d_2 - (2.5 - 2x_2)m \approx 203.68 \text{ (mm)}$$

$$d_{b1} = d_1 \cos \alpha \approx 39.15 \text{ (mm)}$$

$$d_{b2} = d_2 \cos \alpha \approx 195.77 \text{ (mm)}$$

$$\alpha_t = \alpha_{tw} = \arctan \frac{\tan \alpha}{\cos \beta_w} \approx 20.53^\circ$$

$$v = \frac{\pi d_1 n_{sh1}}{6 \times 10^4} \approx 6.39 \text{ (m/s)}$$

2.4.4 Find $[\sigma_{Hw}]$, $[\sigma_{Fw1}]$ and $[\sigma_{Fw2}]$

In this section, we will try to approximate these parameters based on the factors Z_R , Z_V , K_{xH} and Y_R , Y_S , K_{xF} to substitute to equation (6.1) and (6.2):

$$[\sigma_{Hw}] = [\sigma_H] Z_R Z_V K_{xH}$$

$$[\sigma_{Fw}] = [\sigma_F] Y_R Y_S K_{xF}$$

Assuming smooth surface condition, $Z_R = 1$.

$$Z_V = 0.85v^{0.1} \approx 1.02 \text{ with } H \leq 350.$$

In case of $v > 5$ (m/s), $K_{xH} = 1$.

The pair of gears are properly polished, which makes $Y_R = 1.1$

$$Y_S = 1.08 - 0.0695 \ln(m) \approx 1.05$$

Since $d_{a1}, d_{a2} \leq 400$ (mm), $K_{xF} = 1$, which leads to:

$$[\sigma_{Hw}] = 520.93 \text{ (MPa)}$$

$$[\sigma_{Fw1}] = 297.51 \text{ (MPa)}$$

$$[\sigma_{Fw2}] = 285.61 \text{ (MPa)}$$

2.4.5 Contact stress analysis

From section 6.3.3. in the text, contact stress applied on a gear surface must satisfy the condition below:

$$\sigma_H = z_M z_H z_\varepsilon \sqrt{2T_{sh1} K_H \frac{u_{hg} + 1}{b u_{hg} d_{w1}^2}} \leq [\sigma_{Hw}]$$

Find z_M $z_M = 274$, according to table (6.5)

Find z_H $\beta_b = \arctan(\cos \alpha_t \tan \beta_w) \approx 12.76^\circ \Rightarrow z_H = \sqrt{2 \frac{\cos \beta_b}{\sin(2\alpha_{tw})}} \approx 1.72$

Find z_ε Obtaining z_ε through calculations:

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin \alpha_{tw}}{2\pi m \frac{\cos \alpha_t}{\cos \beta_w}} \approx 1.41$$

$$\varepsilon_\beta = b \frac{\sin \beta_w}{m\pi} \approx 3.12 > 1 \Rightarrow z_\varepsilon = \varepsilon_\alpha^{-0.5} \approx 0.86$$

Find K_H We find K_H using equation $K_H = K_{H\beta} K_{H\alpha} K_{Hv}$

From table (6.13), $v \leq 10$ (m/s) \Rightarrow AG = 8

From table (P2.3), using interpolation, we approximate:

$$K_{Hv} \approx 1.07, K_{Fv} \approx 1.18$$

From table (6.14), using interpolation, we approximate:

$$K_{H\alpha} \approx 1.1, K_{F\alpha} \approx 1.29$$

$$\Rightarrow K_H \approx 1.3$$

Find σ_H After calculating $z_M, z_H, z_\varepsilon, K_H$, we get the following result:

$$\sigma_H \approx 477.51 \text{ (MPa)} \leq [\sigma_{Hw}] \approx 509.09 \text{ (MPa)}$$

2.4.6 Bending stress analysis

For safety reasons, the following conditions must be met:

$$\sigma_{F1} = 2 \frac{T_{sh1} K_F Y_\varepsilon Y_\beta Y_{F1}}{b d_{w1} m_n} \leq [\sigma_{Fw1}]$$

$$\sigma_{F2} = \frac{\sigma_{F1} Y_{F2}}{Y_{F1}} \leq [\sigma_{Fw2}]$$

Find Y_ε Knowing that $\varepsilon_\alpha \approx 1.41$, we can calculate $Y_\varepsilon = \varepsilon_\alpha^{-1} \approx 0.71$

Find Y_β $Y_\beta = 1 - \frac{\beta_w}{140} \approx 0.9$

Find Y_F Using formula $z_v = z \cos^{-3}(\beta_w)$ and table (6.18):

$$z_{v1} = z_1 \cos^{-3}(\beta_w) \approx 29.4 \Rightarrow Y_{F1} \approx 3.54$$

$$z_{v2} = z_2 \cos^{-3}(\beta_w) \approx 147.01 \Rightarrow Y_{F2} \approx 3.63$$

Find K_F Using $K_{F\beta}$, $K_{F\alpha}$, K_{Fv} calculated from the sections above, we derive:

$$K_F = K_{F\beta} K_{F\alpha} K_{Fv} \approx 1.91$$

Find σ_F Since $m_n = m \cos \beta_w \approx 1.46$, substituting all the values, we find out that:

$$\sigma_{F1} \approx 114.11 \text{ (MPa)} \leq [\sigma_{Fw1}] \approx 297.51 \text{ (MPa)}$$

$$\sigma_{F2} \approx 117.01 \text{ (MPa)} \leq [\sigma_{Fw2}] \approx 285.61 \text{ (MPa)}$$

2.4.7 Force on shafts

$$F_t = \frac{2T_{sh1}}{d_{w1}} \approx 2402.28 \text{ (N)}$$

$$F_r = F_t \tan \alpha_{tw} \approx 899.55 \text{ (N)}$$

$$F_a = F_t \tan \beta_w \approx 580.75 \text{ (N)}$$

In summary, we have the following table:

	pinion	driving gear
H (HB)	250	240
$[\sigma_F]$ (MPa)	257.14	246.86
$[\sigma_H]$ (MPa)	509.09	
$[\sigma_H]_{max}$ (MPa)	1540	
$[\sigma_F]_{max}$ (MPa)	440	
a_w (mm)	100	
b (mm)	50	
m (mm)	1.5	
d_w (mm)	33.33	166.67
d_a (mm)	37.23	168.77
d_f (mm)	30.48	162.02
d_b (mm)	31.32	156.62
u_{hg}	5	
v (m/s)	5	
x (mm)	0.3	-0.3
z	21	105
α_{tw} ($^\circ$)	20.65	
β_w ($^\circ$)	19.09	

Table 2.1: Gearbox specifications

Chapter 3

Shaft Design

3.1 Nomenclature

$[s]$	permissible safety factor	h_n	distance between bearing lid and bolt, mm
$[\sigma]$	permissible static strength, MPa	hr	tooth direction
$[\tau]$	permissible torsion, MPa	K_x	surface tension concentration factor
a_w	shaft distance, mm	K_y	diminish factor
b_O	rolling bearing width, mm	K_σ	combined influence factor in tension
cb	role of gear on the shaft (active or passive)	K_τ	combined influence factor in shear
cq	rotational direction of the shaft	\tilde{k}_1	distance between elements, mm
d	base shaft diameter, mm	\tilde{k}_2	distance between bearing surface and inner walls of the gearbox, mm
d_w	gear diameter, mm		
F_a	axial force, N		
F_r	radial force, N		
F_t	tangential force, N		
F_x	applied force, N		

\tilde{k}_3	distance between element surface and bearing lid, mm	W	section modulus, mm ³
k_σ	fatigue stress concentration factor in tension	W_O	polar section modulus, mm ³
k_τ	fatigue stress concentration factor in shear	α_{tw}	traverse meshing angle, °
		β	helix angle, °
		ψ_σ	mean stress influence factor
		ψ_τ	mean shear influence factor
l	length (general), mm	σ_{-1}	endurance limit at stress ratio of -1, MPa
l_m	hub length (general), mm		
M	moment at the cross section, N · mm	σ_a	tensile stress amplitude, MPa
		σ_b	ultimate strength, MPa
M_e	equivalent moment, N · mm	σ_{ch}	yield limit, MPa
l_m	hub diameter, mm	σ_m	mean tensile stress, MPa
q	standardized coefficient of shaft diameter	σ_{td}	static strength, MPa
		τ_{-1}	endurance limit at shear ratio of -1, MPa
R	reaction force, N		
r	shoulder fillet radius, mm	τ_a	shear stress amplitude, MPa
\bar{r}	position of applied force on the shaft, mm	τ_m	mean shear stress, MPa
		1	subscript for shaft 1
S	length defined by table (6.1), mm	2	subscript for shaft 2
		max	subscript for maximum value
s	calculated safety factor	$sh1$	subscript for shaft 1
s_σ	safety factor in tensile stress	$sh2$	subscript for shaft 2
s_τ	safety factor in shear stress	x	subscript for x-axis
T	torque at the cross section, N · mm	y	subscript for y-axis
		z	subscript for z-axis

3.2 Choose material

For moderate load, we will use quenched 45X steel to design the shafts. From table (6.1), the specifications are as follows: $S \leq 100$ (mm), HB260, $\sigma_b = 850$ (MPa), $\sigma_{ch} = 650$ (MPa).

3.3 Transmission Design

3.3.1 Load on shafts

Applied forces from Gears

Following p.186, the subscript convention of the book will be used in this chapter. If a symbol has 2 numeric subscripts, the first one is the ordinal number of shafts while the second one is used for machine elements.

On shaft 1, the motor is labeled 1 and the pinion is labeled 2. On shaft 2, the driven gear is labeled 1 and the driving sprocket is labeled 2. Therefore, we obtain:

$$\bar{r}_{12} = -d_{w12}/2 \approx -20.83 \text{ (mm)}, \text{hr}_{12} = +1, \text{cb}_{12} = +1, \text{cq}_1 = +1$$

$$\bar{r}_{21} = +d_{w21}/2 \approx +104.17 \text{ (mm)}, \text{hr}_{21} = -1, \text{cb}_{21} = -1, \text{cq}_2 = -1$$

Find magnitude of F_t , F_r , F_a Using the results from the previous chapter: , $\beta_w = 13.59^\circ$, $d_{w12} \approx 41.67$ (mm)

$$\begin{cases} F_{t12} = F_{t21} = \frac{2T_{sh1}}{d_{w12}} \approx 2402.28 \text{ (N)} \\ F_{r12} = F_{r21} = \frac{F_{t12} \tan \alpha}{\cos \beta_w} \approx 925.46 \text{ (N)} \\ F_{a12} = F_{a21} = F_{t12} \tan \beta_w \approx 580.75 \text{ (N)} \end{cases}$$

Find direction of F_t , F_r , F_a Following the sign convention, we obtain the forces:

$$\begin{cases} F_{x12} = \frac{\bar{r}_{12}}{|\bar{r}_{12}|} c_{q1} c_{b12} F_{t12} \approx -2402.28 \text{ (N)} \\ F_{y12} = -\frac{\bar{r}_{12}}{|\bar{r}_{12}|} \frac{\tan \alpha}{\cos \beta_w} F_{t12} \approx 925.46 \text{ (N)} \\ F_{z12} = c_{q1} c_{b12} h_{r12} F_{t12} \tan \beta_w \approx 580.75 \text{ (N)} \end{cases}$$

$$\begin{cases} F_{x21} = \frac{\bar{r}_{21}}{|\bar{r}_{21}|} c_{q2} c_{b21} F_{t21} \approx 2402.28 \text{ (N)} \\ F_{y21} = -\frac{\bar{r}_{21}}{|\bar{r}_{21}|} \frac{\tan \alpha_{tw}}{\cos \beta_w} F_{t21} \approx -925.46 \text{ (N)} \\ F_{z21} = c_{q2} c_{b21} h_{r21} F_{t21} \tan \beta_w \approx -580.75 \text{ (N)} \end{cases}$$

Applied forces from Chain drives

Assuming the angle between x-axis and F_r is 210° and $F_r \approx 2678.96 \text{ (N)}$ (chapter 2), we get the direction of F_r on shaft 2:

$$\begin{cases} F_{x22} = F_{r22} \cos 210^\circ \approx -2320.05 \text{ (N)} \\ F_{y22} = F_{r22} \sin 210^\circ \approx -1339.48 \text{ (N)} \end{cases}$$

3.3.2 Preliminary calculations

Since shaft 1 and shaft 2 receive input torques T_{sh1} and T_{sh2} , respectively, $[\tau_1] = 15 \text{ (MPa)}$ and $[\tau_2] = 30 \text{ (MPa)}$. Using equation (10.9), we can approximate the base shaft diameters d_1 and d_2 :

$$d_1 \geq \sqrt[3]{\frac{T_{sh1}}{0.2[\tau_1]}} \approx 25.55 \text{ (mm)}$$

$$d_2 \geq \sqrt[3]{\frac{T_{sh2}}{0.2[\tau_2]}} \approx 34.1 \text{ (mm)}$$

Recall that our motor is 4A160M2Y3, inspecting table P1.7 we obtain the motor's output shaft diameter is 42 (mm). According to the recommendations on p.189, we limit the chosen range of $d_1 \geq (0.8 \div 1.2) \times 42 \text{ (mm)}$. For d_2 , the chosen range

must be around $(0.3 \div 0.35) \times a_w$ (mm). Thus, $d_1 = 35$ (mm), $d_2 = 40$ (mm). Consulting table (10.2) gives $b_{O1} \approx 21$ (mm) and $b_{O2} \approx 23$ (mm)

3.3.3 Identify the distance between bearings and applied forces

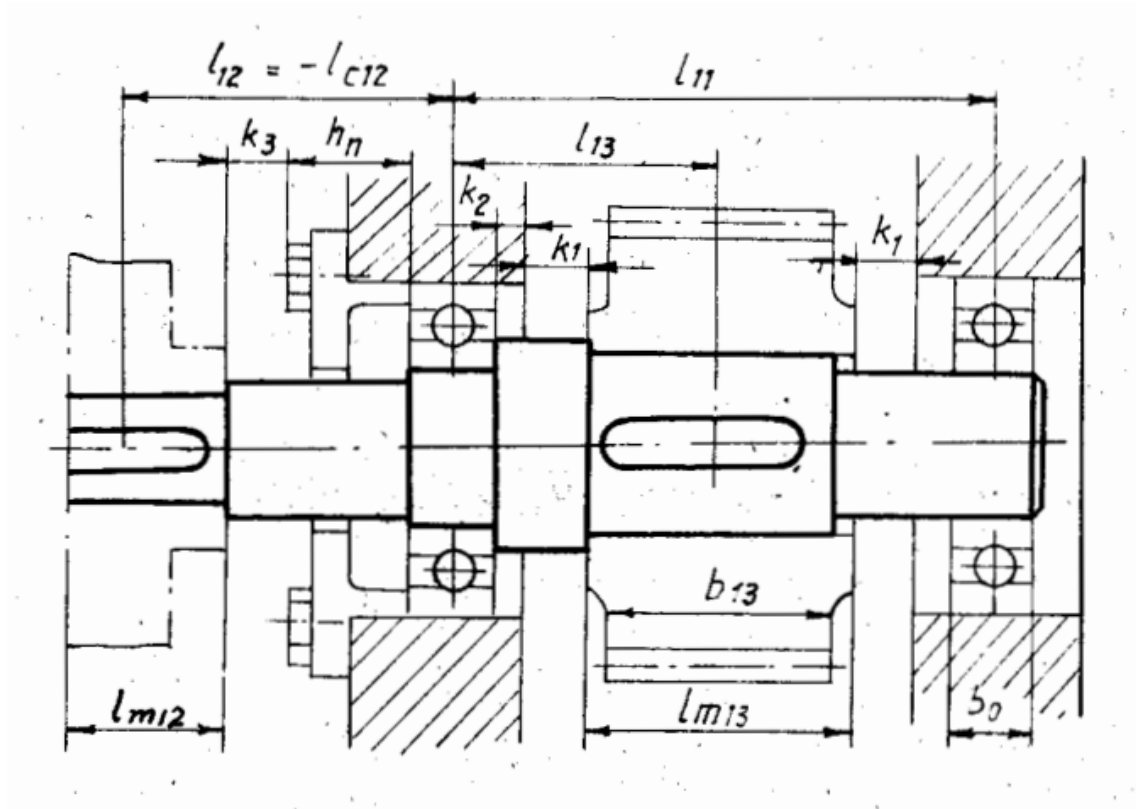


Figure 3.1: Shaft design and its dimensions

In this section, we will find all the parameters in Figure 3.1. However, if a parameter has 2 numeric subscripts, the first one will denote the ordinal number of shafts.

Using equation (10.10), the gear hubs are $l_{m13} = l_{m12} = 1.5d_1 = 45$ (mm), $l_{m23} = l_{m22} = 1.5d_2 = 52.5$ (mm), where l_{m22} is the chain hub.

From table (10.3), we choose $\tilde{k}_1 = 10$ (mm), $\tilde{k}_2 = 8$ (mm), $\tilde{k}_3 = 15$ (mm), $h_n = 18$ (mm). This parameters apply for both shafts in the system.

Table (10.4) introduces the formulas for several types of gearbox. Since our system only concerns about 1-level gear reducer, the ones below are used:

On shaft 1:

$$l_{12} = -l_{c12} = -[0.5(l_{m12} + b_{O1}) + \tilde{k}_3 + h_n] = -71.5 \text{ (mm)}$$

$$l_{13} = 0.5(l_{m13} + b_{O1}) + \tilde{k}_1 + \tilde{k}_2 = 59.5 \text{ (mm)}$$

$$l_{11} = 2l_{13} = 119 \text{ (mm)}$$

On shaft 2:

$$l_{22} = -l_{c22} = -[0.5(l_{m22} + b_{O2}) + \tilde{k}_3 + h_n] = -66.75 \text{ (mm)}$$

$$l_{23} = 0.5(l_{m23} + b_{O2}) + \tilde{k}_1 + \tilde{k}_2 = 54.75 \text{ (mm)}$$

$$l_{21} = 2l_{23} = 109.5 \text{ (mm)}$$

3.3.4 Determine shaft diameters and lengths

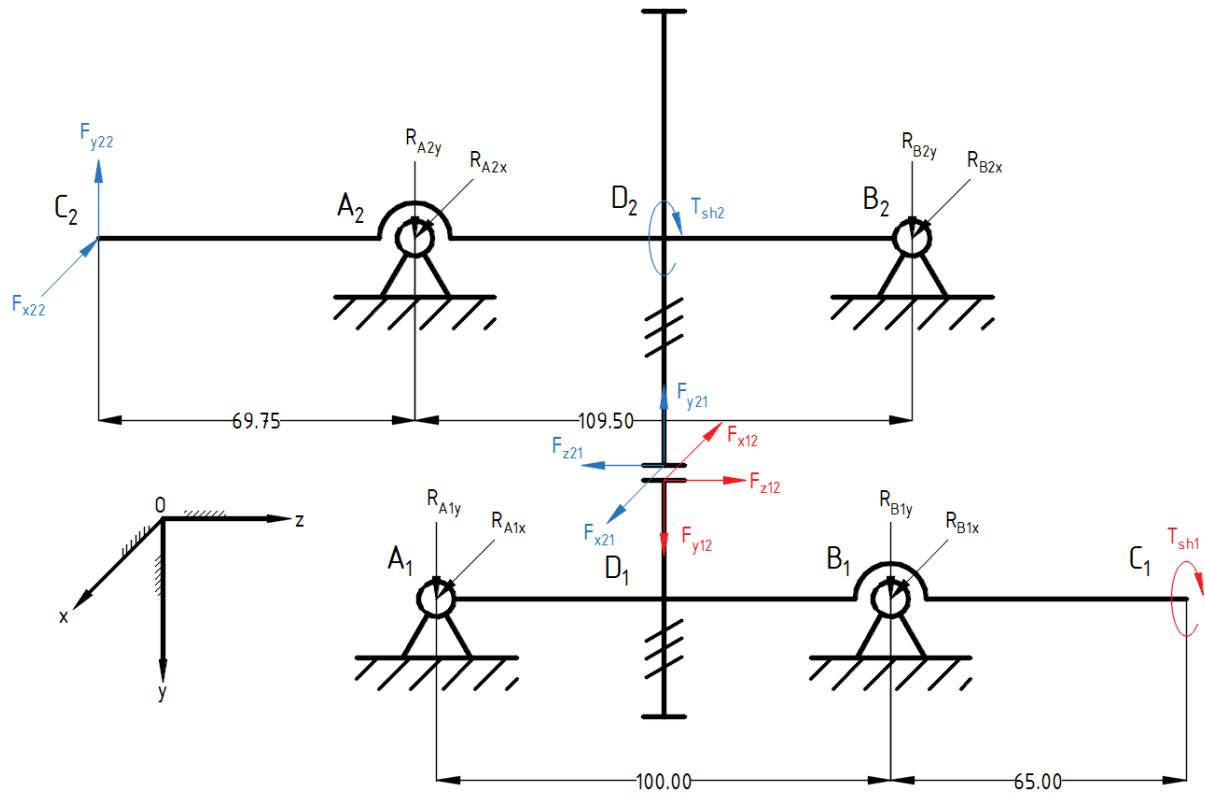


Figure 3.2: Force analysis of 2 shafts

Find reaction forces From the diagram, we solve for the reaction forces at A_1 , A_2 , B_1 , B_2 , which are R_{A1x} , R_{A1y} , R_{B1x} , R_{B1y} , R_{A2x} , R_{A2y} , R_{B2x} , R_{B2y} . Using

equilibrium conditions

$$\begin{cases} \sum_i \mathbf{F}_i = 0 \\ \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0 \end{cases}$$

we obtain the results:

$$\begin{cases} R_{A1x} \approx 1201.14 \text{ (N)} \\ R_{A1y} \approx -352.24 \text{ (N)} \\ R_{B1x} \approx 1201.14 \text{ (N)} \\ R_{B1y} \approx -573.22 \text{ (N)} \end{cases} \quad \begin{cases} R_{A2x} \approx 943.15 \text{ (N)} \\ R_{A2y} \approx 3668.4 \text{ (N)} \\ R_{B2x} \approx -2005.96 \text{ (N)} \\ R_{B2y} \approx -422.89 \text{ (N)} \end{cases}$$

The total bending moments at 8 critical cross sections are also calculated (we use the formula (10.15) to derive $M = \sqrt{M_x^2 + M_y^2}$ at each section):

$$\begin{cases} M_{A1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{D1}^- \approx 68531.85 \text{ (N} \cdot \text{mm)} \\ M_{D1}^+ \approx 72867.4 \text{ (N} \cdot \text{mm)} \\ M_{B1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{C1} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases} \quad \begin{cases} M_{C2} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{A2} \approx 191545.76 \text{ (N} \cdot \text{mm)} \\ M_{D2}^- \approx 146910 \text{ (N} \cdot \text{mm)} \\ M_{D2}^+ \approx 121977.78 \text{ (N} \cdot \text{mm)} \\ M_{B2} \approx 0 \text{ (N} \cdot \text{mm)} \end{cases}$$

Draw bending moment - torque diagrams Knowing the reaction forces, we can easily draw bending moment and torque diagram for both shafts on 2 major planes (xOz) and (yOz).

Find equivalent moments Knowing T_{sh1} and T_{sh2} , we calculate equivalent moment M_e at the 8 cross sections specified using the formula below:

$$M_e = \sqrt{M_x^2 + M_y^2 + 0.75T_{sh}^2}$$

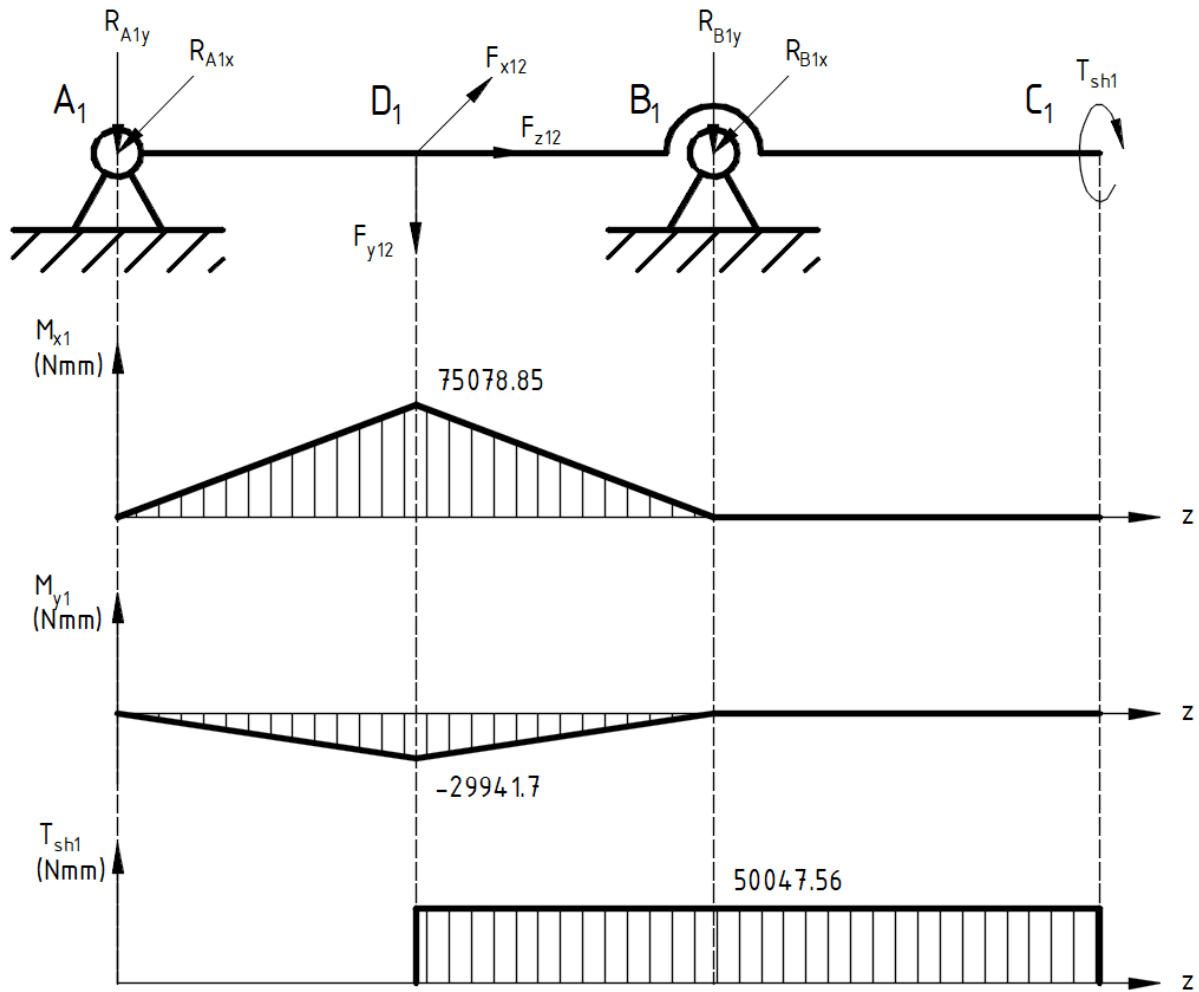


Figure 3.3: Bending moment-torque diagram of shaft 1

$$\left\{ \begin{array}{l} M_{eA1} \approx 0 \text{ (N} \cdot \text{mm)} \\ M_{eD1}^- \approx 81087.5 \text{ (N} \cdot \text{mm)} \\ M_{eD1}^+ \approx 84783.4 \text{ (N} \cdot \text{mm)} \\ M_{eB1} \approx 43342.46 \text{ (N} \cdot \text{mm)} \\ M_{eC1} \approx 43342.46 \text{ (N} \cdot \text{mm)} \end{array} \right. \quad \left\{ \begin{array}{l} M_{eC2} \approx 205963.35 \text{ (N} \cdot \text{mm)} \\ M_{eA2} \approx 281266.2 \text{ (N} \cdot \text{mm)} \\ M_{eD2}^- \approx 252989.03 \text{ (N} \cdot \text{mm)} \\ M_{eD2}^+ \approx 239373.1 \text{ (N} \cdot \text{mm)} \\ M_{eB2} \approx 0 \text{ (N} \cdot \text{mm)} \end{array} \right.$$

Find permissible stress $[\sigma_1]$ and $[\sigma_2]$ are determined by table (10.5). Since we use quenched 45X steel, $[\sigma_1] = 67 \text{ (MPa)}$ and $[\sigma_2] = 64 \text{ (MPa)}$ ($[\sigma_2]$ is achieved using interpolation).

Find standardized diameters at specific locations on the shaft Having M_e and $[\sigma]$, the next step is to estimate specific diameter at the key points mentioned

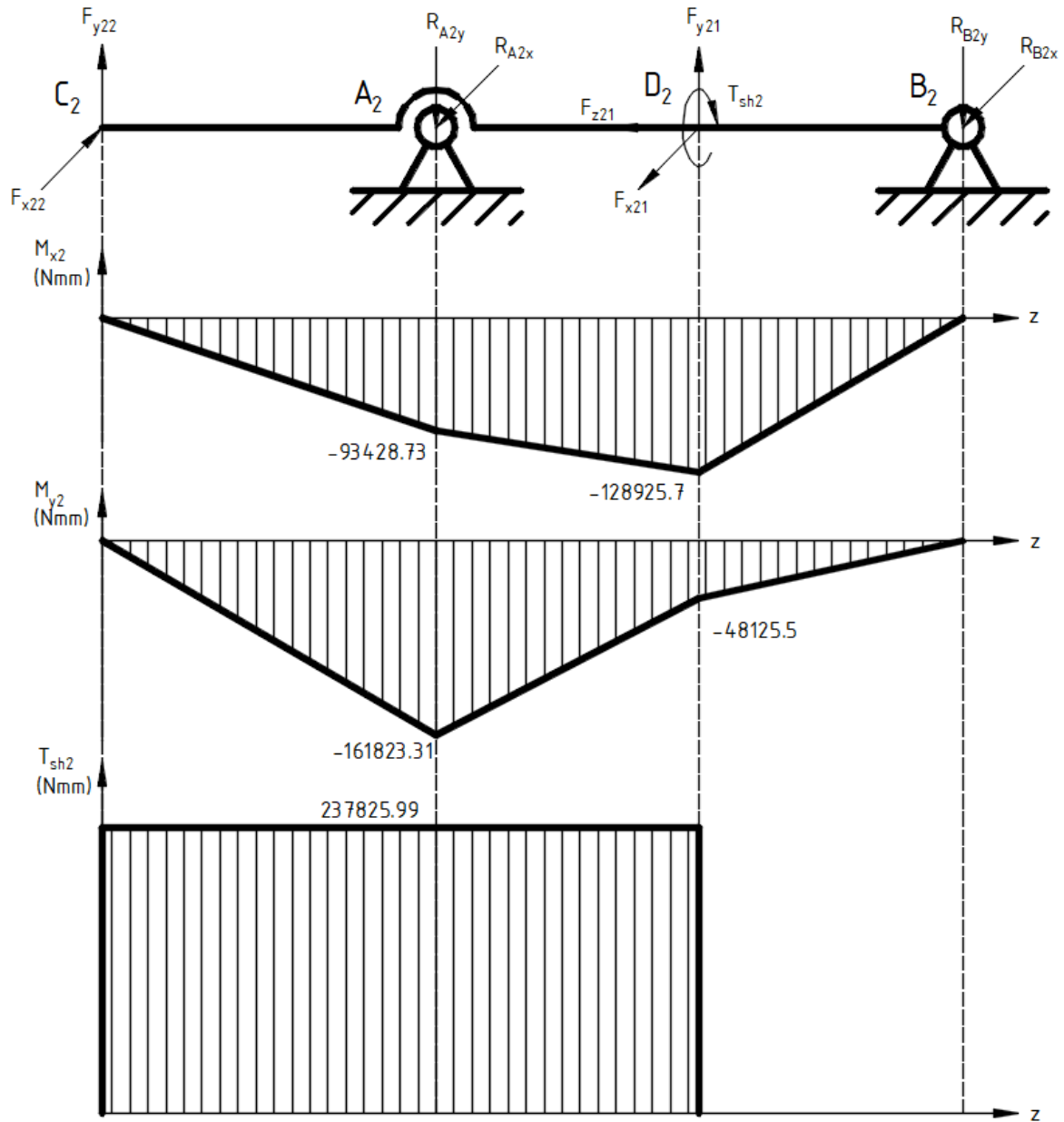


Figure 3.4: Bending moment-torque diagram of shaft 2

above using equation (10.17) on p.194, which only applies for rigid shafts:

$$d \geq \sqrt[3]{\frac{M_e}{0.1[\sigma]}}$$

$$\left\{ \begin{array}{l} d_{A1} \approx 0 \text{ (mm)} \\ d_{D1} \approx 23.66 \text{ (mm)} \\ d_{B1} \approx 18.92 \text{ (mm)} \\ d_{C1} \approx 18.92 \text{ (mm)} \end{array} \right. \quad \left\{ \begin{array}{l} d_{C2} \approx 32.32 \text{ (mm)} \\ d_{A2} \approx 35.86 \text{ (mm)} \\ d_{D2} \approx 34.61 \text{ (mm)} \\ d_{B2} \approx 0 \text{ (mm)} \end{array} \right.$$

Through rough calculations, we will choose the diameters according to standards given on p.195 (one applies for bearings while the other is used for the remaining machine elements):

$$\left\{ \begin{array}{l} d_{A1} = 35 \text{ (mm)} \\ d_{D1} = 24 \text{ (mm)} \\ d_{B1} = 35 \text{ (mm)} \\ d_{C1} = 19 \text{ (mm)} \end{array} \right. \quad \left\{ \begin{array}{l} d_{C2} = 34 \text{ (mm)} \\ d_{A2} = 40 \text{ (mm)} \\ d_{D2} = 36 \text{ (mm)} \\ d_{B2} = 40 \text{ (mm)} \end{array} \right.$$

3.4 Fatigue Strength Analysis

For each critical point, the fatigue strength there must satisfy this condition:

$$s = \frac{s_\sigma s_\tau}{\sqrt{s_\sigma^2 + s_\tau^2}} \geq [s]$$

$$\text{where } s_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m}$$

$$s_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m}$$

Assuming the surfaces are smooth, properly ground and quenched by high frequency voltage, we obtain $K_x = 1$ from table (10.8) and $K_y = 1.4$ from table (10.9), where $[\sigma_b] = 850 \text{ (MPa)}$ is the property of quenched 45X steel.

Find σ_{-1}, τ_{-1} Using formulas on p.196:

$$\sigma_{-1} = 0.35[\sigma_b] + 120 \approx 417.5 \text{ (MPa)}$$

$$\tau_{-1} \approx 0.58\sigma_{-1} \approx 242.15 \text{ (MPa)}$$

Find $\sigma_a, \tau_a, \sigma_m, \tau_m$ For this part, we divide into 3 key points:

1. For rotating shaft, $\sigma_m = 0$, $\sigma_a = \frac{\sqrt{M_x^2 + M_y^2}}{W}$ (equation (10.22)), where M_x and M_y are at the cross section of interest.
2. By design, the shafts only rotate in one direction, thus $\tau_m = \tau_a = \frac{T_{sh}}{2W_O}$ (equation (10.23)).
3. We also assume the shafts have circular cross section, which makes $W = \frac{\pi d^3}{32}$ and $W_O = \frac{\pi d^3}{16}$ according to table (10.6), where d is the diameter of a cross section of the shaft.

The table below shows the results after calculation: Since $\sigma_b = 850$ (MPa) for

	d (mm)	W (mm ³)	W_O (mm ³)	σ_m (MPa)	σ_a (MPa)	τ_m (MPa)	τ_a (MPa)
A_1	20	785.4	1570.8	0	0	15.93	15.93
D_1	24	1357.17	2714.34	0	49.2	9.22	9.22
B_1	20	785.4	1570.8	0	0	15.93	15.93
C_1	19	673.38	1346.76	0	0	18.58	18.58
C_2	32	3216.99	6433.98	0	0	18.48	18.48
A_2	40	6283.19	12566.37	0	29.74	9.46	9.46
D_2	34	3858.66	7717.32	0	36.67	15.41	15.41
B_2	35	4209.24	8418.49	0	0	14.13	14.13

Table 3.1: Calculated variables for $\sigma_a, \tau_a, \sigma_m, \tau_m$

both shafts, $\psi_\sigma = 0.1$ and $\psi_\tau = 0.05$

Find K_σ, K_τ We calculate K_σ using formula:

$$K_\sigma = \left(\frac{k_\sigma}{\varepsilon_\sigma} + K_x - 1 \right) K_y^{-1}$$

and K_τ with:

$$K_\tau = \left(\frac{k_\tau}{\varepsilon_\tau} + K_x - 1 \right) K_y^{-1}$$

Table (10.10), (10.11) and (10.13) are examined to find $\frac{k_\sigma}{\varepsilon_\sigma}$ ratio. Given $[\sigma_H] = 850$ (MPa) base shaft diameters d_1 and d_2 are compared to the diameters at critical locations A, B, C, D . If the base shaft is smaller, table (10.10) and (10.11) are used. If it is larger, we will use table (10.13) instead; the concentration stress factor in this case is demonstrated in the figure:

Final calculation is provided in the table:

	d (mm)	r	k_σ	k_τ	ε_σ	ε_τ	$\frac{k_\sigma}{\varepsilon_\sigma}$	$\frac{k_\tau}{\varepsilon_\tau}$	K_x	K_y	K_σ	K_τ
A_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
D_1	24	0.48	3	1.95	0.81	0.85	3.7	2.29	1	1.4	2.65	1.64
B_1	20	0.4	3	1.95	0.83	0.89	3.61	2.19	1	1.4	2.58	1.57
C_1	19	0.38	3	1.95	0.84	0.89	3.57	2.19	1	1.4	2.55	1.57
C_2	32	0.64	3	1.95	0.76	0.80	3.95	2.44	1	1.4	2.82	1.74
A_2	40	-	-	-	-	-	3.34	2.46	1	1.4	2.39	1.76
D_2	34	0.68	3	1.95	0.74	0.80	4	2.44	1	1.4	2.86	1.75
B_2	35	-	-	-	-	-	3.3	2.44	1	1.4	2.36	1.74

Table 3.2: Calculated variables in K_σ and K_τ

Find s_σ, s_τ and s Combining the results altogether, we obtain the following table:

	s_σ	s_τ	s
A_1	$\gg s_\tau$	9.41	9.41
D_1	3.21	16	3.14
B_1	$\gg s_\tau$	9.41	9.41
C_1	$\gg s_\tau$	8.07	8.07
C_2	$\gg s_\tau$	7.32	7.32
A_2	5.88	14	5.43
D_2	3.99	8.77	3.63
B_2	$\gg s_\tau$	9.56	9.56

Table 3.3: Safety factor at critical cross sections

Since the smallest safety factor is at the cross section D_1 , which has the value

of $3.14 > [s] = 1.5 \div 2.5$, we can neglect rigidity analysis according to the conclusion on p.195.

3.5 Static Strength Analysis

Along with fatigue strength, static strength is also considered and every shaft must satisfy the following condition at critical cross sections (equation (10.27)):

$$\sigma_e = \sqrt{\left(\frac{M_{max}}{0.1d^3}\right)^2 + 3\left(\frac{T_{max}}{0.2d^3}\right)^2} \leq [\sigma]$$

where M_{max}, T_{max} are the largest bending moment and torque at the cross section, respectively. Let $[\sigma] \approx 0.8\sigma_{ch} = 520$ (MPa), the results are in the table below:

	A_1	D_1	B_1	C_1	C_2	A_2	D_2	B_2
σ_e (MPa)	54.18	57.59	54.18	63.19	62.86	43.45	63.58	48.04

Table 3.4: Calculated static strength at critical cross sections

which satisfy the given condition.

Chapter 4

Bearing Design

4.1 Nomenclature

$[s]$	permissible safety factor	h_n	distance between bearing lid and bolt, mm
$[\sigma]$	permissible static strength, MPa	hr	tooth direction
$[\tau]$	permissible torsion, MPa	K_x	surface tension concentration factor
a_w	shaft distance, mm	K_y	diminish factor
b_O	rolling bearing width, mm	K_σ	combined influence factor in tension
C_d	basic dynamic load rating, N	K_τ	combined influence factor in shear
cb	role of gear on the shaft (active or passive)		
cq	rotational direction of the shaft		
d	base shaft diameter, mm		
d_w	gear diameter, mm		
F_a	axial force, N		
F_r	radial force, N		
F_t	tangential force, N		
F_x	applied force, N		

k_d	temperature factor	W	section modulus, mm^3
k_t	load condition factor	W_O	polar section modulus, mm^3
k_σ	fatigue stress concentration factor in tension	X	dynamic radial load factor
		Y	dynamic axial load factor
k_τ	fatigue stress concentration factor in shear	α	contact angle, $^\circ$
		α_{tw}	traverse meshing angle, $^\circ$
L	rated life in million revolutions, million rev	β	helix angle, $^\circ$
		ψ_σ	mean stress influence factor
L_h	rated life in hours, h	ψ_τ	mean shear influence factor
l	length (general), mm	σ_{-1}	endurance limit at stress ratio of -1, MPa
l_m	hub length (general), mm		
M	moment, $\text{N} \cdot \text{mm}$	σ_a	tensile stress amplitude, MPa
M_e	equivalent moment, $\text{N} \cdot \text{mm}$	σ_b	ultimate strength, MPa
M_{max}	maximum moment at the cross section, $\text{N} \cdot \text{mm}$	σ_{ch}	yield limit, MPa
		σ_m	mean tensile stress, MPa
m	load-life exponent	σ_{td}	static strength, MPa
Q	equivalent dynamic load, kN	τ_{-1}	endurance limit at shear ratio of -1, MPa
q	standardized coefficient of shaft diameter		
		τ_a	shear stress amplitude, MPa
R	reaction force, N	τ_m	mean shear stress, MPa
r	shoulder fillet radius, mm	$sh1$	subscript for shaft 1
\bar{r}	position of applied force on the shaft, mm	$sh2$	subscript for shaft 2
		x	subscript for x-axis
S	length defined by table (6.1), mm	y	subscript for y-axis
		z	subscript for z-axis
s	calculated safety factor		
s_σ	safety factor in tensile stress		
s_τ	safety factor in shear stress		
T	torque at the cross section, $\text{N} \cdot \text{mm}$		

4.2 Choose bearing type

As for the types, we will examine $\frac{F_a}{F_r}$ at A_1, B_1, A_2 and B_2 in the 2 shafts from the previous chapter, where F_a is the output axial force $|F_{z12}| = |F_{z21}| \approx 580.75$ (N); F_r is the magnitude of combined reaction force $\sqrt{R_x^2 + R_y^2}$ from the shaft onto the bearing, which is essentially a radial load. The larger ratio in a shaft will be our ratio of choice to determine the bearing type.

Taking our results from the chapter 4:

$$\left\{ \begin{array}{l} F_{rA1} = \sqrt{R_{A1x}^2 + R_{A1y}^2} \approx 1.2 \text{ (kN)} \\ F_{rB1} = \sqrt{R_{B1x}^2 + R_{B1y}^2} \approx 1.33 \text{ (kN)} \\ F_{aA1} = |F_{z12}| \approx 0.58 \text{ (kN)} \\ F_{aB1} = |F_{z12}| \approx 0.58 \text{ (kN)} \end{array} \right\} \left\{ \begin{array}{l} F_{rA2} = \sqrt{R_{A2x}^2 + R_{A2y}^2} \approx 3.79 \text{ (kN)} \\ F_{rB2} = \sqrt{R_{B2x}^2 + R_{B2y}^2} \approx 2.05 \text{ (kN)} \\ F_{aA2} = |F_{z21}| \approx 0.58 \text{ (kN)} \\ F_{aB2} = |F_{z21}| \approx 0.58 \text{ (kN)} \end{array} \right.$$

yields

$$\left\{ \begin{array}{l} \frac{F_{aA1}}{F_{rA1}} \approx 0.46 \\ \frac{F_{aB1}}{F_{rB1}} \approx 0.44 \end{array} \right\} \left\{ \begin{array}{l} \frac{F_{aA2}}{F_{rA2}} \approx 0.15 \\ \frac{F_{aB2}}{F_{rB2}} \approx 0.28 \end{array} \right.$$

Since $0.46 > 0.3$ and $0.28 \leq 0.3$, the pair of bearings on shaft 1 is single-row angular contact ball bearings with $\alpha_{sh1} = 12^\circ$ and the remaining pair is single-row deep-groove bearings ($\alpha_{sh2} = 0^\circ$); AG = 0 according to the recommendations on p.212 and p.213.

We also have dimensions at the cross sections A_1, B_1, A_2, B_2 from the previous chapter:

$$\left\{ \begin{array}{l} b_{O1} = 21 \text{ (mm)} \\ d_{A1} = 35 \text{ (mm)} \\ d_{B1} = 35 \text{ (mm)} \end{array} \right\} \left\{ \begin{array}{l} b_{O2} = 23 \text{ (mm)} \\ d_{A2} = 40 \text{ (mm)} \\ d_{B2} = 40 \text{ (mm)} \end{array} \right.$$

From these parameters, we will look up the tables at the end of the text. The pair

of single-row angular contact ball bearings of choice is 46307, which is suitable for shaft 1 and $C_{o1} = 25.2$ (kN). On shaft 2, the pair of single-row deep-groove bearings are type 308, where $C_{o2} = 21.7$ (kN).

4.3 Bearing dimensions

4.3.1 Calculate basic dynamic load rating

$$C_d = Q \sqrt[p]{L}$$

Find equivalent dynamic load

Since we only use angular contact ball bearings, the following formula applies:

$$Q = (XVF_r + YF_{at})k_tk_d$$

Since the inner ring rotates, $V = 1$ and the $\frac{F_a}{VF_r} = \frac{F_a}{F_r}$, meaning that the ratios in the section above will be used to examine X and Y .

The design problem also does not give any further information about operating temperature, which gives $k_t = 1$. In addition, we get $k_d = 1$ from table (11.3) based on the machine's condition (low load and power rating).

Find the ratio $\frac{iF_a}{C_o}$ This ratio is calculated and applied for 2 shafts ($i = 1$ for single-row bearings in our case):

$$\text{For shaft 1, } \frac{F_a}{C_{o1}} \approx 0.027$$

$$\text{For shaft 2, } \frac{F_a}{C_{o2}} \approx 0.023$$

Compare with e From the previous section, $\alpha_{sh1} = 12^\circ$, $\alpha_{sh2} = 0^\circ$. Inspecting table (11.4) and by interpolation, $e_{sh1} \approx 0.33$, $e_{sh2} \approx 0.22$. These values are then

compared to $\left| \frac{F_a}{VF_r} \right|$ to look up the correct column.

Find X, Y Table (11.3) and interpolation are used in finding these values:

For shaft 1, $\left| \frac{F_a}{VF_r} \right| = \left| \frac{F_{z12}}{VR_{A1y}} \right| \approx 0.46 > e_1 \Rightarrow X_1 = 0.56, Y_1 = 2.1.$

For shaft 2, $\left| \frac{F_a}{VF_r} \right| = \left| \frac{F_{z21}}{VR_{B2y}} \right| \approx 0.28 > e_2 \Rightarrow X_2 \approx 0.45, Y_2 \approx 1.64.$

Find F_{at} For shaft 1, additional radial forces are also applied to the pair of angular contact ball bearings. From table (11.5), the first arrangement is used in the gearbox. Therefore, $\mathbf{F}_{sA1} \uparrow \uparrow \mathbf{F}_{z12}$ and $\mathbf{F}_{sB1} \uparrow \downarrow \mathbf{F}_{z12}$ (the direction of \mathbf{F}_{z12} can be found at Figure 3.2) Following the sign convention on p.218 and combining with equation (11.8), (11.10), (11.11a) and (11.11b):

At cross section A_1 :

$$F_{sB1} = e_1 \sqrt{R_{B1x}^2 + R_{B1y}^2} \approx 0.43 \text{ (kN)}$$

$$\sum F_{aA1} = F_{sB1} - F_{z12} \approx -0.15 \text{ (kN)}$$

At cross section B_1 :

$$F_{sA1} = e_1 \sqrt{R_{A1x}^2 + R_{A1y}^2} \approx 0.41 \text{ (kN)}$$

$$\sum F_{aB1} = F_{sA1} + F_{z12} \approx 0.99 \text{ (kN)}$$

From equation (11.11a) and (11.11b):

$$\sum F_{aA1} \leq F_{sA1} \Rightarrow F_{atA1} = F_{sA1} \approx 0.41 \text{ (kN)}$$

$$\sum F_{aB1} > F_{sB1} \Rightarrow F_{atB1} = \sum F_{aB1} \approx 0.99 \text{ (kN)}$$

In contrast, shaft 2 does not have such additional forces since $\alpha_{sh2} = 0^\circ$.

Therefore, $F_{atA2} = F_{atB2} = |F_{z21}| = 0.58 \text{ (kN)}.$

Find Q_1, Q_2 Recall that the forces are in absolute values, the parameters are substituted to the formula to obtain:

$$Q_{A1} \approx 1.56 \text{ (kN)}$$

$$Q_{B1} \approx 2.82 \text{ (kN)}$$

$$Q_{A2} \approx 2.66 \text{ (kN)}$$

$$Q_{B2} \approx 1.87 \text{ (kN)}$$

We will compare these values and choose the larger load (according to the recommendation on p.219):

$$Q_{A1} < Q_{B1} \Rightarrow Q_1 = Q_{B1} \approx 2.82 \text{ (kN)}$$

$$Q_{A2} > Q_{B2} \Rightarrow Q_2 = Q_{A2} \approx 2.66 \text{ (kN)}$$

Find rated life

Equation (11.2) is rearranged to calculate L :

$$L = L_h 60 n_{sh} \times 10^{-6}$$

In our transmission system, since the gearbox is a speed reducer working 2 shifts daily, we approximate $L_h \approx 32000$ (hours) according to table (11.2), which gives:

$$L_1 \approx 5274 \text{ (million rev)}$$

$$L_2 \approx 1054.8 \text{ (million rev)}$$

Combining the results and letting $m = 3$ (ball bearings are used in this case) yield:

$$C_{d1} = 14385.63 \text{ (N)}$$