VIETNAM NATIONAL UNIVERSITY HO CHI MINH CITY
HCM UNIVERSITY OF TECHNOLOGY
FACULTY OF MECHANICAL ENGINEERING - MECHATRONICS DEPARTMENT

ENGINEERING INTERNSHIP REPORT

Air Compressor Modeling using MATLAB

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Acknowledgement

To complete the internship report topic, first of all I would like to send my sincere thanks and deep gratitude to the lecturers of Ho Chi Minh City University of Technology for providing great opportunities so that I have more time for technical internships.
In particular, I would like to thank Dr. Nguyen Tan Tien, who wholeheartedly guided and helped me complete this internship report topic.
Finally, I also would like to thank my teammates, Nguyen Dang Hung and Nguyen Khanh Trung and the seniors at DCSELab for their help and support.
At the same time, the school has given me the opportunity to practice where I love, let me step into real life to apply the knowledge that my teachers have taught. Through this internship, I realize many new and useful things in the preliminary design of practical machines to help me in my future work.
Due to my lack of knowledge and experience regarding the modeling skills, this report is very likely be prone to mistakes. I am looking forward to your response for future improvements.

Chapter 1

Overview of the Company

1.1 History of the Organization

Figure 1.1: DCSELab main entrance

DCSELab (National key Laboratory of Digital Control and System Engineering), founded on 04/08/2003 under Decision number 590/QD/DHQG/TCCB of Vietnam National Unversity Ho Chi Minh City. It is an independent organization which has a status of legal person; conducts technological advancement on its own accord based on the Regulation for developing and functioning of National key Laboratory.

1.2 Working Objectives

Experimenting, designing, installing, maintaining and transferring equipment (hardware and software) regarding digital control, system engineering; mechatronics, telecommunication, information technology, automation and industrial process.

1.3 The Organizational Structure

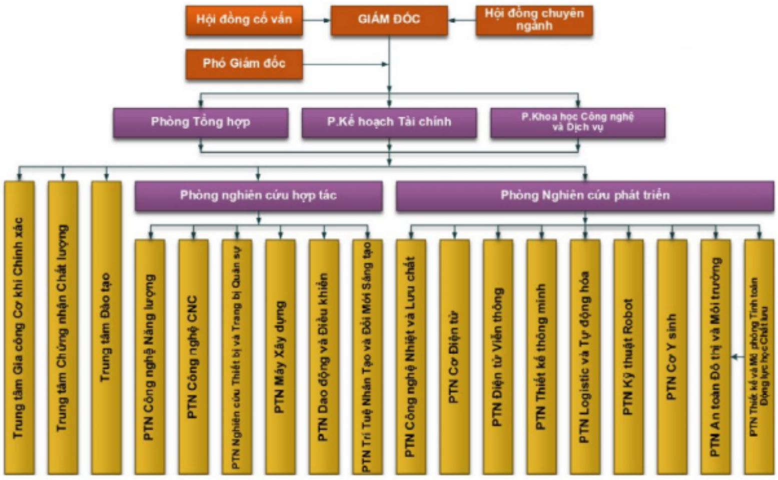

Figure 1.2: The organizational structure of DCSELab

Chapter 2

Internship tasks

2.1 Problem

At the moment, the light duty air compressor market mainly use electricity as the power source, which can be inconvenient at remote locations where power grid is difficult to reach. To solve this problem, we will analyze and model a V-twin air compressor with a compression end and a combustion end. The model is programmed using MATLAB® R2019a.

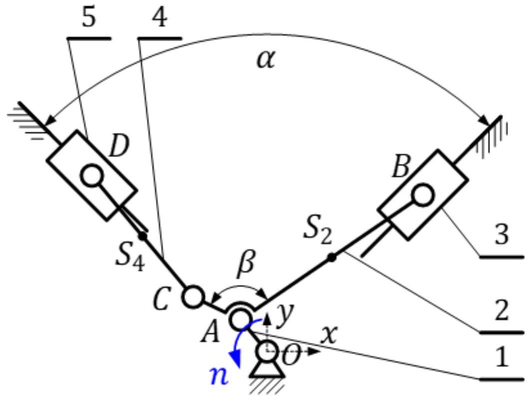

Figure 2.1: Mechanism of the air compressor

2.2 Parameters

Piston lift	$H_C = 39\,\text{mm}$
V-angle	$\alpha = 90°$
Connecting rod dimensions	$\frac{l_{AB}}{l_{OA}} = 5.372, l_{AC} = 25mm,$ $\beta = 120°$
Crankshaft 1 weight	$m_1 = 1.6262\,\text{kg}$
Center of gravity of link 1	$S_1 \equiv O$
Moment of inertia of link 1	$J_{s1} = 0.0045585\,\text{kg}\cdot\text{m}^2$
Connecting rod 2 weight	$m_2 = 0.29597\,\text{kg}$
Center of gravity of link 2	$l_{AS2} = l_{S2B}$
Moment of inertia of link 2	$J_{S2} = 0.004874\,\text{kg}\cdot\text{m}^2$
Connecting rod 4 weight	$m_2 = 0.13865\,\text{kg}$
Center of gravity of link 4	$l_{CS4} = l_{S4D}$
Moment of inertia of link 4	$J_{S4} = 0.00011121\,\text{kg}\cdot\text{m}^2$
Piston weight	$m_3 = m_5 = 0.33513\,\text{kg}$
Center of gravity of link 3 and 5	$S_3 \equiv B, S_5 \equiv D$
Piston crown area	$A = 0.01\,\text{m}^2$
Maximum operating pressure (The graphs given in figure 2.2 and 2.3)	$p_{max} = 23\,\text{bar}$
Allowable tolerance factor	$\delta = 1/80$
Average rotational velocity	$n_1 = 500\,\text{rpm}$
Valve lift (trapezoidal acceleration)	$s_0 = 2\,\text{mm}$
Early opening and late closing of combustion end	25°
Early opening of compression end	40°
Pressure angle of cam follower	$\alpha_2 = 6°$
Periodic angles	$\phi_{rise} = \phi_{fall}, \phi_{rise,comb} = 5°,$ $\phi_{rise,comp} = 20°$

2.3 Objectives

1. Mechanism synthesis [1]
2. Kinematic analysis of the mechanism
3. Force analysis of the mechanism
4. Identify energy relation and calculate the flywheel weight
5. Combining the motion
6. Model cam mechanism for compression piston

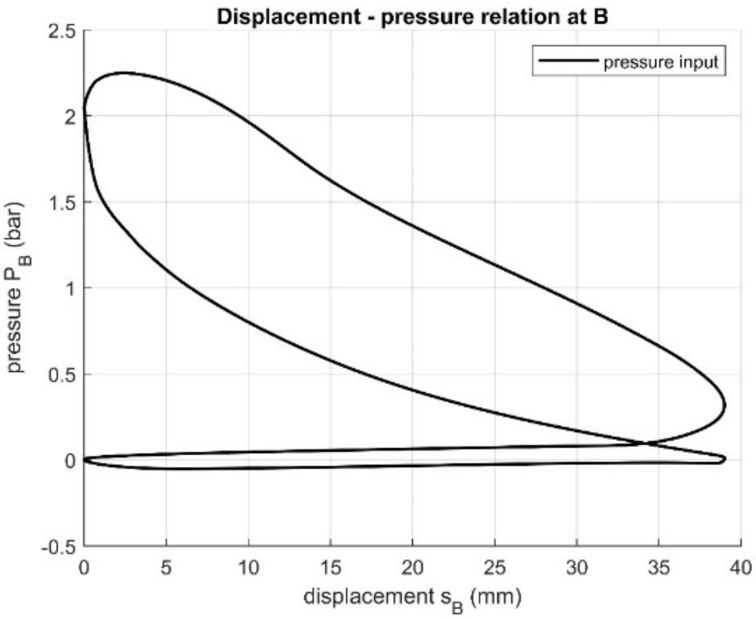

Figure 2.2: Pressure graph at link 3 (not to scale)

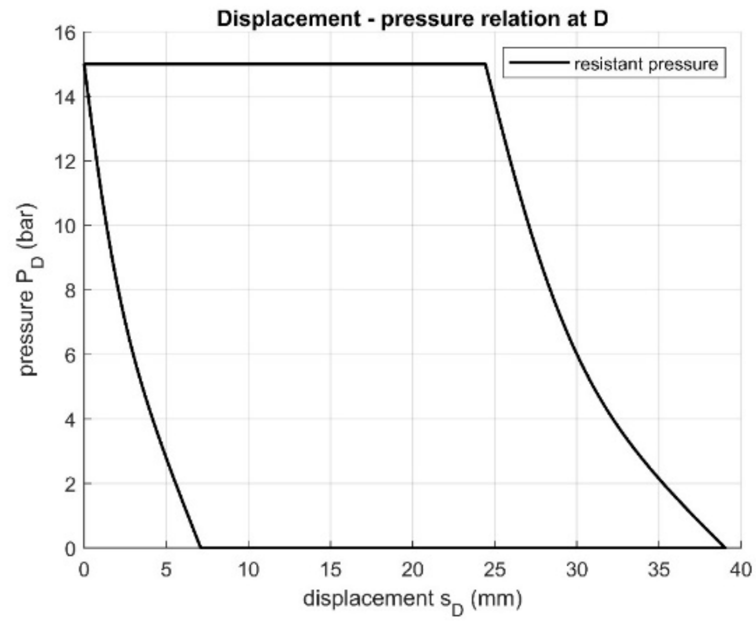

Figure 2.3: Pressure graph at link 5 (not to scale)

2.4 Displacement analysis

Position of A

$$\vec{r}_A = \begin{bmatrix} x_A \\ y_A \\ 0 \end{bmatrix} = \begin{bmatrix} l_{OA}\cos\phi \\ l_{OA}\sin\phi \\ 0 \end{bmatrix} \tag{2.1}$$

where $\phi = \widehat{xOA}$

Position of B

$$\begin{cases} (x_A - x_B)^2 + (y_A - y_B)^2 & = l_{AB}^2 \\ \dfrac{y_B}{x_B} & = \tan\widehat{xOB} \end{cases} \tag{2.2}$$

Solve equation (2.2) yields 2 positions B_1, B_2. Combining with condition $x_B, y_B > 0$ to find the correct solution.

$$\vec{r}_B = \begin{bmatrix} x_B \\ y_B \\ 0 \end{bmatrix}$$

Position of C

$$\begin{cases} (x_A - x_C)^2 + (y_A - y_C)^2 & = l_{AC}^2 \\ (x_B - x_C)^2 + (y_B - y_C)^2 & = l_{BC}^2 \end{cases} \tag{2.3}$$

Solve equation (2.3) yields 2 sets of positions C_1, C_2, one of which is the correct solution. This can be programmed using MATLAB® to try both sets.

$$\vec{r}_C = \begin{bmatrix} x_C \\ y_C \\ 0 \end{bmatrix}$$

Position of D

$$\begin{cases} (x_C - x_D)^2 + (y_C - y_D)^2 & = l_{CD}^2 \\ \dfrac{y_D}{x_D} & = \tan\widehat{xOD} \end{cases} \tag{2.4}$$

Solve equation (2.4) yields 2 positions D_1, D_2. Combining with condition $x_D < 0, y_D > 0$ to find the correct solution.

$$\vec{r}_D = \begin{bmatrix} x_D \\ y_D \\ 0 \end{bmatrix}$$

Using MATLAB® to plot the positions of the mechanism in figure 2.5

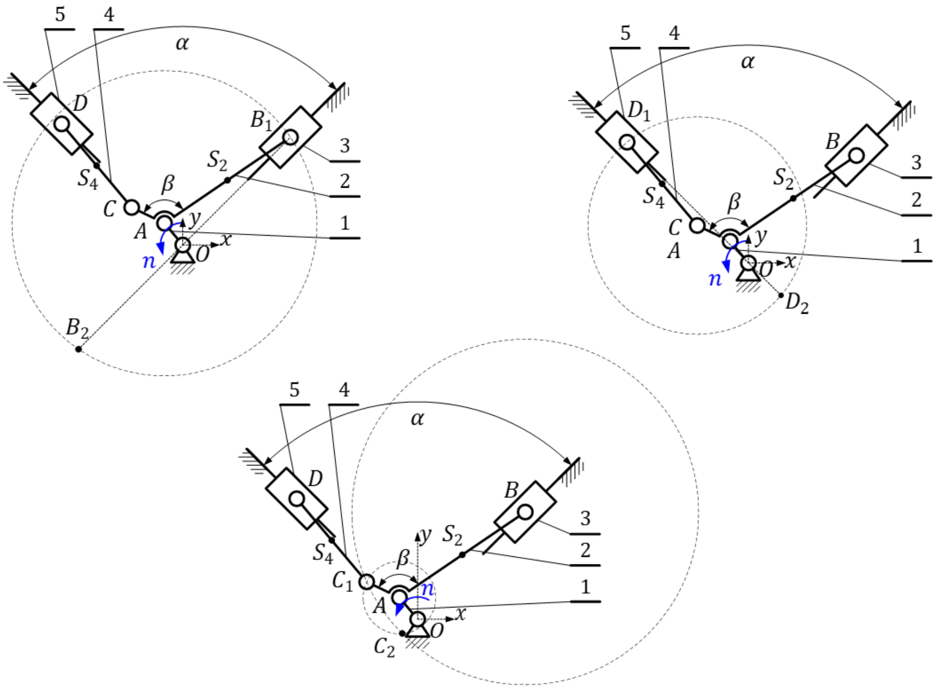

Figure 2.4: Position analysis of B, C and D

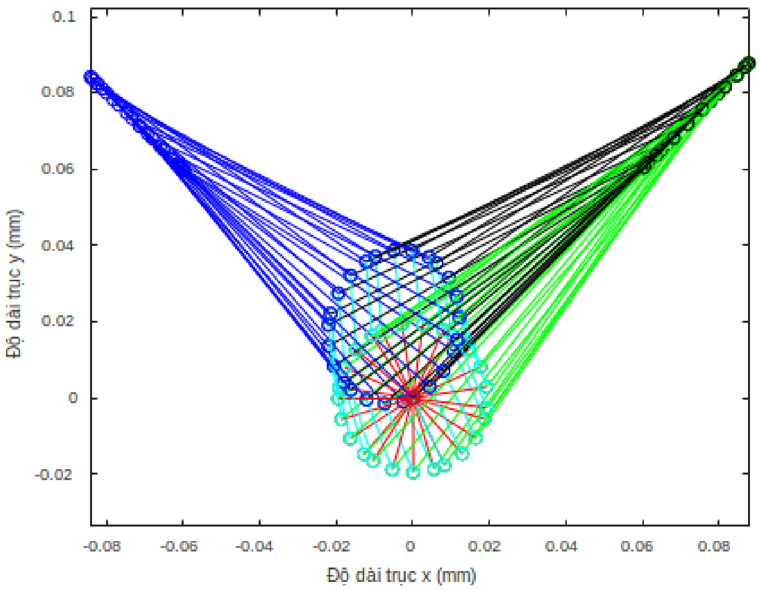

Figure 2.5: Locus of the mechanism using MATLAB®

2.5 Velocity and Acceleration analysis

With the analytical solutions of the points A, B, C, D, we can easily find the corresponding velocities and accelerations using derivative with respect to time t. Also, we need to remember that $\phi(t)$ is a time-dependent variable.

2.5.1 Velocity analysis

Velocity of A

$$\vec{v}_A = \dot{\vec{r}}_A = \frac{d\vec{r}_A}{dt} \tag{2.5}$$

where $\phi = \widehat{xOA}$

Velocity of B

$$\vec{v}_B = \dot{\vec{r}}_B = \frac{d\vec{r}_B}{dt} = \vec{v}_A + \vec{\omega}_2 \times (\vec{r}_B - \vec{r}_A) \tag{2.6}$$

From equation (2.6), we solve analytically for $\vec{\omega}_2$

$$\vec{\omega}_2 = \begin{bmatrix} 0 \\ 0 \\ \omega_2 \end{bmatrix}$$

Velocity of C

$$\vec{v}_C = \dot{\vec{r}}_C = \frac{d\vec{r}_C}{dt} \tag{2.7}$$

Velocity of D

$$\vec{v}_D = \dot{\vec{r}}_D = \frac{d\vec{r}_D}{dt} = \vec{v}_C + \vec{\omega}_4 \times (\vec{r}_D - \vec{r}_C) \tag{2.8}$$

From equation (2.8), we solve analytically for $\vec{\omega}_4$

$$\vec{\omega}_4 = \begin{bmatrix} 0 \\ 0 \\ \omega_4 \end{bmatrix}$$

Using MATLAB® to plot the velocity graph of the mechanism in figure 2.6 [2].

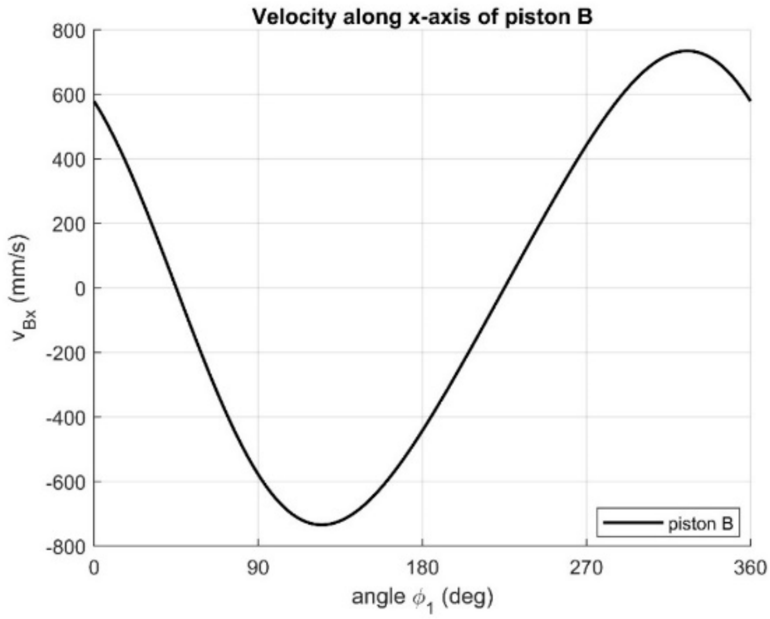

Figure 2.6: Velocity of link 3

2.5.2 Acceleration analysis

Acceleration of *A*

$$\vec{a}_A = \dot{\vec{v}}_A = \frac{d\vec{v}_A}{dt} \tag{2.9}$$

where $\phi = \widehat{xOA}$

Acceleration of *B*

$$\vec{a}_B = \dot{\vec{v}}_B = \frac{d\vec{v}_B}{dt} \tag{2.10}$$

$$\vec{\alpha}_2 = \frac{d\vec{\omega}_2}{dt}$$

Acceleration of *C*

$$\vec{a}_C = \dot{\vec{v}}_C = \frac{d\vec{v}_C}{dt} \tag{2.11}$$

Acceleration of *D*

$$\vec{a}_D = \dot{\vec{v}}_D = \frac{d\vec{v}_D}{dt} \tag{2.12}$$

$$\vec{\alpha}_4 = \frac{d\vec{\omega}_4}{dt}$$

Using MATLAB® to plot the acceleration graph of the mechanism in figures 2.7

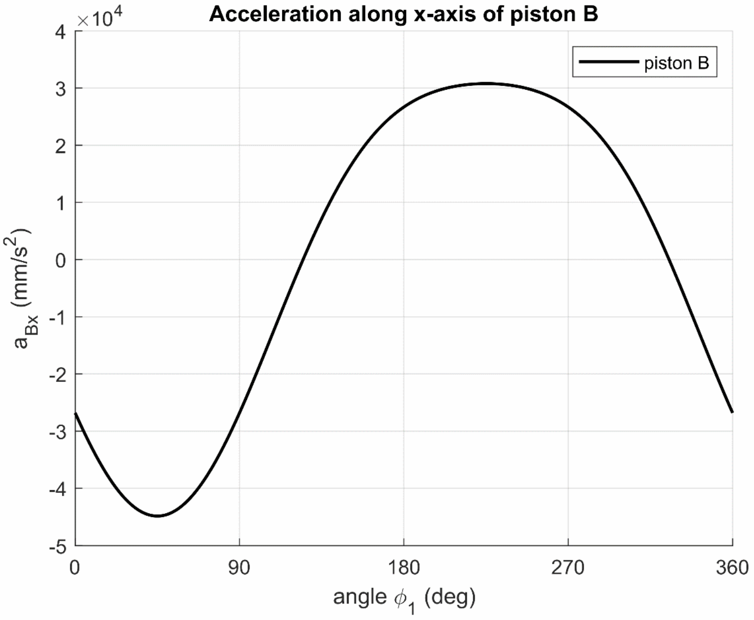

Figure 2.7: Acceleration of link 3

2.6 Force analysis

To find the reaction forces, we separate the links and solve the D'Alembert equations analytically.

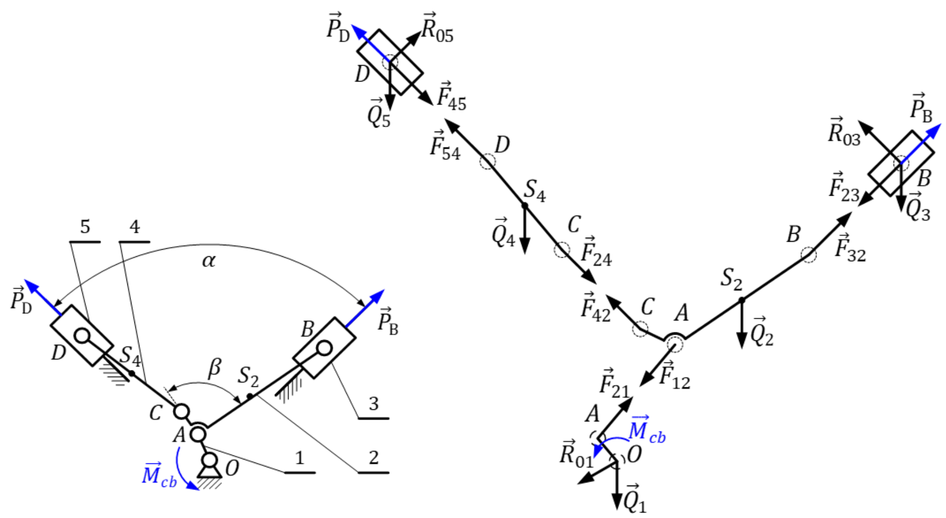

Figure 2.8: Force analysis of the mechanism

Position and acceleration equations of the links, located at their centers of gravity:

$$\vec{r}_{S1} = \vec{0}, \vec{r}_{S2} = \frac{\vec{r}_A + \vec{r}_B}{2}, \vec{r}_{S4} = \frac{\vec{r}_D + \vec{r}_C}{2} \tag{2.13}$$

$$\vec{a}_{S1} = \vec{0}, \vec{a}_{S2} = \frac{\vec{a}_A + \vec{a}_B}{2}, \vec{a}_{S4} = \frac{\vec{a}_D + \vec{a}_C}{2} \tag{2.14}$$

The equations for 5 links are systemized as follows:

Link 5

$$\begin{cases} \vec{Q}_5 + \vec{F}_{45} + \vec{P}_D + \vec{R}_{05} & = m_5\vec{a}_5 \qquad (\vec{a}_5 = \vec{a}_D) \\ \left|\vec{R}_{05x}\right| & = \left|\vec{R}_{05y}\right| \end{cases} \tag{2.15}$$

Link 4

$$\begin{cases} \vec{Q}_4 + \vec{F}_{24} + \vec{F}_{54} & = m_4\vec{a}_{S4} \qquad (\vec{F}_{54} = -\vec{F}_{45}) \\ (\vec{r}_D - \vec{r}_{S4}) \times \vec{F}_{54} + (\vec{r}_C - \vec{r}_{S4}) \times \vec{F}_{24} & = J_{S4}\vec{\alpha}_4 \end{cases} \tag{2.16}$$

Link 3

$$\begin{cases} \vec{Q}_3 + \vec{F}_{23} + \vec{P}_B + \vec{R}_{03} & = m_3\vec{a}_3 \\ \left|\vec{R}_{03x}\right| & = -\left|\vec{R}_{03y}\right| \end{cases} \tag{2.17}$$

Link 2

$$\begin{cases} \vec{Q}_2 + \vec{F}_{12} + \vec{F}_{42} & = m_2\vec{a}_{S2} \\ (\vec{r}_C - \vec{r}_{S2}) \times \vec{F}_{42} + (\vec{r}_B - \vec{r}_{S2}) \times \vec{F}_{32} + (\vec{r}_A - \vec{r}_{S2}) \times \vec{F}_{12} & = J_{S2}\vec{\alpha}_2 \end{cases} \tag{2.18}$$

where $\vec{F}_{42} = -\vec{F}_{24}, \vec{F}_{32} = -\vec{F}_{23}$

Link 1

$$\begin{cases} \vec{Q}_1 + \vec{F}_{21} + \vec{R}_{01} & = m_1\vec{a}_{S1} \\ \vec{r}_A \times \vec{F}_{21} + \vec{M}_{cb} & = 0 \end{cases} \tag{2.19}$$

Solving for system of equations from (2.13) to (2.19) by rearranging them into matrix form, we obtain $\vec{F}_{45}, \vec{R}_{05}, \vec{F}_{24}, \vec{F}_{23}, \vec{F}_{12}, \vec{R}_{01}, \vec{R}_{03}, \vec{M}_{cb}$.
Using MATLAB® to plot the reaction force $\vec{F}_{23}, \vec{R}_{03}$ of the mechanism in figures 2.9 and 2.10

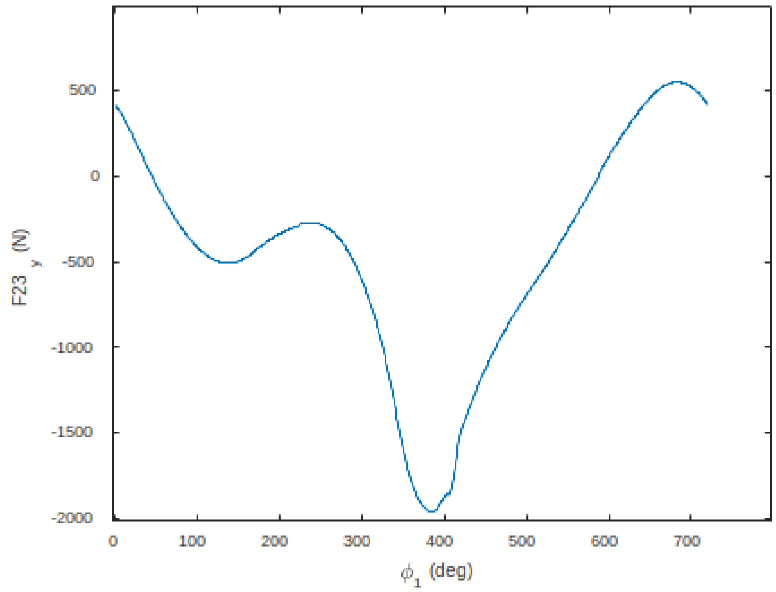

Figure 2.9: Reaction force $\vec{F}_{23}$ along y axis

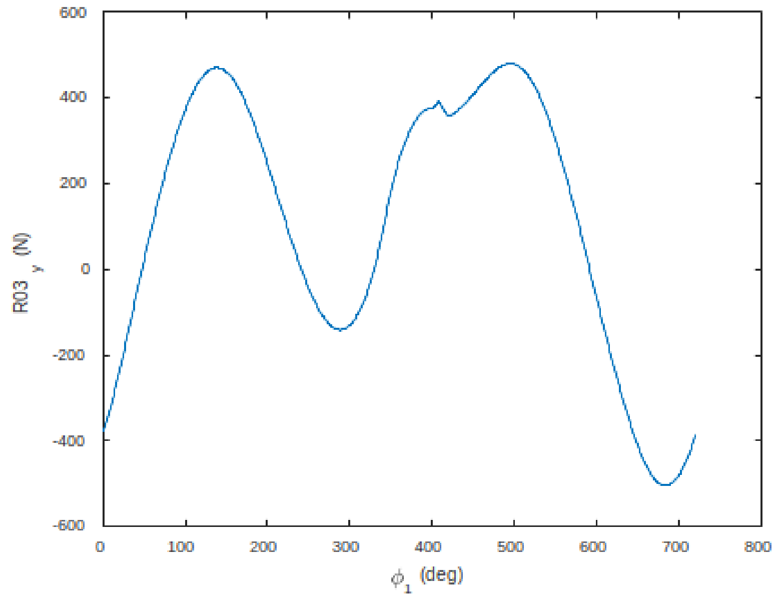

Figure 2.10: Reaction force $\vec{R}_{03}$ along y axis

2.7 Energy relation analysis

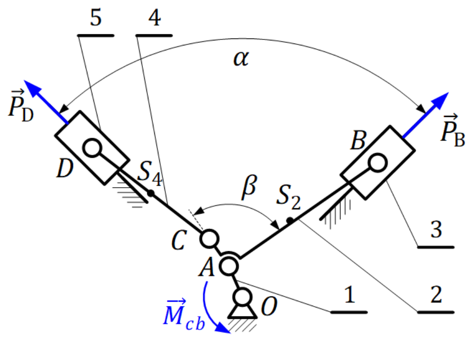

Figure 2.11: Pressure on both ends of the system

The energy relation of the machine must satisfy the condition such that the motion of the system is regulated after each cycle. To put it another way, the dynamic work is offset by the resistance work:

$$\begin{aligned} M_c(0) &= M_d(0) \\ M_c(2\pi) &= M_d(2\pi) \end{aligned} \tag{2.20}$$

Let us choose the driving link as the equivalent link of the system: $\omega_{tt} = \omega_1$.

2.7.1 Find equivalent dynamic moment and dynamic work

From the displacement - pressure relation graph (figure 2.2), we convert it to the external force acting on the system $F_B(\phi) = P_B(\phi)A$ in a cycle (each displacement data corresponds to a specific position and angle $\phi(t)$ of the system).
The equivalent dynamic moment on link 1 is calculated as:

$$M_d(\phi) = \frac{1}{\omega_1}\left(\vec{F}_B \cdot \vec{v}_B + \vec{Q}_2 \cdot \vec{v}_{S2} + \vec{Q}_4 \cdot \vec{v}_{S4} + \vec{Q}_3 \cdot \vec{v}_B + \vec{Q}_5 \cdot \vec{v}_D\right) \tag{2.21}$$

From the moment, we integrate to obtain the dynamic work:

$$A_d(\phi) = \int_{\phi_i}^{\phi_f} M_d d\phi \tag{2.22}$$

2.7.2 Find equivalent resistant moment and resistant work

From the displacement - pressure relation graph (figure 2.3), we convert it to the external force acting on the system $F_D(\phi) = P_D(\phi)A$ in a cycle (each

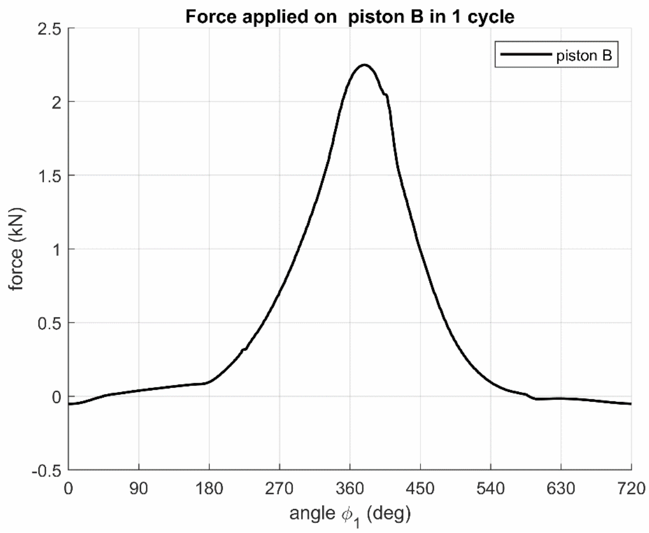

Figure 2.12: Force applied on piston 3 corresponding to angle of crankshaft 1

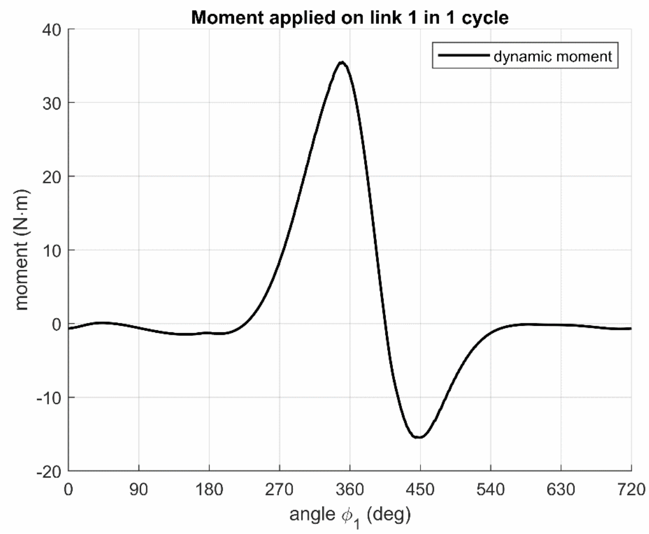

Figure 2.13: Equivalent dynamic moment applied on crankshaft 1

displacement data corresponds to a specific position and angle $\phi(t)$ of the system).
The equivalent resistant moment on link 1 is calculated as:

$$M_c(\phi) = \frac{1}{\omega_1}\vec{F}_D \cdot \vec{v}_D \tag{2.23}$$

From the moment, we integrate to obtain the resistant work:

$$A_c(\phi) = \int_{\phi_i}^{\phi_f} M_c d\phi \tag{2.24}$$

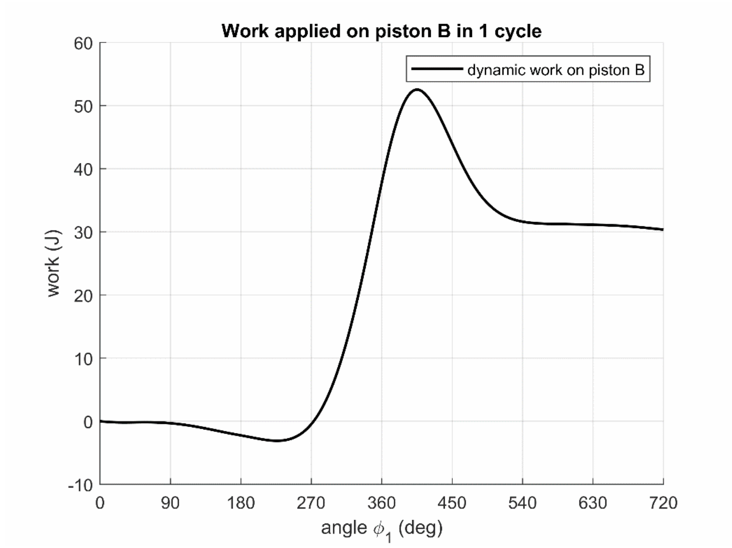

Figure 2.14: Dynamic work of the system in 1 cycle

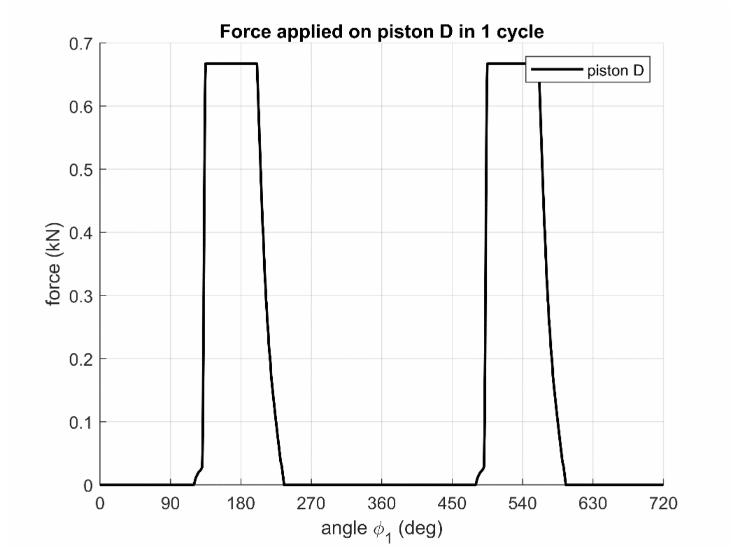

Figure 2.15: Force applied on piston 3 corresponding to angle of crankshaft 1

However, the value of M_c in equation 2.23 still does not satisfy condition (2.20). To compensate for this, we multiply the force F_D by a ratio $\dfrac{A_d}{A_c}$ and recalculate M_c, A_c:

$$\begin{aligned} F_{D,new}(\phi) &= \frac{A_d}{A_c} F_D \\ M_{c,new}(\phi) &= \frac{1}{\omega_1} \vec{F}_{D,new} \cdot \vec{v}_D \\ A_{c,new}(\phi) &= \int_{\phi_i}^{\phi_f} M_{c,new} d\phi \end{aligned} \tag{2.25}$$

We then obtain the figure of equivalent resistant moment and work:

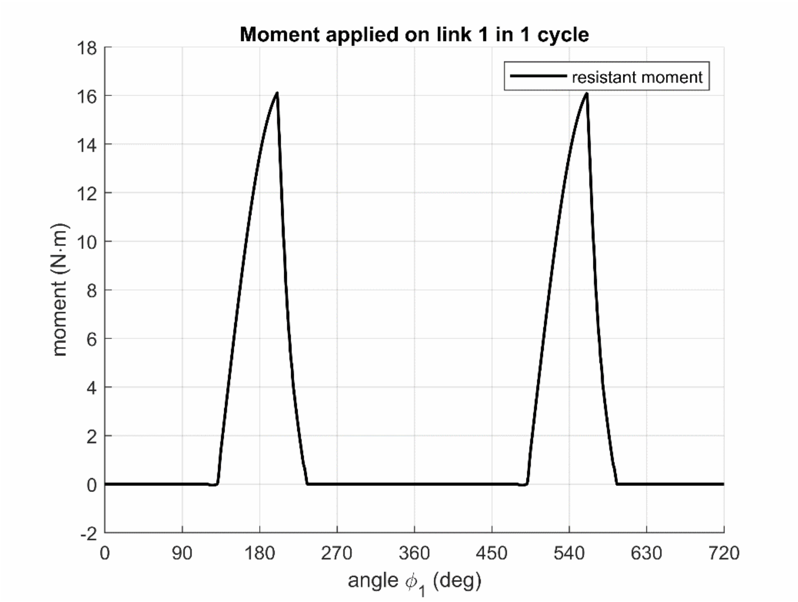

Figure 2.16: Equivalent resistant moment applied on crankshaft 1

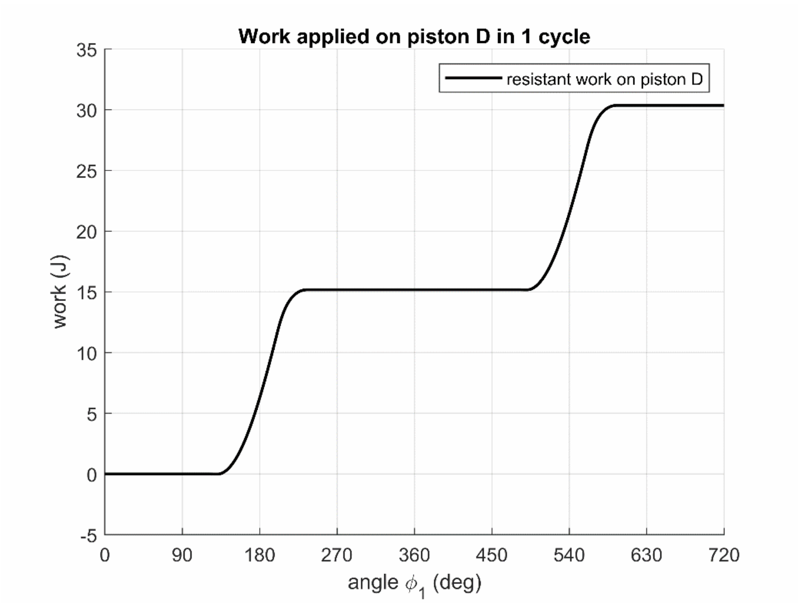

Figure 2.17: Resistant work of the system in 1 cycle

Combining the 2 figures 2.14 and 2.17, we obtain the works applied onto the system in 1 cycle:

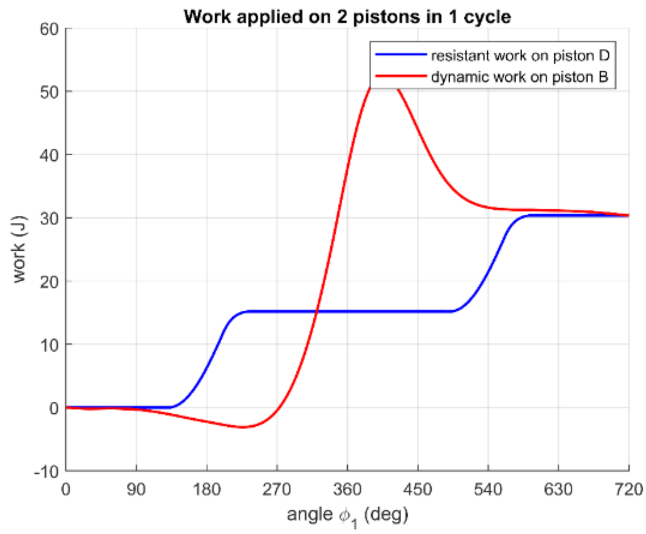

Figure 2.18: Resistant and dynamic moment of the system in 1 cycle

2.8 Find the energy equation and calculate the flywheel weight

Since the crankshaft is chosen to be the equivalent link, we obtain the equivalent moment of inertia as follows:

$$J(\phi) = J_1 + J_2\left(\frac{\omega_2}{\omega_1}\right)^2 + m_2\left(\frac{v_{S2}}{\omega_1}\right)^2 + m_3\left(\frac{v_B}{\omega_1}\right)^2 + J_4\left(\frac{\omega_4}{\omega_1}\right)^2 + m_5\left(\frac{v_D}{\omega_1}\right)^2 \quad (2.26)$$

where m_k, ω_k, v_k are the weight, rotational speed and instantaneous speed at point k respectively.

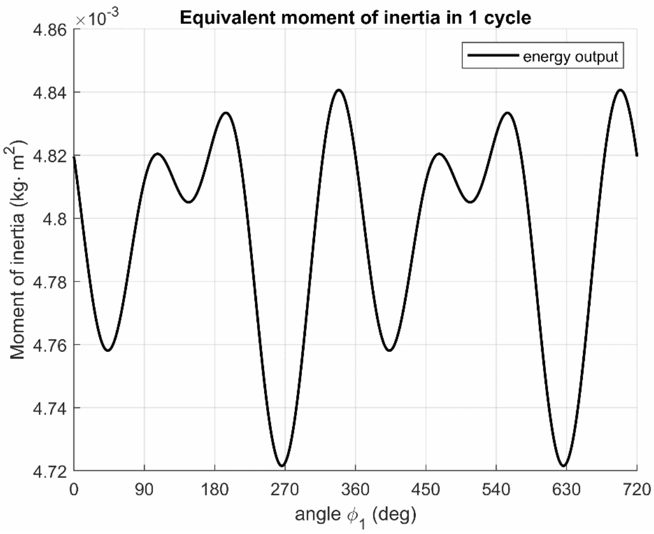

Figure 2.19: Equivalent moment of inertia of the system

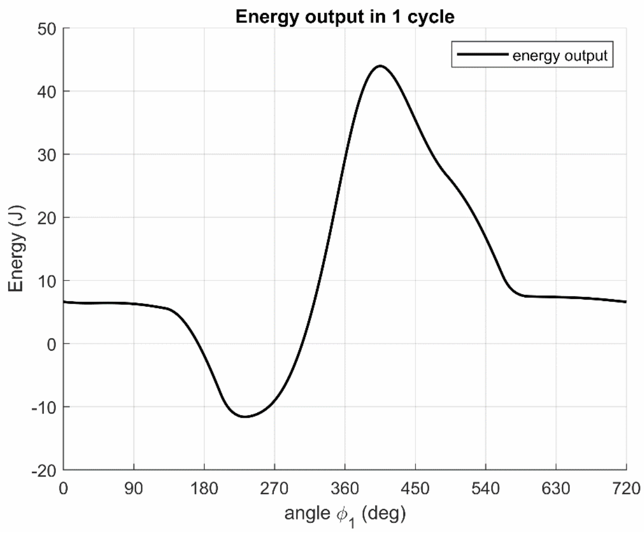

Figure 2.20: Energy output of the system in 1 cycle

Then, we calculate the total energy output of the system:

$$E(\phi) = \Delta E + E_0 = (A_c + A_d) + \frac{1}{2} J_0 \omega_1^2 \tag{2.27}$$

where ΔE is the total equivalent work and $J_0 = J(0)$
From the equivalent energy equation $E(\phi)$ and moment of inertia $J(\phi)$ we plot the graph of $E(J)$

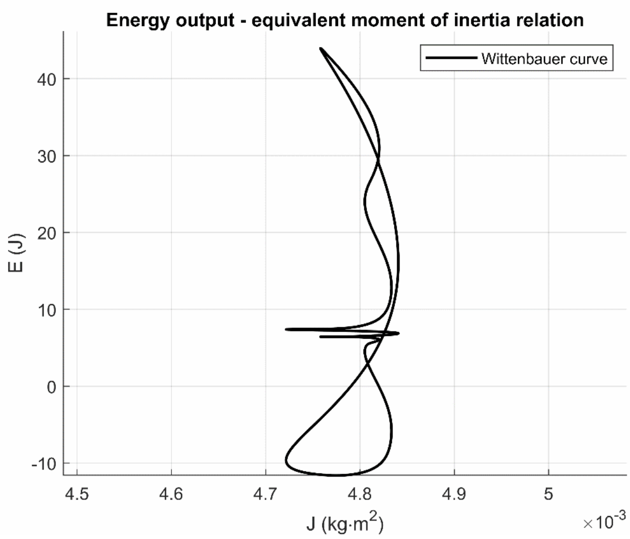

Figure 2.21: Equivalent moment of inertia - energy output relations

From figure 2.21 we draw 2 tangent lines at the boundaries. Let the slope of the lower tangent line be ψ_{min} and the upper one be ψ_{max}. The slope of the lines at

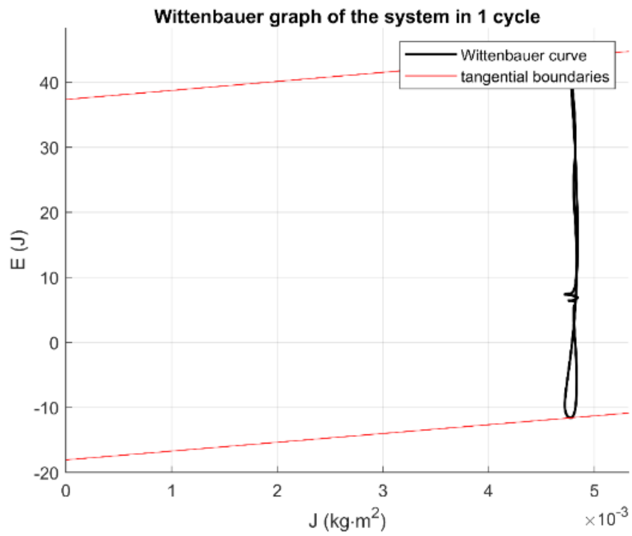

Figure 2.22: Wittenbauer curve with tangent boundaries

calculated numerically as follows:

$$\begin{aligned}\psi_{min} &= \frac{\mu_E \omega_1^2}{2\mu_J}\left(1 - \frac{[\delta]}{2}\right)^2 \\ \psi_{max} &= \frac{\mu_E \omega_1^2}{2\mu_J}\left(1 + \frac{[\delta]}{2}\right)^2\end{aligned} \tag{2.28}$$

where $[\delta] = 1/80$ is given above
$\mu_E = \mu_J = 1$ are the scale of the figure (MATLAB® or similar programs always understands these values as 1)

The lines cross the ordinate $E(J)$ at a, b respectively. We then derive the equations describing these lines and draw them in figure 2.21:

$$\begin{aligned}y_{min} &= \psi_{min} x + a \\ y_{max} &= \psi_{max} x + b\end{aligned} \tag{2.29}$$

Calculating the moment of inertia of the flywheel using:

$$J_d = \frac{\mu_J ab}{\psi_{max} - \psi_{min}} \tag{2.30}$$

Programming with MATLAB®, we estimate $J_d = 1.63\,\mathrm{kg \cdot m^2}$. Assuming the cross section area is $100\,\mathrm{cm^2}$, the weight of the flywheel will be about 16.3 g.

2.9 Combining motion of the system

From the given parameters, we derive the following table:

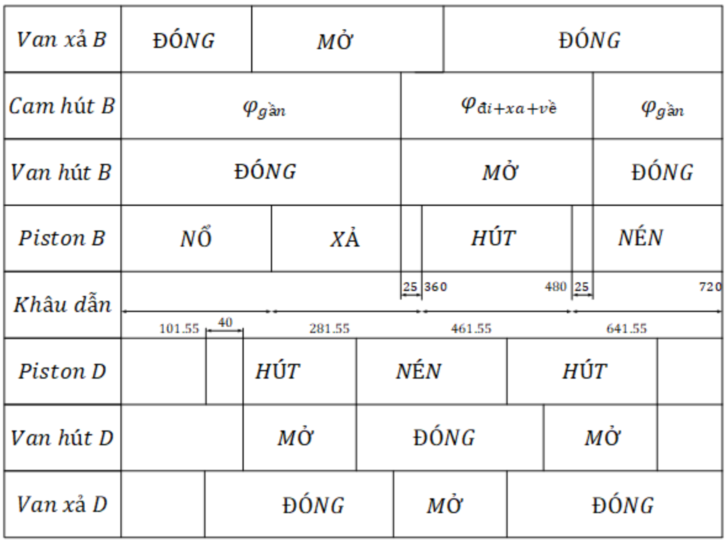

Figure 2.23: Timing chart of the system

2.10 Cam mechanism

2.10.1 Tasks

- Modeling 4 cams for 4 valves, 2 of which are intake-outtake of the combustion end, and the remaining are for the compression end.
- The intake cams are identical.
- The outtake cams are identical.

2.10.2 Cam profile determination

For combustion ends, we will find the rise, dwell, fall of the motion:

$$\phi_{rise,comb} + \phi_{dwell,comb} + \phi_{fall,comb} = \frac{230^\circ}{2} = 115^\circ$$

$$\Rightarrow \begin{cases} \phi_{rise,comb} & = 55^\circ \\ \phi_{dwell,comb} & = 5^\circ \\ \phi_{fall,comb} & = 55^\circ \end{cases}$$

Knowing the angles of each interval and the form of acceleration (modified trapezoidal), we can integrate to find velocity and displacement of the cam follower. For vibration safety, jerk is included using derivative with respect to $\phi(t)$

For flat faced follower, the pressure angle is constant. This leads to the condition of convex cam profile $R_0 + Y + \frac{dY^2}{d\phi} > 0$ or:

$$R_0 > h_{max} \tag{2.31}$$

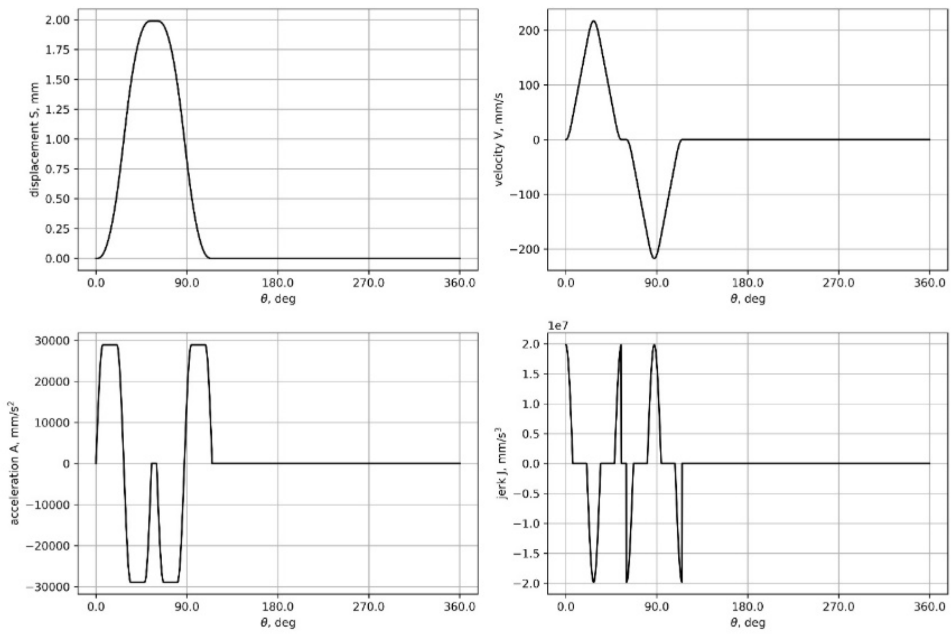

Figure 2.24: Displacement, velocity, acceleration and jerk of the cam follower

where h_{max} is the minimum value of the sum $Y + \frac{dY^2}{d\phi}$.
From the figure, $h_{max} = 9.096\,\text{mm}$. Then, arbitrarily choose $R_0 = 12\,\text{mm}$ to satisfy the condition (2.31).

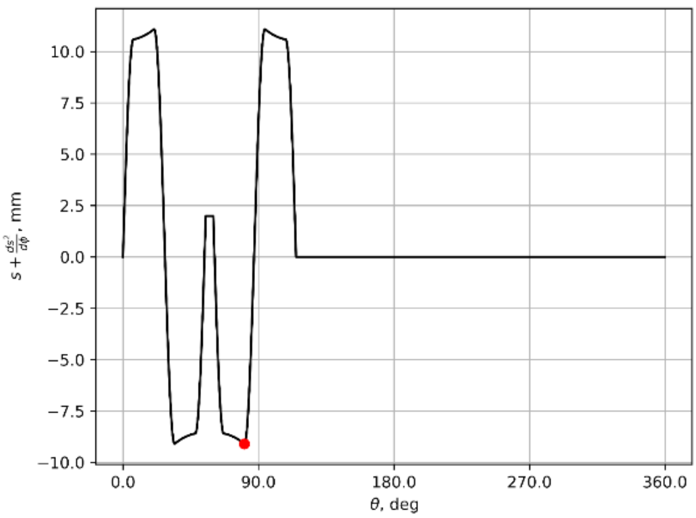

Figure 2.25: Displacement - acceleration diagram of the cam

From the pressure angle $\alpha_2 = 6°$, we use superposition to create an equivalent cam profile, namely $Y = Y \cos \alpha_2$. Then, apply the following formula to find cam profile:

$$\begin{cases} u &= (R_0 + Y) \sin \phi + Y' \cos \phi \\ v &= (R_0 + Y) \cos \phi - Y' \sin \phi \end{cases} \tag{2.32}$$

where $\phi(t) = \widehat{xOA}$; u, v are the position of cam along x, y-axes respectively; Y, Y' are the displacement and velocity of the cam follower as shown in figure 2.24.

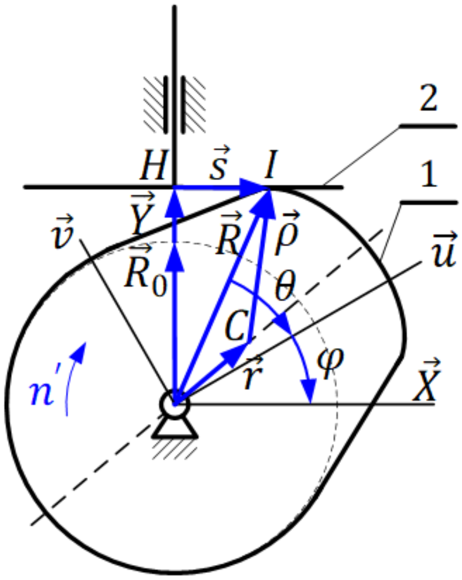

Figure 2.26: Position vectors of the cam

Using MATLAB®, we plot the cam profile numerically. Applying this process to the compression end, we also plot the cam profile in figure 2.28:

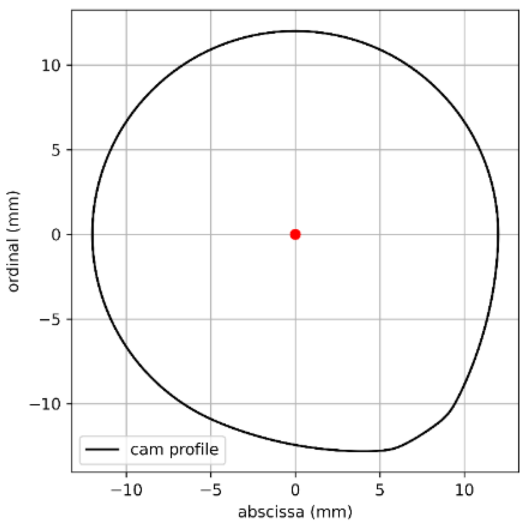

Figure 2.27: Cam profile of the combustion end

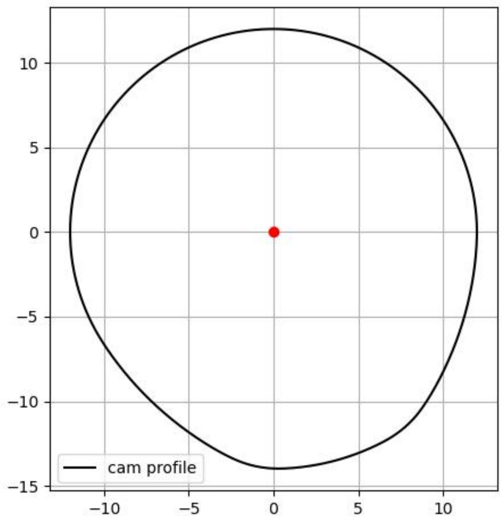

Figure 2.28: Cam profile of the compression end

Chapter 3

Summary and Conclusions

During the internship, I had a chance to get acquainted with a new working environment. I have accumulated experience in industry knowledge as well as experience skills above.
I was trained in problem solving skills in many stages, trying to complete the job in the shortest time, boldly exchanging and sharing knowledge. At the same time, it also fosters a lot of knowledge about graphic software as well as programming skills to solve the problems learned in school but professionally and saves more time.
Shortcomings: the skill is not mature which still takes a long time to execute; the ability to think and propose design plans is limited due to the lack of practical experience and in-depth knowledge.
Solution: practice more software skills, add additional specialized knowledge that is lacking.

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