Q1: Find the sample size necessary to detect a difference in mean response of 50 units between two treatments with 85% power using a t-test (one-sided) at the 0.05 level of significance. Assume the population standard deviation = 75. What's the relationship between sample size and power?

$$\begin{cases} 1-\beta=0.85 \\ \alpha=0.05 \\ \text{one-sided test} \end{cases} \qquad \begin{cases} \sigma_Y=75 \\ n=4 \\ \Delta_A=50 \end{cases} \qquad n=\frac{(Z_\alpha+Z_\beta)^2\cdot\sigma_Y^2\cdot 4}{\Delta_A^2}=\frac{(1.645+1.036)^2\times 75^2\times 4}{50^2}=64.7$$

therefore $n_1=n_2=33$

$\beta\downarrow$ $Z_\alpha\nearrow$

$$\Rightarrow\begin{cases} Z_\alpha=Z_{0.05}=1.645 \\ Z_\beta=Z_{0.15}=1.036 \end{cases} \qquad \begin{cases} H_0: \Delta=0 \\ H_a: \Delta=\Delta_A \end{cases}$$

when the power increasing, $\beta\downarrow$, $Z_\beta\nearrow$ $n\nearrow$

n will increasing

Q2: Suppose there are two treatments, and treatment 1 is the traditional treatment with mortality rate 0.25, and a new treatment is treatment 2 and can reduce the even rate to 0.15. If we are to conduct a clinical trial where equal allocation to either the treatment 1 or treatment 2. How large a sample size is necessary to detect a clinically important difference with 80% power using a two-sided test at the 0.05 level of significance?

$$\begin{cases} 1-\beta=0.8 \\ \alpha=0.05 \\ \text{two-sided} \end{cases} \Rightarrow \begin{cases} Z_\beta=Z_{0.2}=0.841 \\ Z_{\frac{\alpha}{2}}=Z_{0.05}=1.96 \end{cases}$$

$\Delta_A=\pi_1-\pi_2=0.25-0.15=0.1$

for $n_1=n_2 \Rightarrow \bar{\pi}=\frac{\pi_1+\pi_2}{2}=0.2$

$$n=\frac{\left[Z_{\frac{\alpha}{2}}+Z_\beta\sqrt{\frac{\pi_1(1-\pi_1)+\pi_2(1-\pi_2)}{2\bar{\pi}(1-\bar{\pi})}}\right]^2\cdot 4\bar{\pi}(1-\bar{\pi})}{\Delta_A^2}$$

$$=\frac{\left[1.96+0.841\sqrt{\frac{0.25\times0.75+0.15\times0.85}{2\times0.2\times0.8}}\right]^2\times 4\times 0.2\times 0.8}{(0.1)^2}$$

$\doteq 499.8 \approx 500$ total sample size should be at least 500