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13.8 In problem 5.8, suppose that there are only four machines of interest, but the operators were selected at random.

(a) What type of model is appropriate?

(b) Perform the analysis and estimate the model components using the ANOVA Method.

```
data machine;
do operator = 1 to 3; A
do machine = 1 to 4; B
do rep = 1 to 2;
input y @@; output; end; end; end;
cards;
109 110 110 115 108 109 110 108
110 112 110 111 111 109 114 112
116 114 112 115 114 119 120 117
;
proc print;
run;
proc glm;
class operator machine;
model y = operator | machine;
random operator operator*machine/test;
output out=check r=resid p=yhat;
run;
proc univariate normal plot data=check;
var resid;
proc plot data=check;
plot resid*yhat/vref=0;
proc print;
run;
proc mixed data=machine method=typel;
class operator machine;
model y = machine;
random operator operator*machine;
run;
```

The SAS System

The GLM Procedure

Tests of Hypotheses for Mixed Model Analysis of Variance

Source	DF	Type III SS	Mean Square	F Value	Pr > F
operator	2	160.333333	80.166667	10.77	0.0103
machine	3	12.458333	4.152778	0.56	0.6619
Error	6	44.666667	7.444444		
Error: MS(operator*machine)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
operator*machine	6	44.666667	7.444444	1.96	0.1507
Error: MS(Error)	12	45.500000	3.791667		

using the mixed factors model.
in this case machines is fixed.

$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$$

$$\begin{cases} E(MSA) = \sigma^2 + nb\sigma_{\epsilon}^2 + n\sigma_{\tau\beta}^2 \\ E(MSAB) = \sigma^2 + n\sigma_{\tau\beta}^2 \\ E(MSE) = \sigma^2 \end{cases} \Rightarrow$$

$$\begin{aligned} \begin{cases} n=2 \\ a=3 \\ b=4 \end{cases} & \begin{cases} MSE = 3.79167 \\ MSA = 80.1667 \\ MSAB = 7.4444 \end{cases} \\ \begin{cases} \sum \beta_j = 0 \\ \tau_i \sim NID(0, \sigma_{\epsilon}^2) \end{cases} & \\ \hat{\sigma}_{\tau\beta}^2 &= \frac{MSA - MSAB}{nb} = 9.09 \\ \hat{\sigma}_{\tau\beta}^2 &= \frac{MSAB - MSE}{n} = 1.83 \\ \hat{\sigma}^2 &= MSE = 3.79 \end{aligned}$$

Above results are all based on the unrestricted method test.

Using the Mixed Model analysis method, the result is the same as the above estimators.

Type 1 Analysis of Variance								
Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
machine	3	12.458333	4.152778	Var(Residual) + 2 Var(operator*machine) + Q(machine)	MS(operator*machine)	6	0.56	0.6619
operator	2	160.333333	80.166667	Var(Residual) + 2 Var(operator*machine) + 8 Var(operator)	MS(operator*machine)	6	10.77	0.0103
operator*machine	6	44.666667	7.444444	Var(Residual) + 2 Var(operator*machine)	MS(Residual)	12	1.96	0.1507
Residual	12	45.500000	3.791667	Var(Residual)

Covariance Parameter Estimates	
Cov Parm	Estimate
operator	9.0903
operator*machine	1.8264
Residual	3.7917

If we use the restricted method, the results are as below.

$$\frac{MSA - MSB}{nb} = \frac{80.1667 - 3.792}{2 \times 4}$$

$$s_{CB}^2 = \frac{M_{AB} - M_{SE}}{n} = 1.83$$

$$L_{G^2} = m \Delta E = 3.79$$