

13.15 Consider the experiment in example 13.6 Analyze the data for the case where A, B and C are random.

```
data gas;
do TempA = 60 to 90 by 15;
do OperB = 1 to 4;
do PressC = 1 to 3;
do rep = 1 to 2;
input y @@; output; end; end; end; end;
cards;
-2 -3 -6 4 -1 -2
0 -9 -5 -1 -4 -8
-1 -8 -8 -2 0 -7
4 4 -3 -7 -2 4
14 14 22 24 20 16
6 0 8 6 2 0
1 2 6 2 3 0
-7 6 -5 2 -5 -1
-8 -8 -8 3 -2 -1
-2 20 1 -7 -1 -2
-1 -2 -9 -8 -4 -7
-2 1 -8 3 1 3
;
proc print;
run;
proc mixed data=gas method=typel;
class TempA OperB PressC;
model y=;
random TempA|OperB|PressC;
run;
proc mixed data=gas method=reml;
class TempA OperB PressC;
model y=;
random TempA|OperB|PressC;
run;
```

⇒ restricted





Using the unrestricted model, the output is as below!

Type I Analysis of Variance

Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
TempA	2	1023.361111	511.680556	Var(Residual) + 2 Var(TempA*OperB*PressC) + 8 Var(TempA*PressC) + 6 Var(TempA*OperB) + 24 Var(TempA)	MS(TempA*OperB) + MS(TempA*PressC) - MS(TempA*OperB*PressC)	6.9674	2.30	0.1712
OperB	3	423.819444	141.273148	Var(Residual) + 2 Var(TempA*OperB*PressC) + 6 Var(OperB*PressC) + 6 Var(TempA*OperB) + 18 Var(OperB)	MS(TempA*OperB) + MS(OperB*PressC) - MS(TempA*OperB*PressC)	7.0886	0.63	0.6164
TempA*OperB	6	1211.972222	201.995370	Var(Residual) + 2 Var(TempA*OperB*PressC) + 6 Var(TempA*OperB)	MS(TempA*OperB*PressC)	12	14.59	<.0001
PressC	2	7.194444	3.597222	Var(Residual) + 2 Var(TempA*OperB*PressC) + 6 Var(OperB*PressC) + 8 Var(TempA*PressC) + 24 Var(PressC)	MS(TempA*PressC) + MS(OperB*PressC) - MS(TempA*OperB*PressC)	5.9762	0.06	0.9379
TempA*PressC	4	137.888889	34.472222	Var(Residual) + 2 Var(TempA*OperB*PressC) + 8 Var(TempA*PressC)	MS(TempA*OperB*PressC)	12	2.49	0.0991
OperB*PressC	6	209.472222	34.912037	Var(Residual) + 2 Var(TempA*OperB*PressC) + 6 Var(OperB*PressC)	MS(TempA*OperB*PressC)	12	2.52	0.0814
TempA*OperB*PressC	12	166.111111	13.842593	Var(Residual) + 2 Var(TempA*OperB*PressC)	MS(Residual)	36	0.65	0.7882
Residual	36	770.500000	21.402778	Var(Residual)				



Estimators:

**Covariance Parameter Estimates**

Cov Parm	Estimate
TempA	12.0440
OperB	-4.5440 *
TempA*OperB	31.3588
PressC	-2.1644 *
TempA*PressC	2.5787
OperB*PressC	3.5116
TempA*OperB*PressC	-3.7801 *
Residual	21.4028

using the unrestricted model.

The variance of some parameters are negative, that's unreasonable.

Using the restricted model, the output is as below:

**Covariance Parameter Estimates**

Cov Parm	Estimate
TempA	13.5653
OperB	0
TempA*OperB	26.7438
PressC	0
TempA*PressC	0.7072
OperB*PressC	1.2678
TempA*OperB*PressC	0
Residual	19.9924

From the output of restricted model.

The reduced model in this case:

$$Y_{ijk} = \mu + \tau_i + (\tau\beta)_{ij} + (\tau\sigma)_{ik} + (\beta\sigma)_{jk} + \epsilon_{ijk}$$