Analog Computer Simulation of a Mass-Spring-Damper System

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Abstract

This project explores the use of an analog computer to simulate the behavior of a second-order mechanical system under the influence of gravity, damping, and spring forces. The system is modeled using operational amplifier circuits including summers, integrators, and buffers. The Deboo integrator, based on a Howland current source, is employed to overcome limitations of conventional RC integrators. Simulation results for three physical scenarios — free fall, damped motion, and spring oscillation — are analyzed and interpreted.

1 Introduction

Analog computers offer a unique way to simulate dynamic systems using continuous electrical signals. This project demonstrates the analog simulation of vertical motion governed by Newton's second law using op-amp-based circuits. The primary objective is to understand and implement a second-order differential equation using analog computation.

Objectives

- Understand the functionality of analog computers using operational amplifiers.
- Simulate the motion of a mass under gravity, damping, and spring force.
- Implement the system using summers, integrators, and buffers.
- Utilize the Deboo integrator for improved performance.
- Analyze the system response for various parameter configurations.

2 Theoretical Background

Governing Equation

The equation of motion under gravity (g), damping (b), and spring force (k) is:

$$F = ma = -mg - bv - ky \tag{1}$$

Rewriting this as a differential equation:

$$\frac{d^2y}{dt^2} = -g - \frac{b}{m}\frac{dy}{dt} - \frac{k}{m}y \tag{2}$$

Circuit Blocks

- **Buffer Circuit**: Prevents loading from potentiometers.
- **Summation Circuit**: Combines force components: gravity, damping, and spring.
- **Integration Circuit**: Two-stage integration converts acceleration to displacement.

Voltage Mapping

Voltages are mapped to physical quantities:

- $Va \propto a$, $Vv \propto v$, $Vy \propto y$
- Normalized so that $1V \equiv 1$ unit of the corresponding quantity.

3 Deboo Integrator

Motivation

Conventional RC integrators suffer from polarity and initialization issues. The Deboo integrator overcomes these with a grounded capacitor and stable current source.

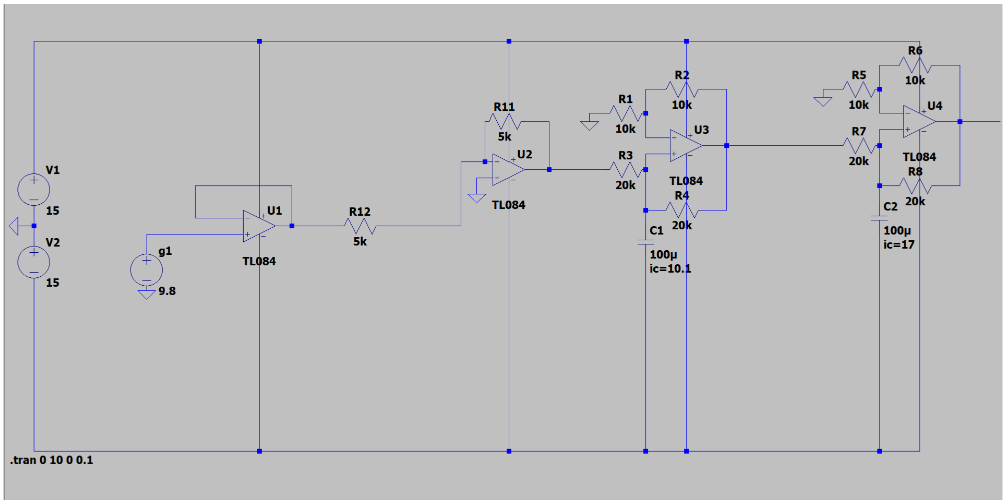
Advantages

- Easy initialization
- Better linearity and stability
- Compatibility with polarized capacitors

4 Simulation Designs

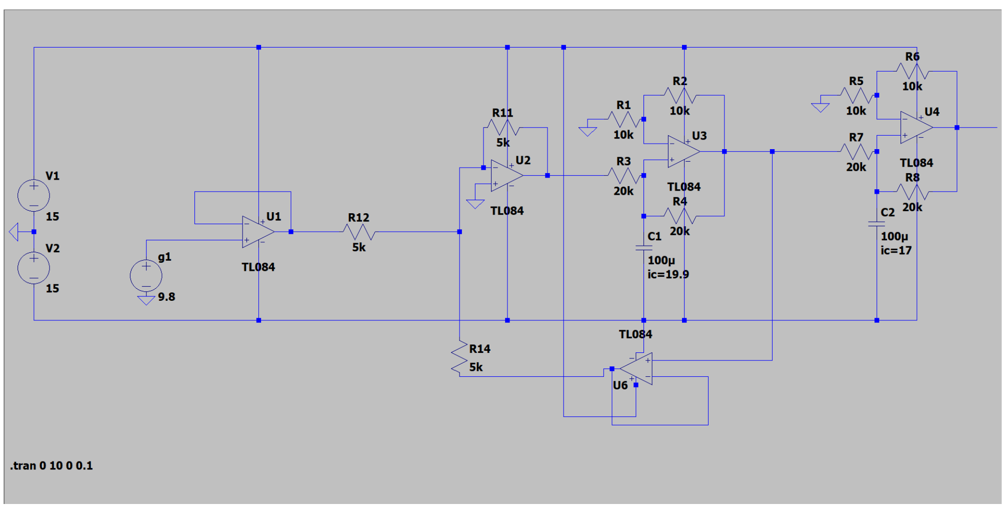
Case 1: Free Fall

Gravity-only system: $\frac{b}{m} = 0$, $\frac{k}{m} = 0$



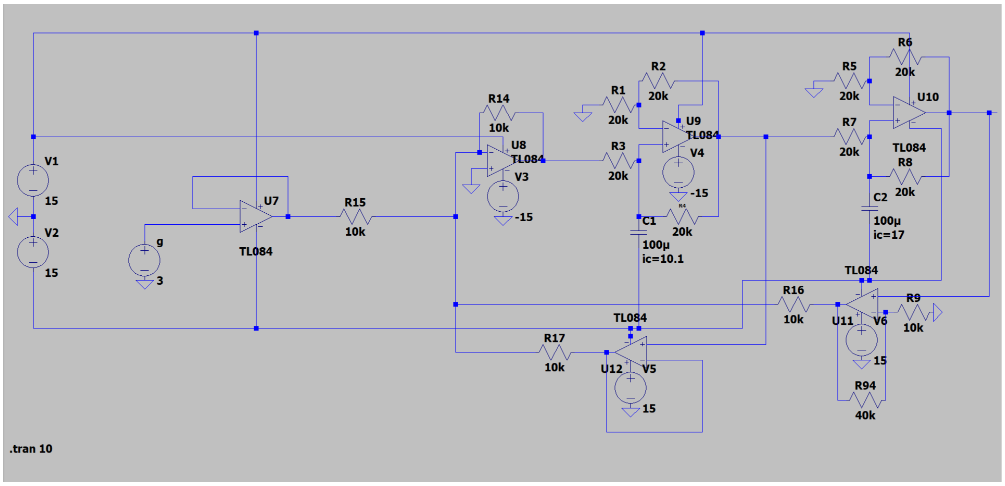
Case 2: Damped Motion

Damping included: $\frac{b}{m} = 1$, $\frac{k}{m} = 0$



Case 3: Spring Oscillation

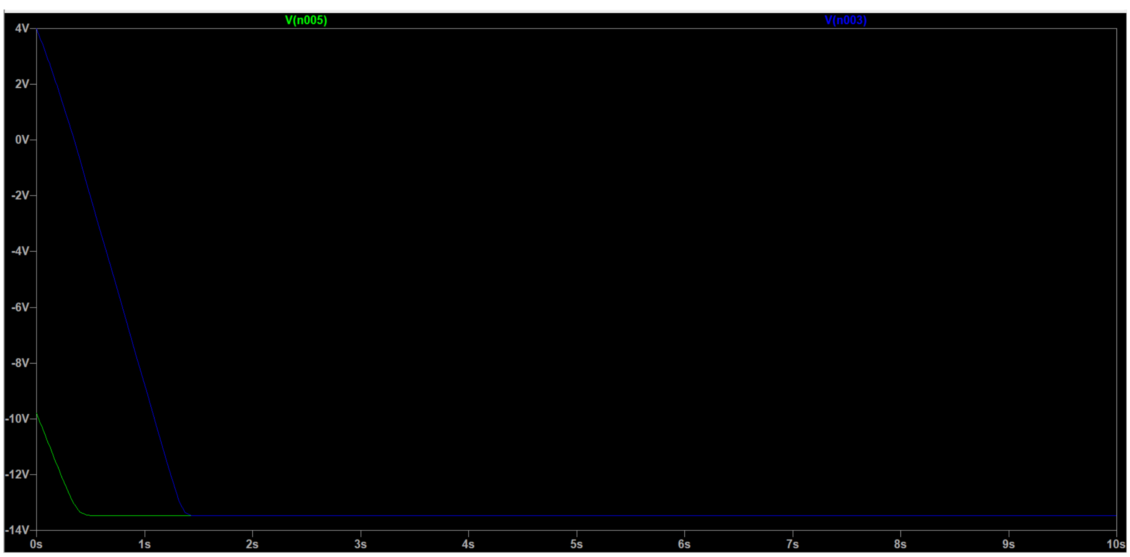
Mass-spring-damper system: $\frac{b}{m} = 1$, $\frac{k}{m} = 5$, $g = 3.0\,\mathrm{m/s^2}$



5 Simulation Results

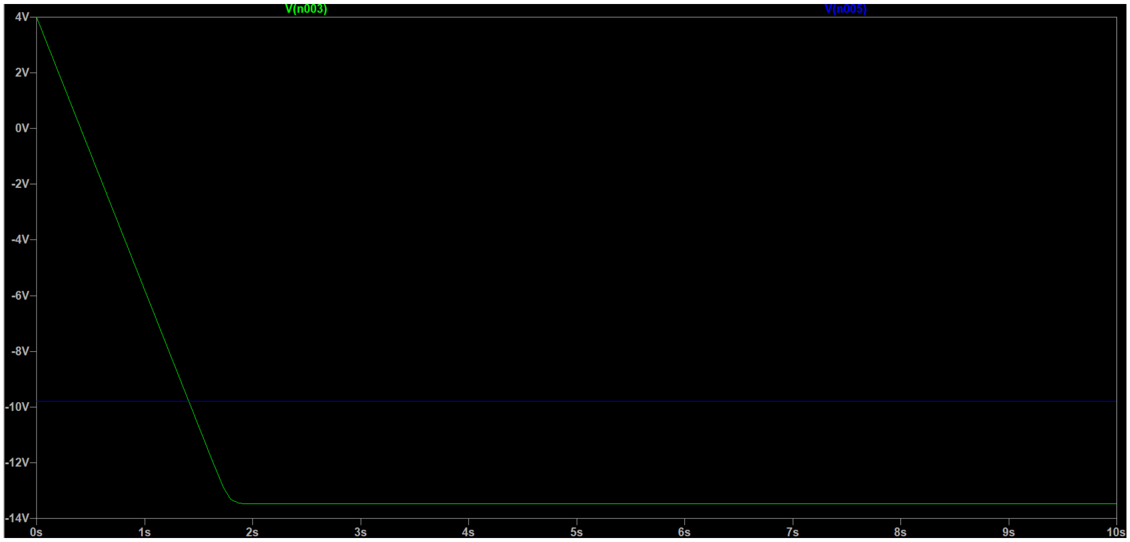
Case 1: Free Fall

- Velocity decreases linearly before getting saturated
- Displacement increases quadratically and finally it saturates



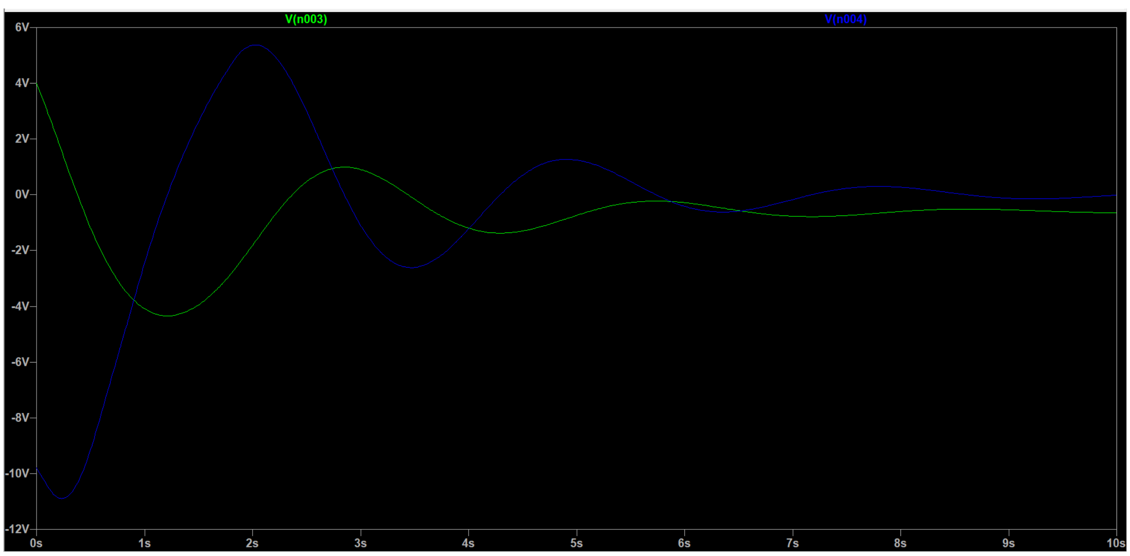
Case 2: Damped Motion

- Velocity is constant as the object is thrown with the terminal velocity itself and there is no net force acting on it
- Displacement decreases linerly as the velocity is constant



Case 3: Spring Oscillation

- Output waveforms show sinusoidal oscillations in their transients which saturate to a steady state value
- Acceleration shows the least damping
- Displacement is the most damped



6 Discussion

Each simulation corresponds well with theoretical expectations. Cascaded integration accurately models physical quantities over time. The use of the Deboo integrator ensures better performance in circuit stability and initialization.

7 Answers to Conceptual Questions

Why two integrators?

To simulate displacement from acceleration, two stages of integration are required:

- Acceleration $\rightarrow$ Velocity (1st integrator)
- Velocity $\rightarrow$ Displacement (2nd integrator)

Why Deboo integrator?

The Howland current source based integrator (also known as the Deboo integrator) offers several advantages over a conventional RC integrator:

- Improved Initialization: The grounded capacitor allows easy setting of initial conditions.
- Better Linearity and Stability: The current source ensures a regulated and linear charging of the capacitor, reducing integration drift.
- Electrolytic Capacitor Compatibility: With a slight modification, it can safely use polarized capacitors by ensuring unidirectional voltage across them.

These features make the Howland integrator more suitable for accurate

Effect of damping coefficient

- Low damping: sustained oscillations
- High damping: faster decay
- Increasing $\frac{b}{m}$ slows response and reduces terminal velocity

8 Experimental Results

8.1 Case 1: Motion Under Gravity

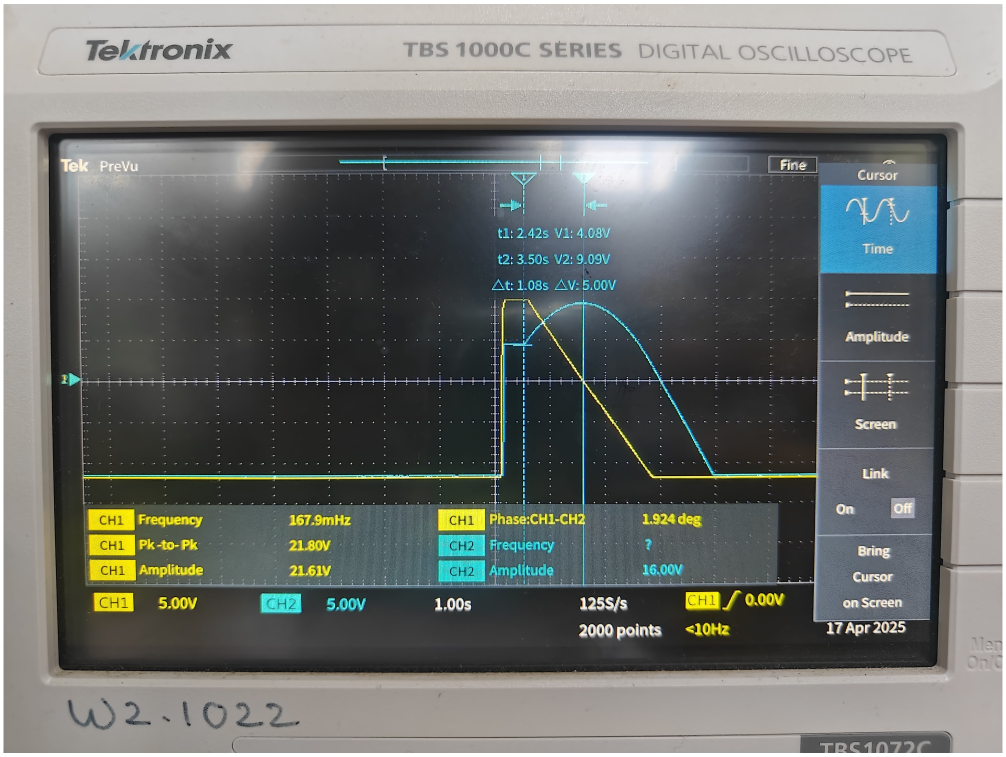


Figure 1: Position and Velocity v/s Time: Motion Under Gravity

Table 1: Time for Velocity to Reach Saturation: Motion Under Gravity

Parameter	Theoretical (s)	Simulated (s)	Experimental (s)
Time for velocity to reach saturation	2.22	2.30	2.38

8.2 Case 2: Motion Under Gravity considering Viscous Drag

Figure 2: Position and Velocity v/s Time: Motion Under Gravity considering drag

Table 2: Time for Position to Reach Saturation: Motion Under Gravity with Drag

Parameter	**Theoretical (s)**	**Simulated (s)**	**Experimental (s)**
Time for position to reach saturation	3.90	3.92	3.98

8.3 Case 3: Damped Spring Oscillation

Table 3: Comparison of Theoretical, Simulated, and Experimental Oscillation Frequency

Parameter	**Theoretical**	**Simulated**	**Experimental**
T (from position curve)	2.88 s	3.20 s	3.25 s
$3 \times T/4$ (from velocity curve)	2.16 s	2.24 s	2.26 s

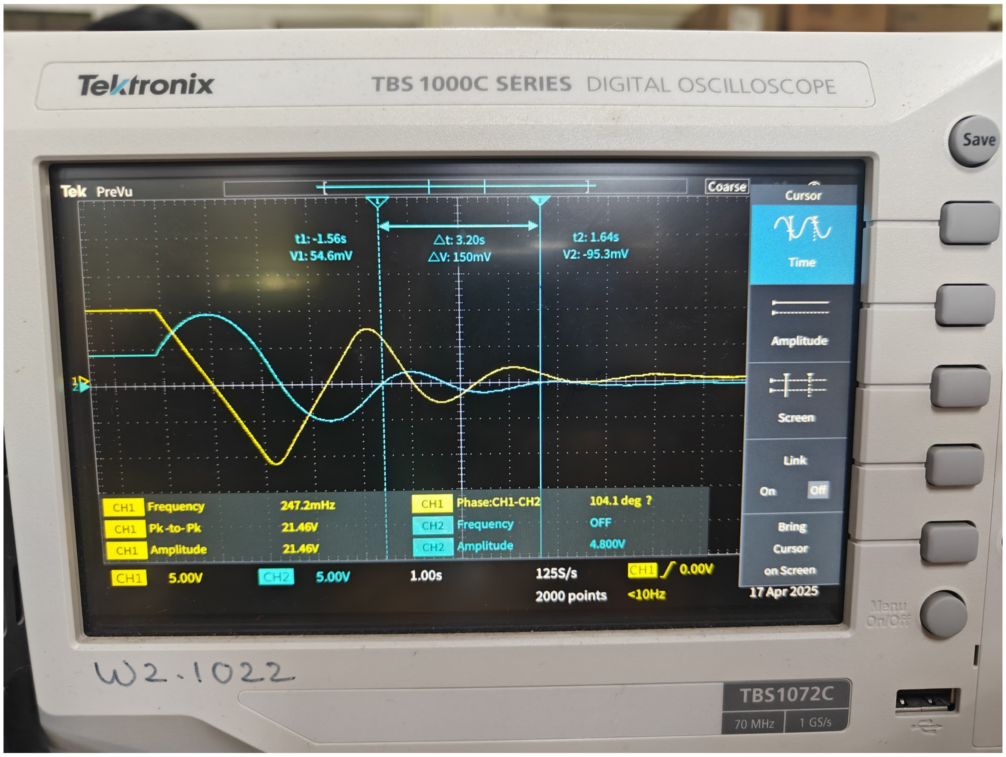


Figure 3: Position and Velocity v/s Time: Damped Oscillation

9 Work Division

- Chinmay was responsible for performing the hand calcualtions and also preparing the Latex report.
- LTspice schematics were preapred by Ishan and Chinmay together.
- During the hardware implementation, Ishan prepared the cirucit, both of them worked to debug the circuit to get the expected output.

10 Conclusion

This project demonstrates that analog computing using op-amps is a powerful method to simulate second-order differential equations. With proper scaling and circuit design, it is possible to visualize and analyze physical systems in real time. The Deboo integrator provides notable advantages in precision and setup ease.