LD. for reference and a uniformly scaled query.

- When considering a reference OD matrix, consider the following:

1. The order of choices will not change since all entries are scaled by the same factor.

$$(A > B) \implies (k * A > k * B) \rightarrow (1)$$

> where A and B are two entries on the same row of the OD matrix, and k is the scaling factor.

Hence, we can simplify the analysis to just two observations.

For an OD matrix as below:

	O1	O2	O3
O1	0	A	B
O2	D	0	C
O3	E	F	0

We can generate a uniformly scaled query matrix

	O1	O2	O3
O1	0	k*A	k*B
O2	k*D	0	k*C
O3	k*E	k*F	0

Hence, since scaling preserves order of magnitude (1),
Without loss of generality, the skeletal-matrix with mass flow values of both the reference and query can be assumed to be:

	C1	C2	C3
O1	(O2,A)	(O3,B)	(O1,0)
O2	(O1,D)	(O3,C)	(O2,0)
O3	(O1,E)	(O2,F)	(O3,0)

	C1	C2	C3
O1	(O2,k*A)	(O3,k*B)	(O1,0)
O2	(O1,k*D)	(O3,k*C)	(O2,0)
O3	(O1,k*E)	(O2,k*F)	(O3,0)

* C# = choice of order '#'

$$LDOD = (k-1)*(A+B+C+D+E+F)$$

$$\boxed{LDOD = (k-1)*(\sum(mass\ flows))}$$

LD. for Uniformly shifted query.

Here too, the order of choice shall not change in the reference and query since

$$A > B \implies A+n > B+n$$

So, with the same line of reasoning;
reference:

	O1	O2	O3
O1	0	A	B
O2	D	0	C
O3	E	F	0

query:

	O1	O2	O3
O1	0	A+n	B+n
O2	D+n	0	C+n
O3	E+n	F+n	0

Hence the Levenshtein Distance shall be

$$\begin{aligned} & LDOD = 6 * n \\ \implies & LDOD = (3^2 - 3) * n \end{aligned}$$

The $d = 3$ is the dimension or the number of origins of the OD matrix.
Hence,

$$\boxed{LDOD = (d^2 - d) * n}$$

L.D. for non-uniformly shifted query matrix but preserved choice-order

In this case, again the reference matrix shall remain the same but $n_{i,j}$ shall be different. then the Levenshtein distance shall be:

$$\boxed{LDOD = \sum_{i,\,j} n_{i,j}}$$