Linear Algebra
Module 12 Assignment

Due Sunday at 11:59 PM

1 Directions

Complete the following problems showing all your work. You may use a calculator to check your work, but should write out (or TeX up) all the steps of your solution. Unless otherwise specified, you may skip some steps of row reduction with a calculator, but state that you did so. Please upload your work as a single .pdf file in the course.

2 Problems

1. If W is a subspace of $\mathbb{R}^n$, and if $\vec{v}$ is in both W and $W^\perp$, give an argument to show that $\vec{v}$ must be the zero vector.

2. In each part, apply Gram-Schmidt to the basis of S to obtain an orthogonal basis for S. Then normalize the vectors in this basis to obtain an orthonormal basis β for S. Then for the vector $\vec{x} \in S$ given, compute the coefficients of $\vec{x}$ relative to β. Use Theorem 5 on pg 341 to verify your answers.

   (a) Let $S = span\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} \right\}$ in $\mathbb{R}^3$, and $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

   (b) Let $S = span\left\{ \begin{bmatrix} 2 \\ -1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -5 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 7 \\ 11 \end{bmatrix} \right\}$ in $\mathbb{R}^3$, $\vec{x} = \begin{bmatrix} -11 \\ 8 \\ -4 \\ 18 \end{bmatrix}$.

3. Find all the least-squares solutions of the equation $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$.