

Self Check 9: Bayesian Classifier

- 1) Calculate the probabilities required for a Naive Bayes Classifier.

1. Prior Probabilities:

$$P(\text{Safe} = \text{yes}) = \frac{7}{15}, \quad P(\text{Safe} = \text{no}) = \frac{8}{15}$$

2. Likelihood for Shape:

$$P(\text{Shape} = \text{square} | \text{Safe} = \text{yes}) = \frac{5}{7}, \quad P(\text{Shape} = \text{square} | \text{Safe} = \text{no}) = \frac{3}{8}$$

$$P(\text{Shape} = \text{round} | \text{Safe} = \text{yes}) = \frac{2}{7}, \quad P(\text{Shape} = \text{round} | \text{Safe} = \text{no}) = \frac{5}{8}$$

3. Likelihood for Size:

$$P(\text{Size} = \text{large} | \text{Safe} = \text{yes}) = \frac{4}{7}, \quad P(\text{Size} = \text{large} | \text{Safe} = \text{no}) = \frac{4}{8}$$

$$P(\text{Size} = \text{small} | \text{Safe} = \text{yes}) = \frac{3}{7}, \quad P(\text{Size} = \text{small} | \text{Safe} = \text{no}) = \frac{4}{8}$$

4. Likelihood for Color:

$$P(\text{Color} = \text{red} | \text{Safe} = \text{yes}) = \frac{2}{7}, \quad P(\text{Color} = \text{red} | \text{Safe} = \text{no}) = \frac{4}{8}$$

$$P(\text{Color} = \text{green} | \text{Safe} = \text{yes}) = \frac{3}{7}, \quad P(\text{Color} = \text{green} | \text{Safe} = \text{no}) = \frac{1}{8}$$

$$P(\text{Color} = \text{blue} | \text{Safe} = \text{yes}) = \frac{2}{7}, \quad P(\text{Color} = \text{blue} | \text{Safe} = \text{no}) = \frac{3}{8}$$

- 2) What is the normalized probability distribution over the possible class labels for square/large/red?

5. Posterior Probability for Safe = yes:

$$\begin{aligned} & P(\text{Safe} = \text{yes} | \text{square, large, red}) \\ &= P(\text{square} | \text{Safe} = \text{yes}) \times P(\text{large} | \text{Safe} = \text{yes}) \times P(\text{red} | \text{Safe} = \text{yes}) \\ & \quad \times P(\text{Safe} = \text{yes}) \end{aligned}$$

$$= \frac{5}{7} \times \frac{4}{7} \times \frac{2}{7} \times \frac{7}{15} = 0.0187$$

6. Posterior Probability for Safe = no:

$$\begin{aligned} & P(\text{Safe} = \text{no} \mid \text{square, large, red}) \\ &= P(\text{square} \mid \text{Safe} = \text{no}) \times P(\text{large} \mid \text{Safe} = \text{no}) \times P(\text{red} \mid \text{Safe} = \text{no}) \\ &\quad \times P(\text{Safe} = \text{no}) \\ &= \frac{3}{8} \times \frac{4}{8} \times \frac{4}{8} \times \frac{8}{15} = 0.03125 \end{aligned}$$

7. Normalized Posterior Probabilities:

$$P(\text{Safe} = \text{yes}) = \frac{0.0187}{0.0187 + 0.03125} = 0.374$$

$$P(\text{Safe} = \text{no}) = \frac{0.03125}{0.0187 + 0.03125} = 0.626$$