Andrew Taylor
atay1136

Linear Algebra Module 5 Assignment

1) $A^{-1} = \begin{bmatrix} 7 & 8 \\ 1 & 2 \end{bmatrix}$, $(A^{-1})^{-1} = A =$

$$\frac{1}{2\cdot7-1\cdot8}\begin{bmatrix} 2 & -8 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 2/6 & -8/6 \\ -1/6 & 7/6 \end{bmatrix} = \begin{bmatrix} 1/3 & -4/3 \\ -1/6 & 7/6 \end{bmatrix}$$

2) Suppose A is invertible.

a) from the supposition we get access to the Invertible Matrix Theorem, which tells us it is equivalent to $(A^T)^{-1} = (A^{-1})^T$ from the properties of transposition. This tells us A^T is invertible. Now we know ~~both~~ matrices are invertible. Using the first circle of implication the supposition also entails that an ~~invertible~~ matrix is row equivalent to the Identity matrix. Thus, both A^T and A are equivalent to I_n, and so their product would be row equivalent to I_n (itself when reduced) as well, because there is no row operation being performed. A^TA implies the supposition that the product is invertible this way. This assumes A is an $n \times n$ matrix.

2) b) Assuming A is $n \times n$, we can show that $A^{-1} = (A^TA)^{-1}A^T$ through computation of the product of A and $(A^TA)^{-1}A^T$ by assigning A but first let's do some algebra to show why:

$$A^{-1} = (A^TA)^{-1}A^T = A^{-1}(A^T)^{-1}A^T = A^{-1}I$$
$$AA^{-1} = AA^{-1}I$$
$$I = II$$
$$I = I$$

and

$$A(A^TA)^{-1}A^T = AA^{-1}(A^T)^{-1}A^T = II = I$$

So for A a 2×2 matrix we can assign it the value of I_2: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and compute; the computation goes in a circle, in both cases we start with I_2 and end with I_2, they are equivalent statements.

$$(A^TA)^{-1}A^T = A(A^TA)^{-1}A^T.$$

Andrew Taylor

3) The question is: find the value of b for inverses:

$$A = \begin{bmatrix} 3 & -2 & -2 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{bmatrix}$$

In order to find the inverse of only one column we augment by only that column of I_n, and perform Gauss-Jordan. So, working from $A\bar{x} = \bar{e}_2$, since the other columns can be found we find B_{22}:

$$\left[\begin{array}{ccc|c} 3 & -2 & -2 & 0 \\ -1 & 1 & 1 & 1 \\ 3 & -1 & -2 & 0 \end{array}\right] R_1 = R_1 + 2R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ -1 & 1 & 1 & 1 \\ 3 & -1 & -2 & 0 \end{array}\right] R_3 = R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 1 \end{array}\right] R_2 = R_2 + R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 2 & 0 & -1 & 1 \end{array}\right] R_3 = R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & -3 \end{array}\right] R_2 = R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -3 \end{array}\right] R_3 = (-1)R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \therefore b = 0.$$