Time Allowed: 120 minutes
Total Weight: 100 points

Name: Andrew Taylor

Introduction to Probability and Statistics Final Exam

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10) What is the value of k in the following CDF?

$$F(x) = \begin{cases} 0 & \\ (10 - kx)x^9 & 0 < x < 1 \\ 1 & \end{cases}$$

$$\int_0^1 (10-kx)x^9\,dx = \int_0^1 10x^9 - kx^{10}\,dx =$$

$$= \int_0^1 10x^9\,dx - \int_0^1 kx^{10}\,dx = 10\int_0^1 x^9\,dx - \int_0^1 kx^{10}\,dx$$

$$10\left[\frac{x^{10}}{10}\right|_0^1 = 10\cdot\frac{1}{10} = \underline{1}$$

$$\int_0^1 kx^{10}\,dx = k\int_0^1 x^{10}\,dx = k\left[\frac{x^{11}}{11}\right|_0^1 = \frac{k}{11}$$

$$= \underline{1} - \frac{k}{11} \qquad \frac{k}{11} = 0 \therefore k = 0$$

compare to 1

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2. (10) Find the expected value of the following pdf

$$f(x) = 2\left(1 - \frac{1}{x^2}\right) \qquad 1 < x < 2$$

$$E[x] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$E[x] = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right) dx = 2\int_1^2 \left(x - \frac{1}{x}\right) dx$$

$$= 2\left[\frac{x^2}{2} - \log x \Big|_1^2\right] = 2\left[\frac{1}{2}(4-1) - (\log 2 - \log 1)\right]$$

$$= 3 - 2\log 2$$

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3. (10) Students arrive to class with an inter-arrival time that is exponentially distributed with a mean of 1.5min. Data is collected for two classes each day (30 inter-arrival times). How are the average inter-arrival times for a class distributed?

$\mu = 1.5, \; \sigma = 1.5$

From CLT, we know that a sample will be normally distributed as long as $n \geq 30$, so we are ok.

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$= \bar{X} \sim N\left(1.5, \frac{1.5^2}{30}\right)$

$\sigma^2/n =$

$= \cancel{\bar{X} \sim N(1.5,} \; 0.075$

$= \bar{X} \sim N(1.5, 0.27386...^2)$

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4. (10) Coca-Cola bottling company must put 20oz. of soda in each bottle. If there is too little, customers are upset. If there is too much, the company loses money. A quality engineer takes a random sample of 30 bottles with mean 20.2 and std dev of 0.2. Are there any problems with the bottling process?

We need to see if they aren't bottling 20oz on average.

$H_0: \mu = 20$

$H_a: \mu \neq 20$

$$t = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{20.2 - 20}{\frac{0.2}{\sqrt{30}}}$$

$$= \sqrt{30} = 5.47722\ldots$$

using t-table and this test stat, $df = 29$

two-tailed (could be either), P-value $= 0.0000\ldots$

$\therefore$ We reject H_0, there is sufficient evidence to find a problem with the bottling process.

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5. (20) Fill in the three red blanks with the correct values

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.914467103
R Square	[redacted]
Adjusted R Square	0.825333421
Standard Error	2.051510168
Observations	17

ANOVA

	df	*SS*	*MS*	*F*
Regression	1	322.3990022	322.399	76.6031
Residual	15	63.13040953	4.208694	
Total	16	385.5294118		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.493174557	0.961315755	-1.55326	0.141202
X Variable 1	0.630845193	0.072077486	[redacted]	

Figure 1: Regression Stat Output

1) $R^2 = \frac{SSR}{SST} = \frac{322.3990022}{385.5294118} = 0.836250081919\ldots$

2) $\text{t-stat} = \frac{0.630845193}{0.072077486} = 8.752319593\ldots$

3) $\text{P-valve} = 2*(\text{T.DIST.RT}(8752319593, 16)) = -1.69562 \times 10^{-07}$

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6. (10) A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years, with a sample standard deviation of 8.9 years. Does the sample seem to indicate that the average life span today is greater than 70 years?

$H_0: \mu = 70$, $H_a: \mu > 70$, $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{71.8 - 70}{8.9/\sqrt{100}} = \frac{1.8}{0.89} = 2.02$$

From z-table, @ 95%, z critical is 1.96

$2.02 > 1.96 \quad \therefore H_0$ rejected.

The sample mean indicates average life span today is greater than 70 years. (reasonable evidence exists)

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7. (20) Weights for a certain rodent population are normally distributed with an unknown mean μ. A random sample of size 20 yields a sample mean weight of 18.5 ounces and a sample standard deviation of 16 ounces. Find a 96% CI for the true mean rodent weight. Include a sketch of the appropriate probability distribution and critical p-values with your answer.

ok $n = 20, \bar{x} = 18.5, s = 16$

sig level $= (1 - 0.96) = 0.04$, $df = 19$

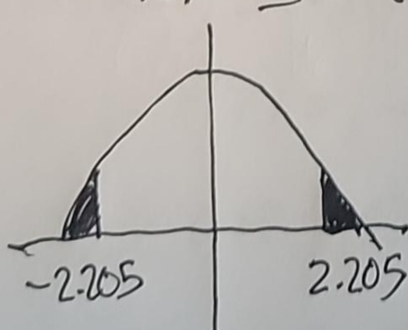
$t_{\alpha/2} =$ T.INV.2T(0.04, 19) $= 2.205$

$CI = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

$= 18.5 \pm 2.205 \frac{16}{\sqrt{20}}$

$= 18.5 \pm 7.88884\ldots$

$\therefore$ 96% CI $= (10.61, 26.39)$



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8. (10) The time in days between breakdowns of the company's e-mail server is exponentially distributed with $\lambda = 0.2$. What is the expected time between server breakdowns? What is the probability that the server will break down within 5 days of its last failure? What is the probability that after the server is repaired it lasts a week before failing again? If the server has performed satisfactorily for six days, what is the probability that it lasts at least two additional days before breaking down?

$\underline{X} \sim exp(\lambda = 0.2)$

a) $E[x] = 1/\lambda = 5$ days expected between server breakdowns.

b) $P(\underline{X} < 5) = \int_0^5 0.2e^{-0.2x} dx = 0.2\int_0^5 e^{-0.2x} dx =$

$-0.2x = u, \frac{du}{dx} = -0.2$

$= \int_0^{-1} e^u (-\frac{1}{0.2}) du \cdot 0.2$

$= 0.2(-(-\frac{1}{0.2} \cdot \int_{-1}^0 e^u du))$

$= 0.2(-(-5[e^u]_{-1}^0))$

$= [e^u|_{-1}^0 = 1 - \frac{1}{e}$

c) $P(\underline{X} \geq 7) = 1 - P(x < 7) = 1 - [P(x<7) + \cancel{P(x=7)}^{=0}] = 1 - P(x<7)$

$= 1 - F(7) = 1 - (1 - e^{-0.2 \times 7}) = 1 - 0.7534 = 0.2466$

d) $P(x \geq 6+2 \mid x \geq 6)$ is $P(x \geq 2)$ because memoryless.

$P(x \geq 2) = 1 - P(x<2) = 1 - F(2) = 1 - (1 - e^{-0.2 \times 2}) =$

$= 1 - 0.3297 = 0.6703$