

**Multivariable and Complex Analysis Midterm****SHOW ALL WORK TO RECEIVE FULL CREDIT**

1. (15) Show that the form under the integral sign is exact in the path independent plane from point  $a : (\pi/2, \pi)$  to  $b : (\pi, 0)$ , then evaluate the integral.

$$\int_a^b \left( \frac{1}{2} \cos \frac{1}{2}x \cos 2y \, dx - 2 \sin \frac{1}{2}x \sin 2y \, dy \right)$$

2. (10) Find the eigenvalues and corresponding eigenvectors of  $\mathbf{A}$ . *To receive full credit, you must show all work.*  
Is  $\mathbf{A}$  diagonalizable?

$$\mathbf{A} = \begin{bmatrix} 13 & 4 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{bmatrix}$$

3. (10) Find and sketch the parametric representation of a circle in the  $yz$ -plane with center  $(4, 0)$  and passing through the point  $(0, 3)$ .

4. (10) Find basis for null space of  $\mathbf{A}$ , where

$$A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3] = \begin{bmatrix} -1 & 2 & 3 \\ -4 & 1 & 1 \end{bmatrix}$$

5. (15) Evaluate the surface integral  $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA$  directly for

$$\mathbf{F} = [-13 \sin y, 3 \sinh z, x],$$

and  $S$  is the rectangle with vertices  $(0, 0, 2)$ ,  $(4, 0, 2)$ ,  $(4, \pi/2, 2)$ ,  $(0, \pi/2, 2)$ .