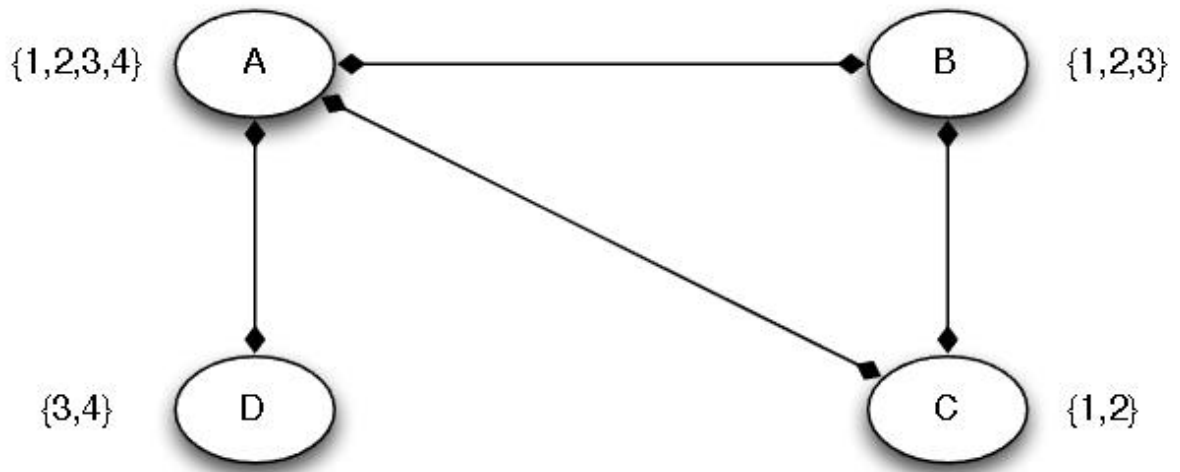## Self Check 6: CSPs

Ok we're considering this CSP:

1) To demonstrate backtracking and forward checking on a four-node constraint graph with the static ordering A,B,C,D for variables and static ordering 1,2,3,4 for the values, we can create a table, examining the choices and applying forward checking as we go.

Constraint Satisfaction Problem Overview:

- **Nodes**: A, B, C, D

- **Domains**:

    o   A: {1, 2, 3, 4}

    o   B: {1, 2, 3}

    o   C: {1, 2}

    o   D: {3, 4}

- **Constraints**:

    o   A ≠ B, A ≠ C, A ≠ D

    o   B ≠ C

    o   C ≠ A, C ≠ B

    o   D ≠ A

Step	Variable	Value Chosen	Domains After Forward Checking	Backtrack?	Explanation
1	A	1	B: {2, 3}, C: {2}, D: {3, 4}	No	A is assigned 1. We apply forward checking and remove 1 from the domains of B, C, and D.
2	B	2	C: {}, D: {3, 4}	Yes	B is assigned 2. After forward checking, C has no available values left, so we backtrack.
3	B	3	C: {2}, D: {4}	No	B is reassigned to 3. Forward checking shows C can still be 2 and D can be 4.
4	C	2	D: {4}	No	C is assigned 2. Forward checking confirms that D still has 4 available.
5	D	4	-	No	D is assigned 4. No further variables need to be assigned.
					Solution found: A = 1, B = 3, C = 2, D = 4.

2) To demonstrate the use of the **Minimum Remaining Values (MRV)** heuristic for variable selection and static value ordering (1, 2, 3, 4), we make another table. The MRV heuristic selects the variable with the fewest remaining valid values in its domain. If there's a tie, the variables are selected alphabetically.

Step	Variable (MRV)	Value Chosen	Domains After Forward Checking	Backtrack?	Explanation
1	C	1	A: {2, 3, 4}, B: {2, 3}, D: {3, 4}	No	C is selected first due to MRV (only 2 values). C is assigned 1. After forward checking, 1 is removed from A and B.
2	B	2	A: {3, 4}, D: {3, 4}	No	B is selected next (domain size 2). B is assigned 2. After forward checking, 2 is removed from A's domain.
3	A	3	D: {4}	No	A is selected next (domain size 2). A is assigned 3. After forward checking, 3 is removed from D's domain.
4	D	4	-	No	D is selected (only remaining variable). D is assigned 4.
					Solution found: C = 1, B = 2, A = 3, D = 4.

3) To demonstrate the **Degree Heuristic** for variable selection with a static value ordering (1, 2, 3, 4), we change the table in the way we start out. The **Degree Heuristic** selects the variable involved in the largest number of constraints with unassigned variables. If there's a tie, we select alphabetically.

Step	Variable (Degree Heuristic)	Value Chosen	Domains After Forward Checking	Backtrack?	Explanation
1	A	1	B: {2, 3}, C: {2}, D: {3, 4}	No	A is selected first because it has the highest degree (3 constraints). A is assigned 1. Forward checking removes 1 from B, C, and D.
2	B	2	C: {}, D: {3, 4}	Yes	B is selected next (degree 2). B is assigned 2. After forward checking, C has no available values, so we backtrack.
3	B	3	C: {2}, D: {4}	No	B is reassigned to 3. After forward checking, C can be 2 and D can be 4.
4	C	2	D: {4}	No	C is selected next (degree 2). C is assigned 2. Forward checking shows D still has 4 available.
5	D	4	-	No	D is selected and assigned 4. No further variables need to be assigned.
					Solution found: A = 1, B = 3, C = 2, D = 4.

4) To demonstrate **Minimum Remaining Values (MRV)** for selecting variables and **Least Constraining Value (LCV)** for selecting values, we will follow the same step-by-step approach. The **MRV heuristic** selects the variable with the fewest remaining values in its domain, and the **LCV heuristic** selects the value that imposes the fewest constraints on the remaining unassigned variables. We will break any ties alphabetically for variables and ascending for values.

Step	Variable (MRV)	Value Chosen (LCV)	Domains After Forward Checking	Backtrack?	Explanation
1	C	1	A: {2, 3, 4}, B: {2, 3}, D: {3, 4}	No	C is selected first using MRV (smallest domain size of 2). LCV selects 1 because it constrains A and B the least.
2	B	2	A: {3, 4}, D: {3, 4}	No	B is selected next using MRV (domain size of 2). LCV selects 2 because it leaves A with more remaining values.

3	A	3	D: {4}	No	A is selected next using MRV (domain size of 2). LCV selects 3 because it leaves D with a valid value.
4	D	4	-	No	D is selected and assigned 4 as the last remaining variable.
					Solution found: C = 1, B = 2, A = 3, D = 4.