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# Feed Forward Back Propagation, version 2: 2 inputs -> 2 hidden -> 1 output ,no biases)
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```
# by Andrew Taylor
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```
import numpy as np
```

```
def sigmoid(a):  
    return 1.0 / (1.0 + np.exp(-a))
```

```
def d_sigmoid_from_y(y):  
    return y * (1.0 - y) # derivative of sigmoid
```

```
class Network:
```

```
    def __init__(self, W_in_h, W_h_out, eta=0.1):  
        self.W_in_h = np.array(W_in_h, dtype=float) # (2,2)  
        self.W_h_out = np.array(W_h_out, dtype=float) # (2,)  
        self.eta = float(eta)
```

```
        self.x = None  
        self.Ah = None  
        self.h = None  
        self.Ay = None  
        self.y = None
```

```
    def forward(self, x):  
        self.x = np.array(x, dtype=float)  
        self.Ah = self.x @ self.W_in_h # (2,) = (2,)@(2,2)  
        self.h = sigmoid(self.Ah) # (2,)  
        self.Ay = float(self.h @ self.W_h_out) # scalar  
        self.y = sigmoid(self.Ay) # a scalar  
        return self.Ah, self.h, self.Ay, self.y
```

```
    def train_pass(self, x, d):  
        self.forward(x)
```

```
        e = d - self.y  
        dE_de = e  
        de_dy = -1.0  
        dy_dAy = d_sigmoid_from_y(self.y)  
        common_out = dE_de * de_dy * dy_dAy # shared chain-rule at output
```

```
        # grads wrt hidden->out weights  
        dE_dW_h_out = common_out * self.h # delA_y/delw_j = h_j
```

```
        # Hidden node gradients
```

```
        # For each hidden unit j:  
        # delE/delh_j = common_out * (delA_y/delh_j) * (delh_j/delA_hj) * (delA_hj/delw_ij)  
        # = common_out * W_h_out[j] * h_j*(1-h_j) * x_i  
        back_to_hidden = common_out * self.W_h_out * d_sigmoid_from_y(self.h) # shape (2,)  
        dE_dW_in_h = np.outer(self.x, back_to_hidden) # (2,2)
```

```
    print("\nForward")  
    print(f"x = {self.x}")  
    print(f"Hidden activations Ah = {self.Ah}")  
    print(f"Hidden activations h = {self.h}")  
    print(f"Output activity Ay = {self.Ay:.6f}")  
    print(f"Output activation y = {self.y:.6f}")  
    print("\noutput node grads")  
    print(f"[delE/del{dE_de:.6f}, delE/del{de_dy:.1f}, delE/del{dAy:.6f}]")  
    print(f"dE/dW_h_out (h1->y, h2->y) = {dE_dW_h_out}")  
    print("\nHidden node grads")  
    print(f"back_to_hidden (each hidden unit) = {back_to_hidden}")  
    print("dE/di in h matrix =")  
    print(dE_dW_in_h)
```

```
    # Gradient descent updates  
    self.W_h_out -= self.eta * dE_dW_h_out  
    self.W_in_h -= self.eta * dE_dW_in_h
```

```
    print("\nUpdated weights")  
    print(f"W_h_out = {self.W_h_out}")  
    print("W_in_h =")  
    print(self.W_in_h)
```

```
    def one_bp_update(self, x, d):  
        Ah, h, Ay, y = self.forward(x)
```

```
        # output grads  
        common_out = (d - y) * (-1.0) * (y * (1 - y))  
        dE_dW_h_out = common_out * h
```

```
        # hidden grads  
        back_to_hidden = common_out * self.W_h_out * (h * (1 - h))  
        dE_dW_in_h = np.outer(np.array(x, float), back_to_hidden)
```

```
        # updates  
        W_in_h_old = self.W_in_h.copy()  
        self.W_h_out -= self.eta * dE_dW_h_out  
        self.W_in_h -= self.eta * dE_dW_in_h  
        return (Ah, h, Ay, y, W_in_h_old)
```

```
# Run one pass with the values x1=1, x2=3 , desired output d=0.95 ; eta=0.1 , no biases
```

```
if __name__ == "__main__":
```

```
    x = np.array([1.0, 3.0], dtype=float)  
    d = 0.95  
    eta = 0.1
```

```
    # weights  
    W_in_h = [[0.8, 0.5],  
              [0.1, 0.2]]
```

```
    # hidden to output weights  
    W_h_out = [0.2, 0.7]
```

```
    net = Network(W_in_h, W_h_out, eta)  
    net.train_pass(x, d)
```

```
    # 1 "3) Initial activations  
    net = Network(W_in_h, W_h_out, eta)
```

```
    Ah, h, Ay, y = net.forward(x)  
    print("Answer to #1:", f"{h[0]:.4g}") # Node 1 activation  
    print("Answer to #2:", f"{h[1]:.4g}") # Node 2 activation  
    print("Answer to #3:", f"{y:.4g}") # output activation
```

```
    # 4 "5) After one backprop update toward d=0.95  
    net = Network(W_in_h, W_h_out, eta)
```

```
    Ah, h, Ay, y, W_in_h_old = net.one_bp_update(x, d)  
    print("Answer to #4 (w h1->y):", f"{net.W_h_out[0]:.4g}")  
    print("Answer to #5 (w h2->y):", f"{net.W_h_out[1]:.4g}")
```

```
    # delta for x1->h2  
    delta_w_x1_h2 = net.W_in_h[0,1] - W_in_h_old[0,1]
```

```
    print("Answer to #6 (delta w x1->h2):", f"{delta_w_x1_h2:.4g}")
```