Project Group 8 - Project 1 – Air Pollution: Particulate Matter

The focus of this report is to elucidate how fine, airborne, particulate matter is measured by industry and regulators to provide for cleaner air. This matter in the air is referred to as "aerosols". The modeling of such concentrations using functions and integrals will be presented, along with the numerical computation of integrals related to aerosol regulation. The report answers questions from an information sheet and the answers have been divided into respective groups (the 1's, 2's and 3's have been grouped together).

Aerosol particles are measured in a typical range of 10 angstroms to 100 microns. Due to the causal relationship of diameter to effects on health, diameter is used to group particles into different strata for measurement and analysis. The assumption is that they are spherical, and where they are not they are taken to be equivalent to a spherical particle of the same volume. So, for a cubic particle, 1.2 microns on each side, having a volume of (1.2)^3 µm = 1.728 microns, it is taken to have a "diameter" of a spherical particle of the same volume which would have a true diameter of 1.48884118 µm.

The measurements of aerosols are taken in intervals, denoted as *p*, and the number of particles at each interval follows from a function *N* so it is denoted as *N(p).* The derivative of this function is called the *size distribution function* $(\frac{dN}{dp})$, and an integral of the form:

$$\int_{p_1}^{p_2} \frac{dN}{dp}\, dp$$

gives the accumulated rate of change of the number of particles *N(p)* from p_1 to p_2.

The size distribution function is approximated as $\Delta N/\Delta p$ through measurements at distinct intervals. The table below shows such an observation at a time in Pasadena, California. The left column, Δp is the size of diameter measured and the approximation is shown next to it. The next column is Δx, which is merely the increment of diameter change from measurement to measurement. The last column is the product of this column and the approximation of the size distribution function. An integral of the form $\int_0^{5.22\ \mu m} \frac{\Delta N}{\Delta p}\, dp$ can be approximated from this data using the table below. The result is an accumulation of the total aerosol particles measured on that day. This is done with a Riemann sum approach. Graphically, we have a decreasing function and such a function would be most closely estimated by a Riemann sum using midpoints. However, in the interest of good safety and public policy, left endpoints, providing an overestimation should be used with this table to give a value for that sum as

p	ΔN/Δp	Δx	Trapezoid
0			0.00E+00
0.0088	1.57E+07	0.00875	1.37E+05
0.0125	5.78E+06	0.00375	2.17E+04
0.0175	2.58E+06	0.005	1.29E+04
0.025	1.15E+06	0.0075	8.63E+03
0.035	6.01E+05	0.01	6.01E+03
0.05	2.87E+05	0.015	4.31E+03
0.07	1.39E+05	0.02	2.78E+03
0.09	8.90E+04	0.02	1.78E+03
0.112	7.02E+04	0.022	1.54E+03
0.137	4.03E+04	0.025	1.01E+03
0.175	2.57E+04	0.038	9.77E+02
0.25	9.61E+03	0.075	7.21E+02
0.35	2.15E+03	0.1	2.15E+02
0.44	9.33E+02	0.09	8.40E+01
0.55	2.66E+02	0.11	2.93E+01
0.66	1.08E+02	0.11	1.19E+01
0.77	5.17E+01	0.11	5.69E+00
0.88	2.80E+01	0.11	3.08E+00
1.05	1.36E+01	0.17	2.31E+00
1.27	5.82E+00	0.22	1.28E+00
1.48	2.88E+00	0.21	6.05E-01
1.82	1.25E+00	0.34	4.25E-01
2.22	4.80E-01	0.4	1.92E-01
2.75	2.17E-01	0.53	1.15E-01
3.3	1.18E-01	0.55	6.49E-02
4.12	6.27E-02	0.82	5.14E-02
5.22	3.03E-02	1.1	
Total			**2.0005317252E+05**

$2.0005317252 \times 10^{5}$ when carried out ten decimal places.

The mass of a pollutant is also of great interest to scientists looking to protect our air. Density is mass over volume. The total mass of particulates can be found by multiplying the mass density of the particle by the number of particles. This is clearly of interest because health is directly affected depending on how much mass is inhaled into the lungs. This is a hot-button issue because of microplastics found in the lungs during recent studies. Microplastics were detected in human blood for the first time in March of this year.[1]

The typical density of a particle is 1.5 g/cm^3. It follows an expression for the mass of a single particle of diameter p in μm would be:

$$Mass = \frac{\pi p^3}{6} * 1.5\, g/cm^3$$

Where one cm^{3} equals $1 \times 10^{12} \mu m$.

When the EPA sets a standard for fine particulate matter emitted into the atmosphere, the Agency requires filtering of any particles with a diameter greater than or equal to p. The EPA's current standard for diameter p is $10\mu m$. This means that all particles with a diameter $p \geq 10\mu m$ will be filtered. The maximum diameter p of a particle is understood to be $p \leq 100\ \mu m$. Therefore, the measure of the number of particles that would be filtered from air with the current EPA standard can be expressed by the integral:

$$\int_{10\mu m}^{100\ \mu m} \frac{dN}{dp} dp$$

To determine the amount of particles per cubic centimeter between $.05\mu m$ and $.55\mu m$, the definite integral must be estimated via a Reimann sum using the empirical values of p and $\frac{\Delta N}{\Delta p}$ provided in the table below. The values provided for $\frac{\Delta N}{\Delta p}$ are approximately equal to $\frac{dN}{dp}$ for each value of p. Per the data in the provided table, this function is decreasing between $.05\mu m$ and $.55\mu m$.

p	ΔN/Δp	Δx	Total
0.050	2.87E+05	0.02	
0.070	1.39E+05	0.02	
0.090	8.90E+04	0.022	
0.112	7.02E+04	0.025	
0.137	4.03E+04	0.038	
0.175	2.57E+04	0.075	
0.250	9.61E+03	0.1	
0.350	2.15E+03	0.09	
0.440	9.33E+02	0.11	
0.550	2.66E+02	0.11	
Total			**1.69490300000E+04**

The most accurate Reimann sum of rectangles for this dataset would be estimated using midpoints for the rectangles. That said, in the interest of public safety and improvement, left endpoints were used to generate an overestimate of the actual amount of particles within this cubic centimeter. The result of the left endpoint Reimann sum show that there are an estimated 16,949 particles per cubic centimeter between $.05\mu m$ and $.55\mu m$.

Above, we determined that the mass of a single particle can be measured using the following function:

$$Mass = \frac{\pi p^3}{6} * 1.5\ g/cm^3$$

This function can be expressed as a definite integral over the interval $[0\mu, 5.22\mu]$ to represent the total mass of particles per cubic centimeter, where one cm^3 is equivalent to $1x10^{12}\ \mu m$. The integral representation of the function over this interval can be seen below.

$$Total\ Mass\ = \int_{0\mu}^{5.22\mu} \frac{\pi p^3}{6} (1.5\ x\ 10^{12}) dp$$

This integral was estimated using a Reimann sum with left endpoints based upon the data provided in the table. This Reimann sum is an underestimate for the amount of mass measured by the integral, as the function is increasing. This calculation may differ in actuality, as one of the most commonly used diameters is that mass median diameter (MMD). The MMD defines the particle diameter as half of the aerosol mass in small particles, and half the aerosol mass in larger particles. If the MMD is used, the result would be half of the total actual mass[2]. The Reimann sum estimate is 100.58 grams, to two significant figures. . The actual amount of mass is 145.78 grams, to two significant figures.

In the Spring of 1997 the EPA lowered the regulatory standard from 10 μm to 2.5 μm. The number of additional particles that would be removed if the standard were changed is given by the integral:

$$\int_{2.5\mu m}^{10\mu m} \frac{dN}{dp} dp$$

Under the proposed 2.5 μm standard, all particulate matter of diameter 2.5 μm and above should be filtered out of the atmosphere. The percentage of particles removed under this new standard is

$$\frac{\int_{2.5}^{5.22} \frac{\Delta N}{\Delta p} dp}{\int_{0.00875}^{5.22} \frac{\Delta N}{\Delta p} dp} \times 100 = \frac{0.4020669999999999}{103696.727867} \times 100 = 0.0003877335459569038\%$$

The standard required if we want to reduce the number of particulates by 10% can be found by solving the equation below:

$$\frac{\int_{x}^{5.22} \frac{\Delta N}{\Delta p} dp}{\int_{0.00875}^{5.22} \frac{\Delta N}{\Delta p} dp} \times 100 = 10$$

$$\frac{\int_{x}^{5.22} \frac{\Delta N}{\Delta p} dp}{103696.727867} = 0.1$$

$$\int_{x}^{5.22} \frac{\Delta N}{\Delta p} dp = 10369.6727867$$

Here, we would need to set a standard of 0.07 μm to remove 10% of the particulates.

The integral that represents the additional mass of particulate matter that would be removed from the air if the standard were changed from 10 µm to 2.5 µm can be written as the equation shown below,

$$\int_{2.5}^{\infty} \frac{1}{4}\pi p^3 * 10^{-12} \times \frac{\Delta N}{\Delta p} dp - \int_{10}^{\infty} \frac{1}{4}\pi p^3 * 10^{-12} \times \frac{\Delta N}{\Delta p} dp$$

If we wanted to reduce the mass of the particulates by 10%, we only need to set the standard to be at $p = 3.3$ µm.

References

1) Microplastics found deep in the lungs of living people for the first time, The Guardian.
https://www.theguardian.com/environment/2022/apr/06/microplastics-found-deep-in-lungs-of-living-people-for-first-time

2) Particle Size Distributions, Journal of Aerosol Medicine and Pulmonary Drug Delivery.
https://www.liebertpub.com/doi/10.1089/jamp.2020.29028.whf