Linear Algebra
Module 13 Assignment

Due Sunday at 11:59 PM

1 Directions

Complete the following problems showing all your work. You may use a calculator to check your work, but should write out (or TeX up) all the steps of your solution. Unless otherwise specified, you may skip some steps of row reduction with a calculator, but state that you did so. Please upload your work as a single .pdf file in the course.

2 Problems

1. (pg 401, #14) Orthogonally diagonalize $A = \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}$ giving an orthogonal matrix P and a diagonal matrix D. Show all work.
2. Answer the following :
   (a) Show that if A is a symmetric matrix, then so is A^2.
   (b) Show that if A is orthogonally diagonalizable, then so is A^2
3. Answer the following:
   (a) Let $Q(\vec{x})$ be the quadratic form with associated matrix $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$. Compute $Q\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$.
   (b) Let $Q(\vec{x})$ be the quadratic form with associated matrix $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Compute $Q\left(\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\right)$.
   (c) Find the matrix A of the quadratic form $Q(\vec{x}) = 6x_1^2 + 3x_2^2 + 4x_3^2 + 8x_1x_2 - 2x_2x_3 + 12x_1x_3$.
   (d) Let $Q(\vec{x}) = \vec{x}^T \begin{bmatrix} 19 & 2 \\ 2 & 16 \end{bmatrix} \vec{x}$ and $R(\vec{x}) = \vec{x}^T \begin{bmatrix} -34 & 12 \\ 12 & -41 \end{bmatrix} \vec{x}$. Are Q and R positive definite, negative definite, or indefinite?
4. Answer the following:
   (a) Find the singular values of $A = \begin{bmatrix} \sqrt{5} & 1 \\ 0 & \sqrt{5} \end{bmatrix}$.
   (b) Find a singular value decomposition (SVD) of A $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$.