Time Allowed: 120 minutes
Total Weight: 80 points

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Multivariable and Complex Analysis Quiz II

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (20) Show that 7 is an eigenvalue of A, and find the corresponding eigenvectors.

$$\det(A-\lambda I)=0:$$

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 6 \\ 5 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)-30 \rightarrow \lambda^2-3\lambda+\overset{-30}{2}=0 = \lambda^2-3\lambda-28=0$$

$$\lambda = \frac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot(-28)}}{2} = \frac{3\pm\sqrt{121}}{2} = \frac{3\pm11}{2} = 7,-4$$

$$\begin{array}{l}(1-7)x_1+6x_2=0 \\ 5x_1+(2-7)x_2=0\end{array} \Rightarrow \begin{array}{l}-6x_1+6x_2=0 \\ 5x_1-5x_2=0\end{array} \Rightarrow x_1=x_2,\ \text{pick } x_1=1$$

$$\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix} \text{ for } 7=\lambda_1.$$

2. (20) Is matrix B diagonalizable? *To receive full credit, you must give a compelling reason as to why or why not.*

$$\lambda_1=0,\ \lambda_2=-2,\ \lambda_3=5 \qquad B = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix} \qquad D=X^{-1}AX \quad (1)$$

eigenvectors: $\begin{pmatrix} 8 \\ 5 \\ 0 \end{pmatrix}, \begin{pmatrix} -58 \\ -49 \\ 14 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \leftarrow$ where X is the matrix with the eigenvectors as column vectors.

Rearranging equation (1) we find that A is diagonizable if $A=XDX^{-1}$

Let $D_B \cong$ the diagonalization of $B = \begin{bmatrix} 0&0&0 \\ 0&-2&0 \\ 0&0&5 \end{bmatrix}$, then we can see:

$$\underset{B}{\begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}} = \underset{X}{\begin{bmatrix} 8 & -58 & 1 \\ 5 & -49 & 0 \\ 0 & 14 & 0 \end{bmatrix}} \underset{D_B}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}} \underset{X^{-1}}{\begin{bmatrix} 0 & 1/5 & 7/10 \\ 0 & 0 & 1/14 \\ 1 & -8/5 & -\frac{51}{35} \end{bmatrix}}$$

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3. (20) Find the characteristic equation for matrix C and determine how many distinct eigenvalues matrix C possess.

$$C = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det(C-\lambda I)=0$

the determinant of the matrix equals the diagonal product.

$$(C-\lambda I)=\begin{bmatrix} 5-\lambda & -2 & 6 & -1 \\ 0 & 3-\lambda & -8 & 0 \\ 0 & 0 & 5-\lambda & 4 \\ 0 & 0 & 0 & 1-\lambda \end{bmatrix},\quad \det(C-\lambda I)=(5-\lambda)(3-\lambda)(5-\lambda)(1-\lambda) \quad ①$$

$=\lambda^4-14\lambda^3+68\lambda^2-130\lambda+75$ this is the characteristic equation.

If we set ① equal to zero we get three distinct roots $\lambda_1=1, \lambda_2=3, \lambda_3=5$

4. (20) Which term(s) best describe matrix D; Hermitian, Skew-Hermitian, or Unitary? Find its eigenvalues and eigenvectors.

$$D = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

D is skew-Hermitian because $\bar{D}^T=-D$.

$\det\left(\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}-\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right): \lambda^2+1$

for $\lambda^2+1=0: \lambda^2=-1, \pm\lambda=\sqrt{-1}=i,-i$

for i: $(0-i)x_1+ix_2=0$, $ix_2=ix_1$, $x_2=x_1$, pick $x_1=1$

$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\begin{bmatrix} i \\ i \end{bmatrix}$

for $-i$: $(0+i)x_1+ix_2=0$, $ix_2=-ix_1$, $x_2=-x_1$, pick $x_1=1$

$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}=\begin{bmatrix} -i \\ i \end{bmatrix}$ are the eigenvectors.