

Now, turning to the advanced economies of the world, we see a far different picture. First let's reiterate, the data used for this comparison was drawn from Lee and Lee, "Regional Female to Male Years Schooling" (2016), Human capital in the long run. Journal of Development Economics.", where the "gender ratio" was defined as the regional average of total years of schooling for females (15-64 years of age) divided by the corresponding regional average of total years of schooling for males (15-64 years of age). Individual data points therefore represent a ratio of "female schooling to male schooling" where 1 represents educational equality, or when converted to percent, that is 100%. In the data these ratios have been converted to a percentage. Gender rights, economic and educational opportunities, and social acceptance all had to evolve over time from ancient days where women were not allowed to be educated in society. Let's turn to some analysis of the data. There are also charts in our Excel file.

Fitting a polynomial of degree 2 to the data is a great fit initially and yields  $f(x) = -0.00157x^2 + 0.394119x + 73.07723$  with an R squared of 0.9395. The tangent line is  $g(x) = 0.1426x + 79.779$ . Gender equality in education from 1900 to 1950 exploded, with an average rate of change of 22.6%. In the years following World War II, and the subsequent period to 2010, that rate of change slowed dramatically to 9.7%, but began to approach parity more closely. This means there were great strides in the former period, but comparatively slower growth in the latter period. In particular it took many years to reclaim the highs set in the pre-war period. I believe it's because educated males had an exception to the draft, while women did not have an exception to wartime support roles, away from school.

The instantaneous rate of change at 1990 shows a window into best approximation, for the mean of the average rate of change between 1985 and 1990, and 1990 and 1995 is 5.9%, and the same rate of change calculated as a derivative is only 0.9%. This is due to the drastic difference between the slope that results from the average of the slopes of two secants, and the slope of a true tangent line. The better representation of the instantaneous rate of change is the slope of the tangent to the curve of best fit, because it takes into account only the change at that time, while the mean of the slope of two secants is better understood over the whole period used in the calculation. Historically, we also see some stalling out of the feminist movement of the 1970's and '80's by 1990, so the instantaneous rate of change better reflects that shift.

In predicting future values, the curve of best fit calculated in Excel begins to deteriorate. For 2020, the prediction is a gender ratio of 96.9. The function  $f(x)$  is not a good approximation of the ratio in 2020, because trend of the ratio during the World War II years is not smooth. This drop makes  $f(x)$  a downward facing parabola and throws off prediction. Eyeballing the 2020 value from the tangent line

$g(x)$  a value of 101.3 is closer to the mark. This is a better approximation as the upward trend resumed and made new highs around 1990. Anecdotal evidence is that there are already more females than men in college. A quadratic polynomial may have been a good choice, but the degree could have been larger to reflect the decreasing period in the middle of the data set. As it is, the sign of the second derivative is negative, indicating an accelerating deterioration, while recent statistics from 2021 show women outnumbering men in college, again pointing to the inadequacy of  $f(x)$  as the model for the data.