factorAnalytics Implementation of New Ding-Martin Factor Model Method

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Abstract

We describe the `factorAnalytics` package implementation of the Ding and Martin (2017) new factor model for computing active portfolios with maximum information ratio (IR), where the latter is given by their new fundamental law of active management (FLAM) formula. We do so for the special case of single factor models using 22 DJIA stocks monthly returns from May 2004 to March 2013 and each of the following four factors: enterprise value (ENTVAL), price-to-book (P2B), enterprise-value-to-sales (EV2S), and firm size (SIZE), where the latter is the natural log of market capitalization in millions of dollars.

1 The Analytic Foundation

Here we provide the basic mathematics of Ding and Martin (2017), DM for short, for the special case of a single factor model.

1.1 Residual Returns

The following single index model relative to the benchmark is a standard tool in active portfolios management:

$$r_{it}^{Total} = \beta_i r_{B,t} + r_{it}; \qquad i = 1, 2, \dots, N, \;\; t = 1, 2, \dots, T,$$

where

$$r_{it}^{Total} = \text{Total return of security } i \text{ at time } t \text{ in excess of the risk-free rate}$$
$$\beta_i = \text{Beta of security } i \text{ with respect to the benchmark}$$
$$r_{B,t} = \text{Benchmark return at time } t \text{ in excess of the risk-free rate}$$
$$r_{it} = \text{Residual return of security } i \text{ at time } t$$

and the residual returns are uncorrelated with the benchmark returns. We will use S&P500 as our benchmark.

1.2 The Single Factor Model

For a single factor model equation:

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{r_{it}}} = f_t z_{i,\, t-1} + \epsilon_{it}; \qquad i = 1, 2, \ldots,\ N,\ \ t = 1, 2, \ldots,\ T,$$

where

$$r_{it} = \text{Residual return of security } i \text{ at time } t$$
$$\sigma_{r_{it}} = \text{Conditional volatility for security } i \text{ based on lagged returns } r_{iu};\ u \leq t-1$$
$$f_t = \text{Factor return of the factor at time } t$$
$$z_{i,\, t-1} = \text{Standardized factor exposure of security } i \text{ at time } t-1$$
$$\epsilon_{it} = \text{Error term of security } i \text{ at time } t$$

Note that the standardization of residual returns makes them have zero mean and unit variance, and the standardized exposures $z_{i,\, t-1}$ are assumed to have zero mean and unit variance.

Motivated by the common practice of estimating the factor returns by a cross-sectional regression, we can define the random factor returns in our model as

$$f_t \equiv \frac{cov(\tilde{r}_{it},\ z_{i,\, t-1})}{var(z_{i,\, t-1})} = corr(\tilde{r}_{it},\ z_{i,\, t-1}).$$

Since both the risk-adjusted returns and the factor exposure have unit variance and out factor return is the correlation between the exposures and the standardized residuals, and as such is commonly called the **information coefficient**:

$$IC_t \equiv corr(\tilde{r}_{it},\ z_{i,\,t-1}) = f_t.$$

Then the stationary time series IC_t has unconditional mean value μ_{IC} and variance σ^2_{IC}, and volatility σ_{IC}.

1.3 Conditional Mean Alpha Predictor and Covariance Matrix

In order to derive the formula for the fundamental law for the single factor model we need a vector version of the alpha conditional forecast model and the matrix version of the conditional covariance matrix. Under our factor model assumptions that the conditional mean predictor of r_{it} conditioned on known $z_{i,t-1}$ is:

$$\alpha_{it} = \mu_{IC} \cdot \sigma_{r_{it}} \cdot z_{i,t-1}$$

with vector form

$$\boldsymbol{\alpha}_t = \mu_{IC} \cdot \boldsymbol{\Lambda}_t^{1/2} \cdot \mathbf{z}_{t-1}$$

where $\boldsymbol{\Lambda}_t$ is diagonal with elements $\sigma^2_{r_{it}}$. The conditional covariance matrix for this predictor is

$$\boldsymbol{\Omega}_t = \boldsymbol{\Lambda}_t^{1/2}\left(\sigma^2_{IC}\mathbf{z}_{t-1}\mathbf{z}'_{t-1} + \sigma^2_{\epsilon}\mathbf{I}\right)\boldsymbol{\Lambda}_t^{1/2} \tag{1}$$

where $\sigma^2_{\epsilon} = 1 - (\mu^2_{IC} + \sigma^2_{IC})$.

1.4 Optimal Active Portfolio Weights and IR

We define the active weights in the usual manner as benchmark relative portfolio weights:

$$\Delta w_{it} = w_{P,\,it} - w_{B,\,it}.$$

Then it follows that

$$r_{A,\,t} = \sum_{i=1}^{N} \Delta w_{it} r_{it} = \Delta \mathbf{w}_t' \mathbf{r}_t. \tag{2}$$

It follows that the conditional mean and conditional variance of the active return $r_{A,\,t}$ are given by:

$$\mathrm{E}\left(r_{A,\,t} \middle|\ I_{t-1}\right) = \mathrm{E}\left(\Delta \mathbf{w}_t' \mathbf{r}_t \middle|\ I_{t-1}\right) = \Delta \mathbf{w}_t' \boldsymbol{\alpha}_t,$$

and

$$\mathrm{V}\left(r_{A,\,t} \middle|\ I_{t-1}\right) = \mathrm{V}\left(\Delta \mathbf{w}_t' \mathbf{r}_t \middle|\ I_{t-1}\right) = \Delta \mathbf{w}_t' \boldsymbol{\Omega}_t \Delta \mathbf{w}_t$$

Using $\boldsymbol{\alpha}_t$ and $\boldsymbol{\Omega}_t$ above and recalling that the active weights are dollar neutral, we will have the closed form for the active weights as a solution for the quadratic utility active portfolio optimization problem:

$$\Delta \mathbf{w}_t = \sigma_{A,t} \frac{\boldsymbol{\Omega}_t^{-1}(\boldsymbol{\alpha}_t - \kappa \mathbf{1})}{\sqrt{\boldsymbol{\alpha}_t'(\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t - \kappa \mathbf{1})}} \tag{3}$$

where

$$\kappa = \frac{\boldsymbol{\alpha}_t' \boldsymbol{\Omega}_t \mathbf{1}}{\mathbf{1}' \boldsymbol{\Omega}_t \mathbf{1}}.$$

The portfolio manager typical specifies the value of the tracking error $\sigma_{A,t}$ (we use 6% in our later example).

The portfolio active return conditional mean is

$$\alpha_{A,t} = \Delta \mathbf{w}_t' \boldsymbol{\alpha}_t$$

and the corresponding conditional information ratio is

$$IR_t = \frac{\alpha_{A,t}}{\sigma_{A,t}}.$$

1.5 The New Fundamental Law Ex Ante Formula

The next version of this vignette will include material on the new ex ante fundamental law of active management (FLAM) introduced by Ding and Martin (2017), including its use in determining what IR to expect using their model.

2 The New Model Computational Method

The steps of the computations are as follows for each stock (where i denotes the i-th stock) on each 60 month "training" window, which moves in increments of one month over the interval May 2004 to March 2013.

2.1 Computing Standardized Residual Returns and Exposures

Residual Returns

$$r_{it} = r_{it}^{Total} - \hat{\beta}_i \cdot r_{B,t}$$

where

$$\hat{\beta}_i = \frac{\widehat{cov}(r_{it}^{Total}, r_{B,t})}{\widehat{var}(r_{B,t})}$$

as estimated using ordinary least squares (LS). Alternatively, robust regression may be used here, and is usually advisable.

Standardized Residual Returns

First compute the GARCH(1,1) variances of the residual returns r_{it} using the following formula

$$\sigma_{r_{it}}^2 = (1 - \alpha - \beta)\hat{\sigma}_i^2 + \alpha\, r_{i,t-1}^2 + \beta\, \sigma_{r_{i,t-1}}^2$$

where$\alpha = 0.10$, $\beta = 0.81$, and $\hat{\sigma}_i^2$ is the asset i sample variance on each training window. The initial values of the recursion are $r_{i,0} = 0$ and $\sigma_{r_{i,0}}^2 = 0$. Then compute the standardized residual returns as:

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{r_{it}}}.$$

Standardized Exposures

For each stock exposure, compute the EWMA variances $s_{x_{it}}^2$ of the raw exposures x_{it} as

$$s^2_{x_{i,t}} = 0.1(x_{it} - \bar{x}_i)^2 + 0.9 s^2_{x_{i,t-1}}$$

where $s^2_{x_{i0}}$ is the sample variance of the asset i exposures on each training window. The term $\bar{x}_i$ is the mean of asset i through the whole time period.

Then compute the standardized exposures:

$$z_{it} = \frac{x_{it} - \bar{x}_t}{s_{x_{it}}}.$$

2.2 Estimates of Conditional Mean Alpha and Error Covariance Matrix

Use `fitFfm` to fit a single factor model with standardized residual returns $\tilde{r}_{it}$ and standardized exposures $z_{i,t-1}$ on a moving window of length 60 months, with first month on May 2004 and last month on March 2013. For any such window, `fitFfm` will returns 60 months of factor returns that, due to the residuals returns and exposures standardizations, are equal to the information coefficient time series:

$$f_t = IC_t.$$

For each 60 month window position `fitFfm` will also return an $N \times N$ diagonal matrix of error-variance estimates

$$\hat{\mathbf{\Sigma}}_{\epsilon,t} = diag\left(\hat{\sigma}^2_{\epsilon_{1,t}}, \hat{\sigma}^2_{\epsilon_{2,t}}, \cdots, \hat{\sigma}^2_{\epsilon_{N,t}}\right).$$

where the $\hat{\sigma}^2_{\epsilon_{i,t}}$, $i = 1, \cdots, N$ could be EWMA or GARCH estimates.

From the 60 months of IC_t we compute the sample mean and sample variance estimates

$$\hat{\mu}_{IC} \qquad \text{and} \qquad \hat{\sigma}^2_{IC}$$

Using $\hat{\mu}_{IC}$ you compute the following estimates of the conditional mean alpha vector predictions:

$$\hat{\boldsymbol{\alpha}}_{t_{end}} = \hat{\mu}_{IC} \cdot \boldsymbol{\Lambda}_{t_{end}}^{1/2} \cdot \mathbf{z}_{t_{end}-1}$$

where where $\boldsymbol{\Lambda}^{\mathbf{1/2}}{}_t$ is diagonal with elements $\sigma_{r_{it}}$, and where t_{end} is the end of each moving window of length 60, e.g., for the first window position $t_{end} = T$ and for the next moving window position $t_{end} = T + 1$, etc.

Using $\hat{\sigma}_{IC}^2$ and $\hat{\boldsymbol{\Sigma}}_{\epsilon,t}$ the standard way to estimate the conditional forecast error covariance matrices is with

$$\hat{\boldsymbol{\Omega}}_{t_{end}} = \boldsymbol{\Lambda}_{t_{end}}^{1/2} \left(\hat{\sigma}_{IC}^2 \cdot \mathbf{z}_{t_{end}-1} \mathbf{z}'_{t_{end}-1} + \hat{\boldsymbol{\Sigma}}_{\epsilon,t_{end}} \right) \boldsymbol{\Lambda}_{t_{end}}^{1/2}.$$

Alternatively, the results in Ding and Martin (2017) imply that one should use the conditional error covariance matrix estimates

$$\hat{\boldsymbol{\Omega}}_{DM,t_{end}} = \boldsymbol{\Lambda}_{t_{end}}^{1/2} \left(\sigma_{IC}^2 \cdot \mathbf{z}_{t_{end}-1} \mathbf{z}'_{t_{end}-1} + \hat{\sigma}_{\epsilon}^2 \mathbf{I} \right) \boldsymbol{\Lambda}_{t_{end}}^{1/2}$$

where

$$\hat{\sigma}_{\epsilon}^2 = 1 - (\hat{\mu}_{IC}^2 + \hat{\sigma}_{IC}^2).$$

Our `fitFfm` implementation provides both of the above conditional covariance matrix estimator options.

2.3 Computing Optimal Active Portfolio Quantities

The ex ante conditional mean active portfolio weights, return and information ratio at the end of each period of the moving window are computed by plugging the estimates of Section 2.2 into the formulas of Section 1.4.

3 Industry Standard Model (ISM)

The industry standard model (ISM) is

$$r_{it} = f_t \tilde{z}_{i,\, t-1} + \epsilon_{it}; \qquad i = 1, 2, \ldots,\ N, \ \ t = 1, 2, \ldots,\ T,$$

where r_{it} are the raw residuals and $\tilde{x}_{i,t-1}$ are the cross-section standardized exposures.

Use of `fitFfm` on a moving window of length 60 months results in:

- The factor returns f_t at time t over the 60 month interval
- The residual returns covariance matrix at time t_{end}:

$$\widehat{\mathbf{\Omega}}_{t_{end}} = \hat{\sigma}_f^2 \cdot \tilde{\mathbf{x}}_{t_{end}-1} \tilde{\mathbf{x}}'_{t_{end}-1} + \widehat{\mathbf{\Sigma}}_{\epsilon, t_{end}}$$

where $\hat{\sigma}_f^2$ is the sample variance estimate of the variance of f_t base on a 60 month interval, and $\widehat{\mathbf{\Sigma}}_{\epsilon,t}$ is as given above

Compute the conditional expected value predictor of raw residual returns r_{it} as the mean of r_{it}:

$$\boldsymbol{\alpha}_t^k = \frac{1}{T} \sum_{t=1}^{T} \mathbf{r}_t$$

3. Compute the forecast error covariance matrix:

$$\mathbf{\Omega}_t^k = \sigma_k^2 \cdot \tilde{\mathbf{x}}_{t-1} \tilde{\mathbf{x}}'_{t-1} + \mathbf{\Sigma}_\epsilon^k$$

Remark: $\boldsymbol{\alpha}_t^k$ is equal while $\mathbf{\Omega}_t^k$ is different across the four exposures.

4 Out-of-sample Information Ratio

1. Compute the ex-ante active returns

$$\mathbf{r}_t^{Out} = \Delta w_{t-1} \cdot \mathbf{r}_t$$

2. Compute the out-of-sample information ratio

$$IR_{Out} = \frac{\mu_{\mathbf{r}_t^{Out}}}{\sigma_{\mathbf{r}_t^{Out}}} \tag{4}$$

3. Compute the finite sample standard error

$$\widehat{S.E.}_{Out}^{finite} = \frac{1}{\sqrt{N_{\mathbf{r}_t^{Out}}}} \sqrt{1 - \widehat{sk} \cdot IR_{Out} + \frac{\hat{\kappa}+2}{4} \cdot IR_{Out}^2}, \tag{5}$$

where $\widehat{sk}$ and $\hat{\kappa}$ are the sample skewness and excess kurtosis of $\mathbf{r}_t^{Out}$, respectively.

4.1 Bootstrap Information Ratio and Standard errors

A single bootstrap sample is obtained by sampling the original set of active returns $\mathbf{r} = (r_1, \ldots,\ r_n)$ n times with replacement and call the result $\mathbf{r}^* = (r_1^*, \ldots,\ r_n^*)$. Generate B independent bootstrap samples: $\mathbf{r}_1^*, \ldots,\ \mathbf{r}_B^*$. The bootstrap estimate of the unknown standard error is computed as the sample standard deviation of the B bootstrap estimates:

$$\widehat{S.E.}_{IR}(B) = \frac{1}{B-1} \sum_{b=1}^{B} \left(IR(\mathbf{r}_b^*) - \overline{IR}^* \right) \tag{6}$$

where

$$\overline{IR}^* = \frac{1}{B} \sum_{b=1}^{B} IR(\mathbf{r}_b^*).$$

5 Use of the new functionalities

In this work, we will use the DJIA data set from `factorAnalytics` ranging from May 2004 to March 2013. There are four factors in consideration: ENTVAL, P2B, EV2S, and SIZE. The function `fitFfm` has the following argument:

```r
args(fitFfm)
```

```
## function (data, asset.var, ret.var, date.var, exposure.vars, 
##     weight.var = NULL, fit.method = c("LS", "WLS", "Rob", "W-Rob"), 
##     rob.stats = FALSE, full.resid.cov = FALSE, z.score = c("none", 
##         "crossSection", "timeSeries"), addIntercept = FALSE, 
##     lagExposures = FALSE, resid.EWMA = FALSE, lambda = 0.9, stdReturn = FALSE, 
##     fullPeriod = FALSE, windowLength = 60, analysis = c("none", 
##         "ISM", "NEW"), targetedVol = 0.06, ...) 
## NULL
```

Note that we have a slightly different set of arguments compared to the original **`fitFfm`**. Namely

- stdReturn: logical argument representing if we want to standardize the return variable
- z.score: the standardization method used in the factor exposures. It can be one of "none", "crossSection", or "timeSeries".
- fullPeriod: logical argument representing if we want to train all but the last time period
- windowLength: numerical value representing the length of the moving window
- analysis: the analysis of the conditional mean forecast and its covariance. It can be one of "none", "ISM", or "NEW"
- targetedVol: the target volatility of the portfolio

We show an example of how to use **`fitFfm`** as follows:

```r
p2bFfmNEW <- fitFfm(modData, asset.var = "TICKER", ret.var = "RESIDRETURN",
    exposure.vars = c("P2B"), addIntercept = TRUE, stdReturn = TRUE, z.score = "timeSeri
    date.var = "DATE", lagExposures = TRUE, analysis = "NEW")

sizeFfmNEW <- fitFfm(modData, asset.var = "TICKER", ret.var = "RESIDRETURN",
    exposure.vars = c("SIZE"), addIntercept = TRUE, stdReturn = TRUE, z.score = "timeSer
    date.var = "DATE", lagExposures = TRUE, analysis = "NEW")
```

```
ev2sFfmNEW <- fitFfm(modData, asset.var = "TICKER", ret.var = "RESIDRETURN",
    exposure.vars = c("EV2S"), addIntercept = TRUE, stdReturn = TRUE, z.score = "timeSer
    date.var = "DATE", lagExposures = TRUE, analysis = "NEW")

entvalFfmNEW <- fitFfm(modData, asset.var = "TICKER", ret.var = "RESIDRETURN",
    exposure.vars = c("ENTVAL"), addIntercept = TRUE, stdReturn = TRUE, z.score = "timeS
    date.var = "DATE", lagExposures = TRUE, analysis = "NEW")
```

We employ Equation (4) for the out-of-sample IRs. The following table shows the results we have from the ISM and our NEW model.

	P2B	SIZE	EV2S	ENTVAL
In-Sample IR (ISM)	0.435 (0.0573)	0.458 (0.0540)	0.461 (0.0555)	0.453 (0.0569)
In-Sample IR (NEW)	0.734 (0.0652)	0.255 (0.0826)	0.069 (0.0652)	0.117 (0.0810)
Out-of-Sample IR (ISM)	-0.203	-0.315	-0.027	-0.195
Out-of-Sample IR (NEW)	0.521	0.368	0.034	0.0028

Table 1: In-sample and out-of-sample IR between ISM and NEW model with the standard deviation in parentheses

From Table (1), we can see that the industry standard model has a better performance in all factors except the P2B. However, it is clear that our NEW model has a better performance for the out-of-sample IRs.

We can also look at the standard errors with the direct standard error and finite sample formula for standard error from Equation (5)

	P2B	SIZE	EV2S	ENTVAL
SE (ISM)	0.0573	0.0540	0.0555	0.0569
SE (NEW)	0.0652	0.0826	0.0652	0.0810
SE formula (ISM)	0.141	0.142	0.145	0.144
SE formula (NEW)	0.148	0.155	0.144	0.146

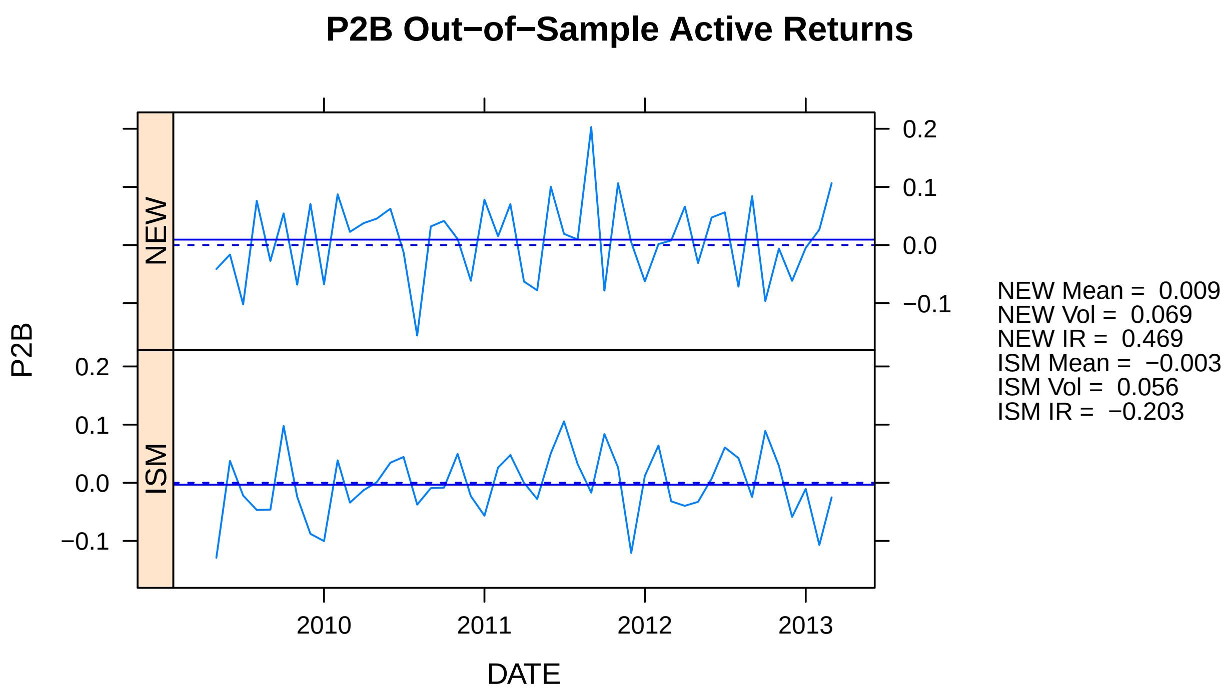
Table 2: Standard Errors of the out-of-sample IR and finite sample formula

We can observe that our NEW model's standard errors are larger than those of the industry standard model. Finally, we bootstrap the out-of-sample IRs and obtain the standard errors by Equation (6) with $B = 100$:

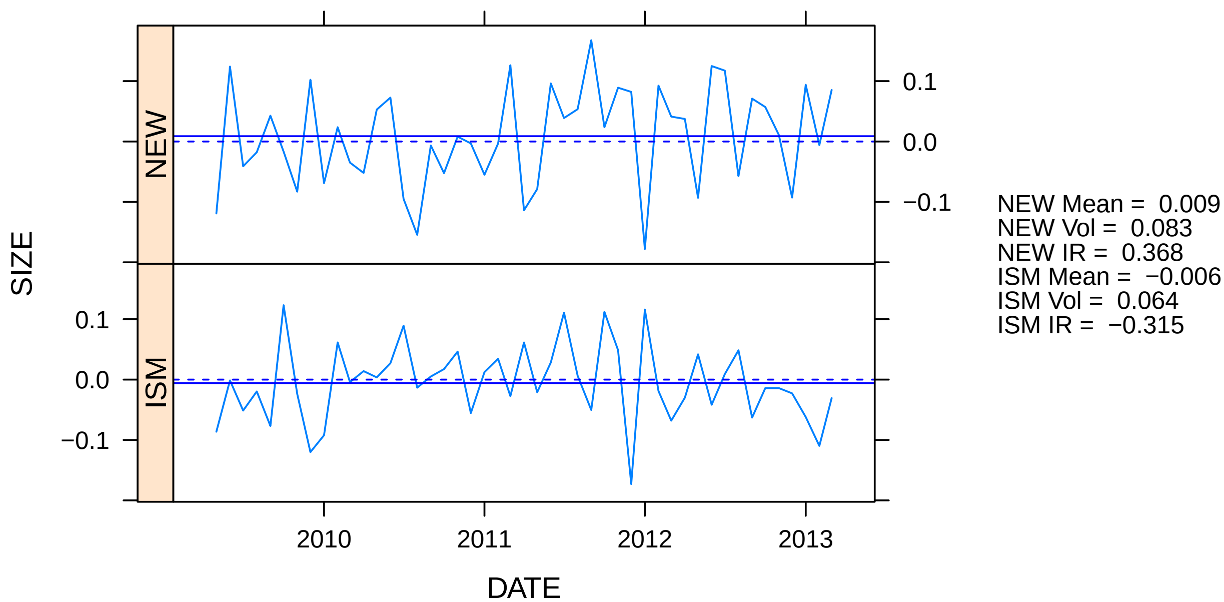
	P2B	SIZE	EV2S	ENTVAL
$\overline{IR}^*$(ISM)	-0.433	-0.280	-0.159	-0.0826
$\overline{IR}^*$(NEW)	0.427	0.382	-0.0085	-0.0904
$\widehat{S.E.}_{IR}$ (ISM)	0.142	0.135	0.140	0.173
$\widehat{S.E.}_{IR}$ (NEW)	0.158	0.150	0.148	0.143

Table 3: Bootstrap out-of-sample IRs and standard errors

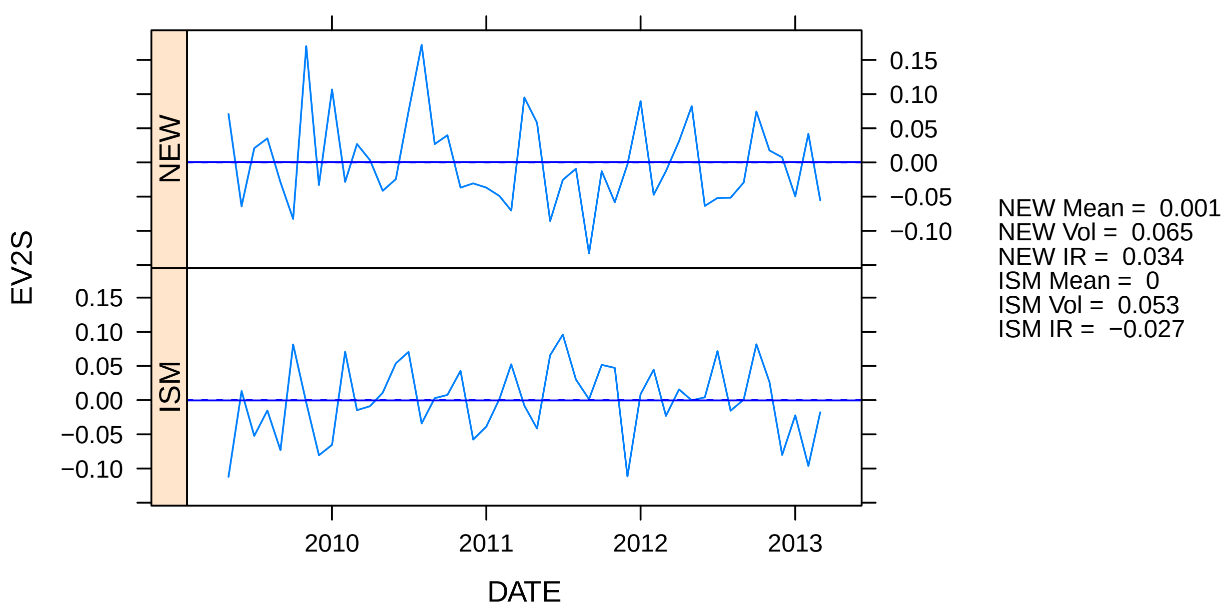
To shed the light of how NEW model outperforms the ISM, we can look and compare their active returns from the optimal active weights.

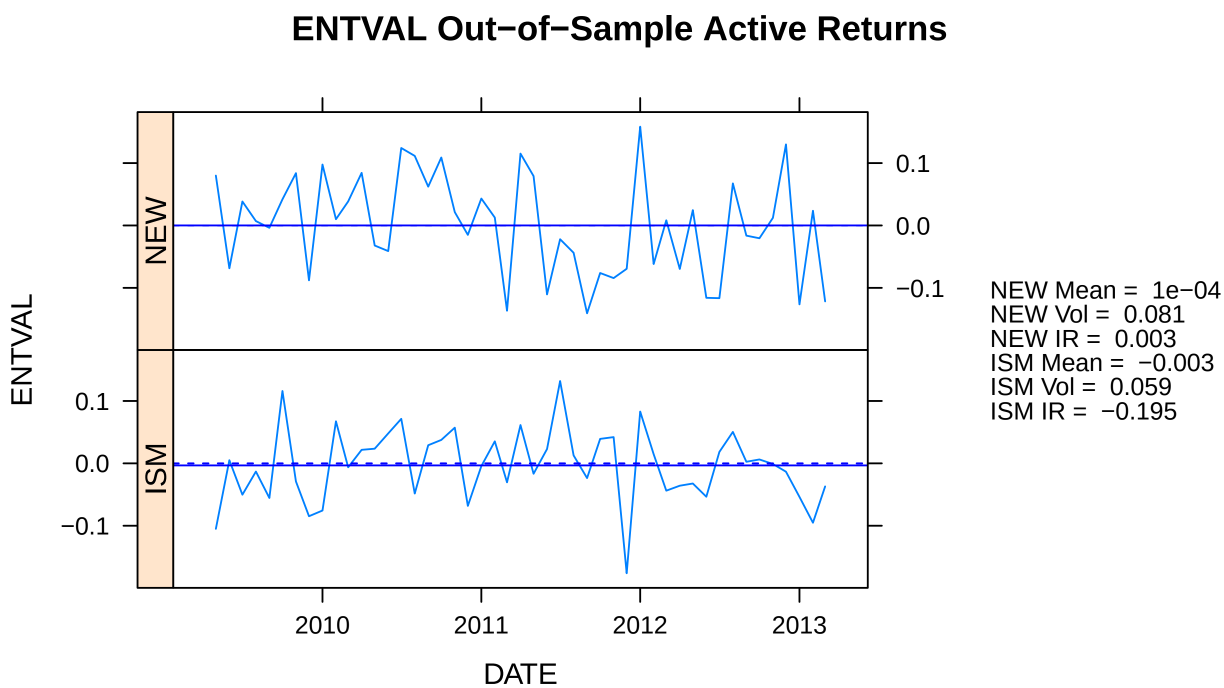


SIZE Out−of−Sample Active Returns



EV2S Out−of−Sample Active Returns




ENTVAL Out−of−Sample Active Returns
ENTVAL
NEW
ISM
0.1
0.0
−0.1
2010
2011
2012
2013
DATE
NEW Mean = 1e−04
NEW Vol = 0.081
NEW IR = 0.003
ISM Mean = −0.003
ISM Vol = 0.059
ISM IR = −0.195