Department of Mathematics
Indian Institute of Technology Guwahati

MA311M: Lab Assignment 4

Date of Submission: 28/11/2020 **Weightage: 20 Marks**

1. Consider the one-dimensional heat equation

$$U_t = U_{xx} \text{ for } (x,t) \in (0,1) \times (0,0.1].$$

Use classical explicit scheme to solve the above equation, where the initial and boundary data are taken from the exact solution $U(x,t) = \exp(-\pi^2 t)\sin \pi x$. Perform the following experiments: (i) Plot the numerical solutions with $h = 0.1$, $k = .005$ against the exact solution; (ii) Study the convergence of numerical solutions when $k/h^2 > 1/2$. **[5 Marks]**

2. Solve the above problems (Q.1) using the Crank-Nicolson method satisfying the following boundary and initial conditions:

$$\begin{aligned} U(0,t) &= U(1,t) = 0, \quad t > 0, \\ U(x,0) &= 2x, \quad 0 \le x \le 1/2, \\ &= 2(1-x), \quad 1/2 \le x \le 1. \end{aligned}$$

Plot numerical solutions with $h = 0.1$ and $k = 0.01$. **[5 Marks]**

3. Consider the following initial-boundary problems

$$U_t = U_{xx} + U_{yy}, \quad 0 < x < 1;\ 0 < y < 1; \quad t \in (0;1]$$

satisfying the initial condition

$$U(x;y;0) = \sin(x)\sin(y); \quad 0 \le x \le 1;\ 0 \le y \le 1,$$

and the boundary conditions

$$U(0;y;t) = U(1;y;t) = U(x;0;t) = U(x;1;t) = 0; \quad \forall t \in (0;1].$$

Solve this problem numerically using the explicit method with $r = k/h^2 = 1/4$ and plot the numerical solution. **[10 Marks]**

Note: The program and output files should be submitted on or before the due date. No submission will be accepted after the due date. Output files should contain your Name and Roll number.