Heat Transfer in a Shallow Fluidized Bed

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1 Introduction

An experiment was run to study heat transfer in a shallow fluidized bed. Data contains four candidate regressors - X1: fluidizing gas flow rate in pounds per hour, X2: supernatant gas flow rate in pounds per hour, X3: supernatant gas inlet nozzle opening in millimeters, X4: supernatant gas inlet temperature ($^{\circ}F$). On the other hand, two measured responses are Y1: heat transfer coefficients, Y2: thermal efficiency. Twenty observations were gathered.

2 Questions of Interest

2.1 Prediction Model for Y1

In this analysis, we build a regression model for predicting heat transfer coefficient Y1 with covariates X1,..., X4. Before implementing the model, we first apply EDA (initial check) for our data. As seen in Figure 1, the correlation between Y1 and other covariates are not significant at all. After digging deeper into our data as Figure 2 and Figure 3, we found there are two potential outliers. Based on the opinions of chemical engineering experts, we decide to exclude these two outliers. Ex: Typically, the temperature lies between 10 and 350 when dealing with gas flows in tubes and between tubes [1]. As in Figure 4, the correlation becomes stronger after we remove the outliers. This result suggests that the relations between Y1 and regressors are mostly linear (Figure 5). Therefore, we decide to fit Bayesian linear regression to our data with non-informative prior N(0,10) for the coefficients. Table 1 implies the regressors X1 (`Fgas_rate`), X2 (`Sgas_rate`), and X3 (`Sgas_size`) are all significant to the predicting the response Y1 while X4 (`Sgas_temp`) might be not relevant to the prediction. Although the predictive check for the model is not very fit as shown in Figure 6, this is mostly because of the current lacking of data and can be further improved as we collect more samples.

2.2 A New Experiment

Suppose we are given a new run of the experiment and we observe $x_1 = 116.9$ and $x_2 = 172.1$ only. Since we want to apply our model in 2.1 without knowing X3 and X4, we first need to impute (give a possible) their values. Here we calculate the sample average of X3 and X4, which equals 43.89 and 220.36 respectively. Using the linear model in 2.1, we get the histogram of 9000 estimated the heat transfer coefficient Y1 distributed as Figure 7. Using these estimated values we can get the estimated Y1 with empirical mean 78.05 and 95% CI

[35.54, 120.03]. Among 9000 estimated Y1, 1746 of them are greater than 100. Therefore, we can also estimate the probability that this coefficient exceeds 100 as $\frac{1746}{9000} = 0.194$.

2.3 Binary Outcome Whether Y1 Exceeds 100

Table 2 represent the result of the model for the indicator variable $Z = 1_{Y_1} > 100$. Since the response variable Z is a binary outcome, we choose a logistic regression model to fit our data. As we can see in Table 2, the credible interval (95% CI) of the first three regressors does not include 0. This implies we are confident that these three covariates have a significant, either positive or negative, impact on the binary response Z. Similarly as in Table 1, the inlet temperature X4 does not have an impact on Z and the posterior predictive check does not fit very well due to lacking data.

2.4 Prediction Model for Y2

An interesting question is that does the regression model for Y1 can still doing a nice job of predicting Y2? To find out the result we use the same linear regression model for Y1 to fit the new response Y2 as shown in Figure 9. Focus again on the CI for each covariate, we found them either irrelevant or just have a minor effect on this new response. Therefore, the prediction is no longer as good as Y1. A possible explanation is the value of thermal efficiency $\frac{Y_2}{100}$ represents the percentage which lies between 0 and 1. Thus, it is inappropriate to still choose Y2 as a response variable with a linear regression model. Typically, when considering building a regression model for predicting response between 0 and 1, one can consider the Beta regression model. Therefore, the future direction can continue to work on Beta regression to predict Y2.

3 Conclusion

This report provides the first attempt to solve the four questions in the heat transfer of a shallow fluidized bed. Some of the questions are standard in statistical analysis and this report offers the corresponding estimates based on the Bayesian model. However, two studies might be further improved. First is the validity of the imputed value of X3 and X4. Here we substitute sample average based only on 18 values. Thus, the prediction might be better if we get more data for this experiment. Next, Beta regression, as mention in Section 2.4, might be a better model for the prediction of Y2. In the end, here we apply the Bayesian model with small data set (18 in total). One may argue the analysis might be better if we can choose a more informative prior instead of the current non-informative prior N(0,10) for the coefficients. Further discussion can be done in the future.

References

[1] Kurganov, *Heat transfer coefficient*, 2011.

4 Appendix

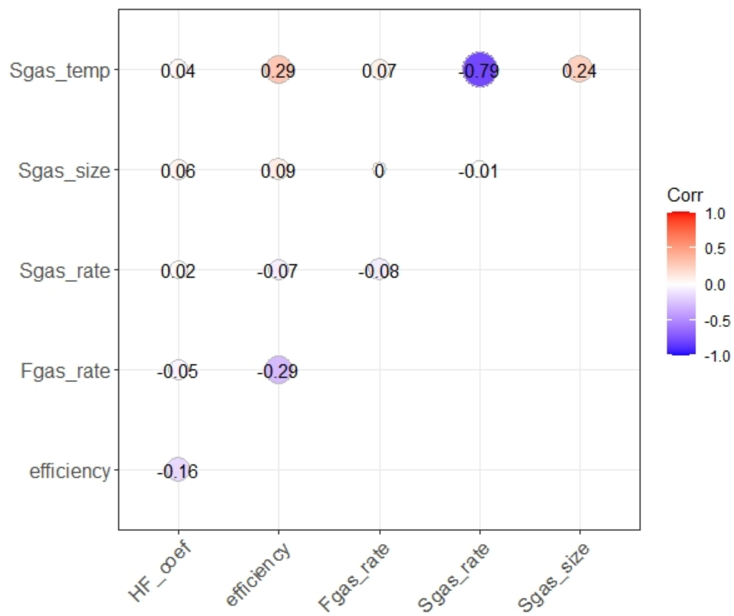

Figure 1: Correlation Plot for Data Containing Outliers

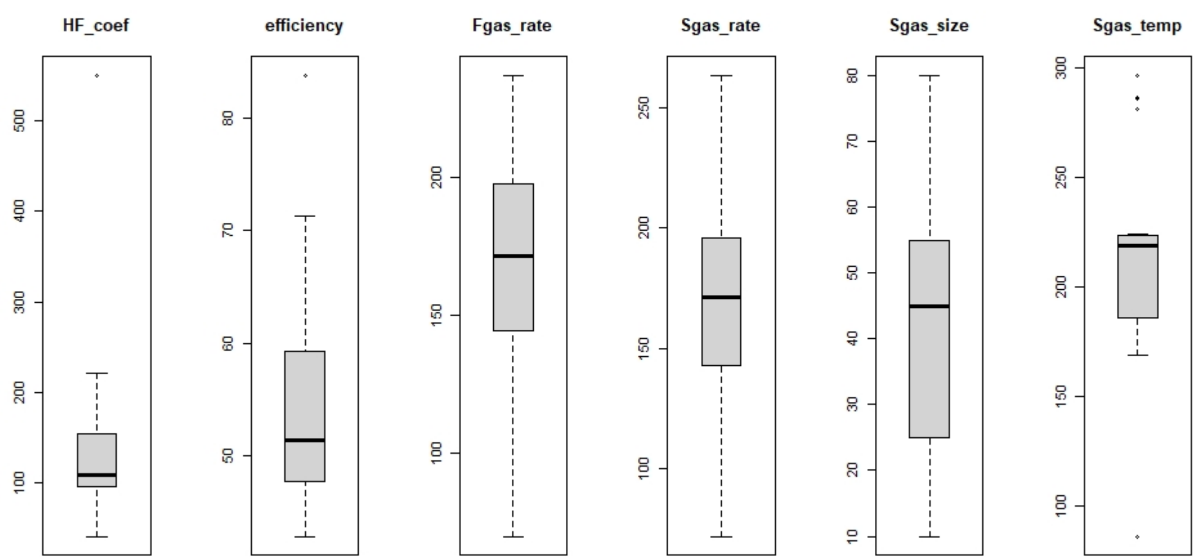

Figure 2: Boxplot for each 6 categories

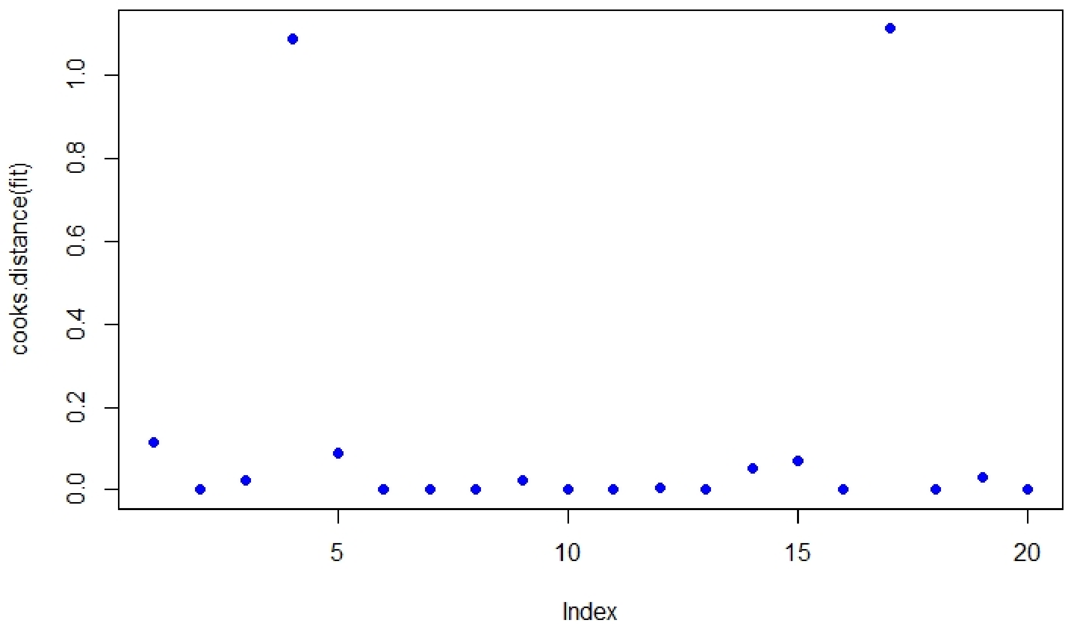

Figure 3: Cook's Distance for Data

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: HF_coef ~ Sgas_rate + Fgas_rate + Sgas_size + Sgas_temp
   Data: data1 (Number of observations: 18)
Samples: 2 chains, each with iter = 5000; warmup = 500; thin = 1;
         total post-warmup samples = 9000

Population-Level Effects:
          Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept   -34.62     92.12  -221.81   145.98 1.00     5392     4741
Sgas_rate     0.70      0.22     0.26     1.16 1.00     5135     4298
Fgas_rate     0.64      0.15     0.34     0.93 1.00     6294     5379
Sgas_size    -1.42      0.32    -2.06    -0.75 1.00     7161     5516
Sgas_temp    -0.08      0.31    -0.68     0.54 1.00     4909     4177

Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma    22.15      4.66    15.03    33.07 1.00     4679     5072

Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
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Table 1: Summary of Linear Regression for Heat Transfer Coefficient Y1

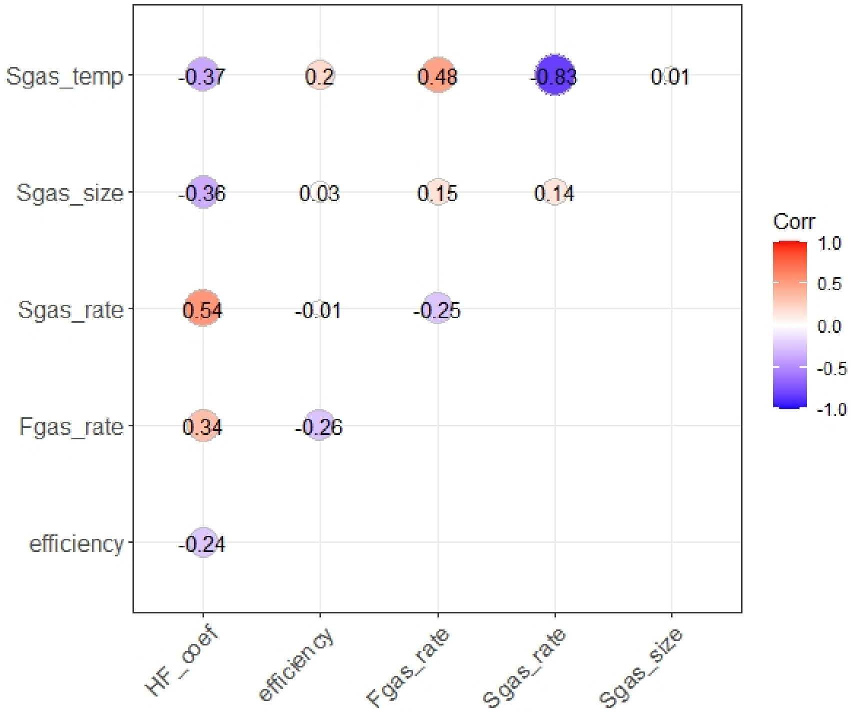

Figure 4: Correlation Plot for Data Without Outliers

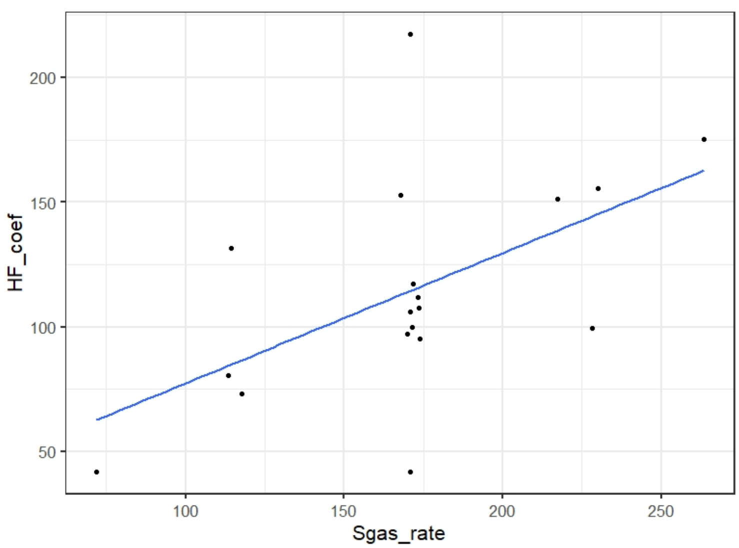

Figure 5: Linear Relation between Rate of Supernatant Gas Rate and Heat Transfer Coefficient

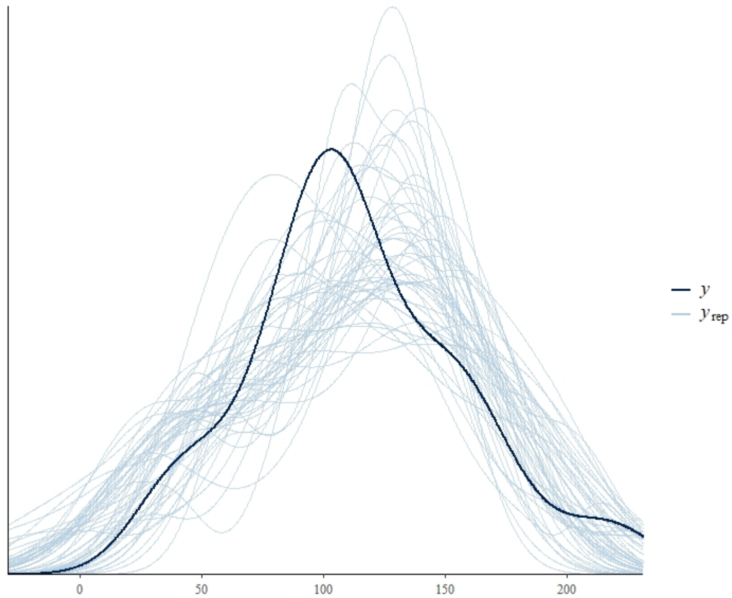

Figure 6: Posterior Predictive Check of Linear Model Without Outliers

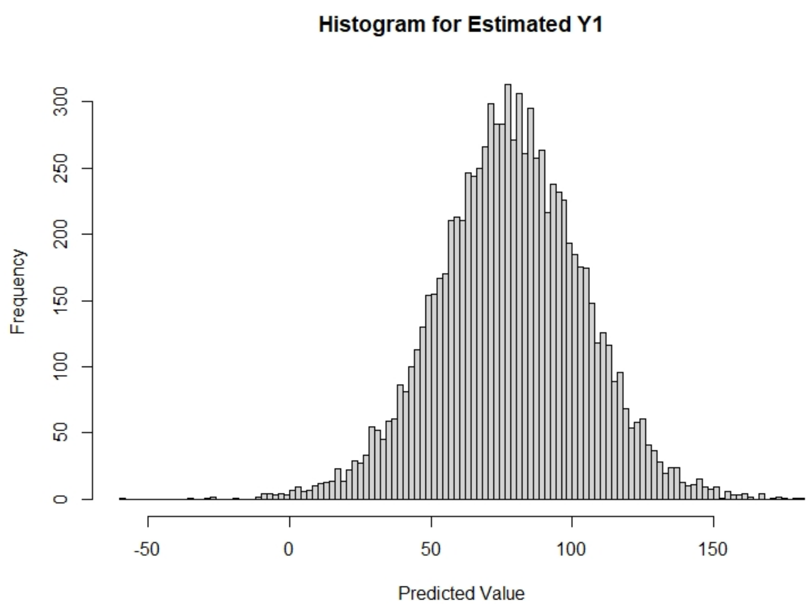

Figure 7: Histogram of Estimated Heat Transfer Coefficient Y1

```
 Family: bernoulli
  Links: mu = logit
Formula: HF_index ~ Sgas_rate + Fgas_rate + Sgas_size + Sgas_temp
   Data: data2 (Number of observations: 18)
Samples: 2 chains, each with iter = 5000; warmup = 500; thin = 1;
         total post-warmup samples = 9000

Population-Level Effects:
          Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept -3609.73   1539.16 -6835.05  -833.21 1.00      411      794
Sgas_rate    11.82      4.76     3.29    21.74 1.00      409      799
Fgas_rate    10.66      4.47     3.28    20.63 1.00      428      951
Sgas_size    -8.63      4.16   -18.15    -2.40 1.00      534     1073
Sgas_temp     0.77      2.40    -4.11     5.30 1.01      584     1039

Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
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Table 2: Summary of Logistic Regression for Heat Transfer Coefficient $Z = 1Y_1 > 100$

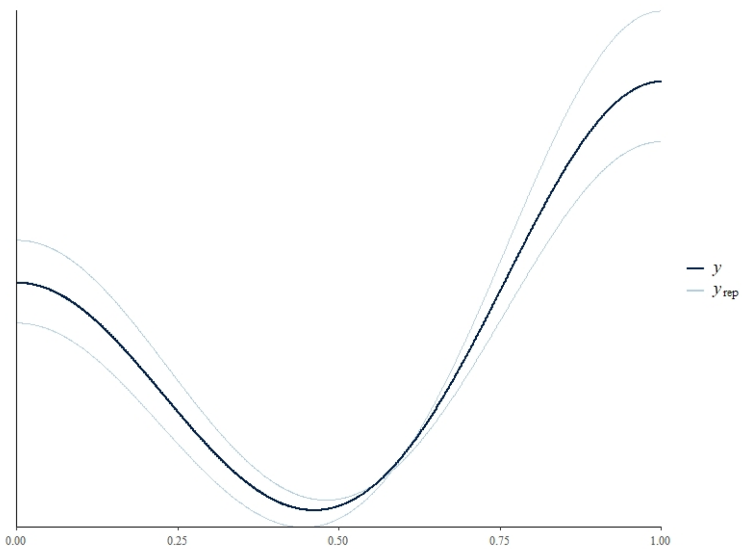

Figure 8: Posterior Predictive Check of Logit Model Without Outliers

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: efficiency ~ Sgas_rate + Fgas_rate + Sgas_size + Sgas_temp
   Data: data1 (Number of observations: 18)
Samples: 2 chains, each with iter = 5000; warmup = 500; thin = 1;
         total post-warmup samples = 9000

Population-Level Effects:
          Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept   -23.45     41.63  -107.84    58.85 1.00     5854     5657
Sgas_rate     0.18      0.10    -0.02     0.38 1.00     5645     5483
Fgas_rate    -0.15      0.07    -0.28    -0.01 1.00     6257     5389
Sgas_size     0.00      0.15    -0.29     0.29 1.00     7878     5991
Sgas_temp     0.33      0.14     0.05     0.60 1.00     5368     5127

Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma     9.96      1.92     6.98    14.47 1.00     5322     6055

Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
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Table 3: Summary of Linear Regression for Thermal Efficiency Y2

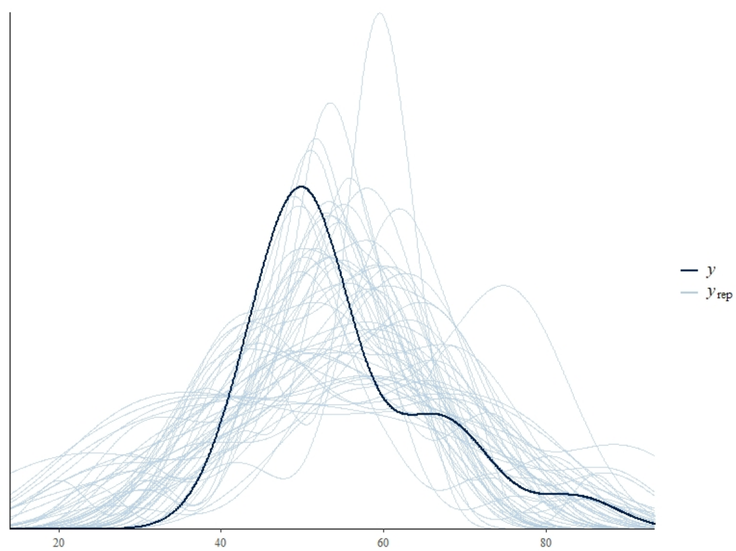

Figure 9: Posterior Predictive Check of Linear Model Without Outliers