SECI1013: DISCRETE STRUCTURES

SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 3 (CHAPTER 3 & 4 [Part 1])

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INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment consists of 7 questions (85 Marks), contributing 5% of overall course marks.

STRUCTURES:

1. Chapter 3 Part 3: Pigeonhole Problem [10 Marks]
2. Chapter 3 Part 4: Probability [25 Marks]
3. Chapter 4 Part 1: Graph Theory (until Path and Cycle) [50 Marks]

Question 1 **[10 marks]**

a. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks)
b. what is the minimum number of students required in a Structure Discrete class so that at least six students will receive the same letter grade (A,B,C,D, or F) (5 marks)

Question 2 **[25 marks]**

The following table gives information on Mobile phone sold by a certain store:

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
Brand 1	70	20
Brand 2	30	40

A purchaser is randomly selected from among all those bought a mobile phone from the store.

Determine the probability that :

a. customer purchased Brand 1. (2 marks)

b. customer purchased Brand 2 (2 marks)
c. customer purchase extended warranty given that purchase brand 1. (2 marks)
d. customer who bought brand 1 and purchased extended warranty. (4 marks)
e. customer purchased brand 2 and extended warranty purchased. (5 marks)
f. extended probability purchased. (5 marks)
g. purchased brand 1 item given that also bought extended warranty (5 marks)

Question 3

Explain the given keyword using your own word and represent your understanding by drawing the graph.

a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. Loop
g. Parallel Edges

(7 Marks)

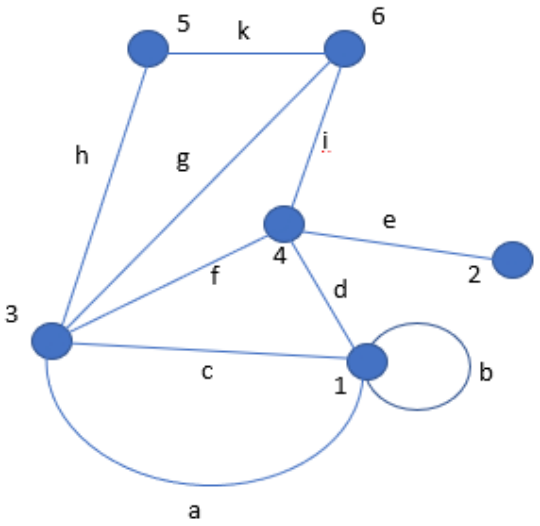
Question 4

Let G = {V, E} be a graph. An undirected graph having V ={v1, v2, v3, v4, v5} and E = {a, b, c, d, e, f}. Where *a* = (*v*1,*v*2), *b* = (*v*1,*v*3), *c* = (*v*2,*v*4), *d* = (*v*1,*v*4), *e* = (*v*3,*v*4) and *f* = (*v*3,*v*5).

Find the degree of each vertex.

(5 Marks)

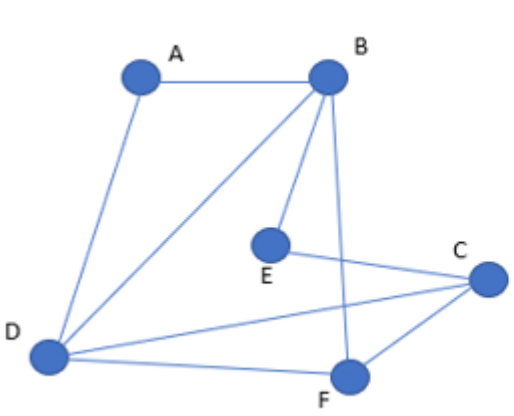
Question 5



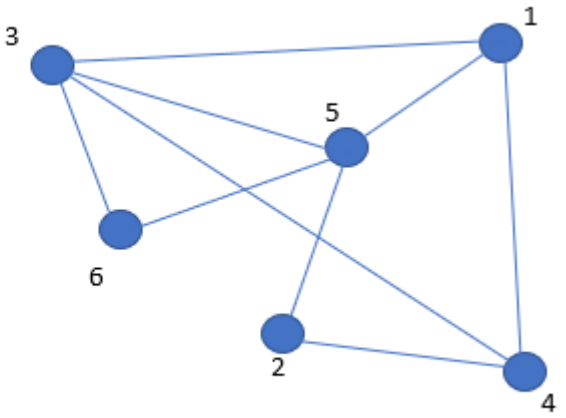
Given the graph shown above, Find:

i. Incidence Matrix (6 Marks)
ii. Adjacency Matrix (6 Marks)

Question 6



Graph Y



Graph Z

Determine whether Graph Y and Z above are isomorphic. If it is proven isomorphic, find their adjacency matrix.

(12 Marks)

Question 7

Consider an undirected graph with vertices V = {p, q, r, s, t} and edges E = {e1, e2, e3, e4, e5, e6, e7}. The edges are defined as follows:

e1 = (p, q); e2 = (q, r); e3 = (r, s); e4 = (s, t); e5 = (t, p); e6 = (q, s); e7 = (r, t)

Draw the graph and from the graph:

i. Find all possible paths from vertex p to vertex t. (5 Marks)
ii. Determine all possible trails from vertex p to vertex t. (5 Marks)
iii. Identify the shortest and longest path from vertex p to vertex t. (2.5 marks)
iv. Find the shortest and longest trail from vertex p to vertex t. (2.5 Marks)

(14 Marks)

Question 1

a. 0-100 ⇒ 101 students

Guarantee that at least two students receive the same score:

101 + 1 = 102 students

∴ To guarantee that at least two students received the same score, there must be 102 students in the class.

b. A : 🧍🧍🧍🧍🧍

B : 🧍🧍🧍🧍🧍

C : 🧍🧍🧍🧍🧍 ← any 🧍 To form at least 6

D : 🧍🧍🧍🧍🧍

F : 🧍🧍🧍🧍🧍

5 × 5 = 25 students (5 students per grade)

25 + 1 = 26 students (n ≥ 26)

∴ The minimum number of students required is 26.

Question 2

a. Total = 70 + 30 = 100

$P(\text{Brand 1}) = \frac{70}{100} = 0.7$

b. $P(\text{Brand 2}) = \frac{30}{100} = 0.3$

c. $P(\text{Warranty} \mid \text{Brand 1}) = 0.2$

d. $P(\text{Brand 1} + \text{Warranty Brand 1}) = 0.7 \times 0.2 = 0.14$

e. $P(\text{Brand 2} + \text{Warranty Brand 2}) = 0.3 \times 0.4 = 0.12$

f. $P(\text{Warranty}) = 0.14 + 0.12 = 0.26$

g. $P(\text{Brand 1} + \text{Warranty Brand 1} \mid \text{Warranty}) = \frac{0.14}{0.26} = 0.5385$

Question 3

a. Vertices : Points in a network A B

b. Edges : Line between Points in a network A e B

c. Adjacent Vertices : 2 vertices connected by an edge A B

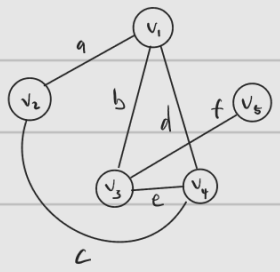
d. Incident Edge : An Edge connected to a certain vertice

e. Isolated Vertex : Alone Vertex C

f. Loop : An edge that starts and ends at the same vertex A

g. Parallel Edges : 2 edges connected to the same pair vertices A B

Question 4



	v_1	v_2	v_3	v_4	v_5
deg	3	2	3	3	1

Question 5

i)

	1	2	3	4	5	6
a	1	0	1	0	0	0
b	2	0	0	0	0	0
c	1	0	1	0	0	0
d	1	0	0	1	0	0
e	0	1	0	1	0	0
f	0	0	1	1	0	0
g	0	0	1	0	0	1
h	0	0	1	0	1	0
i	0	0	0	1	0	1
k	0	0	0	0	1	1

Incidence Matrix:

	1	2	3	4	5	6
a	1	0	1	0	0	0
b	2	0	0	0	0	0
c	1	0	1	0	0	0
d	1	0	0	1	0	0
e	0	1	0	1	0	0
f	0	0	1	1	0	0
g	0	0	1	0	0	1
h	0	0	1	0	1	0
i	0	0	0	1	0	1
k	0	0	0	0	1	1

ii)

	1	2	3	4	5	6
1	1	0	2	1	0	0
2	0	0	0	1	0	0
3	2	0	0	1	1	1
4	1	1	1	0	0	1
5	0	0	1	0	0	1
6	0	0	1	1	1	0

Adjacency Matrix:

	1	2	3	4	5	6
1	1	0	2	1	0	0
2	0	0	0	1	0	0
3	2	0	0	1	1	1
4	1	1	1	0	0	1
5	0	0	1	0	0	1
6	0	0	1	1	1	0

Question 6

Graph Y:

V = A, B, C, D, E, F

E = (A,B), (A,D), (B,D), (B,E), (B,F), (C,E), (C,D), (C,F), (D,F)

Graph Z:

V = 1, 2, 3, 4, 5, 6

E = (1,3), (1,4), (1,5), (3,5), (3,4), (3,6), (5,6), (2,5)(2,4)

∴ Both graphs have the same number of vertices and edges

Compare degrees:

Graph Y A → F [2, 4, 3, 4, 2, 3]

deg(A) = 2 deg(D) = 4

deg(B) = 4 deg(E) = 2

deg(C) = 3 deg(F) = 3

Graph Z 1 → 6 [3, 2, 4, 3, 4, 2]

deg(1) = 3 deg(4) = 3

deg(2) = 2 deg(5) = 4

deg(3) = 4 deg(6) = 2

Verify vertex:

A ↔ 6 D ↔ 3

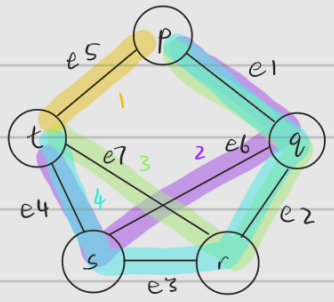
B ↔ 5 E ↔ 2

C ↔ 4 F ↔ 1

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	0	1	1	1
C	0	0	0	1	1	1
D	1	1	1	0	0	1
E	0	1	1	0	0	0
F	0	1	1	1	0	0

	6	5	4	3	2	1
6	0	1	0	1	0	0
5	1	0	0	1	1	1
4	0	0	0	1	1	1
3	1	1	1	0	0	1
2	0	1	1	0	0	0
1	0	1	1	1	0	0

Question 7



i) 1. $p \rightarrow t$ (e5)

2. $p \rightarrow q \rightarrow s \rightarrow t$ (e1, e6, e4)

3. $p \rightarrow q \rightarrow r \rightarrow t$ (e1, e2, e7)

4. $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$ (e1, e2, e3, e4)

ii) 1. $p \rightarrow t$

2. $p \rightarrow q \rightarrow s \rightarrow t$

3. $p \rightarrow q \rightarrow r \rightarrow t$

4. $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$

iii) Shortest Path = $p \rightarrow t$

Longest Path = $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$

iv) Shortest Trail = $p \rightarrow t$

Longest Trail = $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$