Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff x−y = 3n for some n ∈ Z. Find all possible equivalence relations for R.

(5 marks)

$R = \{(2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (8,5), (8,2), (7,4), (6,3), (5,2), (5,8), (2,8), (4,7), (3,6), (2,5)\}$

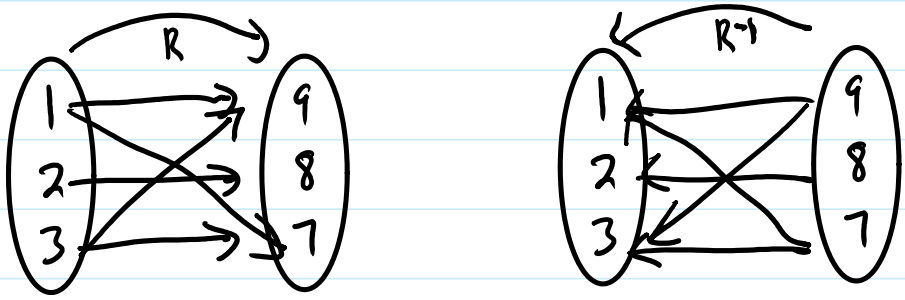
2. Let A = {1, 2, 3} and B = {9, 8, 7}.
Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

   a. Determine R and R^{-1}.
   b. Draw arrow diagrams for both.
   c. Describe R−1 in words.

(10 marks)

(a) $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$
$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

(b)



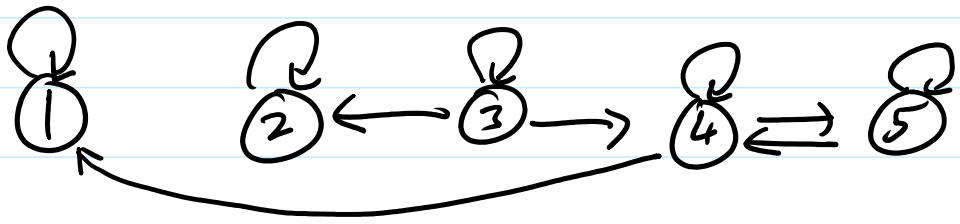
(c) For all $(b,a) \in B \times A$, $(b,a) \in R^{-1} \Rightarrow b+a$ is an even

3. Let $A = \{1, 2, 3, 4, 5\}$, and let R be the relation on A that has the matrix (given below)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Construct the digraph of R, and list in-degrees and out-degrees of all vertices.

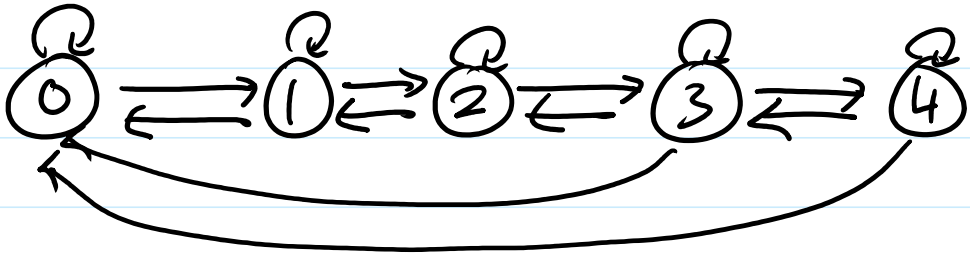
(6 marks)



	1	2	3	4	5
In degree	2	2	1	3	2
Out degree	1	1	3	3	2

4. Given A = {0, 1, 2, 3, 4}, and R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the relation graph and find is R reflexive, symmetric, or transitive?

(12 marks)



Reflexive:

$$\forall x \in A, (x, x) \in R$$

Symmetric:

$$\forall x \in A, \text{if } (x, \quad) \in R \quad (\quad, x) \in R$$

Symmetric:

$\forall x, y \in A$, if $(x, y) \in R$ then $(y, x) \in R$

Not Transitive:

$(0, 1), (1, 2) \in R$ but $(0, 2) \notin R$

5. Relation R in the set A = {1, 2, 3...13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is
   a. Reflexive
   b. Symmetric
   c. Transitive

   Support your answer with the reason.

(9 marks)

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ⓐ Reflexive: The relation is Reflexive because
The diagonal value is 1 and
$\forall x \in A, (x, x) \in R$

ⓑ Symmetric: Yes, because
all value except on the diagonal are 0
and $\forall x, y \in A$, if $(x, y) \in R$ then $(y, x) \in R$

Transitive: Yes. because
all value except on the diagonal are 0

6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetric, Find

a. RS
b. SR

(8 marks)

(a) $RS = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) $SR = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$