机电控制第二次作业

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December 12, 2017

Problem 1:

$$G(s) = \frac{1}{(s+a)^2} \tag{1}$$

计算其 Z 变换

$$G(z) = \frac{z-1}{z} Z\left[\frac{G_p(s)}{s}\right] = \frac{z-1}{z} Z\left[\frac{1}{s(s+a)^2}\right] \tag{2}$$

使用 ztrans 函数计算

$$G(z) = \frac{z-1}{z}\left(\frac{z}{a^2\ (z-1)} - \frac{z}{a^2\ (z-\mathrm{e}^{T\,a})} + \frac{T\,z\,\mathrm{e}^{T\,a}}{a\,(z-\mathrm{e}^{T\,a})^2}\right) \tag{3}$$

$$= (z-1)\left(\frac{1}{a^2\ (z-1)} - \frac{1}{a^2\ (z-\mathrm{e}^{T\,a})} + \frac{T\,\mathrm{e}^{T\,a}}{a\,(z-\mathrm{e}^{T\,a})^2}\right) \tag{4}$$

Problem 2

化简系统方框图，其形式为

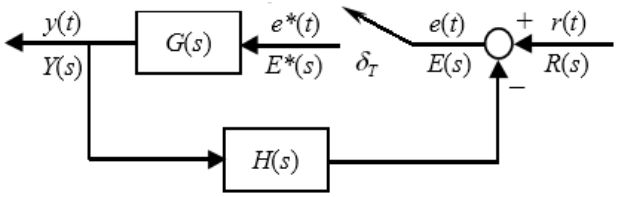

图 1: 系统方框图

其中，

$$G(s) = (1 + \frac{1}{Ts})(G_{ZOH}(s)) \tag{5}$$

$$H(s) = \frac{\frac{1}{Ts}}{1 + \frac{1}{Ts}} = \frac{1}{Ts+1} \tag{6}$$

$$E(s) = R^*(s) - H(s)Y(s) \tag{7}$$

$$Y(s) = G(s) \cdot E^*(s) \tag{8}$$

$$\Rightarrow E(s) = R^*(s) - H(s)G(s) \cdot E^*(s) \tag{9}$$

$$\Rightarrow E^*(s) = R^*(s) - HG^*(s)E^*(s) \tag{10}$$

$$\Rightarrow E^*(s) = \frac{1}{1 + HG^*(s)} \cdot R^*(s) \tag{11}$$

则系统的传递函数为

$$G_{FOH}(s) = \frac{Y(s)}{R^*(s)} = \frac{G(s)}{1 + HG^*(s)} \tag{12}$$

其中，

$$G(s) = (1 + \frac{1}{Ts})\frac{1 - e^{-Ts}}{s} \tag{13}$$

通过 z 变换计算 $HG^*(s)$，

$$HG^*(s) = HG(z) = Z\left[G(s)H(s)\right] = \frac{z-1}{z} Z\left[(1 + \frac{1}{Ts})\frac{1}{Ts+1}\frac{1}{s}\right] \tag{14}$$

$$= \frac{z-1}{z} \cdot \frac{z}{(z-1)^2} = \frac{1}{z-1} = \frac{1}{e^{Ts} - 1} \tag{15}$$

则传递函数为

$$G_{FOH}(s) = \frac{G(s)}{1 + HG^*(s)} = \frac{(1 + \frac{1}{Ts})\frac{1 - e^{-Ts}}{s}}{1 + \frac{1}{e^{Ts} - 1}} = \frac{(1 + \frac{1}{Ts})\frac{(1 - e^{-Ts})(e^{Ts} - 1)}{s}}{(e^{Ts} - 1) + 1} \tag{16}$$

$$= (1 - e^{-Ts})^2 \left(\frac{Ts + 1}{Ts^2}\right) \tag{17}$$

Problem 3

(a)

FOH 系统传递函数为

$$G_{FOH}(s) = (1 - e^{-Ts})^2 \left(\frac{Ts + 1}{Ts^2}\right) \tag{18}$$

傅里叶变换得

$$G_{FOH}(j\omega) = (1 - e^{-Tj\omega})^2 \left(\frac{Tj\omega + 1}{T(j\omega)^2}\right) \tag{19}$$

其中，

$$(1 - e^{-Tj\omega})^2 = 1 + e^{-2Tj\omega} - 2e^{-Tj\omega} = e^{-Tj\omega}(e^{Tj\omega} + e^{-Tj\omega} - 2) \tag{20}$$

$$= e^{-Tj\omega}(\cos T\omega + j\sin T\omega + \cos T\omega - j\sin T\omega - 2) \tag{21}$$

$$= e^{-Tj\omega}(2\cos T\omega - 2) = -e^{-Tj\omega} 4\sin^2(T\omega/2) \tag{22}$$

因此，

$$G_{FOH}(j\omega) = -e^{-Tj\omega} 4\sin^2(T\omega/2) \left(\frac{Tj\omega + 1}{T(j\omega)^2}\right) = \frac{Tj\omega + 1}{T} e^{-Tj\omega} \frac{4\sin^2(T\omega/2)}{\omega^2} \tag{23}$$

(b)

取采样周期 T = 1s，绘制伯德图。

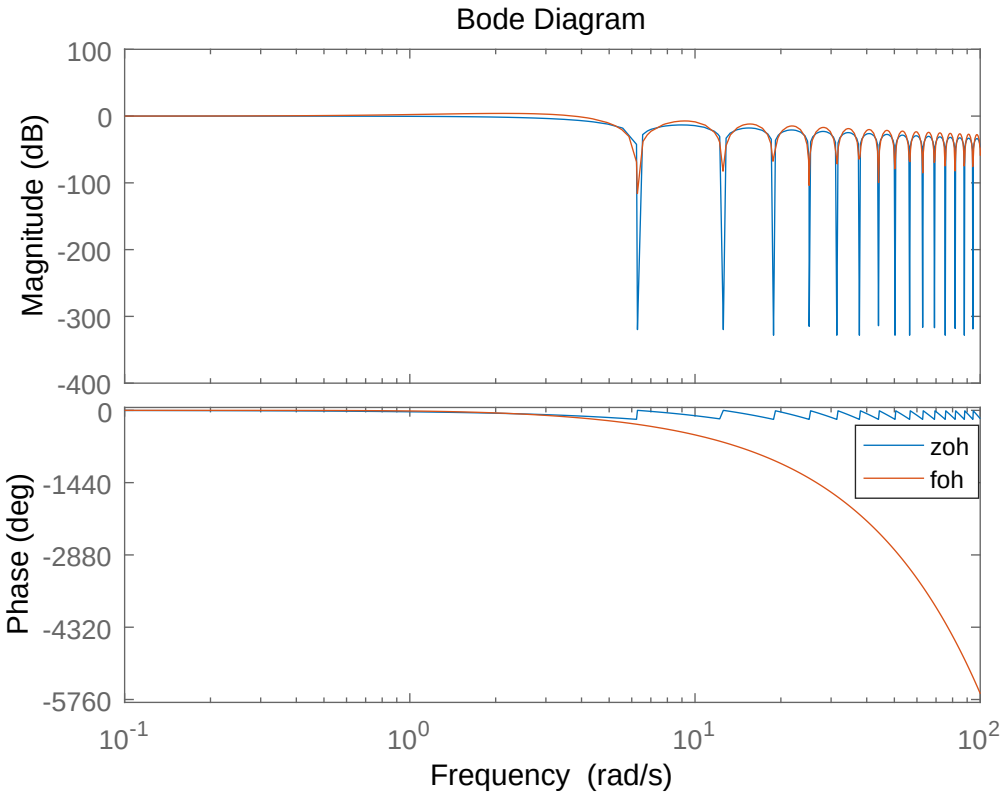

图 2: bode 图

Problem 5

(a)

$$G(z) = \frac{z-1}{z} Z\left[\frac{1}{s^2} e^{-0.4s}\right] = \frac{z-1}{z} \frac{3z+2}{5(z-1)^2} = \frac{3z+2}{5z(z-1)} \tag{24}$$

(b)

延迟环节 $T_D = 0.4s$，由于

$$T_D = nT + T_L \tag{25}$$

系统的阶数为 n+2 = 5，因此 n=3，则

$$\frac{1}{10} \leq T \leq \frac{2}{15} \tag{26}$$