

1. Consider simple network with single ~~input~~ ⁱⁿ and single output $y=1$

$$w^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \quad w^{(2)} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

a) Forward propagation

$$u^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}^T \begin{bmatrix} 1 \\ z \end{bmatrix} \quad \text{Input layer}$$

$$w^{(2)} = \begin{bmatrix} 0.2 & 0.2 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ z \end{bmatrix} = \begin{bmatrix} 0.1 + 0.3z \\ 0.2 + 0.4z \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$$

$$z^{(1)} = \begin{bmatrix} \tanh(0.7) \\ \tanh(1) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix}$$

$$u^{(2)} = \begin{bmatrix} 0.2 & 0.2 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}^T \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix} = \begin{bmatrix} -1.48 \\ -1.48 \end{bmatrix}$$

$$z^{(2)} = \begin{bmatrix} \tanh(-1.48) \\ \tanh(-1.48) \end{bmatrix} = \begin{bmatrix} -0.9 \\ -0.9 \end{bmatrix}$$

$$y^{(3)} = \tanh(1 \times 1 + (-0.9) \times 2) + (-0.9) \times 1 = \tanh(-1.7) = -0.935$$

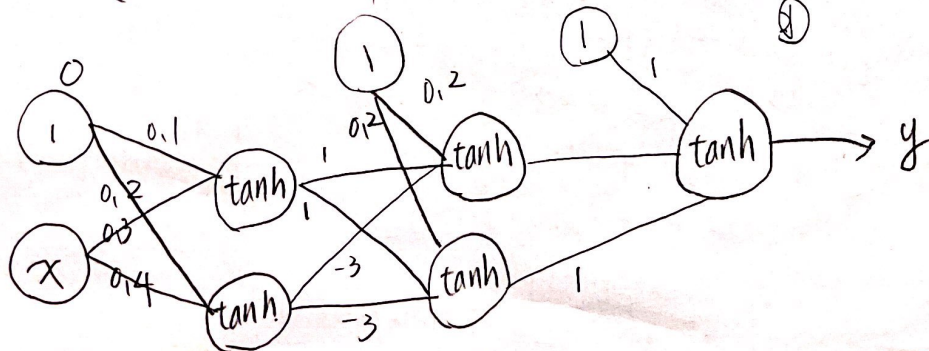
$$\Rightarrow u^{(1)} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}, u^{(2)} = \begin{bmatrix} -1.48 \\ -1.48 \end{bmatrix}, z^{(1)} = \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix}, z^{(2)} = \begin{bmatrix} -0.9 \\ -0.9 \end{bmatrix}, y^{(3)} = -0.935$$

Backward propagation

$$b) \delta^3 = (-0.935 - 1) \cdot [1 - \tanh^2(-1.7)] = -1.935 \cdot [1 - (-0.935)^2] = -0.243$$

$$\delta^2 = -0.243 \begin{bmatrix} 2 \cdot [1 - (-0.9)^2] \\ 1 \cdot [1 - (-0.9)^2] \end{bmatrix} = -0.243 \begin{bmatrix} 2 \cdot 0.19 \\ 0.19 \end{bmatrix} = -0.046 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\delta^1 = \begin{bmatrix} (-0.092) \cdot (1 - 0.6^2) + (-0.046) \cdot 1 \cdot (1 - 0.6^2) \\ (-0.092) \cdot (-3) \cdot (1 - 0.76^2) + (-0.046) \cdot (-3) \cdot (1 - 0.76^2) \end{bmatrix} = \begin{bmatrix} -0.088 \\ -0.175 \end{bmatrix}$$



$$c) \frac{\delta E_n}{\delta W^{(1)}} = X^{(0)} (\delta^{(1)})^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -0.088 & 0.175 \end{bmatrix} = \begin{bmatrix} -0.088 & 0.175 \\ -0.176 & 0.35 \end{bmatrix}$$

$$\frac{\delta E_n}{\delta W^{(2)}} = Z^{(1)} (\delta^{(2)})^T = \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix} \begin{bmatrix} -0.092 & -0.046 \end{bmatrix} = \begin{bmatrix} 0.092 & 0.046 \\ 0.055 & 0.028 \\ 0.07 & 0.035 \end{bmatrix}$$

$$\frac{\delta E_n}{\delta W^{(3)}} = Z^{(2)} (\delta^{(3)})^T = \begin{bmatrix} 1 \\ -0.9 \\ -0.9 \end{bmatrix} \begin{bmatrix} -0.243 \end{bmatrix} = \begin{bmatrix} -0.243 \\ 0.2187 \\ 0.2187 \end{bmatrix}$$

d) learning rate $\Rightarrow \eta = 0.5$

$$W^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} - 0.5 \begin{bmatrix} -0.088 & 0.175 \\ -0.176 & 0.35 \end{bmatrix} = \begin{bmatrix} 0.1+0.044 & 0.2-0.0875 \\ 0.3+0.352 & 0.4-0.175 \end{bmatrix}$$

$$= \begin{bmatrix} 0.144 & 0.1125 \\ 0.652 & 0.225 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} 0.2 & 0.2 \\ 1 & -3 \\ -3 & -3 \end{bmatrix} - 0.5 \begin{bmatrix} -0.092 & 0.046 \\ 0.055 & 0.028 \\ 0.07 & 0.035 \end{bmatrix}$$

$$W^{(3)} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 0.5 \begin{bmatrix} -0.243 \\ -0.2187 \\ 0.2187 \end{bmatrix} = \begin{bmatrix} 1+0.1215 \\ 2+0.10935 \\ 1-0.10935 \end{bmatrix} = \begin{bmatrix} 1.1215 \\ 2.10935 \\ 0.89065 \end{bmatrix} = \begin{bmatrix} 1.122 \\ 1.89 \\ 0.89 \end{bmatrix}$$

$$U^{(1)} = \begin{bmatrix} 0.144 & 0.1125 \\ 0.652 & 0.225 \end{bmatrix}^T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.92 \\ 0.563 \end{bmatrix}$$

$$Z^{(1)} = \begin{bmatrix} 1 \\ \tanh(0.92) \\ \tanh(0.563) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.716 \\ 0.51 \end{bmatrix}$$

$$U^{(2)} = \begin{bmatrix} 0.246 & 0.223 \\ 1.028 & 1.014 \\ -2.965 & -2.183 \end{bmatrix}^T \begin{bmatrix} 0.716 \\ 0.51 \end{bmatrix} = \begin{bmatrix} -0.52 \\ -0.562 \end{bmatrix}$$

$$Z^{(2)} = \begin{bmatrix} 1 \\ \tanh(-0.52) \\ \tanh(-0.562) \end{bmatrix} = \begin{bmatrix} 1 \\ -0.498 \\ -0.509 \end{bmatrix}$$

$$y^{(3)} = \tanh(-0.234) = -0.22788$$

$$U^{(3)} = \begin{bmatrix} 1.122 \\ 1.89 \\ 0.89 \end{bmatrix}^T \begin{bmatrix} -0.498 \\ -0.509 \end{bmatrix} = -0.239$$