More generally, we will need to describe the movement of charge from one place to another. This is captured by a quantity known as the *current density* $\mathbf{J}(\mathbf{x}, t)$, defined as follows: for every surface S, the integral

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

counts the charge per unit time passing through S. (Here $d\mathbf{S}$ is the unit normal to S). The quantity I is called the *current*. In this sense, the current density is the current-per-unit-area.

The above is a rather indirect definition of the current density. To get a more intuitive picture, consider a continuous charge distribution in which the velocity of a small volume, at point $\mathbf{x}$, is given by $\mathbf{v}(\mathbf{x}, t)$. Then, neglecting relativistic effects, the current density is

$$\mathbf{J} = \rho \mathbf{v}$$

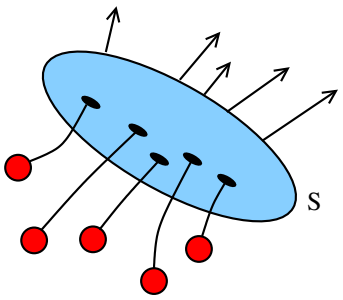

Figure 1: Current flux

In particular, if a single particle is moving with velocity $\mathbf{v} = \dot{\mathbf{r}}(t)$, the current density will be $\mathbf{J} = q\mathbf{v}\delta^3(\mathbf{x} - \mathbf{r}(t))$. This is illustrated in the figure, where the underlying charged particles are shown as red balls, moving through the blue surface S.

As a simple example, consider electrons moving along a wire. We model the wire as a long cylinder of cross-sectional area A as shown below. The electrons move with velocity $\mathbf{v}$, parallel to the axis of the wire. (In reality, the electrons will have some distribution of speeds; we take $\mathbf{v}$ to be their average velocity). If there are n electrons per unit volume, each with charge q, then the charge density is $\rho = nq$ and the current density is $\mathbf{J} = nq\mathbf{v}$. The current itself is $I = |\mathbf{J}|A$.

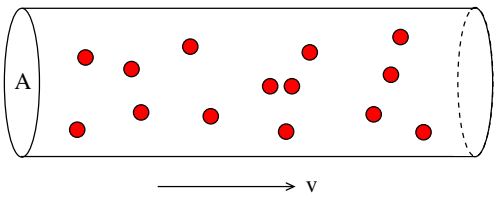

Figure 2: The wire

Throughout this course, the current density $\mathbf{J}$ plays a much more prominent role than the current I. For this reason, we will often refer to $\mathbf{J}$ simply as the "current" although we'll be more careful with the terminology when there is any possibility for confusion.