

More generally, we will need to describe the movement of charge from one place to another. This is captured by a quantity known as the *current density*  $\mathbf{J}(\mathbf{x}, t)$ , defined as follows: for every surface  $S$ , the integral

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

counts the charge per unit time passing through  $S$ . (Here  $d\mathbf{S}$  is the unit normal to  $S$ ). The quantity  $I$  is called the *current*. In this sense, the current density is the current-per-unit-area.

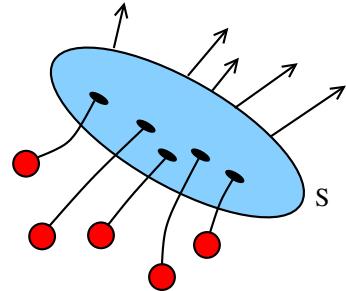
The above is a rather indirect definition of the current density. To get a more intuitive picture, consider a continuous charge distribution in which the velocity of a small volume, at point  $\mathbf{x}$ , is given by  $\mathbf{v}(\mathbf{x}, t)$ . Then, neglecting relativistic effects, the current density is

$$\mathbf{J} = \rho \mathbf{v}$$

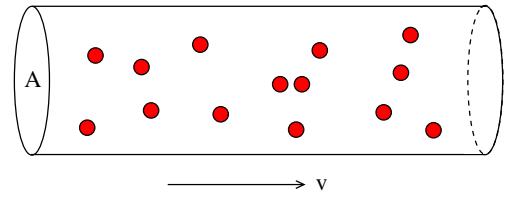
In particular, if a single particle is moving with velocity  $\mathbf{v} = \dot{\mathbf{r}}(t)$ , the current density will be  $\mathbf{J} = q\mathbf{v}\delta^3(\mathbf{x} - \mathbf{r}(t))$ .

This is illustrated in the figure, where the underlying charged particles are shown as red balls, moving through the blue surface  $S$ .

As a simple example, consider electrons moving along a wire. We model the wire as a long cylinder of cross-sectional area  $A$  as shown below. The electrons move with velocity  $\mathbf{v}$ , parallel to the axis of the wire. (In reality, the electrons will have some distribution of speeds; we take  $\mathbf{v}$  to be their average velocity). If there are  $n$  electrons per unit volume, each with charge  $q$ , then the charge density is  $\rho = nq$  and the current density is  $\mathbf{J} = nq\mathbf{v}$ . The current itself is  $I = |\mathbf{J}|A$ .



**Figure 1:** Current flux



**Figure 2:** The wire

Throughout this course, the current density  $\mathbf{J}$  plays a much more prominent role than the current  $I$ . For this reason, we will often refer to  $\mathbf{J}$  simply as the “current” although we’ll be more careful with the terminology when there is any possibility for confusion.