Let's pick our Gaussian surface to be a sphere, centered at the origin, of radius $r < R$. The charge contained within this sphere is $4\pi\rho r^3/3 = Qr^3/R^3$, so Gauss' law gives

$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Qr^3}{\epsilon_0 R^3}$$

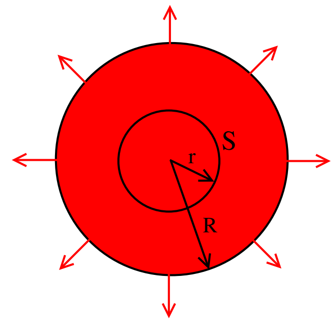

Figure 6:

Again, using the symmetry argument we can write $\mathbf{E}(\mathbf{r}) = E(r)\hat{\mathbf{r}}$ and compute

$$\int_S \mathbf{E} \cdot d\mathbf{S} = E(r) \int_S \hat{\mathbf{r}} \cdot d\mathbf{S} = E(r)\, 4\pi r^2 = \frac{Qr^3}{\epsilon_0 R^3}$$

This tells us that the electric field grows linearly inside the sphere

$$\mathbf{E}(\mathbf{x}) = \frac{Qr}{4\pi\epsilon_0 R^3}\,\hat{\mathbf{r}} \qquad r < R \tag{2.5}$$

Outside the sphere we revert to the inverse-square form (2.4). At the surface of the sphere, $r = R$, the electric field is continuous but the derivative, dE/dr, is not. This is shown in the graph.

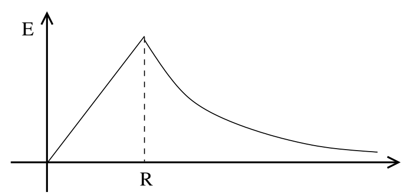

Figure 7:

2.1.3 Line Charges

Consider, next, a charge smeared out along a line which we'll take to be the z-axis. We'll take uniform charge density η per unit length. (If you like you could consider a solid cylinder with uniform charge density and then send the radius to zero). We want to know the electric field due to this line of charge.

Our set-up now has cylindrical symmetry. We take the Gaussian surface to be a cylinder of length L and radius r. We have

$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{\eta L}{\epsilon_0}$$

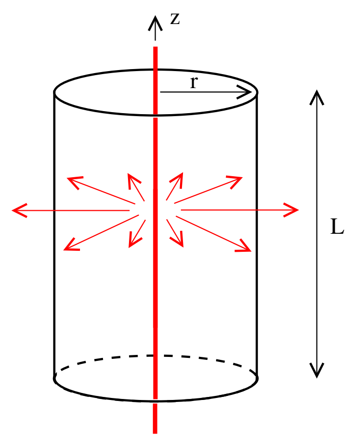

Figure 8:

Again, by symmetry, the electric field points in the radial direction, away from the line. We'll denote this vector in cylindrical polar coordinates as $\hat{\mathbf{r}}$ so that $\mathbf{E} = E(r)\hat{\mathbf{r}}$. The symmetry means that the two end caps of the Gaussian