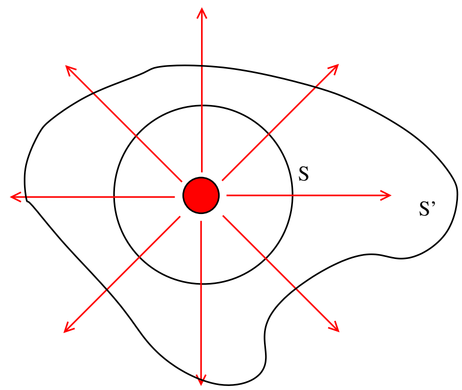


Figure 3: The flux through S and S' is the same.

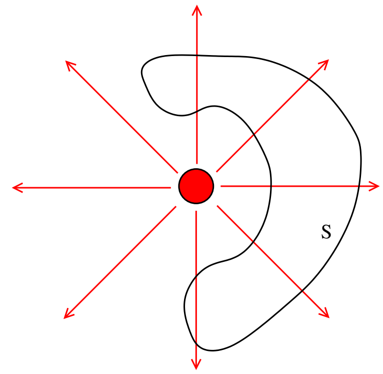


Figure 4: The flux through S vanishes.

Notice that it doesn't matter what shape the surface S takes. As long as it surrounds a total charge Q, the flux through the surface will always be Q/ϵ_0. This is shown, for example, in the left-hand figure above. A fancy way of saying this is that the integral of the flux doesn't depend on the geometry of the surface, but does depend on its topology since it must surround the charge Q. The choice of S is called the *Gaussian surface*; often there's a smart choice that makes a particular problem simple.

Only charges that lie inside V contribute to the flux. Any charges that lie outside will produce an electric field that penetrates through S at some point, giving negative flux, but leaves through the other side of S, depositing positive flux. The total contribution from these charges that lie outside of V is zero, as illustrated in the right-hand figure above.

For a general charge distribution, we'll need to use both Gauss' law (2.1) and the extra equation (2.2). However, for rather special charge distributions – typically those with lots of symmetry – it turns out to be sufficient to solve the integral form of Gauss' law (2.3) alone, with the symmetry ensuring that (2.2) is automatically satisfied. We start by describing these rather simple solutions. We'll then return to the general case in Section 2.2.

2.1.1 The Coulomb Force

We'll start by showing that Gauss' law (2.3) reproduces the more familiar Coulomb force law that we all know and love. To do this, take a spherically symmetric charge distribution, centered at the origin, contained within some radius R. This will be our model for a particle. We won't need to make any assumption about the nature of the distribution other than its symmetry and the fact that the total charge is Q.

Let's pick our Gaussian surface to be a sphere, centered at the origin, of radius $r < R$. The charge contained within this sphere is $4\pi\rho r^3/3 = Qr^3/R^3$, so Gauss' law gives

$$\int_S \mathbf{E}\cdot d\mathbf{S} = \frac{Qr^3}{\epsilon_0 R^3}$$

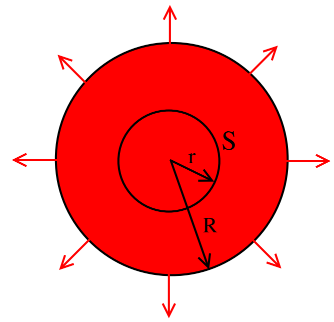


Figure 6:

Again, using the symmetry argument we can write $\mathbf{E}(\mathbf{r}) = E(r)\hat{\mathbf{r}}$ and compute

$$\int_S \mathbf{E}\cdot d\mathbf{S} = E(r)\int_S \hat{\mathbf{r}}\cdot d\mathbf{S} = E(r)\,4\pi r^2 = \frac{Qr^3}{\epsilon_0 R^3}$$

This tells us that the electric field grows linearly inside the sphere

$$\mathbf{E}(\mathbf{x}) = \frac{Qr}{4\pi\epsilon_0 R^3}\,\hat{\mathbf{r}} \qquad r < R \tag{2.5}$$

Outside the sphere we revert to the inverse-square form (2.4). At the surface of the sphere, $r = R$, the electric field is continuous but the derivative, dE/dr, is not. This is shown in the graph.

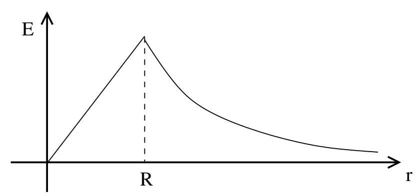


Figure 7:

2.1.3 Line Charges

Consider, next, a charge smeared out along a line which we'll take to be the z-axis. We'll take uniform charge density η per unit length. (If you like you could consider a solid cylinder with uniform charge density and then send the radius to zero). We want to know the electric field due to this line of charge.

Our set-up now has cylindrical symmetry. We take the Gaussian surface to be a cylinder of length L and radius r. We have

$$\int_S \mathbf{E}\cdot d\mathbf{S} = \frac{\eta L}{\epsilon_0}$$

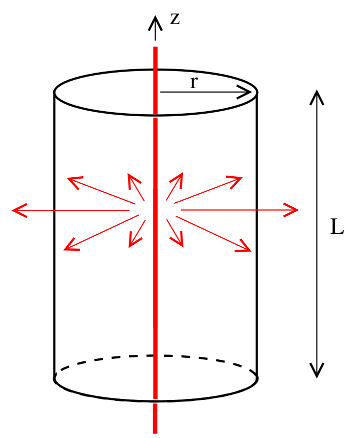
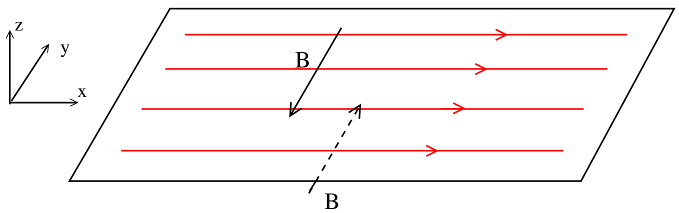


Figure 8:

Again, by symmetry, the electric field points in the radial direction, away from the line. We'll denote this vector in cylindrical polar coordinates as $\hat{\mathbf{r}}$ so that $\mathbf{E} = E(r)\hat{\mathbf{r}}$. The symmetry means that the two end caps of the Gaussian

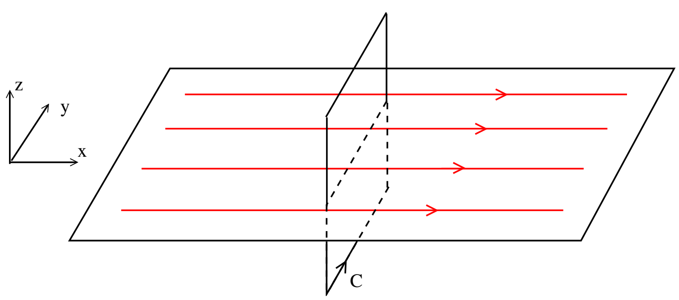
B is oriented along the **y** direction. In fact, from the symmetry of the problem, it must look like



with **B** pointing in the $-\hat{\mathbf{y}}$ direction when $z > 0$ and in the $+\hat{\mathbf{y}}$ direction when $z < 0$. We write

$$\mathbf{B} = -B(z)\hat{\mathbf{y}}$$

with $B(z) = -B(-z)$. We invoke Ampère's law using the following open surface:



with length L in the y direction and extending to $\pm z$. We have

$$\oint_C \mathbf{B} \cdot d\mathbf{r} = LB(z) - LB(-z) = 2LB(z) = \mu_0 KL$$

so we find that the magnetic field is constant above an infinite plane of surface current

$$B(z) = \frac{\mu_0 K}{2} \qquad z > 0$$

This is rather similar to the case of the electric field in the presence of an infinite plane of surface charge.

The analogy with electrostatics continues. The magnetic field is not continuous across a plane of surface current. We have

$$B(z \to 0^+) - B(z \to 0^-) = \mu_0 K$$

In fact, this is a general result that holds for any surface current **K**. We can prove this statement by using the same curve that we used in the Figure above and shrinking it