
Script to run the EM algorithm for Q2

```
clear; close all;
% initialize the label image and parameters.
[X, Y, mean_init, std_init] = init();
```

Motivation -

Since we are dealing with a clustering problem, a good initialization to the labels can be achieved using K-means clustering on the noisy image. Based on the initially assigned label, the initial means and standard deviations are computed.

```
mu = mean_init; sigma = std_init;
X_inp = X;

figure;
imshow(Y,[]);
colorbar;
title('Original Noisy Image');

figure;
imshow(X,[]);
colorbar;
title('Initialized Image using K means');

% initial estimate of parameters -:
disp('initialized means'); disp(mu);
disp('initialized sigmas'); disp(sigma);

% EM Loop
for i = 1:50
    [mem,X_map,post_before, post_after] = Estep(X_inp,Y,mu,sigma,0.8);
    [mu,sigma] = Mstep(mem,Y);
    prob_arr(i) = post_after;
    X_inp = X_map;
    fprintf('iteration %f. | log posterior before ICM beta = 0.8 %f. \n',i,post_before);
    fprintf('log posterior after ICM beta = 0.8 %f. \n',post_after);
end

% The optimal estimates for class means are -:
disp('optimal class mean estimates for beta = 0.8'); disp(mu);
[~, label_img] = max(mem);
label_img = squeeze(label_img); label_img(X==0) = 0;

figure;
subplot(2,2,1)
imshow(squeeze(mem(1,:,:)),[]);
colorbar;
title('Membership for class 1');
subplot(2,2,2)
imshow(squeeze(mem(2,:,:)),[]);
```

```

colorbar;
title('Membership for class 2');
subplot(2,2,3)
imshow(squeeze(mem(3,:,:)),[]);
colorbar;
title('Membership for class 3');
subplot(2,2,4)
imshow(label_img,[]);
colorbar;
title('Label Image');
suptitle('beta = 0.8');

figure;
plot(prob_arr);
xlabel('iteration number'); ylabel('log Posterior'); title('log
    Posterior Probability vs Iterations beta = 0.8');

mu = mean_init; sigma = std_init;
X_inp = X;

for i = 1:50
    [mem,X_map,post_before, post_after] = Estep(X_inp,Y,mu,sigma,0);
    [mu,sigma] = Mstep(mem,Y);
    prob_arr(i) = post_after;
    X_inp = X_map;
    fprintf('iteration %f. | log posterior before ICM beta = 0 %f.
    \n',i,post_before);
    fprintf('log posterior after ICM beta = 0 %f. \n',post_after);
end

[~, label_img] = max(mem);
label_img = squeeze(label_img); label_img(X==0) = 0;

figure;
subplot(2,2,1)
imshow(squeeze(mem(1,:,:)),[]);
colorbar;
title('Membership for class 1');
subplot(2,2,2)
imshow(squeeze(mem(2,:,:)),[]);
colorbar;
title('Membership for class 2');
subplot(2,2,3)
imshow(squeeze(mem(3,:,:)),[]);
colorbar;
title('Membership for class 3');
subplot(2,2,4)
imshow(label_img,[]);
colorbar;
title('Label Image');
suptitle('beta = 0');

```

```

figure;
plot(prob_arr);
xlabel('iteration number'); ylabel('log Posterior'); title('log
  Posterior Probability vs Iterations beta = 0');

initialized means
    0.6304
    0.5101
    0.2931

initialized sigmas
    0.0358
    0.0413
    0.0655

iteration 1.000000. | log posterior before ICM beta = 0.8
-14574.400000.
log posterior after ICM beta = 0.8 -10649.377478.
iteration 2.000000. | log posterior before ICM beta = 0.8
-10511.684591.
log posterior after ICM beta = 0.8 -10366.823182.
iteration 3.000000. | log posterior before ICM beta = 0.8
-10340.570802.
log posterior after ICM beta = 0.8 -9747.911148.
iteration 4.000000. | log posterior before ICM beta = 0.8
-9821.239864.
log posterior after ICM beta = 0.8 -10000.088788.
iteration 5.000000. | log posterior before ICM beta = 0.8
-9855.452703.
log posterior after ICM beta = 0.8 -10024.469071.
iteration 6.000000. | log posterior before ICM beta = 0.8
-9873.956538.
log posterior after ICM beta = 0.8 -10035.035460.
iteration 7.000000. | log posterior before ICM beta = 0.8
-9884.543767.
log posterior after ICM beta = 0.8 -10040.464224.
iteration 8.000000. | log posterior before ICM beta = 0.8
-9890.732671.
log posterior after ICM beta = 0.8 -10045.439734.
iteration 9.000000. | log posterior before ICM beta = 0.8
-9894.381679.
log posterior after ICM beta = 0.8 -10048.361996.
iteration 10.000000. | log posterior before ICM beta = 0.8
-9896.541180.
log posterior after ICM beta = 0.8 -10050.089336.
iteration 11.000000. | log posterior before ICM beta = 0.8
-9897.821426.
log posterior after ICM beta = 0.8 -10051.112585.
iteration 12.000000. | log posterior before ICM beta = 0.8
-9898.581083.
log posterior after ICM beta = 0.8 -10051.719181.
iteration 13.000000. | log posterior before ICM beta = 0.8
-9899.032051.
log posterior after ICM beta = 0.8 -10052.078835.

```

```
iteration 14.000000. | log posterior before ICM beta = 0.8
-9899.299838.
log posterior after ICM beta = 0.8 -10052.292096.
iteration 15.000000. | log posterior before ICM beta = 0.8
-9899.458874.
log posterior after ICM beta = 0.8 -10052.418756.
iteration 16.000000. | log posterior before ICM beta = 0.8
-9899.553333.
log posterior after ICM beta = 0.8 -10052.493988.
iteration 17.000000. | log posterior before ICM beta = 0.8
-9899.609438.
log posterior after ICM beta = 0.8 -10052.538675.
iteration 18.000000. | log posterior before ICM beta = 0.8
-9899.642765.
log posterior after ICM beta = 0.8 -10052.565219.
iteration 19.000000. | log posterior before ICM beta = 0.8
-9899.662561.
log posterior after ICM beta = 0.8 -10052.580986.
iteration 20.000000. | log posterior before ICM beta = 0.8
-9899.674320.
log posterior after ICM beta = 0.8 -10052.590352.
iteration 21.000000. | log posterior before ICM beta = 0.8
-9899.681305.
log posterior after ICM beta = 0.8 -10052.595916.
iteration 22.000000. | log posterior before ICM beta = 0.8
-9899.685454.
log posterior after ICM beta = 0.8 -10052.599220.
iteration 23.000000. | log posterior before ICM beta = 0.8
-9899.687918.
log posterior after ICM beta = 0.8 -10052.601183.
iteration 24.000000. | log posterior before ICM beta = 0.8
-9899.689382.
log posterior after ICM beta = 0.8 -10052.602349.
iteration 25.000000. | log posterior before ICM beta = 0.8
-9899.690252.
log posterior after ICM beta = 0.8 -10052.603042.
iteration 26.000000. | log posterior before ICM beta = 0.8
-9899.690769.
log posterior after ICM beta = 0.8 -10052.603454.
iteration 27.000000. | log posterior before ICM beta = 0.8
-9899.691076.
log posterior after ICM beta = 0.8 -10052.603698.
iteration 28.000000. | log posterior before ICM beta = 0.8
-9899.691258.
log posterior after ICM beta = 0.8 -10052.603843.
iteration 29.000000. | log posterior before ICM beta = 0.8
-9899.691366.
log posterior after ICM beta = 0.8 -10052.603930.
iteration 30.000000. | log posterior before ICM beta = 0.8
-9899.691430.
log posterior after ICM beta = 0.8 -10052.603981.
iteration 31.000000. | log posterior before ICM beta = 0.8
-9899.691469.
log posterior after ICM beta = 0.8 -10052.604011.
```

```
iteration 32.000000. | log posterior before ICM beta = 0.8
-9899.691491.
log posterior after ICM beta = 0.8 -10052.604029.
iteration 33.000000. | log posterior before ICM beta = 0.8
-9899.691505.
log posterior after ICM beta = 0.8 -10052.604040.
iteration 34.000000. | log posterior before ICM beta = 0.8
-9899.691513.
log posterior after ICM beta = 0.8 -10052.604046.
iteration 35.000000. | log posterior before ICM beta = 0.8
-9899.691518.
log posterior after ICM beta = 0.8 -10052.604050.
iteration 36.000000. | log posterior before ICM beta = 0.8
-9899.691520.
log posterior after ICM beta = 0.8 -10052.604052.
iteration 37.000000. | log posterior before ICM beta = 0.8
-9899.691522.
log posterior after ICM beta = 0.8 -10052.604054.
iteration 38.000000. | log posterior before ICM beta = 0.8
-9899.691523.
log posterior after ICM beta = 0.8 -10052.604055.
iteration 39.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604055.
iteration 40.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604055.
iteration 41.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604055.
iteration 42.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 43.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 44.000000. | log posterior before ICM beta = 0.8
-9899.691524.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 45.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 46.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 47.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 48.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
iteration 49.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
```

```

iteration 50.000000. | log posterior before ICM beta = 0.8
-9899.691525.
log posterior after ICM beta = 0.8 -10052.604056.
optimal class mean estimates for beta = 0.8
    0.6321
    0.5218
    0.3192

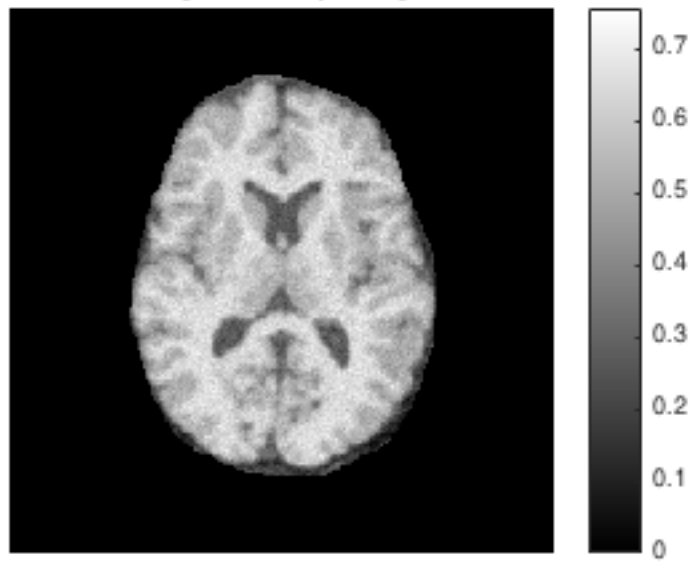
iteration 1.000000. | log posterior before ICM beta = 0
-10488.000000.
log posterior after ICM beta = 0 -10013.797551.
iteration 2.000000. | log posterior before ICM beta = 0 -8751.659950.
log posterior after ICM beta = 0 -8428.806551.
iteration 3.000000. | log posterior before ICM beta = 0 -8042.707405.
log posterior after ICM beta = 0 -7706.506309.
iteration 4.000000. | log posterior before ICM beta = 0 -7579.314267.
log posterior after ICM beta = 0 -7154.198419.
iteration 5.000000. | log posterior before ICM beta = 0 -7237.143053.
log posterior after ICM beta = 0 -6841.205302.
iteration 6.000000. | log posterior before ICM beta = 0 -7339.740868.
log posterior after ICM beta = 0 -6842.443802.
iteration 7.000000. | log posterior before ICM beta = 0 -7189.309011.
log posterior after ICM beta = 0 -6905.084400.
iteration 8.000000. | log posterior before ICM beta = 0 -7115.826190.
log posterior after ICM beta = 0 -7051.430925.
iteration 9.000000. | log posterior before ICM beta = 0 -7177.764002.
log posterior after ICM beta = 0 -7156.742216.
iteration 10.000000. | log posterior before ICM beta = 0
-7237.827079.
log posterior after ICM beta = 0 -7229.448614.
iteration 11.000000. | log posterior before ICM beta = 0
-7285.411048.
log posterior after ICM beta = 0 -7279.772521.
iteration 12.000000. | log posterior before ICM beta = 0
-7319.122953.
log posterior after ICM beta = 0 -7314.762600.
iteration 13.000000. | log posterior before ICM beta = 0
-7342.469422.
log posterior after ICM beta = 0 -7339.009538.
iteration 14.000000. | log posterior before ICM beta = 0
-7357.996616.
log posterior after ICM beta = 0 -7355.198992.
iteration 15.000000. | log posterior before ICM beta = 0
-7367.770640.
log posterior after ICM beta = 0 -7364.845211.
iteration 16.000000. | log posterior before ICM beta = 0
-7372.020110.
log posterior after ICM beta = 0 -7369.651608.
iteration 17.000000. | log posterior before ICM beta = 0
-7373.256399.
log posterior after ICM beta = 0 -7370.926119.
iteration 18.000000. | log posterior before ICM beta = 0
-7371.536698.
log posterior after ICM beta = 0 -7368.873235.

```

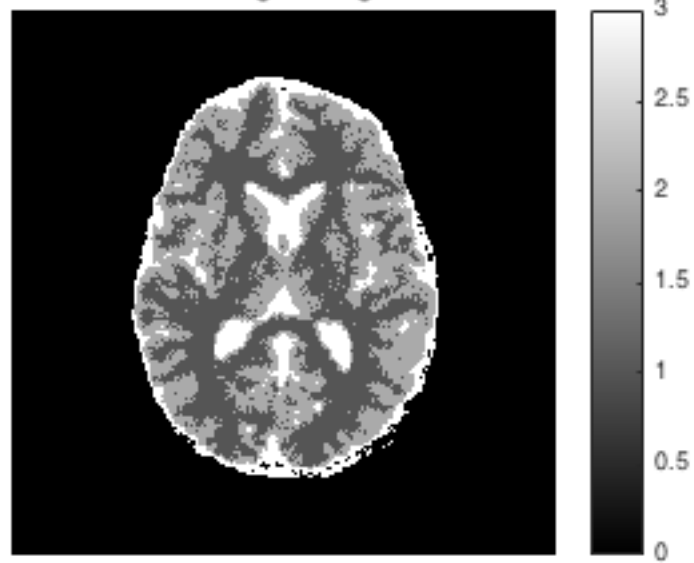
```
iteration 19.000000. | log posterior before ICM beta = 0
-7366.726517.
log posterior after ICM beta = 0 -7363.838625.
iteration 20.000000. | log posterior before ICM beta = 0
-7359.026699.
log posterior after ICM beta = 0 -7356.549981.
iteration 21.000000. | log posterior before ICM beta = 0
-7350.404666.
log posterior after ICM beta = 0 -7347.550008.
iteration 22.000000. | log posterior before ICM beta = 0
-7339.250446.
log posterior after ICM beta = 0 -7336.901191.
iteration 23.000000. | log posterior before ICM beta = 0
-7327.730667.
log posterior after ICM beta = 0 -7324.739382.
iteration 24.000000. | log posterior before ICM beta = 0
-7313.783595.
log posterior after ICM beta = 0 -7311.409538.
iteration 25.000000. | log posterior before ICM beta = 0
-7299.784956.
log posterior after ICM beta = 0 -7297.340046.
iteration 26.000000. | log posterior before ICM beta = 0
-7284.858835.
log posterior after ICM beta = 0 -7282.646446.
iteration 27.000000. | log posterior before ICM beta = 0
-7269.714383.
log posterior after ICM beta = 0 -7267.288929.
iteration 28.000000. | log posterior before ICM beta = 0
-7253.426194.
log posterior after ICM beta = 0 -7251.652420.
iteration 29.000000. | log posterior before ICM beta = 0
-7237.854366.
log posterior after ICM beta = 0 -7236.334801.
iteration 30.000000. | log posterior before ICM beta = 0
-7222.874430.
log posterior after ICM beta = 0 -7220.992208.
iteration 31.000000. | log posterior before ICM beta = 0
-7207.110127.
log posterior after ICM beta = 0 -7205.684651.
iteration 32.000000. | log posterior before ICM beta = 0
-7192.151834.
log posterior after ICM beta = 0 -7190.747135.
iteration 33.000000. | log posterior before ICM beta = 0
-7177.665991.
log posterior after ICM beta = 0 -7176.445030.
iteration 34.000000. | log posterior before ICM beta = 0
-7163.539711.
log posterior after ICM beta = 0 -7162.444500.
iteration 35.000000. | log posterior before ICM beta = 0
-7149.961671.
log posterior after ICM beta = 0 -7149.169687.
iteration 36.000000. | log posterior before ICM beta = 0
-7137.556035.
log posterior after ICM beta = 0 -7136.915398.
```

```
iteration 37.000000. | log posterior before ICM beta = 0
-7126.302655.
log posterior after ICM beta = 0 -7125.820804.
iteration 38.000000. | log posterior before ICM beta = 0
-7116.063872.
log posterior after ICM beta = 0 -7115.636154.
iteration 39.000000. | log posterior before ICM beta = 0
-7106.715794.
log posterior after ICM beta = 0 -7106.302923.
iteration 40.000000. | log posterior before ICM beta = 0
-7098.132301.
log posterior after ICM beta = 0 -7097.838029.
iteration 41.000000. | log posterior before ICM beta = 0
-7090.434430.
log posterior after ICM beta = 0 -7090.231806.
iteration 42.000000. | log posterior before ICM beta = 0
-7083.630597.
log posterior after ICM beta = 0 -7083.476081.
iteration 43.000000. | log posterior before ICM beta = 0
-7077.549755.
log posterior after ICM beta = 0 -7077.436509.
iteration 44.000000. | log posterior before ICM beta = 0
-7072.196226.
log posterior after ICM beta = 0 -7072.122208.
iteration 45.000000. | log posterior before ICM beta = 0
-7067.523004.
log posterior after ICM beta = 0 -7067.442279.
iteration 46.000000. | log posterior before ICM beta = 0
-7063.411597.
log posterior after ICM beta = 0 -7063.362482.
iteration 47.000000. | log posterior before ICM beta = 0
-7059.839039.
log posterior after ICM beta = 0 -7059.771346.
iteration 48.000000. | log posterior before ICM beta = 0
-7056.659469.
log posterior after ICM beta = 0 -7056.623011.
iteration 49.000000. | log posterior before ICM beta = 0
-7053.915469.
log posterior after ICM beta = 0 -7053.891895.
iteration 50.000000. | log posterior before ICM beta = 0
-7051.542786.
log posterior after ICM beta = 0 -7051.527628.
```

Original Noisy Image

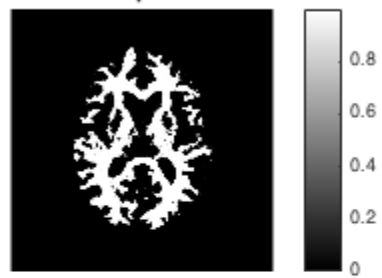


Initialized Image using K means

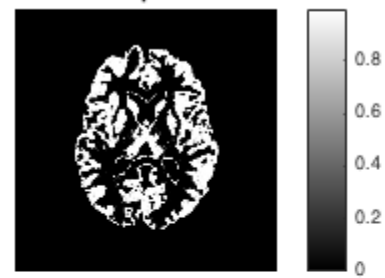


beta = 0.8

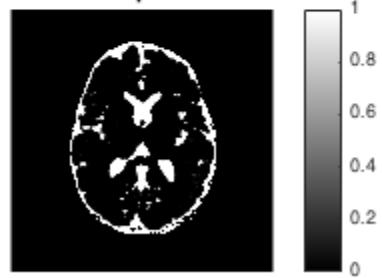
Membership for class 1



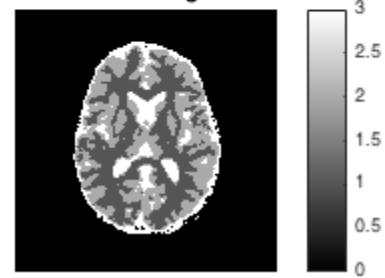
Membership for class 2

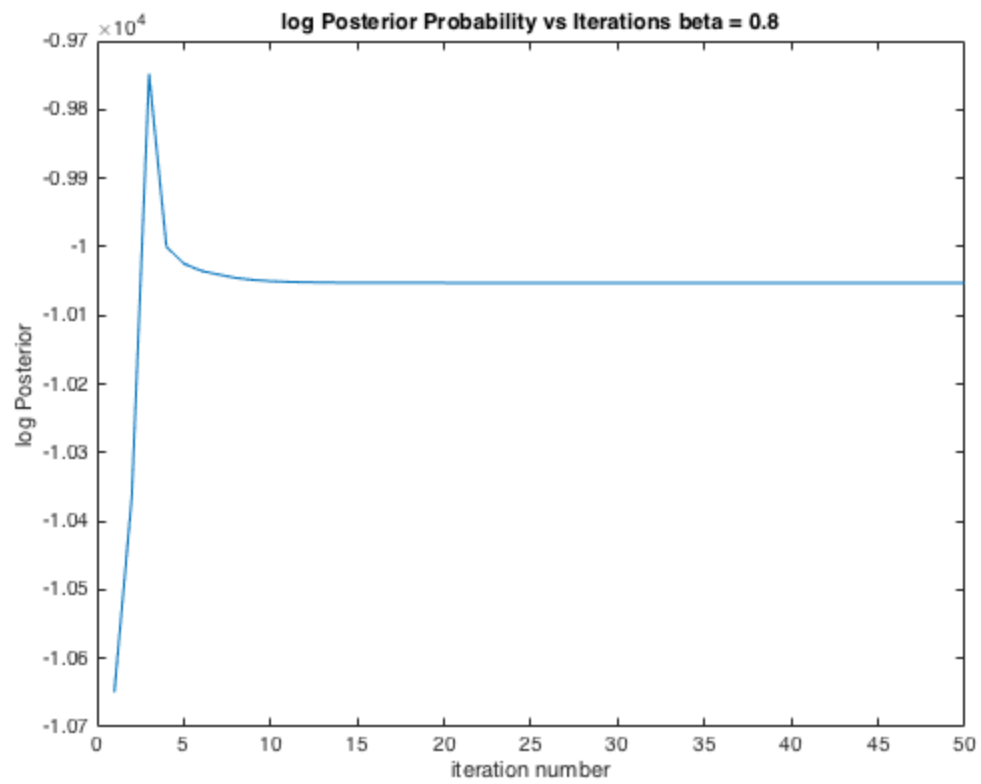


Membership for class 3



Label Image



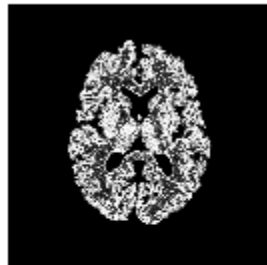


beta = 0

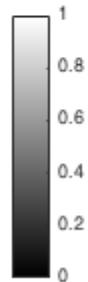
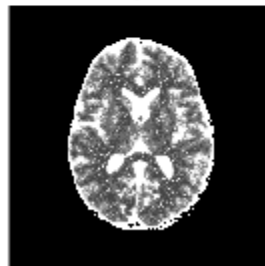
Membership for class 1



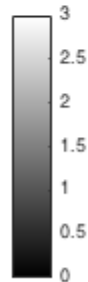
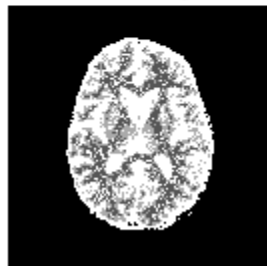
Membership for class 2

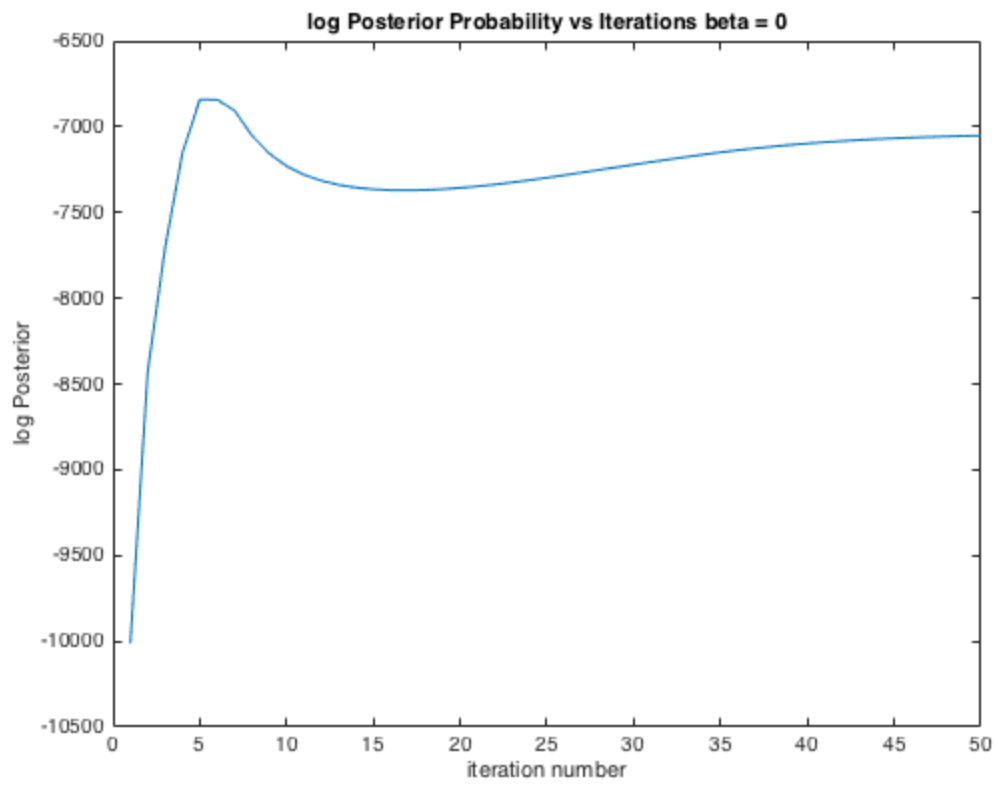


Membership for class 3



Label Image





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