Single Image Super Resolution using sparse linear representations for medical imaging

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Super Resolution Imaging

- Generating a magnified high resolution image from a low resolution input.
- Applications in various fields like Medical imaging, remote sensing etc.
- The problem can be posed in two scenarios - Multi-image SR and single image SR
- Multi-image SR doesn't scale very well, since blurring effects and registering multiple noisy images is a hard task.
- We focus on a single image SR method in this project in context of medical imaging

Challenges in Super Resolution for medical imaging

- Noisy data
- Spatial smoothness
- Repetition of local structures

Super Resolution using sparse weights - joint denoising and super resolution

- Dataset construction
- Patch SR
- Reconstruction of entire HR image

Database construction

- $$\mathbf{p}_k^l = D_s H \mathbf{p}_k^h$$

$$(\mathbf{P}_l, \mathbf{P}_h) = \left\{ (\mathbf{u}_k^l, \mathbf{u}_k^h) = \left(\frac{\mathbf{p}_k^l}{\|\mathbf{p}_k^l\|}, \frac{\mathbf{p}_k^h}{\|\mathbf{p}_k^l\|} \right), \quad k \in \mathcal{I} \right\}$$

- Generate a dataset from the original high resolution image images. The dataset consists of 2 matrices of corresponding vectorized LR and HR image.

Patch super resolution -:

- Estimate a high resolution output patch as a sparsely weighted non-negative linear combination of the high resolution patches in the constructed dataset.

$$\mathbf{x}_i^h = \sum_{k \in \mathcal{I}} \alpha_{ik} \mathbf{u}_k^h$$

- The same sparse weights can be used to represent the low resolution input as a sparsely weighted non-negative linear combination of the low resolution patches in the constructed dataset.

$$\mathbf{y}_i^l = D_s H \mathbf{x}_i^h + \boldsymbol{\eta}_i = \mu_{ik} \mathbf{u}_k^l + \boldsymbol{\eta}_i$$

$$\begin{aligned} \boldsymbol{\alpha}^i = \arg\min_{\boldsymbol{\alpha} \geq 0} \quad & \|\boldsymbol{\alpha}\|_0 + \sum_{k \in \mathcal{I}} w_{ik} \alpha_{ik} \\ \text{subject to} \quad & \|\mathbf{y}_i^l - \sum_{k \in \mathcal{I}} \alpha_{ik} \mathbf{u}_k^l\|_2^2 \leq \epsilon \sigma_i^2, \end{aligned}$$

$$w_{ik} = \Phi_i(d(\mathbf{y}_i^l, \mathbf{u}_k^l))$$

$$\mu_{ik} = \frac{\mathrm{E}(\mathbf{y}_i^l)}{\mathrm{E}(\mathbf{u}_k^l)}$$

$$a_{ik} = \left|\mathrm{E}(\mathbf{y}_i^l - \mu_{ik} \mathbf{u}_k^l)\right| + \left|\mathrm{Var}(\mathbf{y}_i^l - \mu_{ik} \mathbf{u}_k^l) - \sigma_i^2\right| \simeq 0.$$

$$d(\mathbf{y}_i^l, \mathbf{u}_k^l) = \|\mathbf{y}_i^l - \mu_{ik} \mathbf{u}_k^l\|_2^2 + a_{ik}.$$

$$\Phi_i(t) = \begin{cases} e^t & \text{if } t > \rho_i \\ t & \text{if } t \leq \rho_i \end{cases}$$

$$\boldsymbol{\alpha}^i = \arg\min_{\boldsymbol{\alpha} \geq 0} \frac{1}{2} \|\mathbf{y}_i^l - \sum_{k \in \mathcal{I}_i} \alpha_{ik} \mathbf{u}_k^l\|_2^2 + \lambda \sum_{k \in \mathcal{I}_i} (1 + w_{ik}) \alpha_{ik}$$

$$\boldsymbol{\alpha}^i = \arg\min_{\boldsymbol{\alpha} \geq 0} \frac{1}{2} \|\mathbf{y}_i^l - \mathbf{U}_i \boldsymbol{\alpha}\|_2^2 + \mathbf{w}_i^T \boldsymbol{\alpha}$$

Algorithm 1 Multiplicative Updates Algorithm for NQP

Input: $\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 > 0$, number of iterations T.

Updating: $t = 0$

While $t < T$ & $\left\|\mathbf{y}_i^l - \mathbf{U}_i \boldsymbol{\alpha}_t\right\|_2^2 > m\sigma_i^2$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\alpha}_t .^* (\mathbf{U}_i^T \mathbf{y}_i^l) ./ (\mathbf{U}_i^T \mathbf{U}_i \boldsymbol{\alpha}_t + \mathbf{w}_i);$$

$$t = t + 1;$$

End

Output: $\boldsymbol{\alpha}^i = \boldsymbol{\alpha}_t$.

SR estimates using sparse coefficients

$$\hat{\mathbf{x}}_i^h = \sum_{k \in \mathcal{I}_i} \alpha_{ik} \mathbf{u}_k^h$$

$$\hat{\mathbf{y}}_i^l = \mathbf{U}_i \boldsymbol{\alpha}^i = \sum_{k \in \mathcal{I}_i} \alpha_{ik} \mathbf{u}_k^l$$

Reconstruction of entire HR image

$\hat{\mathbf{X}}^{coarse}$ - Estimate of HR image by averaging the overlapping high resolution patches estimated.

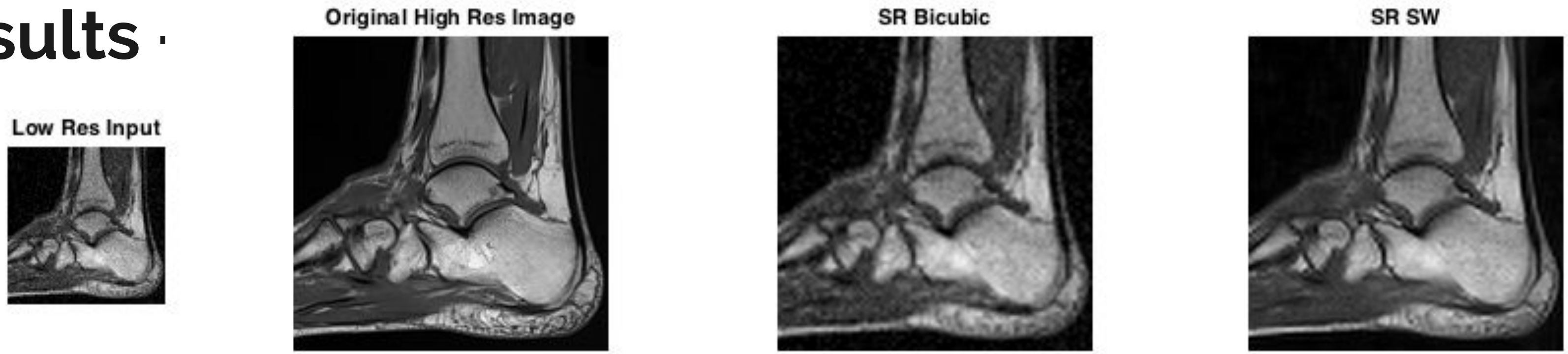
$\mathbf{Y}^{denoise}$: Estimate of denoised LR image by averaging the overlapping high resolution patches estimated.

- Iterative back projection to estimate the final high resolution output image

$$\min_{\mathbf{X}} \|\mathbf{X} - \hat{\mathbf{X}}^{coarse}\|_2^2 \quad \text{subject to} \quad D_s H\mathbf{X} = \mathbf{Y}^{denoise}$$

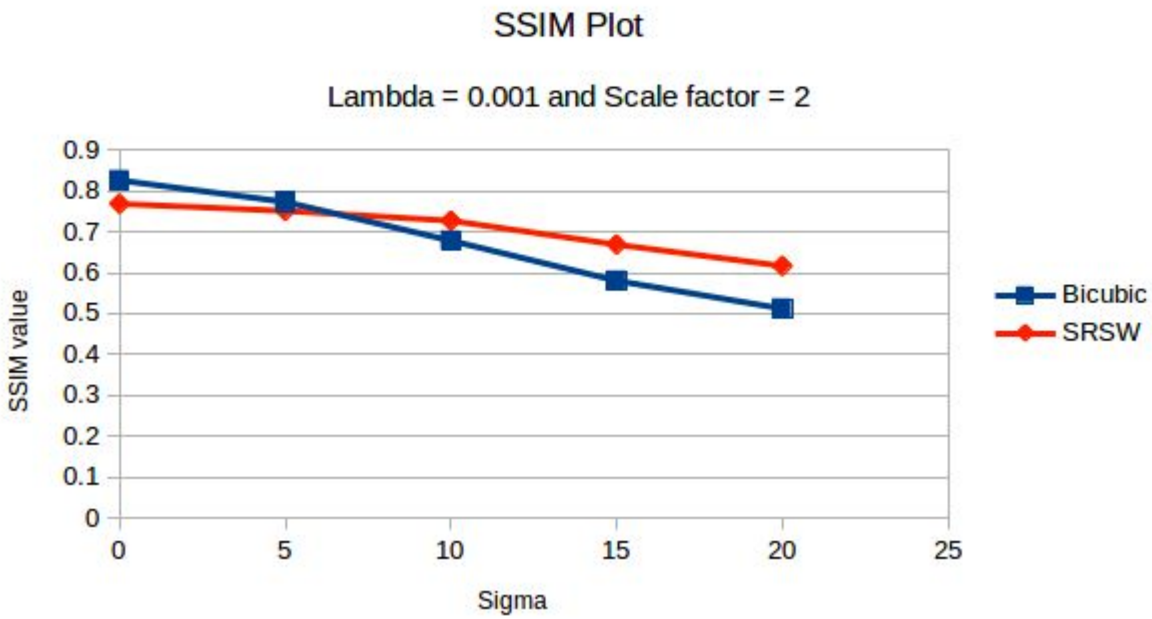
$$\mathbf{X}_{t+1} = \mathbf{X}_t + ((\mathbf{Y}^{denoise} - D_s H\mathbf{X}_t) \uparrow_s) * p$$

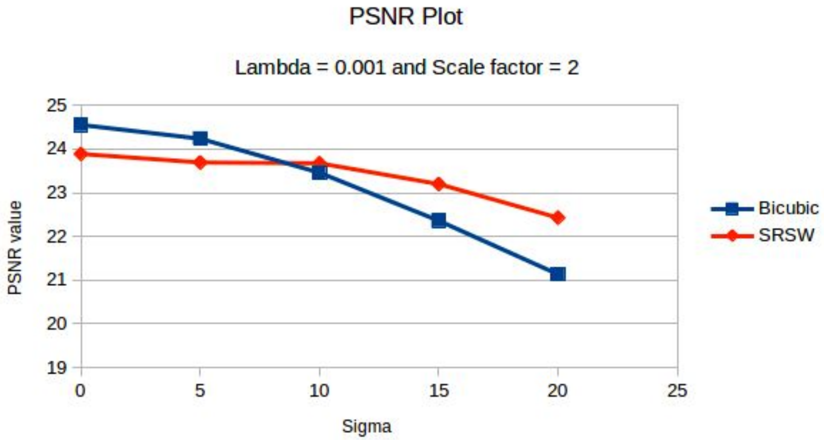
Results

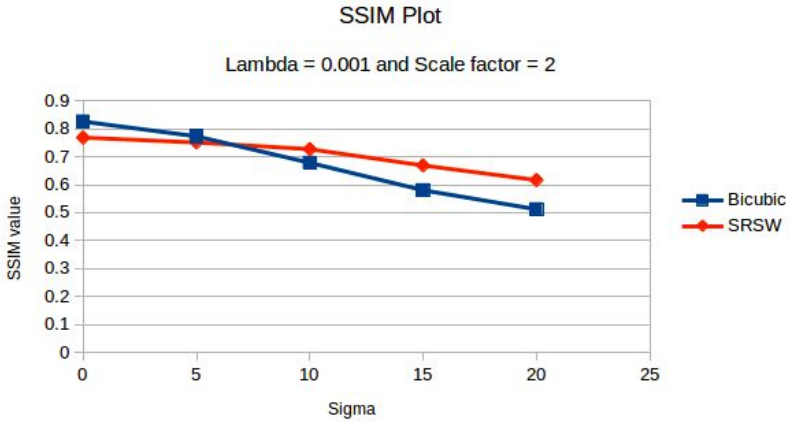


The low resolution image has a noise standard deviation of 10, and is converted to a 2-x high resolution image

Comparing with bicubic interpolation




PSNR Plot
Lambda = 0.001 and Scale factor = 2
PSNR value
Sigma
Bicubic
SRSW
25
24
23
22
21
20
19
0
5
10
15
20
25


SSIM Plot
Lambda = 0.001 and Scale factor = 2
SSIM value
Sigma
Bicubic
SRSW
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25

Thank you