PRACTICAL-1

Ex1 $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\cancel{4}^{2}\sqrt{a}}{\cancel{2}\sqrt{3}\sqrt{a}}$$

$$= \frac{2}{3\sqrt{3}}$$

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2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$\frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})} = \frac{1}{2a}$

3) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\,\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

using

$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$\cos\frac{\pi}{6} = \cos 30^\circ = \sqrt{3}/2$

$\sin\frac{\pi}{6} = \sin 30^\circ = 1/\sqrt{2}$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos \frac{\sqrt{3}}{2}h \cdot \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}h}{-6h}$$

$$\lim_{h\to 0} \frac{+\sin\frac{4h}{2}}{+6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{3/2\, h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

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4) $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

By rationalizing Numerator & Denominator both

$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$

$\lim_{x \to \infty} \left[\frac{(\cancel{x^2}+5-\cancel{x^2}+3)}{(\cancel{x^2}+3-\cancel{x^2}-1)} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$

$\lim_{x \to \infty} \frac{\cancel{8}\,4}{\cancel{2}} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$

$4 \lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

After applying limit
we get
$= 4.$

5) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$
$= \frac{\cos x}{\pi - 2x}$, ~~for $\frac{\pi}{2} < x < \pi$~~ } at $x = \pi/2$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0.$

for $x = \pi/2$ define

11) $\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

when $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - \cancel{2}\left(\frac{2h + \pi}{\cancel{2}}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$ using $\cos(A+B) = \cos A \cos B - \sin A$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h \to 0} \frac{\cancel{\cos h \cdot 0} - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{\cancel{-} \sin h}{\cancel{-} 2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}.$$

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$$\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$ using $\sin 2x = 2\sin x \cdot \cos x$

$$\lim_{x \to \pi/2^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \pi/2^-} \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2}\, \cancel{\sin x}}$$

$$\lim_{x \to \pi/2^-} \frac{2 \cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

7. $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3 \quad 3 \le x \le 6$

$= \dfrac{x^2-9}{x+3} \quad 6 \le x < 9$

at $x=3$ & $x=6$

at $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

f is continuous at $x=3$

for $x=6$

$f(6) = \cancel{\frac{x^2-9}{x+3}} \quad \frac{=36-9}{6+3} = \frac{\cancel{27}\,3}{\cancel{9}\,1} = 3$

2) $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

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$\lim_{x \to 6^-} (x-3) = 6-3 = 3$

$\lim_{x \to 6^+} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continuous

6

i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $x<0$, $= k$ $x=0$ } at $x=0$

soln:- f is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \dfrac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \dfrac{2\sin^2 2x}{x^2} = k$

$2\lim_{x \to 0} \dfrac{\sin^2 x}{x^2} = k$

$2\lim_{x \to 0} \left(\dfrac{\sin \cdot 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\therefore k = 8$

ii) $f(x) = (\sec^2 x)^{\cot 2x}$ $x \neq 0$, $= k$ $x=0$ } at $x=0$

soln: $f(x) = (\sec^2 x)^{\cot 2x}$

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using

$\sec^2 x - \tan^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

& $\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

We know that

$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$

$\therefore = e$

$\therefore k = e$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \frac{\pi}{3}$

$= k$ $\quad x = \frac{\pi}{3}$ $\Big\}$ at $x = \pi/3$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$f\left(\frac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

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using

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

using $\tan \pi/3 = \tan 60 = \sqrt{3}$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} \cdot \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \cdot \tan h)(\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3} \tan h}$$

$$\lim_{h \to 0} \frac{-4 \tan h}{-3h(1 - \sqrt{3} \tan h)}$$

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$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)}$$

$$=\frac{4}{3}\frac{1}{(1-\sqrt{3}(0)}$$

$$\frac{\tan h}{h}=1$$

$$=\frac{4}{3}\left(\frac{1}{1}\right)=\frac{4}{3}$$

7)

i) $f(x)=\dfrac{1-\cos 3x}{x\tan x}$ $\quad x\neq 0$

$=q$ $\quad x=0$ $\Big\}$ at $x=0$

$$f(x)=\frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0}=\frac{2\sin^2 3/2\,x}{x\tan x}$$

$$\lim_{x\to 0}=\frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2}\times x^2}{\dfrac{x\cdot\tan x}{x^2}\times x^2}$$

$$=2\lim_{x\to 0}\frac{\left(\frac{3}{2}\right)^2}{1}=\frac{\cancel{2}\times 9}{\cancel{4}\,2}=\frac{9}{2}$$

$\lim_{x \to 0} f(x) = \frac{9}{2}$ $\quad g = f(0)$

$\therefore f$ is not continuous at $x=0$

redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x \tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity

7)

(ii) $$f(x) = \begin{cases} \frac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$\lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x \to 0} \frac{3 \cdot e^{3x}-1}{3x} \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

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CEO

8 loge $\frac{\pi}{\cancel{180}\ 60} = \frac{\pi}{60} = f(0)$

f is continuous at $x=0$

8b $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x=0$

is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 in Num & denominator

$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$ //

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Q7 $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(x)$ is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \cdot \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{1 - \sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)(1+\cancel{\sin x})(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

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100

Practical - 2

TOPIC : Derivative

Q1 Show that the following function defined from R to R are differentiable

1) $\cot x$ 2) $\operatorname{cosec} x$ 3) $\sec x$

Q2) If $f(x) = 4x+1 \quad x \le 2$
$= x^2+5 \quad x > 0$ at $x = 2$, then find f is differentiable or not.

Q3) If $f(x) = 4x+1 \quad x < 3$
$= x^2+3x+1 \quad x \geq 3$ at $x = 3$ then find f is differentiable or not

Q4) If $f(x) = 8x-5 \quad x \le 2$
$= 3x^2-4x+1 \quad x < 2$ at $x = 2$ then find f is differentiable or not

Solution

1) $\cot x$

$f(x) = \cot x$

$$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$\lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

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$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x - a = h$ $\quad x = a + h \quad$ at $x \to a$, $h \to 0$

$$Df(a) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos 2a$

$\therefore f$ is differentiable $\forall a \in R$

2) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$\operatorname{cosec} a$

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SPD

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x-a=h$ $\quad x = a+h \quad$ as $x \to a \quad h \to 0$

$$D f(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$D f(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

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3) sec x

$f(x) = \sec x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x - a}$

$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$

$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$ put $x - a = h$, $x = a + h$

as $x \to a$ $h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

formula - $2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\cos a \cos(a+h) \times -h/2}$

$= -1/2 \times -2 \frac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$

02) sol^n:-

LHD

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$

$= \lim \frac{4x+1-9}{x-2}$

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$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$Df(2^-) = 4$

RHD

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$ RHD = LHD

f is differentiable at $x = 2$

Q3) Soln: RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3-3+1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$=$ lim ...

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$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

$LHD = Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$$

$Df(3^+) = 4$

$RHD \neq LHD$

f is not differentiable at $x = 3$.

Q4) Soln:-

$f(2) = 8 \times 2 - 5 = 16 - 15 = 11$

$$RHD = f'(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

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$f'(2^+) = 8$

$$LHD = f'(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 8$$

$f'(2^-) = 8$

LHD = RHD

f is differentiable at $x = 3$

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PRACTICAL - 3

TOPIC :- Application of Derivative

1) Find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which func^n is concave upwards

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 4$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

1) a)

i) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

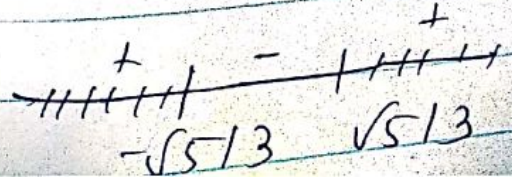
f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$x \in (-\infty, -\sqrt{5/3}) \cup (-\sqrt{5/3}, \infty)$



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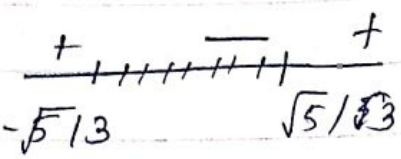
and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$



$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore$ f(x) is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

f is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f'(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

$= 6x^2 + 2x - 20 > 0$

$= 2(3x^2 + x - 10) > 0$

$= 3x^2 + x - 10 > 0$

$= 3x^2 + 6x - 5x - 10 >$

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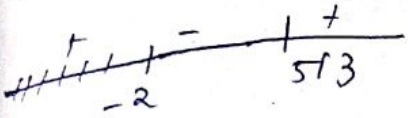
$3x(x+2)-5(x+2)>0$

$(x+2)(3x-5)>0$



$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$= 6x^2+2x-20 < 0$

$= 2(3x^2+x-10) < 0$

$= 3x^2+x-10 < 0$

$= 3x^2+6x-5x-10 < 0$

$= 3x(x+2)-5(x+2) < 0$

$= (x+2)(3x-5) < 0$

$x \in (-2, 5/3)$

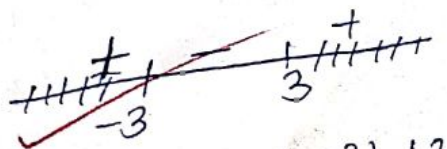
(4) $f(x) = x^3-27x+5$

$f'(x) = 3x^2-27$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2-9) > 0$

$\therefore (x-3)(x+3) > 0$



$\therefore x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x) < 0$.

$\therefore 3x^2-27 < 0$

$\therefore 3(x^2-9) < 0$

$\therefore (x^2-3)(x+3) < 0$

$- \quad +$ $\quad \therefore x \in (-3, 3)$

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5) $f(x) = 2x^3 - 9x^2 - 24x + 67$

$f'(x) = 6x^2 - 18x - 24$

$\therefore$ f is increasing iff $f'(x) > 0$

$= 6x^2 - 18x - 24 > 0$

$= 6(x^2 - 3x - 4) > 0$

$= (x^2 - 3x - 4) > 0$

$= x^2 - 4x + x - 4 > 0$

$= x(x-4) + 1(x-4) > 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

If f is decreasing iff $f'(x) < 0$

$= 6x^2 - 18x - 24 < 0$

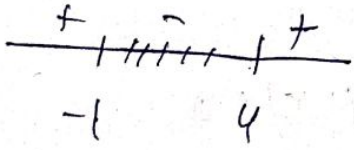
$= 6(x^2 - 3x - 4) < 0$

$= x^2 - 3x - 4 < 0$

$= x^2 - 4x + x - 4 < 0$

$= x(x-4) + 1(x-4) < 0$

$= (x+1)(x-4) < 0$



$= x \in (-1, 4)$

2)1) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$= (6 - 12x) > 0$

$= 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$= 12x^2 - 36x + 24 > 0$

$= 12(x^2 - 3x + 2) > 0$

$= x^2 - 3x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$= (x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$= 12x - 18 > 0$

$= 12(x - 18/12) > 0$

$= x - 3/2 > 0$ $\quad \therefore x > 3/2 \quad \therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$= x + 1/6 > 0$

$= x > 1/6$

$x \in (-1/6, \infty)$

21/12/17

Practical-4

Topic:- Application of derivative & Newton's method

Q1) Find maximum & minimum value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$. $[-1/2; 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$. $[-2, 3]$

Q2) Find the root of the following equation by Newton's (take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

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Q1)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, & $x = -2$.

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ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

iii) $f(x) = x^3 - 3x^2 + 1$.

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore f$ has max value at $x = 0$

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$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore f$ has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has max value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has max value 8 at $x = -1$ and f has min value -19 at

02)
1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= 55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0899 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of the eq is 0.1712

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ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4 = 17.9851$

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$= 0.0181$

$= 0.0181 = 2.7015$

$(2.7015)^2 - 6(2.7015) + 9$

$= 7.3018 - 16.2090 + 9 = -0.0901$

$3(2.7015)^2 - 6 = 21.8943 - 6 = 17.8943$

$= 2.7015 + 0.0901 / 17.8943 = 2.7015 + 0.0050$

$= 2.7065$

$f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

$x_0 = 2$ is initial approximation By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 2 - 2.2/13.2$

$= 2 - 0.4230 = 1.577$

$f(x) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4605 - 5.6772 - 10$

$= -8.2164$

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$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5577 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.6592 + 0.0204/7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.618 + \frac{0.0004}{7.6977}$

$= 1.6618$

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PRACTICAL-5

Topic 1: Integration

Q1) Solve the following integration

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\int x)\,dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t + \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x}\sin 2x\,dx$

x) $\int \left(\frac{x^2-2x}{x^2-3x^2+1}\right)dx$

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i) $\int \frac{1}{x^2+2x-3} dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}} dx$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

substituting $x+1=t$

$= dx = \frac{1}{t} dt$

when $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2+a^2}|)$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c$

2] $\int (4e^{3x}+1) dx$

$= \int 4e^{3x} dx + \int 1 dx$

$= 4\int e^{3x} dx + \int 1 dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

$\int e^{2x} dx = \frac{1}{a} x e^{ax}$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\,dx - \int 3\sin x\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{-2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$\int \frac{x^3 + 3x + 4}{x^{1/2}}\,dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx.$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

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$\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 8 \times t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{8 \times t^3}\,du$

$= \int t^4 \times \sin(2t^4) \times \frac{1}{8 \times t}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)\,du}{8}$

substitute t^4 with $\frac{u}{2}$

$= \int \frac{4/2 \times \sin(u)}{8}\,du$

$= \int \frac{4 \times \sin(u)}{16}\,du$

$= \frac{1}{16} \int u \times \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin u \times du$

$du = 1\,du \quad v = -\cos(u)$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\,du$

$\int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

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vi) $\int \sqrt{x}(x^2-1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= I_1 = \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x}^7}{7} = \dfrac{2x^3\sqrt{x}}{7}$

$I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3} = \dfrac{2\sqrt{x}^3}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x}^3}{3} + C$

vii) $\int \dfrac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \dfrac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$dt = \cos x$

$= \int \dfrac{\cos(x)}{\sin(x)^{3/2}} \times \dfrac{1}{\cos(x)}\,dt$

$= \dfrac{1}{\sin x^{3/2}}\,dt$

$= \dfrac{1}{t^{2/3}}\,dt$

$I = \int \dfrac{1}{t^{2/3}}\,dt = -\dfrac{1}{(2/3-1)\,t^{2/3-1}} = \dfrac{-1}{(2/3-1)(t^{2/3-1})}$

$= \dfrac{-1}{-1/3\,t^{2/3-1}} = \dfrac{1}{1/3\,t^{-1/3}} = \dfrac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

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Substituting $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2 6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2 6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$

$= 1/3 \int 1/t\,dt = \int \frac{1}{x}\,dx = \ln(x)$

$= 1/3\, x \ln|t| + C$

$= \frac{1}{3} x \ln(|x^3-3x^2+1|) + C$

04/01/2020

PRACTICAL - 6

TOPIC - Application of Integration & Numerical Integration

Q1) Find the length of the following curve

1) $x = 1 - \sin$; $y = 1 - \cos t \in [0, 2\pi]$

2) $y = \sqrt{y - x}$; $x \in [-2, 2]$

3) $y = x^{3/2}$; $[0, 2\pi]$

4) $x = 3 \sin t$; $y = 3 \cos t \quad t \in [0, 2\pi]$

5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$; $y \in [1, 2]$

Q2) Using Simpson's Rule solve the following

1) $\int_0^2 e^{x^2} dx$ with $n = 4$

2) $\int_0^n x^2 dx$ with $n = 4$

3) $\int_0^{\pi/4} \sqrt{\sin x}\, dx$ with $n = 4$

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1) $x = t\sin t \qquad y = 1-\cos t \qquad t \in [0, 2\pi]$

$$l = \int_0^{t_1} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$\frac{dy}{dt} = \sin t \qquad \frac{dx}{dt} = 1-\cos t$$

$$C = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1-\cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1-\cos t}\, dt$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2\left(\frac{t}{2}\right)}\, dt$$

$$= \sqrt{2} \times \sqrt{2}$$

$$= 2 \times 2 \left[-\cos\left(\frac{t}{2}\right)\right]_0^{2\pi}$$

$$= 4[-1-1]$$

$$= 8$$

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$\sqrt{4-x^2}$ $\quad x \in [-2,2]$

$$c = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{1(-2x)}{2\sqrt{4-x}}$$

$$= \frac{-4}{\sqrt{4-x}}$$

$$= L = \int_{-2}^{2} \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\,dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\,dx$$

$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\pi/2 - (-\pi/2)\right]$$

$$= 2\pi$$

3) $y = x^{3/2} \quad x \in [0,4]$

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$$

$$L = \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx$$

$$= \frac{1}{2}\int^{4} \sqrt{4+9x}\,dx$$

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$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]^{4}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} \times \frac{1}{9}\left[(4+9x)^{3/2}\right]_{0}^{4}$$

$$= \frac{1}{27}\left[40^{3/2} - 8\right]$$

4) $x = 3\sin t \quad y = 3\cos t \quad t \in [0, 2\pi]$

$$L = \int_{a}^{3} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$\frac{dy}{dt} = -3\sin t \qquad \frac{dx}{dt} = 3\cos t$$

$$L = \int_{0}^{2\pi} \sqrt{-(3\sin^2 t) + (3\cos^2 t)}\, dt$$

$$L = 3\int_{0}^{2\pi} \sqrt{\sin^2 t + \cos^2 t}$$

$$= 3\int_{0}^{2\pi} \sqrt{1}\, dt$$

$$= 3\left[t\right]_{0}^{2\pi}$$

$$= 6\pi$$

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$$x = \frac{1}{6}y^3 + \frac{1}{2y} \qquad y \in [1,2]$$

$$L = \int_a^b \sqrt{1 + \frac{dx}{dy}}\, dy$$

$$\frac{dx}{dy} = \frac{3}{2}y^2 - \frac{1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}y^2 - \frac{1}{2y}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{2y^2}\right)^2}\, dy$$

$$= \int_1^2 \frac{\sqrt{(y^4-1)^2 + 4y^4 \times 1}}{4y^2}\, dy$$

$$= \int_1^2 \frac{\sqrt{(y^4+1)^2}}{(2y^2)^2}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 \frac{y^2}{2}\, dy + \frac{1}{2}\int_1^2 \frac{1}{y^2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^5}{5}\right]_1^2 + \frac{1}{2}\left[\frac{1}{y}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{5}{3} - \frac{1}{3}\right] + \frac{1}{2}\left[\frac{1}{2} - 1\right]$$

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$= \frac{7}{6} - \frac{1}{4}$

$= \frac{14-3}{12}$

$= \frac{11}{12}$

Q2] i] $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0$, $b = 2$, $n = 4$.

$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

x	0	0.5	1	1.5	2
y	1	1.2540	2.7182	9.4877	54.5981

By simpson's rule:

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1+54.5981) + 4(1.2840 + 9.4877) + 2(2.7182)\right]$$

$= 17.3535$

ii] $\int_0^4 x^2 dx$ with $n = 4$

$a = 0$, $b = 4$, $n = 4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

By simpsons rule

$$\int_0^4 x^2\,dx = \frac{1}{3}\left[(0+16)+4(1+9)+2(4)\right]$$

$$= \frac{1}{3}(16+40+8)$$

$$= \frac{1}{3}\times 64$$

$$= \frac{64}{3}$$

iii) $\int_0^{\pi/3} \sin x\,dx$ with $n=6$

$a=0$, $b=\pi/3$, $n=6$

$$h = \frac{b-a}{n} = \frac{\pi/3-0}{6} = \pi/18.$$

$x=$	0	$\pi/18$	$3\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
$y=$	0	0.9167	0.9848	0.7071	0.8017	0.8752	0.9306

by simpson's rule

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{\pi}{54}\left[(0.09306)+4(0.4167+0.7071+0.8752)+2(0.9848+0.5017)\right]$$

$$= \pi/54 \times 11.6996$$

$$= 0.6806$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.6806$$

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130

PRACTICAL - 7

* Differential equation

Q1) solve the following differentiatial equation

i) $x\frac{dy}{dx} + y = e^x$

ii) $e^x\frac{dy}{dx} + 2e^x y = 1$.

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$.

vi) $\sec^2 x \tan y + \sec^2 y \tan x \, dy = 0$

vii) $\frac{dy}{dx} = \sin^2(x^2 - y + 1)$

viii) $\frac{dy}{dx} = \frac{2y + 3y - 1}{6x + 9y + 6}$

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$x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

comparing with $\frac{dy}{dx} + P(x) y = Q(x)$

$P(x) = \frac{1}{x}$, $Q(x) = \frac{e^x}{x}$

$I.f = e^{\int \frac{1}{x} dx}$

$= e^{\log x}$

$I.f. = x$

$y(I.f) = \int y(x)(I.f) dx$

$y(x) = \int \frac{e^x}{x} \cdot x dx$

$y(x) = \int \frac{e^x}{x} \cdot x dx$

$y(x) = \int e^x dx$

$xy = e^x + c$

$e^x \frac{dy}{dx} + 2e^x y = 1$

$e^x \left(\frac{dy}{dx} + 2y\right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

Comparing with $\frac{dy}{dx} + P(x) y = Q(x)$

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$= P(x) = 2 \qquad Q(x) = \frac{1}{e^{x}}$

$I.f = e^{\int 2 dx}$

$= e^{2x}$

$y(I.f) = \int Q(x)(I.f)dx$

$y e^{2x} = \int \frac{1}{e^{x}} e^{2x} dx$

$y e^{2x} = \int e^{-x} \cdot e^{2x} dx$

$y e^{2x} = \int e^{x} dx$

$y e^{2x} = e^{x} + c$

iii) $\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{x\,dy}{dx} + 2y = \frac{\cos x}{x}$

$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = \frac{\cos x}{x^2}$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{2}{x}$; $Q(x) = \frac{\cos x}{x}$

$I.f = e^{\int \frac{2}{x} dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$I.f = x^2$

$y(I.f) = \int Q(x)\ (I.f)\, dx$

$y(x)^2 = \int \frac{\cos x}{x^2}(x^2)\,dx$

$x^2 y = \sin x + C$

v) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$ $Q(x) = \frac{\sin x}{x^3}$

$If = e^{\int \frac{3}{x}dx}$

$= e^{3\log x}$

$= e^{\log x^3}$

$= x^3$

$y(If) = \int Q(x)(If)\,dx$

$y(x^3) = \int \frac{\sin x}{x^3}(x^3)\,dx$

$x^3 y = -\cos x + C$

$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$\frac{dy}{dx} + P(x)y = Q(x)$

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$\therefore P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}}$

$I.f = e^{\int 2\,dx}$

$= e^{2x}$

$y(I.f) = \int Q(x)\,(I.f)\,dx$

$y(e^{2x}) = \int \frac{2x}{e^{2x}} \cdot (e^{2x})\,dx$

$ye^{2x} = \frac{\cancel{2}x^2}{\cancel{2}} + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$\frac{\sec^2 x}{\tan x}\,dx = -\frac{\sec^2 y}{\tan y}\,dy$

$\therefore \int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$

$= \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x| + \log|\tan y| = c$

$\log|\tan x \cdot \tan y| = c$

$\Rightarrow \tan x \cdot \tan y = e^c$

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$$\frac{dy}{dx} = \sin^2(x-y+1)$$

Put $x-y+1 = v$

$$1-\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1-\frac{dv}{dx}$$

$$1-\frac{dv}{dx} = \sin^2 v$$

$$1-\sin^2 v = \frac{dv}{dx}$$

$$1x = \frac{dv}{1-\sin^2 v}$$

$$\int dx = \int \sec^2 v\, dv$$

$$x = \tan v + C$$

But $v = x+y-1$

$$x = \tan(x+y-1)+C$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$$

Put $2x+3y = v$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

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$$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int\left(\frac{v+1+1}{v+1}\right)dv = \int dx$$

$$\frac{1}{3}\int 1+\frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v+\log(v+1)\right) = x+c$$

Put $v = 2x+3y$

$$\therefore 2x+3y+\log|2x+3y+1| = 3x+c$$

$$\therefore 3y-x-\log|2x+3y+1| + c$$

11/01/2020

PRACTICAL - 8

* Using Euler's method find the following

1) $\frac{dy}{dx} + y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$ find $y(2)$

$x_0 = 2$

$f(x) = y + e^{x} - 2$

$y(0)\ h = 0.5$

$y(0.2) = ?$

n	x_n	y_0	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.0487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	5.2021	9.8215
4	2	9.8215		

$5.7205 + e^{1.5} - 2$

$y(2) = 9.8215$

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$\frac{dy}{dx} = 1+y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

$y_2 = 0$ $\quad$ $y_0 = 0$

$h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2937
5	0.1	1.2939		

$y(1) = 1.2939$

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5) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $h = 0.2$ find $y(1)$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.5059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$.

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$\frac{dy}{dx} = 3x^2 + 1 \quad y(1) = 2$ find $y(2)$

$h = 0.5 \qquad 2h = 0.25$

$y_0 = 2 \qquad x_0 = 1$

x_n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.9966		

$y(2) = 299.9966$

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330

$y(1.2)$

5) $\frac{du}{dx} = \sqrt{xy} + 2$

$y(1) = 1$ find $h = 0.2$

$y_0 = 1$ $x_0 = 1$ $n = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

11/01/2020

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PRACTICAL 9

Aim: Limits & partial and derivatives

1) $\lim_{(x,y)\to(-4,7)} \frac{x^3-3y+y^2-1}{x+y+5}$

$\lim_{(x,y)\to(-4,7)} \frac{x^3-3y+y^2-1}{x+y+3}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$\frac{(-4)^3-3(-4)+(-4)^2-1}{-4(7)+5}$

$= 6/17$

2) $\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(-2,0)$ denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(0+1)(4+0-8)}{2}$

$= -4/2 = -2$

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iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^3-y^3z^2}{x^3-x^2y^2}$

At $(1,1,1)$ Denominator $\neq 0$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^3-y^2z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x+y^2)}{x^2}$

on applying limit

$\frac{1+1(1)}{1^2}$

$= 1$

(v) $f(x,y) = xye^{x^2+y^2}$

$f(x) = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial(xye^{x^2+y^2})}{\partial x}$

$= ye^{x^2+y^2}(2x)$

$= f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial(xye^{x^2+y^2})}{\partial x}$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

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11) $f(x,y) = e^x \cos y$

$\Rightarrow f_x = \frac{\partial x}{\partial x} f(x,y)$

$= \frac{\partial (e^x \cos y)}{\partial x}$

$f_x = e^x \cos y$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial (e^x \cos y)}{\partial y}$

$f_y = -e^x \sin y$

1) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial f(x,y)}{\partial x}$

$= \frac{\partial f(x^3y^3 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 6xy$

$\Rightarrow f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial f(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

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830

Q3) i) $f(x,y) = \frac{2x}{1+y^2}$

$\Rightarrow f_x = \frac{\delta f(x,y)}{\delta x}$

$= \frac{\delta f(2x)(1+y^2)}{\delta x}$

$= \frac{1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y^2)^2}$

$= 2/1+y^2$

At $f(0,0)$

$= \frac{2}{1+0} = 2$

$f_y = \frac{\delta f(2x)(1+y^2)}{\delta y}$

$= \frac{1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

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At (0,0)

$$= \frac{-4(0)(0)}{(1+0)^2} = 0$$

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\delta}{\delta x}(y^2 - xy) - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\delta}{\delta x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4(\delta/\delta x(-x^2 y - 2xy^2 + 2x^2 y)) - (-x^2 y - 2xy^2 + 2x^2 y)\,\delta/\delta x(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad \text{---(I)}$$

$$f_{yy} = \frac{\delta}{\delta y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad \text{---(II)}$$

$$f_{yx} = \frac{\delta}{\delta x}\left(\frac{2y - x}{x^2}\right)$$

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$$= \frac{x^2 \frac{\delta}{\delta x}(xy-x) - (2y-x)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy - 2x^2}{x^4} \quad \cdots (iv)$$

from (iii) & (iv)

$f_{xy} = f_{yx}$

Q) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\Rightarrow f_x = \frac{\delta}{\delta x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x+1}$

$f_y = \frac{\delta}{\delta y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 6x^2y$

$$f_{xx} = 6x + 6y^2 - \frac{(x^2+1)\frac{\delta}{\delta x}(2x) - 2x\left(\frac{\delta}{\delta x}(x^2+1)\right)}{(x^2+1)^2}$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \cdots (i)$$

$f_{yy} = \frac{\delta}{\delta y}(6x^2y)$

$= 6x^2 \quad \cdots (ii)$

$f_{xy} = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy + 0$

$= 12xy \quad \cdots (iii)$

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$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy \quad \cdots (ii)$

from (ii) & (iv)

$f_{xy} = f_{yx}$

Q. $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{1+0}$

$= -y^2\sin(xy)(y) + e^{1+x}$

$= -y^2\sin(xy) + e^{x+y} \quad \cdots (i)$

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y} \quad \cdots (ii)$

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y} \quad \cdots (iii)$

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050

$f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ --- (III)

from (III) & (IV)

$f_{xy} = f_{yx}$

Q5) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

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$\sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

$f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$\Rightarrow f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0)$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin \pi/2$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a-b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-b)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

$f(x,y) = \log x + \log y$ at $(1,1)$

$\Rightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x}$ $\quad f_y = 1/y$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

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$= x-1+y-1$

$= x+y-2$.

7/10/2020

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1) Find the directional derivative of the following function at given pt & in the direction of a given vector

1) $f(x,y) = x+2y-3 \qquad a=(1,-1) \qquad u=3i-j$

Here $u=3i-j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|\bar{u}|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1)+2(-1)-3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1)+h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)-3$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right)-3$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right)+2\left(-1-\frac{h}{\sqrt{10}}\right)-3$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}}$

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$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\pm k/\sqrt{10} + k}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

11) $f(x) = y^2 - 4x + 1$ $\quad a = (3, 4)$ $\quad u = (-1, 5)$ then $u = (-1, 5)$ not a unit vector

$$|\bar{v}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{v}{|\bar{v}|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$

$$f(a + hv) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f = \left(\frac{3+h}{\sqrt{26}}, \frac{4+5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(\frac{4+5h}{\sqrt{26}}\right)^2 - 4\left(\frac{3+h}{\sqrt{26}}\right) + 1$$

$f(x, y)$

$$= \frac{16 + 25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

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$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$\frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36h}{\sqrt{26}}$$

iii) 2x+3y $a = (1,2)$ $u = (3i+4j)$

given $u = 3i + 4j$ is not a unit vector

$|u| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 18 + \frac{8h}{5}$

$D_u f(a) = \frac{18}{5}$

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Q2) Find gradient vector for the following functions given pt

i) $f(x,y) = x^y + y^x$ $a = (1,1)$

$f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y, x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) - y^2$, $a = (1,-1)$

$f_x = \frac{1}{1+x^2} - y^2$

$f_y = 2y \tan^{-1} x$

$\therefore f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \frac{1}{2}, \frac{\pi}{4}(-2)$

$= \frac{1}{2}, -\frac{\pi}{2}$.

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$f(x,y,z) = xyz - e^{x+y+z}$

$f_x = yz - e^{x+y+z}$, at $(1,-1,0)$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\therefore f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(-1,0) = ((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0}$

$= (-1, -1, -2)$

3) Find the equation of tangent & normal to each of the following moving curves at given points

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(x) = \cos y^2 x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0) \therefore x_0 = 1, y_0 = 0$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$= (1)(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

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eq^n of Normal

$= ax+by+c=0$

$= bx+ay+d=0$

$1(1)+2(y)+d=0$

$1+2y+d=0$ at $(1,0)$

$= 1+2(0)+d=0$

$d+1=0$

$\therefore d=-1$

i) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f(x) = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0=2, y_0=-2$

$f_x(x_0, y_0) = 2(2)-2=2$

$f_y(x_0, y_0) = 2(-2)+3=-1$

eq^n of tangent

$2(x-2)+(-1)(y+2)=0$

$2x-2-y-2=0$

$2x-y-4=0$

eq^n of normal

$= ax+by+c=0$

$bx+ay+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2-4+d=0$

$-6+d=0$

$d=6$

Find the eqn of tangent & normal line to each of the following surface

i) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

$f_x = 2x-0+0+z$

$f_x = 2x+z$

$f_y = 0-2z+3+0$

$= 2z+3$

$f_z = 0-2y+0+0$

$= -2y+x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0=2, y_0=1, z_0=0$

$f_x(x_0, y_0, z_0) = 2(2)+0 = 4$

$f_y(x_0, y_0, z_0) = 2(0)+3 = 3$

$f_z(x_0, y_0, z_0) = -2(1)+2 = 0$

eqn of tangent

$4(x-2)+3(y-1)+0(z-0)=0$

$4x+3y-11=0$ ⟶ This is req eqn of tangent

$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

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$\therefore \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{8}$

11) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$\delta x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$\delta y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$\delta z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1,-1,2) = x_0 = 1, y_0 = -1, z_0 = 2$

$\delta x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$\delta y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$\delta z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqn of tangent

$-7x + 5y - 2z + 11 = 0$ → Ans 4 eq. eqn of tangent

eqn of normal.

$\frac{x-1}{-2} = \frac{y+1}{5} = \frac{z-2}{-2}$

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Find the local maxima & minima for the following eq

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ --(1)

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ --(II)

Multiply eq (1) with 2
$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Put the value of x in eq^n (1)
$2(0) - y = -2$
$-y = +2$
$y = 2$

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$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$rt - s^2$

$6(2) - (-3)^2$

$12 - 9$

$3 > 0$

f has maximum at (0,2)

$3x^2 + y^2 - 3xy + 8x - 4y$ at (0,2)

$0 + 4 - 0 + 0 - 8$

$= -4$

11) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ --- (1)

$f_y = 0$

$3x^2 - 2y = 0$ --- (11)

multiply eqn (1) with 3

(11) with 4.

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$18y = 0 \quad y = 0$

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putting value of y in eqn (1)

$4x^2+3(0)=0$

$4x^2=0$

$x=0$

critical point is (0,0)

$r=f_{xx}=24x^2+6x$

$t=f_{yy}=0-2=-2$

$s=f_{xy}=6x-0$ $6x=6(0)=0$

at (0,0)

$r(0)+6(0)=0$

$r=0$

$rt-s^2=0(-2)-s^2$

$=0-0=0$

(nothing to say)

$f(x,y)=x^2-y^2+2x+8y-10$

$f_x=2x+2$

$f_y=-2y+8$

$f_x=0$ $\therefore 2x+2=0$

$x=-1$

$f_y=0$ $-2y+8=0$

$y=\frac{-8}{-2}=4$

$y=4$

critical pt is $(-1,4)$

$f(x,y)$ at (0,0)

$2(0)^4+3(0)^2-(0)-0$

$=0+0-0$

$=0$

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850

$h = 6 \times 8 = 2$

$l = 6 \times 4 = -2$

$s = 6 \times 4 = 0$

$h > 0$

$ax^2 + by^2 = a(-2)^2 + b(0)^2$

$= -4 - 0$

$= -4 < 0$

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$f(x, y) \text{ at } (-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 18(4) - 70$

$= 1 + 16 - 2 + 72 - 70$

$= 17 + 70 - 70$

$= 87 - 70 = 17$

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