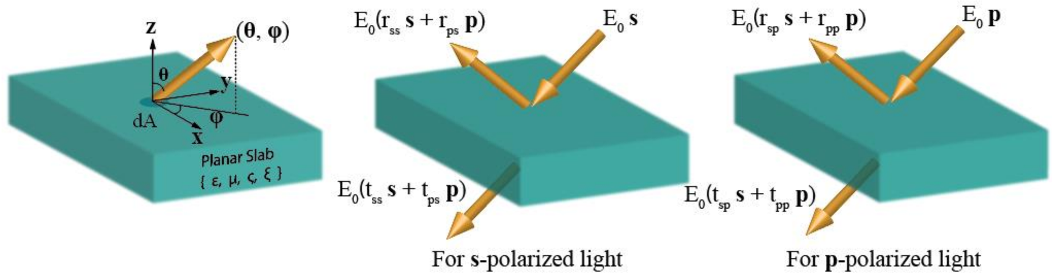
Fresnel coefficients for generic bianisotropic planar media

Introduction

The reflection and transmission of s-polarized or p-polarized light incident on a planar surface is described using the Fresnel interconversion coefficients as shown in the figure below. For instance, incident unit-amplitude s-polarized light gets reflected as (r_{ss} **s** + r_{ps} **p**) and transmitted as (t_{ss} **s** + t_{ps} **p**). The leftmost figure denotes the notation for directions and co-ordinate axes used in this work.



We are interested in finding these coefficients for very generic, bianisotropic (linear, time-invariant) media (multilayered or composite planar geometries) that are described using the following constitutive relations in Maxwell's equations:

$$\begin{bmatrix}\mathbf{D}\\ \mathbf{B}\end{bmatrix} = \begin{bmatrix}\bar{\bar{\epsilon}}\epsilon_0 & \bar{\bar{\xi}}/c\\ \bar{\bar{\zeta}}/c & \bar{\bar{\mu}}\mu_0\end{bmatrix} \times \begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix}$$

Here, $\bar{\bar{\epsilon}}, \bar{\bar{\mu}}, \bar{\bar{\xi}}, \bar{\bar{\zeta}}$ are dimensionless permittivity, permeability and magneto-electric coupling tensors. They are scalars for isotropic materials. ϵ_0 is vacuum permittivity, μ_0 is vacuum permeability and c_0 is vacuum speed of light. We assume the above equation is written in the frequency domain with fields $\mathbf{E}(\omega), \mathbf{H}(\omega)$ and dispersive material parameters $\bar{\bar{\epsilon}}(\omega), \bar{\bar{\xi}}(\omega)$ and so on. We will denote the 6×6 material matrix as $\mathbf{M}$ in the subsequent discussion for convenience and this is used as an input to the code. Examples of various materials studied in the literature are given in the table below:

No.	Material Class	Description	Examples
1	Reciprocal isotropic	$\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}$ are scalars, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$	Common dielectric and metallic materials
2	Reciprocal anisotropic	Diagonal $\bar{\bar{\varepsilon}}$ with unequal entries, scalar $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Uniaxial and biaxial crystals
3	Nonreciprocal Gyroelectric	$\bar{\bar{\varepsilon}} = -\bar{\bar{\varepsilon}}^T$, scalar $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Weyl Semimetals, Metals and semiconductors in magnetic field
4	Nonreciprocal Gyromagnetic	$\bar{\bar{\mu}} = -\bar{\bar{\mu}}^T$, scalar $\bar{\bar{\varepsilon}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Ferromagnets, Ferrites
5	Reciprocal Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, nonzero $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}^T$.	Chiral media, Artificial metamaterials, Pasteur media (diagonal $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}$)
6	Nonreciprocal Isotropic Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, diagonal or scalar coupling $\bar{\bar{\xi}} = \bar{\bar{\zeta}}^T$.	Topological insulators, Tellegen media
7	Nonreciprocal Anisotropic Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}}^T \neq 0$ have nonzero off-diagonal entries.	Multiferroic media, Engineered heterostructures

How to use?

For all purposes, we enforce the passivity constraint on the material parameters i.e. dimensionless tensors $\bar{\epsilon}, \bar{\mu}, \bar{\bar{\xi}}, \bar{\bar{\zeta}}$ which define the material matrix **M**. They cannot be arbitrary. For instance, one example is given below:

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

This example represents a gyroelectric (magneto-optic) medium. Note that a small amount of loss [Im{μ}~ 10^{-6}] is introduced above. This passivity constraint ensures that the power dissipation inside a medium given below is always positive semi-definite.

$$q = \frac{1}{2}\mathrm{Re}\left\{\left(\mathbf{E}^* \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}^* \cdot \frac{\partial \mathbf{B}}{\partial t}\right)\right\} \geq 0$$

<u>We hope that the user keeps this passivity constraint in mind. The code cannot be readily used for non-passive media.</u> If the passivity constraint is violated, the code will give the reflection coefficients whose magnitude is larger than 1 implying the presence of active/gain medium.

Having said that, we describe its application below for various configurations.

A. Semi-infinite half-space.

There are only 4 Fresnel reflection coefficients which can be found using 'fresnel.m'
First define the material matrix as given below (satisfying the passivity constraint)

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

Step 1

Define the angles (Θ, φ) that denote direction of incidence. Θ is the angle made by the incident light with z-axis while φ is the angle with x-axis.

```
>> theta=pi/3; phi=pi/6;
>> [rss rps rsp rpp]=fresnel_halfspace(theta,phi,MM);
```

Step 2

B. Finite-thickness slab.

There are total 8 Fresnel coefficients. For a finite thickness slab, the light can be incident from top (+z) or from the bottom (-z) side of the slab. Depending on this incidence, we have two different functions 'fresnel_top.m' and 'fresnel_bottom.m'. The thickness of the slab is given in units of (c/ω) where ω is the frequency in rad/s at which the material parameters are measured.

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

Step 1

Define the angles (Θ, φ) that denote direction of incidence. Θ is angle with z-axis, φ is angle with x-axis. Give thickness (d) in units of (c/ω). And find the Fresnel coefficients as the following.

```
>> theta=pi/3; phi=pi/6; d=0.5;
>> [rss rps rsp rpp tss tps tsp tpp]=fresnel_film_top(theta,phi,MM,d);
>> [rss1 rps1 rsp1 rpp1 tss1 tps1 tsp1 tpp1]=fresnel_film_bottom(theta,phi,MM,d);
```

Step 2

Note that for the incidence of light from the bottom hemisphere, the angle ϴ is the angle with either +z or -z axis. This distinction does not matter because this angle is used to calculate the component of the wavevector parallel to the surface which is the same for both cases.

The anlge φ however denotes the angle made by the parallel component of the wavevector (projection of momentum onto the surface of the slab) with x-axis of the geometry.

The Fresnel coefficients are obtained by solving the boundary conditions (continuity of tangential **E**, **H** fields) for each interface in the planar geometry. For details regarding the method, we encourage the reader to refer to the supplement section of the following reference.

[1] Thermal spin photonics in the near-field of nonreciprocal media, C. Khandekar and Z. Jacob, New. J. Phys. **21**, 103030, 2019. [link]