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Fresnel Coefficients

This script provies Fresnel reflection and transmission coefficients for light incident on a planar slab of thickness 'd' in arbitrary directions characterized by (θ, ϕ). Here θ denotes the angle made by the incident light with z-axis while ϕ denotes the angle made by the in-plane wavevector of the incident light with x-axis. The thickness 'd' is in units of c/ω where 'c' is vacuum speed of light and ω is the frequency.

For each example, a material matrix 'MM' is defined which contains dimensionless permeability tensor (ϵ), permeability tensor (μ), and magneto-electric coupling tensors (ξ, ζ). For each material example, these response tensors are not arbitrary but also satisfy passivity constaint. We ensure that this passivity is satisfied since our theory is not applicable for optically active gain media. If the medium is not passive, an error message will be displayed.

As discussed in our manuscript, the spin-resolved emissivity along the incidence direction (θ, ϕ) requires the calculation of Fresnel reflection and transmission coefficients for light incident along the reflection direction ($\theta, \phi + \pi$) and the transmission direction ($\pi - \theta, \phi + \pi$) on the other side of the slab. We calculate all these coefficients for each example. For simplicity, we focus on $\theta \in [0, \pi/2]$ and $\phi \in= [0, 2\pi]$ which denotes light incident in the top hemisphere.

The user can specify the parameters θ, ϕ, thickness 'd', and the material matrix MM in this script. The spin-resolved Kirchhoff's laws are derived based on certain relations between the Fresnel coefficients. These can be verified for various material (MM) and thickness (d) parameters, and indcidence directions (θ, ϕ). The user only needs to choose the parameters and run the command 'publish('make_report.m','pdf')' to produce a report on Fresnel coefficients for several examples. On a standard computer with MATLAB

2018, it takes approximately 10 minutes to put together all results and provide this universal perspective of Fresnel coefficients for several material classes. Below, we fix some of the parameters.

```
theta=pi/3;  % angle between incident light and z-axis (normal to
 slab)
phi=pi/6; % angle between parallel wavevector and x-axis
d=0.4; % thickness of slab in units of c/w.
```

Below, we first consider reciprocal media that satisfy SKL-1 and then nonreciprocal media which satisfy SKL-2 or SKL-3. We also provide these parameters for a double layered thin film.

Uniaxial anisotropic (reciprocal SKL-1) medium

```
nf=1;  % example 1 and figure 1
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 2+0.1i]; % ep_xx not equal to ep_yy
mu=(1+1e-6*1i)*eye(3);   % small loss added for passivity
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

       %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.3502 - 0.4783i	-0.3502 - 0.4783i	-0.3502 - 0.4783i
rps	-0.1058 - 0.2251i	-0.1058 - 0.2251i	0.1058 + 0.2251i
rsp	0.1058 + 0.2251i	0.1058 + 0.2251i	-0.1058 - 0.2251i
rpp	0.0341 + 0.1054i	0.0341 + 0.1054i	0.0341 + 0.1054i
tss	0.6306 - 0.2847i	0.6306 - 0.2847i	0.6306 - 0.2847i
tps	0.1038 + 0.2301i	0.1038 + 0.2301i	-0.1038 - 0.2301i
tsp	0.1038 + 0.2301i	0.1038 + 0.2301i	-0.1038 - 0.2301i
tpp	0.8331 + 0.3625i	0.8331 + 0.3625i	0.8331 + 0.3625i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Biaxial anisotropic (reciprocal SKL-1) medium

```
nf=nf+1;
```

```
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 3+0.1i];
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

%
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.3494 - 0.4777i	-0.3494 - 0.4777i	-0.3494 - 0.4777i
rps	-0.1060 - 0.2253i	-0.1060 - 0.2253i	0.1060 + 0.2253i
rsp	0.1060 + 0.2253i	0.1060 + 0.2253i	-0.1060 - 0.2253i
rpp	0.0138 + 0.1471i	0.0138 + 0.1471i	0.0138 + 0.1471i
tss	0.6314 - 0.2841i	0.6314 - 0.2841i	0.6314 - 0.2841i
tps	0.1035 + 0.2312i	0.1035 + 0.2312i	-0.1035 - 0.2312i
tsp	0.1035 + 0.2312i	0.1035 + 0.2312i	-0.1035 - 0.2312i
tpp	0.8120 + 0.4046i	0.8120 + 0.4046i	0.8120 + 0.4046i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Isotropic magnetoelectric (reciprocal SKL-1) medium (Pasteur medium)

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=0.2i*eye(3);   % nonzero magnetoelectric coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);


%
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, $d\omega/c$=0.4

	Incidence	Reflection	Transmission
rss	-0.6312 + 0.3563i	-0.6312 + 0.3563i	-0.6312 + 0.3563i
rps	0.0008 + 0.0456i	0.0008 + 0.0456i	0.0008 + 0.0456i
rsp	-0.0008 - 0.0456i	-0.0008 - 0.0456i	-0.0008 - 0.0456i
rpp	0.0508 - 0.0521i	0.0508 - 0.0521i	0.0508 - 0.0521i
tss	0.3460 + 0.5630i	0.3460 + 0.5630i	0.3460 + 0.5630i
tps	0.0379 + 0.0659i	0.0379 + 0.0659i	0.0379 + 0.0659i
tsp	-0.0379 - 0.0659i	-0.0379 - 0.0659i	-0.0379 - 0.0659i
tpp	0.7384 + 0.6469i	0.7384 + 0.6469i	0.7384 + 0.6469i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Anisotropic magnetoelectric (reciprocal SKL-1) medium [xy-coupling]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0.2i 0; 0 0 0; 0 0 0;];  % Ex-Hy, Ey-Hx coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667) thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6270 + 0.3694i	-0.6270 + 0.3694i	-0.6397 + 0.3488i
rps	-0.0139 - 0.0227i	-0.0139 - 0.0227i	-0.0133 - 0.0231i
rsp	0.0139 + 0.0227i	0.0139 + 0.0227i	0.0133 + 0.0231i
rpp	0.0094 - 0.0823i	0.0094 - 0.0823i	0.0908 - 0.0110i
tss	0.3446 + 0.5657i	0.3446 + 0.5657i	0.3446 + 0.5657i
tps	0.0006 - 0.0162i	0.0006 - 0.0162i	0.0007 - 0.0141i
tsp	-0.0007 + 0.0141i	-0.0007 + 0.0141i	-0.0006 + 0.0162i
tpp	0.7386 + 0.6504i	0.7386 + 0.6504i	0.7386 + 0.6504i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Anisotropic magnetoelectric (reciprocal SKL-1) medium [xz-coupling]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2i; 0 0 0; 0 0 0;];  % Ex-Hz, Ez-Hx coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667) thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6321 + 0.3595i	-0.6321 + 0.3595i	-0.6321 + 0.3595i
rps	-0.0062 - 0.0163i	0.0023 + 0.0173i	-0.0023 - 0.0173i
rsp	-0.0023 - 0.0173i	0.0062 + 0.0163i	-0.0062 - 0.0163i
rpp	0.0482 - 0.0436i	0.0482 - 0.0436i	0.0482 - 0.0436i
tss	0.3460 + 0.5663i	0.3460 + 0.5663i	0.3460 + 0.5663i
tps	0.0069 + 0.0165i	-0.0029 - 0.0176i	0.0029 + 0.0176i
tsp	-0.0029 - 0.0176i	0.0069 + 0.0165i	-0.0069 - 0.0165i
tpp	0.7413 + 0.6502i	0.7413 + 0.6502i	0.7413 + 0.6502i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Anisotropic magnetoelectric (reciprocal SKL-1) medium [yz-coupling]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2i; 0 0.1i 0;];  % Ey-Hz, Ez-Hy coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6293 + 0.3605i	-0.6293 + 0.3605i	-0.6293 + 0.3605i
rps	0.0015 - 0.0098i	0.0025 + 0.0084i	-0.0025 - 0.0084i
rsp	-0.0025 - 0.0084i	-0.0015 + 0.0098i	0.0015 - 0.0098i
rpp	0.0499 - 0.0458i	0.0499 - 0.0458i	0.0499 - 0.0458i
tss	0.3488 + 0.5671i	0.3488 + 0.5671i	0.3488 + 0.5671i
tps	0.0031 + 0.0113i	-0.0068 - 0.0106i	0.0068 + 0.0106i
tsp	-0.0068 - 0.0106i	0.0031 + 0.0113i	-0.0031 - 0.0113i
tpp	0.7404 + 0.6513i	0.7404 + 0.6513i	0.7404 + 0.6513i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Composite anisotropic magnetoelectric (reciprocal SKL-1) medium

```
nf=nf+1;
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 2+0.1i]; % uniaxial anisotropy
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2i; 0 0.1i 0;];  % magnetoelectric anisotropy
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.3565 - 0.4799i	-0.3565 - 0.4799i	-0.3565 - 0.4799i
rps	-0.1148 - 0.2191i	-0.1007 - 0.2319i	0.1007 + 0.2319i
rsp	0.1007 + 0.2319i	0.1148 + 0.2191i	-0.1148 - 0.2191i
rpp	0.0355 + 0.1039i	0.0355 + 0.1039i	0.0355 + 0.1039i
tss	0.6243 - 0.2866i	0.6243 - 0.2866i	0.6243 - 0.2866i
tps	0.1211 + 0.2258i	0.0909 + 0.2338i	-0.0909 - 0.2338i
tsp	0.0909 + 0.2338i	0.1211 + 0.2258i	-0.1211 - 0.2258i
tpp	0.8329 + 0.3614i	0.8329 + 0.3614i	0.8329 + 0.3614i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Multilayered anisotropic magnetoelectric (reciprocal SKL-1) medium

```
nf=nf+1;
% Material 1
ep1=[4+0.1i 0 0; 0 -3+0.1i 0; 0 0 4+0.1i];  % uniaxial anisotropic
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
xi2=0.2i*eye(3);   % isotropic magnetoelectric coupling
zeta2=-transpose(xi2);
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
D_layer_table(theta,phi,MM1,MM2,td,nf);

 %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)

	Incidence	Reflection	Transmission
rss	-0.8684 - 0.1887i	-0.8684 - 0.1887i	-0.7462 + 0.3581i
rps	-0.0317 - 0.0374i	-0.0317 - 0.0374i	-0.1065 - 0.1941i
rsp	0.0317 + 0.0374i	0.0317 + 0.0374i	0.1065 + 0.1941i
rpp	0.1474 + 0.2261i	0.1474 + 0.2261i	0.1767 - 0.1082i
tss	0.0462 + 0.4248i	0.0462 + 0.4248i	0.0462 + 0.4248i
tps	-0.0679 + 0.0774i	-0.0679 + 0.0774i	0.1766 + 0.0622i
tsp	-0.1766 - 0.0622i	-0.1766 - 0.0622i	0.0679 - 0.0774i
tpp	-0.1806 + 0.9036i	-0.1806 + 0.9036i	-0.1806 + 0.9036i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Gyroelectric (nonreciprocal SKL-2) medium [gyrotropy axis along z]

```
nf=nf+1;
ep=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i;]; % must be anti-symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

(θ,ϕ)/π= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6334 + 0.3589i	-0.6334 + 0.3589i	-0.6334 + 0.3589i
rps	-0.0056 - 0.0223i	-0.0056 - 0.0223i	0.0056 + 0.0223i
rsp	-0.0056 - 0.0223i	-0.0056 - 0.0223i	0.0056 + 0.0223i
rpp	0.0501 - 0.0451i	0.0501 - 0.0451i	0.0501 - 0.0451i
tss	0.3447 + 0.5657i	0.3447 + 0.5657i	0.3447 + 0.5657i
tps	0.0064 + 0.0227i	0.0064 + 0.0227i	-0.0064 - 0.0227i
tsp	-0.0064 - 0.0227i	-0.0064 - 0.0227i	0.0064 + 0.0227i
tpp	0.7393 + 0.6519i	0.7393 + 0.6519i	0.7393 + 0.6519i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Gyroelectric (nonreciprocal SKL-3) medium [gyrotropy axis along x]

```
nf=nf+1;
ep=[4+0.1i 0 0; 0 4+0.1i 0.2i; 0 -0.2i 4+0.1i;]; % must be anti-
symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

(θ,ϕ)/π= (0.33333 ,0.16667) thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6326 + 0.3596i	-0.6326 + 0.3596i	-0.6326 + 0.3596i
rps	-0.0025 - 0.0086i	0.0034 + 0.0082i	0.0025 + 0.0086i
rsp	0.0025 + 0.0086i	-0.0034 - 0.0082i	-0.0025 - 0.0086i
rpp	0.0560 - 0.0410i	0.0440 - 0.0510i	0.0560 - 0.0410i
tss	0.3455 + 0.5664i	0.3455 + 0.5664i	0.3455 + 0.5664i
tps	-0.0038 - 0.0084i	0.0028 + 0.0088i	0.0038 + 0.0084i
tsp	0.0028 + 0.0088i	-0.0038 - 0.0084i	-0.0028 - 0.0088i
tpp	0.7394 + 0.6524i	0.7394 + 0.6524i	0.7394 + 0.6524i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Gyroelectric (nonreciprocal SKL-3) medium [gyrotropy axis along y]

```
nf=nf+1;
ep=[4+0.1i 0 0.2i; 0 4+0.1i 0; -0.2i 0 4+0.1i;]; % must be anti-
symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6333 + 0.3594i	-0.6333 + 0.3594i	-0.6333 + 0.3594i
rps	0.0012 + 0.0050i	-0.0022 - 0.0047i	-0.0012 - 0.0050i
rsp	-0.0012 - 0.0050i	0.0022 + 0.0047i	0.0012 + 0.0050i
rpp	0.0601 - 0.0369i	0.0393 - 0.0542i	0.0601 - 0.0369i
tss	0.3448 + 0.5663i	0.3448 + 0.5663i	0.3448 + 0.5663i
tps	0.0024 + 0.0048i	-0.0014 - 0.0052i	-0.0024 - 0.0048i
tsp	-0.0014 - 0.0052i	0.0024 + 0.0048i	0.0014 + 0.0052i
tpp	0.7397 + 0.6521i	0.7397 + 0.6521i	0.7397 + 0.6521i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Gyromagnetic (nonreciprocal SKL-2) medium [gyrotropy axis along z]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=[-2+0.1i 0.2i 0; -0.2i -2+0.1i 0; 0 0 -2+0.1i]; % must be anti-
symmetric
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667) thickness, $d\omega/c$=0.4

	Incidence	Reflection	Transmission
rss	-0.6669 + 0.6113i	-0.6669 + 0.6113i	-0.6669 + 0.6113i
rps	0.0159 - 0.0212i	0.0159 - 0.0212i	-0.0159 + 0.0212i
rsp	0.0159 - 0.0212i	0.0159 - 0.0212i	-0.0159 + 0.0212i
rpp	-0.2659 - 0.7767i	-0.2659 - 0.7767i	-0.2659 - 0.7767i
tss	0.2545 + 0.2707i	0.2545 + 0.2707i	0.2545 + 0.2707i
tps	0.0172 - 0.0142i	0.0172 - 0.0142i	-0.0172 + 0.0142i
tsp	-0.0172 + 0.0142i	-0.0172 + 0.0142i	0.0172 - 0.0142i
tpp	0.4925 - 0.1515i	0.4925 - 0.1515i	0.4925 - 0.1515i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Gyromagnetic (nonreciprocal SKL-3) medium [gyrotropy axis along x]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=[2+0.1i 0 0; 0 2+0.1i 0.2i; 0 -0.2i 2+0.1i]; % must be anti-
symmetric
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

(θ,ϕ)/π= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6514 + 0.2347i	-0.6592 + 0.2172i	-0.6514 + 0.2347i
rps	0.0021 - 0.0151i	-0.0002 + 0.0128i	-0.0021 + 0.0151i
rsp	-0.0021 + 0.0151i	0.0002 - 0.0128i	0.0021 - 0.0151i
rpp	-0.2232 + 0.0948i	-0.2232 + 0.0948i	-0.2232 + 0.0948i
tss	0.2290 + 0.6371i	0.2290 + 0.6371i	0.2290 + 0.6371i
tps	0.0011 - 0.0162i	0.0017 + 0.0138i	-0.0011 + 0.0162i
tsp	0.0017 + 0.0138i	0.0011 - 0.0162i	-0.0017 - 0.0138i
tpp	0.4316 + 0.8171i	0.4316 + 0.8171i	0.4316 + 0.8171i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Isotropic magnetoelectric (nonreciprocal SKL-2) medium (Tellegen medium)

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=0.2*eye(3);  %  nonzero magnetoelectric coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6312 + 0.3588i	-0.6312 + 0.3588i	-0.6312 + 0.3588i
rps	-0.0445 + 0.0286i	-0.0445 + 0.0286i	-0.0445 + 0.0286i
rsp	-0.0445 + 0.0286i	-0.0445 + 0.0286i	-0.0445 + 0.0286i
rpp	0.0508 - 0.0487i	0.0508 - 0.0487i	0.0508 - 0.0487i
tss	0.3462 + 0.5650i	0.3462 + 0.5650i	0.3462 + 0.5650i
tps	-0.0264 - 0.0047i	-0.0264 - 0.0047i	-0.0264 - 0.0047i
tsp	-0.0264 - 0.0047i	-0.0264 - 0.0047i	-0.0264 - 0.0047i
tpp	0.7397 + 0.6492i	0.7397 + 0.6492i	0.7397 + 0.6492i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Anisotropic magnetoelectric (nonreciprocal SKL-3) medium [xz-coupling]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2; 0 0 0; 0 0 0;]; % Ex-Hz, Ez-Hx coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

      %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6061 + 0.3658i	-0.6561 + 0.3522i	-0.6561 + 0.3522i
rps	-0.0192 + 0.0055i	0.0145 - 0.0033i	-0.0145 + 0.0033i
rsp	0.0192 - 0.0055i	-0.0145 + 0.0033i	0.0145 - 0.0033i
rpp	0.0483 - 0.0435i	0.0482 - 0.0438i	0.0482 - 0.0438i
tss	0.3721 + 0.5721i	0.3219 + 0.5595i	0.3219 + 0.5595i
tps	0.0195 - 0.0062i	-0.0147 + 0.0038i	0.0147 - 0.0038i
tsp	0.0195 - 0.0062i	-0.0147 + 0.0038i	0.0147 - 0.0038i
tpp	0.7413 + 0.6501i	0.7413 + 0.6503i	0.7413 + 0.6503i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Anisotropic magnetoelectric (reciprocal SKL-3) medium [yz-coupling]

```
nf=nf+1;
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2; 0 0.1 0;]; % Ey-Hz, Ez-Hy coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, dω/c=0.4

	Incidence	Reflection	Transmission
rss	-0.6701 + 0.3476i	-0.5825 + 0.3706i	-0.5825 + 0.3706i
rps	-0.0067 - 0.0007i	0.0116 - 0.0020i	-0.0116 + 0.0020i
rsp	0.0067 + 0.0007i	-0.0116 + 0.0020i	0.0116 - 0.0020i
rpp	0.0579 - 0.0573i	0.0417 - 0.0345i	0.0417 - 0.0345i
tss	0.3079 + 0.5550i	0.3958 + 0.5764i	0.3958 + 0.5764i
tps	0.0087 - 0.0041i	-0.0134 + 0.0061i	0.0134 - 0.0061i
tsp	0.0087 - 0.0041i	-0.0134 + 0.0061i	0.0134 - 0.0061i
tpp	0.7494 + 0.6389i	0.7310 + 0.6634i	0.7310 + 0.6634i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Composite gyroelectric magnetoelectric (non-reciprocal SKL-2) medium

```
nf=nf+1;
ep=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i];  % gyrotropy axis
 along z
mu=(1+1e-6*1i)*eye(3);
xi=0.2*eye(3);  % Isotropic magnetoelectric
zeta=transpose(xi);  % rotational symmetry preserved
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

    %
```

$(\theta,\phi)/\pi$= **(0.33333 ,0.16667)   thickness, dω/c=0.4**

	Incidence	Reflection	Transmission
rss	-0.6319 + 0.3585i	-0.6319 + 0.3585i	-0.6300 + 0.3580i
rps	-0.0502 + 0.0063i	-0.0502 + 0.0063i	-0.0388 + 0.0510i
rsp	-0.0502 + 0.0063i	-0.0502 + 0.0063i	-0.0388 + 0.0510i
rpp	0.0536 - 0.0477i	0.0536 - 0.0477i	0.0478 - 0.0479i
tss	0.3465 + 0.5645i	0.3465 + 0.5645i	0.3465 + 0.5645i
tps	-0.0199 + 0.0180i	-0.0199 + 0.0180i	-0.0329 - 0.0274i
tsp	-0.0329 - 0.0274i	-0.0329 - 0.0274i	-0.0199 + 0.0180i
tpp	0.7398 + 0.6483i	0.7398 + 0.6483i	0.7398 + 0.6483i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Composite gyroelectric magnetoelectric (non-reciprocal SKL-3) medium

```
nf=nf+1;
ep=[4+0.1i 0 0; 0 4+0.1i 0.2i; 0 -0.2i 4+0.1i];  % gyrotropy axis alon
 x
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2; 0 0 0; 0.1 0 0;];  % Ex-Hz, Ez-Hx coupling
zeta=transpose(xi);
% rotational symmetry not preserved for this nonreciprocal medium
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);

%
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)   thickness, $d\omega/c$=0.4

	Incidence	Reflection	Transmission
rss	-0.6043 + 0.3669i	-0.6545 + 0.3534i	-0.6556 + 0.3519i
rps	-0.0204 - 0.0059i	0.0169 + 0.0090i	-0.0106 + 0.0085i
rsp	0.0204 + 0.0059i	-0.0169 - 0.0090i	0.0106 - 0.0085i
rpp	0.0485 - 0.0324i	0.0459 - 0.0557i	0.0598 - 0.0452i
tss	0.3734 + 0.5721i	0.3230 + 0.5595i	0.3230 + 0.5595i
tps	0.0174 - 0.0178i	-0.0141 + 0.0167i	0.0203 + 0.0009i
tsp	0.0244 - 0.0010i	-0.0203 - 0.0009i	0.0141 - 0.0167i
tpp	0.7362 + 0.6567i	0.7468 + 0.6426i	0.7468 + 0.6426i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Multilayered gyroelectric magnetoelectric (non-reciprocal SKL-2) medium

```
nf=nf+1;
% Material 1
ep1=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i];  % gyrotropy axis
 along z
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
xi2=0.2*eye(3);  % isotropic magnetoelectric
zeta2=transpose(xi2); % nonreciprocity
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
D_layer_table(theta,phi,MM1,MM2,td,nf);

    %
```

(θ,ϕ)/π= (0.33333 ,0.16667)

	Incidence	Reflection	Transmission
rss	-0.8620 - 0.1891i	-0.8620 - 0.1891i	-0.8779 - 0.0579i
rps	-0.0336 - 0.0327i	-0.0336 - 0.0327i	-0.0530 + 0.0195i
rsp	-0.0336 - 0.0327i	-0.0336 - 0.0327i	-0.0530 + 0.0195i
rpp	0.1679 + 0.1606i	0.1679 + 0.1606i	0.2249 - 0.0292i
tss	-0.0606 + 0.4462i	-0.0606 + 0.4462i	-0.0606 + 0.4462i
tps	-0.0087 - 0.0172i	-0.0087 - 0.0172i	0.0235 - 0.0201i
tsp	0.0235 - 0.0201i	0.0235 - 0.0201i	-0.0087 - 0.0172i
tpp	-0.2731 + 0.9102i	-0.2731 + 0.9102i	-0.2731 + 0.9102i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

Multilayered gyroelectric magnetoelectric (non-reciprocal SKL-3) medium

```
nf=nf+1;
% Material 1
ep1=[4+0.1i 0 0.2i; 0 4+0.1i 0; -0.2i 0 4+0.1i];  % gyrotropy axis
 along y
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
xi2=[0 0 0; 0 0 0.2; 0 0.1 0;];  % Ey-Hz, Ez-Hy coupling
zeta2=transpose(xi2);
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
D_layer_table(theta,phi,MM1,MM2,td,nf);
```

$(\theta,\phi)/\pi$= (0.33333 ,0.16667)

	Incidence	Reflection	Transmission
rss	-0.8616 - 0.2083i	-0.8625 - 0.1672i	-0.8753 - 0.0542i
rps	0.0027 - 0.0052i	-0.0016 + 0.0078i	0.0017 - 0.0034i
rsp	-0.0027 + 0.0052i	0.0016 - 0.0078i	-0.0017 + 0.0034i
rpp	0.1852 + 0.1712i	0.1474 + 0.1417i	0.1992 - 0.0172i
tss	-0.0667 + 0.4409i	-0.0547 + 0.4563i	-0.0547 + 0.4563i
tps	0.0067 + 0.0027i	-0.0084 - 0.0046i	0.0076 - 0.0015i
tsp	0.0075 - 0.0026i	-0.0076 + 0.0015i	0.0084 + 0.0046i
tpp	-0.2584 + 0.9110i	-0.2914 + 0.9128i	-0.2914 + 0.9128i

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

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