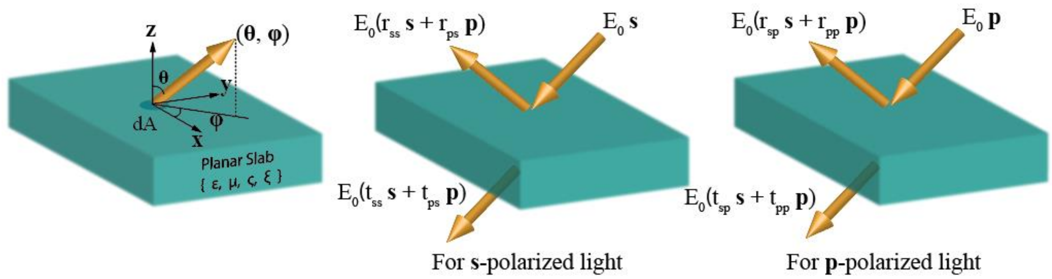
Fresnel coefficients for generic bianisotropic planar media

Introduction

The reflection and transmission of s-polarized or p-polarized light incident on a planar surface is described using the Fresnel interconversion coefficients as shown in the figure below. For instance, incident unit-amplitude s-polarized light gets reflected as (r_{ss} **s** + r_{ps} **p**) and transmitted as (t_{ss} **s** + t_{ps} **p**). The leftmost figure denotes the notation for directions and co-ordinate axes used in this work.



We are interested in finding these coefficients for very generic, bianisotropic (linear, time-invariant) media (multilayered or composite planar geometries) that are described using the following constitutive relations in Maxwell's equations:

$$\begin{bmatrix}\mathbf{D}\\ \mathbf{B}\end{bmatrix} = \begin{bmatrix}\bar{\bar{\epsilon}}\epsilon_0 & \bar{\bar{\xi}}/c\\ \bar{\bar{\zeta}}/c & \bar{\bar{\mu}}\mu_0\end{bmatrix} \times \begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix}$$

Here, $\bar{\bar{\epsilon}}, \bar{\bar{\mu}}, \bar{\bar{\xi}}, \bar{\bar{\zeta}}$ are dimensionless permittivity, permeability and magneto-electric coupling tensors. They are scalars for isotropic materials. ϵ_0 is vacuum permittivity, μ_0 is vacuum permeability and c_0 is vacuum speed of light. We assume the above equation is written in the frequency domain with fields $\mathbf{E}(\omega), \mathbf{H}(\omega)$ and dispersive material parameters $\bar{\bar{\epsilon}}(\omega), \bar{\bar{\xi}}(\omega)$ and so on. We will denote the 6×6 material matrix as $\mathbf{M}$ in the subsequent discussion for convenience and this is used as an input to the code. Examples of various materials studied in the literature are given in the table below:

No.	Material Class	Description	Examples
1	Reciprocal isotropic	$\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}$ are scalars, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$	Common dielectric and metallic materials
2	Reciprocal anisotropic	Diagonal $\bar{\bar{\varepsilon}}$ with unequal entries, scalar $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Uniaxial and biaxial crystals
3	Nonreciprocal Gyroelectric	$\bar{\bar{\varepsilon}} = -\bar{\bar{\varepsilon}}^T$, scalar $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Weyl Semimetals, Metals and semiconductors in magnetic field
4	Nonreciprocal Gyromagnetic	$\bar{\bar{\mu}} = -\bar{\bar{\mu}}^T$, scalar $\bar{\bar{\varepsilon}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}} = 0$.	Ferromagnets, Ferrites
5	Reciprocal Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, nonzero $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}^T$.	Chiral media, Artificial metamaterials, Pasteur media (diagonal $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}$)
6	Nonreciprocal Isotropic Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, diagonal or scalar coupling $\bar{\bar{\xi}} = \bar{\bar{\zeta}}^T$.	Topological insulators, Tellegen media
7	Nonreciprocal Anisotropic Magneto-electric	scalar $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, $\bar{\bar{\xi}} = \bar{\bar{\zeta}}^T \neq 0$ have nonzero off-diagonal entries.	Multiferroic media, Engineered heterostructures

How to use?

For all purposes, we enforce the passivity constraint on the material parameters i.e. dimensionless tensors $\bar{\bar{\epsilon}}, \bar{\bar{\mu}}, \bar{\bar{\xi}}, \bar{\bar{\zeta}}$ which define the material matrix **M**. They cannot be arbitrary. For instance, one example is given below:

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

This example represents a gyroelectric (magneto-optic) medium. Note that a small amount of loss [Im{μ}~ 10^{-6}] is introduced above. This passivity constraint ensures that the power dissipation inside a medium given below is always positive semi-definite.

$$q = \frac{1}{2}\mathrm{Re}\left\{\left(\mathbf{E}^* \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}^* \cdot \frac{\partial \mathbf{B}}{\partial t}\right)\right\} \geq 0$$

<u>We hope that the user keeps this passivity constraint in mind. The code cannot be readily used for non-passive media.</u> If the passivity constraint is violated, the code will produce an error. Having said that, we describe its application below for various configurations.

A. Semi-infinite half-space.

There are only 4 Fresnel reflection coefficients which can be found using 'fresnel.m'
First define the material matrix as given below (satisfying the passivity constraint)

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

Step 1

Define the angles (Θ, φ) that denote direction of incidence. Θ is the angle made by the incident light with z-axis while φ is the angle with x-axis.

```
>> theta=pi/3; phi=pi/6;
>> [rss rps rsp rpp]=fresnel(theta,phi,MM);
```

Step 2

B. Finite-thickness slab.

There are total 8 Fresnel coefficients. For a finite thickness slab, the light can be incident from top (+z) or from the bottom (-z) side of the slab. Depending on this incidence, we have two different codes 'fresnel_top.m' and 'fresnel_bottom.m'. The thickness of the slab is given in units of (c/ω) where ω is the frequency in rad/s where the material parameters are measured.

```
>> ep=[4+0.1i 0.1i 0; -0.1i 4+0.1i 0; 0 0 4+0.1i;];
>> mu=(1+1e-6*1i)*eye(3); xi=zeros(3); zeta=zeros(3);
>> MM=[ep xi; zeta mu;];
```

Step 1

Define the angles (Θ, φ) that denote direction of incidence. Θ is angle with z-axis, φ is angle with x-axis. And find the Fresnel coefficients as the following.

```
>> theta=pi/3; phi=pi/6; d=0.2;
>> [rss rps rsp rpp tss tps tsp tpp]=fresnel_top(theta,phi,MM,d);
>> [rss1 rps1 rsp1 rpp1 tss1 tps1 tsp1 tpp1]=fresnel_bottom(theta,phi,MM,d);
```

Step 2

C. Multi-layered medium

A multi-layered finite-thickness slab can consist of generic bianisotropic materials. Same as the previous case of finite-thickness slab of a single (/composite) material, here we find the coefficients separately for light incident either from the top (+z) or bottom (-z) side of the slab using separate codes 'multilayer_top.m' and 'multilayer_bottom.m' respectively.

Please refer to the code to find how the geometry is set up. For the given geometry (layers and various materials), one can find the reflection and transmission coefficients for specific directions.

Define the materials and the layer thicknesses etcetera. **Step 1**

```
>> [rss rps rsp rpp tss tps tsp tpp]=multilayer_top(theta,phi);
>> [rss1 rps1 rsp1 rpp1 tss1 tps1 tsp1 tpp1]=multilayer_bottom(theta,phi)
```

Step 2

The Fresnel coefficients are obtained by solving the boundary conditions (continuity of tangential **E**, **H** fields) for each interface in the planar geometry. For details regarding the method, we encourage the reader to refer to the supplement section of the following reference.

[1] Thermal spin photonics in the near-field of nonreciprocal media, C. Khandekar and Z. Jacob, New. J. Phys. **21**, 103030, 2019. [link]