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# Fresnel Coefficients

This script provies Fresnel reflection and transmission coefficients for light incident on a planar slab of thickness 'd' in arbitrary directions characterized by ($\theta, \phi$). Here $\theta$ denotes the angle made by the incident light with $z$-axis while $\phi$ denotes the angle made by the in-plane wavevector of the incident light with $x$-axis. The thickness 'd' is in units of $c/\omega$ where 'c' is vacuum speed of light and $\omega$ is the frequency.

For each example, a material matrix 'MM' is defined which contains dimensionless permeability tensor ($\epsilon$), permeability tensor ($\mu$), and magneto-electric coupling tensors ($\xi, \zeta$). For each material example, these response tensors are not arbitrary but also satisfy passivity constaint. We ensure that this passivity is satisfied since our theory is not applicable for optically active gain media. If the medium is not passive, an error message will be displayed.

As discussed in our manuscript, the spin-resolved emissivity along the incidence direction ($\theta, \phi$) requires the calculation of Fresnel reflection and transmission coefficients for light incident along the reflection direction ($\theta, \phi + \pi$) and the transmission direction ($\pi - \theta, \phi + \pi$) on the other side of the slab. We calculate all these coefficients for each example. For simplicity, we focus on $\theta \in [0, \pi/2]$ and $\phi \in= [0, 2\pi]$ which denotes light incident in the top hemisphere.

The user can specify the parameters $\theta, \phi$, thickness 'd', and the material matrix MM in this script. The spin-resolved Kirchhoff's laws are derived based on certain relations between the Fresnel coefficients. These can be verified for various material (MM) and thickness (d) parameters, and indcidence directions ($\theta, \phi$). The user only needs to choose the parameters and run the command 'publish('make_report.m','pdf')' to produce a report on Fresnel coefficients for several examples. On a standard computer with MATLAB

2018, it takes approximately 10 minutes to put together all results and provide this universal perspective of Fresnel coefficients for several material classes. Below, we fix some of the parameters.

```
theta=pi/3;  % angle between incident light and z-axis (normal to
 slab)
phi=pi/6; % angle between parallel wavevector and x-axis
d=0.4; % thickness of slab in units of c/w.
```

Below, we first consider reciprocal media that satisfy SKL-1 and then nonreciprocal media which satisfy SKL-2 or SKL-3. We also provide these parameters for a double layered thin film.

# Uniaxial anisotropic (reciprocal SKL-1) medium

```
nf=1  % example 1 and figure 1
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 2+0.1i]; % ep_xx not equal to ep_yy
mu=(1+1e-6*1i)*eye(3);   % small loss added for passivity
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     1
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.3502 - 0.4783i | -0.3502 - 0.4783i | -0.3502 - 0.4783i |
| rps | -0.1058 - 0.2251i | -0.1058 - 0.2251i | 0.1058 + 0.2251i |
| rsp | 0.1058 + 0.2251i | 0.1058 + 0.2251i | -0.1058 - 0.2251i |
| rpp | 0.0341 + 0.1054i | 0.0341 + 0.1054i | 0.0341 + 0.1054i |
| tss | 0.6306 - 0.2847i | 0.6306 - 0.2847i | 0.6306 - 0.2847i |
| tps | 0.1038 + 0.2301i | 0.1038 + 0.2301i | -0.1038 - 0.2301i |
| tsp | 0.1038 + 0.2301i | 0.1038 + 0.2301i | -0.1038 - 0.2301i |
| tpp | 0.8331 + 0.3625i | 0.8331 + 0.3625i | 0.8331 + 0.3625i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=-t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Biaxial anisotropic (reciprocal SKL-1) medium

```
nf=nf+1
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 3+0.1i];
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

*nf =*

*2*

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667) thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.3494 - 0.4777i | -0.3494 - 0.4777i | -0.3494 - 0.4777i |
| rps | -0.1060 - 0.2253i | -0.1060 - 0.2253i | 0.1060 + 0.2253i |
| rsp | 0.1060 + 0.2253i | 0.1060 + 0.2253i | -0.1060 - 0.2253i |
| rpp | 0.0138 + 0.1471i | 0.0138 + 0.1471i | 0.0138 + 0.1471i |
| tss | 0.6314 - 0.2841i | 0.6314 - 0.2841i | 0.6314 - 0.2841i |
| tps | 0.1035 + 0.2312i | 0.1035 + 0.2312i | -0.1035 - 0.2312i |
| tsp | 0.1035 + 0.2312i | 0.1035 + 0.2312i | -0.1035 - 0.2312i |
| tpp | 0.8120 + 0.4046i | 0.8120 + 0.4046i | 0.8120 + 0.4046i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi) = r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi) = -r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi) = t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi) = -t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Isotropic magnetoelectric (reciprocal SKL-1) medium (Pasteur medium)

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=0.2i*eye(3);   % nonzero magnetoelectric coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     3
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6312 + 0.3563i | -0.6312 + 0.3563i | -0.6312 + 0.3563i |
| rps | 0.0008 + 0.0456i | 0.0008 + 0.0456i | 0.0008 + 0.0456i |
| rsp | -0.0008 - 0.0456i | -0.0008 - 0.0456i | -0.0008 - 0.0456i |
| rpp | 0.0508 - 0.0521i | 0.0508 - 0.0521i | 0.0508 - 0.0521i |
| tss | 0.3460 + 0.5630i | 0.3460 + 0.5630i | 0.3460 + 0.5630i |
| tps | 0.0379 + 0.0659i | 0.0379 + 0.0659i | 0.0379 + 0.0659i |
| tsp | -0.0379 - 0.0659i | -0.0379 - 0.0659i | -0.0379 - 0.0659i |
| tpp | 0.7384 + 0.6469i | 0.7384 + 0.6469i | 0.7384 + 0.6469i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi) = r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi) = -r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi) = t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi) = -t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Anisotropic magnetoelectric (reciprocal SKL-1) medium [xy-coupling]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0.2i 0; 0 0 0; 0 0 0;];  % Ex-Hy, Ey-Hx coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     4
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6270 + 0.3694i | -0.6270 + 0.3694i | -0.6397 + 0.3488i |
| rps | -0.0139 - 0.0227i | -0.0139 - 0.0227i | -0.0133 - 0.0231i |
| rsp | 0.0139 + 0.0227i | 0.0139 + 0.0227i | 0.0133 + 0.0231i |
| rpp | 0.0094 - 0.0823i | 0.0094 - 0.0823i | 0.0908 - 0.0110i |
| tss | 0.3446 + 0.5657i | 0.3446 + 0.5657i | 0.3446 + 0.5657i |
| tps | 0.0006 - 0.0162i | 0.0006 - 0.0162i | 0.0007 - 0.0141i |
| tsp | -0.0007 + 0.0141i | -0.0007 + 0.0141i | -0.0006 + 0.0162i |
| tpp | 0.7386 + 0.6504i | 0.7386 + 0.6504i | 0.7386 + 0.6504i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=-t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Anisotropic magnetoelectric (reciprocal SKL-1) medium [xz-coupling]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2i; 0 0 0; 0 0 0;];  % Ex-Hz, Ez-Hx coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     5
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6321 + 0.3595i | -0.6321 + 0.3595i | -0.6321 + 0.3595i |
| rps | -0.0062 - 0.0163i | 0.0023 + 0.0173i | -0.0023 - 0.0173i |
| rsp | -0.0023 - 0.0173i | 0.0062 + 0.0163i | -0.0062 - 0.0163i |
| rpp | 0.0482 - 0.0436i | 0.0482 - 0.0436i | 0.0482 - 0.0436i |
| tss | 0.3460 + 0.5663i | 0.3460 + 0.5663i | 0.3460 + 0.5663i |
| tps | 0.0069 + 0.0165i | -0.0029 - 0.0176i | 0.0029 + 0.0176i |
| tsp | -0.0029 - 0.0176i | 0.0069 + 0.0165i | -0.0069 - 0.0165i |
| tpp | 0.7413 + 0.6502i | 0.7413 + 0.6502i | 0.7413 + 0.6502i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=-t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Anisotropic magnetoelectric (reciprocal SKL-1) medium [yz-coupling]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2i; 0 0.1i 0;];  % Ey-Hz, Ez-Hy coupling
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     6
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6293 + 0.3605i | -0.6293 + 0.3605i | -0.6293 + 0.3605i |
| rps | 0.0015 - 0.0098i | 0.0025 + 0.0084i | -0.0025 - 0.0084i |
| rsp | -0.0025 - 0.0084i | -0.0015 + 0.0098i | 0.0015 - 0.0098i |
| rpp | 0.0499 - 0.0458i | 0.0499 - 0.0458i | 0.0499 - 0.0458i |
| tss | 0.3488 + 0.5671i | 0.3488 + 0.5671i | 0.3488 + 0.5671i |
| tps | 0.0031 + 0.0113i | -0.0068 - 0.0106i | 0.0068 + 0.0106i |
| tsp | -0.0068 - 0.0106i | 0.0031 + 0.0113i | -0.0031 - 0.0113i |
| tpp | 0.7404 + 0.6513i | 0.7404 + 0.6513i | 0.7404 + 0.6513i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=-t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Composite anisotropic magnetoelectric (reciprocal SKL-1) medium

```
nf=nf+1
ep=[2+0.1i 0 0; 0 -2+0.1i 0; 0 0 2+0.1i]; % uniaxial anisotropy
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2i; 0 0.1i 0;];  % magnetoelectric anisotropy
zeta=-transpose(xi);  % reciprocity condition
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     7
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.3565 - 0.4799i | -0.3565 - 0.4799i | -0.3565 - 0.4799i |
| rps | -0.1148 - 0.2191i | -0.1007 - 0.2319i | 0.1007 + 0.2319i |
| rsp | 0.1007 + 0.2319i | 0.1148 + 0.2191i | -0.1148 - 0.2191i |
| rpp | 0.0355 + 0.1039i | 0.0355 + 0.1039i | 0.0355 + 0.1039i |
| tss | 0.6243 - 0.2866i | 0.6243 - 0.2866i | 0.6243 - 0.2866i |
| tps | 0.1211 + 0.2258i | 0.0909 + 0.2338i | -0.0909 - 0.2338i |
| tsp | 0.0909 + 0.2338i | 0.1211 + 0.2258i | -0.1211 - 0.2258i |
| tpp | 0.8329 + 0.3614i | 0.8329 + 0.3614i | 0.8329 + 0.3614i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along

these directions satisfy the relations: $r_{ss,pp}(\theta,\phi) = r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi) = -r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi) = t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi) = -t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Multilayered anisotropic magnetoelectric (reciprocal SKL-1) medium

```
nf=nf+1
% Material 1
ep1=[4+0.1i 0 0; 0 -3+0.1i 0; 0 0 4+0.1i];  % uniaxial anisotropic
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
xi2=0.2i*eye(3);   % isotropic magnetoelectric coupling
zeta2=-transpose(xi2);
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
make_2layered_table(theta,phi,MM1,MM2,td,nf);
```

```
nf =

     8
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.8684 - 0.1887i | -0.8684 - 0.1887i | -0.7462 + 0.3581i |
| rps | -0.0317 - 0.0374i | -0.0317 - 0.0374i | -0.1065 - 0.1941i |
| rsp | 0.0317 + 0.0374i | 0.0317 + 0.0374i | 0.1065 + 0.1941i |
| rpp | 0.1474 + 0.2261i | 0.1474 + 0.2261i | 0.1767 - 0.1082i |
| tss | 0.0462 + 0.4248i | 0.0462 + 0.4248i | 0.0462 + 0.4248i |
| tps | -0.0679 + 0.0774i | -0.0679 + 0.0774i | 0.1766 + 0.0622i |
| tsp | -0.1766 - 0.0622i | -0.1766 - 0.0622i | 0.0679 - 0.0774i |
| tpp | -0.1806 + 0.9036i | -0.1806 + 0.9036i | -0.1806 + 0.9036i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incidence along these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=-t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-1.

```
%
```

# Gyroelectric (nonreciprocal SKL-2) medium [gyrotropy axis along z]

```
nf=nf+1
ep=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i;]; % must be anti-
symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

     9
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6334 + 0.3589i | -0.6334 + 0.3589i | -0.6334 + 0.3589i |
| rps | -0.0056 - 0.0223i | -0.0056 - 0.0223i | 0.0056 + 0.0223i |
| rsp | -0.0056 - 0.0223i | -0.0056 - 0.0223i | 0.0056 + 0.0223i |
| rpp | 0.0501 - 0.0451i | 0.0501 - 0.0451i | 0.0501 - 0.0451i |
| tss | 0.3447 + 0.5657i | 0.3447 + 0.5657i | 0.3447 + 0.5657i |
| tps | 0.0064 + 0.0227i | 0.0064 + 0.0227i | -0.0064 - 0.0227i |
| tsp | -0.0064 - 0.0227i | -0.0064 - 0.0227i | 0.0064 + 0.0227i |
| tpp | 0.7393 + 0.6519i | 0.7393 + 0.6519i | 0.7393 + 0.6519i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta, \phi)$, reflection direction is $(\theta, \phi + \pi)$, and transmission direction is $(\pi - \theta, \phi + \pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{ss,pp}(\theta, \phi) = r_{ss,pp}(\theta, \phi + \pi)$, $r_{sp}(\theta, \phi) = r_{ps}(\theta, \phi + \pi)$, $t_{ss,pp}(\theta, \phi) = t_{ss,pp}(\pi - \theta, \phi + \pi)$, $t_{sp}(\theta, \phi) = t_{ps}(\pi - \theta, \phi + \pi)$. These relations lead to SKL-2.

```
%
```

# Gyroelectric (nonreciprocal SKL-3) medium [gyrotropy axis along x]

```
nf=nf+1
ep=[4+0.1i 0 0; 0 4+0.1i 0.2i; 0 -0.2i 4+0.1i;]; % must be anti-symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

    10
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6326 + 0.3596i | -0.6326 + 0.3596i | -0.6326 + 0.3596i |
| rps | -0.0025 - 0.0086i | 0.0034 + 0.0082i | 0.0025 + 0.0086i |
| rsp | 0.0025 + 0.0086i | -0.0034 - 0.0082i | -0.0025 - 0.0086i |
| rpp | 0.0560 - 0.0410i | 0.0440 - 0.0510i | 0.0560 - 0.0410i |
| tss | 0.3455 + 0.5664i | 0.3455 + 0.5664i | 0.3455 + 0.5664i |
| tps | -0.0038 - 0.0084i | 0.0028 + 0.0088i | 0.0038 + 0.0084i |
| tsp | 0.0028 + 0.0088i | -0.0038 - 0.0084i | -0.0028 - 0.0088i |
| tpp | 0.7394 + 0.6524i | 0.7394 + 0.6524i | 0.7394 + 0.6524i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi)=t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi)=-t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-2.

```
%
```

# Gyroelectric (nonreciprocal SKL-3) medium [gyrotropy axis along y]

```
nf=nf+1
ep=[4+0.1i 0 0.2i; 0 4+0.1i 0; -0.2i 0 4+0.1i;]; % must be anti-
symmetric
mu=(1+1e-6*1i)*eye(3);
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

    11
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, $d\omega/c$=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6333 + 0.3594i | -0.6333 + 0.3594i | -0.6333 + 0.3594i |
| rps | 0.0012 + 0.0050i | -0.0022 - 0.0047i | -0.0012 - 0.0050i |
| rsp | -0.0012 - 0.0050i | 0.0022 + 0.0047i | 0.0012 + 0.0050i |
| rpp | 0.0601 - 0.0369i | 0.0393 - 0.0542i | 0.0601 - 0.0369i |
| tss | 0.3448 + 0.5663i | 0.3448 + 0.5663i | 0.3448 + 0.5663i |
| tps | 0.0024 + 0.0048i | -0.0014 - 0.0052i | -0.0024 - 0.0048i |
| tsp | -0.0014 - 0.0052i | 0.0024 + 0.0048i | 0.0014 + 0.0052i |
| tpp | 0.7397 + 0.6521i | 0.7397 + 0.6521i | 0.7397 + 0.6521i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi) = -r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi) = t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi) = -t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

```
%
```

# Gyromagnetic (nonreciprocal SKL-2) medium [gyrotropy axis along z]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=[-2+0.1i 0.2i 0; -0.2i -2+0.1i 0; 0 0 -2+0.1i]; % must be anti-
symmetric
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

*nf =*

*12*

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6669 + 0.6113i | -0.6669 + 0.6113i | -0.6669 + 0.6113i |
| rps | 0.0159 - 0.0212i | 0.0159 - 0.0212i | -0.0159 + 0.0212i |
| rsp | 0.0159 - 0.0212i | 0.0159 - 0.0212i | -0.0159 + 0.0212i |
| rpp | -0.2659 - 0.7767i | -0.2659 - 0.7767i | -0.2659 - 0.7767i |
| tss | 0.2545 + 0.2707i | 0.2545 + 0.2707i | 0.2545 + 0.2707i |
| tps | 0.0172 - 0.0142i | 0.0172 - 0.0142i | -0.0172 + 0.0142i |
| tsp | -0.0172 + 0.0142i | -0.0172 + 0.0142i | 0.0172 - 0.0142i |
| tpp | 0.4925 - 0.1515i | 0.4925 - 0.1515i | 0.4925 - 0.1515i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-2.

```
%
```

# Gyromagnetic (nonreciprocal SKL-3) medium [gyrotropy axis along x]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=[2+0.1i 0 0; 0 2+0.1i 0.2i; 0 -0.2i 2+0.1i]; % must be anti-symmetric
xi=zeros(3); zeta=zeros(3);
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

*nf =*

*13*

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6514 + 0.2347i | -0.6592 + 0.2172i | -0.6514 + 0.2347i |
| rps | 0.0021 - 0.0151i | -0.0002 + 0.0128i | -0.0021 + 0.0151i |
| rsp | -0.0021 + 0.0151i | 0.0002 - 0.0128i | 0.0021 - 0.0151i |
| rpp | -0.2232 + 0.0948i | -0.2232 + 0.0948i | -0.2232 + 0.0948i |
| tss | 0.2290 + 0.6371i | 0.2290 + 0.6371i | 0.2290 + 0.6371i |
| tps | 0.0011 - 0.0162i | 0.0017 + 0.0138i | -0.0011 + 0.0162i |
| tsp | 0.0017 + 0.0138i | 0.0011 - 0.0162i | -0.0017 - 0.0138i |
| tpp | 0.4316 + 0.8171i | 0.4316 + 0.8171i | 0.4316 + 0.8171i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi)=t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi)=-t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

```
%
```

# Isotropic magnetoelectric (nonreciprocal SKL-2) medium (Tellegen medium)

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=0.2*eye(3);  %  nonzero magnetoelectric coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

*nf =*

*14*

$(\theta,\phi)/\pi$= (0.33333 ,0.16667) thickness, d$\omega$/c=0.4

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6312 + 0.3588i | -0.6312 + 0.3588i | -0.6312 + 0.3588i |
| rps | -0.0445 + 0.0286i | -0.0445 + 0.0286i | -0.0445 + 0.0286i |
| rsp | -0.0445 + 0.0286i | -0.0445 + 0.0286i | -0.0445 + 0.0286i |
| rpp | 0.0508 - 0.0487i | 0.0508 - 0.0487i | 0.0508 - 0.0487i |
| tss | 0.3462 + 0.5650i | 0.3462 + 0.5650i | 0.3462 + 0.5650i |
| tps | -0.0264 - 0.0047i | -0.0264 - 0.0047i | -0.0264 - 0.0047i |
| tsp | -0.0264 - 0.0047i | -0.0264 - 0.0047i | -0.0264 - 0.0047i |
| tpp | 0.7397 + 0.6492i | 0.7397 + 0.6492i | 0.7397 + 0.6492i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta, \phi)$, reflection direction is $(\theta, \phi + \pi)$, and transmission direction is $(\pi - \theta, \phi + \pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{ss,pp}(\theta, \phi) = r_{ss,pp}(\theta, \phi + \pi)$, $r_{sp}(\theta, \phi) = r_{ps}(\theta, \phi + \pi)$, $t_{ss,pp}(\theta, \phi) = t_{ss,pp}(\pi - \theta, \phi + \pi)$, $t_{sp}(\theta, \phi) = t_{ps}(\pi - \theta, \phi + \pi)$. These relations lead to SKL-2.

```
%
```

# Anisotropic magnetoelectric (nonreciprocal SKL-3) medium [xz-coupling]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2; 0 0 0; 0 0 0;]; % Ex-Hz, Ez-Hx coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

    15
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6061 + 0.3658i | -0.6561 + 0.3522i | -0.6561 + 0.3522i |
| rps | -0.0192 + 0.0055i | 0.0145 - 0.0033i | -0.0145 + 0.0033i |
| rsp | 0.0192 - 0.0055i | -0.0145 + 0.0033i | 0.0145 - 0.0033i |
| rpp | 0.0483 - 0.0435i | 0.0482 - 0.0438i | 0.0482 - 0.0438i |
| tss | 0.3721 + 0.5721i | 0.3219 + 0.5595i | 0.3219 + 0.5595i |
| tps | 0.0195 - 0.0062i | -0.0147 + 0.0038i | 0.0147 - 0.0038i |
| tsp | 0.0195 - 0.0062i | -0.0147 + 0.0038i | 0.0147 - 0.0038i |
| tpp | 0.7413 + 0.6501i | 0.7413 + 0.6503i | 0.7413 + 0.6503i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta, \phi)$, reflection direction is $(\theta, \phi+\pi)$, and transmission direction is $(\pi-\theta, \phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi) = -r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi) = t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi) = -t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

```
%
```

# Anisotropic magnetoelectric (nonreciprocal SKL-3) medium [yz-coupling]

```
nf=nf+1
ep=(4+0.1i)*eye(3);
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0; 0 0 0.2; 0 0.1 0;]; % Ey-Hz, Ez-Hy coupling
zeta=transpose(xi);  % non-reciprocity
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =

    16
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, $d\omega/c$=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6701 + 0.3476i | -0.5825 + 0.3706i | -0.5825 + 0.3706i |
| rps | -0.0067 - 0.0007i | 0.0116 - 0.0020i | -0.0116 + 0.0020i |
| rsp | 0.0067 + 0.0007i | -0.0116 + 0.0020i | 0.0116 - 0.0020i |
| rpp | 0.0579 - 0.0573i | 0.0417 - 0.0345i | 0.0417 - 0.0345i |
| tss | 0.3079 + 0.5550i | 0.3958 + 0.5764i | 0.3958 + 0.5764i |
| tps | 0.0087 - 0.0041i | -0.0134 + 0.0061i | 0.0134 - 0.0061i |
| tsp | 0.0087 - 0.0041i | -0.0134 + 0.0061i | 0.0134 - 0.0061i |
| tpp | 0.7494 + 0.6389i | 0.7310 + 0.6634i | 0.7310 + 0.6634i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi)=t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi)=-t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

%

# Composite gyroelectric magnetoelectric (non-reciprocal SKL-2) medium

```
nf=nf+1
ep=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i];  % gyrotropy axis
 along z
mu=(1+1e-6*1i)*eye(3);
xi=0.2*eye(3);  % Isotropic magnetoelectric
zeta=transpose(xi);  % rotational symmetry preserved
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

*nf =*

*17*

$$(\theta,\phi)/\pi = (0.33333, 0.16667) \quad \text{thickness, } d\omega/c = 0.4$$

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6319 + 0.3585i | -0.6319 + 0.3585i | -0.6300 + 0.3580i |
| rps | -0.0502 + 0.0063i | -0.0502 + 0.0063i | -0.0388 + 0.0510i |
| rsp | -0.0502 + 0.0063i | -0.0502 + 0.0063i | -0.0388 + 0.0510i |
| rpp | 0.0536 - 0.0477i | 0.0536 - 0.0477i | 0.0478 - 0.0479i |
| tss | 0.3465 + 0.5645i | 0.3465 + 0.5645i | 0.3465 + 0.5645i |
| tps | -0.0199 + 0.0180i | -0.0199 + 0.0180i | -0.0329 - 0.0274i |
| tsp | -0.0329 - 0.0274i | -0.0329 - 0.0274i | -0.0199 + 0.0180i |
| tpp | 0.7398 + 0.6483i | 0.7398 + 0.6483i | 0.7398 + 0.6483i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{ss,pp}(\theta,\phi) = r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi) = r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi) = t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi) = t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-2.

```
%
```

# Composite gyroelectric magnetoelectric (non-reciprocal SKL-3) medium

```
nf=nf+1
ep=[4+0.1i 0 0; 0 4+0.1i 0.2i; 0 -0.2i 4+0.1i];  % gyrotropy axis alon
 x
mu=(1+1e-6*1i)*eye(3);
xi=[0 0 0.2; 0 0 0; 0.1 0 0;];  % Ex-Hz, Ez-Hx coupling
zeta=transpose(xi);
% rotational symmetry not preserved for this nonreciprocal medium
MM=[ep xi; zeta mu];
make_table(theta,phi,MM,d,nf);
```

```
nf =
```

*18*

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)    thickness, d$\omega$/c=0.4**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.6043 + 0.3669i | -0.6545 + 0.3534i | -0.6556 + 0.3519i |
| rps | -0.0204 - 0.0059i | 0.0169 + 0.0090i | -0.0106 + 0.0085i |
| rsp | 0.0204 + 0.0059i | -0.0169 - 0.0090i | 0.0106 - 0.0085i |
| rpp | 0.0485 - 0.0324i | 0.0459 - 0.0557i | 0.0598 - 0.0452i |
| tss | 0.3734 + 0.5721i | 0.3230 + 0.5595i | 0.3230 + 0.5595i |
| tps | 0.0174 - 0.0178i | -0.0141 + 0.0167i | 0.0203 + 0.0009i |
| tsp | 0.0244 - 0.0010i | -0.0203 - 0.0009i | 0.0141 - 0.0167i |
| tpp | 0.7362 + 0.6567i | 0.7468 + 0.6426i | 0.7468 + 0.6426i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi)=t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi)=-t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

%

# Multilayered gyroelectric magnetoelectric (non-reciprocal SKL-2) medium

```
nf=nf+1
% Material 1
ep1=[4+0.1i 0.2i 0; -0.2i 4+0.1i 0; 0 0 4+0.1i];  % gyrotropy axis
 along z
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
```

```
xi2=0.2*eye(3);  % isotropic magnetoelectric
zeta2=transpose(xi2); % nonreciprocity
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
make_2layered_table(theta,phi,MM1,MM2,td,nf);
```

```
nf =

    19
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.8620 - 0.1891i | -0.8620 - 0.1891i | -0.8779 - 0.0579i |
| rps | -0.0336 - 0.0327i | -0.0336 - 0.0327i | -0.0530 + 0.0195i |
| rsp | -0.0336 - 0.0327i | -0.0336 - 0.0327i | -0.0530 + 0.0195i |
| rpp | 0.1679 + 0.1606i | 0.1679 + 0.1606i | 0.2249 - 0.0292i |
| tss | -0.0606 + 0.4462i | -0.0606 + 0.4462i | -0.0606 + 0.4462i |
| tps | -0.0087 - 0.0172i | -0.0087 - 0.0172i | 0.0235 - 0.0201i |
| tsp | 0.0235 - 0.0201i | 0.0235 - 0.0201i | -0.0087 - 0.0172i |
| tpp | -0.2731 + 0.9102i | -0.2731 + 0.9102i | -0.2731 + 0.9102i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{ss,pp}(\theta,\phi)=r_{ss,pp}(\theta,\phi+\pi)$, $r_{sp}(\theta,\phi)=r_{ps}(\theta,\phi+\pi)$, $t_{ss,pp}(\theta,\phi)=t_{ss,pp}(\pi-\theta,\phi+\pi)$, $t_{sp}(\theta,\phi)=t_{ps}(\pi-\theta,\phi+\pi)$. These relations lead to SKL-2.

```
    %
```

# Multilayered gyroelectric magnetoelectric (non-reciprocal SKL-3) medium

```
nf=nf+1
% Material 1
```

```
ep1=[4+0.1i 0 0.2i; 0 4+0.1i 0; -0.2i 0 4+0.1i];  % gyrotropy axis
 along y
mu1=(1+1e-6*1i)*eye(3);
xi1=zeros(3); zeta1=zeros(3);
MM1=[ep1 xi1; zeta1 mu1];
% Material 2
ep2=(6+0.1i)*eye(3);
mu2=(1+1e-6*1i)*eye(3);
xi2=[0 0 0; 0 0 0.2; 0 0.1 0;];  % Ey-Hz, Ez-Hy coupling
zeta2=transpose(xi2);
MM2=[ep2 xi2; zeta2 mu2];
% Geometry
td=[0.4 0.5];  % thicknesses of layers.
make_2layered_table(theta,phi,MM1,MM2,td,nf);
```

```
nf =

    20
```

**$(\theta,\phi)/\pi$= (0.33333 ,0.16667)**

| | Incidence | Reflection | Transmission |
|---|---|---|---|
| rss | -0.8616 - 0.2083i | -0.8625 - 0.1672i | -0.8753 - 0.0542i |
| rps | 0.0027 - 0.0052i | -0.0016 + 0.0078i | 0.0017 - 0.0034i |
| rsp | -0.0027 + 0.0052i | 0.0016 - 0.0078i | -0.0017 + 0.0034i |
| rpp | 0.1852 + 0.1712i | 0.1474 + 0.1417i | 0.1992 - 0.0172i |
| tss | -0.0667 + 0.4409i | -0.0547 + 0.4563i | -0.0547 + 0.4563i |
| tps | 0.0067 + 0.0027i | -0.0084 - 0.0046i | 0.0076 - 0.0015i |
| tsp | 0.0075 - 0.0026i | -0.0076 + 0.0015i | 0.0084 + 0.0046i |
| tpp | -0.2584 + 0.9110i | -0.2914 + 0.9128i | -0.2914 + 0.9128i |

For deriving SKL-1 and SKL-2, A) compare relations between rss, rps, rsp, rpp in incidence and reflection directions, and B) compare relations between tss, tps, tsp, tpp in incidence and transmission directions

For deriving SKL-3, compare relations between coefficients in reflection and transmission directions

In the table, incidence direction is $(\theta,\phi)$, reflection direction is $(\theta,\phi+\pi)$, and transmission direction is $(\pi-\theta,\phi+\pi)$. The reflection and transmission coefficients for light incident along these directions satisfy the relations: $r_{sp}(\theta,\phi)=-r_{ps}(\theta,\phi)$, $t_{ss,pp}(\pi-\theta,\phi+\pi)=t_{ss,pp}(\theta,\phi+\pi)$, $t_{sp}(\pi-\theta,\phi+\pi)=-t_{ps}(\theta,\phi+\pi)$. These relations lead to SKL-3.

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