

Assignment 1

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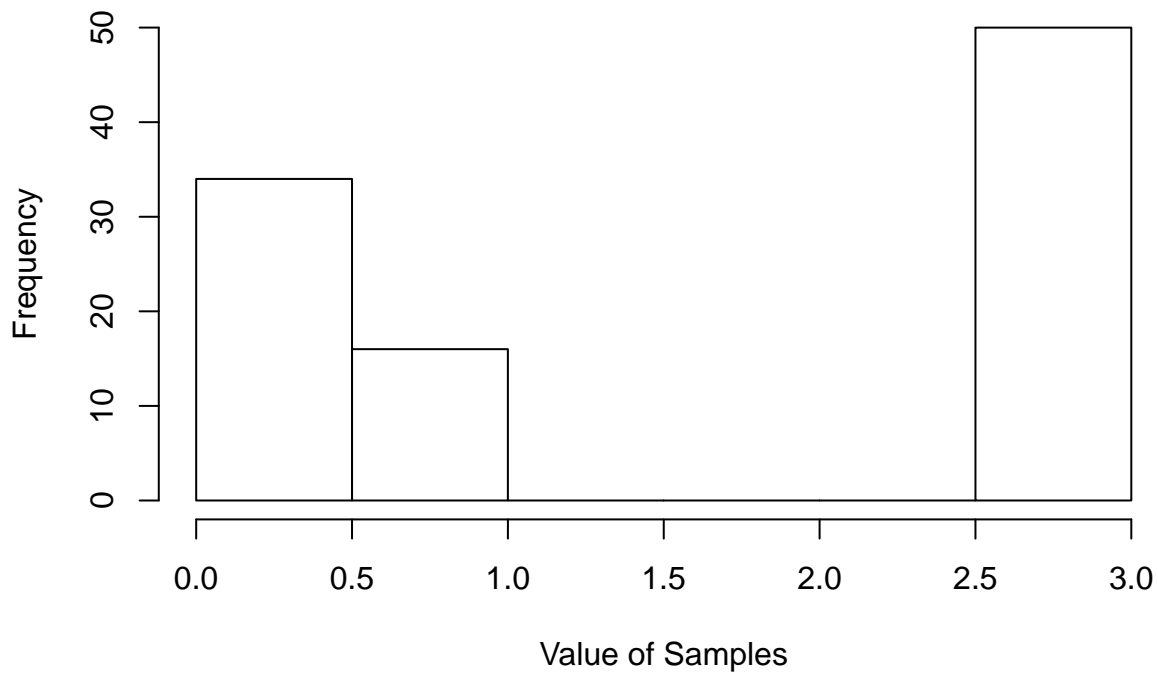
4 September 2018

Question 1:

Consider the probability density function $p(x) = (c/x^4)$ for $x \geq 1$, where c is a constant. Generate 100 random samples from this distribution and plot a histogram. How close is the average of the samples to the expected value of X ?

Answer 1:

Histogram of Samples



Average value of samples is

```
## [1] 1.681075
```

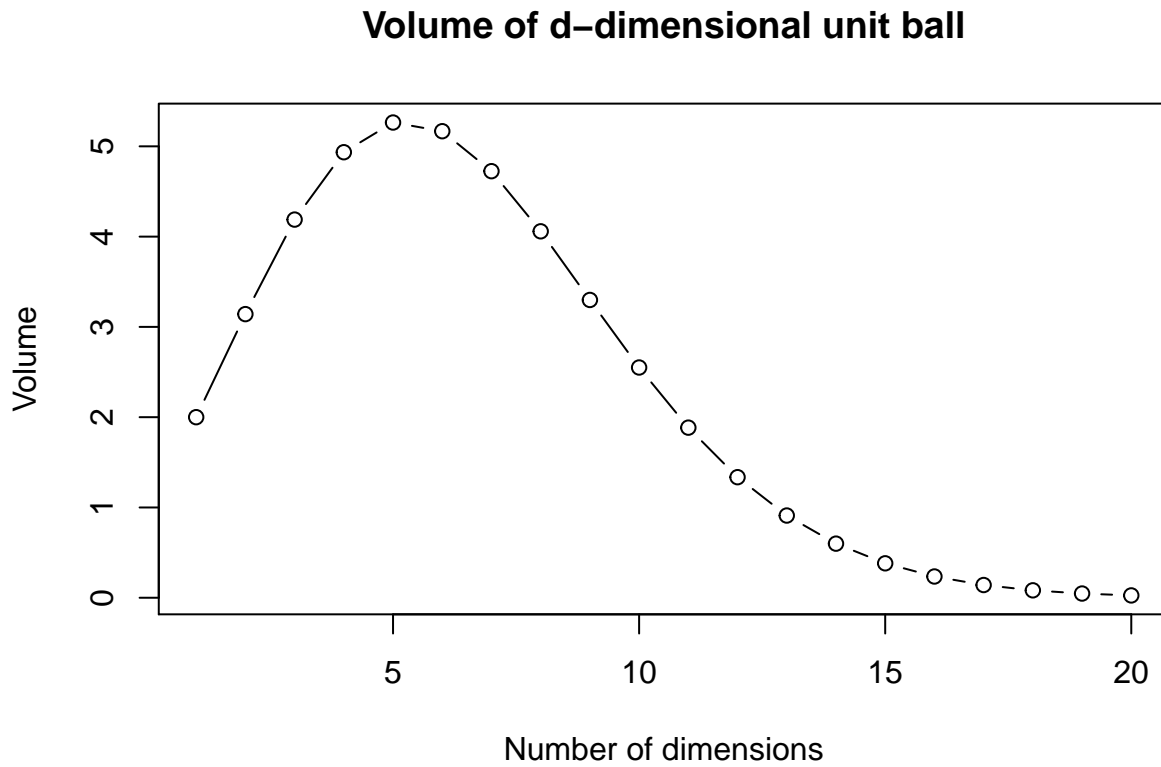
Expected value of X is

```
## 1.5 with absolute error < 1.7e-14
```

Question 2:

Draw a 2-D plot in which the Y-axis represents $V(d)$, the volume of a d -dimensional unit ball, and the X-axis represents $d = 1, 2, 3, \dots$. State your observations

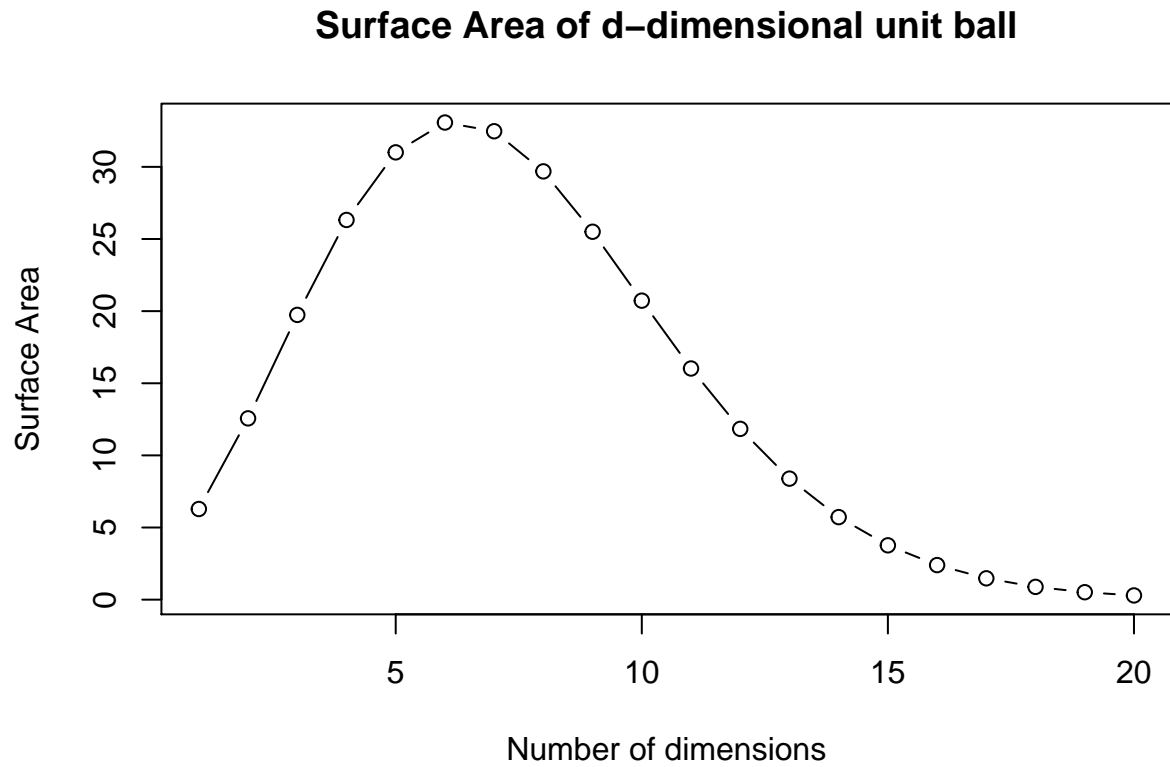
Answer 2:



Question 3:

Draw a 2-D plot in which the Y-axis represents $S(d)$, the surface area of a d -dimensional unit ball, and the X-axis represents $d = 1, 2, 3, \dots$. State your observations.

Answer 3:

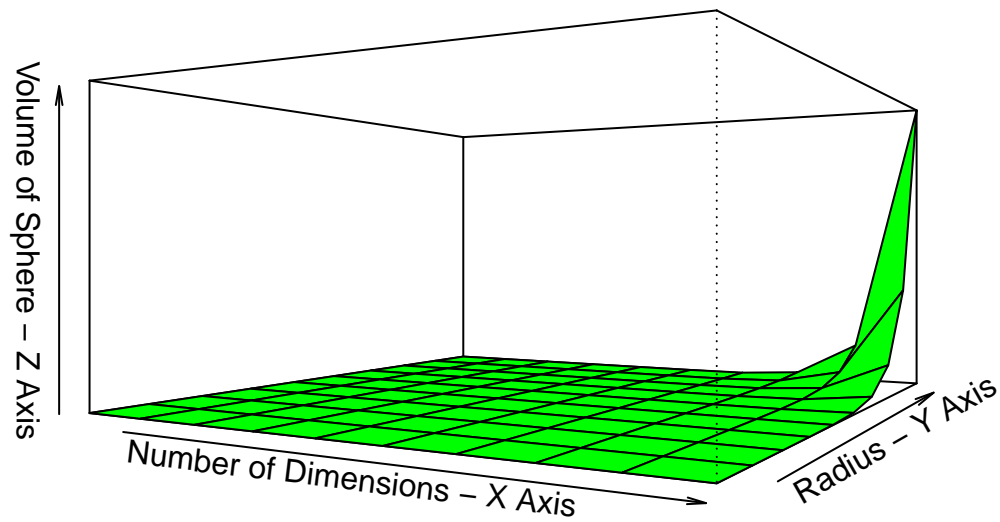


Question 4:

Draw a 3-D plot in which the Z-axis represents $V(d)$, the volume of a d-dimensional ball of radius R , and the X-axis represents d , and the Y-axis represents the radius R . State your observations.

Answer 4:

Plot of Volume

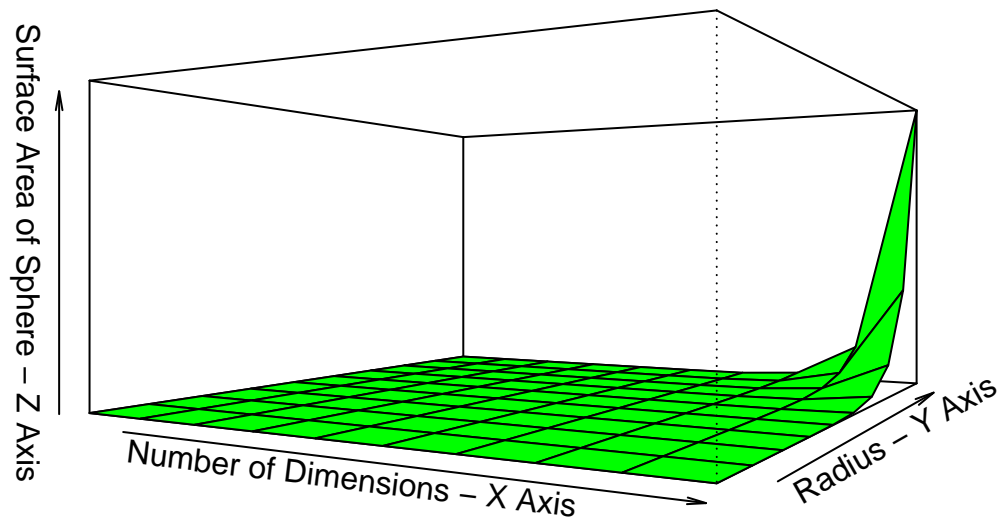


Question 5:

Draw a 3-D plot in which the Z-axis represents $S(d)$, the surface area of a d -dimensional ball of radius R , and the X -axis represents d , and the Y-axis represents the radius R . State your observations.

Answer 5:

Plot of Surface Area



Question 6:

Generate 20 points uniformly at random on a 900 -dimensional sphere of radius 30. Calculate the distance between each pair of points. Then, select a method of projection and project the data onto subspaces of dimension $k = 100, 50, 10, 5, 4, 3, 2, 1$ and calculate the difference between $k^{(1/2)}$ times the original distances and the new pair-wise distances. For each value of k what is the maximum difference as a percent of $k^{(1/2)}$.

Answer 6:

The differences in euclidean distances for the subspace projection is as follows:

```
## [1] "Subspace of dimension: 1"
## [1] 42.86912

## [1] "Subspace of dimension: 2"
## [1] 58.67276

## [1] "Subspace of dimension: 3"
## [1] 68.678

## [1] "Subspace of dimension: 4"
## [1] 76.32516

## [1] "Subspace of dimension: 5"
## [1] 83.78994

## [1] "Subspace of dimension: 10"
## [1] 109.1806
```