

# Assignment 1

*Chinmaya Kumar Bansal*

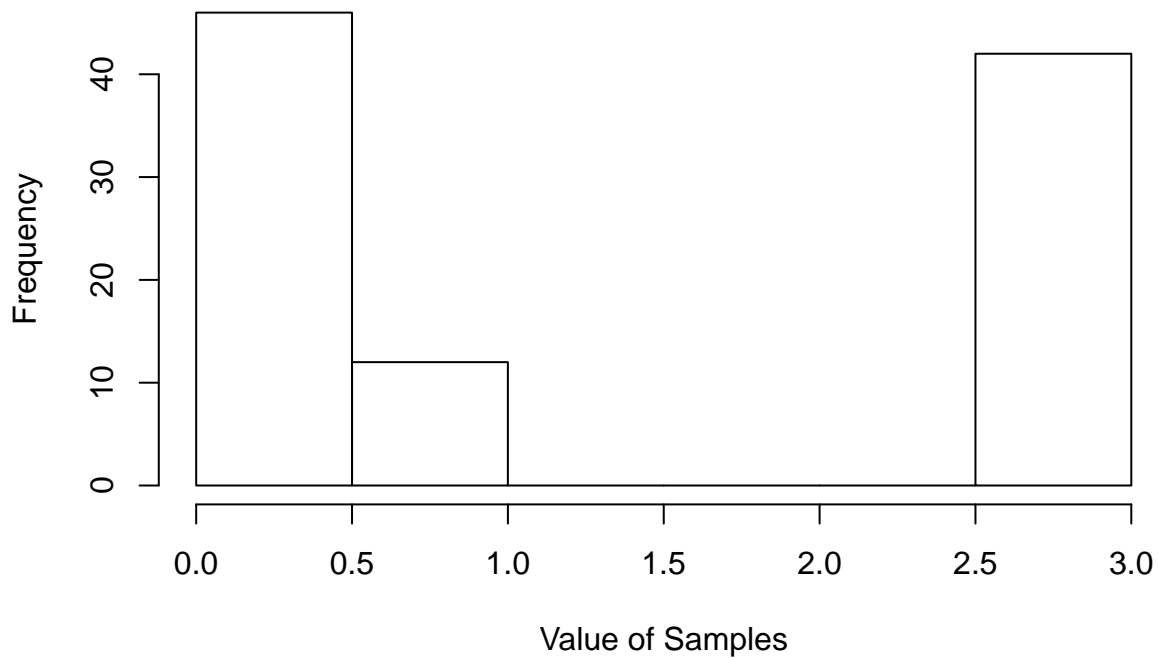
*4 September 2018*

## Question 1:

Consider the probability density function  $p(x) = (c/x^4)$  for  $x \geq 1$ , where  $c$  is a constant. Generate 100 random samples from this distribution and plot a histogram. How close is the average of the samples to the expected value of  $X$ ?

## Answer 1:

### Histogram of Samples



Average value of samples is

```
## [1] 1.413494
```

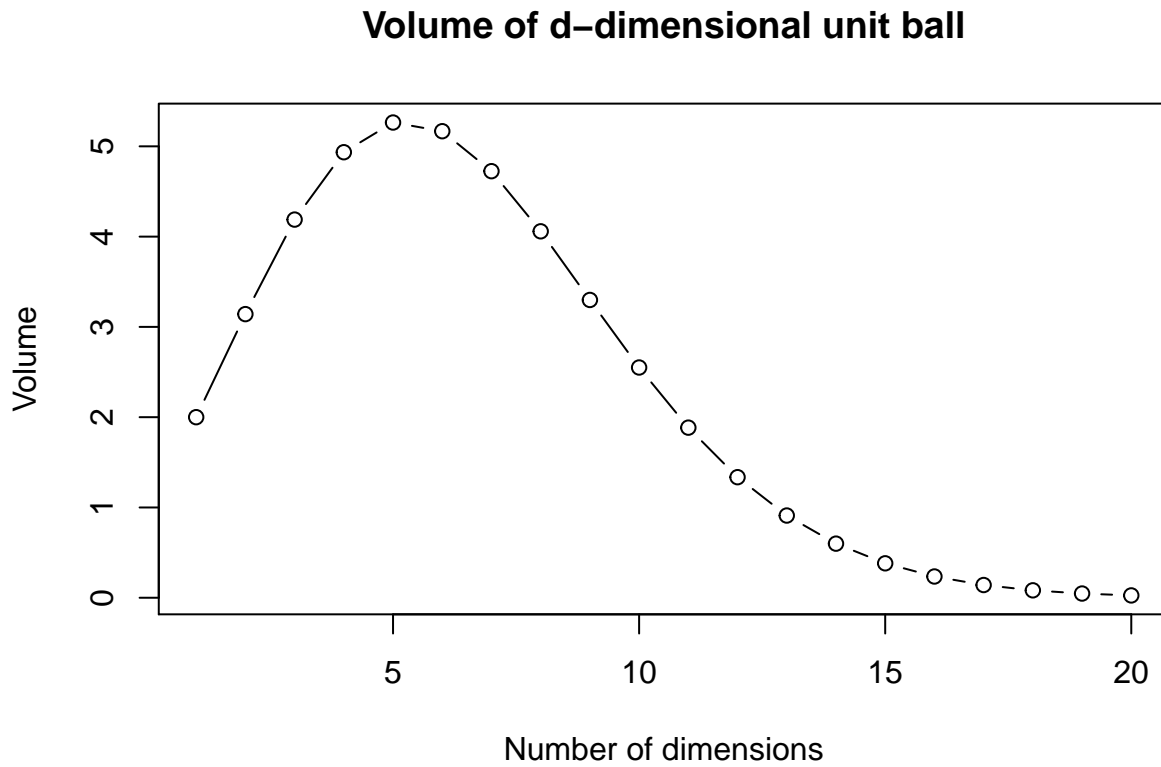
Expected value of  $X$  is

```
## 1.5 with absolute error < 1.7e-14
```

Question 2:

Draw a 2-D plot in which the Y-axis represents  $V(d)$ , the volume of a  $d$ -dimensional unit ball, and the X-axis represents  $d = 1, 2, 3, \dots$ . State your observations

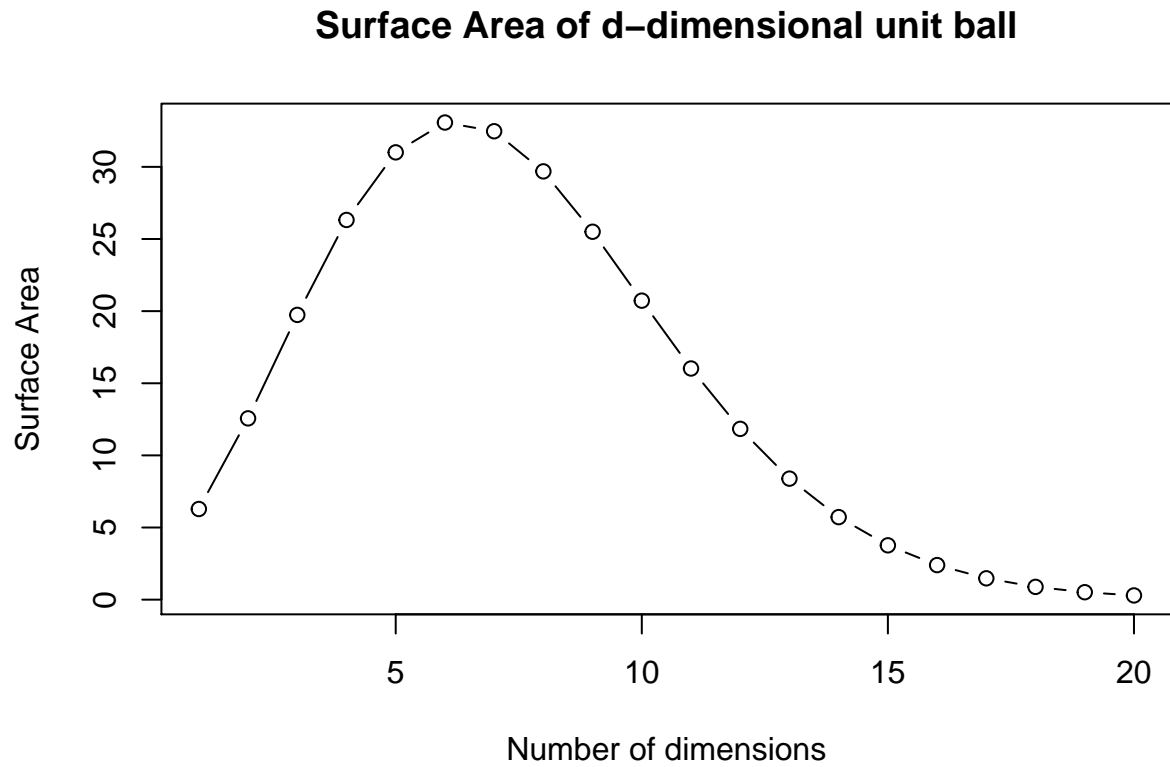
Answer 2:



Question 3:

Draw a 2-D plot in which the Y-axis represents  $S(d)$ , the surface area of a  $d$ -dimensional unit ball, and the X-axis represents  $d = 1, 2, 3, \dots$ . State your observations.

Answer 3:

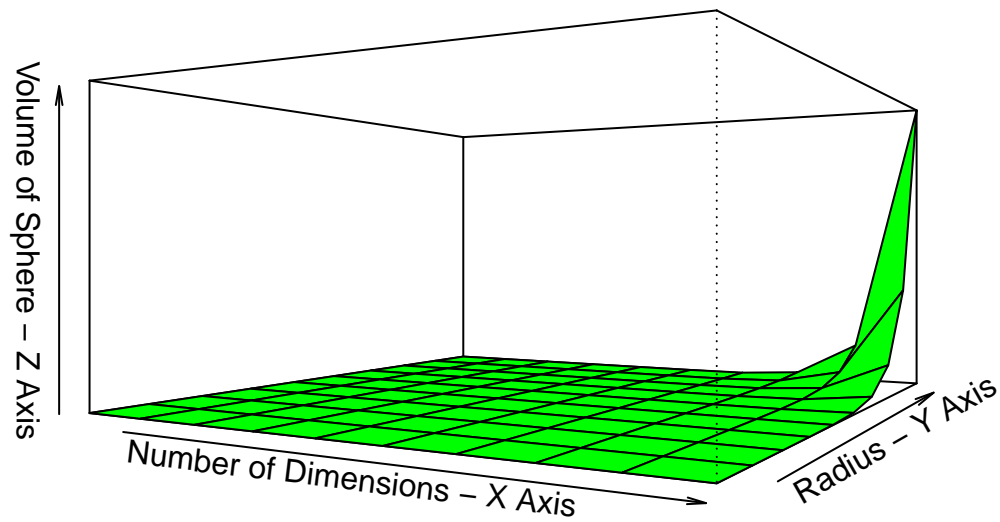


Question 4:

Draw a 3-D plot in which the Z-axis represents  $V(d)$ , the volume of a d-dimensional ball of radius  $R$ , and the X-axis represents  $d$ , and the Y-axis represents the radius  $R$ . State your observations.

Answer 4:

### Plot of Volume

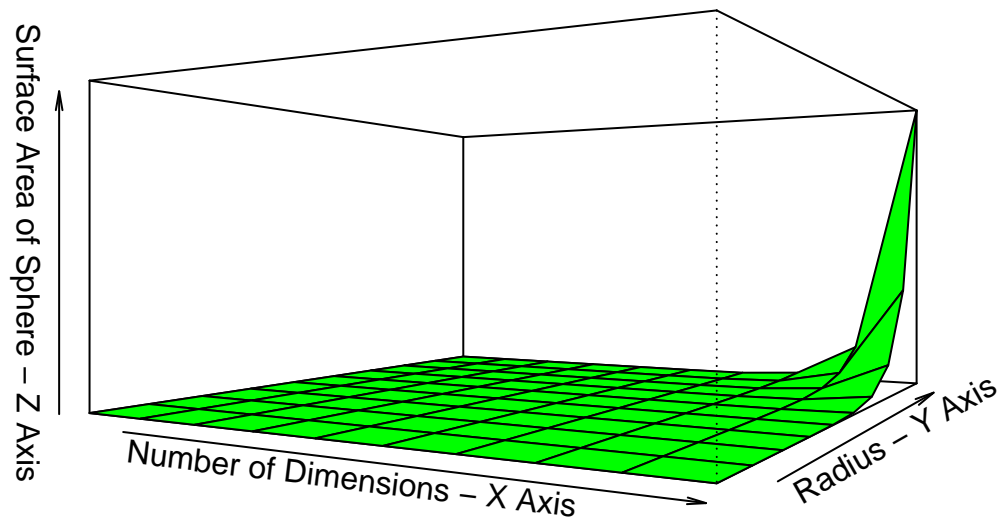


Question 5:

Draw a 3-D plot in which the Z-axis represents  $S(d)$ , the surface area of a  $d$ -dimensional ball of radius  $R$ , and the X -axis represents  $d$ , and the Y-axis represents the radius  $R$ . State your observations.

Answer 5:

### Plot of Surface Area



### Question 6:

Generate 20 points uniformly at random on a 900 -dimensional sphere of radius 30. Calculate the distance between each pair of points. Then, select a method of projection and project the data onto subspaces of dimension  $k = 100, 50, 10, 5, 4, 3, 2, 1$  and calculate the difference between  $k^{(1/2)}$  times the original distances and the new pair-wise distances. For each value of  $k$  what is the maximum difference as a percent of  $k^{(1/2)}$ .

### Answer 6:

The differences in euclidean distances for the subspace projection is as follows:

```
## [1] "Subspace of dimension: 1"
## [1] 42.72793

## [1] "Subspace of dimension: 2"
## [1] 59.07517

## [1] "Subspace of dimension: 3"
## [1] 69.75864

## [1] "Subspace of dimension: 4"
## [1] 80.94912

## [1] "Subspace of dimension: 5"
## [1] 87.05084

## [1] "Subspace of dimension: 10"
## [1] 112.0279
```