Assignment 2.2

Solved by Tamanna and Chinmay

Question 1: Short Forward Contract (10 Marks)

Given:

- Short forward contract to sell **1,000 ounces** of gold.
- Forward price = **$2,050 per ounce**.
- Profit/Loss formula:

$$\text{Profit or Loss} = (\text{Forward Price} - \text{Spot Price}) \times 1,000$$

Step-by-Step Calculation:
For each spot price, compute $(\$2,050 - \text{Spot Price}) \times 1,000$:

1. **Spot Price = $1,400**:
$$(\$2,050 - \$1,400) \times 1,000 = \$650 \times 1,000 = \mathbf{\$650{,}000}$$
2. **Spot Price = $1,500**:
$$(\$2,050 - \$1,500) \times 1,000 = \$550 \times 1,000 = \mathbf{\$550{,}000}$$
3. **Spot Price = $1,560**:
$$(\$2,050 - \$1,560) \times 1,000 = \$490 \times 1,000 = \mathbf{\$490{,}000}$$
4. **Spot Price = $1,600**:
$$(\$2,050 - \$1,600) \times 1,000 = \$450 \times 1,000 = \mathbf{\$450{,}000}$$
5. **Spot Price = $1,800**:
$$(\$2,050 - \$1,800) \times 1,000 = \$250 \times 1,000 = \mathbf{\$250{,}000}$$
6. **Spot Price = $2,050**:
$$(\$2,050 - \$2,050) \times 1,000 = \$0 \times 1,000 = \mathbf{\$0}$$
7. **Spot Price = $2,200**:
$$(\$2,050 - \$2,200) \times 1,000 = (-\$150) \times 1,000 = \mathbf{-\$150{,}000}$$
8. **Spot Price = $2,300**:
$$(\$2,050 - \$2,300) \times 1,000 = (-\$250) \times 1,000 = \mathbf{-\$250{,}000}$$
9. **Spot Price = $2,400**:
$$(\$2,050 - \$2,400) \times 1,000 = (-\$350) \times 1,000 = \mathbf{-\$350{,}000}$$

Completed Table:

May-2024 Spot Price ($)	Profit or Loss ($)
1,400	650,000
1,500	550,000
1,560	490,000
1,600	450,000
1,800	250,000
2,050	0
2,200	-150,000
2,300	-250,000
2,400	-350,000

Question 2: Futures Contract Profit/Loss (40 Marks)

General Formulas

- **Long Position (Buyer)**:

$$\text{Profit/Loss} = (\text{Closing Price} - \text{Initial Price}) \times \text{Contract Size} \times \text{Number of Contracts}$$

- **Short Position (Seller)**:

$$\text{Profit/Loss} = (\text{Initial Price} - \text{Closing Price}) \times \text{Contract Size} \times \text{Number of Contracts}$$

(a) Corn Futures (10 Marks)

Given:

- Position: Long
- Contract Size: 5,000 bushels
- Initial Price: \$5.20 per bushel
- Closing Price: \$5.80 per bushel
- Number of Contracts: 1

Calculation:

$$\begin{aligned}
\text{Profit} &= (\text{Closing Price} - \text{Initial Price}) \times \text{Contract Size} \times \text{Contracts} \\
&= (5.80 - 5.20) \times 5,000 \times 1 \\
&= 0.60 \times 5,000 \\
&= \boxed{\$3,000}
\end{aligned}$$

(b) Coffee Futures (10 Marks)

Given:

- Position: Short
- Contract Size: 37,500 pounds
- Initial Price: \$1.60 per pound
- Closing Price: \$1.40 per pound
- Number of Contracts: 1

Calculation:

$$\begin{aligned}
\text{Profit} &= (\text{Initial Price} - \text{Closing Price}) \times \text{Contract Size} \times \text{Contracts} \\
&= (1.60 - 1.40) \times 37,500 \times 1 \\
&= 0.20 \times 37,500 \\
&= \boxed{\$7,500}
\end{aligned}$$

(c) SPI200 Futures (10 Marks)

Given:

- Position: Short
- Notional Value: A\$25 per index point
- Initial Price: 7,500 index points

- Closing Price: 7,800 index points
- Number of Contracts: 40

Calculation:

$$\begin{aligned}
\text{Profit per contract} &= (\text{Initial Price} - \text{Closing Price}) \times \text{Notional Value} \\
&= (7,500 - 7,800) \times 25 \\
&= (-300) \times 25 \\
&= -\text{A\$}7,500
\end{aligned}$$

$$\begin{aligned}
\text{Total Profit} &= \text{Per Contract Profit} \times \text{Contracts} \\
&= (-7,500) \times 40 \\
&= \boxed{-\text{A\$}300,000} \quad \text{(Loss)}
\end{aligned}$$

(d) Stainless Steel Futures (10 Marks)

Given:

- Position: Long
- Contract Size: 5 tonnes per contract
- Initial Price: RMB 15,000 per tonne
- Closing Price: RMB 13,500 per tonne
- Number of Contracts: 3

Calculation:

$$\begin{aligned}
\text{Profit per contract} &= (\text{Closing Price} - \text{Initial Price}) \times \text{Contract Size} \\
&= (13,500 - 15,000) \times 5 \\
&= (-1,500) \times 5 \\
&= -\text{RMB } 7,500
\end{aligned}$$

$$\begin{aligned}
\text{Total Profit} &= \text{Per Contract Profit} \times \text{Contracts} \\
&= (-7,500) \times 3 \\
&= \boxed{-\text{RMB } 22,500} \quad \text{(Loss)}
\end{aligned}$$

Summary Table

Contract	**Position**	**Result**	**Profit/Loss**
(a) Corn	Long	Profit	+$3,000
(b) Coffee	Short	Profit	+$7,500
(c) SPI200	Short	Loss	-A$300,000
(d) Stainless Steel	Long	Loss	-RMB 22,500

Question 3: Futures vs Spot Contracts (10 marks)

Step 1: Distinguishing Futures Contracts from Spot Contracts

- **Spot Contracts:**
  - *Timing:* Immediate transaction (settlement typically T+0 to T+2 days)
  - *Price:* Determined by current market conditions at trade execution
  - *Transaction:* Direct exchange of asset for payment

  - *Counterparty Risk:* High - direct exposure between buyer and seller
  - *Customization:* Terms negotiated bilaterally (quantity, quality, delivery specifics)
  - *Primary Purpose:* Physical acquisition of assets

- **Futures Contracts:**
  - *Timing:* Agreement for future exchange (weeks/months/years ahead)
  - *Price:* Agreed upfront based on market expectations
  - *Transaction:* Standardized contract traded on exchange
  - *Counterparty Risk:* Eliminated - exchange acts as central counterparty
  - *Standardization:* Fixed contract specifications (size, quality, delivery date/location)
  - *Primary Purpose:* Risk hedging and price speculation

Step 2: How Futures Contracts Work on Commodity Exchanges

1. **Contract Initiation:**
   - Buyer (long position) and seller (short position) agree on future delivery price
   - Standardized terms set by exchange (e.g., 5,000 bushels of corn for December delivery)
2. **Margin Requirements:**
   - *Initial Margin:* Security deposit required to open position (e.g., 5-15% of contract value)
   - *Maintenance Margin:* Minimum account balance to hold position (typically 75-80% of initial margin)
   - *Margin Call:* Requirement to deposit additional funds if account falls below maintenance level
3. **Daily Settlement (Mark-to-Market):**
   - Daily profit/loss calculation based on settlement price
   - Funds transferred between counterparties' accounts each trading day
   - Example: If futures price rises \$1, long gains \$1 $\times$ contract size, short loses equivalent
4. **Position Management:**
   - *Offsetting:* Closing position before expiration with opposite trade (95-98% of contracts)
   - *Delivery:* Physical/virtual settlement at expiration (¡2% of contracts)
5. **Price Discovery:**
   - Continuous bidding/offering reflects market expectations
   - Transparent pricing through exchange order book

Step 3: Role of Commodity Exchanges

- **Trading Facilitation:**
  - Provides electronic/platform-based marketplace
  - Ensures liquidity through market makers and high-frequency traders
  - Enforces standardized contract specifications
- **Counterparty Risk Mitigation:**
  - Acts as central counterparty (CCP) for all trades
  - Guarantees contract performance via clearinghouse
  - Example: CME Group guarantees \$1 quadrillion in annual trades
- **Settlement Mechanisms:**
  - Daily mark-to-market settlements
  - Final settlement at expiration:
    - Physical delivery (commodities - e.g., gold, oil)
    - Cash settlement (financial futures - e.g., indices, interest rates)

  - Manages delivery logistics (warehousing, quality inspections)

- **Risk Management:**
  - Margin system with tiered requirements
  - Default funds (e.g., CME's $10B guaranty fund)
  - Position limits to prevent market manipulation

- **Regulatory Compliance:**
  - Enforces market rules and surveillance
  - Reports trades to regulators (CFTC, ASIC, FCA)
  - Ensures price transparency through public data feeds

Question 4: European Put Option (30 marks)

Given Parameters

- Put option type: **European** (exercisable only at expiration)
- Option premium (price paid): \$3
- Current stock price (S_0): \$42
- Strike price (K): \$40

Part 1: Profit Conditions

Step 1: Define Payoff Function

For a put option, the payoff at expiration is:

$$\text{Payoff} = \max(K - S_T, 0) = \max(40 - S_T, 0)$$

where S_T is the stock price at maturity.

Step 2: Calculate Profit Function

The profit is the payoff minus the premium paid:

$$\text{Profit} = \text{Payoff} - \text{Premium} = \max(40 - S_T, 0) - 3$$

Step 3: Determine Break-even Point

Set profit equal to zero:

$$40 - S_T - 3 = 0 \implies S_T = 37$$

Step 4: Establish Profit Range

$$\begin{aligned} \text{Profit} > 0 \text{ when } & 40 - S_T - 3 > 0 \\ \text{when } & S_T < 37 \end{aligned}$$

$$\boxed{\text{The investor makes a profit when the stock price at maturity } (S_T) \text{ is below \$37.}}$$

Part 2: Exercise Conditions

Step 1: Identify In-the-Money Condition

A put option is exercised when:

$$K > S_T \implies 40 > S_T$$

$$\boxed{\text{The option will be exercised when the stock price at maturity } (S_T) \text{ is below \$40.}}$$

Part 3: Profit Diagram

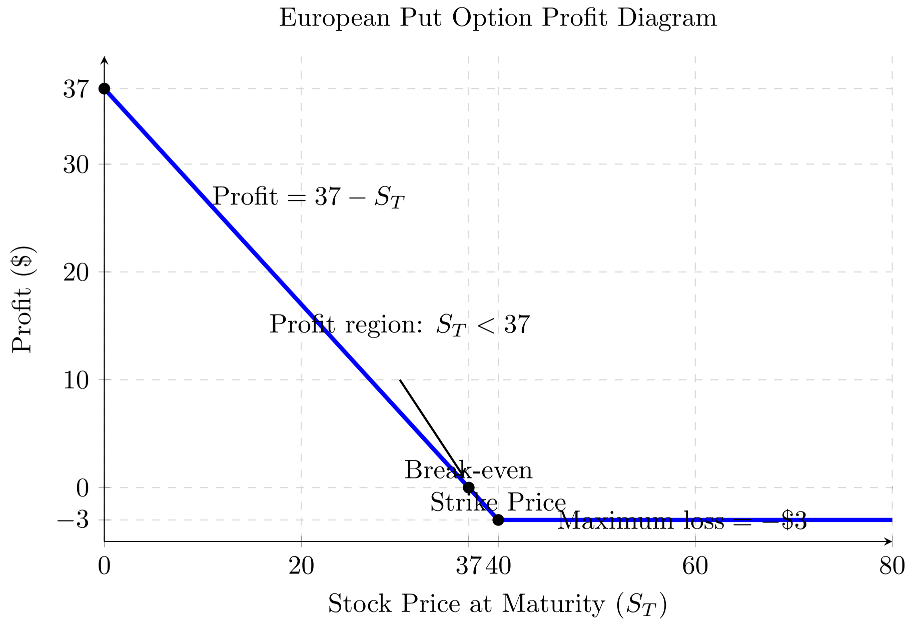
Step 1: Define Piecewise Profit Function

$$\text{Profit} = \begin{cases} 40 - S_T - 3 = 37 - S_T & \text{if } S_T < 40 \\ 0 - 3 = -3 & \text{if } S_T \geq 40 \end{cases}$$

Step 2: Identify Key Points

- At $S_T = 0$: Profit = \$37
- At $S_T = 37$: Profit = \$0 (break-even)
- At $S_T = 40$: Profit = -\$3
- For $S_T > 40$: Profit remains -\$3

Step 3: Plot the Diagram



Key Observations

- **Maximum Profit**: \$37 (if $S_T = 0$)
- **Maximum Loss**: \$3 (premium paid)
- **Break-even Point**: \$37
- **Exercise Threshold**: \$40
- The diagram shows the characteristic **limited loss, unlimited gain** profile of long put positions

Question 5: Portfolio of Forward + Put Option (20 marks)

Problem Statement

Consider a portfolio consisting of:

(i) A newly entered-into **long forward contract** on an asset with delivery price K and maturity T

(ii) A **long European put option** on the same asset with:

- Strike price K (equal to the forward price)
- Maturity T

Part 1: Terminal Portfolio Value

Step 1: Analyze Forward Contract Payoff

The payoff of the long forward at maturity T is:

$$V_{\text{forward}}(T) = S_T - K$$

where:

- S_T = spot price at maturity
- K = delivery price (agreed forward price)

Step 2: Analyze Put Option Payoff

The payoff of the European put option at maturity is:

$$V_{\text{put}}(T) = \max(K - S_T, 0)$$

Step 3: Combine Payoffs

The terminal portfolio value $\Pi(T)$ is the sum of both positions:

$$\Pi(T) = V_{\text{forward}}(T) + V_{\text{put}}(T) = (S_T - K) + \max(K - S_T, 0)$$

Step 4: Case Analysis

- **Case 1: $S_T \geq K$ (Spot price above strike)**

$$\max(K - S_T, 0) = 0$$
$$\Pi(T) = (S_T - K) + 0 = S_T - K$$

- **Case 2: $S_T < K$ (Spot price below strike)**

$$\max(K - S_T, 0) = K - S_T$$
$$\Pi(T) = (S_T - K) + (K - S_T) = 0$$

Step 5: Unified Expression

The portfolio payoff can be expressed as:

$$\Pi(T) = \max(S_T - K, 0)$$

which is identical to the payoff of a European call option with strike K.

Part 2: Put-Call Equivalence Proof

Step 1: Initial Portfolio Value

At contract initiation ($t = 0$):

- The forward contract has zero value (fair price):

$$V_{\text{forward}}(0) = 0$$

- The put option has value:

$$V_{\text{put}}(0) = P(0)$$

- Thus, the portfolio's initial value is:

$$\Pi(0) = P(0)$$

Step 2: Call Option Valuation

Let $C(0)$ be the value of a European call option with:

- Strike price K
- Maturity T

Step 3: No-Arbitrage Argument

Since the portfolio replicates the call option payoff at maturity:

$$\Pi(T) = C(T)$$

By no-arbitrage, their present values must be equal:

$$\Pi(0) = C(0) \implies P(0) = C(0)$$

Economic Interpretation

This result shows that when:

- The put's strike equals the forward price ($K = F_0$)
- Both options are European with same maturity

then their premiums are equal due to put-call symmetry.

Verification via Put-Call Parity

The general put-call parity relation:

$$C(0) - P(0) = S_0 - Ke^{-rT}$$

When $K = F_0 = S_0 e^{rT}$ (fair forward price), we get:

$$C(0) - P(0) = S_0 - S_0 e^{rT} e^{-rT} = 0 \implies C(0) = P(0)$$

Conclusion

The European put option has the same value as a European call option with identical strike price K and maturity T when the strike equals the forward price:

$$\boxed{P(0) = C(0) \quad \text{when} \quad K = F_0}$$

Instrument	**Initial Value**	**Terminal Payoff**
Long Forward	0	$S_T - K$
Long Put	$P(0)$	$\max(K - S_T, 0)$
Portfolio	$P(0)$	$\max(S_T - K, 0)$
European Call	$C(0)$	$\max(S_T - K, 0)$

Question 6: Put-Call Parity - Risk-Free Rate (10 marks)

Given Parameters

- Current stock price (S_0) = \$130
- Strike price (K) = \$120
- Time to expiration (T) = 12 months = 1 year
- European call price (C) = \$20
- European put price (P) = \$5
- Non-dividend-paying stock

Step 1: Put-Call Parity Formula

For a non-dividend-paying stock:

$$C - P = S_0 - Ke^{-rT}$$

where r is the continuously compounded risk-free rate.

Step 2: Substitute Values

$$20 - 5 = 130 - 120e^{-r\cdot 1}$$

$$15 = 130 - 120e^{-r}$$

Step 3: Solve for e^{-r}

$$120e^{-r} = 130 - 15 = 115$$

$$e^{-r} = \frac{115}{120} = \frac{23}{24} \approx 0.958333$$

Step 4: Solve for r

$$-r = \ln\left(\frac{23}{24}\right)$$

$$r = -\ln\left(\frac{23}{24}\right) = \ln\left(\frac{24}{23}\right)$$

Step 5: Calculate Numerical Value

$$\frac{24}{23} \approx 1.043478$$

$$r = \ln(1.043478) \approx 0.04254$$

Step 6: Convert to Percentage

$$r \approx 0.04254 \times 100\% = \boxed{4.254\%}$$

Verification

Verify put-call parity:

$$S_0 - Ke^{-rT} = 130 - 120e^{-0.04254} \approx 130 - 120 \times 0.95833 = 130 - 115 = 15$$

$$C - P = 20 - 5 = 15$$

Both sides equal, confirming solution.

Conclusion

The implied continuously compounded risk-free rate is $\boxed{4.254\%}$ per annum.