Permutations and Combinations

Dr. Umesh Aundhakar

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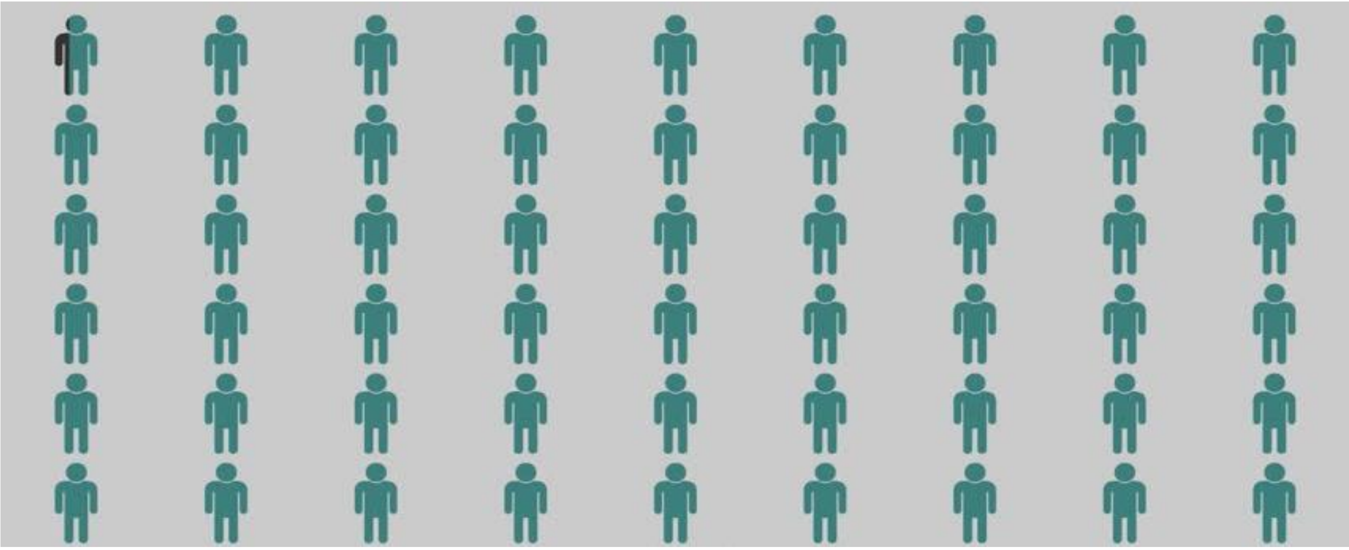
Counting

What is Counting ???

Lets see one trivial example to understand term counting

Counting



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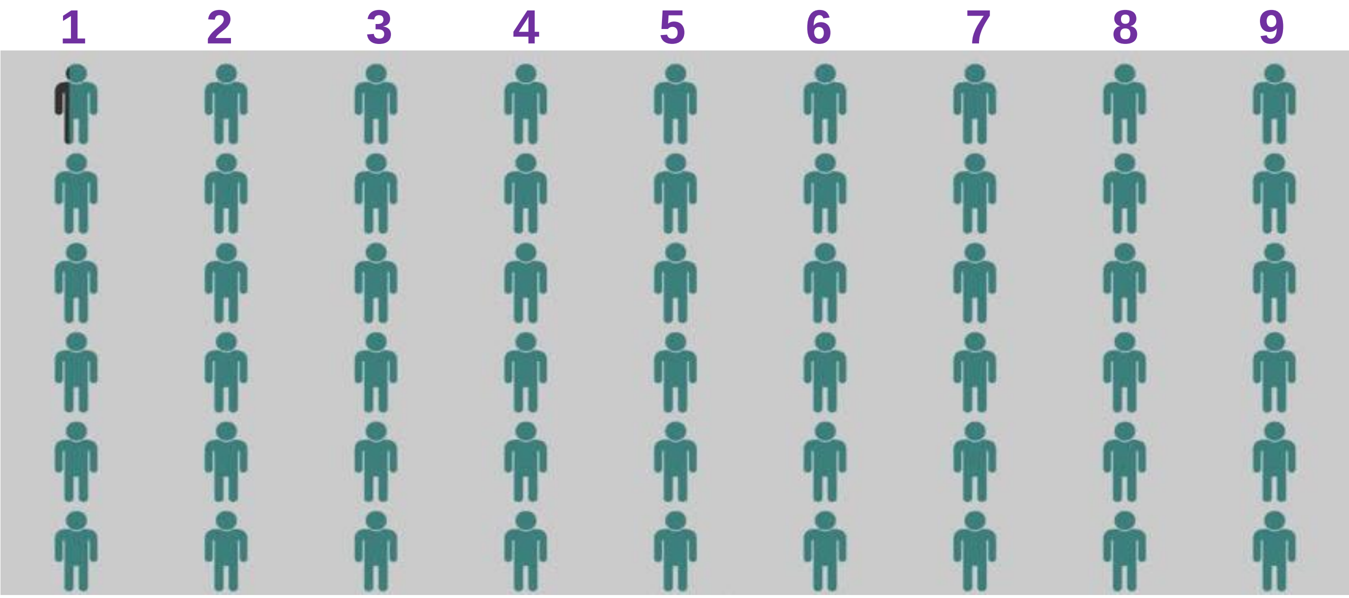

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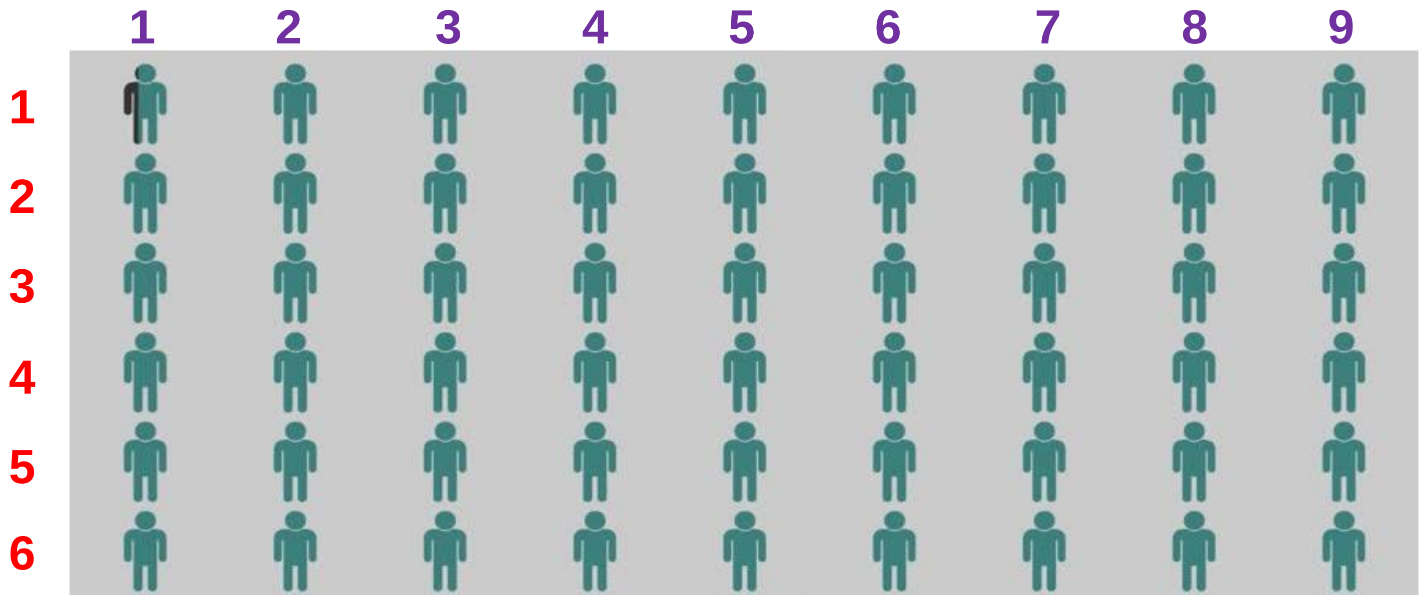

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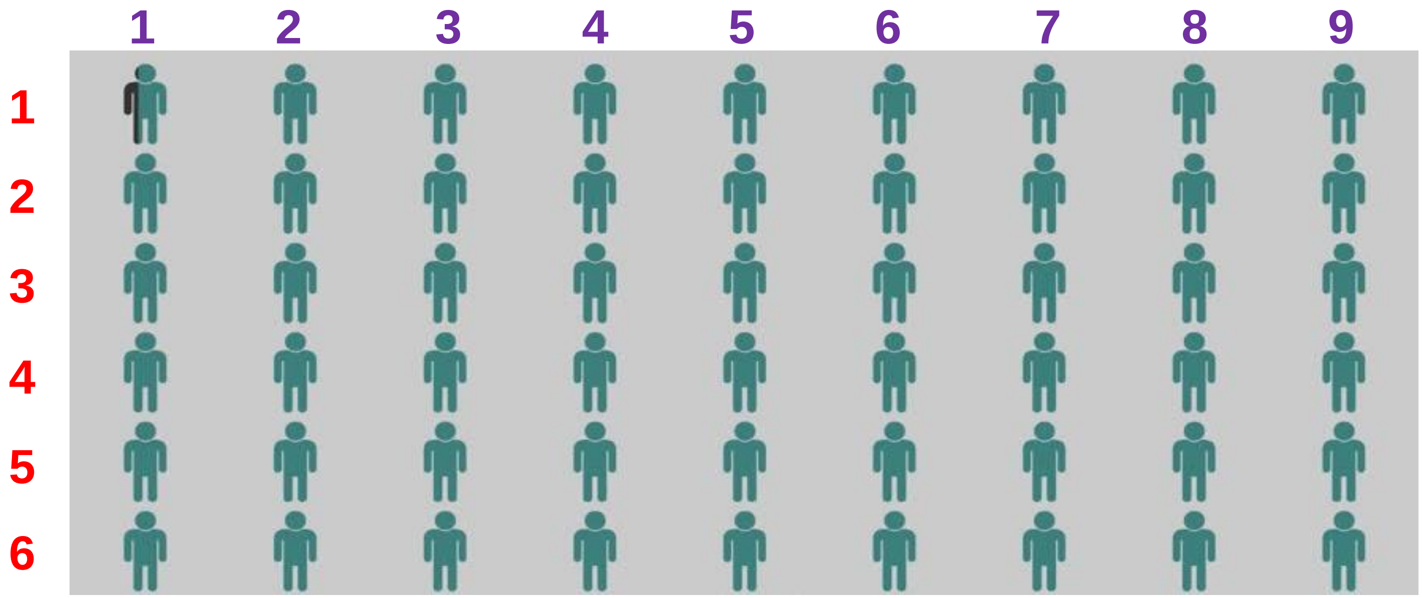

Total number of persons = **9X6**

= **54**

Rules of Counting

- Rule of **AND**
- Rule of **OR**

Arrangement	Selection
Specific Position	Collection
Order is Important	Order is NOT Important

Permutation	Combinations
r out of n -- Arrangements	r out of n -- Selections
${}_nP_r = n!/(n-r)!$	${}_nC_r = n!/\{(n-r)!*r!\}$

Permutation

Permutations

The team selectors needs to choose opener batsman to open in cricket match, one should open the inning and other for non-strike. Rahul, Sachin, Virat and Dhoni are eligible to do so in the team. How many possible ways are there for the selector to select the batsman?

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Sachin		
Virat		
Dhoni		

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	Virat	
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Virat	Rahul	
	Sachin	
	Dhoni	
Dhoni	Rahul	
	Sachin	
	Virat	

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	Virat	RV
	Dhoni	RD
Sachin	Rahul	SR
	Virat	SV
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Virat	Rahul	VR
	Sachin	VS
	Dhoni	VD
Dhoni	Rahul	DR
	Sachin	DS
	Virat	DV

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There are 12 different ways for the 4 batsman to hold the 2 positions.

Permutations

In the previous example, the positions are in specific order, so each arrangement is unique.

The symbol $_4P_2$ denotes the number of permutations when arranging 4 batsman in two positions.

Outcomes

RS
RV
RD

SR
SV
SD

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VD

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The symbol $_4P_2$ denotes the number of permutations when arranging 4 batsman in two positions.

You can also use the Fundamental Counting Principle to determine the number of permutations.

$$\underbrace{\text{ways to choose first employee}} \quad \text{X} \quad \underbrace{\text{ways to choose second employee}}$$

$$_4P_2 = \quad 4 \quad \text{X} \quad 3$$

Outcomes

RS
RV
RD

SR
SV
SD

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VS
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Permutations

$$
{}_4P_2 = \underbrace{4}_{\text{ways to choose first employee}} \times \underbrace{3}_{\text{ways to choose second employee}}
$$

Outcomes

RS
RV
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Permutations

$$ {}_4P_2 = \underbrace{4}_{\text{ways to choose first employee}} \times \underbrace{3}_{\text{ways to choose second employee}} $$

$$ {}_4P_2 = \frac{4!}{2!} \qquad {}_4P_2 = \frac{4*3*2*1}{2*1} \qquad {}_4P_2 = \frac{4*3}{1}\left(\frac{2*1}{2*1}\right) $$

Outcomes

RS
RV
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In general, ${}_nP_r$ is used to denote the number of Permutations

Permutations

Permutation: (Order is important!)

Find ${}_{10}P_6$

Permutations

Permutation: (Order is important!)

Find ${}_{10}P_6$

$${}_{10}P_6 = \frac{10!}{(10-6)!}$$

$${}_{10}P_6 = \frac{10!}{4!} = \frac{10*9*8*7*6*5*4*3*2*1}{4*3*2*1}$$

$${}_{10}P_6 = 10*9*8*7*6*5 \quad \text{or} \quad 151{,}200$$

There are 151,200 permutations

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Permutations

A computer program requires the user to enter a **7-digit** registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9.
Each number has to be used, and repetition is not allowed.

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$$ {}_nP_r = {}_7P_7 $$

$$ {}_7P_7 = \frac{7*6*5*4*3*2*1}{1} $$ = 5040

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$$ {}_{n}P_{r} = {}_{7}P_{7} $$

$$ {}_{7}P_{7} = \frac{7*6*5*4*3*2*1}{1} = 5040 $$

There are 5040 possible codes with the digits 1, 2, 4, 5, 6, 7, and 9.

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Permutations

Permutations without Repetition	Permutations with Repetition
$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$	$${}^{n}P_{r} = n^{r}$$
Example: If three alphabets are to be chosen from A, B, C, D and E such that repetition is not allowed then in how many ways it can be done?	Example: If five digits 1, 2, 3, 4, 5 are being given and a three digit code has to be made from it if the repetition of digits is allowed then how many such codes can be formed.
$${}^{n}p_{r} = \frac{5!}{(5-3)!} = 60$$	${}^{5}P_{3} = 5^{3} = 5\times5\times5 = 125.$

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$$ {}_nC_r = {}_7C_4 $$

Combinations: (Order is not important!)

$$ {}_7C_4 = \frac{7!}{(7-4)!4!} = \frac{7*6*5*\cancel{4*3*2*1}}{3*2*1*\cancel{4*3*2*1}} $$

$$ = \frac{7*6*5}{3*2*1} \quad \text{or} \quad 35 $$

There are 35 different groups of students that could be selected.

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Combinations

Combinations without Repetition	Combinations with Repetition
$^{n}C_{r} = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$	$^{n+r-1}C_{r} = \binom{n+r-1}{r} = \frac{(n+r-1)!}{(n-1)!\,r!}$
Example: In a lucky draw chits of ten names are out in a box out of which three are to be taken out. Find the number of ways in which those three names can be taken out.	Example: There are three flavours of ice-cream: chocolate, lemon and vanilla. You can have three scoops. How many variations will there be? Possible selections are: {c,c,c} {c,c,l} {c,c,v} {c,l,v} {l,l,l} {l,l,c} {l,l,v} {v,v,v} {v,v,c} {v,v,l}
$^{10}C_{3} = \frac{10!}{(10-3)!\,3!} = 120$	$\binom{3+3-1}{3} = \binom{5}{3} = \frac{5!}{3!\cdot 2!} = 10$

Permutations and Combinations

Tip!

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Consider our previous example:

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How many different groups of students could be selected?

The order in which the people are being chosen does not matter because the positions for which they are being chosen are the same. They are all going to be members of the student council, with the same duties. **(Combination)**

Permutations and Combinations

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When working with **permutations** and **combinations**, it is vital that you are able to distinguish when the counting **order** is important, or not.

Consider our previous example:

The students of Mr. Fant's Seminar class had to choose 4 out of the 7 people who were nominated to serve on the Student Council.
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The order in which the people are being chosen does not matter because the positions for which they are being chosen are the same. They are all going to be members of the student council, with the same duties. **(Combination)**

However, if Mr. Fant's class was choosing 4 out of 7 students to be president, vice-president, secretary, and treasurer of the student council, then the order in which they are chosen would matter. **(Permutation)**

Probability

PROBABILITY:

It is a numerical measure which indicates the chance of occurrence.

RANDOM EXPERIMENT:

Random Experiment is an experiment which may not result in the same output when repeated under the same conditions.

OUTCOME:

The result of an experiment is Outcome.

SAMPLE SPACE:

The set of all possible outcomes of a random experiment is called sample space.

Finite Sample Space:

A sample space with finite number of outcomes is called **Finite Sample Space**.

Infinite Sample Space:

A sample space with infinite number of outcomes is called **Infinite Sample Space**.

EVENT:
It is a subset of sample space.

Null/Impossible Event:
An event which does not contain any outcome is a Null event. It is denoted by Φ.

Simple/Elementary Event:
An event which has only one outcome is called Simple event.

Compound Event:
An event which has more than one outcome is a Compound event.

Sure/Certain Event:
An event which contains all the outcomes (It is same as Sample space) is called sure event.

COMPLEMENT OF AN EVENT:

The complement of an **event A** is the event of non-occurrence of **event A**.

If **A** is an event, the **complement of A** is denoted as **A'**.

SUB-EVENT:

Let A and B be two events such that event A occurs whenever event B occurs. Then, event B is sub-event of event A. It is denoted by $B \subset A$.

UNION OF EVENTS:

Union of two or more events is the event of occurrence of at least one of these events. The union of A and B is denoted by $A \cup B$ or AorB or $A+B$.

INTERSECTION OF EVENTS:

Intersection of two or more events is the event of simultaneous occurrence of all these events. The union of A and B is denoted by $A \cap B$ or AandB or AB.

EQUALLY LIKELY EVENTS (Equiprobable events):

Two or more events are equally likely if they have equal chance of occurrence.

MUTUALLY EXCLUSIVE EVENTS (Disjoint events):

Two or more events are mutually exclusive if only one of them can occur at a time. Mutually exclusive events cannot occur together.

Note:

If **A** is an event, **A** and **A'** are mutually exclusive.

If **A** and **B** is an mutually exclusive events, then, $A \cap B = \Phi$.

EXHAUSTIVE EVENTS:

A set of events is exhaustive if one or the other of the events in the set occurs whenever the experiment is conducted.

Probability

If an experiment has n equally likely simple events and if m be the favourable number of outcomes to an event A. Then the probability of A, P(A), is

$$P(A) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{m}{n}$$

Results:

1. $0 \leq P(A) \leq 1$

2. $P(A)=1-P(A')$

3. $P(\Phi)=0$ where Φ is null event.

Examples ! ! !