Maths for Intelligent Systems

Topic 1:

Linear Algebra I

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Scalar : number

Vector : row/column of numbers

Matrix : many rows and columns

Linear Algebra: Vectors

Vectors

A vector is a 1 Dimension array of values

We use the notation $A \in \Re^n$ to denote that x is an n-dimensional vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

We use the notation x_i to denote the i^{th} entry of x

By default, we consider vectors to represent *column vectors*.

Linear Algebra: Matrices

Matrices

A matrix is a 2 Dimension array of values

Notation:$A \in \Re^{m\times n}$(with m rows and n columns)

$$\begin{array}{cc} A = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mn} \end{bmatrix}_{m \times n} & \begin{array}{l} \text{(Row 1)} \\ \\ \text{(Row m)} \end{array} \\ \text{(Column1)} \quad \text{(Column n)} & \end{array}$$

Column Matrix

$$\begin{bmatrix} 2 \\ 3 \\ 1 \\ 5 \end{bmatrix}_{4x1}$$

Only 1 column

Row Matrix

$$\begin{bmatrix} 6 & 1 & 4 \end{bmatrix}_{1x3}$$

Only 1 row

Square Matrix

$$\begin{bmatrix} 4 & -6 & 9 \\ -6 & 0 & -3 \\ 4 & 1 & 1 \end{bmatrix}_{3x3}$$

# of Rows = # of Columns

Diagonal Matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 7 \end{bmatrix}_{3x3}$$

Non-Diagonal elements = 0

Identity (Unit) Matrix

$$I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$$

Diagonal elements =1 & Non-Diagonal elements = 0

Null (Zero) Matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3x3}$$

All elements = 0

Symmetric Matrix

$$\begin{bmatrix} 2 & 6 & -1 \\ 6 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}_{3x3}$$

A' = A

Equality of Matrix

$$\begin{bmatrix} 4 & -6 & 9 \\ -6 & 0 & -3 \\ 4 & 1 & 1 \end{bmatrix}_{3x3} = \begin{bmatrix} 4 & -6 & 9 \\ -6 & 0 & -3 \\ 4 & 1 & 1 \end{bmatrix}_{3x3}$$

Respective elements of both matrix should be same i.e. $A_{ij} = B_{ij}$

Transpose of Matrix:

$$A = \begin{bmatrix} 4 & 2 & 9 \\ -6 & 5 & -3 \\ 4 & 5 & 1 \end{bmatrix}_{3x3}$$

$$A' = \begin{bmatrix} 4 & -6 & 4 \\ 2 & 5 & 5 \\ 9 & -3 & 1 \end{bmatrix}_{3x3}$$

Properties:

1. **(A')'=A**
2. **(kA)'=kA'**
3. **(A+B)'=A'+B'**
4. **(AB) '=B'A'**

Interchange Rows into Column (So that Columns into Rows)

Addition of Matrices:

$$\begin{bmatrix} 1 & 3 & -6 \\ -2 & 1 & 3 \\ 3 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 8 \\ 2 & 5 & 6 \\ 3 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 1+2 & 3+4 & -6+8 \\ -2+2 & 1+5 & 3+6 \\ 3+3 & 0+6 & 1+0 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 2 \\ 0 & 5 & 9 \\ 6 & 6 & 1 \end{bmatrix}$$

Add respective elements of both matrix

Properties:

1. **A+B = B+A (Communicative law)**
2. **(A+B)+C = A+(B+C) = (A+C)+B (Associative Law)**
3. **A+0 = 0+A = A**
4. **A+(-A) = 0**

Subtraction of Matrices:

$$\begin{bmatrix} 1 & 3 & -6 \\ -2 & 1 & 3 \\ 3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 8 \\ 2 & 5 & 6 \\ 3 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 1-2 & 3-4 & -6-8 \\ -2-2 & 1-5 & 3-6 \\ 3-3 & 0-6 & 1-0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -14 \\ -4 & -4 & -3 \\ 0 & -6 & 1 \end{bmatrix}$$

Subtract respective elements of both matrix

Multiplication of Matrices:

$$A=\begin{bmatrix} \boxed{1 \quad 3 \quad 2} \\ 4 \quad 0 \quad 1 \end{bmatrix} 2\text{x}3 \qquad B=\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} 3\text{x}2$$

$$AB=\begin{bmatrix} \boxed{(1*1)+(3*0)+(2*5)} & (1*3)+(3*1)+(2*2) \\ (4*1)+(0*0)+(1*5) & (4*3)+(0*1)+(1*2) \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix} 2\text{x}2$$

Properties:

A(B+C) = AB+AC

(Distributive law)

(AB)C = A(BC) = (AC)B

(Associative Law)

IA = AI = A

A+(-A) = 0

$$A=\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 3 & 0 & 1 \end{bmatrix}_{3\text{x}3} \qquad B=\begin{bmatrix} 2 & 4 & 8 \\ 2 & 5 & 6 \\ 3 & 6 & 0 \end{bmatrix}_{3\text{x}3}$$

$$AB=\begin{bmatrix} (1*2)+(3*2)+(6*3) & (1*4)+(3*5)+(6*6) & (1*8)+(3*6)+(6*0) \\ (2*2)+(1*2)+(3*3) & (2*4)+(1*5)+(3*6) & (2*8)+(1*6)+(3*0) \\ (3*2)+(0*2)+(1*3) & (3*4)+(0*5)+(1*6) & (3*8)+(0*6)+(1*0) \end{bmatrix}_{3\text{x}3}$$

$$=\begin{bmatrix} 26 & 55 & 26 \\ 15 & 31 & 22 \\ 9 & 18 & 24 \end{bmatrix}_{3\text{x}3}$$