Linear Inequalities

Dr. Umesh R Aundhakar

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The sign in linear inequalities are:

$>$	:	greater than
$<$	:	less than
$\geq$	:	greater than or equal to
$\leq$	:	less than or equal to

Introduction

"Is it possible to solve a statement problem which is in the form of equation of inequality type?"

Statement:

For example, the height of all the students in your class is greater than 165 cm.

Similarly, Here we may get certain statements involving a sign '<' (less than), '>' (greater than), '≤' (less than or equal) and ≥ (greater than or equal) which are known as inequalities.

Inequalities

Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.

Note:

- $3 < 5$; $7 > 5$ are the examples of numerical inequalities
- $x < 5$; $y > 2$; $x \geq 3$, $y \leq 4$ are some examples of literal inequalities.
- $3 < 5 < 7$, $3 < x < 5$ and $2 < y < 4$ are the examples of double inequalities.

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Inequalities

Example:

Ravi goes to market with Rs. 200 to buy rice, which is available only in packets of 1 kg. The price of one packet of rice is Rs. 30. If x denotes the number of packets of rice, which he buys, then the total amount spent by him is Rs $(30x)$. Since, he has to buy rice in packets only, he may not be able to spend the entire amount of Rs. 200.

(Why?)

Hence, $30x < 200$

Clearly the above statement is not an equation as it does not involve the sign of equality.

Inequalities

Example:

Reshma has Rs 120 and wants to buy some registers and pens. The cost of one register is Rs 40 and that of a pen is Rs 20. In this case, if x denotes the number of registers and y, the number of pens which Reshma buys, then the total amount spent by her is Rs $(40x + 20y)$ and we have

$$40x + 20y \leq 120$$

Note that the above statement consists of two statements;

$$40x + 20y < 120 \text{ and} \qquad \text{……….(1)}$$

$$40x + 20y = 120 \qquad \text{……….(2)}$$

Statement (1) is not an equation, i.e., it is an inequality while statement (2) is an equation

Types of Inequalities

$ax + b < 0$ ... (1)

$ax + b > 0$ ... (2)

$ax + b \leq 0$ ... (3)

$ax + b \geq 0$ ... (4)

$ax + by < c$ ... (5)

$ax + by > c$ ... (6)

$ax + by \leq c$ ... (7)

$ax + by \geq c$ ... (8)

$ax^2 + bx + c \leq 0$ ... (9)

$ax^2 + bx + c > 0$ ... (10)

Inequalities (1), (2), (5), (6) and (10) are strict inequalities.

Inequalities (3), (4), (7), (8) and (9) are slack inequalities.

Inequalities from (1) to (4) are linear inequalities in one variable.

Inequalities from (5) to (8) are linear inequalities in two variables.

Inequalities (9) and (10) are quadratic inequalities in one variable.

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Examples of Inequalities:

$x < 5$

$x > 3$

$x \leq 5$

$x \geq 6$

$x < -4$

$x \geq -4$

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Examples of Inequalities:

$x < 5$ $x = 4, 3, 2, 1, 0, \ldots$

$x > 3$

$x \leq 5$

$x \geq 6$

$x < -4$

$x \geq -4$

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Examples of Inequalities:

$x < 5$ $\quad x = 4, 3, 2, 1, 0, \ldots$

$x > 3$ $\quad x = 4, 5, 6, 7, 8, \ldots$

$x \leq 5$

$x \geq 6$

$x < -4$

$x \geq -4$

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Examples of Inequalities:

$x < 5$ $x = 4 , 3 , 2 , 1 , 0 , \ldots$

$x > 3$ $x = 4 , 5 , 6 , 7 , 8 , \ldots$

$x \leq 5$ $x = 5 , 4 , 3 , 2 , 1 , \ldots$

$x \geq 6$

$x < -4$

$x \geq -4$

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Examples of Inequalities:

$x < 5$ $\quad x = 4, 3, 2, 1, 0, \ldots$

$x > 3$ $\quad x = 4, 5, 6, 7, 8, \ldots$

$x \leq 5$ $\quad x = 5, 4, 3, 2, 1, \ldots$

$x \geq 6$ $\quad x = 6, 7, 8, 9, \ldots$

$x < -4$

$x \geq -4$

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Examples of Inequalities:

$x < 5$ $\quad x = 4, 3, 2, 1, 0, \ldots$

$x > 3$ $\quad x = 4, 5, 6, 7, 8, \ldots$

$x \leq 5$ $\quad x = 5, 4, 3, 2, 1, \ldots$

$x \geq 6$ $\quad x = 6, 7, 8, 9, \ldots$

$x < -4$ $\quad x = -5, -6, -7, -8, \ldots$

$x \geq -4$

Examples of Inequalities:

$x < 5$ $\quad x = 4, 3, 2, 1, 0, \ldots$

$x > 3$ $\quad x = 4, 5, 6, 7, 8, \ldots$

$x \leq 5$ $\quad x = 5, 4, 3, 2, 1, \ldots$

$x \geq 6$ $\quad x = 6, 7, 8, 9, 10, \ldots$

$x < -4$ $\quad x = -5, -6, -7, -8, \ldots$

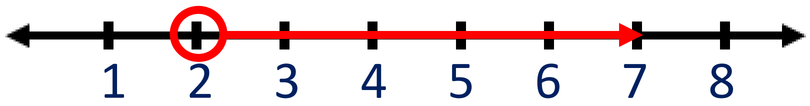
$x \geq -4$ $\quad x = -4, -3, -2, -1, \ldots$

Solution of Linear Inequalities

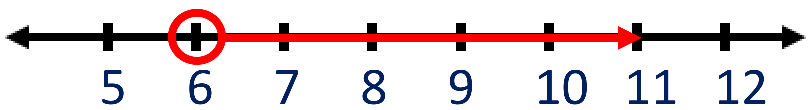
Graphical Solution

Inequality on a Number Line :

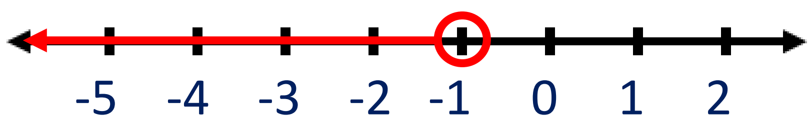
X > 2



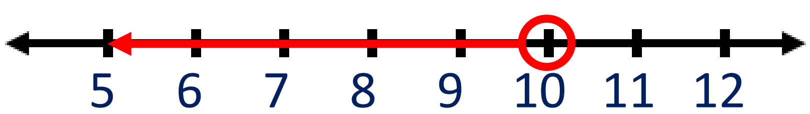
X > 6



X < -1

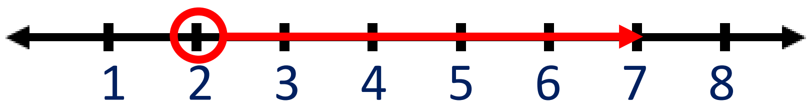


X < 10

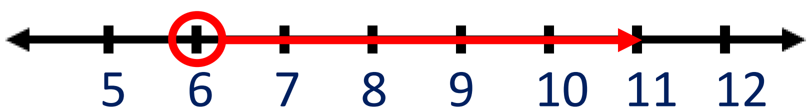


Inequality on a Number Line :

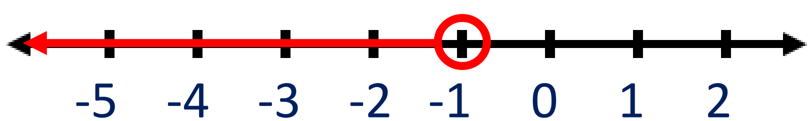
X > 2



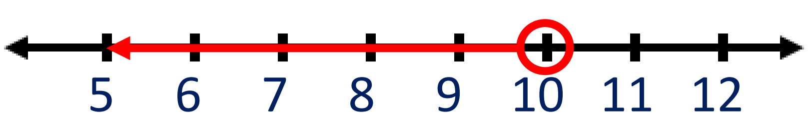
X > 6



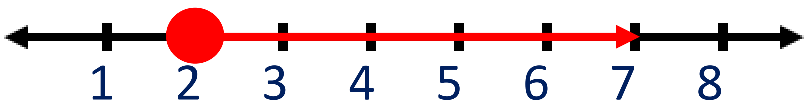
X < -1



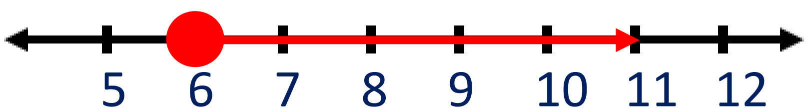
X < 10



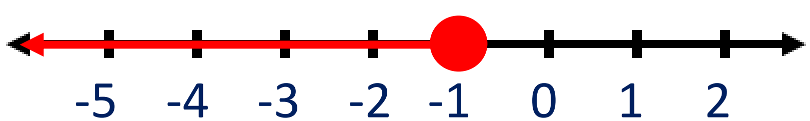
X ≥ 2



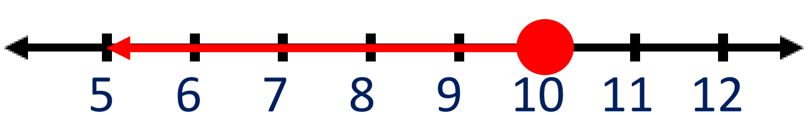
X ≥ 6



X ≤ -1



X ≤ 10



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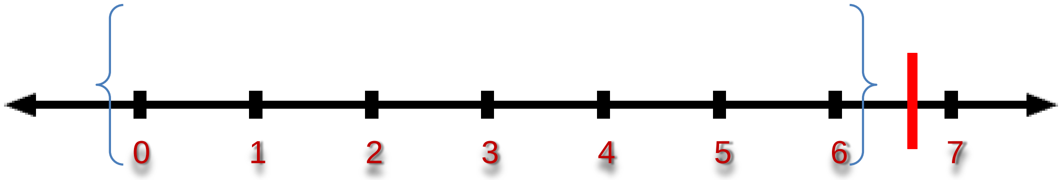
Graphical Solutions

Let us consider the inequality; $30x < 200$

x: denotes the number of packets of rice. (non-negative)

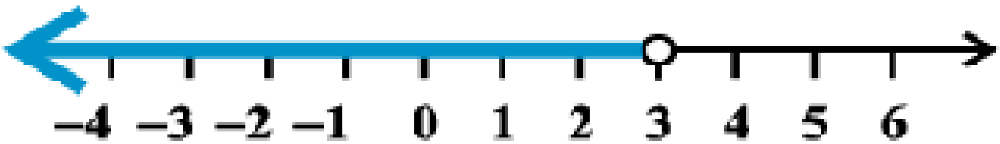


Therefore, $x=\{0,1,2,3,4,5,6\}$

Example:

$x < 3$

The graphical representation of the Solutions

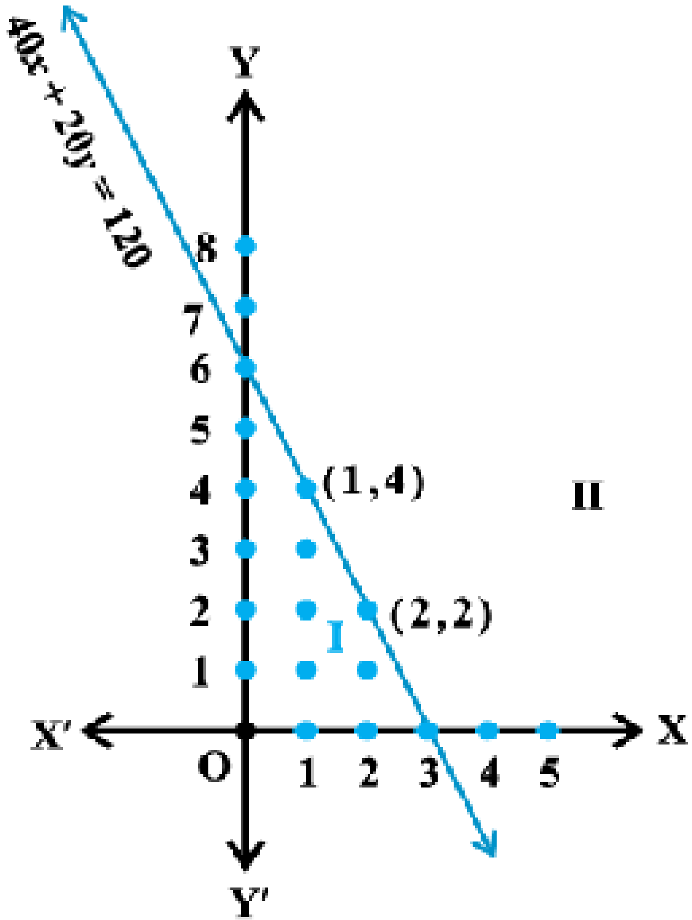


Graphical Solution of Linear Inequalities with Two Variables

Reshma has Rs 120 and wants to buy some registers and pens. The cost of one register is Rs 40 and that of a pen is Rs 20.

we have

$40x + 20y \leq 120$



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Some Important Rules:

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an equation.

Ex. $x+5 < 8$ $\qquad$ $x+5 < 8$

$x+5+5 < 8+5$ $\qquad$ $x+5-5 < 8-5$

$x+10 < 13$ $\qquad$ $x < 3$

Rule 2: Both sides of an equation may be multiplied (or divided) by the <u>same positive number</u> (Obviously, Zero not allowed).

Ex. $2x < 8$ $\qquad$ $2x < 8$

$2x*2 < 8*2$ $\qquad$ $2x/2 < 8/2$

$4x < 16$ $\qquad$ $x < 4$

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Some Important Rules:

Rule 3: Both sides of an equation may be multiplied (or divided) by the same negative number but, <u>the sign of inequality is reversed.</u>

Ex.

$2x < 8$	$2x < 8$
$2x*(-2) > 8*(-2)$	$2x/(-2) > 8/(-2)$
$-4x > -16$	$-x > -4$

Example

- Is $(-1, 9)$ a solution of $2x + y < -3$?
- Is $(2, -2)$ a solution of $x - 3y \geq 8$?
- Is $(-3, 4)$ a solution of $y > -1$?
- Is $(0, 0)$ a solution of $y > x$?
- Is $(-4, -1)$ a solution of $5x - 2y \leq -1$?

Dashed or Solid

- A **dashed** line means points on the line are **not solutions**.
  - Use for “<” and “>”.

- A **solid** line means points on the line **are solutions**.
  - Use for “≤” and “≥”.

Where to Shade?

- The shaded area shows **all** possible points that make the inequality true.
- Test a point.
  - We usually use (0,0).
  - If it is on the line, choose a different point.
- If it is a solution, shade **that side**.
- If it is not a solution, shade the **other side.**

Graphing Simple Linear Inequalities

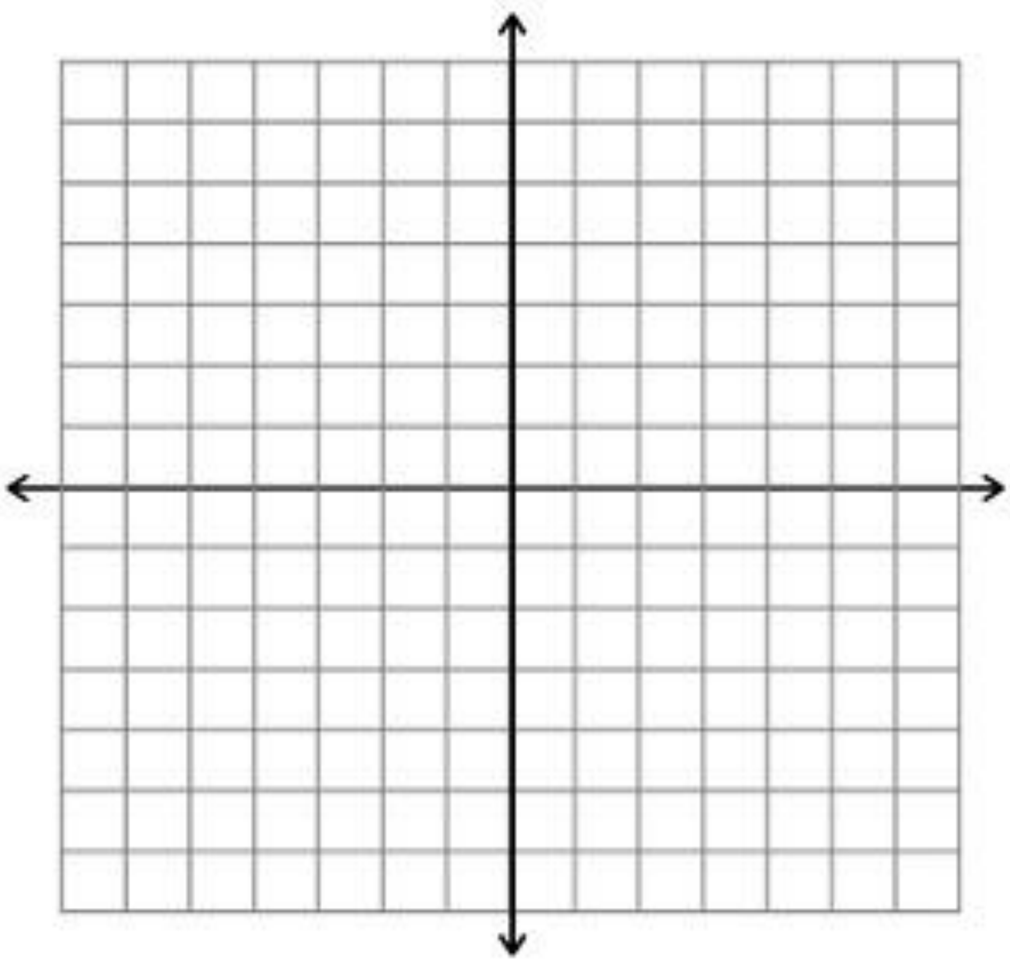
Linear inequalities with only one variable have horizontal or vertical boundary lines.

y: vertical (up and down)

x: horizontal (side to side)

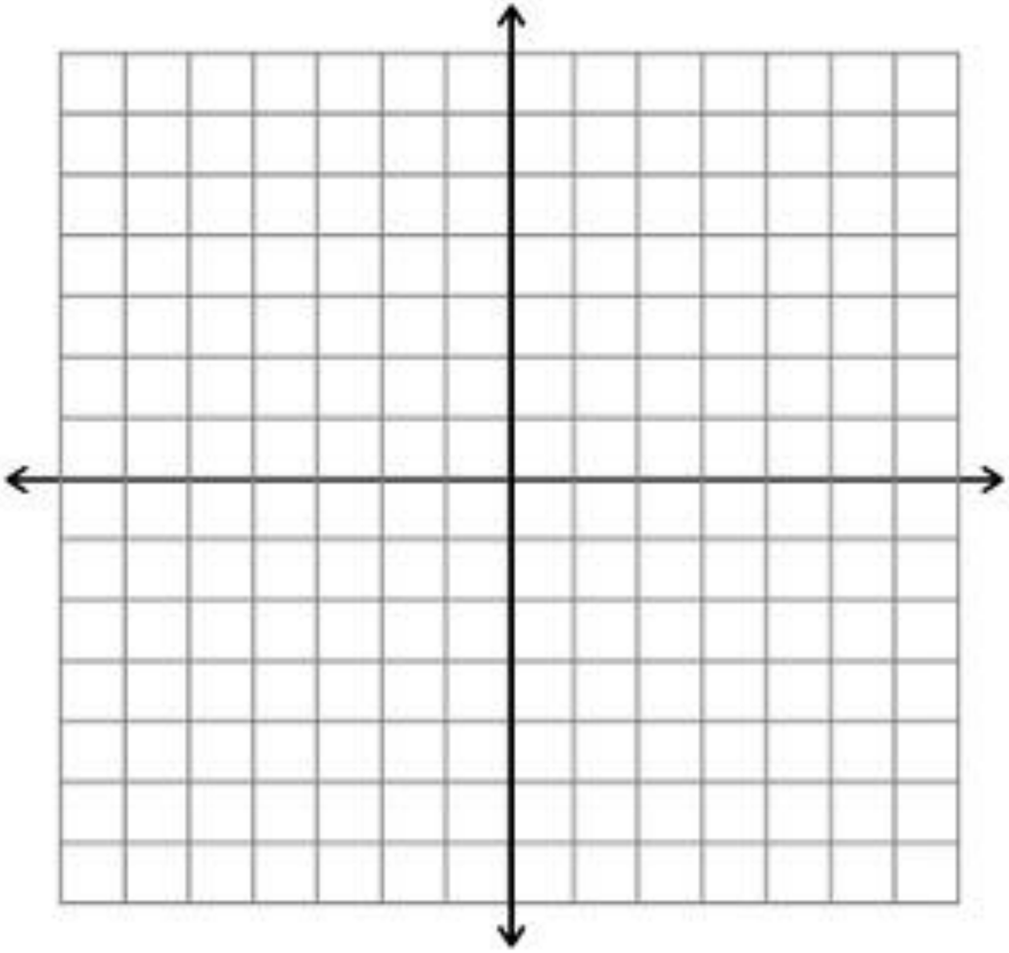
Example:

Graph $y \leq 2$.



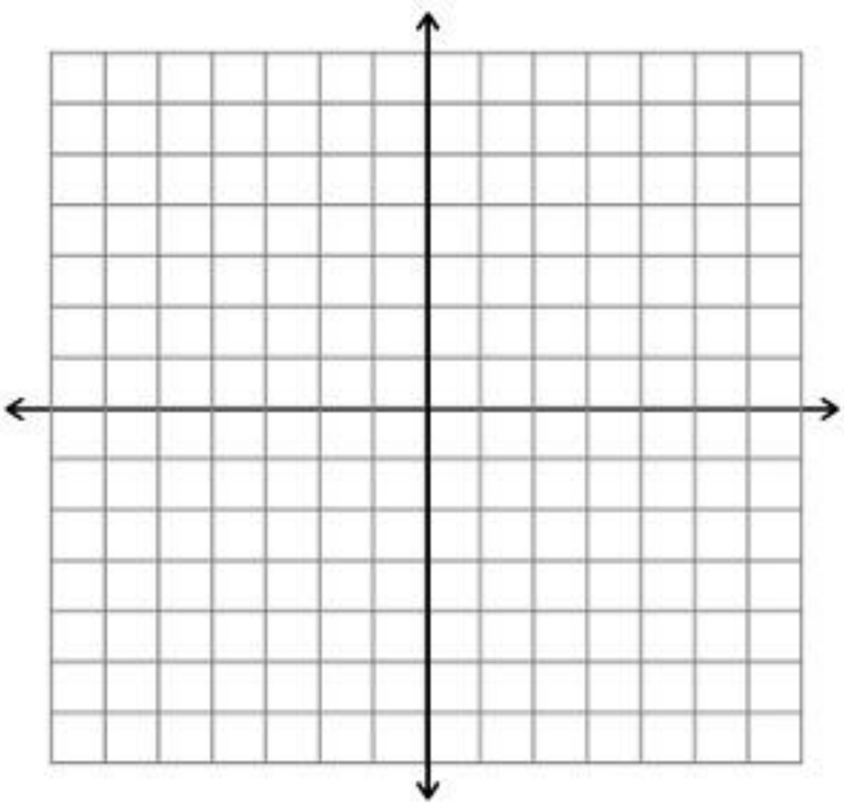
Example:

Graph x > 1.

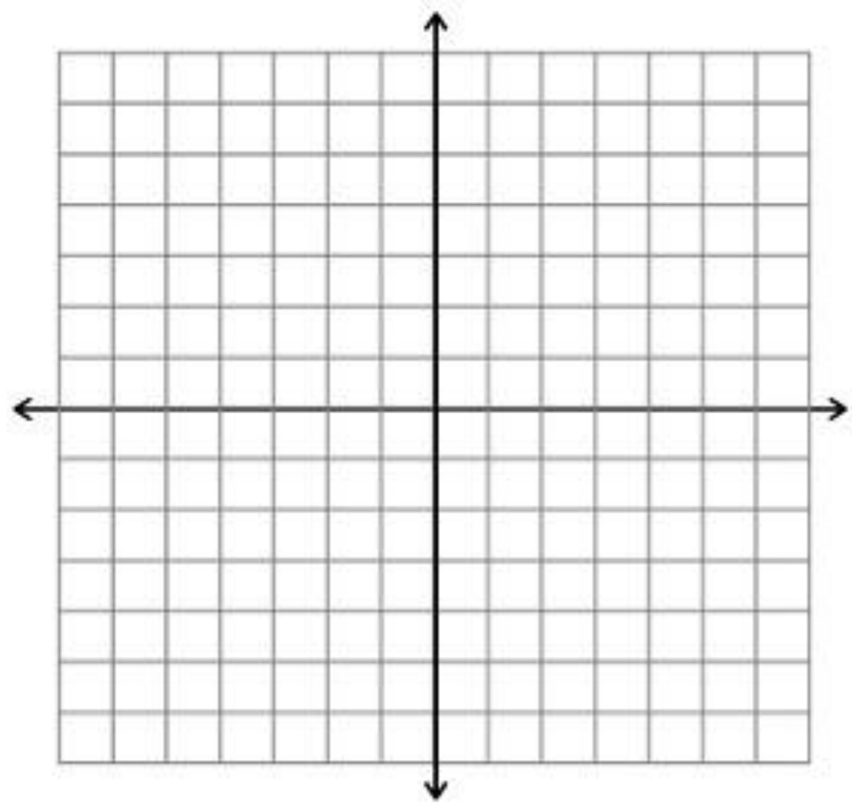


Try!

Graph $y > -1$



Graph $3.5 > x$



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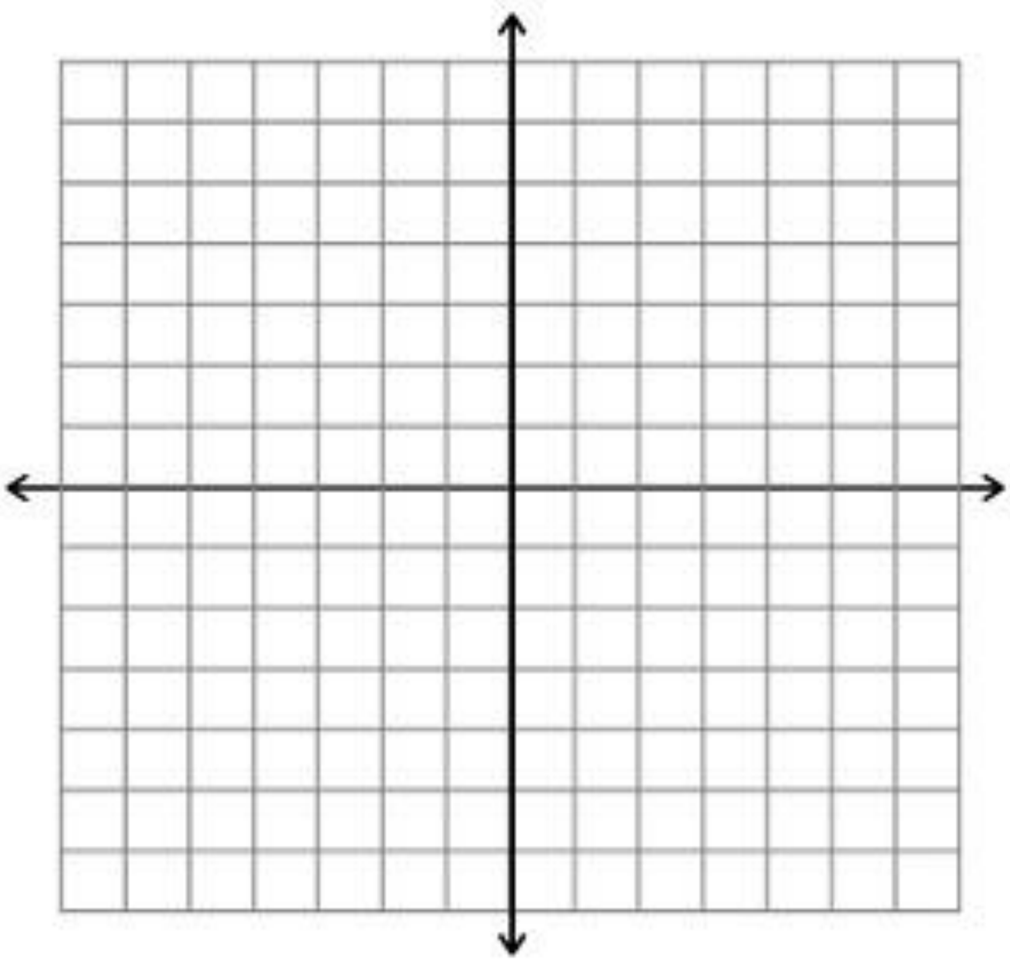
Example

Graph $3x - 2y < 6$



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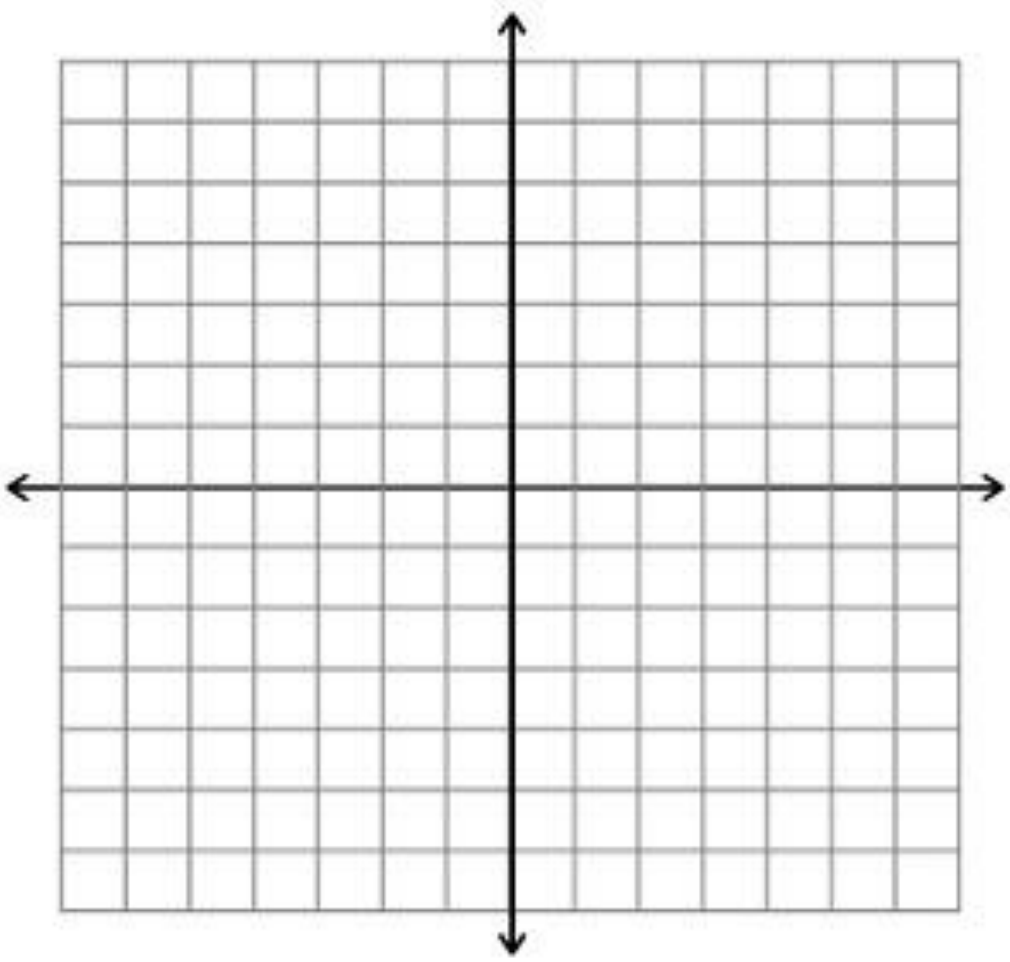
Example

Graph $3x - 2y \geq 0$



Multiple constraint:

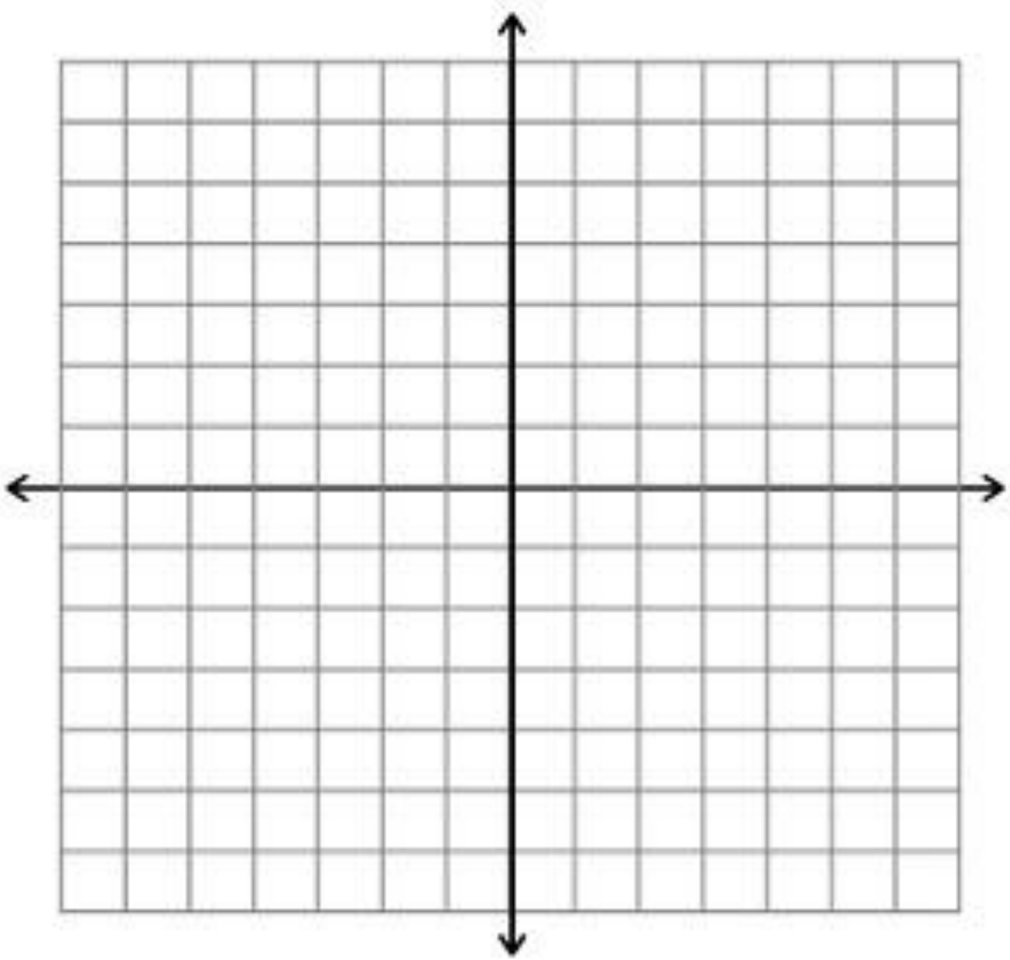
Example:

$6x+y\leq 21$

$x+3y\geq 12$

$x+y \leq 10$

$x,y \geq 0$



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Application: LPP

1. Lets Create and solve 1 example.
2. Solution - Feasible solution - Optimal Solution
3. Types of Solutions for LPP.

END