Hypothesis Testing of the Difference between Two Population Means

Example: 1

Measurement of Serum uric acid level is taken. Make inference w.r.t. following statements; (Use 95% confidence Interval)
A. Is there a difference between the means between individuals with Down's syndrome and normal individuals?

B. Is Serum uric acid level is more in individual with Down's syndrome w.r.t. normal individual.

Data:

Down's syndrome	$\bar{x}_1 = 4.5 \quad n_1 = 12 \quad \sigma_1{}^2 = 1$
Normal Person	$\bar{x}_2 = 3.4 \quad n_2 = 15 \quad \sigma_2{}^2 = 1.5$
	$\alpha = .05$

Ans:

A. z=2.57, z(0.05)=(-1.96, 1.96), H0 Reject, (Two tailed)
B. z=2.57, z(0.05)= 1.64, H0 Rejected, (Upper Tailed Test)

Example: 2

Lung destructive index

We wish to know if we may conclude, at the 95% confidence level, that smokers, in general, have greater lung damage than do non-smokers. (Note: Assume equal population variances)

(1) Data

Smokers	$\bar{x}_1 = 17.5 \quad n_1 = 16 \quad {s_1}^2 = 4.4752$
Non-smokers	$\bar{x}_2 = 12.4 \quad n_2 = 9 \quad {s_2}^2 = 4.8492$
	$\alpha = .05$

Ans:

t=5.7, $t(n_1+n_2-2,0.05)$= 1.71, H0 Rejected, (Upper Tailed Test)

Example: 3

These data were obtained in a study comparing persons with disabilities with persons without disabilities. A scale known as the Barriers to Health Promotion Activities for Disabled Persons (BHADP) Scale gave the data. We wish to know if we may conclude, at the 99% confidence level, that persons with disabilities score higher than persons without disabilities.

Data:

Disabled	$\bar{x}_1 = 31.83 \quad n_1 = 132 \quad {\sigma_1}^2 = 7.93$
Nondisabled	$\bar{x}_2 = 25.07 \quad n_2 = 137 \quad {\sigma_2}^2 = 4.8$
	$\alpha = .01$

Ans:

z=21.9, z(0.01)= 2.33, H0 Rejected, (Upper Tailed Test)

Example: 4

Very-low-calorie diet (VLCD) Treatment: following Table gives B (before) and A (after) treatment data for obese female patients in a weight-loss program. Table gives B (before) and A (after) treatment data for obese female patients in a weight-loss program.

B:	117.3	111.4	98.6	104.3	105.4	100.4	81.7	89.5	78.2
A:	83.3	85.9	75.8	82.9	82.3	77.7	62.7	69.0	63.9

We wish to know if we may conclude, at the 95% confidence level, that the treatment is effective in causing weight reduction in these people.

Ans:

t= - 12.7395, t(0.05, n-1)= -1.8595, H0 Rejected, (Lower Tailed Test)

Example: 5

In order to investigate the relationship between mean job tenure in years among workers who have a bachelor's degree or higher and those who do not, random samples of each type of worker were taken, with the following results.

Bachelors degree or higher	$\bar{x}_1 = 5.2 \quad n_1 = 155 \quad s_1 = 7.93$
No degree	$\bar{x}_2 = 5.0 \quad n_2 = 210 \quad s_2 = 4.8$

a. Construct the 99% confidence interval for the difference in the population means based on these data.
b. Test, at the 1% level of significance, the claim that mean job tenure among those with higher education is greater than among those without, against the default that there is no difference in the means.

Ans:

a)0.2±0.4

b)z = 0.27, z(0.01)=2.326, do not reject H0 (not greater)

Example: 6

In comparing the academic performance of college students who are affiliated with fraternities and those male students who are unaffiliated, a random sample of students was drawn from each of the two populations on a university campus. Summary statistics on the student GPAs are given below.

Fraternity	$\bar{x}_1 = 2.9 \quad n_1 = 645 \quad s_1 = 0.47$
Unaffiliated	$\bar{x}_2 = 2.88 \quad n_2 = 450 \quad s_2 = 0.42$

Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that there is a difference in average GPA between the population of fraternity students and the population of unaffiliated male students on this university campus.

Ans:

$Z = 0.738$, $\pm z_{0.025} = \pm 1.960$, do not reject H_0 (no difference)

Example: 7

A university administrator asserted that upperclassmen spend more time studying than underclassmen.

1. Test this claim against the default that the average number of hours of study per week by the two groups is the same, using the following information based on random samples from each group of students. Test at the 1% level of significance.

Upper Class	$\bar{x}_1 = 15.6 \quad n_1 = 35 \quad s_1 = 2.9$
Lower Class	$\bar{x}_2 = 12.3 \quad n_2 = 35 \quad s_2 = 4.1$

Ans:
Z = 3.888, z0.01=2.326, reject *H*0 (upperclassmen study more)

Example: 8

A county environmental agency suspects that the fish in a particular polluted lake have elevated mercury level. To confirm that suspicion, five striped bass in that lake were caught and their tissues were tested for mercury. For the purpose of comparison, four striped bass in an unpolluted lake were also caught and tested. The fish tissue mercury levels in mg/kg are given below.

Sample 1 (from polluted lake)	Sample 2 (from unpolluted lake)
0.580	0.382
0.711	0.276
0.571	0.570
0.666	0.366
0.598	

Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that fish in the polluted lake have elevated levels of mercury in their tissue.

Ans:

$T = 3.635$, df=5, $t_{0.05}=2.015$, reject $H0$ (elevated levels)

Example: 9

A university administrator wishes to know if there is a difference in average starting salary for graduates with master's degrees in engineering and those with master's degrees in business. Fifteen recent graduates with master's degree in engineering and 11 with master's degrees in business are surveyed and the results are summarized below.

Engineering	$\bar{x}_1 = 68535 \quad n_1 = 15 \; s_1 = 1627$
Buisness	$\bar{x}_2 = 63230 \quad n_2 = 11 \quad s_2 = 2033$

Test, at the 10% level of significance, whether the data provide sufficient evidence to conclude that the average starting salaries are different.

Ans:

T = 7.395, df=24, ±t0.05=±1.711, reject *H*0 (different)

Example: 10

Use the following paired sample data for this exercise.

Population 1:	35	32	35	35	36	35	36
Population 2:	28	26	27	26	29	27	29

Test, at the 10% level of significance, the hypothesis that $\mu_1-\mu_2>7$ as an alternative to the null hypothesis that $\mu_1-\mu_2=7$.

Ans:

$T = 1.162$, df=6, $t_{0.10}=1.44$, do not reject H_0

Example: 11

Each of five laboratory mice was released into a maze twice. The five pairs of times to escape were:

Mouse	1	2	3	4	5
First release	129	89	136	163	118
Second release	113	97	139	85	75

Test, at the 10% level of significance, the hypothesis that it takes mice less time to run the maze on the second trial, on average.

Ans:

T = 1.580, df=4, t0.10=1.533, reject H0 (takes less time)

Example: 12

Voters in a particular city who identify themselves with one or the other of two political parties were randomly selected and asked if they favor a proposal to allow citizens with proper license to carry a concealed handgun in city parks. The results are:

	Party A	Party B
Sample size, n	150	200
Number in favor, x	90	140

Test, at the 5% level of significance, the hypothesis that the proportion of all members of Party A who favor the proposal is less than the proportion of all members of Party B who do. Also, compute the p-value of the test.

Ans:

$Z=-1.943$, $-z_{0.05}=-1.645$, reject H_0 (fewer in Party A favor)

p-value = 0.0262

Example: 13

A local school board member randomly sampled private and public high school teachers in his district to compare the proportions of National Board Certified (NBC) teachers in the faculty. The results were:

	Private Schools	Public Schools
Sample size, n	80	520
Proportion of NBC teachers, $\hat{p}$	0.175	0.15

1. Construct the 90% confidence interval for the difference, based on these data.
2. Test, at the 10% level of significance, the hypothesis that the proportion of all public school teachers who are National Board Certified is less than the proportion of private school teachers who are. Also, compute the p-value of the test.

Ans:

0.025 ± 0.0745

$Z = 0.552$, $z_{0.10} = 1.282$, do not reject H_0

p-value = 0.2912

Example: 14

Randomly selected middle-aged people in both China and the United States were asked if they believed that adults have an obligation to financially support their aged parents. The results are summarized below.

	China	USA
Sample size, n	1300	150
Number of yes, x	1170	110

Test, at the 1% level of significance, whether the data provide sufficient evidence to conclude that there exists a cultural difference in attitude regarding this question.

Ans:

$Z = 4.498$, $\pm z_{0.005} = \pm 2.576$, reject H_0 (exists a cultural different)

Example: 15

The data set contains 480 ceramic strength measurements for two batches of material. The summary statistics for each batch are shown below.

BATCH 1:

NUMBER OF OBSERVATIONS	=	240
MEAN	=	688.9987
STANDARD DEVIATION	=	65.54909

BATCH 2:

NUMBER OF OBSERVATIONS	=	240
MEAN	=	611.1559
STANDARD DEVIATION	=	61.85425

We are testing the null hypothesis that the variances for the two batches are equal.

H0: $\sigma_1^2 = \sigma_2^2$

Ha: $\sigma_1^2 \neq \sigma_2^2$

Ans:

Test statistic: $F = 1.123037$

Numerator degrees of freedom: $n_1 - 1 = 239$

Denominator degrees of freedom: $n_2 - 1 = 239$

Significance level: $\alpha = 0.05$

Critical values: $F(1-\alpha/2, n_1-1, n_2-1) = 0.7756$

$F(\alpha/2, n_1-1, n_2-1) = 1.2894$

Rejection region: Reject H0 if $F < 0.7756$ or $F > 1.2894$

The F test indicates that there is not enough evidence to reject the null hypothesis that the two batch variance's are equal at the 0.05 significance level.