Examples: Normal Distribution

1. X is a normally normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

   a) $P(x < 40)$

   b) $P(x > 21)$

   c) $P(30 < x < 35)$

2. A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

3. For a certain type of computers, the length of time bewteen charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

4. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

5. The length of similar components produced by a company are approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random

   a) what is the probability that the length of this component is between 4.98 and 5.02 cm?

   b) what is the probability that the length of this component is between 4.96 and 5.04 cm?

6. The length of life of an instrument produced by a machine has a normal ditribution with a mean of 12 months and standard deviation of 2 months. Find the probability that an instrument produced by this machine will last

   a) less than 7 months.

   b) between 7 and 12 months.

7. The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time

   a) less than 19.5 hours?

   b) between 20 and 22 hours?

8. A large group of students took a test in Physics and the final grades have a mean of 70 and a standard deviation of 10. If we can approximate the distribution of these grades by a normal distribution, what percent of the students

a) scored higher than 80?

b) should pass the test (grades$\geq$60)?

c) should fail the test (grades$<$60)?

9. The annual salaries of employees in a large company are approximately normally distributed with a mean of \$50,000 and a standard deviation of \$20,000.

   a) What percent of people earn less than \$40,000?

   b) What percent of people earn between \$45,000 and \$65,000?

   c) What percent of people earn more than \$70,000?

10. Suppose the diameter of a certain car component follows the normal distribution with $X \sim N(10, 9)$. Find the proportion of these components that have diameter larger than 13.4 mm. Or, if we randomly select one of these components, find the probability that its diameter will be larger than 13.4 mm.

11. The lengths of the sardines received by a certain cannery is normally distributed with mean 4.62 inches and a standard deviation 0.23 inch. What percentage of all these sardines is between 4.35 and 4.85 inches long?

12. A baker knows that the daily demand for apple pies is a random variable which follows the normal distribution with mean 43.3 pies and standard deviation 4.6 pies. Find the demand which has probability 5% of being exceeded.

13. Suppose that the height of UCLA female students has normal distribution with mean 62 inches and standard deviation 8 inches.

    a. Find the height below which is the shortest 30% of the female students.

    b. Find the height above which is the tallest 5% of the female students.

14. A firm's marketing manager believes that total sales for next year will follow the normal distribution, with mean of \$2.5 million and a standard deviation of \$300, 000.

    a. What is the probability that the firm's sales will fall within \$150000 of the mean?

    b. Determine the sales level that has only a 9% chance of being exceeded next year.

15. Suppose that the height (X) in inches, of a 25-year-old man is a normal random variable with mean $\mu = 70$ inches. If $P(X > 79) = 0.025$ what is the standard deviation of this random normal variable?

16. Suppose that the weight (X) in pounds, of a 40-year-old man is a normal random variable with standard deviation $\sigma = 20$ pounds. If 5% of this population is heavier than 214 pounds what is the mean μ of this distribution?

Answers to the Above Questions

1. Note: What is meant here by area is the area under the standard normal curve.

   a) For $x = 40$, the z-value $z = (40 - 30) / 4 = 2.5$
   Hence $P(x < 40) = P(z < 2.5)$ = [area to the left of 2.5] = 0.9938

   b) For $x = 21$, $z = (21 - 30) / 4 = -2.25$
   Hence $P(x > 21) = P(z > -2.25)$ = [total area] - [area to the left of -2.25]
   = 1 - 0.0122 = 0.9878

   c) For $x = 30$, $z = (30 - 30) / 4 = 0$ and for $x = 35$, $z = (35 - 30) / 4 = 1.25$
   Hence $P(30 < x < 35) = P(0 < z < 1.25)$ = [area to the left of z = 1.25] - [area to the left of 0]
   = 0.8944 - 0.5 = 0.3944

2. Let x be the random variable that represents the speed of cars. x has $\mu = 90$ and $\sigma = 10$. We have to find the probability that x is higher than 100 or $P(x > 100)$

   For $x = 100$, $z = (100 - 90) / 10 = 1$

   $P(x > 90) = P(z >, 1)$ = [total area] - [area to the left of z = 1]

   = 1 - 0.8413 = 0.1587

   The probability that a car selected at a random has a speed greater than 100 km/hr is equal to 0.1587

3. Let x be the random variable that represents the length of time. It has a mean of 50 and a standard deviation of 15. We have to find the probability that x is between 50 and 70 or $P(50< x < 70)$

   For $x = 50$, $z = (50 - 50) / 15 = 0$

   For $x = 70$, $z = (70 - 50) / 15 = 1.33$ (rounded to 2 decimal places)

   $P(50< x < 70) = P(0< z < 1.33)$ = [area to the left of z = 1.33] - [area to the left of z = 0]

   = 0.9082 - 0.5 = 0.4082

   The probability that John's computer has a length of time between 50 and 70 hours is equal to 0.4082.

4. Let x be the random variable that represents the scores. x is normally ditsributed with a mean of 500 and a standard deviation of 100. The total area under the normal curve represents the total number of students who took the test. If we multiply the values of the

areas under the curve by 100, we obtain percentages.

For $x = 585$, $z = (585 - 500) / 100 = 0.85$

The proportion P of students who scored below 585 is given by

P = [area to the left of $z = 0.85$] = 0.8023 = 80.23%

Tom scored better than 80.23% of the students who took the test and he will be admitted to this University.

5. a) $P(4.98 < x < 5.02) = P(-1 < z < 1) = 0.6826$

   b) $P(4.96 < x < 5.04) = P(-2 < z < 2) = 0.9544$

6. a) $P(x < 7) = P(z < -2.5) = 0.0062$

   b) $P(7 < x < 12) = P(-2.5 < z < 0) = 0.4938$

7. a) $P(x < 19.5) = P(z < -0.25) = 0.4013$

   b) $P(20 < x < 22) = P(0 < z < 1) = 0.3413$

8. a) For $x = 80$, $z = 1$
   Area to the right (higher than) $z = 1$ is equal to 0.1586 = 15.87% scored more that 80.

   b) For $x = 60$, $z = -1$
   Area to the right of $z = -1$ is equal to 0.8413 = 84.13% should pass the test.

   c)100% - 84.13% = 15.87% should fail the test.

9. a) For $x = 40000$, $z = -0.5$
   Area to the left (less than) of $z = -0.5$ is equal to 0.3085 = 30.85% earn less than $40,000.

   b) For $x = 45000$, $z = -0.25$ and for $x = 65000$, $z = 0.75$
   Area between $z = -0.25$ and $z = 0.75$ is equal to 0.3720 = 37.20 earn between $45,000 and $65,000.

   c)For $x = 70000$, $z = 1$
   Area to the right (higher) of $z = 1$ is equal to 0.1586 = 15.86% earn more than $70,000.

10. $P(X > 13.4) = P(Z > 1.13) = 1 - 0.8708 = 0.1292$. Therefore the probability that the diameter is larger than 13.4 mm is 12.92%.

11. Example 1 We are given $X \sim N(4.62, 0.23)$. We want to compute
$P(4.35 < X < 4.85) = P(-1.17 < z < 1) = 0.8413 - 0.1210 = 0.7203$.

12. We are given $X \sim N(43.3, 4.6)$. We want to find the demand d such that $P(X > d) = 0.05$. From the standard normal table this corresponds to $z = 1.645$. Therefore $d = 50.9$ pies.

13. We are given $X \sim N(62, 8)$. a. We want to find the height h such that $P(X < h) = 0.30$. From the standard normal table this corresponds to $z = -0.525$. Therefore $h = 57.8$ inches. b. We want to find the height h such that $P(X > h) = 0.05$. From the standard normal table this corresponds to $z = 1.645$. Therefore $h = 75.16$ inches.

14. We are given $X \sim N(2500000, 300000)$. a. $P(2350000 < X < 2650000) = P(-0.5 < z < 0.5) = 0.6915 - 0.3085) = 0.3830$. b. We want to find the sales level s such that $P(X > s) = 0.09$. This corresponds to $z = 1.345$. Therefore $s = 2903500$.

15. We are given $X \sim N(70, \sigma)$. From $P(X > 79) = 0.025$ we find the corresponding z-value: $z = 1.96$. Therefore $\sigma = 4.59$ inches.

16. We are given $X \sim N(\mu, 20)$. From $P(X > 214) = 0.05$ we find the corresponding z-value: $z = 1.645$. Therefore $1.645 = 214{-}\mu\ 20 \Rightarrow \mu = 181.1$ pounds.