Probability

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Why to Learn Probability?

- Nothing in life is certain.
- In everything we guess the chances of successful outcomes, from business to medicine to the weather.
- A probability provides a quantitative description of the chances or likelihoods associated with various outcomes
- It provides a bridge between descriptive and inferential statistics

PROBABILITY:

It is a numerical measure which indicates the chance of occurrence.

RANDOM EXPERIMENT:

Random Experiment is an experiment which may not result in the same output when repeated under the same conditions.

OUTCOME:

The result of an experiment is Outcome.

SAMPLE SPACE:

The set of all possible outcomes of a random experiment is called sample space.

Finite Sample Space:

A sample space with finite number of outcomes is called **Finite Sample Space**.

Infinite Sample Space:

A sample space with infinite number of outcomes is called **Infinite Sample Space**.

EVENT:
It is a subset of sample space.

Null/Impossible Event:
An event which does not contain any outcome is a Null event. It is denoted by **Φ**.

Simple/Elementary Event:
An event which has only one outcome is called Simple event.

Compound Event:
An event which has more than one outcome is a Compound event.

Sure/Certain Event:
An event which contains all the outcomes (It is same as Sample space) is called sure event.

COMPLEMENT OF AN EVENT:

The complement of an **event A** is the event of non-occurrence of **event A**.

If **A** is an event, the **complement of A** is denoted as **A'.**

SUB-EVENT:

Let A and B be two events such that event A occurs whenever event B occurs. Then, event B is sub-event of event A. It is denoted by $\mathbf{B \subset A}$.

UNION OF EVENTS:

Union of two or more events is the event of occurrence of at least one of these events. The union of A and B is denoted by $A \cup B$ or AorB or A+B.

INTERSECTION OF EVENTS:

Intersection of two or more events is the event of simultaneous occurrence of all these events. The union of A and B is denoted by $A \cap B$ or AandB or AB.

EQUALLY LIKELY EVENTS (Equiprobable events):

Two or more events are equally likely if they have equal chance of occurrence.

MUTUALLY EXCLUSIVE EVENTS (Disjoint events):

Two or more events are mutually exclusive if only one of them can occur at a time. Mutually exclusive events cannot occur together.

Note:

If **A** is an event, **A** and **A’** are mutually exclusive.

If **A** and **B** is an mutually exclusive events, then, $A \cap B = \Phi$.

EXHAUSTIVE EVENTS:

A set of events is exhaustive if one or the other of the events in the set occurs whenever the experiment is conducted.

Probability

If an experiment has ***n*** equally likely simple events and if ***m*** be the favorable number of outcomes to an event A. Then the probability of A, *P*(A), is

$$P(A) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{m}{n}$$

Results:

1.0 ≤ P(A) ≤1

2.P(A)=1-P(A')

3.P(Φ)=0 where Φ is null event.

EXPERIMENTS AND EVENTS

Experiment: Record an age

A: person is 30 years old

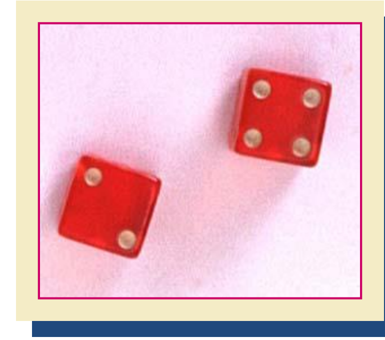
B: person is older than 65

Experiment: Toss a die

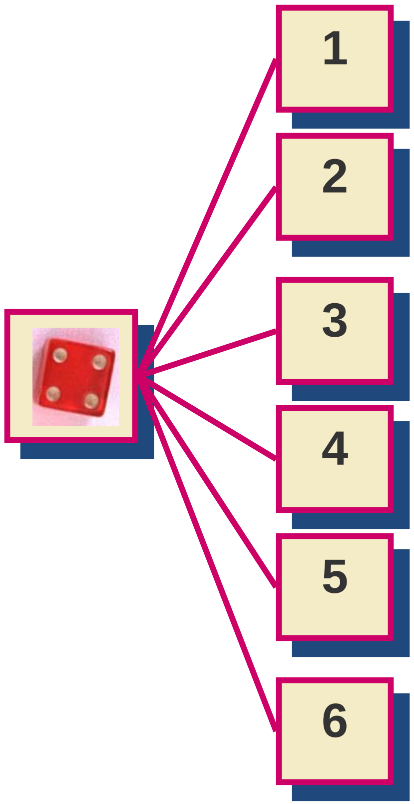
A: observe an odd number

B: observe a number greater than 2

EXAMPLE: THE DIE TOSS



Simple events:



E_1

E_2

E_3

E_4

E_5

E_6

Sample space:

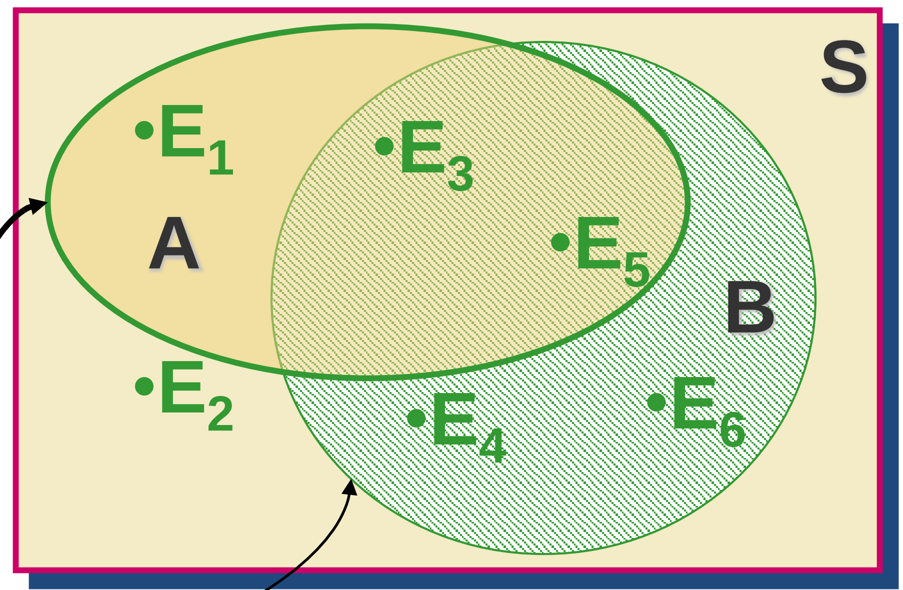
$$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$$

S

- E_1
- E_3
- E_5
- E_2
- E_4
- E_6

BASIC CONCEPTS

An **event** is a collection of one or more **simple events (outcomes).**



- **The die toss:**
  - –A: an odd number
  - –B: a number > 2

A ={E_1, E_3, E_5}

B ={E_3, E_4, E_5, E_6}

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EMPIRICAL APPROACH

Let a random experiment be repeated n times essentially under identical conditions. Let m of these repetition result in the occurrence of an event A. Then probability of event A

$$\boxed{P(A) = \lim_{n \to \infty} \frac{m}{n}}$$

PROBABILITY OF AN EVENT

$P(A)$ must be between 0 and 1.

If event A can never occur, $P(A) = 0$. If event A always occurs when the experiment is performed, $P(A) =1$.

The sum of the probabilities for all simple events in S equals 1.

EXAMPLE 1

Toss a fair coin twice. What is the probability of observing at least one head?

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

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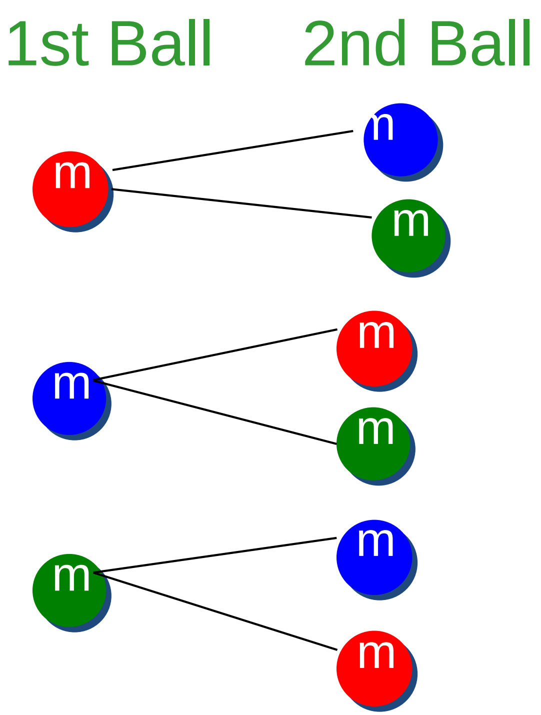
EXAMPLE 2

A bowl contains three marbles, one red, one blue and one green. A child selects two balls at random. What is the probability that at least one is red?



E_i	$P(E_i)$
RB	1/6
RG	1/6
BR	1/6
BG	1/6
GB	1/6
GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

CONDITIONAL PROBABILITIES

The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0}$$

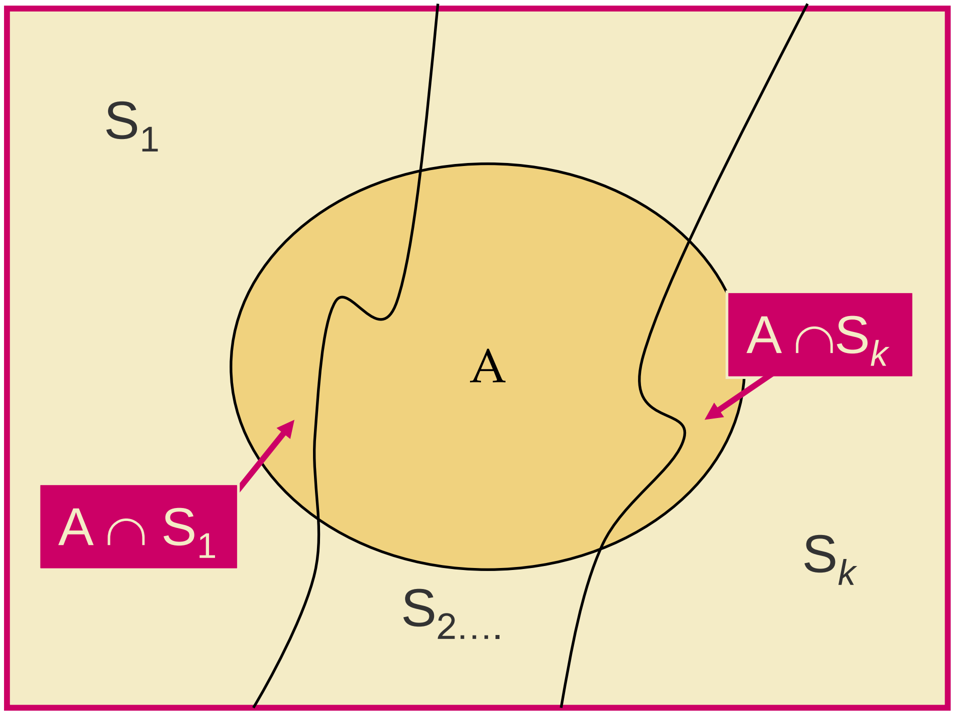
"given"

THE LAW OF TOTAL PROBABILITY

Let S_1 , S_2 , S_3 ,..., S_k be mutually exclusive and exhaustive events (that is, one and only one must happen). Then the probability of any event A can be written as

$$P(A) = P(A \cap S_1) + P(A \cap S_2) + \dots + P(A \cap S_k)$$
$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots + P(S_k)P(A|S_k)$$

The Law of Total Probability



$$P(A) = P(A \cap S_1) + P(A \cap S_2) + \ldots + P(A \cap S_k)$$

$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)$$

BAYES' RULE

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Let S_1 , S_2 , S_3 ,..., S_k be mutually exclusive and exhaustive events with prior probabilities $P(S_1)$, $P(S_2)$,...,$P(S_k)$. If an event A occurs, the posterior probability of S_i, given that A occurred is

$$P(S_i \mid A) = \frac{P(S_i)P(A|S_i)}{\sum P(S_i)P(A|S_i)} \text{ for } i = 1, 2, \ldots k$$

Probability

Examples !

Next Lecture

Topic: **Random Variables and Mathematical Expectations**

Discrete and continuous random variable

Mathematical expectations

Background material to study

Business Statistics, Chpt3, pp.92

We will have MCQ on this lec: 10-15 Questions

And will have recap of the lec one.

Discuss on the assignment of this lec.