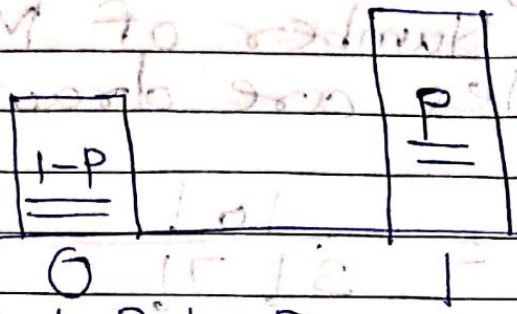
Bernouli Distribution



$$\mu = (1-P)\cdot 0 + P\cdot 1 = P$$

$$\sigma^2 = (1-P)(0-P)^2 + P(1-P)^2$$
$$= (1-P)\cdot P^2 + P(1-2P+P^2)$$
$$= P^2 - P^3 + P - 2P^2 + P^3$$
$$= P - P^2 = P(1-P) = Pq$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{P(1-P)}$$

1) $X \rightarrow$ Success $= 1$ $\quad P(X=1) = P$

   $\rightarrow$ Failure $= 0$ $\quad P(X=0) = q = (1-P)$

2) $0 \leq P \leq 1$

   $P + q = 1 \Rightarrow$ L.H.S $= P + (1-P) = 1$

Examples:-

Coin
- $\rightarrow H$
- $\rightarrow T$

* A Fair Coin, probability of occuring Head

$P(\text{Head}) = \frac{1}{2}$ $\quad P(\text{Tail}) = \frac{1}{2}$

if we want head when coin is tossed ?

Success $\rightarrow$ Head

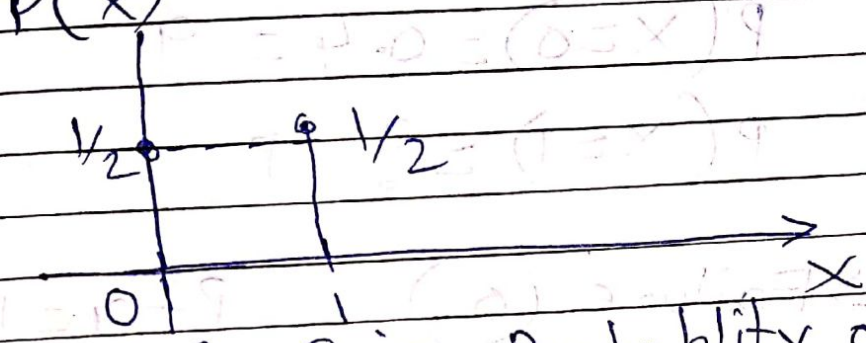
Failure $\rightarrow$ Tails

$P(X=1) = \frac{1}{2} = P$

$P(X=0) = q = 1-P$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$



2] UnFair Coin Probablity of occurence of Head is 0.4, So What is Probablity of occurence of Tail?

Unfair Coin

$P = \frac{1}{2}$ $q = \frac{1}{2}$ (is wrong)

// Head

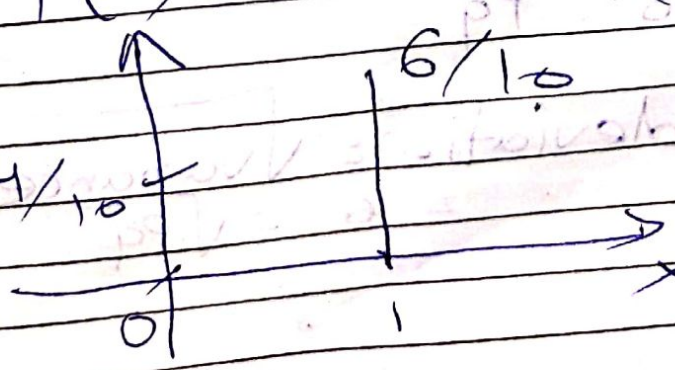
$P(X=0) = \frac{4}{10}$

$P(X=1)$ Tails $= 1 - P(X=0) = 1 - \frac{4}{10} = \frac{6}{10}$

$P + q = 1$ → Failure (H)

↳ Success (T)

$= 1 - (P(H))$



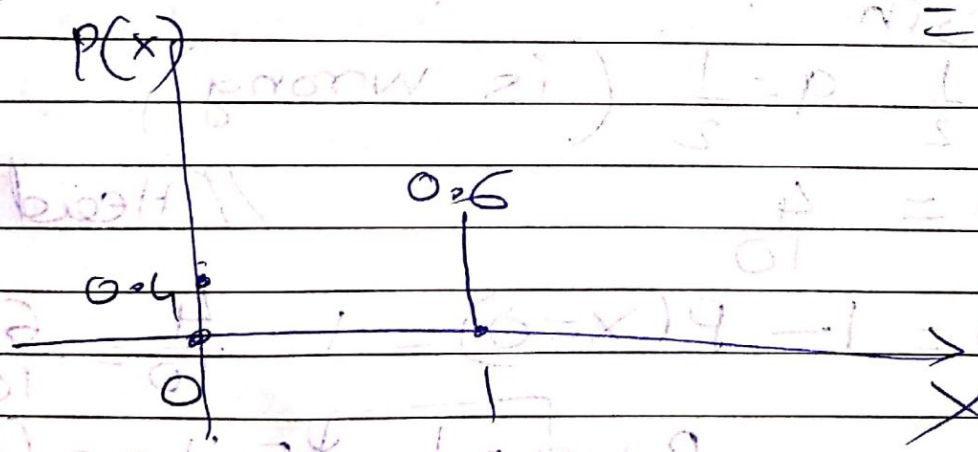
3) A bulb, Probablity that bulb is defective is 0.4.
What is the Probablity that bulb is Not defective?

$P(X=0) = 0.4 = q$

$P(X=1) = ?\ p$

Def ⇒ Failure (0) $\quad p+q=1$

Not Def ⇒ Success (1) $\quad p = 1-q =$
$= 1-0.4$
$= 0.6$



Properties

1) Expected value = $E(X) = p$

2) Variance = $\sigma^2 = pq$

3) Standard deviation = $\sqrt{\text{Variance}}$
$= \sigma = \sqrt{pq}$

PAGE No
DATE / /

Binomial distribution

1] $P[X=x] = {}^{n}C_x \, p^x \, q^{n-x}$

2] n · finite value, total

e.g. Say 1000 books, 10 pens, 15 bulbs

p → success } p+q=1

q → Failure

X → Q : + last line of Q

Example:-

i) If 10% of pens manufactured by Company are defective. Find Probablity that a box containing 12 pens Contains

i) exactly 2 defective pens.

ii) At least 2 defective pens.

Soln:- 1) $n = 12$

2) P = defective $= \frac{10}{100} = \frac{1}{10}$

3) $q = 1 - P = \frac{9}{10}$

i) $\rightarrow P[X=2] = {}^{n}C_x \, P^x \, q^{n-x}$

$= {}^{12}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{10}$

ii) $\rightarrow P[X \geq 2] = 1 - P[X < 2]$

$= 1 - [P(X=0) + P(X=1)]$

2] Probablity that at any moment one telephone line out of 10 will be busy is 0.2

Find Probablity that 5 lines are busy.

PAGE No.
DATE 1/9/

Soln:- $n = 10$

p = busy $= 0.2$

$q = (1-p) = 0.8$

2) $P[x=5] = {}^{n}C_x \, p^x q^{n-x}$

$= {}^{10}C_5 (0.2)^5 (0.8)^5 = !!$

Properties of Binomial distribution

1) Expected value = mean $= np$

2) Variance $= \sigma^2 = npq$

Standard deviation $= \sqrt{npq}$

Difference Between Bernouli and Binomial Distribution.

Bernouli ⟹ 1 Coin, 1 die

Binomial ⟹ n, 10 books
1 10 pens.

Poisson Distribution

1) $P[x=x] = \frac{e^{-m} m^x}{x!}$

2) (m) = mean
= Average
$m = np$

n = Total no.
P = Probablity

DATE / /

(B) X :- x ⇒ the last line of Question

e.g. 1) A hospital switch board receives an avg of 4 emergency calls in 10 minutes interval. what is the Probablity that

(i) There are at most 2 emergency calls.

Soln :- 1) m = 4
x ≤ 2

(2) Poisson formula

$$P[X \le 2] = \frac{e^{-m} m^{x}}{x!}$$

$$P[X=0) + P(X=1) + P(X=2) \quad m=4, \; x=0, 1, 2$$

(2) A Transmission channel has per digit error probablity P = 0.01 : calculate Probablity of more than 1 error in 10 received digits.

Soln :- 1) m = np = (10)(0.01) = 0.1

2] X > 1

PAGE No.
DATE / /

3] $P[X>1] = \frac{e^{-m} \cdot m^{x}}{x!}$ ... (1)

$= 1 - (P[X=0] + P[X=1])$

$= 1 - \quad x=0 \qquad x=1$

$m = 0.1 \qquad m = 0.1$

Alternate Method:-

$n = 10$

$p = 0.01$

$q = (1-p) = (1-0.01)$

$nC_x p^x q^{n-x} = P[X>1]$

$1 - P[X=0] + P(X=1)$

n

p

q

Properties of Poisson distribution

1) Expected value - m = mean = np (Avg)

2) Variance = $\sigma^2 = m$ { Binomial: $E[X] = np$, $V[X] = npq$ }

Standard deviation = $\sigma = \sqrt{m}$

DATE / /

Q.3] If X follows Poisson distribution, such that P[X=1] & P[X=2]. Find mean & Variance

Soln:- 1) P.D → $P[X=x] = \frac{e^{-m} \cdot m^x}{x!}$

2) $\frac{e^{-m} m^1}{1!} = 2 \cdot \frac{e^{-m} \cdot m^2}{2!}$

$m = m^2 \Rightarrow m^2 - m = 0$

$m(m-1) = 0$

$m = 0$ or $m = 1$

3) Mean = Variance = $m = 1$