Testing of Hypothesis (1 sample problem)

A. Large Sample Tests for a Population Mean

Standardized Test Statistics for Large Sample Hypothesis Tests Concerning a Single Population Mean

$$\text{If } \sigma \text{ is known:} \quad Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$\text{If } \sigma \text{ is unknown:} \quad Z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

The test statistic has the standard normal distribution.

Systematic Hypothesis Testing Procedure: Critical Value Approach

1. Identify the null and alternative hypotheses.
2. Identify the relevant test statistic and its distribution.
3. Compute from the data the value of the test statistic.
4. Construct the rejection region.
5. Compare the value computed in Step 3 to the rejection region constructed in Step 4 and make a decision. Formulate the decision in the context of the problem, if applicable.

Problems:

Solved Problem:

It is hoped that a newly developed pain reliever will more quickly produce perceptible reduction in pain to patients after minor surgeries than a standard pain reliever. The standard pain reliever is known to bring relief in an average of 3.5 minutes with standard deviation 2.1 minutes. To test whether the new pain reliever works more quickly than the standard one, 50 patients with minor surgeries were given the new pain reliever and their times to relief were recorded. The experiment yielded sample mean *x*bar=3.1 minutes and sample standard deviation s = 1.5 minutes. Is there sufficient evidence in the sample to indicate, at the 5% level of significance, that the newly developed pain reliever does deliver perceptible relief more quickly?

Solution:

Step 1. The natural assumption is that the new drug is no better than the old one, but must be proved to be better. Thus if μ denotes the average time until all patients who are given the new drug experience pain relief, the hypothesis test is

$H0{:}\mu=3.5$ vs. $H1{:}\mu<3.5$ @ $\alpha=0.05$

Step 2. The sample is large, but the population standard deviation is unknown (the 2.1 minutes pertains to the old drug, not the new one). Thus the test statistic is

$$Z=\frac{\overline{x}-\mu_0}{s/\sqrt{n}}$$

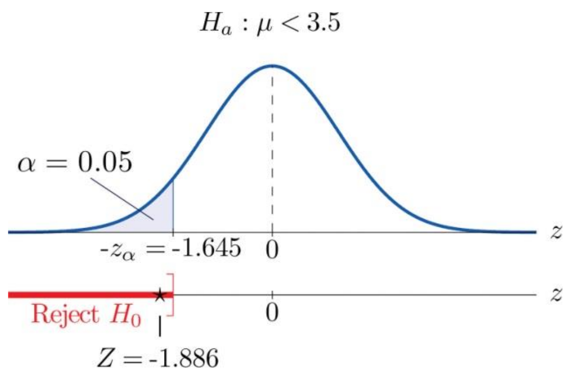
and has the standard normal distribution.

Step 3. Inserting the data into the formula for the test statistic gives

$$Z = \frac{\bar{x} - \mu_0}{s \big/ \sqrt{n}} = \frac{3.1 - 3.5}{1.5 \big/ \sqrt{50}} = -1.886$$

Step 4. Since the symbol in H_1 is "<" this is a left-tailed test, so there is a single critical value, $-z\alpha = -z0.05$, we read off as -1.645. The rejection region is $(-\infty, -1.645]$.

qnorm(0.05)



Step 5. As shown in FIGURE the test statistic falls in the rejection region. The decision is to reject H_0. In the context of the problem our conclusion is:

The data provide sufficient evidence, at the 5% level of significance, to conclude that the average time until patients experience perceptible relief from pain using the new pain reliever is smaller than the average time for the standard pain reliever.

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Exercise:

1. In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager wishes to check whether that average has decreased after the introduction of policy changes. A sample of 100 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds. Perform the relevant test at the 1% level of significance.

Ans: $Z = -2.86$, $-z(0.01) = -2.33$, reject H_0

2. The government of an impoverished country reports the mean age at death among those who have survived to adulthood as 66.2 years. A relief agency examines 30 randomly selected deaths and obtains a mean of 62.3 years with standard deviation 8.1 years. Test whether the agency's data support the alternative hypothesis, at the 1% level of significance, that the population mean is less than 66.2.

Ans: Z=-2.64, $-z(0.01)=-2.33$, reject H_0

3. The average household size in a certain region several years ago was 3.14 persons. A sociologist wishes to test, at the 5% level of significance, whether it is different now. Perform the test using the information collected by the sociologist: in a random sample of 75 households, the average size was 2.98 persons, with sample standard deviation 0.82 person.

Ans: $Z=-1.69$, $+-z(0.025)=+-1.96$, do not reject H_0.

4. The recommended daily calorie intake for teenage girls is 2,200 calories/day. A nutritionist at a state university believes the average daily caloric intake of girls in that state to be lower. Test that hypothesis, at the 5% level of significance, against the null hypothesis that the population average is 2,200 calories/day using the following sample data: n = 36, $x$bar= 2,150, s = 203.

Ans: Z=-1.47783, $-z0.05$=-1.644854, do not reject H_0.

5. An automobile manufacturer recommends oil change intervals of 3,000 miles. To compare actual intervals to the recommendation, the company randomly samples records of 50 oil changes at service facilities and obtains sample mean 3,752 miles with sample standard deviation 638 miles. Determine whether the data provide sufficient evidence, at the 5% level of significance, that the population mean interval between oil changes exceeds 3,000 miles.

Ans: $Z = 8.33$, $z0.05=1.645$, reject H_0.

6. A grocery store chain has as one standard of service that the mean time customers wait in line to begin checking out not exceed 2 minutes. To verify the performance of a store the company measures the waiting time in 30 instances, obtaining mean time 2.17 minutes with standard deviation 0.46 minute. Use these data to test the null hypothesis that the mean waiting time is 2 minutes versus the alternative that it exceeds 2 minutes, at the 10% level of significance.

Ans: Z =2.02, z0.10=1.28, reject H_0.

7. Authors of a computer algebra system wish to compare the speed of a new computational algorithm to the currently implemented algorithm. They apply the new algorithm to 50 standard problems; it averages 8.16 seconds with standard deviation 0.17 second. The current algorithm averages 8.21 seconds on such problems. Test, at the 1% level of significance, the alternative hypothesis that the new algorithm has a lower average time than the current algorithm.

Ans: Z=−2.08, −z0.01=−2.33, do not reject H_0.

8. The mean household income in a region served by a chain of clothing stores is \$48,750. In a sample of 40 customers taken at various stores the mean income of the customers was \$51,505 with standard deviation \$6,852.

a. Test at the 10% level of significance the null hypothesis that the mean household income of customers of the chain is \$48,750 against that alternative that it is different from \$48,750.

b. The sample mean is greater than \$48,750, suggesting that the actual mean of people who patronize this store is greater than \$48,750. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)

Ans: a) Z = 2.54, z0.05=1.645 , reject H_0; b) Z = 2.54, z0.10=1.28, reject H_0.

Systematic Hypothesis Testing Procedure: *p*-Value Approach

1. Identify the null and alternative hypotheses.
2. Identify the relevant test statistic and its distribution.
3. Compute from the data the value of the test statistic.
4. Compute the *p*-value of the test.
5. Compare the value computed in Step 4 to significance level α and make a decision: reject H_0 if $p \leq \alpha$ and do not reject H_0 if $p > \alpha$. Formulate the decision in the context of the problem, if applicable.

Problems:

Solved Problem:

The total score in a professional basketball game is the sum of the scores of the two teams. An expert commentator claims that the average total score for NBA games is 202.5. A fan suspects that this is an overstatement and that the actual average is less than 202.5. He selects a random sample of 85 games and obtains a mean total score of 199.2 with standard deviation 19.63. Determine, at the 5% level of significance, whether there is sufficient evidence in the sample to reject the expert commentator's claim.

Solution:

Step 1. Let μ be the true average total game score of all NBA games. The relevant test is

$H0{:}\mu$ vs. $Ha{:}\mu$=<202.5 @ α=0.05

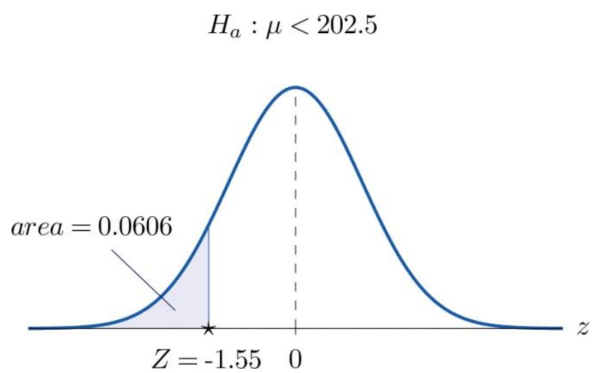
Step 2. The sample is large and the population standard deviation is unknown. Thus the test statistic is

$$Z = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

and has the standard normal distribution.

Step 3. Inserting the data into the formula for the test statistic gives

$$Z = \frac{\overline{x} - \mu_0}{s / \sqrt{n}} = \frac{199.2 - 202.5}{19.63 / \sqrt{85}} = -1.55$$



Step 4. The area of the left tail cut off by $z=-1.55$ is, by, 0.0606, as illustrated in FIGURE. Since the test is left-tailed, the p-value is just this number, $p = 0.0606$.

Step 5. Since $p=0.0606>0.05=\alpha$, the decision is not to reject H_0. In the context of the problem our conclusion is: The data do not provide sufficient evidence, at the 5% level of significance, to conclude that the average total score of NBA games is less than 202.5.

pnorm(-1.55)

(For more Practice, Use above exercise)

B. Small Sample Tests for a Population Mean

Standardized Test Statistics for small Sample Hypothesis Tests Concerning a Single Population Mean

$$\text{If } \sigma \text{ is known:} \quad Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$\text{If } \sigma \text{ is unknown:} \quad T = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

The first test statistic (σ known) has the standard normal distribution.

The second test statistic (σ unknown) has Student's t-distribution with $n-1$ degrees of freedom.

The population must be normally distributed.

Problems:

Solved Problem:

The price of a popular tennis racket at a national chain store is $179. Portia bought five of the same racket at an online auction site for the following prices:

155 179 175 175 161

Assuming that the auction prices of rackets are normally distributed, determine whether there is sufficient evidence in the sample, at the 5% level of significance, to conclude that the average price of the racket is less than $179 if purchased at an online auction.

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A<-c(155,    179,    175,    175,    161)
Mean(A)
Sd(A)
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Solution: We will use the critical value approach to perform the test.

Step 1. The assertion for which evidence must be provided is that the average

online price μ is less than the average price in retail stores, so the hypothesis test is

$$H0{:}\mu \quad \text{vs.} \quad Ha{:}\mu{=}{<}179 \qquad @\ \alpha{=}0.05$$

Step 2. The sample is small and the population standard deviation is unknown. Thus the test statistic is

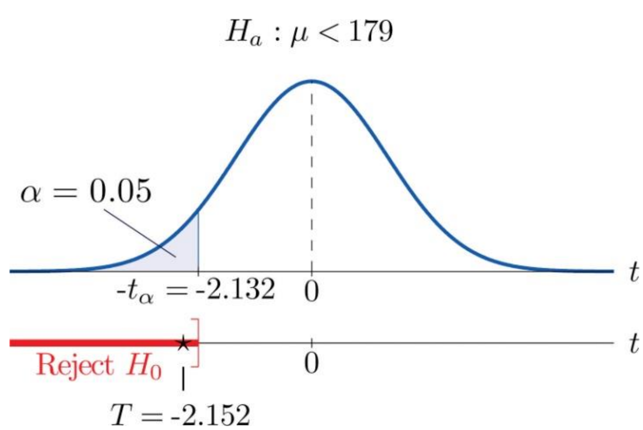
$$T = \frac{\bar{x} - \mu_0}{s \big/ \sqrt{n}}$$

and has the Student t-distribution with $n-1=5-1=4$ degrees of freedom.

Step 3. From the data we compute *xbar*=169 and s = 10.39. Inserting these values into the formula for the test statistic gives

$$T = \frac{\bar{x} - \mu_0}{s \big/ \sqrt{n}} = \frac{169 - 179}{10.39 \big/ \sqrt{5}} = -2.152$$

Step 4. Since the symbol in H_a is "<" this is a left-tailed test, so there is a single critical value, $-t_\alpha=-t_{0.05}[df=4]$. Reading from the row labeled df=4 in its value is -2.132. The rejection region is $(-\infty,-2.132]$.



Step 5. As shown in FIGURE the test statistic falls in the rejection region. The decision is to reject H_0. In the context of the problem our conclusion is:

The data provide sufficient evidence, at the 5% level of significance, to conclude that the average price of such rackets purchased at online auctions is less than \$179.

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Exercise:

1. A small component in an electronic device has two small holes where another tiny part is fitted. In the manufacturing process the average distance between the two holes must be tightly controlled at 0.02 mm, else many units would be defective and wasted. Many times throughout the day quality control engineers take a small sample of the components from the production line, measure the distance between the two holes, and make adjustments if needed. Suppose at one time four units are taken and the distances are measured as

   0.021   0.019   0.023   0.020

   Determine, at the 1% level of significance, if there is sufficient evidence in the sample to conclude that an adjustment is needed. Assume the distances of interest are normally distributed.

Ans: T=0.877, $\pm t\alpha/2$=-+5.84 [df=3], do not reject H_0

2. Researchers wish to test the efficacy of a program intended to reduce the length of labor in childbirth. The accepted mean labor time in the birth of a first child is 15.3 hours. The mean length of the labors of 13 first-time mothers in a pilot program was 8.8 hours with standard deviation 3.1 hours. Assuming a normal distribution of times of labor, test at the 10% level of significance test whether the mean labor time for all women following this program is less than 15.3 hours.

Ans: $T=-7.560$, $df=12$, $-t_{0.10}=-1.356$, reject H_0.

3. Six coins of the same type are discovered at an archaeological site. If their weights on average are significantly different from 5.25 grams then it can be assumed that their provenance is not the site itself. The coins are weighed and have mean 4.73 g with sample standard deviation 0.18 g. Perform the relevant test at the 0.1% (1/10th of 1%) level of significance, assuming a normal distribution of weights of all such coins.

Ans: $T=-7.076$, $df=5$, $-t_{0.0005}=-6.869$, reject H_0.

4. The recommended daily allowance of iron for females aged 19–50 is 18 mg/day. A careful measurement of the daily iron intake of 15 women yielded a mean daily intake of 16.2 mg with sample standard deviation 4.7 mg.

a)Assuming that daily iron intake in women is normally distributed, perform the test that the actual mean daily intake for all women is different from 18 mg/day, at the 10% level of significance.

b)The sample mean is less than 18, suggesting that the actual population mean is less than 18 mg/day. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)

Ans: a)$T=-1.483$, $df=14$, $-t_{0.05}=-1.761$, do not reject H_0;

b)$T=-1.483$, $df=14$, $-t_{0.10}=-1.345$, reject H_0;

5. Pasteurized milk may not have a standardized plate count (SPC) above 20,000 colony-forming bacteria per milliliter (cfu/ml). The mean SPC for five samples was 21,500 cfu/ml with sample standard deviation 750 cfu/ml. Test the null hypothesis that the mean SPC for this milk is 20,000 versus the alternative that it is greater than 20,000, at the 10% level of significance. Assume that the SPC follows a normal distribution.

Ans: $T = 4.472$, $df=4$, $t0.10=1.533$, reject H_0.

C. LARGE SAMPLE TESTS FOR A POPULATION PROPORTION

Standardized Test Statistics for Large Sample Hypothesis Tests Concerning a Single Population Proportion

$$Z=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0q_0}{n}}}$$

The test statistic has the standard normal distribution.

Problems:

Solved Problem:

A soft drink maker claims that a majority of adults prefer its leading beverage over that of its main competitor's. To test this claim 500 randomly selected people were given the two beverages in random order to taste. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Determine whether there is sufficient evidence, at the 5% level of significance, to support the soft drink maker's claim against the default that the population is evenly split in its preference.

Solution:

We will use the critical value approach to perform the test.

Here, $p\hat{}$=270/500=0.54

Hence

Step 1. The relevant test is

$H0{:}p$ vs. $Ha{:}p$=>0.50 @ α=0.05

where p denotes the proportion of all adults who prefer the company's beverage over that of its competitor's beverage.

Step 2. The test statistic is

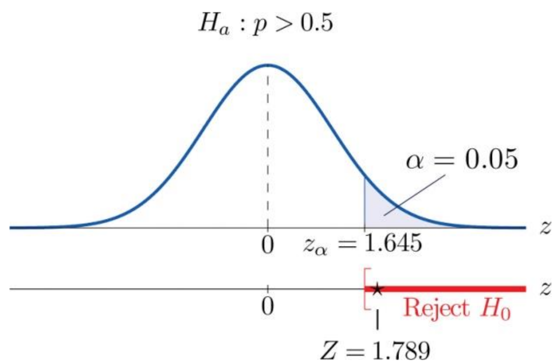
$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

and has the standard normal distribution.

Step 3. The value of the test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.54 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{500}}} = 1.789$$

Step 4. Since the symbol in H_a is ">" this is a right-tailed test, so there is a single critical value, $z\alpha$=z0.05. Reading from the last line in its value is 1.645. The rejection region is [1.645,∞).



Step 5. As shown in FIGURE the test statistic falls in the rejection region. The decision is to reject H_0. In the context of the problem our conclusion is:

The data provide sufficient evidence, at the 5% level of significance, to conclude that a majority of adults prefer the company's beverage to that of their competitor's.

1. Five years ago 3.9% of children in a certain region lived with someone other than a parent. A sociologist wishes to test whether the current proportion is different. Perform the relevant test at the 5% level of significance using the following data: in a random sample of 2,759 children, 119 lived with someone other than a parent.

Ans: $Z = 1.11$, $z0.025=1.96$, do not reject H_0.

2. Two years ago 72% of household in a certain county regularly participated in recycling household waste. The county government wishes to investigate whether that proportion has increased after an intensive campaign promoting recycling. In a survey of 900 households, 674 regularly participate in recycling. Perform the relevant test at the 10% level of significance.

Ans: $Z = 1.93$, $z0.10=1.28$, reject H_0.

3. A report five years ago stated that 35.5% of all state-owned bridges in a particular state were “deficient.” An advocacy group took a random sample of 100 state-owned bridges in the state and found 33 to be currently rated as being “deficient.” Test whether the current proportion of bridges in such condition is 35.5% versus the alternative that it is different from 35.5%, at the 10% level of significance.

Ans: $Z=-0.523$, $\pm z0.05=\pm 1.645$, do not reject H_0.

4. According to the Federal Poverty Measure 12% of the U.S. population lives in poverty. The governor of a certain state believes that the proportion there is lower. In a sample of size 1,550, 163 were impoverished according to the federal measure.
   a) Test whether the true proportion of the state’s population that is impoverished is less than 12%, at the 5% level of significance.

b) Compute the observed significance of the test.

Ans: *a)* $Z = -1.798$, $-z0.05 = -1.645$, reject H_0;

b) $p-$ -value=0.0359.

D. Chi-Square Test for the Variance

Test Statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

$n =$ sample size

$s^2 =$ ***sample*** variance

$\sigma^2 =$ ***population*** variance

Exercise:

1. A manufacturer of candy must monitor the temperature at which the candies are baked. Too much variation will cause inconsistency in the taste of the candy. Past records show that the standard deviation of the temperature has been 1.2°F. A random sample of 30 batches of candy is selected, and the sample standard deviation of the temperature is 2.1°F.
   a. At the 0.05 level of significance, is there evidence that the population standard deviation has increased above
   b. Compute the p-value in (a) and interpret its meaning.

Ans:a) chisq(cal)=88.85, k2=42.55 (at 5%, df=30-1=29), Rej H0
b)p-val=0.00000001, Rej H0

2. A market researcher for an automobile dealer intends to conduct a nationwide survey concerning car repairs. Using his past experience and judgment, he estimates that the standard deviation of the amount of repairs is $200. Suppose that a small-scale study of 25 auto owners selected at random indicates a sample standard deviation of $237.52.
   a. At the 0.05 level of significance, is there evidence that the population standard deviation is different from $200?
   b. Compute the p-value in part (a) and interpret its meaning.

Ans:

Reference website:
https://saylordotorg.github.io/text_introductory-statistics/s12-05-large-sample-tests-for-a-popul.html