Testing of Hypothesis

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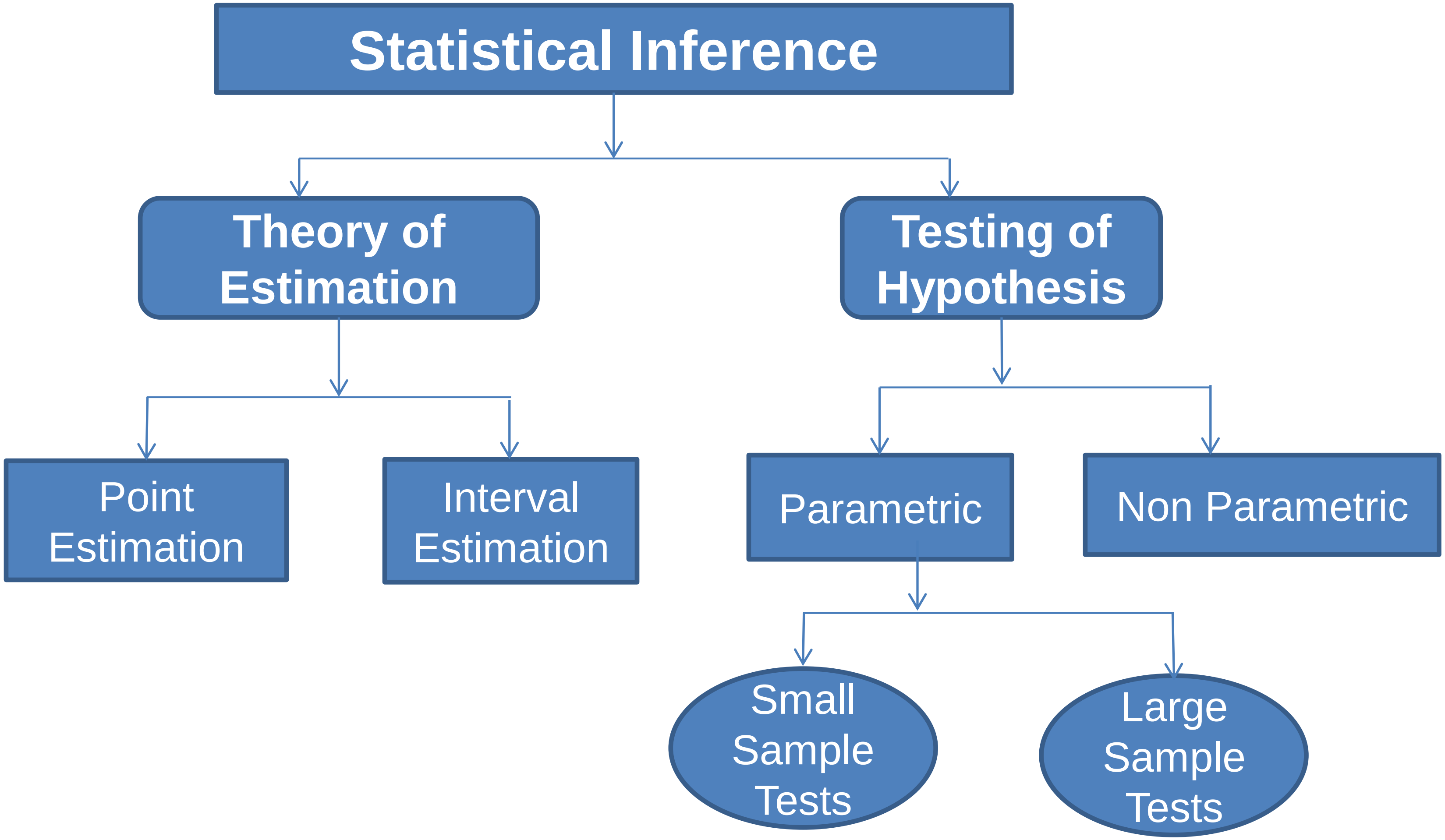
- Statistical inference:



Testing of Hypothesis

Statistical Hypothesis

- A statistical hypothesis is an assertion regarding the statistical distribution of the population. It is statement regarding the parameters of the population.
- It is denoted by **H**

  Example:

  – **H:** population has mean height 175

  – **H:** Population is normally distributed with mean 25 and SD 2

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Hypothesis

- **Null hypothesis:** The hypothesis which is being tested for possible rejection is called Null hypothesis.

  -It is denoted by $\mathbf{H_0}$

- **Example:**

  $\mathbf{H_0 : \mu=150\ cm}$

Hypothesis

- **Alternative hypothesis:** The hypothesis is accepted when the null hypothesis is rejected is called Alternative Hypothesis.

  -It is denoted by $\mathbf{H_1}$

- **Example:**

  $\mathbf{H_1 : \mu > 150\ cm}$ **or**

  $\mathbf{H_1 : \mu < 150\ cm}$ **or**

  $\mathbf{H_1 : \mu \neq 150\ cm}$

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Test Procedure:

(Steps Involving in Testing of Hypothesis)

1. Setting up the **Null Hypothesis** (i.e. $\mathbf{H_0}$)
2. Setting up the **Alternative Hypothesis** (i.e. $\mathbf{H_1}$)
3. Identifying the **test statistics** and its null distribution.
4. Identifying the **critical region**.
5. Drawing a random sample and actually **conducting the test**.
6. Making the **decision** (Giving the inference)

IMP Terms:

1. **Test Statistic**: It is the statistic based on whose distribution the test is conducted.
2. **Null Distribution**: The statistical distribution of the test statistic $\mathbf{H_0}$ is called null distribution.
3. **Critical Region (or Rejection Region)**: The set of those values of the test statistic which leads to the rejection of the null hypothesis is called critical region.
4. **Acceptance Region**: The set of those values of the test statistic which leads to the acceptance of the null hypothesis is called acceptance region.
5. **Level of Significance**: It is a pre-decided upper limit for the probability of rejection of null hypothesis when it is actually true. It is denoted by $\boldsymbol{\alpha}$.

- **Level of Significance :**

- The pre assigned low probability of Type I error is called **los.** It is denoted by $\boldsymbol{\alpha}$
- It is taken as 5% or 1%.
- los 5% means probability of rejecting null hypothesis is 0.05
- i.e. out of 100 chances we are correct about 95times in taking decisions. And 5 times wrong in taking decisions.
- If we want greater accuracy then $\alpha = 1\%$.

Degrees of Freedom:

- It means freedom to vary.
- It is number of independent observations available for estimating the true parameter of the population.

Example:

Choose three numbers such as 5,6 and 8. now choose three numbers under the constraint that the sum is 20. here first two numbers we can write independently but not third number, so out of three possible ways of writing, one degree of freedom is lost. Thus every constraint (restrictions) imposed on data reduces one degree of freedom.

Errors of the First Kind and the Second Kind
(Type I Error and Type I Error)

Decision based on sample ⇩	Actual Fact	
	H_0 True	H_0 False
H_0 Accept	✓ . Correct Decision	Type II Error Wrong Decision
H_0 Reject	Type I Error Wrong Decision	✓ . Correct Decision

Type I Error: It is taking wrong decision to reject H_0 when it is actually true. Also called as **Error of the First Kind**.

Type II Error: It is taking wrong decision to accept H_0 when it is actually not true. Also called as **Error of the Second Kind**.

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Population:

- In the statistical sense a *population* is a theoretical concept used to describe an entire group of individuals in whom we are interested.
- Examples
  - population of all patients with diabetes mellitus
  - population of all middle-aged men with diabetes .

Parameter:

- Examples: Parameters are quantities used to describe characteristics of such population.
- Mean, SD of the population these constants are called **parameters** of the population.
- If we find the mean, SD etc. for the sample then these are called as **Statistic.**

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Properties of Standard Error:

- It is measure of variability of the statistic. It is useful in estimation and testing of hypothesis.
- In the theory of estimation, SE is used to decide the efficiency and consistency of the statistic as an estimator.
- In interval estimation, SE is used to write down the confidence interval.
- In testing of hypothesis, SE of test statistic is used to standardize the distribution of test statistic.
- Difference between SD and SE
  - SE refers to an estimate of the parameter.
  - SD refers to estimate of the observations.

Steps in Hypotheses Testing

Classical Method

1. Determine and state H_0 and H_1
2. Decide the significance level α and the critical region
3. Based on the parameter, choose the test statistic
4. Using available data compute the test statistic
5. Make the statistical **Accept** or **Reject** decision based on
   a) Computed value of the test statistic
   b) The critical region identified in step 2

Steps in Hypotheses Testing

p-value method

1. Determine and state H_0 and H_1
2. Decide the significance level α
3. Based on the parameter, choose the test statistic
4. Using available data compute the test statistic and the p-value
   - How to calculate p-value?
5. Make the statistical **Accept** or **Reject** decision based on
   - α and p-value
     a) p-value less than α should reject H_0
     b) p-value greater than α should not reject H_0

P value

- The computed probability of getting the observed result, or any result at least as extreme in its difference from what the null hypothesis would imply – is called the *p-value*
- A *p-value of 0.05 is the de facto standard cut-off between significant and non-significant results*
- If this de facto value is used as the critical value, it will result in wrong results 5% of the time

P value

- p value tells us the probability that the results occurred by chance. It is the probability of wrongly rejecting null hypothesis . If $p < 0.05$, we say that test result is significant at 5% level of significance.

- The observed significance level is a criterion that can be computed from a sample data. It is a probability
- If $p\ value \leq \alpha$; reject the null hypothesis. There is statistical significant difference between two treatments.
- If $p\ value > \alpha$; accept the null hypothesis.

Z- tests (Large Sample Tests)

1. Test for Mean
2. Test for equality of Means
3. Test for Proportion
4. Test for equality of Proportions

Remember!

Important table values (commonly used)

	k_{α}	$k_{\alpha/2}$
$\alpha = 0.05$ (i.e. 5%)	1.645	1.959
$\alpha = 0.01$ (i.e. 1%)	2.326	2.576

Z- test: Test for Mean

- The null hypothesis is,

  $H_0: \mu = \mu_0$ (μ_0 is population Mean)

- Under H_0 the Test Statistic is,

  $$Z = \frac{(\overline{x} - \mu_0)}{\sigma / \sqrt{n}}$$ (follows N(0,1) distribution)

- The alternative hypothesis is,

  $H_1: \mu \neq \mu_0$ (two tailed test)

  $H_1: \mu > \mu_0$ (upper tail test)

  $H_1: \mu < \mu_0$ (lower tail test)

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When σ (Population SD) is unknown the Test Statistic is,

$$Z = \frac{(\overline{x} - \mu_0)}{s/\sqrt{n}}$$ (follows N(0,1) distribution)

Here, ***s*** is SD of Sample observations.

Z- test: Test for equality of Means

- The null hypothesis is,

$$H_0: \mu_1 = \mu_2$$

- Under $\mathbf{H_0}$ the Test Statistic is,

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$ (follows N(0,1) distribution)

- The alternative hypothesis is,

$H_1: \mu_1 \neq \mu_2$ (two tailed test)

$H_1: \mu_1 > \mu_2$ (upper tail test)

$H_1: \mu_1 < \mu_2$ (lower tail test)

When σ (Population SD) is unknown the Test Statistic is,

$$Z = \frac{(\overline{x}_1 - \overline{x}_2)}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$$ (follows N(0,1) distribution)

Here, s_1 and s_2 is SD of both Sample observations.

Z- test: Test for Proportion

- The null hypothesis is,

$$H_0: P = P_0$$

- Under $\mathbf{H_0}$ the Test Statistic is,

$$Z = \frac{(p - P_0)}{\sqrt{\frac{P_0 Q_0}{n}}}$$ (follows N(0,1) distribution)

here, $p = \frac{x}{n}$

- The alternative hypothesis is,

$H_1: P \neq P_0$ (two tailed test)

$H_1: P > P_0$ (upper tail test)

$H_1: P < P_0$ (lower tail test)

Z- test: Test for equality of Proportion

- The null hypothesis is,

  $H_0: P_1 = P_2$

- Under $\mathbf{H_0}$ the Test Statistic is,

$$Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$ (follows N(0,1) distribution)

here, P be the common proportion and

$$Q = 1 - P \quad p_1 = \frac{x_1}{n_1} \quad p_2 = \frac{x_2}{n_2}$$

- The alternative hypothesis is,

  $H_1: P_1 \neq P_2$ (two tailed test)

  $H_1: P_1 > P_2$ (upper tail test)

  $H_1: P_1 < P_2$ (lower tail test)

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Generally in common proportion *P* will not be known. And so, it is estimated from the samples.

The estimates is

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

And so,

$$\hat{Q} = 1 - \hat{P}$$

$$Z = \frac{(p_1 - p_2)}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$ (follows N(0,1) distribution)

t- tests

1. Small sample test for Mean
2. Small sample test for equality of Means
3. Paired *t*-Test

t- test: Small sample test for Mean

- σ is unknown
- The null hypothesis is,

  $H_0: \mu = \mu_0$ (${\mu_0}$ is population Mean)

- Under **H_0** the Test Statistic is,

$$t = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}}$$

(follows t-distribution)

- Degrees of Freedom = ***(n – 1)***
- The alternative hypothesis is,

  $H_1: \mu \neq \mu_0$ (two tailed test)

  $H_1: \mu > \mu_0$ (upper tail test)

  $H_1: \mu < \mu_0$ (lower tail test)

(Note: This test is based on assumption that the population is Normal)

t- test: Small sample test for equality of Means

- The null hypothesis is,

  $H_0: \mu_1 = \mu_2$

- Under **H_0** the Test Statistic is,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(follows N(0,1) distribution)

- Degrees of Freedom =

$$d.f. = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

- The alternative hypothesis is,

  $H_1: \mu_1 \neq \mu_2$ (two tailed test)

  $H_1: \mu_1 > \mu_2$ (upper tail test)

  $H_1: \mu_1 < \mu_2$ (lower tail test)

t- test: Small sample test for equality of Means equal variance

- The null hypothesis is,

  $H_0: \mu_1 = \mu_2$

- Under H_0 the Test Statistic is,

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ (follows t- distribution)

- Degrees of Freedom = $(n_1 + n_2 - 2)$
- The alternative hypothesis is,

  $H_1: \mu_1 \neq \mu_2$ (two tailed test)

  $H_1: \mu_1 > \mu_2$ (upper tail test)

  $H_1: \mu_1 < \mu_2$ (lower tail test)

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Paired *t*-Test

- The null hypothesis is,

  $H_0: \mu_1 = \mu_2$

- Under $\mathbf{H_0}$ the Test Statistic is,

$$t = \frac{\bar{d}}{s_d / \sqrt{n}}$$ (follows N(0,1) distribution)

- Degrees of Freedom = $(n - 1)$
- The alternative hypothesis is,

$H_1: \mu_1 \neq \mu_2$	(two tailed test)
$H_1: \mu_1 > \mu_2$	(upper tail test)
$H_1: \mu_1 < \mu_2$	(lower tail test)

Here, X_i and Y_i are paired observations.

$$d_i = X_i - Y_i$$

$$\bar{d} = \frac{\sum d_i}{n}$$

$$s_d = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$$

Thank you