Chinmayee COE18 102103498

Predictive Analysis using stats

Assignment - Parameter Estimation

Sol^n 1. Given a random Sample $(x_1, \ldots x_n)$ from normal distribution.

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\left(\frac{(x_i - \mu)^2}{2\sigma^2}\right)\right)$$

Taking natural logarithm of likelihood f^xn.

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i - \mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\right)$$

To find MLEs, we differentiate log-likelihood f^xn with respect to θ_1, θ_2.

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^2}\right) = 0.$$

This implies,

$$\sum_{i=1}^{n} x_i - n\mu = 0$$

$$\theta_1 / \mu = \frac{1}{n} \sum_{i=1}^{n} x_i .$$

for θ_2,

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i - \theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0.$$

This implies,

$$\sum_{i=1}^{n} \frac{(x_i - \theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0.$$

$$\frac{1}{\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = \frac{n}{\theta_2}.$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2.$$

sample variance.

Sol^n 2. To find the MLE of parameter θ for a binomial distribution $B(m, \theta)$, where m is a known positive integer.

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log,

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m-x_i) \ln(1-\theta) \right)$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0.$$

solving for θ:

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}.$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

MLE of θ is sample mean of observations.