Chapter 7

INTEGRALS

EXERCISE 7.1

Find an anti-derivative (or integral) of the following functions by the method of inspection.

1. $\sin 2x$

2. $\cos 3x$

3. e^{2x}

4. $(ax + b)^2$

5. $\sin 2x - 4e^{3x}$

Find the following integrals in Exercises 6 to 20.

6. $\int (4e^{3x} + 1)\, dx$

7. $\int x^2 \left(1 - \dfrac{1}{x^2} \right) dx$

8. $\int (ax^2 + bx + c)\, dx$

9. $\int (2x^2 + e^x)\, dx$

10. $\int \left(\sqrt{x} - \dfrac{1}{\sqrt{x}} \right)^2 dx$

11. $\int \dfrac{x^3 + 5x^2 - 4}{x^2}\, dx$

12. $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

13. $\int \dfrac{x^3 - x^2 + x - 1}{x - 1}\, dx$

14. $\int (1 - x)\sqrt{x}\, dx$

15. $\int \sqrt{x}(3x^2 + 2x + 3)\, dx$

16. $\int (2x - 3\cos x + e^x)\, dx$

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

18. $\int \sec x(\sec x + \tan x)\, dx$

19. $\int \dfrac{\sec^2 x}{\csc^2 x}\, dx$

20. $\int \dfrac{2 - 3\sin x}{\cos^2 x}\, dx$

EXERCISE 7.2

Integrate the functions in Exercises 1 to 17.

1. $\dfrac{2x}{1+x^2}$

2. $\dfrac{(\log x)^2}{x}$

3. $\dfrac{1}{x + x \log x}$

4. $\sin x(\sin x + \cos x)$

5. $\sin(ax + b)\cos(ax + b)$

6. $ax + b$

7. $\sqrt{x} + 2$

8. $\sqrt{1 + 2x}$

9. $4x + \sqrt{x^2 + x + 1}$

10. $\dfrac{1}{\sqrt{1 - x}}$

11. $\dfrac{x}{\sqrt{x^2 + 4}}$, $x > 0$

12. $(x^2 + 1)^{3/2}$

13. $\dfrac{x^2}{2 + 3x^2}$

14. $\log(x^2 - x)$, $x > 0$, $x \neq 1$

15. $\dfrac{x}{9 - 4x^2}$

16. e^{2x+3}

17. $\dfrac{x}{e^x}$

SUMMARY

Integration is the inverse process of differentiation. In integral calculus, a function whose derivative is given is to be found.

If $F'(x) = f(x)$, then $\int f(x)\, dx = F(x) + C$, where C is a constant of integration.

Properties of Integrals:

1. $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$

2. $\int k f(x) dx = k \int f(x) dx$

Some Standard Integrals:
$\int e^x dx = e^x + C$, $\int \sin x\, dx = -\cos x + C$, $\int \cos x\, dx = \sin x + C$, $\int \dfrac{1}{x} dx = \log |x| + C$.

Integration by Parts:
$\int u\, dv = uv - \int v\, du$.